# ESTIMATION OF MINIMUM SPOUTING VELOCITY IN A RECTANGULAR SPOUTED BED

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## ABSTRACT

A study was conducted to explore the effects of static bed height, nozzle diameter, cone angle, and particle properties on the minimum spouting velocity in a 4” x 1” rectangular spouted bed. Tests were conducted with various solids materials (including 871 µm HPDE pellets, 3.2mm nylon beads, 707 µm glass beads, and 1.5mm alumina spheres), two gas inlet nozzle diameters, and 2 cone angles. Experimentally obtained minimum spouting velocities were compared to existing published correlations developed for cylindrical spouted beds. In each case, it was determined that the existing correlations did not adequately predict the minimum spouting velocity for a rectangular spouted bed. A new correlation is proposed.

***Keywords: Fluidization, Spouted Beds, Minimum Spouting Velocity***

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## INTRODUCTION

Many industrial processes involve the conversion or reduction of solid materials via non-homogeneous reactions between the solid material and a surrounding gaseous or liquid medium. In the case of solid-gas reactions, fluidized beds are perhaps one of the most popular reactors because individual particles are suspended within the gaseous phase, which provides excellent surface contact for the desired reactions to take place. However, not all types of solid particles can be easily fluidized. For example, very fine particles (Geldart class C particles) are more susceptible to inter-particle cohesion forces, such as Van der Waals, capillary, and electrostatic forces, and tend not to fluidize. Instead, dense beds of these cohesive particles tend to agglomerate and form cracks or channels that allows the gas phase to bypass the solids with very little contact between the two. At the other end of the particle size spectrum are the Geldart class D particles, which are coarse (< 1mm) and do undergo little bed expansion and mix when fluidized via traditional means [1,2].

In 1955, Mathur and Gishler [3] proposed a new method for processing coarse materials. This method, known as spouting, involves introducing a jet of gas into the bottom of a densely-packed bed of solids through a small orifice, or nozzle. The gas jet pushes up through the densely-packed particles, forming an upwards moving core of entrained particles. These entrained particles are eventually ejected out of the top of the dense bed and fall into an annular region outside of the central core area, forming a fountain-like structure above the bed. The particles located within the annular region recirculate downwards towards the bottom of the bed, where they are eventually re-entrained into the upwards moving gas core. Figure 1 provides a conceptual diagram of this process. The gas velocity at which this spout begins to form at the bottom of the bed is known as the minimum spouting velocity, $U_{ms}$, and is analogous to the minimum fluidization velocity, $U_{mf}$, in a traditional fluidized bed.

Mathur and Gishler proposed the following expression for estimation of the minimum spouting velocity, derived from experiments conducted with cylindrical bed with diameters ranging from 6-12 inches:

$$U_{ms} = \left(\frac{d_p}{d_c}\right)\left(\frac{d_i}{d_c}\right)^{1/3}\sqrt{\frac{2gH(\rho_s-\rho_g)}{\rho_g}} \tag{1}$$

Where $d_p$ is the particle diameter, $d_c$ is the bed diameter, $d_i$ is the nozzle diameter, and $H$ is the static bed height. The expression provided in Eqn. (1) was obtained for a cone geometry resulting in an included angle of 85°, but does not consider the effects of differing cone angles.

Similarly, Smith and Reddy [4] conducted a study of minimum fluidization velocity in a 6-inch diameter spouted bed using different gas inlet sizes and static bed heights. The results of their study were an alternate expression for the minimum spouting velocity, as shown in Eqn. (2).

$$\sqrt{\left(\frac{U_{ms}}{gd_p}\right)\left(\frac{d_c}{d_p}\right)\frac{\rho_g}{(\rho_s-\rho_g)}} = \left[0.64-26.8\left(\frac{d_i}{d_c}\right)^2\right]\left(\frac{H}{d_c}\right)^{0.5-1.76(d_i/d_c)} \tag{2}$$

Brunello et al. [5] studied minimum fluidization in for mixtures of soybean and sorghum in a 12-inch diameter cylindrical spouted bed and proposed the expression in Eqn. (3).

$$U_{ms} = 0.0143 d_p^{0.741} H^{0.592}\sqrt{\frac{2g(\rho_s-\rho_g)}{\rho_g}} \tag{3}$$

Of the expressions for minimum spouting velocity presented in equations (1) – (3), all of them were based upon experimental data obtained from cylindrical beds with fixed cone angles, and thus do not incorporate the effects of non-cylindrical geometries or of varying cone angles.

