

# 1                   The effect of nearby fractures on hydraulically induced fracture 2                   propagation and permeability changes

3                   Bruno Figueiredo<sup>1</sup>, Chin-Fu Tsang<sup>1,2</sup>, Jonny Rutqvist<sup>2</sup> and Auli Niemi<sup>1</sup>  
4

5                   <sup>1</sup>Uppsala University, Villavägen 16, Uppsala, Sweden

6                   e-mail: bruno.figueiredo@geo.uu.se

7                   Phone number: +46739735500

8                   <sup>2</sup>Lawrence Berkeley National Laboratory, Berkeley, California

9  
10                   **Abstract:** Fracture propagation caused by a hydraulic fracturing operation can be significantly influenced by nearby fractures. This paper presents a detailed coupled hydro-mechanical analysis to study the effect of nearby fractures on hydraulically induced fracture propagation and changes in fracture permeability. Two fractured rock domains were considered in comparison: FD1, with one single fracture, and FD2, with two adjacent parallel and non-parallel fractures. It is assumed that water injection occurs in a borehole that intersects the single fracture in FD1, and one of the two fractures, in FD2. Simulations were made for a time period of 3 hours with an injection period of 2 hours followed by 1 hour of shut-in. An elastic-brittle model based on material properties degradation, was implemented in a 2D finite-difference scheme, and used for elements of the intact rock subjected to tension failure. The intact rock was considered to have a low but non-negligible permeability. A verification study against analytical solutions showed that the fracture propagation and stress concentrations due to differential boundary stresses can be accurately represented by our model. Then, a base case is considered, in which the ratio  $SR$  between the magnitudes of the maximum and minimum boundary stresses, the permeability  $k_R$  of the intact rock and the initial permeability  $k_{TF}$  of the tension failure regions, are fixed. In FD2, the distance  $d$  between the two fractures defined as the closest distance is also fixed. Results show that in both fractured rock domains, the fracture starts to propagate when the pore pressure is approximately 85% of the magnitude of minimum boundary stress. The propagation of a single fracture is significantly larger than the propagation of a double fracture system, because in the latter case, the pore pressure decreases when the two fractures connect. As a result, changes in permeability in FD2 were found to be smaller than in FD1. At 2 hours of injection the maximum ratio between the final and initial permeability of the fractures was found to be approximately 3 and 2 for fractured rock domains FD1 and FD2, respectively. For non-parallel fractures, the controlling factor is the separation between the tips of pressurised fracture to the neighbouring non-pressurised fracture. A sensitivity analysis was done to study the influence of the key parameters  $d$ ,  $SR$ ,  $k_R$  and  $k_{TF}$  on the simulation results. Fracture propagation showed more sensitivity to  $d$  and  $SR$  than to the other parameters.

36  
37                   **Keywords:** hydraulic fracturing stimulation, coupled hydro-mechanical effects, fractures propagation  
38                   and connectivity, elastic-brittle model

39 **1. INTRODUCTION**

40 Hydraulic fracturing is a method used routinely in oil and gas exploitation and in enhanced geo-  
41 thermal systems. This is a technique which creates fractures in deep-rock formations by mean of high  
42 pressure fluid injection and thus increases flow permeability in the injection region. Hydraulic fracturing  
43 stimulation leads to changes in pore pressure and effective normal stress across the created fractures,  
44 which in turn, leads to consequent fracture propagation. Hence, the fracture permeability depends on  
45 the *in situ* stress conditions and on the pressure of the flowing fluid [1]. Hydro-mechanical coupling is  
46 an important issue that needs to be taken into account [2,3].

47 In order to understand the fracturing processes, several laboratory experiments and 2D and 3D  
48 numerical studies have been made by many researchers. In those studies, the fracture closure, exten-  
49 sion and mechanical interactions for parallel and quasi-parallel fractures have been analysed [4, 5, 6].  
50 Laboratory experiments were done in gypsum ([7], [8]) and gypsum and marble ([9], [10]) to under-  
51 stand the fracture propagation caused by differential boundary stresses. In [11] and [12], samples of  
52 granite with single and double flaw geometries under quasi-static vertical compressive loads were  
53 tested. In [13] fractures were created by compressing granite cores uniaxially. In [14] the fracture  
54 propagation in sandstones induced by the confining stresses and increase pore pressure was studied.  
55 Fracture propagation based on application of dynamic loads was studied in [15], [16] and [17].

56 Numerical continuum and discrete based models have been developed to study the fracture prop-  
57 agation induced by hydraulic injection pressure under confining stresses. Continuum based models  
58 have used the finite element method ([18], [19], [20], [21], [22]), the extended finite element method  
59 ([23], [24], [25], [26]) and the explicit finite differences method ([27], [28], [29]). Discrete based models  
60 used the boundary element method ([30], [31], [32]), the particle flow method ([33], [34], [35], [36],  
61 [37]), the bonded particle model ([38]), the distinct element code ([39]) and the discontinuum defor-  
62 mation analysis method ([40]).

63 Discrete based models are more realistic for discontinuous media, but they have the limitation of  
64 not considering the intact rock permeability and are time consuming for modelling the hydro-  
65 mechanical behaviour of fractured rock domains with curved or dead-end fractures. Continuum based  
66 models require a representation of discrete fracture behaviour in an element cell by appropriate equiv-  
67 alent hydro-mechanical properties [41]. Compared with discontinuous approaches, they have as main  
68 advantages, the representation of complex fracture networks without the need of update their topology  
69 and the modelling with high accuracy of the hydro-mechanical behaviour of both rock matrix and the  
70 fractures which can be sealed or filled with mineral materials. Thus, once the fracture propagates into  
71 the continuum medium, stress-induced changes in permeability and porosity can be included ([42],  
72 [43], [44], [45], [46]). By using an elastic-plastic and strain softening model, a continuum based model  
73 may not be very effective in simulating fracture propagation because of large plastic zones around the  
74 fracture tips. However, it has been shown that a model based on degradation of the mechanical prop-  
75 erties and stress distribution for the failure elements of the intact rock by tension and shear, is effective  
76 for this purpose ([27], [28], [29]).

77 To the authors' knowledge, no continuum based model was used in a detailed coupled hydro-  
78 mechanical study to understand the difference between the propagation in a low permeable medium of

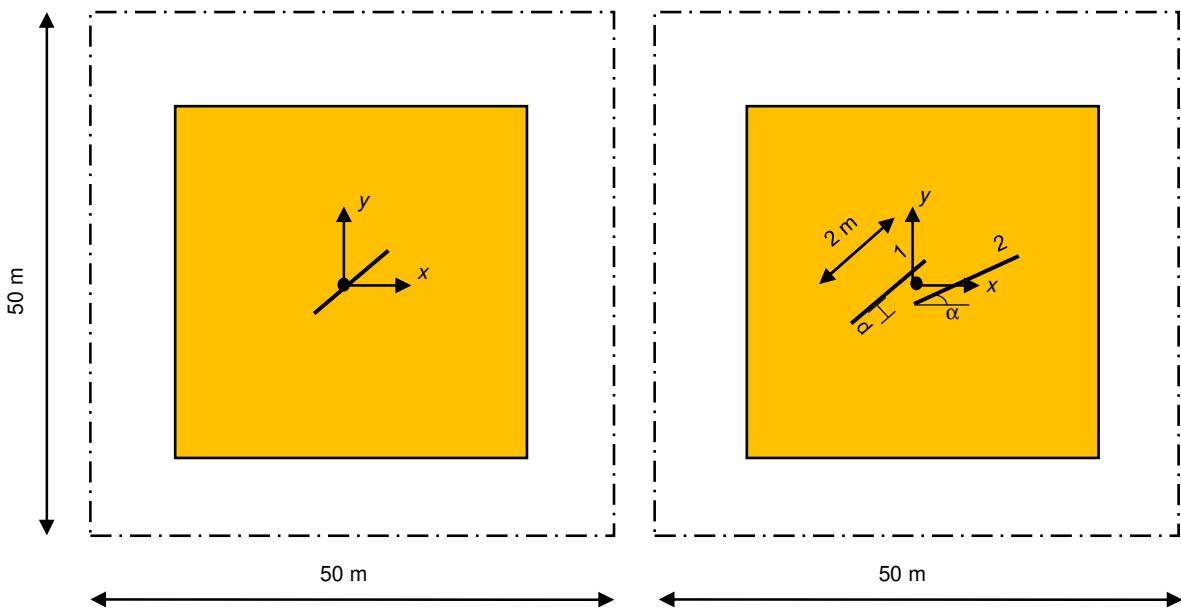
79 a single fracture and double parallel and non-parallel fractures under various stress conditions and  
80 different levels of fluid injection pressure. For our study of coupled hydro-mechanical effects as a func-  
81 tion of increases in pore pressure, we consider two fractured rock domains: the first with one single  
82 fracture, and the second with double adjacent parallel and non-parallel fractures. Changes in fluid pore  
83 pressure are assumed to be caused by constant injection flow rate in a well that intersects one of the  
84 fractures.

85 The main objectives of the paper are (1) firstly to verify or demonstrate the effectiveness of using a  
86 continuum mechanics based model with an implemented elastic-brittle stress relation to simulate the  
87 fracture propagation and stress concentrations around fracture tips, (2) to study how a single fracture  
88 propagates when it is subjected to hydraulic fracturing stimulation (3) to evaluate changes in the pore  
89 pressure field and fracture permeability induced by coupled hydro-mechanical processes (4) to ana-  
90 lyse how the results are influenced by a nearby parallel and non-parallel fracture and (5) to conduct a  
91 sensitivity analysis to determine the key parameters with significant influence on fracture propagation  
92 and linkage between nearby fractures during hydraulic fracturing process. The paper is completed with  
93 some concluding remarks.

94

## 95 **2. PROBLEM DEFINITION**

96 For our study, we choose to consider two fractured rock domains, FD1 and FD2, each with dimen-  
97 sions  $50 \text{ m} \times 50 \text{ m}$ , which allow us to conduct a large number of simulations in order to explore the  
98 detailed coupled hydro-mechanical processes involved. The fractured rock domains FD1 and FD2  
99 consider one single and two fractures, respectively (Fig. 1). In FD2, the left and right hand-side frac-  
100 tures are identified as fractures 1 and 2. In both rock domains, the length  $2f$  of the fractures is 2 m.

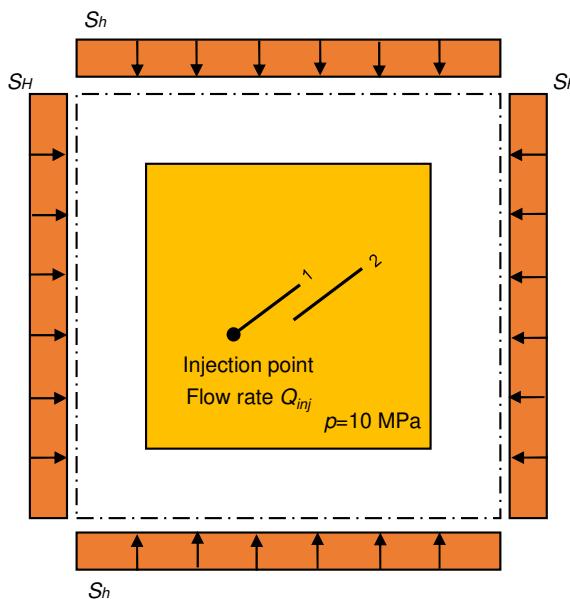


102  
103 Fig. 1: Geometry of the fractured rock domains FD1 (left) and FD2 (right)  
104

105 To study in detail the linkage between the two fractures, in FD2, parallel and non-parallel fractures  
 106 with different angles between those and the maximum horizontal boundary stress direction, were con-  
 107 sidered. Thus, for parallel fracture case, the fractures were assumed to be at angles  $\alpha_1$  and  $\alpha_2$  equal  
 108 to  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ , and for non-parallel fracture case, the angle  $\alpha_1$  of fracture 1 is taken to be equal to  
 109  $45^\circ$ , and the angle  $\alpha_2$  of fracture 2 to be equal to  $30^\circ$  and  $60^\circ$ . The origin of the  $x$  and  $y$ -axis system is  
 110 located in the centre of the studied regions. In FD2, the closest distance  $d$  (see Fig. 1) between the  
 111 fractures is 0.25 m, with a sensitivity analysis conducted to study the influence of  $d$  on the obtained  
 112 results (see section 6.1).

113 Let us now assume that these fractured rock domains are located at 1000 m depth. By assuming a  
 114 vertical gradient of 0.027 MPa/m, the magnitude of the vertical stress component ( $S_v$ ) at 1000 m depth  
 115 below the surface is 27 MPa. A loading case was considered, in which the minimum horizontal bound-  
 116 ary stress magnitude ( $S_h$ ) is equal to the vertical stress magnitude ( $\sigma_v$ ) and the ratio  $SR$  between the  
 117 maximum horizontal  $S_H$  and minimum horizontal  $S_h$  boundary stresses is 2 (Fig. 2). Further, a sensitiv-  
 118 ity analysis is made to study the influence of  $SR$  on the simulation results (see section 6.2). Because  
 119 the vertical dimension of the model is only 50 m, the vertical gradient of all stress components was  
 120 neglected. The stresses are applied normal to the boundaries which are free to move. No shear  
 121 stresses are considered at the boundaries (see Fig. 2). Results of our simulations showed that be-  
 122 cause the boundary conditions are imposed far enough, they do not influence the stresses around  
 123 fractures as well as their propagation in the intact rock.

124



125  
 126 Fig. 2: Boundary loading and pore pressure conditions:  $S_H$  and  $S_h$  are the maximum and minimum  
 127 horizontal boundary stresses, respectively;  $p$  is the initial fluid pore pressure;  $Q_{inj}$  is the constant flow  
 128 rate

129  
 130 By assuming that the water table is located at the land surface and a fluid pore pressure vertical  
 131 gradient of 0.01 MPa/m, the fluid pore pressure  $p$  at 1000 m depth below the surface is 10 MPa. The

132 pore pressure gradient in the  $x$  and  $y$ -axis directions was neglected. All the boundaries were considered to be closed to flow. Results of our simulations showed that the results are not influenced by the flow boundary conditions.

135 We simulate a water injection at a constant rate  $Q_i$  for two hours in one borehole penetrating the  
136 only fracture in FD1 and the fracture 1 in FD2 (see Fig. 2). The borehole is assumed to be vertical  
137 (perpendicular to FD1 and FD2). In this way, hydraulic fracturing was imposed in the single fracture in  
138 FD1 and in the fracture 1 in FD2. After two hours, water injection is stopped, and the simulation con-  
139 tinues for another hour.

140

### 141 **3. NUMERICAL APPROACH**

#### 142 **3.1 Finite-difference numerical model**

143 To study the fracture propagation due to coupled hydro-mechanical effects as a result of hydraulic  
144 fracturing stimulation, a 3D model is desirable if at all possible. However a global 3D model would be  
145 very large and the necessary fine refinement close to the fracture would require a great computational  
146 effort. A 2D model is adequate from a mechanical perspective, particularly for investigating the fracture  
147 propagation, because this is driven by the pore pressure build-up at the tip of the fractures that can be  
148 simulated explicitly with a 2D model and an adequate injection rate. This should lead to a pore pres-  
149 sure at the fracture tip necessary to start fracture propagation, as observed in field experiments. A 2D  
150 finite-difference model was developed in FLAC3D ([47]). This code was chosen because we want to  
151 have the possibility to consider multi-phase flow in the future studies, and we have already the rou-  
152 tines to couple FLAC3D with TOUGH2 [48], which is a leading multiphase flow and transport simula-  
153 tor. The model is a square region with 50 m side, with a thickness of 1 m (Fig. 3). A plane strain analy-  
154 sis was carried out. The mesh consists of 56000 elements and is more refined in a square region 10 m  
155 by 10 m around the fractures, where the elements are squares with each side 0.05 m (Fig. 3).

