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Muonic Atom Lamb Shifts via Simple Means and the Proton Radius Problem

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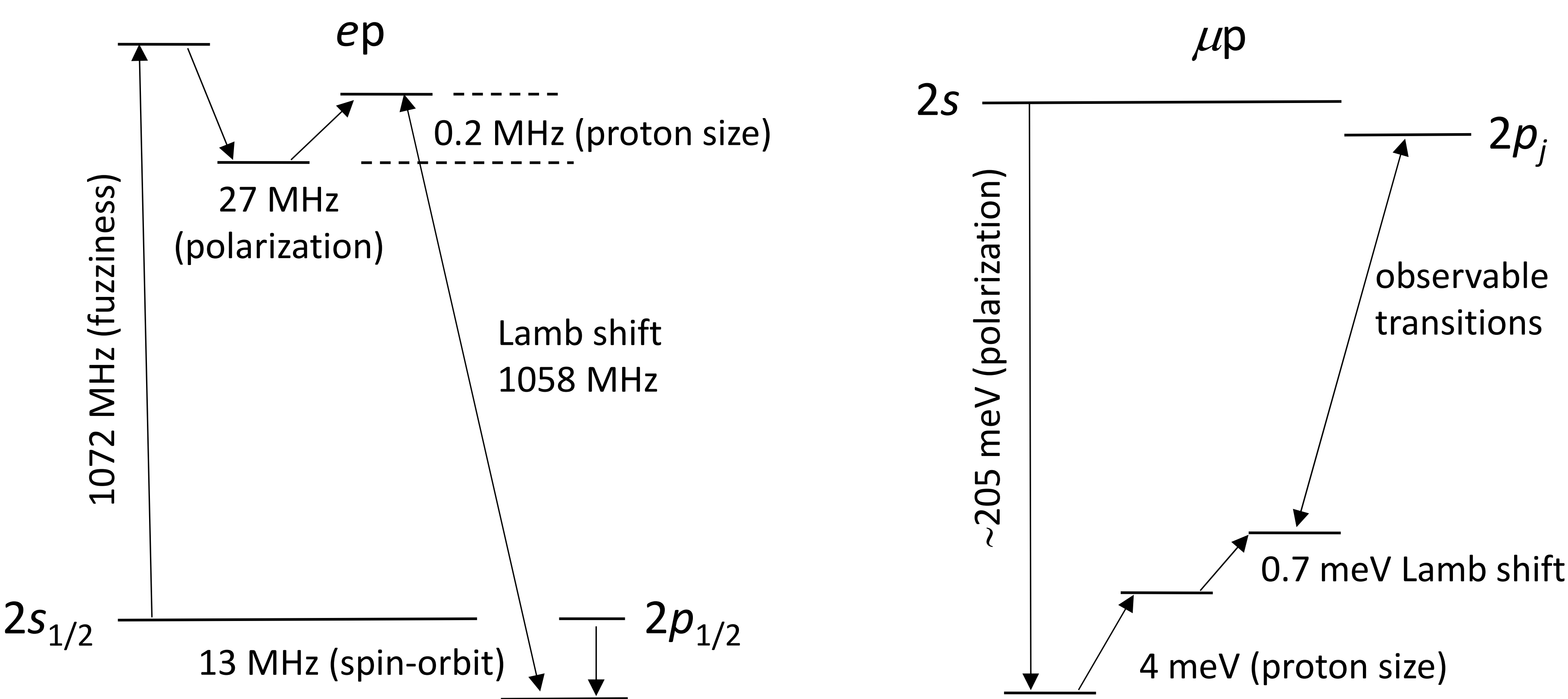
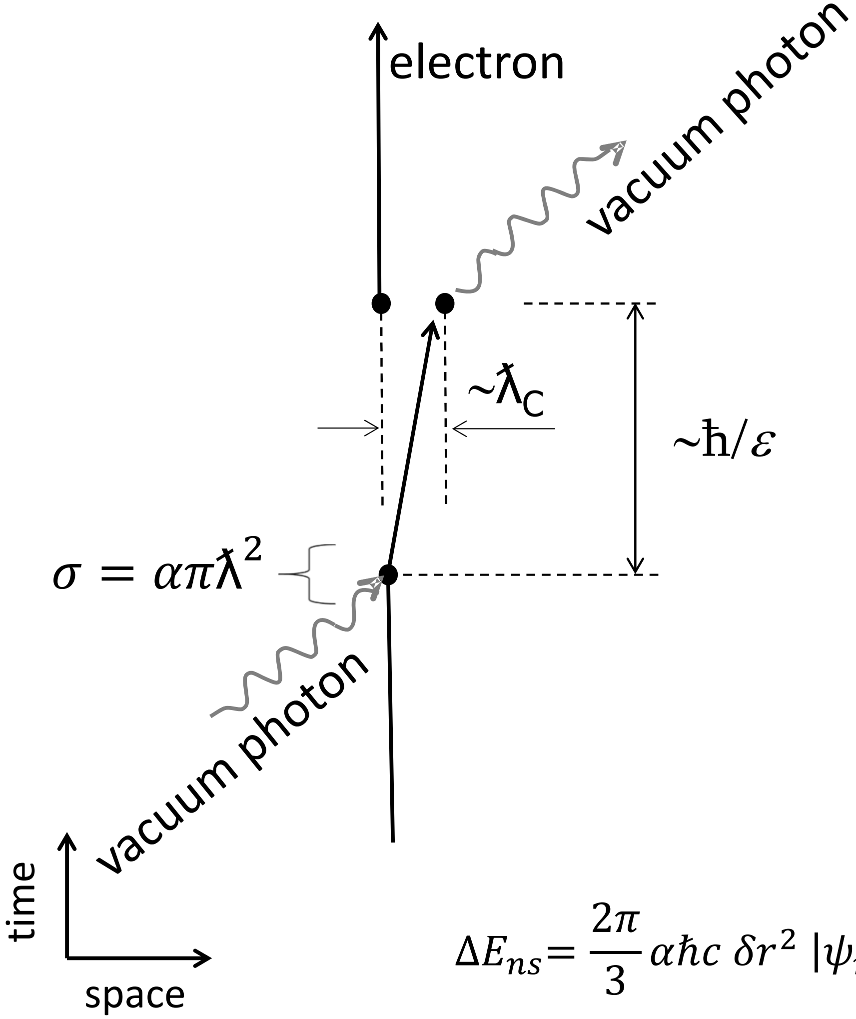
Abstract