Anabtawi [6] studied minimum fluidization velocities for several mixtures of polystyrene with a variety of beds with square cross sections with at least 5 different nozzle geometries, developing the expression given in Eqn. (4):

$$U_{ms} = 0.25\left(\frac{d_p}{d_c}\right)^{0.65}\left(\frac{d_n}{d_c}\right)^{0.312}\left(\frac{H}{d_c}\right)^{0.254}\sqrt{\frac{2gH\left(\rho_s-\rho_f\right)}{\rho_g}} \tag{4}$$

Where $d_c$ is an effective bed diameter corresponding to the diameter of a circle with the same area as the bed cross section.

The objective of the current study is to study the effects of varying cone angles on the minimum spouting velocity for rectangular spouted beds, and to obtain an expression for the prediction of the minimum spouting velocity in terms of cone angle, static bed height, particle and nozzle diameter, and a characteristic bed length.

## EXPERIMENTAL SETUP

A small-scale cold flow spouted bed unit is shown in Fig. 2. Fig. 2(a) provides a process flow diagram, and the actual experimental unit is depicted in figure 2(b). the experimental unit has a cross section of 4 inches wide by 1 inch in depth, with a total height of approximately 32 inches. The lower section of the bed is flanged to allow for interchanging different sloped cone sections. For the current testing, cone angles of 60° and 75° were used, providing included angles of 60° and 30°, respectively. Solids are loaded into the unit via a port located at the top of the bed, as well as through the side via a small plunger-like feed device (not shown). The spouting gas (air) enters the bed via an interchangeable, 3d printed nozzle assembly located at the bottom of the cone section, and exits through a pair of outlet ports located at the top of the unit. For the current study, rectangular slotted nozzles with areas corresponding to 3/8-inch, and ½-inch diameter circular nozzles were used. The rate of airflow into the unit is controlled via an alicat mass flow controller with a range of 0-1500 slpm. The gas exits the unit via exhaust ports located at the top of the unit, where any entrained solids are separated via filters. For collection of differential pressure data, pressure taps are located at multiple locations along the height of the unit. These are connected to a series of Setra model 239 differential pressure transducers with ranges of 0-30 in-$H_2O$. The signals from these transducers are sampled and recorded via Labview at a sample rate of 100hz.

Experiments were carried out using a combination of different static bed heights (measured from the bottom of the conical section), nozzle sizes, cone angles, and solid particles. Table 1 provides the material properties of the solids materials used.

## RESULTS AND DISCUSSION

### Nozzle Size Effects

Figure 3(a), 3(b) and 3(c) show comparisons between the experimentally determined minimum spouting velocities for three different solids (nylon, glass beads, and alumina spheres) for both the 0.5-inch and 0.375-inch equivalent rectangular nozzles using the 75-degree cone geometry. As shown in the figures, the two nozzle sizes resulted in nearly identical values for the minimum spouting velocity, with the minor variations falling within experimental error.