156 Fractures can be modelled as an equivalent solid material, in which the elastic modulus  $E_F$  of the  
157 elements intersected by a fracture trace is calculated according with the following equation ([41], [49]):

158

$$159 \frac{1}{E_F} = \frac{1}{E_R} + \frac{1}{k_n d}, \quad (1)$$

160

161 where  $d$  is the element size (0.05 m).

162 The hydraulic behaviour of the fractures may be described in terms of the flow transmissivity and  
163 the normal and shear stiffness of the fractures. Laboratory experiments on single fractures show that  
164 the fracture transmissivity can be very sensitivity to changes in stress normal to the fractures as well  
165 as to shear displacement. Thus, mechanically induced changes in the fracture's ability to conduct fluid  
166 may be estimated using the cubic relations between flow along an open fracture and fractures aper-  
167 ture ([2], [50]):

168

169

$$T = \frac{b_h^3 \rho g}{12\mu}, \quad (2)$$

170

171 where  $T$  is the fracture transmissivity,  $b_h$  is fracture aperture,  $\rho$  and  $\mu$  are fluid density and viscosity, respectively, and  $g$  is the acceleration of gravity.

172

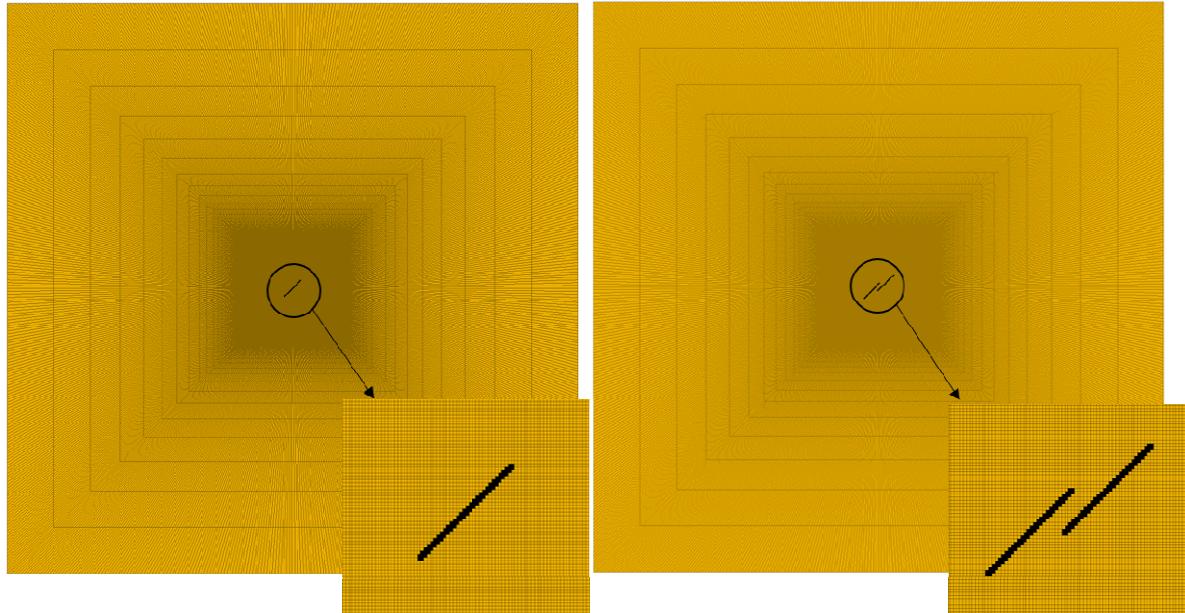
173 The permeability  $k_F$  of an element containing by a fracture trace is related to the fracture aperture  $b_h$  by the cubic law:

174

$$k_F = \frac{b_h^3}{12d}, \quad (3)$$

175

176 where  $d$  is element size (0.05 m).



180

181 Fig. 3: Detail of the mesh of the finite-difference model to study the hydro-mechanical behaviour of the  
182 fractured rock domains FD1 (left) and FD2 (right), for the case of parallel fractures

183

184 3.2 Model parameters

185 Necessary model parameters are listed in Table 1. For the intact rock, a Mohr-Coulomb model with  
186 tension cut-off was used, in which the mechanical properties (elastic modulus  $E_R$ , Poisson's ratio  $\nu_R$ ,  
187 cohesion  $c_R$ , friction angle  $\phi_R$ ) are characteristic of limestone rocks ([51], [52]). An elastic-brittle model  
188 was implemented in FLAC3D to describe the behaviour of the failure elements in the intact rock. This  
189 model is described in the next section. A tensile strength  $\sigma_{IR}$  of 5 MPa for the intact rock was as-  
190 sumed. In the regions of the intact rock where the tensile stress exceeds the tensile strength, tension  
191 failure occurs. A sensitivity analysis was done to study the influence of this parameter on the results.  
192 An additional value of 10 MPa was considered which is acceptable for intact limestone at 1000 m

193 depth. Results showed a decreased fracture extension when the tensile strength increases. It was  
 194 found that when the tensile strength increases from 5 to 10 MPa, in FD1 the fracture propagation de-  
 195 creases 0.7 m. In FD2, this decrease is 10 cm and 25 cm, for fractures 1 and 2, respectively. Howev-  
 196 er, the conclusions are similar to those reported in this paper. Regarding the hydraulic properties, the  
 197 values of  $10^{-18} \text{ m}^2$  and 0.001 were assigned to the permeability  $k_R$  and porosity  $e_R$  of the intact rock,  
 198 which are typical of limestone rocks. Further, a sensitivity analysis is done to evaluate the influence of  
 199 the permeability of intact rock on the simulation results (see section 6.3).

200

201

Table 1: FLAC3D model parameters

|             |                                      |                       |
|-------------|--------------------------------------|-----------------------|
| Intact rock | Elastic modulus $E_R$ (GPa)          | 20                    |
|             | Poisson's ratio $\nu_R$              | 0.2                   |
|             | Tensile strength $\sigma_{tR}$ (MPa) | 5                     |
|             | Cohesion $c_R$ (MPa)                 | 30                    |
|             | Friction angle $\phi_R$ (°)          | 25                    |
|             | Permeability $k_R$ ( $\text{m}^2$ )  | $10^{-18}$            |
|             | Porosity $e_R$                       | 0.001                 |
| Fractures   | Elastic modulus $E_F$ (GPa)          | 14.3                  |
|             | Poisson's ratio $\nu_F$              | 0.2                   |
|             | Tensile strength $\sigma_{tF}$ (MPa) | 0                     |
|             | Friction angle $\phi_F$ (°)          | 25                    |
|             | Dilation angle $\psi_F$ (°)          | 5                     |
|             | Normal stiffness $k_n$ (GPa/m)       | 1000                  |
|             | Cohesion $c_F$ (MPa)                 | 0                     |
|             | Aperture $b_h$ ( $\mu\text{m}$ )     | 30                    |
|             | Permeability $k_F$ ( $\text{m}^2$ )  | $4.5 \times 10^{-14}$ |
|             | Porosity $e_F$                       | 0.01                  |

202

203 The mechanical fracture behaviour is modelled with continuum elasto-plasticity using a Mohr-  
 204 Coulomb constitutive model with tension cut-off. The mechanical properties of the fractures (Poisson's  
 205 ratio  $\nu_F$ , friction angle  $\phi_F$ , dilation angle  $\psi_F$ , cohesion  $c_F$ , fractures aperture  $b_h$ ) were extracted from [1].  
 206 When the Mohr-Coulomb criterion is exceeded, plastic shear strain (and corresponding shear dis-  
 207 placement) occurs along the fractures. The tensile strength  $\sigma_{tF}$  for fractures was assumed to be zero.  
 208 Results of our simulations showed low sensitivity to this parameter, because tension failure occurs in  
 209 the intact rock and in the fractures, shear failure is the dominant mechanism. The fracture normal stiff-  
 210 ness  $k_n$  was assumed to be 1000 GPa/m ([49]). Based on a fractures aperture of 30  $\mu\text{m}$  (see Table 1),  
 211 Eqs. (2) and (3) lead to a fracture transmissivity  $T$  of  $2.2 \times 10^{-8} \text{ m}^2/\text{s}$  and permeability of fractures  $k_F$  of  
 212  $4.5 \times 10^{-14} \text{ m}^2$ , respectively. The porosity  $e_F$  of an element representing a fracture was assumed to be  
 213 equal to 0.01 ([49]).

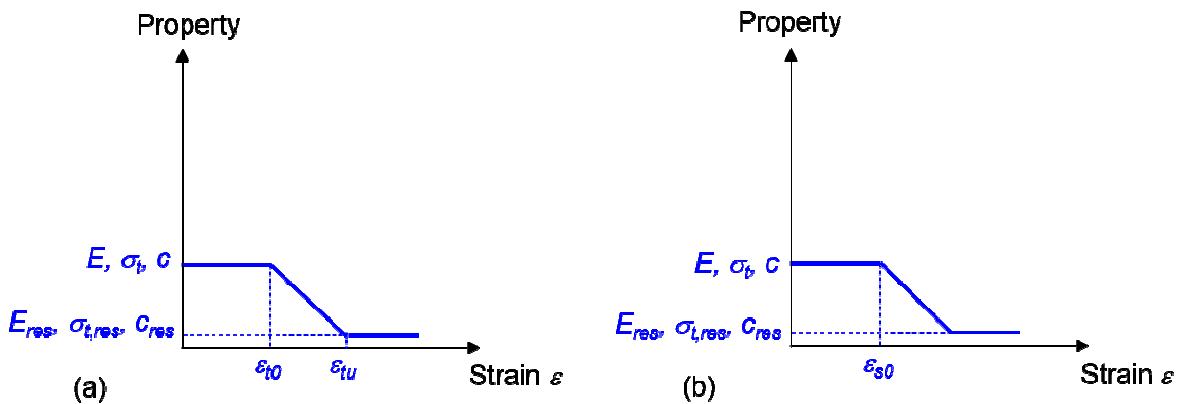
214 In the theoretical study presented in this paper, we simulated water injection as representative of  
 215 conditions that lead to propagation of existing fractures. We assumed a 2D injection rate of about  $4.0 \times 10^{-4} \text{ m}^3/\text{s}$  into a  $0.0025 \text{ m}^3$  grid block. This pressurization rate enables to reach to a maximum injection pressure that is about 2.5 times the initial pore pressure.  
 216

218

### 219 3.3 Elastic-brittle model in the failure regions in the intact rock

220 The behaviour of the intact rock undergoing tension or shear failure may be simplified to be represented by an elastic-brittle, elastic-strain softening (a combination of brittle and ductile) or elastic-  
 221 ductile (plastic) mechanisms. An elastic-plastic and strain softening model cannot effectively simulate  
 222 the fractures propagation because a large plastic zones appear around the fracture tips. An elastic-  
 223 brittle stress-strain relation, based on degradation of the mechanical properties and consequent stress  
 224 distribution for the failure elements by tension and shear (Fig. 4) has been shown to be more effective  
 225 for this purpose ([27], [28], [29]). In this model, failure of an element causes disturbance of the local  
 226 stress field, which may lead to progressive failure of surrounding elements.  
 227

228



229

230 Fig. 4: Degradation of the stiffness and strength properties for the failure elements of the intact rock by  
 231 (a) tension and (b) shear:  $E$ ,  $\sigma_t$  and  $c$  are the initial values for elastic modulus, tensile strength and  
 232 cohesion, respectively;  $E_{\text{res}}$ ,  $\sigma_{t,\text{res}}$  and  $c_{\text{res}}$  are their residual values, respectively,  $\varepsilon_{t0}$  is the strain thresh-  
 233 old of tension damage,  $\varepsilon_{tu}$  is the limit strain of tensile strength and  $\varepsilon_{s0}$  is the strain threshold of shear  
 234 damage.  
 235

236

237 In this model, for the elements in the intact rock that undergo yield tensile strength (Fig. 4a), stiff-  
 238 ness and strength properties are degraded, according to a damage variable  $D$ . This variable can be  
 239 expressed by the following equations ([29]):

240

$$D = \begin{cases} 0, & \varepsilon < \varepsilon_{t0} \\ 1 - \frac{\sigma_{t,\text{res}}}{E \cdot \varepsilon}, & \varepsilon_{t0} \leq \varepsilon \leq \varepsilon_{tu} \\ 1, & \varepsilon > \varepsilon_{tu} \end{cases}, \quad (4)$$

241  $\sigma_{t,res} = \eta \sigma_t ,$  (5)

243  $\varepsilon = \sqrt{(\varepsilon_1)^2 + (\varepsilon_2)^2 + (\varepsilon_3)^2} ,$  (6)

244 where  $\sigma_{t,res}$  is the residual tensile strength,  $E$  and  $\sigma_t$  are the elastic modulus and tensile strength of the  
 245 intact rock (Table 1),  $\eta$  is the residual strength coefficient,  $\varepsilon_{t0}$  is the initial damage threshold,  $\varepsilon_{tu}$  is the  
 246 limit of tensile strength, and  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  are the three principal strains.

247 For the elements subjected to shear failure (Fig. 4b), the damage variable  $D$  can be expressed as  
 248 follows ([29]):

250

$$251 D = \begin{cases} 0, \varepsilon_s < \varepsilon_{s0} \\ 1 - \frac{\tau_{s,res}}{\varepsilon_s \cdot E}, \varepsilon_s \geq \varepsilon_{s0} \end{cases} , \quad (7)$$

252 where  $E$  is the elastic modulus,  $\tau_{s,res}$  is the residual strength of shear damage,  $\varepsilon_{s0}$  is the strain thresh-  
 253 old of shear damage, and  $\varepsilon_s$  is the shear strain.

254 This model was implemented in a finite difference scheme. In our case, it was found that shear  
 255 failure does not occur in the intact rock where tension failure is the dominant mechanism. In those  
 256 regions, the stiffness and strength properties were degraded. Stiffness degradation is implemented by  
 257 simply updating elastic modulus  $E$  in the stress-strain calculations, and strength degradation is mod-  
 258 elled by reducing the tensile strength  $\sigma_t$  and the cohesion  $c$  of the intact rock. The friction angle was  
 259 kept invariant ([29]). The corrected values for the elastic modulus  $E_{corr}$ , tensile strength  $\sigma_{t,corr}$  and co-  
 260 hesion  $c_{corr}$  are given by the following equations:

262

$$263 E_{corr} = E - (E - E_{res}) \times D , \quad (8)$$

264

$$265 \sigma_{t,corr} = \sigma_t - (\sigma_t - \sigma_{t,res}) \times D , \quad (9)$$

266

$$267 c_{corr} = c - (c - c_{res}) \times D , \quad (10)$$

268 where  $E_{res}$ ,  $\sigma_{t,res}$  and  $c_{res}$  are the residual values of the elastic modulus, tensile strength and cohesion  
 269 (Fig. 3), respectively. In our simulations, the initial values of the elastic modulus, tensile strength and  
 270 cohesion (Table 1) were reduced to one percent of the original values ([29]). This enabled our model  
 271 to obtain a good fit for fracture extension with the analytical solutions when the rock domain is subject-  
 272 ed to differential boundary stresses, as it will be shown in section 4.

274 In the original fractures, shear failure is the dominant mechanism. The elements that represent  
275 them get into shear failure for very small shear strains because they have null cohesion. Consequent-  
276 ly, for those elements, the stiffness is not degraded and the elastic modulus is given by equation (1).

277

### 278 3.4 Permeability changes in the natural fractures and tension failure regions

279 In fractured rock masses, effective stresses (which include effect of fluid pore pressure) induce  
280 changes in hydraulic properties, such as the permeability and porosity. In natural fractures the initial  
281 values of porosity and permeability were corrected by taking into account changes in volumetric  
282 strains ([50]), which are defined as the ratio of the change in volume of the fracture elements to its  
283 original volume. For this purpose, a model developed and applied by [53] to consider permeability  
284 changes in petroleum reservoirs was used. This model first relates the porosity  $\phi$  at a given stress to  
285 the isotropic volumetric strain variation  $\varepsilon_v$  in the fracture elements and then the permeability  $k$  at a  
286 given stress to changes in porosity, according to the following equations:

287

$$288 \phi = 1 - (1 - \phi_i) \exp(-\varepsilon_v), \quad (11)$$

289

$$290 k = k_i \left( \frac{\phi}{\phi_i} \right)^n, \quad (12)$$

291 where  $\phi_i$  is the initial porosity,  $k_i$  is the initial permeability and  $n$  is a power law exponent.