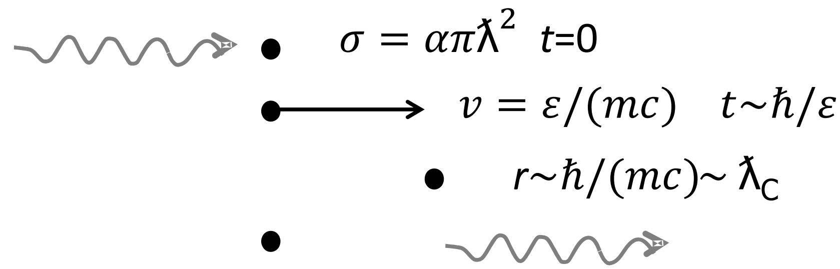
A new simple method for understanding and calculating Lamb shifts to better than 1% is introduced and applied to muonic hydrogen and muonic deuterium systems. The hydrogen and deuterium Lamb shifts are due to an intrinsic fuzziness for the bound electron and muon probing the electric field near the proton, associated with their interaction with the vacuum, of 74 fm and 0.36 fm respectively. The smallness of the muon fuzziness suggests that the associated Lamb shifts need to be calculated including some aspects of the internal degrees of freedom of the proton. If the charge of the proton is assumed to be contained within a quasi-free π^+ for half of the time, then the calculated μp and μd Lamb shifts are consistent with experiment without a need for a change in the proton radius.



ep
0.2 MHz (proton size)
27 MHz
(polarization)
1072 MHz (fuzziness)
Lamb shift
1058 MHz
$2s_{1/2}$
$2p_{1/2}$
13 MHz (spin-orbit)
μp
$2s$
$2p_j$
~205 meV (polarization)
observable
transitions
0.7 meV Lamb shift
4 meV (proton size)

A simple electron virtual-photon interaction mechanism can be used to understand and calculate Lamb shifts





$$R_a(\varepsilon) = \alpha\pi\lambda\!\!\!^{-2}\frac{\varepsilon^2 d\varepsilon}{\pi^2\hbar^3c^3}c = \frac{\alpha d\varepsilon}{\pi\hbar}$$

$$\delta r^2(\varepsilon) = \frac{\int_{r=0}^{\infty}\frac{r^2}{3}\frac{rmc}{\varepsilon}\frac{\alpha d\varepsilon}{\pi\hbar}\exp(-r/\lambda\!\!\!^{-}_{C})dr}{\int_{r=0}^{\infty}\exp(-r/\lambda\!\!\!^{-}_{C})dr}$$