### Cone Angle Effects

Given the results of the previous section illustrates that the minimum spouting velocity values were only minimally effected by the size of the gas inlet nozzle located at the bottom of the bed, the following discussion focuses primarily on the results for the 0.375-inch area-equivalent nozzle cases. Figure 4(a) and 4(b) show comparisons between the experimentally determined minimum spouting velocities for the HDPE cases against the predicted values of Eqns. (1) - (3) using the bed width and hydraulic bed diameters, respectively. The Mathur-Gishler (Eqn. 1) and Smith and Reddy (Eqn. 2) correlations provide reasonably close predictions of Ums for one, but not both, of the cone angles when using the rectangular bed width as the bed diameter. However, when the hydraulic diameter is used with these equations, the Mathur-Gishler equation significantly overshoots the experimental data, while the Smith and Reddy equation more closely approximates the 60-degree cone values, whereas it more closely approximated the 75-degree experimental values when calculated with the bed width. Similarly, when using the bed width as the characteristic dimension of the rectangular bed, the expression proposed by Brunello et al. results in an under-prediction of Ums, while closely approximating the 75° cone case when the hydraulic diameter is used. In each case, the three models are unable to predict the effects of cone angle on the minimum spouting velocities.

Figure 5(a) and 5(b) show similar comparisons for the 3.2mm nylon beads. In this case, if the bed width is used as the diameter in Eqns. (1) – (3), the result is an under-prediction if Ums for all three models. If the hydraulic diameter is used, the result is an over-prediction from Eqn. (1) and under-predictions from Eqns. (2) and (3).

It can be concluded from Figures 4 and 5 that the previously published expressions for estimation of the minimum spouting velocity that were derived for cylindrical spouted beds are not appropriate in the case of a rectangular bed. Anabtawi et al. proposed Eqn. 4 for the estimation of minimum spouting velocities for rectangular beds, using the equivalent, or hydraulic diameter, as the characteristic size of the bed. However, as was the case with the previous models, Eqn. 4 also lacks the ability to predict the effects of differing cone angles on the minimum spouting velocity. This is shown in figure 6, which compared the nylon bead data to the that predicted by Eqn. 4. When the equivalent (or hydraulic) diameter is used, as suggested by Anabtawi et al., the result is a large over-prediction of Ums. Using the bed width as the characteristic length results in a much closer prediction at lower bed heights, but not for higher bed heights.

**Correlation**

To obtain an expression that more closely matched the experimental data obtained for the minimum spouting velocity, a power law of the form shown in Eqn. 5 was applied to the experimental data. The unknowns (a, b, c, d, e, f) where obtained via a non-linear regression analysis. The final form of the new expression is shown in Eqn. 6.

$$U_{ms} = a\theta^{b}\left(\frac{d_p}{L_c}\right)^{c}\left(\frac{H}{L_c}\right)^{d}\left(\frac{2gH\left(\rho_p - \rho_g\right)}{\rho_g}\right)^{e} \quad (5)$$

Where θ is the included cone angle (in radians) and $L_c$ is a characteristic length for the bed. Here, $L_c$ is taken to be the diagonal dimension of the rectangular cross section.

$$U_{ms} = 0.178\theta^{0.219}\left(\frac{d_p}{L_c}\right)^{1.304}\left(\frac{H}{L_c}\right)^{-0.468}\left(\frac{2gH\left(\rho_p - \rho_g\right)}{\rho_g}\right)^{0.837} \quad (6)$$

Figure 7 is a cross-plot of the experimental data and the model predictions based upon Eqn. (6). As can be seen, the model is a close fit to the experimental data ($R^2$ value of 0.995). The primary benefit of this model over Eqn. (1) – (4) is that it accounts for variations in cone angle. It should be noted that nozzle diameter does not appear in Eqns. (5) or (6) due to the earlier determination that it did not have an appreciable effect on $U_{ms}$.

## CONCLUSIONS

A series of experiments was conducted to examine the effects of bed geometry, particle properties, and static bed height on the minimum spouting velocity in a 4-inch x 1-inch rectangular spouted bed. Four different solids materials, two different cone angles, two different nozzle sizes, and up to three static bed heights were tested. The resulting minimum fluidization velocities were compared to previously published models based upon cylindrical spouted beds, and it was determined that these models did not provide good predictions. Additionally, a comparison was made between the experimental data and a previously published model for rectangular bed. This model more closely matched the experimental data when the bed width was used as the characteristic length; however, it did so primarily at lower bed heights, and did not include effects of cone angle variations.