292 With the changes in volumetric strains resulting from changes in the fractures normal stress,  
293 changes in fractures aperture are considered: if the compressive stress normal to the fractures de-  
294 creases, the fractures aperture increases and the compressive volumetric strains decrease. Volumet-  
295 ric strains include elastic and plastic components. Elastic component is originated by elastic shear  
296 deformation until the Mohr-Coulomb criterion is reached. After this criterion is reached, shear failure  
297 occurs and variations in the volumetric strains include effects of plastic shear deformation and associ-  
298 ated shear dilation. Shear dilation leads to an increase in the fractures aperture and a subsequent  
299 increase in the initial values of porosity and permeability of the fractures.

300 The empirical relation between permeability and porosity expressed in Eq. (12) has been shown to  
301 be widely applicable to geological materials. Even though the exponent  $n$  could vary between 3 and 25  
302 for consolidated geological materials ([54]), we have set the exponent to 3, based on a cubic variation  
303 of the permeability with the aperture and porosity of the elements intersected by the fracture trace  
304 ([55]):

305

$$306 \frac{k}{k_i} = \left( \frac{b_h}{b_{hi}} \right)^3 = \left( \frac{\phi \cdot d}{\phi_i \cdot d} \right)^3 = \left( \frac{\phi}{\phi_i} \right)^3, \quad (13)$$

307 where  $b_{hi}$  is the initial aperture of the fractures.

310 The failure regions by tension in the intact rock are considered to be similar to natural fractures.  
311 When the elements of the intact rock get into failure by tension, they are assigned with the same initial  
312 values for porosity and permeability as those of natural fracture elements. Then, these initial values  
313 were updated, according to Eqs. (11) and (12), to take into account the stress induced changes in  
314 porosity and permeability ([50]). In this way, extension of fractures is modelled. It was found that the  
315 maximum increase in the initial permeability of the tension failure regions is two orders of magnitude.  
316 Further, a sensitivity analysis is done to evaluate the influence of the permeability of the tension failure  
317 regions on the obtained results (see section 6.4).

318

### 319 3.5 Coupled hydro-mechanical calculation

320 A mechanical analysis is carried out by considering the boundary stresses  $S_H$  and  $S_h$  and the initial  
321 fluid pore pressure  $p$  of 10 MPa. After the mechanical equilibrium is reached, a flow analysis is made  
322 to calculate changes in pore pressure field resultant from water injection into the fracture (see Fig. 2)  
323 with a constant flow rate  $Q_{inj}$  during a 2 hours period. At 2 hours of injection, water injection is stopped.  
324 Increase in fluid pore pressure in the fracture and surrounding intact rock leads to a decrease in the  
325 effective stress. In the regions of intact rock where the tensile stresses exceed the tensile strength,  
326 tension failure occurs. Then, a mechanical analysis is made to calculate stress field induced changes  
327 in porosity and permeability. The post-failure values of porosity and permeability of those tension fail-  
328 ure regions are set to the respective values considered for natural fractures. Then, changes in porosity  
329 and permeability in the natural fractures and tension failure regions were considered as a function of  
330 the volumetric strains, as described in section 3.4. The coupled hydro-mechanical analysis is sequen-  
331 tial and stepped forward in time. In each time step of transient flow calculation, a quasi-static mechani-  
332 cal analysis is conducted to calculate stress-induced changes in permeability. The analysis is done for  
333 a period of 3 hours (shut-in occurs after 2 hours of injection).

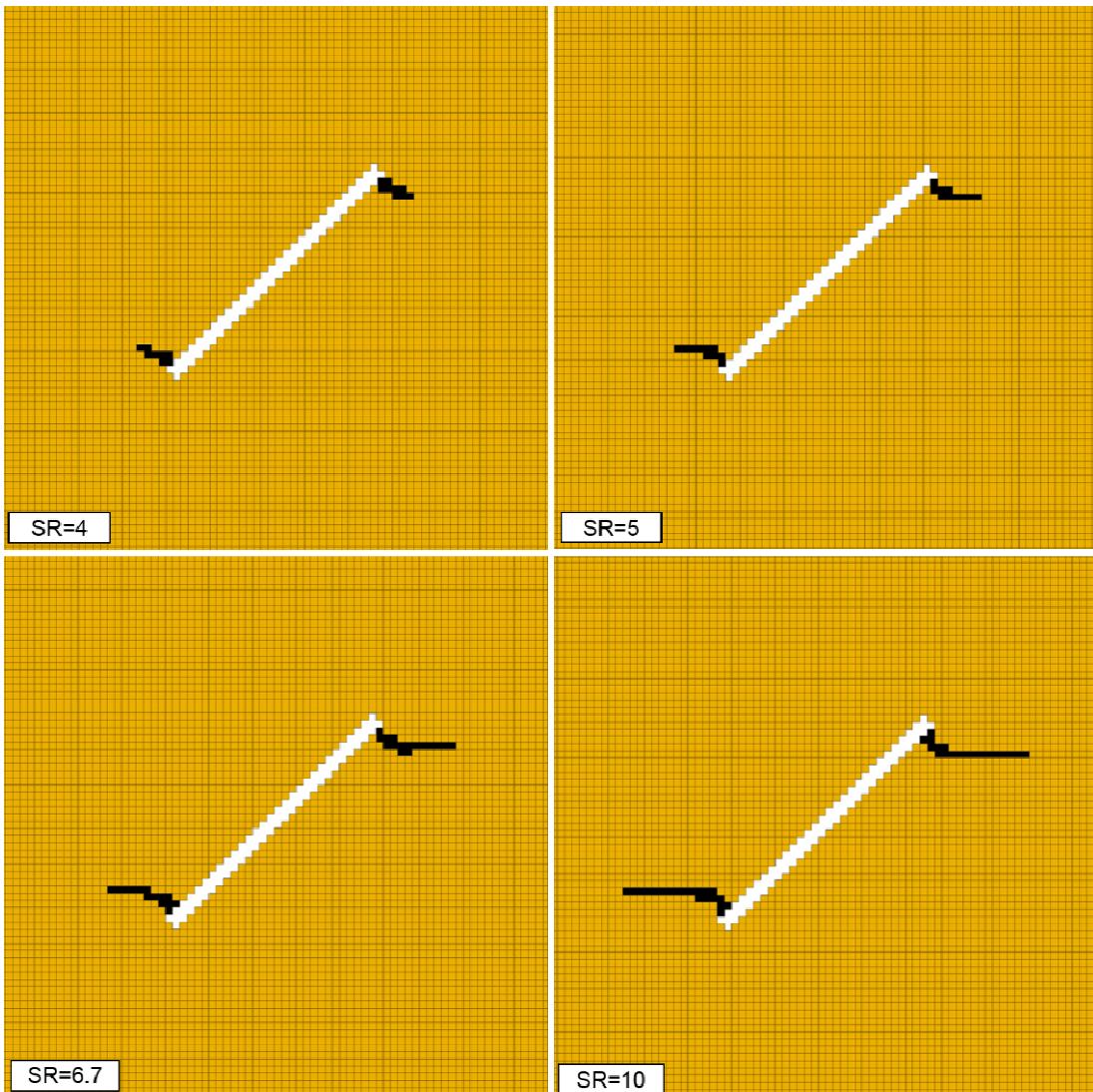
334

## 335 4. VERIFICATION OF THE MODEL DUE TO MECHANICAL LOADING

336 To check if this model enables to simulate properly the fractures propagation in intact rock due to  
337 differential boundary stresses, the fractured rock domain FD1 was considered. For intact rock, model  
338 parameters presented in Table 1 were used. It was assumed that the fracture has no filling material  
339 and is completely open with no fracture surface contacts (no stiffness or stress transfer through sur-  
340 face contacts). The compressive maximum boundary stress  $S_H$  is 40 MPa (see Fig. 2), and the ratio  
341  $SR$  between the maximum horizontal  $S_H$  and minimum horizontal  $S_h$  boundary stresses was consid-  
342 ered to have four alternative values: 4, 5, 6.7 and 10. Fig. 5 shows the fracture propagation obtained  
343 with the FLAC3D model. Results obtained with different degrees of refinement showed that the frac-  
344 ture propagation trajectories are not mesh dependent. The figure shows that as expected the fracture  
345 propagation increases with the ratio between the maximum and minimum boundary stresses. At the tip  
346 of natural fractures, the fracture propagation is not confined to a single row of fractures, because of  
347 formation of wing cracks. At a certain distance away from the fracture tips, the fracture propagates in a  
348 direction perpendicular to the minimum principal stress direction. These results for fracture propaga-

349 tion were verified against those estimated by analytical solutions obtained for an infinite elastic medi-  
 350 um ([56]). Fig 6 shows a comparison of the length  $w$  of the fracture extension (wing cracks by tension),  
 351 normalised by the half-length  $f$  of the fracture, obtained with FLAC3D and that obtained with analytical  
 352 solutions. Results of this comparison showed that the difference between the solution provided by  
 353 analytical solutions and FLAC3D is reasonable with the largest difference at  $SR$  equal to 10, but it is  
 354 smaller than 15 cm, which is acceptable for this very high stress ratio, where the fracture propagation  
 355 is approximately 1 m. Additional values of 100 and 500 GPa/m were considered for the fracture normal  
 356 stiffness  $k_n$ . An additional value of 10 MPa was considered for the tensile strength of  $\sigma_t$ . For mechani-  
 357 cal loading, results showed low sensitivity for these two parameters.

358



359

360

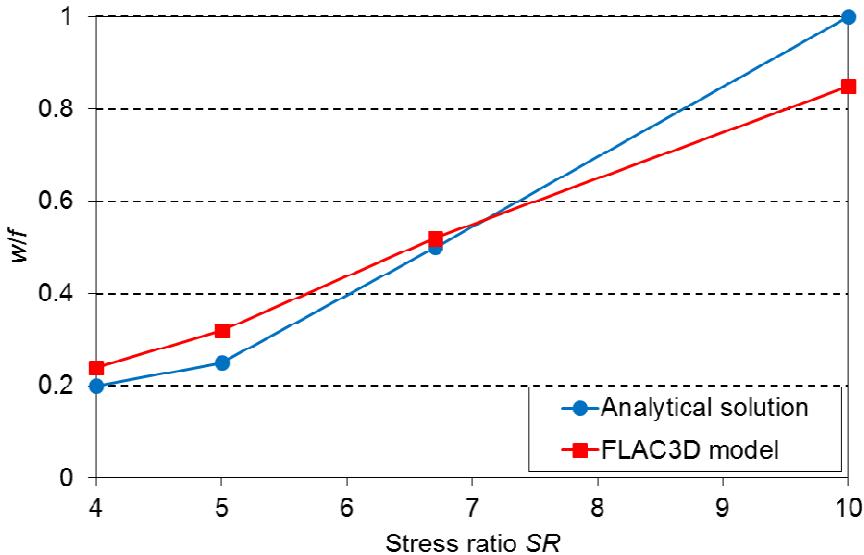
Fig. 5: Results for the fractures propagation obtained with the FLAC3D model

361

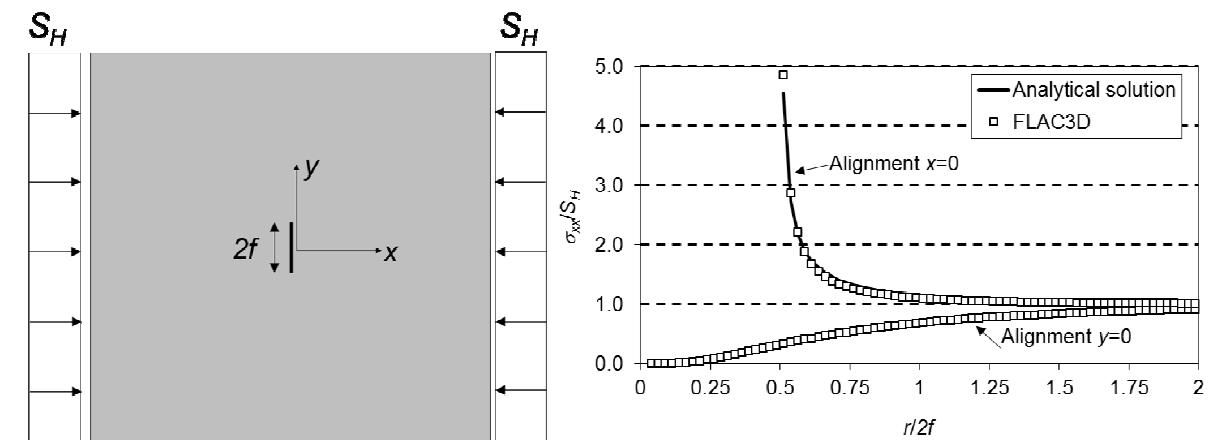
362 To check if the mesh resolution is sufficient to obtain a good estimate of the stresses close to the  
 363 fractures obtained in elastic regime, a very simple model with one vertical fracture and with a length  $2f$   
 364 of 2 m, was considered. A stress  $S_H$  of 40 MPa was applied in the boundaries perpendicular to the x-  
 365 axis (Fig. 7a). The variation of the ratio between fracture normal stress  $\sigma_{xx}$  and boundary stress  $S_H$  as

366 a function of distance  $r/2f$  along the lines  $x=0$  and  $y=0$  away from the fracture was obtained and com-  
 367 pared with the analytical solution presented in [57]. Results of this comparison are shown in Fig. 7b.  
 368 The figure shows that, even close to the fracture, the difference between the solution provided by [57]  
 369 and FLAC3D is smaller than 5%, which enables us to conclude that the calculated stress redistribution  
 370 and concentrations around fractures are accurately represented in our model.

371



372  
 373  
 374  
 375



376  
 377  
 378  
 379

Fig. 7: (a) Geometry and boundary conditions of the model used to study the behaviour of a single fracture with length  $2f$  (b) variation of the fracture normal stress as a function of the distance  $r$  away from the fracture, along the lines  $x=0$  and  $y=0$

380

381 In contrast to this verification study, for our present investigation, fractures with filling material or  
 382 with stress transfer through surface contacts were considered. This is a more realistic scenario be-  
 383 cause it enables the possibility considering changes in fractures aperture caused by changes in the  
 384 stress normal to the fractures.

385 **5. RESULTS**

386 In this section, results on tension failure regions, changes in fluid pore pressure, and in fracture  
387 permeability are presented for a base case, in which the ratio  $SR$  between the magnitudes of the max-  
388 imum and minimum boundary stresses, the permeability  $k_R$  of the intact rock, the initial permeability  
389  $k_{TF}$  of the tension failure regions and the closest distance  $d$  between fractures in FD2, are fixed. A  
390 sensitivity study to analyse the influence of these key parameters on the simulation results, is present-  
391 ed in section 6.