$$= \frac{\alpha mc^2}{3\lambda\!\!\!^{-}_{C}\pi\hbar c}\frac{d\varepsilon}{\varepsilon}\int_{r=0}^{\infty} r^3\exp\left(-\frac{r}{\lambda\!\!\!^{-}_{C}}\right)dr = \frac{2\alpha\lambda\!\!\!^{-2}_{C}}{\pi}\frac{d\varepsilon}{\varepsilon}.$$

$$\delta r^2 = \frac{2\alpha\lambda\!\!\!^{-2}_{C}}{\pi}\int_{\alpha/\pi}^{2\pi}\frac{d\varepsilon}{\varepsilon} = \frac{2\alpha\ln(2\pi^2/\alpha)\lambda\!\!\!^{-2}_{C}}{\pi}$$

$$\Delta E_{ns} = \frac{2\pi}{3}\alpha\hbar c\,\delta r^2\,|\psi_{ns}(0)|^2 \qquad \Delta E_{2s} = \frac{mc^2\alpha^4}{12}\frac{2\alpha\ln(2\pi^2/\alpha)}{\pi} = 1072\text{ MHz}$$

$$\Delta E_{2s} \sim \underbrace{\frac{m_l c^2 \alpha^4}{12 \times 5.2192^2}} \left(\frac{m_+}{m_l + m_+}\right)^3 + \text{other term} + f(R) \qquad 5.2192^2 = \frac{\pi}{2\alpha \ln(2\pi^2/\alpha)}$$ particle variance in units of $(\lambda\!\!\!^{-}_C)^2$

m_e=0.511 keV $\Rightarrow$ 1072 MHz

m_μ=106 MeV $\Rightarrow$ 0.917 meV $\quad m_+ = m_\mathrm{p} \Rightarrow 0.666$ meV $\quad \Delta E(\mu\mathrm{P}) = 0.30(6)$ meV

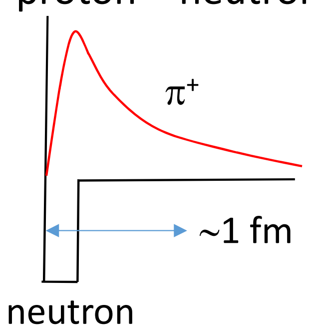
$m_+ = m_\mathrm{d} \Rightarrow 0.778$ meV $\quad \Delta E(\mu\mathrm{D}) = 0.41(6)$ meV

$\Delta E(\mu\mathrm{He}) \sim 0.0$ meV (preliminary)

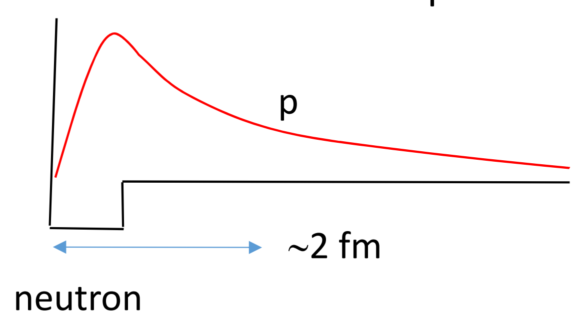
r_e(Lamb shift) ~74 fm $\quad r_\mu$(Lamb shift) ~0.36 fm $\quad r_\mathrm{p}$~0.9 fm $\quad r_\mathrm{D}$ ~2.14 fm ~ r_α

Electron Lamb-shift interactions will see the proton and deuteron as points (at least displaceable ridged bodies)
Muon Lamb-shift interactions might see the internal structure of the proton and deuteron.