A new model is proposed for rectangular spouted beds using the diagonal (of the rectangular cross section) dimension in place of bed diameter. This new model incorporates the effects of cone angle variations (in the form of the included angle), and provides a 0.995 $r^2$ correlation to experimental data.

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The authors declare no competing financial interest.

## NOMENCLATURE

| | |
|---|---|
| $U_{ms}$ | Minimum spouting velocity, m/s |
| $dp$ | Particle diameter, m |
| $d_i$ | Gas inlet diameter, m (inches) |
| $d_c$ | Characteristic length/diameter of bed, m (inches) |
| H | Static bed height, m (inches) |
| $g$ | Gravitational constant, $m/s^2$ |
| $\rho_s$ | Solids density, $kg/m^3$ |
| $\rho_g$ | Gas density, $kg/m^3$ |

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| | |
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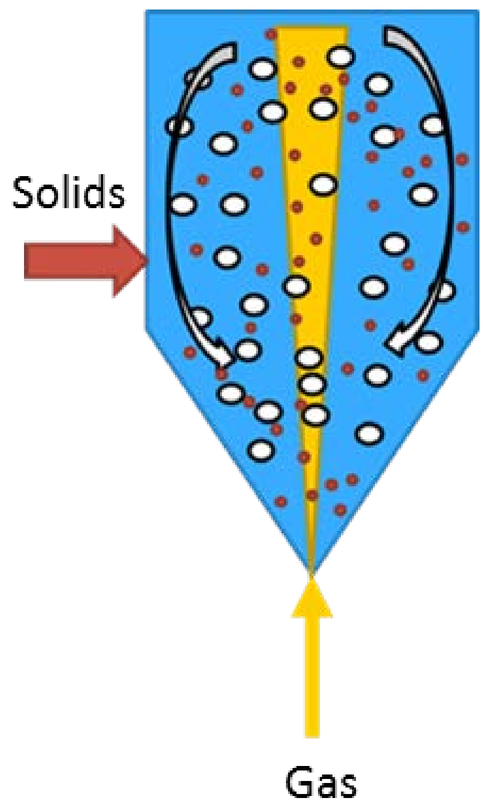
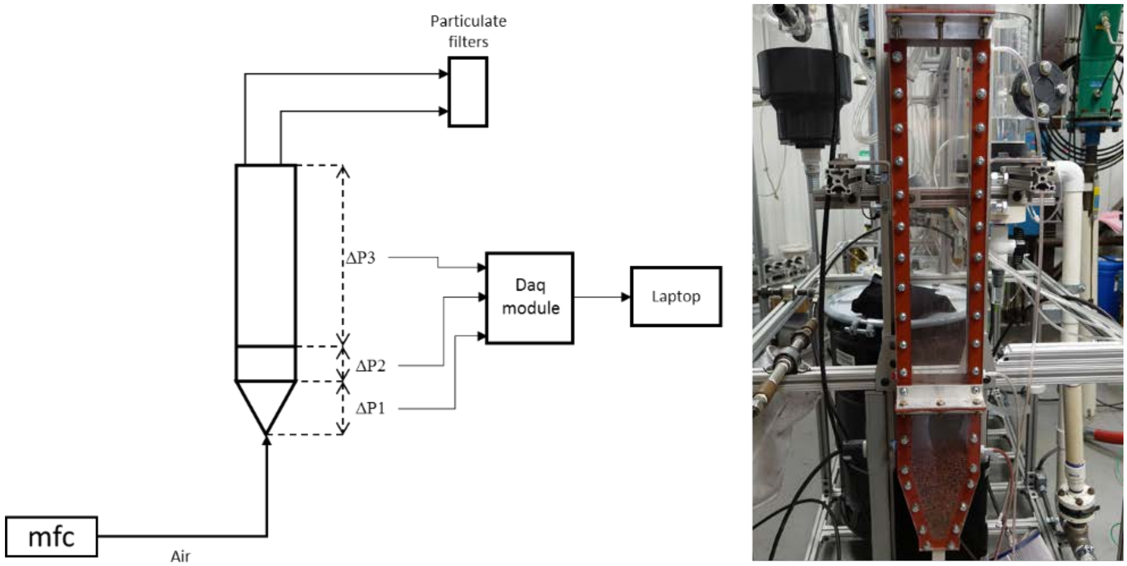