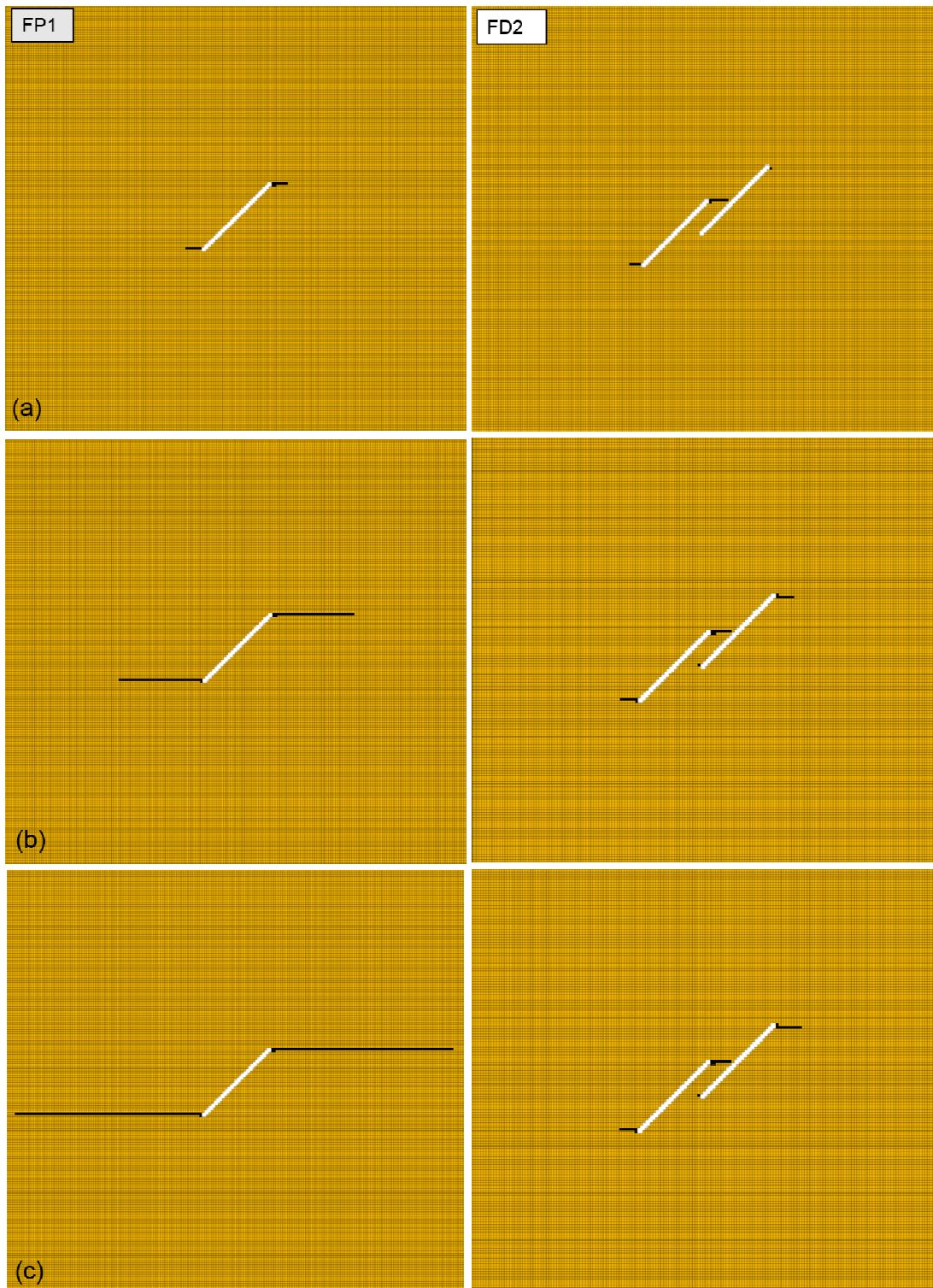
392

393 **5.1 Results for tension failure regions**

394 Fig. 8 shows the failure regions by tension in the intact rock for fractured rock domains FD1 and  
395 FD2, obtained at 1.5, 2 and 3 hours for fracture angles  $\alpha_1=\alpha_2=45^\circ$ . An interesting aspect is that the  
396 ratio of 2 between the magnitudes of boundary stresses is too small to lead to formation of wing cracks  
397 from mechanical effect alone, as it is observed in Fig. 5. In addition, in our study, the fractures have  
398 stiffness or shear transfer through fracture surface contacts, and consequently their extension is paral-  
399 lel to the maximum principal stress direction from the moment of crack initiation, as it is shown by [31].  
400 At 1.5 and 2 hours of injection, results show that in FD1, the fracture extension is approximately 0.35  
401 and 1.75 m, respectively. In FD2, fractures 1 and 2 (see Fig. 1) are already connected at 1.5 hours of  
402 injection. At 2 hours of injection, their extension (away from the connected region) is 35 cm which is  
403 significantly smaller than in FD1. In addition, fracture 1 does not extend beyond fracture 2. This is  
404 because in FD2, when the two fractures connect, the pore pressure decreases and becomes smaller  
405 than the minimum pressure necessary to initiate the propagation of the fracture (see section 5.2). In  
406 FD1, at 3 hours, the fracture propagates approximately 2.25 m longer and is still propagating. In FD2  
407 case, the two fractures only propagate 0.15 m after the shut-in at 2 hours of injection. This is because  
408 in FD1 the fluid pore pressure is larger than in FD2. This will be explained with more detail in the next  
409 section.

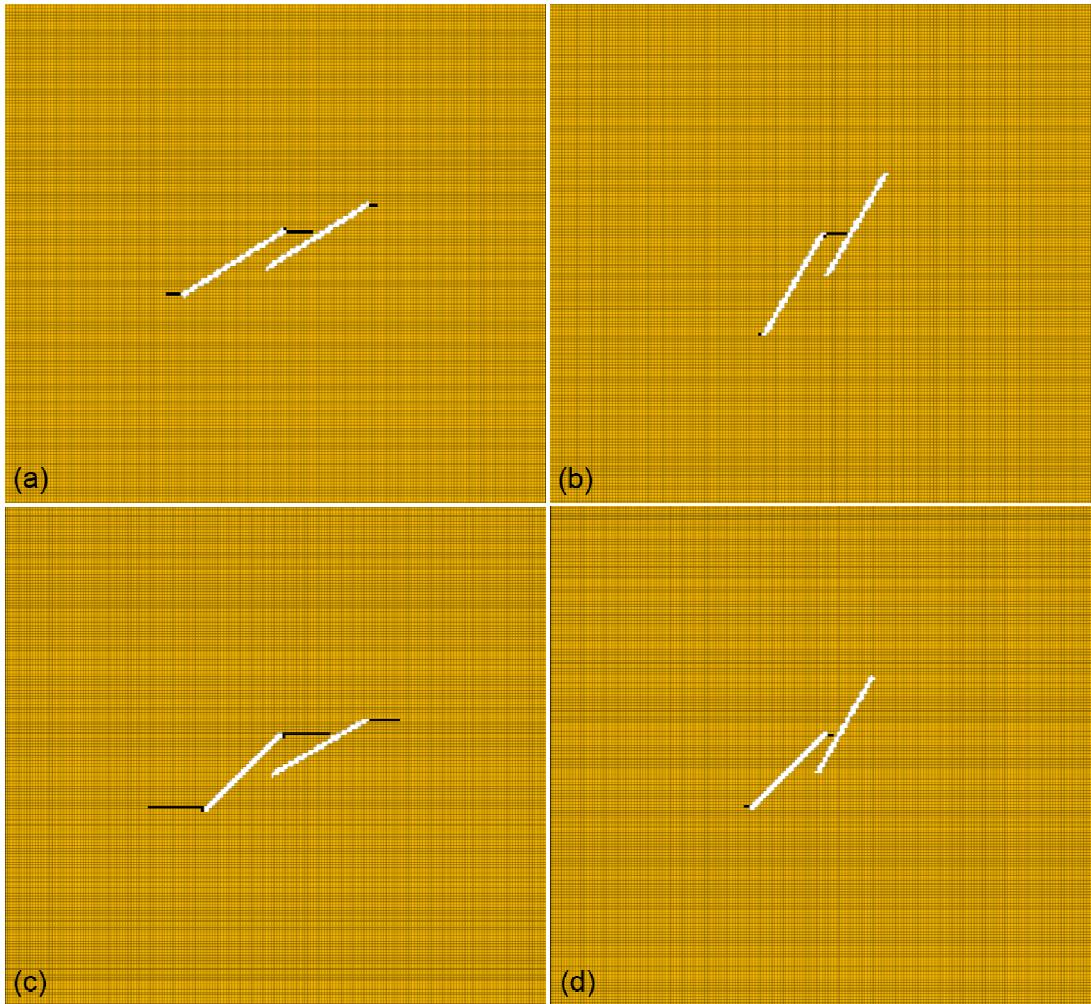
410 Let us now consider the fractured rock domain FD2 and the 2 hours of injection period. Fig. 9  
411 shows those failure regions for parallel fractures inclined at angles  $\alpha_1=\alpha_2=30^\circ$  and  $\alpha_1=\alpha_2=60^\circ$  and non-  
412 parallel fractures inclined at angles  $\alpha_1=45^\circ$ ,  $\alpha_2=30^\circ$  and  $\alpha_1=45^\circ$ ,  $\alpha_2=60^\circ$ . Results obtained for parallel  
413 fractures with  $\alpha_1=\alpha_2=30^\circ$ , the extension of both fractures (away from the connection zone) is 30 cm,  
414 which is less 5 cm than that obtained for fracture angles  $\alpha_1=\alpha_2=45^\circ$ . When  $\alpha_1=\alpha_2=60^\circ$ , the fractures do  
415 connect but their propagation is small because for this geometry, the fractures are sub-perpendicular  
416 to the horizontal direction, which is the maximum principal stress direction. For the non-parallel frac-  
417 tures case, when  $\alpha_2$  decreases from  $45^\circ$  to  $30^\circ$ , the extension (away from the connected region) of the  
418 pressurized fracture (fracture 1) increases approximately 0.5 m. When  $\alpha_2$  increases from  $45^\circ$  to  $60^\circ$ ,  
419 that fracture extension decreases approximately 0.4 m. For  $\alpha_2$  equal to  $60^\circ$ , the non-pressurised frac-  
420 ture (fracture 2) does not propagate. The justification will be explained in sections 5.2 and 5.3.

421



422

423 Fig. 8: Tension failure regions in fractured rock domains FD1 (left) and FD2 with parallel fractures  
 424 (right) at (a) 1.5 hours (b) 2 hours and (c) 3 hours (results obtained for fracture angles  $\alpha_1=\alpha_2=45^\circ$ )



425

426 Fig. 9: Tension failure regions in fractured rock domain FD2 for fracture angles (a)  $\alpha_1=\alpha_2=30^\circ$  (parallel)  
 427 (b)  $\alpha_1=\alpha_2=60^\circ$  (parallel) (c)  $\alpha_1=45^\circ$ ,  $\alpha_2=30^\circ$  (non-parallel) (d)  $\alpha_1=45^\circ$ ,  $\alpha_2=60^\circ$  (non-parallel) (results  
 428 obtained at 2 hours of injection)

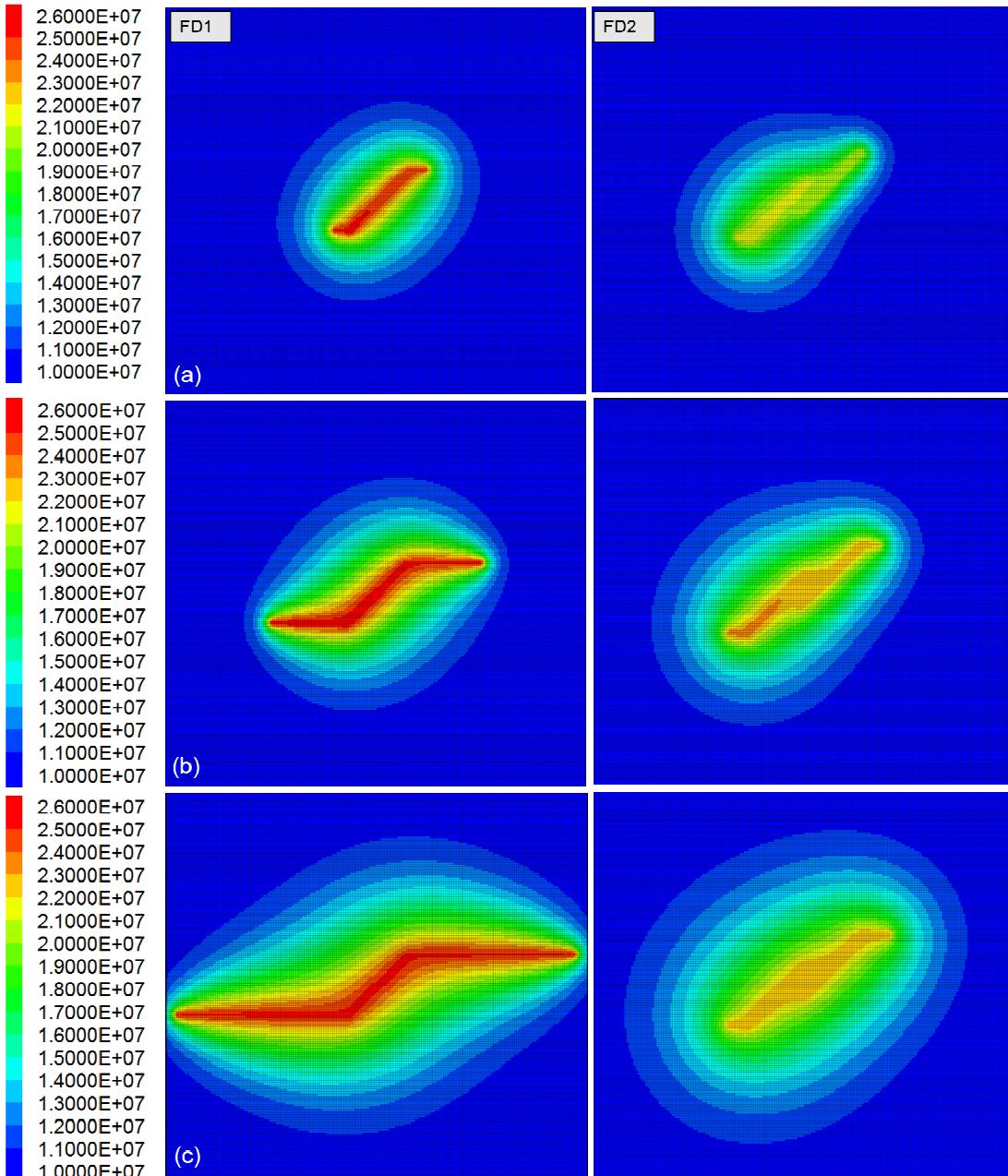
429

### 430 5.2 Changes in fluid pore pressure

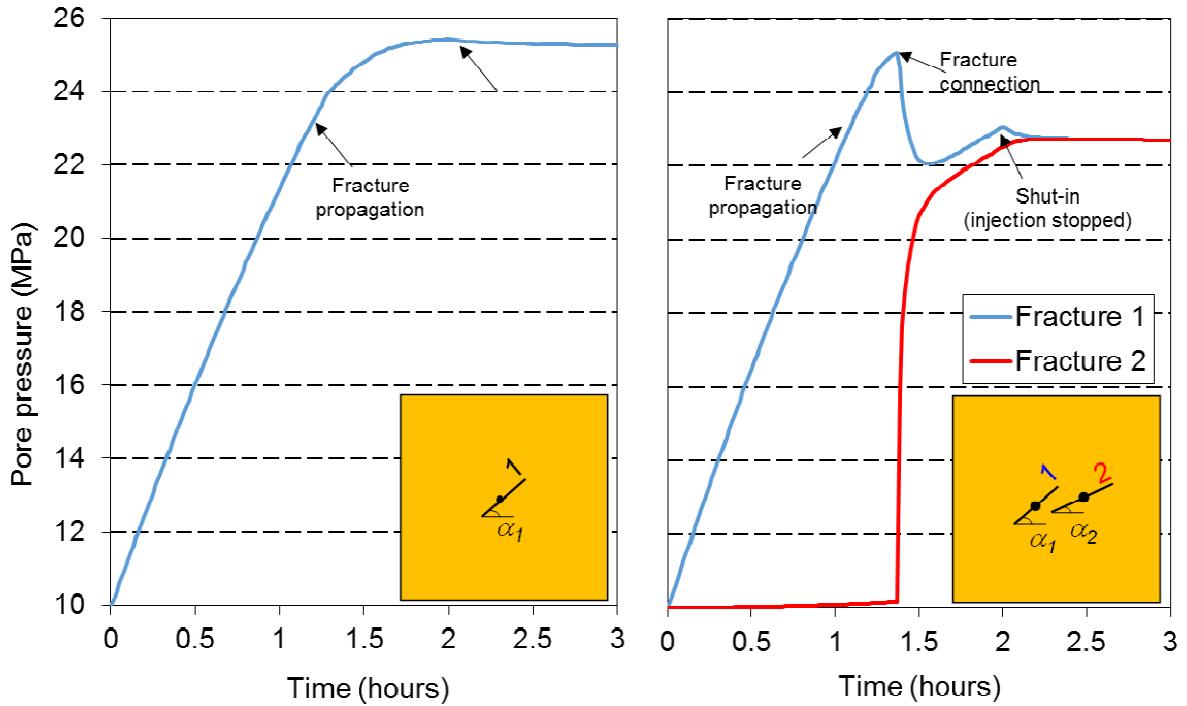
431 In this section, changes in the fluid pore pressure due to coupled hydro-mechanical effects are  
 432 analysed. Fig. 10 shows the contour of the fluid pore pressure field obtained in fractured rock domains  
 433 FD1 and FD2 after 1.5 hours, 2 hours (end of injection) and 3 hours (after 1 hour of shut-in), when the  
 434 two fractures are parallel and have angles  $\alpha_1=\alpha_2=45^\circ$ . Fig. 11 shows the variation of the fluid pore  
 435 pressure with time in the centre of the fractures for the fractured rock domains FD1 and FD2.