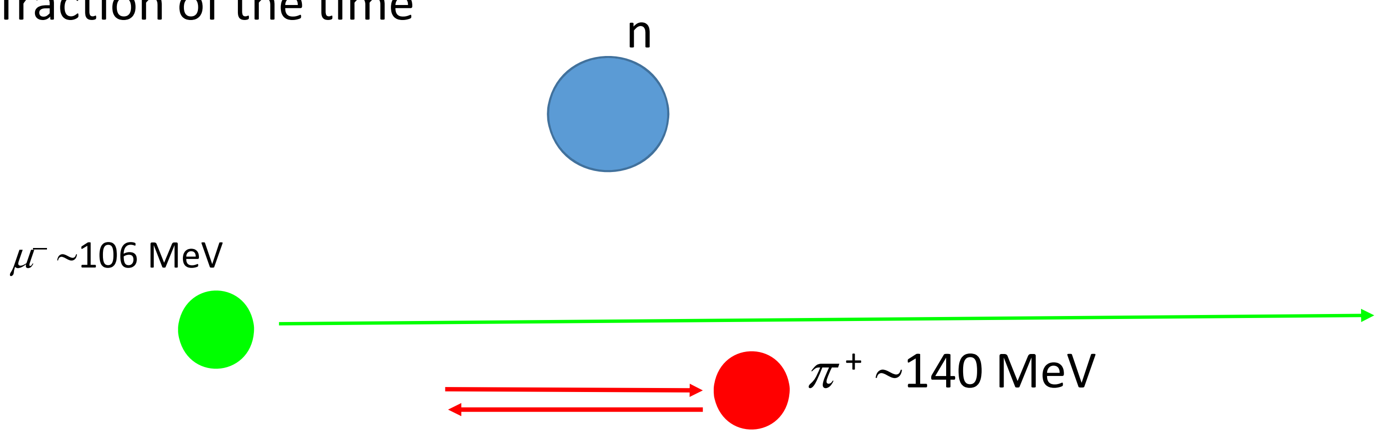
proton = neutron + quasi-free π^+



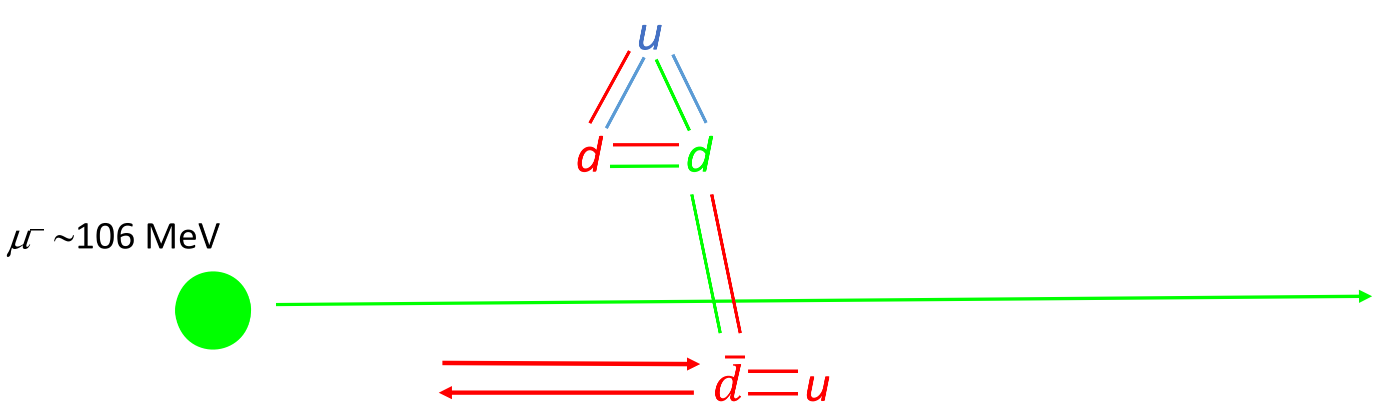
deuteron = neutron + quasi-free proton



Hypothesis : What if the proton behaves like a neutron plus a quasi-free π^+ for a significant fraction of the time



Can QCD theorists perform time-dependent lattice QCD with a bound lepton?



μP

$$\left.\begin{array}{l} m_+ = m_\mathrm{p} \Rightarrow 0.666\ \mathrm{meV}\ \ (50\%) \\ m_+ = m_\pi \Rightarrow 0.170\ \mathrm{meV}\ (50\%) \end{array}\right] \text{—}\ 0.418\ \mathrm{meV}$$

$$\begin{array}{ll} \Delta E(\mathrm{model}) & = 0.25\ \ \ \ \ \mathrm{meV} \\ \Delta E(\mathrm{exp}) & = 0.30(6)\ \mathrm{meV} \end{array}$$

proton has a quasi-free π^+ > 38% (is this possible?)

μD

$$m_+ = m_\mathrm{d} \Rightarrow 0.778\ \mathrm{meV}$$

$$\left.\begin{array}{l} m_+ = m_\mathrm{p} \Rightarrow 0.666\ \mathrm{meV}\ \ (50\%) \\ m_+ = m_\pi \Rightarrow 0.170\ \mathrm{meV}\ (50\%) \end{array}\right] \text{—}\ 0.418\ \mathrm{meV}$$

$$\begin{array}{ll} \Delta E(\mathrm{model}) & = 0.36\ \ \ \ \ \mathrm{meV} \\ \Delta E(\mathrm{exp}) & = 0.41(6)\ \mathrm{meV} \end{array}$$

deuteron has a quasi-free proton > 80% (is this possible?)

Can the suggestion be dismissed?

Is it worth trying to perform the QCD calculations to test the suggestion?