Figure 1: Conceptual diagram of a spouted bed




(a) (b)

Figure 2: (a) Process and instrumentation diagram (b) cold flow spouted bed unit.

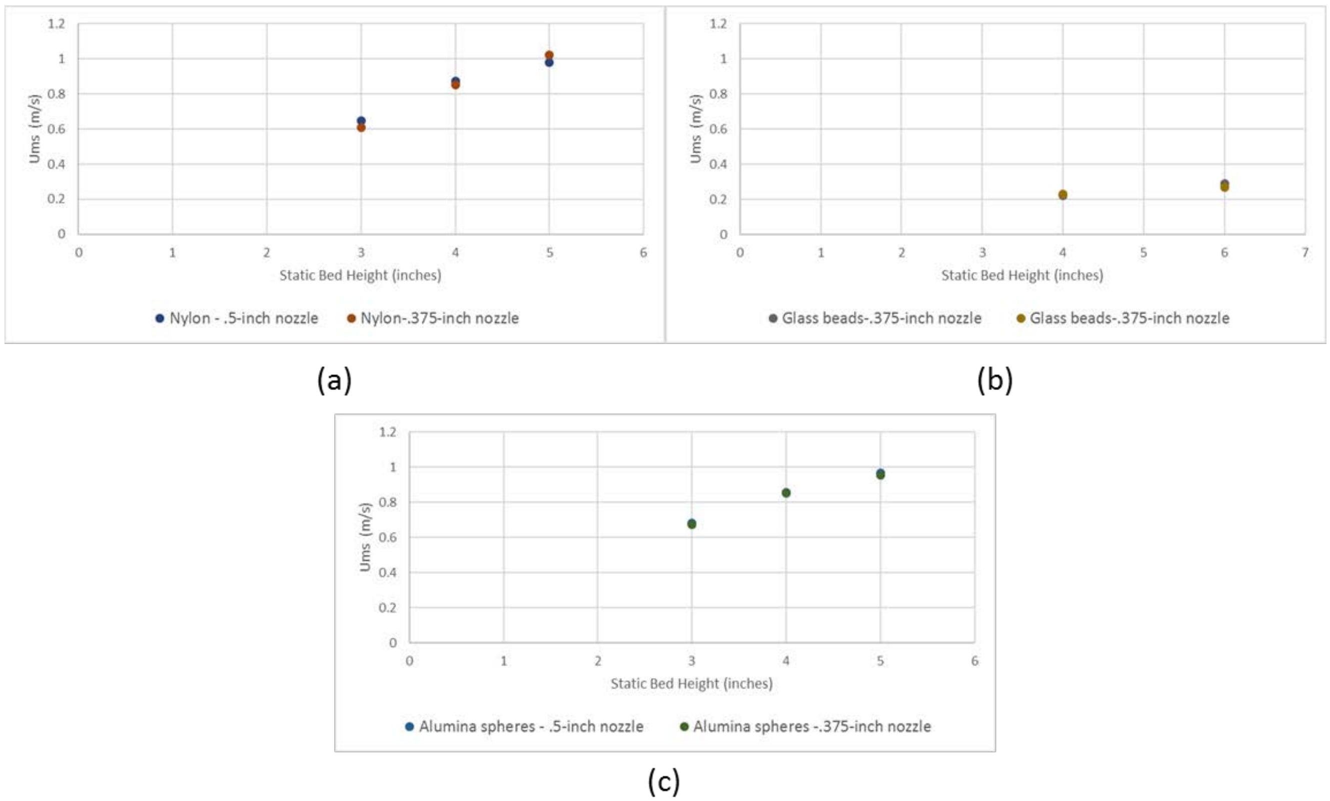



Figure 3: Comparison of nozzle size effects on Ums values as a function of static bead height for (a) Nylon beads, (b) Glass beads, and (c) Alumina spheres.