436

437  
438  
439  
440  
441



442  
443 Fig. 10: Fluid pore pressure field (Pa) obtained over fractured rock domains FD1 (left) and FD2 with  
444 parallel fractures (right) for at (a) 1.5 hours (b) 2 hours and (c) 3 hours (results obtained for fracture  
445 angles  $\alpha_1=\alpha_2=45^\circ$ )  
446  
447  
448  
449



450  
451 Fig. 11: Variation with time (hours) of the fluid pore pressure (MPa) in the centre of fractures for the  
452 fractured rock domains (left) FD1 and (right) FD2 with parallel fractures (results obtained for fracture  
453 angles  $\alpha_1=\alpha_2=45^\circ$ )  
454

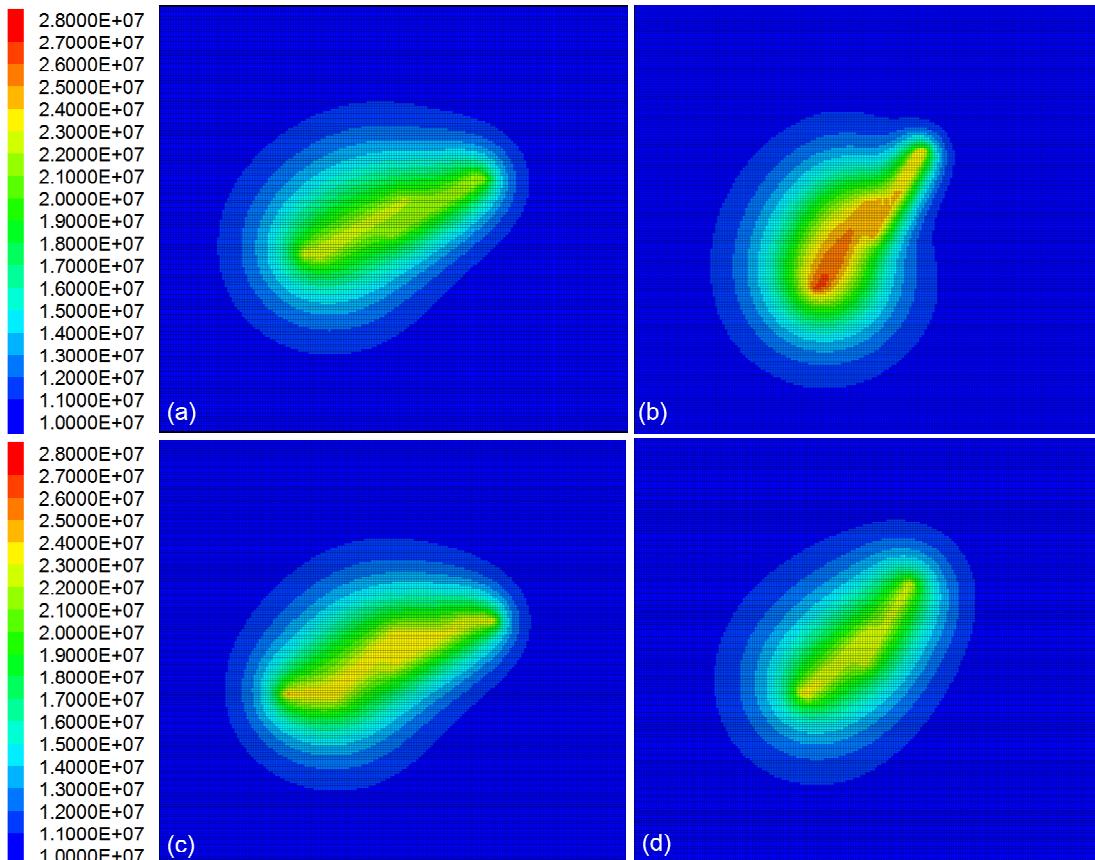
455 Results show that the variation of fluid pore pressure with time in fractured rock domains FD1 and  
456 FD2 is different. In FD1, this relation is approximately linear until approximately 1 hour. The reason  
457 why this relation is not perfectly linear is because of the fluid pore pressure diffusion into the intact  
458 rock (see Fig. 10). For a fluid pore pressure of approximately 23 MPa, the fracture starts to propagate  
459 and the rate of fluid pore pressure build-up decreases with time. The fluid pore pressure necessary to  
460 initiate the fracture propagation is smaller than the minimum boundary stress magnitude (27 MPa).  
461 This is because the fracture has different properties from those of the surrounding intact rock, so that  
462 the minimum principal stress around the fracture tip gets slightly smaller than the minimum boundary  
463 stress magnitude, when differential boundary stresses are applied. The fracture starts to propagate  
464 when the tensile stress caused by pore pressure increase exceeds the tensile strength of the intact  
465 rock around the fracture tip. At that instant, the pore pressure at the fracture centre is larger than that  
466 at the tips. As the fracture propagates, the permeability of the elements failed by tension increases  
467 (see section 5.3). This leads to fluid penetration in the adjacent elements and a consequent increase  
468 in fluid pore pressure (see Fig. 10), which in turn, leads to tension failure in those elements. In this  
469 way, during the hydro-mechanical calculation, the fluid pore pressure diffusion follows the extension of  
470 the fractures (see Figs 8 and 10). At 1.5 and 2 hours of injection, the maximum fluid pore pressure in  
471 the fracture is 24.8 and 25.4 MPa. After shut-in, the pressure decrease is very small (less than 0.1  
472 MPa) and the fracture propagates significantly at 3 hours (see Fig. 8).

473 In FD2, the fractures start to propagate approximately at the same fluid pore pressure as observed  
474 in FD1 (23 MPa). In fracture 1, the fluid pore pressure increases with time until the two fractures con-

nect, which occurs after approximately 80 minutes and for a fluid pore pressure of about 25.5 MPa (see Fig. 11). Before the two fractures connect, there is a small increase of fluid pore pressure in fracture 2 (less than 0.2 MPa), due to the permeability of the intact rock. When the fractures connect, the fluid pore pressure in fractures 1 and 2 suddenly decreases and increases, respectively. At 1.5 hours of injection (10 minutes after the fractures connect), the fluid pore pressure in fractures 1 and 2 is approximately 22 and 21 MPa, respectively. After that, the fluid pore pressure continues to increase in both fractures because the increase in fractures permeability is not significant (see section 5.3) and the fracture 2 starts to propagate (see Fig. 8). At 2 hours of injection, the fluid pore pressure in fractures 1 and 2 is approximately 23 and 22.5 MPa. One hour after shut-in, the pore pressure in both fractures is significantly smaller than that observed in FD1 (approximately 22.8 MPa) and the fractures propagate only a further 0.15 m.

Fig. 12 shows the pore pressure field obtained at 2 hours of injection, for parallel fractures having angles  $\alpha_1=\alpha_2=30^\circ$  and  $\alpha_1=\alpha_2=60^\circ$ , and non-parallel fractures in which  $\alpha_1=45^\circ$ ,  $\alpha_2=30^\circ$  and  $\alpha_1=45^\circ$ ,  $\alpha_2=60^\circ$ . Fig. 13 shows the time evolution of pore pressure for those analysed four cases.

489



490

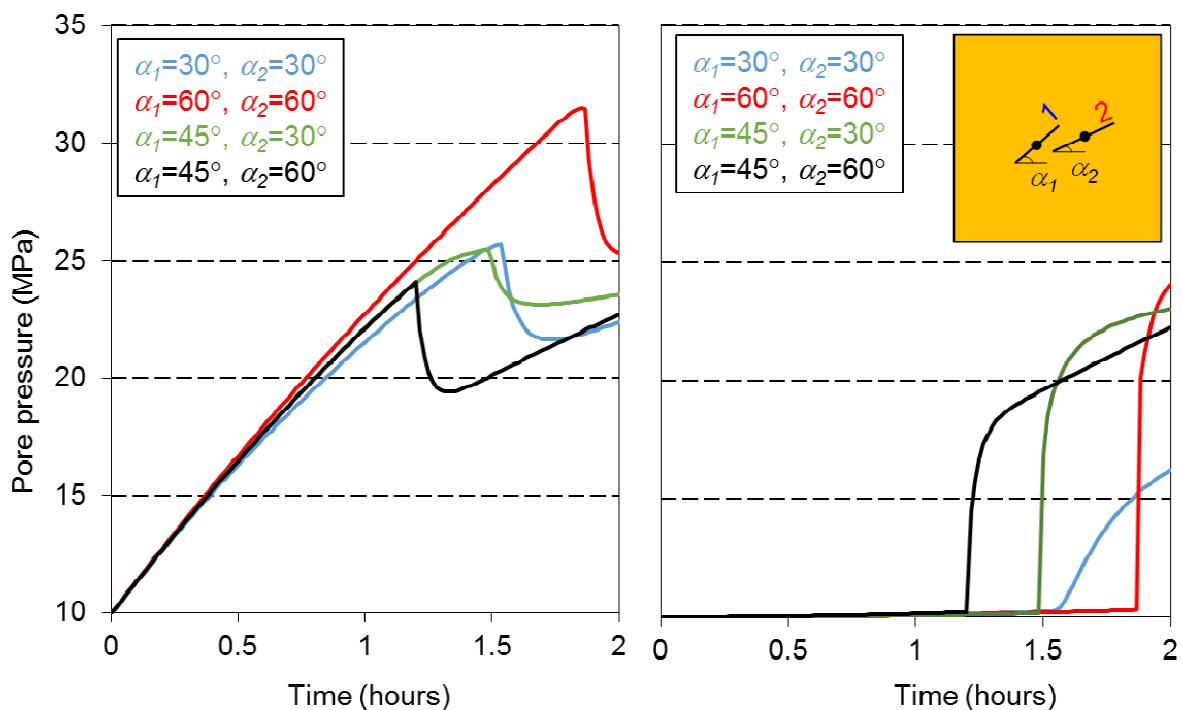
Fig. 12: Fluid pore pressure field (Pa) obtained over fractured rock domain FD2 for fracture angles (a)  $\alpha_1=\alpha_2 = 30^\circ$  (parallel) (b)  $\alpha_1=\alpha_2=60^\circ$  (parallel) (c)  $\alpha_1=45^\circ$ ,  $\alpha_2=30^\circ$  (non-parallel) (d)  $\alpha_1=45^\circ$ ,  $\alpha_2=60^\circ$  (non-parallel) (results obtained at 2 hours of injection)

494

For parallel fractures, results obtained for  $\alpha_1=\alpha_2=30^\circ$  show that the fractures connect when fluid pore pressure is similar to that obtained for  $\alpha_1=\alpha_2=45^\circ$  (25.5 MPa). At 2 hours of injection, the fluid

497 pore pressure in the two fractures is approximately equal to 23 MPa. When  $\alpha_1=\alpha_2=60^\circ$ , the fluid pore  
 498 pressure increases mainly in fracture 1 until the fractures connect for a maximum value of approxi-  
 499 mately 31.5 MPa, which is 6 MPa larger than that obtained when  $\alpha_1=\alpha_2=30^\circ$  or  $\alpha_1=\alpha_2=45^\circ$  (see Figs  
 500 11 and 13). For non-parallel fractures, results show that when the angle between the non-pressurised  
 501 fracture (fracture 2) and the maximum boundary stress direction increases, the distance between the  
 502 tip of the pressurised fracture and neighbouring fracture decreases and as a consequence, the time  
 503 necessary for fractures to connect decreases. In this way, when the fractures connect at an earlier  
 504 instant of time, the increase in the initial pore pressure is less and the fractures propagate less (see  
 505 Fig. 9). The controlling factor appears to be the separation between the tips of the pressurised fracture  
 506 1 to the neighbouring fracture 2. This will be more discussed in section 6.1.

507

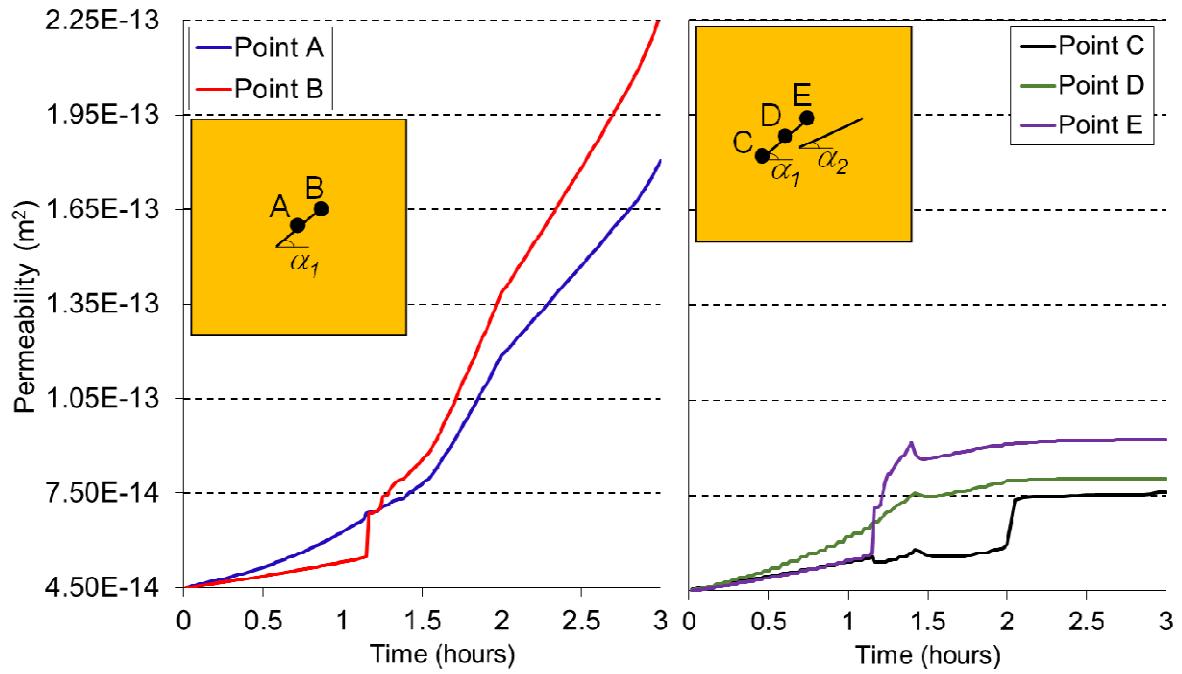


508  
 509 Fig. 13: Variation with time (hours) of the pore pressure (MPa) in the centre of the fractures 1 (left) and  
 510 2 (right) in fractured rock domain FD2, obtained for fracture angles  $\alpha_1$  and  $\alpha_2$  (results obtained at 2  
 511 hours of injection)

512

### 513 5.3 Changes in fracture permeability

514 In this section, changes in permeability of the fractures are analysed. Fig. 14 shows the variation  
 515 with time of the permeability in the centre (points A and D) and tips (points B, C and E) of the frac-  
 516 tures. These results were obtained for parallel fractures with angles  $\alpha_1=\alpha_2=45^\circ$ .



517

518 Fig. 14: Variation with time (hours) of the fracture permeability ( $\text{m}^2$ ) in the centre (points A and D) and  
 519 tips (points B, C and E) of the fractures for the fractured rock domains FD1 (left) and FD2 (right) (re-  
 520 sults obtained for fracture angles  $\alpha_1=\alpha_2=45^\circ$ )

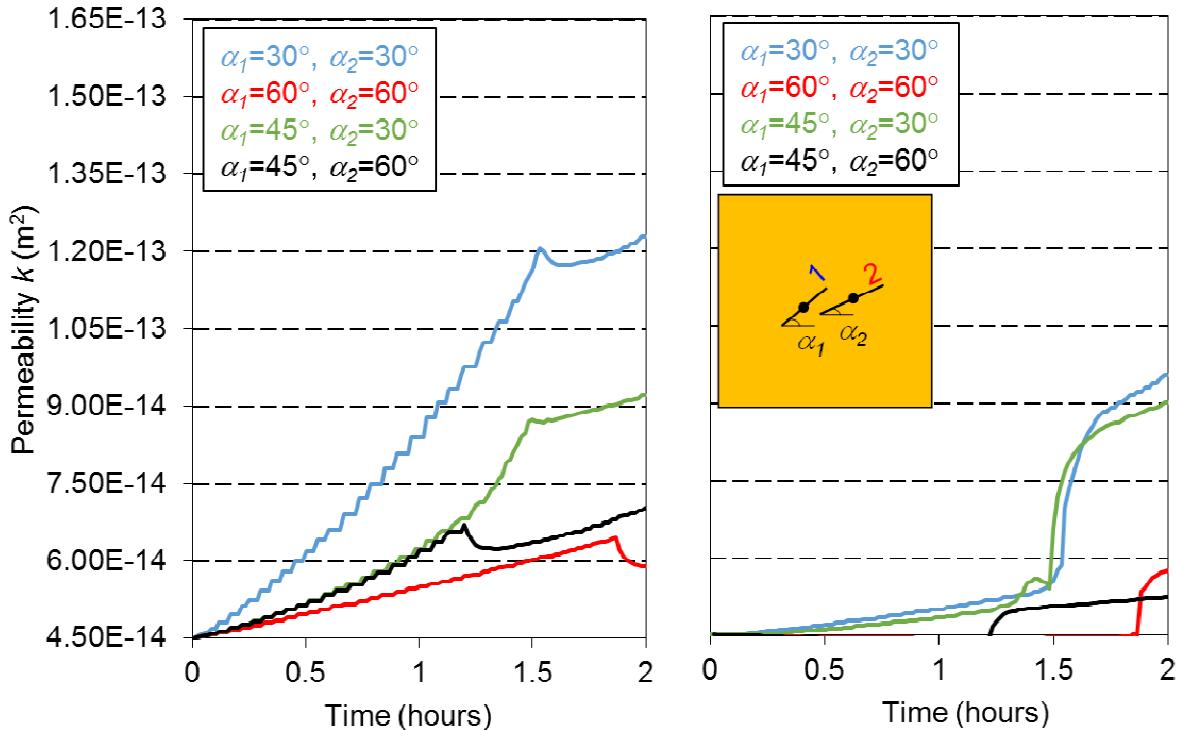
521

522 In fractured rock domain FD1, it was found that before the fracture propagation initiates, changes in  
 523 permeability in the centre of the fracture (point A) are slightly higher than those at the tip (point B).  
 524 When the fracture starts to propagate, at approximately 70 minutes after injection is started, the in-  
 525 crease in permeability at the fracture tip (point B) is larger than in the centre (point A). At 1.5 hours of  
 526 injection, the permeability of the centre of the fracture (point A) is about 1.7 times the initial value. At 2  
 527 hours of injection, the maximum fracture permeability (point A) is approximately 3 times the initial val-  
 528 ue. After shut-in, the permeability of the fracture continues to increase because of changes in volumet-  
 529 ric strains caused by fracture propagation. At 3 hours, the maximum fracture permeability (point B) is  
 530 approximately 5 times the initial value. In FD2, changes in fracture permeability are less significant  
 531 than in FD1 because when the two fractures become connected, the fluid pore pressure decreases  
 532 (see Fig. 11) and as a result, changes in fractures aperture are smaller. After the fractures connect,  
 533 the permeability at point E, located at the tip of the fracture, is slightly higher than at the centre (point  
 534 D), as observed in FD1 case. At 2 hours of injection, in points D and E, the permeability is about 1.8  
 535 and 2.0 times the initial value, respectively. The fracture permeability remains practically constant after  
 536 shut-in, because fracture propagation is very small (see Fig. 11).

537 Fig. 15 shows the time evolution of permeability in the centre of the fractures, for parallel fractures  
 538 having angles  $\alpha_1=\alpha_2=30^\circ$  and  $\alpha_1=\alpha_2=60^\circ$  with the maximum boundary stress direction, and non-  
 539 parallel fractures in which  $\alpha_1=45^\circ$ , and  $\alpha_2=30^\circ$  and  $\alpha_2=60^\circ$ . For parallel fractures, results obtained for  
 540  $\alpha_1=\alpha_2=30^\circ$  show that the permeability changes are similar to those obtained with  $\alpha_1=\alpha_2=45^\circ$  (see Fig.  
 541 14). As a result, the time evolution of pore pressure obtained with  $\alpha_1=\alpha_2=30^\circ$  and  $\alpha_1=\alpha_2=45^\circ$  is similar  
 542 (see Figs 11 and 13). When  $\alpha_1=\alpha_2=60^\circ$ , changes in permeability are the smaller than the other ana-

543 lysed cases. Although the injection pressure is larger than that in the other cases (see Fig. 13), the  
 544 fracture propagates less because it is sub-perpendicular to the maximum principal direction (see Fig.  
 545 9). For the non-parallel fractures case, when  $\alpha_2$  increases from  $30^\circ$  to  $60^\circ$ , the permeability decreases  
 546 because the fractures connect at an earlier time and as a result, the increase in the injection pore  
 547 pressure is less (see Fig. 13), and the fracture propagation decreases.

548



549  
 550 Fig. 15: Variation with time (hours) of the permeability ( $\text{m}^2$ ) of in the centre of the fractures 1 (left) and  
 551 2 (right) in fractured rock domain FD2, obtained for fracture angles  $\alpha_1$  and  $\alpha_2$  (results obtained at 2  
 552 hours of injection)

553

554 **6. SENSITIVITY ANALYSIS**

555 This section presents the results of a sensitivity analysis to study the influence of the distance  $d$   
 556 between the fractures, the ratio  $SR$  between the magnitude of the maximum and minimum boundary  
 557 stresses, the permeability  $k_R$  of the intact rock and the initial permeability  $k_{TF}$  of the tension failure re-  
 558 gions, on the simulation results. The values of these key parameters used in the sensitivity study are  
 559 presented in Table 2 together with those used for the base case. In this analysis, only fractured rock  
 560 domain FD2 with parallel fractures inclined at an angle of  $45^\circ$  and the period up to shut-in (2 hours)  
 561 were considered.

562

563

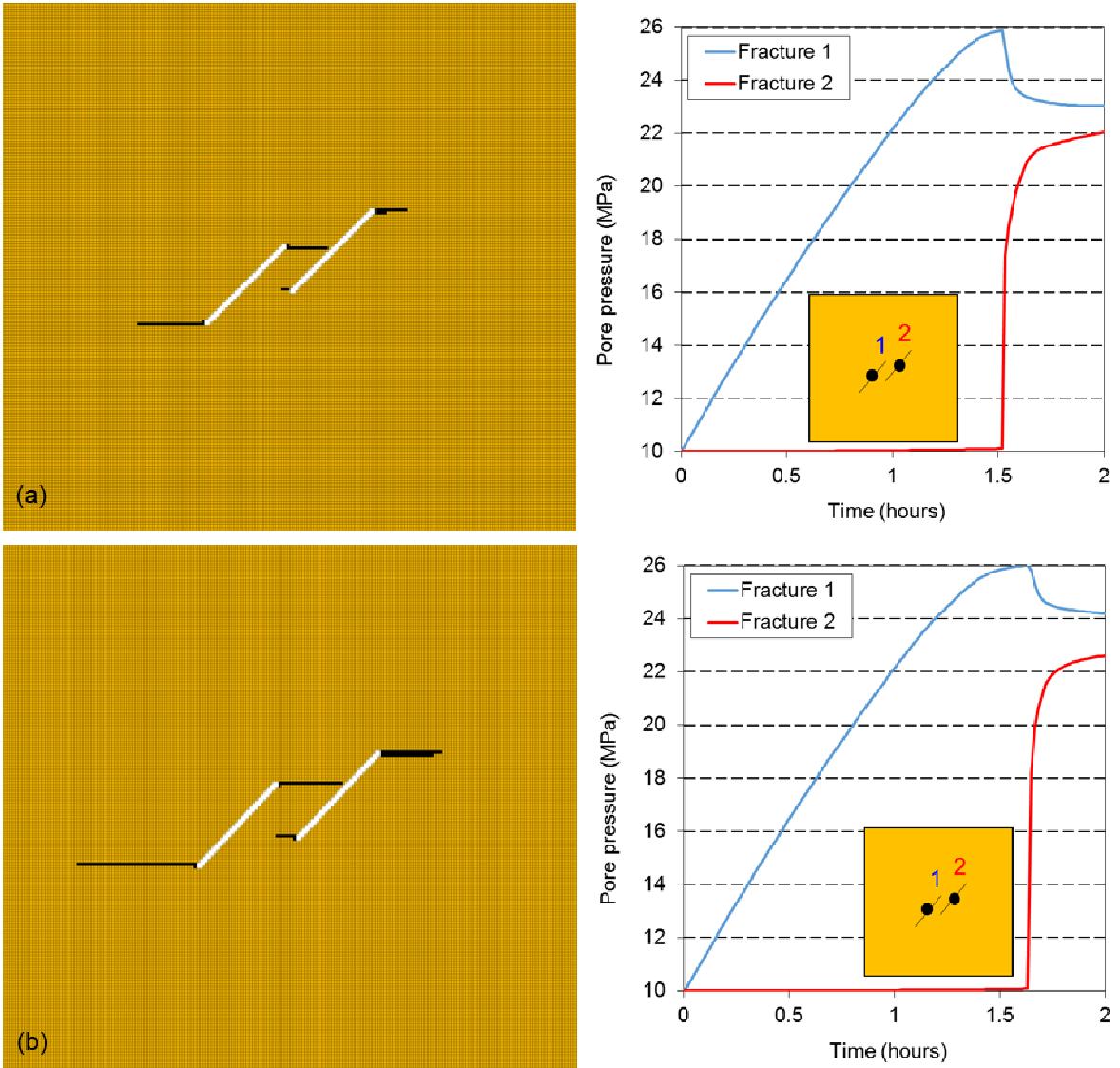
Table 2: Values of the key parameters considered in the base case and sensitivity study

| Key parameter   | Parameter value       |   |
|---|-----------------------|---|
|   | Base case             | Sensitivity study   |
| Distance $d$ (m)  | 0.25                  | 0.50, 0.75  |
| Stress ratio $SR$                                       | 0.7                   | 0.6, 0.8  |
| Permeability $k_R$ ( $\text{m}^2$ )                     | $10^{-18}$            | $10^{-17}$<br>$10^{-16}$  |
| Initial fracture permeability $k_{TF}$ ( $\text{m}^2$ ) | $4.5 \times 10^{-14}$ | $4.5 \times 10^{-15}$<br>$4.5 \times 10^{-16}$<br>$4.5 \times 10^{-18}$ |

564

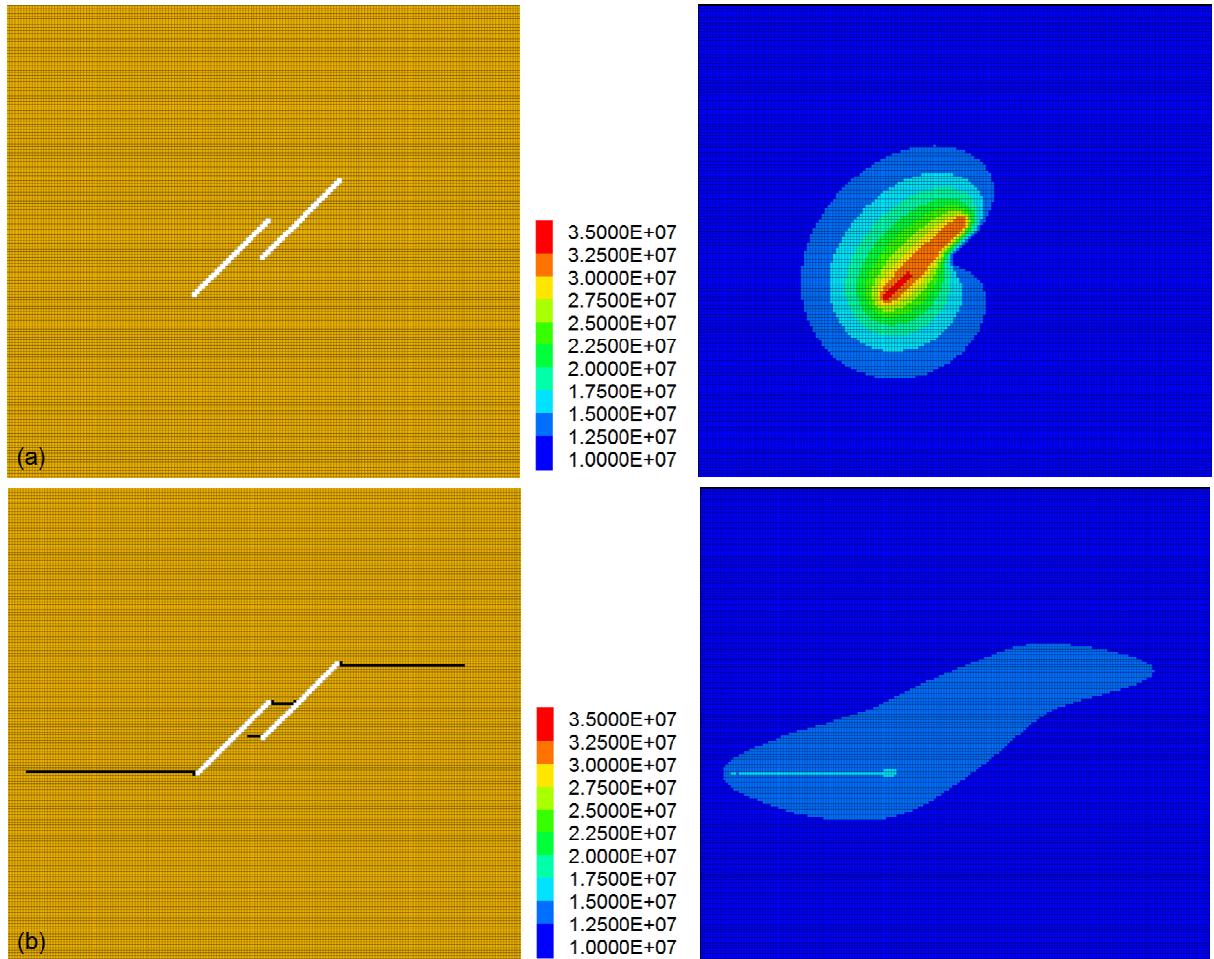
565 6.1 Effect of the distance  $d$  between the fractures

566 Fig. 16 shows the fractures extension and the curves of variation of fluid pore pressure with time in  
 567 the centre of fractures 1 and 2, obtained for a distance  $d$  between fractures of 0.5 m and 0.75 m. Re-  
 568 sults were compared with those presented in section 5, obtained for  $d$  equal to 0.25 m. When  $d$  in-  
 569 creases from 0.25 m to 0.5 m, the fracture extension (away from the connection region) increases  
 570 approximately 0.8 m. When  $d$  is 0.75 m, the propagation of fracture 1 is similar to that obtained in the  
 571 single fracture case. This shows that as  $d$  increases the time necessary for fractures to connect slight-  
 572 ly increases which leads to a major increase in pore pressure and propagation of the pressurised frac-  
 573 ture. This results in a larger difference in fracture propagation and fluid pore pressure in the two frac-  
 574 tures. In addition, as  $d$  increases the effects caused by the linkage between the two fractures de-  
 575 crease.