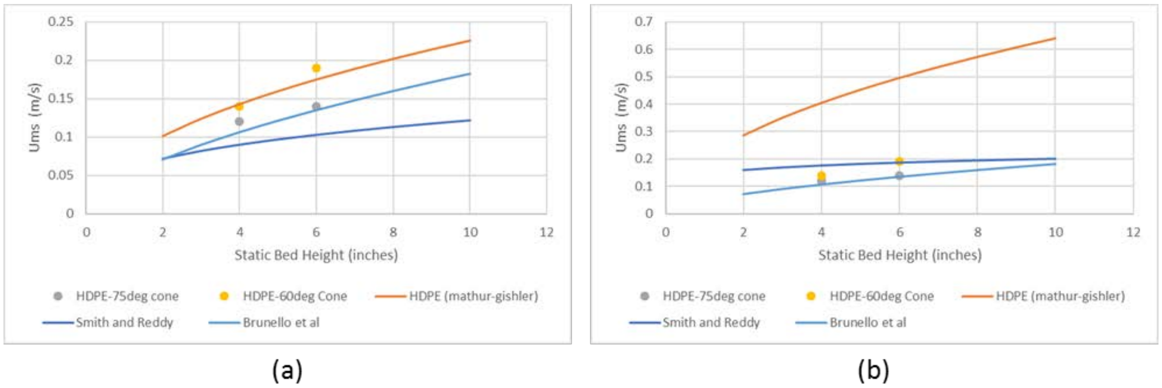



Figure 4: Comparison of experimental data to predicted Ums values from Eqns. (1) – (3) for HDPE cases when (a) dc = rectangular bed width, and (b) dc = hydraulic diameter.

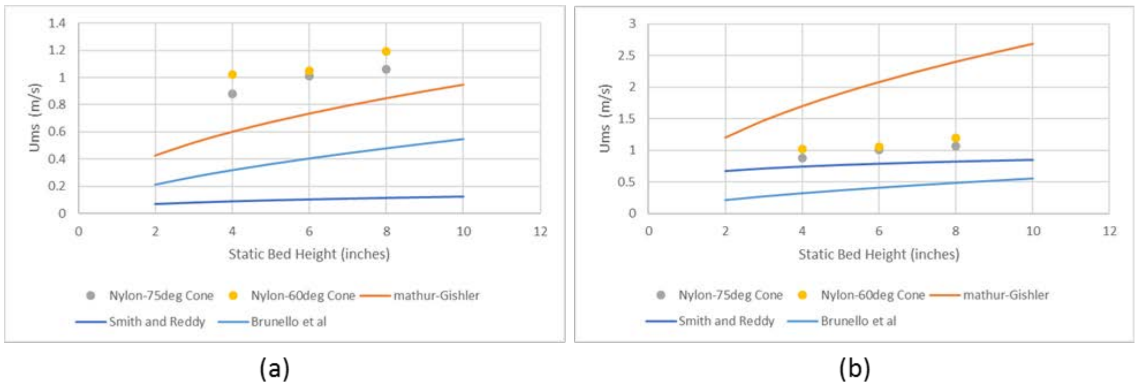



Figure 5: Comparison of experimental data to predicted Ums values from Eqns. (1) – (3) for Nylon Beads cases when (a) dc = rectangular bed width, and (b) dc = hydraulic diameter.