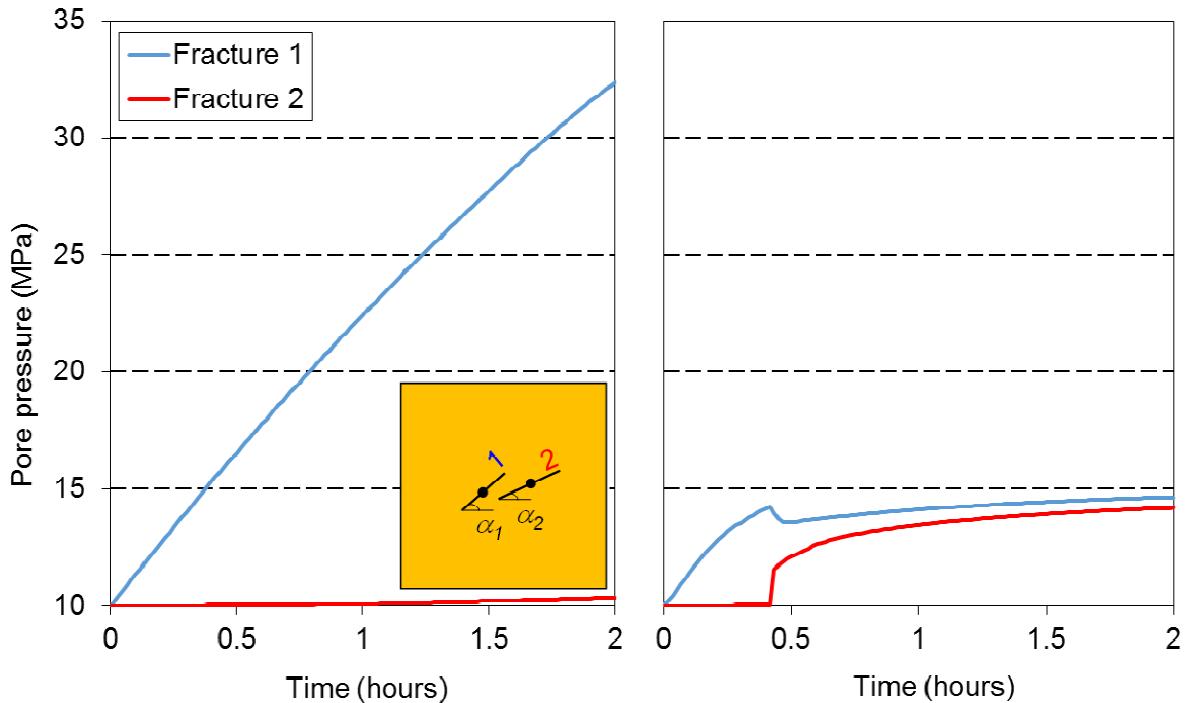
576  
577 Fig. 16: Tension failure regions (left) and variation with time (hours) of fluid pore pressure (MPa) (right)  
578 obtained with  $d$  equal to (a) 0.50 m and (b) 0.75 m (results obtained at 2 hours of injection for fractured  
579 rock domain FD2 with fracture angles  $\alpha_1=\alpha_2=45^\circ$ )

580  
581 6.2 Effect of the ratio  $SR$  between the magnitudes of maximum and minimum horizontal stresses  
582 Fig. 17 shows fracture extensions and fluid pore pressure field, obtained for  $SR$  equal to 1 and 3.  
583 Fig. 18 shows the variation with time of the fluid pore pressure in the centre of the fractures. Results  
584 obtained for  $SR$  equal to 1 show that the fractures do not propagate and hence, there is no interaction  
585 between the two fractures. Consequently, because the intact rock is less permeable than the fractures,  
586 the increase in fluid pore pressure in fracture 2 is negligible (less than 0.1 MPa). In fracture 1, the fluid  
587 pore pressure increases almost linearly with time during the injection period (see Fig. 18). At 2 hours  
588 of injection, the fluid pore pressure is approximately 32 MPa. Comparison with results presented for  
589  $SR$  equal 2 enable us to conclude that the minimum fluid pore pressure necessary to extend the frac-  
590 tures increases when the boundary stresses have equal magnitude. In this case, to observe fracture  
591 propagation, the injection period has to be greater than 2 hours.



592  
593 Fig. 17: Tension failure regions (left) and fluid pore pressure field (Pa) (right) obtained with a stress  
594 ratio  $SR$  equal to (a) 1 and (b) 3 (results obtained at 2 hours of injection for fractured rock domain FD2  
595 with fracture angles  $\alpha_1=\alpha_2=45^\circ$ )  
596

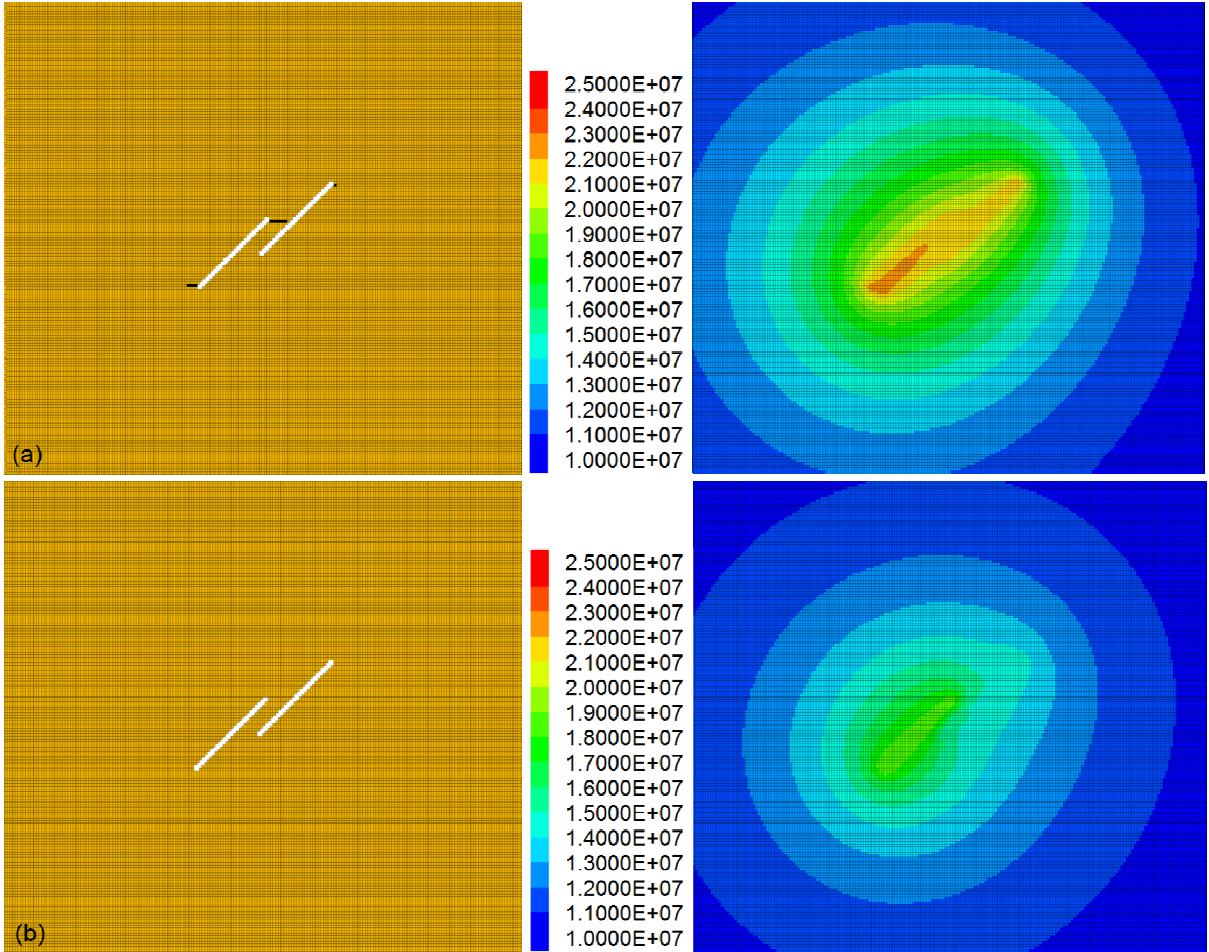
597 When  $SR$  is equal to 3, the fractures propagate approximately 25 cm even before hydraulic fracturing  
598 stimulation starts. This fracture extension results already from stresses applied at the boundaries  
599 and an initial fluid pore pressure of 10 MPa. As a result, the rate of increase in fluid pore pressure with  
600 time is slower than that observed for  $SR$  equal to 1 and 2. In addition, because the principal stresses  
601 magnitude around the fracture tips decreases more for larger differential boundary stresses, the  
602 fractures connect for an injection pressure of significantly smaller (14.2 MPa) than the pressure of 25.5  
603 MPa necessary for fractures to connect when  $SR$  is equal to 2 (see Fig. 11). After the fractures con-  
604 nect, the curves of the variation of fluid pore pressure with time are similar to those obtained for  $SR$   
605 equal to 2. At 2 hours of injection, the fluid pore pressure in fractures 1 and 2 is approximately 14.6  
606 and 14.2 MPa, respectively, and the fracture extension is approximately 3.65 m, which is 1.85 m more  
607 than that obtained for a stress ratio of 2 (see Figs 8 and 10). This shows the important role of the max-  
608 imum principal stress magnitude on the propagation of existing fractures when they are stimulated by  
609 hydraulic fracturing.  
610  
611



612  
613 Fig. 18: Variation with time (hours) of the fluid pore pressure (MPa) in the centre of the fractures for  
614 fracture rock domain FD2 by considering a stress ratio  $SR$  of (left) 1 and (right) 3 (results obtained at 2  
615 hours of injection for fractured rock domain FD2 with fracture angles  $\alpha_1=\alpha_2=45^\circ$ )

616  
617 6.3 Effect of permeability  $k_R$  of the intact rock

618 In this section, the influence of the permeability  $k_R$  of the intact rock on the simulation results, is  
619 analysed by considering two additional values for  $k_R$ :  $10^{-17}$  and  $10^{-16}$  m<sup>2</sup>. Results for tension failure  
620 regions and fluid pore pressure field are presented in Fig. 19 and they were compared with those ob-  
621 tained for  $k_R$  equal to  $10^{-18}$  m<sup>2</sup>. Results show that when  $k_R$  increases by one order of magnitude, the  
622 fracture propagation decreases 15 cm. When  $k_R$  increases by two orders of magnitude, no fracture  
623 propagation is observed. The injection pressure decreases approximately 1 and 4.5 MPa when  $k_R$   
624 increases by one and two orders of magnitude, respectively. This shows that as  $k_R$  increases the frac-  
625 ture propagation decreases because of the dissipating of pressure into the rock matrix and the result-  
626 ing decrease in the pressure build-up around the fractures tip.



628

629 Fig. 19: Tension failure regions (left) and fluid pore pressure field (Pa) (right) obtained with a permea-  
 630 bility  $k_R$  of the intact rock equal to (a)  $10^{-17} \text{ m}^2$  (b)  $10^{-16} \text{ m}^2$  (results obtained at 2 hours of injection for  
 631 fractured rock domain FD2 with fracture angles  $\alpha_1=\alpha_2=45^\circ$ )

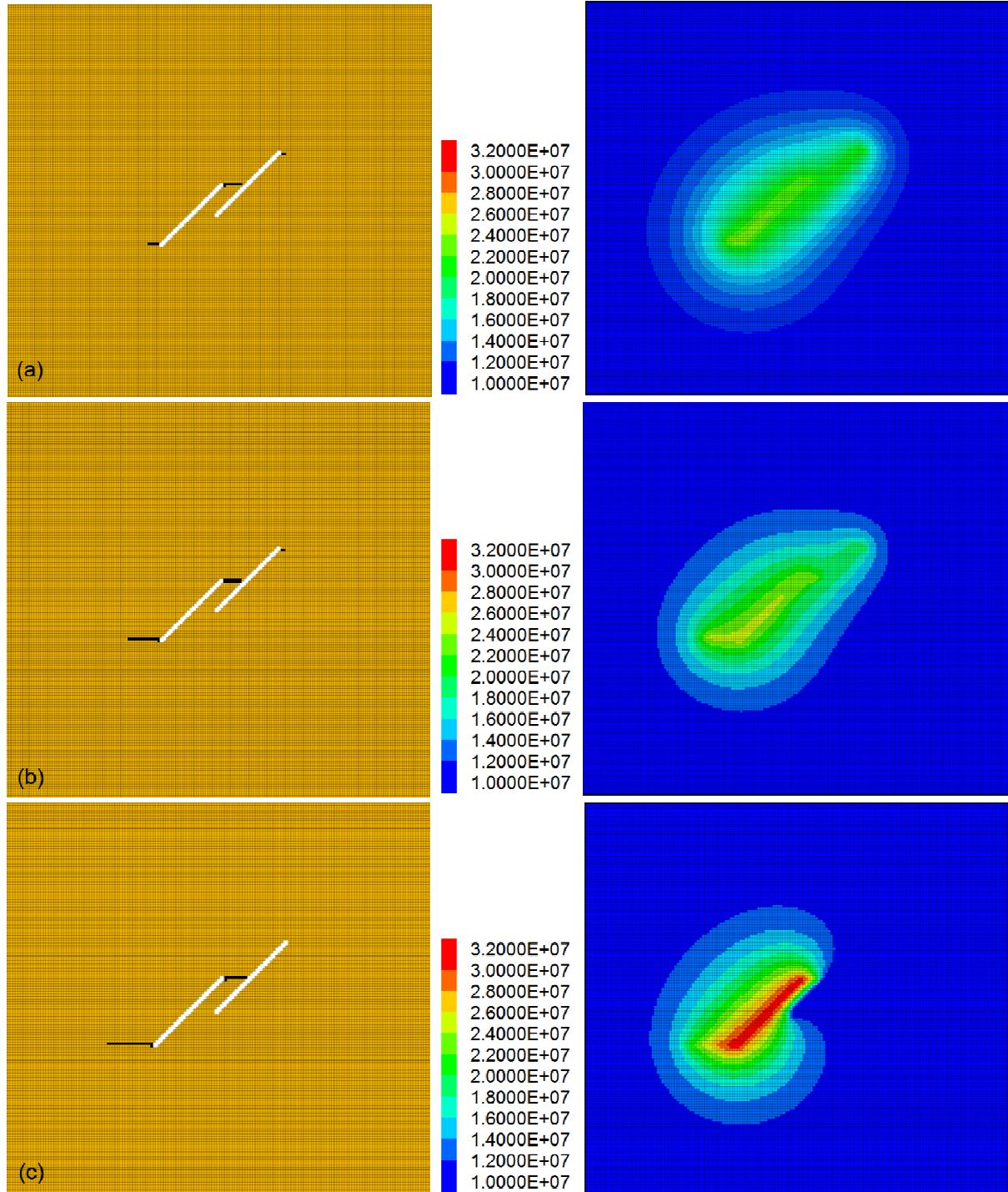
632

#### 633 6.4 Effect of the initial permeability $k_{TF}$ of the tension failure regions

634 Fig. 20 shows the fractures extension and fluid pore pressure field, obtained for an initial permea-  
 635 bility  $k_{TF}$  of the failure regions equal to  $4.5 \times 10^{-15} \text{ m}^2$ ,  $4.5 \times 10^{-16} \text{ m}^2$  and  $4.5 \times 10^{-18} \text{ m}^2$ . Results show that  
 636 when  $k_{TF}$  decreases one order of magnitude from  $4.5 \times 10^{-14} \text{ m}^2$  to  $4.5 \times 10^{-15} \text{ m}^2$ , the extension of frac-  
 637 tures 1 and 2 decreases approximately 10 and 25 cm, respectively. The difference between the maxi-  
 638 mum values in fluid pore pressure observed at the centre of the two fractures increases from 0.6 to 2.1  
 639 MPa. This is because as  $k_{TF}$  decreases, it is more difficult for fluid to penetrate into the recently creat-  
 640 ed fracture and hence, the fluid pore pressure increases more in the fracture where hydraulic fracturing  
 641 occurs. When  $k_{TF}$  decreases by two orders of magnitude, this difference in fluid pore pressure in  
 642 the two fractures increases to 4.7 MPa, and the propagation of fracture 2 is only 10 cm. In contrast,  
 643 fracture 1 propagation increases by 0.55 m. When  $k_{TF}$  is set to  $4.5 \times 10^{-18} \text{ m}^2$ , fracture 1 propagates 15  
 644 cm more than in the case of  $k_{TF}$  equal to  $4.5 \times 10^{-16} \text{ m}^2$ , but fracture 2 does not propagate. Because of  
 645 the changes in volumetric strains, the permeability of the tension failure elements increases by two  
 646 orders of magnitude with respect to their initial permeability  $k_{TF}$ . This is not enough to lead to signifi-  
 647 cant flow in fracture 2, and the difference in fluid pore pressure between fractures 1 and 2 increases to

648 20 MPa. In this case, practically there is no fluid pore pressure build-up in fracture 2. This concludes  
649 that results are more sensitivity to the initial permeability  $k_{TF}$  of the failure regions than to changes in  
650 volumetric strains in the elements subjected to tension failure.