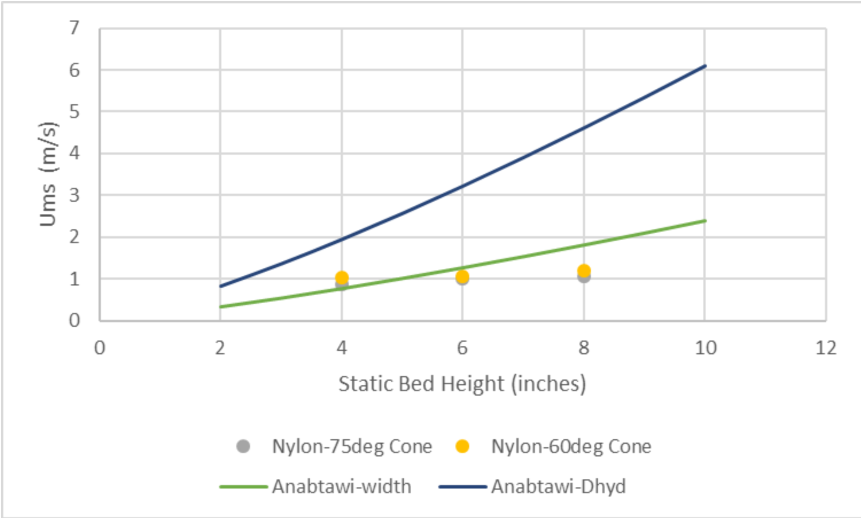



Figure 6: Comparison of experimental data to predicted Ums values from Eqn. (4) for Nylon Beads cases.

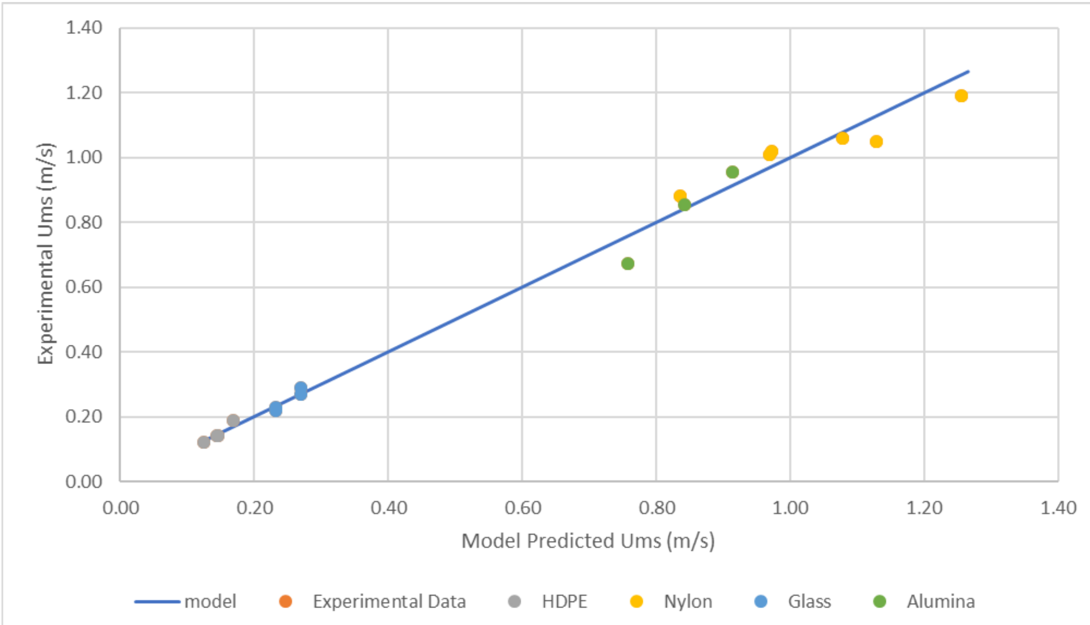


Figure 7: Comparison of experimental data to predicted Ums values from Eqn. (6)

Table 1: Material properties

| | HDPE | Glass Beads | Nylon Beads | Alumina Spheres |
|---|---|---|---|---|
| Particle Diameter (μm) | 871 | 707 | 3190 | 1500 |
| Density ($kg/m^3$) | 860 | 2500 | 1100 | 3600 |