651



652

653 Fig. 20: Tension failure regions (left) and fluid pore pressure field (Pa) (right) obtained with a perme-  
654 ability  $k_{TF}$  of the tension failure regions equal to (a)  $4.5 \times 10^{-15} \text{ m}^2$  (b)  $4.5 \times 10^{-16} \text{ m}^2$  and (c)  $4.5 \times 10^{-18} \text{ m}^2$   
655 (results obtained at 2 hours of injection for fractured rock domain FD2 with fracture angles  $\alpha_1=\alpha_2=45^\circ$ )  
656

657 **7. CONCLUDING REMARKS**

658 The focus of the present study is on the influence of a neighbouring fracture on a fracture subjected  
659 to increased fluid pressure during hydraulic fracturing stimulation. This is accomplished by a compara-  
660 tive coupled hydro-mechanical analysis of two fractured rock domains: FD1, with one single fracture;  
661 and FD2, with two adjacent parallel and non-parallel fractures. Hydraulic fracturing was assumed to  
662 occur by high-pressure injection in the single fracture in FD1 and one of the two fractures in FD2. Sim-  
663 ultations were made for a time period of 3 hours with an injection period of 2 hours. A base case was  
664 considered in which the closest distance  $d$  between the two fractures is 0.25 m, the ratio  $SR$  between  
665 the magnitude of the maximum and minimum boundary stresses is set to be 2, the permeability  $k_R$  of  
666 the intact rock is set to be  $10^{-18} \text{ m}^2$  and the initial permeability  $k_{TF}$  of the tension failure regions is con-  
667 sidered to be equal to the initial permeability of the natural fractures. The conclusions from the ob-  
668 tained results may be summarized as follows:

669 Firstly, the minimum fluid pore pressure necessary to initiate fractures propagation is smaller than  
670 the minimum boundary stress magnitude (approximately 85%). This is because the fractures have  
671 softer properties than the intact rock, and due to boundary stresses, the minimum principal stress at  
672 the tips is smaller than the minimum boundary stress magnitude. The fractures start to propagate  
673 when the local tensile stress around the tip of the fracture, induced by increase in pore pressure, is  
674 larger than the tensile strength of the intact rock. At that instant, the pore pressure at its centre is larg-  
675 er than that at the tip of the fracture. It was found that until the fractures start to propagate, the injec-  
676 tion pressure increases with time, but this relation is not perfectly linear because of fluid pore pressure  
677 diffusion into the permeable intact rock. In contrast with the FD1 case, in the case of FD2 the pressure  
678 in the pressurised fracture decreases significantly (approximately 15%) after it connects to the second  
679 fracture.

680 Secondly, in a double fractures case, with parallel fractures, the results obtained with an angle  
681 between the two fractures and the maximum principal stress direction (horizontal) of  $30^\circ$  and  $45^\circ$ , were  
682 found to be similar. However, when the fractures are inclined at an angle of  $60^\circ$ , they are sub-  
683 perpendicular to the maximum principal stress direction, and hence, the propagation of fractures is  
684 smaller. The fractures will connect after an injection period larger than that observed for an angle  
685 equal to  $30^\circ$  or  $45^\circ$ , and as a result, in the former case, the fluid pore pressure increases more in the  
686 fracture subjected to water injection. When the fractures are non-parallel, it was found that, as the  
687 angle between the non-pressurised fracture and the maximum boundary stress direction increases,  
688 the time necessary for fractures to connect decreases, the increase in pore pressure is less, and  
689 hence the fracture propagation decreases.

690 Thirdly, the propagation of a single fracture caused by water injection is larger than that obtained  
691 with the presence of a neighbouring second fracture. This is because in the latter case, the pore pres-  
692 sure decreases when the two fractures connect and the tensile stress at the tip of fracture is then not  
693 enough exceed the tensile strength of the intact rock.

694 Fourthly, in a single fracture case, changes in fracture permeability were found to be larger than  
695 those obtained in a double fractures case. This is because in the latter case, the injection pressure  
696 decreases when the fractures connect, and thus changes in the fractures aperture are smaller than in

697 the former case. However, changes in fracture permeability were found to be not very significant. At 2  
698 hours of injection, the maximum ratio between the final and initial fracture permeability was approxi-  
699 mately 3. Consequently, after the fractures start to propagate, the injection pressure still increases.

700 A sensitivity study was made to analyse the influence of the key parameters referred above on the  
701 obtained results for the particular case of FD2 with parallel fractures inclined at an angle of 45°. It is  
702 found that:

- 703 1. When the distance  $d$  between the two fractures increases, the pressurised fracture extends  
704 more and the effect of linkage between fractures on their propagation decreases. At the limit, for  
705 a very large  $d$  value, the results obtained for the single and double fractures case are expected  
706 to be very similar.
- 707 2. When the ratio  $SR$  between the boundary stresses magnitude increases, the minimum fluid pore  
708 pressure value necessary to initiate fracture propagation decreases. This is because when  $SR$   
709 increases, the minimum principal stress around the fracture tip decreases.
- 710 3. When the permeability  $k_R$  of the intact rock increases, the pore pressure around the fracture tip  
711 decreases, which results in a decrease in the fracture propagation. On the other hand, it was  
712 found that when  $k_{TF}$  decreases, the fractures may connect but the difference in fluid pore pres-  
713 sure observed in the two fractures increases. This is because when the tension failure regions  
714 are more impermeable, the flow between the two fractures is less. This shows that differences  
715 in results are more sensitive to the initial permeability of the tension failure regions than to  
716 stress induced changes in their apertures.

717 To summarise, fracture propagation was found to be more sensitive to  $d$  and  $SR$  than to the other  
718 parameters. The conclusions from the present study bring out some interesting aspects of fracture  
719 rock hydromechanics that deserve further studies and these need to be accounted for in modelling  
720 hydro-mechanical behaviour of fractured rocks during a hydraulic fracturing operation. As further work  
721 it is proposed to apply the methodology presented in this paper to study the fractured propagation in  
722 the three dimensional space and also when the multi-phase fluid is involved.

## 724 ACKNOWLEDGMENTS

725 The authors gratefully acknowledge the Swedish Geological Survey (SGU), grant number 1724,  
726 and the EU project, grant number 640979, for providing financial support to research reported in this  
727 paper. Additional support was provided by the U.S. Department of Energy under contract No. DE-  
728 AC02-05CH11231.

729

## 730 REFERENCES

- 731 [1] Min KB, Rutqvist J, Tsang CF, Jing L. Stress-dependent permeability of fractured rock masses: a  
732 numerical study. *Int. J. Rock Mech. Min. Sci.* 2004; 41: 1191-1210.
- 733 [2] Rutqvist J, Stephansson O. The role of hydro-mechanical coupling in fractured rock engineering.  
734 *Hydrogeology J.* 2013; 11: 7-40.

735 [3] Tsang CF, Niemi A. Deep hydrogeology: a discussion of issues and research needs. *Hydrogeology*  
736 J. 2013; 21: 1687-90.

737 [4] Germanovich LN, Astakhov DK. Stress-dependent permeability and fluid flow through parallel  
738 joints. *J. Geophys. Res.* 2004; 108: doi:10.1029/2002JB002133, in press.

739 [5] Germanovich L, Askakov D. Fracture closure in extension and mechanical interaction of parallel  
740 joints. *J. Geophys. Res.* 2004; 109: B02208, doi: 10.1029/2002JB002131.

741 [6] Germanovich LN, Ring LM, Astakhov DK, Shlyapobersky J, Mayerhofer MJ. Hydraulic fracture with  
742 multiple segments, part II: Effect of interaction. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 1997;  
743 34 (3): 98.e1–98.e15.

744 [7] Reyes O. Experimental Study and Analytical Modeling of Compressive Fracture in Brittle Materials  
745 (Ph.D. thesis). Massachusetts Institute of Technology, 1991.

746 [8] Bobet A, Einstein HH. Fracture coalescence in rock-type materials under uniaxial and biaxial com-  
747 pression. *Int. J. Rock Mech. Min. Sci.* 1998; 35 (7): 863–88.

748 [9] Wong LNY, Einstein HH. Crack coalescence in molded gypsum and Carrara marble: part 1 – Mac-  
749 roscopic observations and interpretation. *Rock Mech. Rock Eng.* 2009a; 42 (3): 475–511.

750 [10] Wong LNY, Einstein HH. Crack coalescence in molded gypsum and Carrara marble: part 2 –  
751 macroscopic observations and interpretation. *Rock Mech. Rock Eng.* 2009b; 42 (3): 513–45.

752 [11] Miller JT. Crack coalescence in granite (S.M. thesis). Massachusetts Institute Technology, 2008.

753 [12] Morgan S, Johnson C, Einstein H. Cracking processes in Barre granite: fracture process zones  
754 and crack coalescence. *Int. J. Fract.* 2013; 180: 177–204.

755 [13] Zang A, Wagner C, Stanchits S, Dresen G, Andresen R, Haidekker M. Source analysis of acous-  
756 tic emissions in Aue granite cores under symmetric and asymmetric compressive loads. *Int. J. Ge-  
757 ophys.* 1998; 135: 1113–30.

758 [14] Mayer SI, Stanchits S, Langenbruch C, Dresen G, Shapiro S. Acoustic emission induced by pore-  
759 pressure changes in sandstone samples. *Geophysics* 2011; 76 (3): MA21–MA32.

760 [15] Ramulu M, Kobayashi AS. Mechanics of crack curving and branching – a dynamic fracture analy-  
761 sis. *Int. J. Fract.* 1985; 27: 187–201.

762 [16] Guo ZK, Kobayashi AS. Dynamic mixed mode fracture of concrete. *Int. J. Solids Struct.* 1995; 32:  
763 2591–07.

764 [17] Ravi-Chandar K. *Fracture Dynamics*. Elsevier B.V, Amsterdam, 2004.

765 [18] Moes N, Belytschko T. A finite element method for growth without remeshing. *Int. J. Numer. Meth.  
766 Eng.* 1999; 46 (1): 131–50.

767 [19] Mughieda O, Omar MT. Stress analysis for rock mass failure with offset joints. *Geotech. Geol.  
768 Eng.* 2008; 26: 543–52.

769 [20] Gonçalves da Silva B, Einstein HH. Modeling of crack initiation, propagation and coalescence in  
770 rocks. *Int. J. Fract.* 2013; 182 (2): 167–86.

771 [21] Li HQ, Wong LNY. Influence of flaw inclination angle and loading condition on crack initiation and  
772 propagation. *Int. J. Solids Struct.* 2012; 49: 2482–99.

773 [22] Yao Y. Linear elastic and cohesive fracture analysis to model hydraulic fracture in brittle and duc-  
774 tile rocks. *Rock Mech. Rock Eng.* 2012; 45: 375–87.

775 [23] Lecampion B. An extended finite element method for hydraulic fracture problems. *Commun. Nu-  
776 mer. Meth. Engng* 2009; 25: 121-33.

777 [24] Rannou J, Limodin N, Réthoré J, Gravouil A, Ludwig W, Baïetto-Dubourg MC, Buffière JY,  
778 Combescure A, Hild F, Roux S. Three dimensional experimental and numerical multiscale analysis  
779 of a fatigue crack. *Comput. Meth. Appl. Mech. Eng.* 2010; 199 (21–22): 1307– 25.

780 [25] Liu ZL. An XFEM/Spectral element method for dynamic crack propagation. *Int. J. Fract.* 2011;  
781 169: 183–98.

782 [26] Xu Y, Yuan H. Applications of normal stress dominated cohesive zone models for mixed-mode  
783 crack simulation based on the extended finite element methods. *Eng. Fract. Mech.* 2011; 78: 544–  
784 58.

785 [27] Fang Z, Harrison JP. Development of a local degradation approach to the modeling of brittle frac-  
786 ture in heterogeneous rocks. *Int. J. Rock Mech. Min. Sci.* 2002; 39: 443–57.

787 [28] Fang Z, Harrison JP. Application of a local degradation model to the analysis of brittle fracture of  
788 laboratory scale rock specimens under triaxial conditions. *Int. J. Rock Mech. Min. Sci.* 2002; 39:  
789 459–76.

790 [29] Li Y, Zhou H, Zhu W, Li S, Liu J. Numerical study of crack propagation in brittle joined rock mass  
791 influenced by fracture water pressure. *Materials* 2015; 8: 3364-76.

792 [30] Bobet A, Einstein HH. Numerical modeling of fracture coalescence in a model rock material. *Int. J.  
793 Fract.* 1998a; 92: 221–52.

794 [31] Vásárhelyi B, Bobet A. Modelling of crack initiation, propagation and coalescence in uniaxial com-  
795 pression. *Rock Mech. Rock Eng.* 2000; 33 (2): 119-39.

796 [32] Gonçalves da Silva B, Einstein HH. Modeling of crack initiation, propagation and coalescence in  
797 rocks. *Int. J. Fract.* 2013; 182 (2): 167–86.

798 [33] Potyondy DO, Cundall PA. A bonded-particle model for rock. *Int. J. Rock Mech. Min. Sci.* 2004;  
799 41: 1329–64.

800 [34] Yoon J. Application of experimental design and optimization to PFC model calibration in uniaxial  
801 compression simulation. *Int. J. Rock Mech. Min. Sci.* 2007; 44: 871–89.

802 [35] Lee H, Jeon S. An experimental and numerical study of fracture coalescence in pre-cracked spec-  
803 imens under uniaxial compression. *Int. J. Solids Struct.* 2011; 48: 979-99.

804 [36] Jia L, Chen M, Zhang W, Xu T, Zhou Y, Hou B, Yan J. Experimental study and numerical model-  
805 ing of brittle fracture of carbonate rock under uniaxial compression. *Mech. Res. Commun.* 2013;  
806 50: 58–62.

807 [37] Yang SQ, Huang YH, Jing HW, Liu XR. Discrete element modeling on fracture coalescence be-  
808 havior of red sandstone containing two unparallel fissures under uniaxial compression. *Eng. Geol.*  
809 2014; 178: 28–48.

810 [38] Manouchehrian A, Sharifzadeh M, Marji MF, Gholamnejad J. A bonded particle model for analysis  
811 of the flaw orientation effect on crack propagation mechanism in brittle materials under compres-  
812 sion. *Arch. Civ. Mech. Eng.* 2014; 14: 40–52.

813 [39] Zhang X and Sanderson D. Evaluation of instability in fractured rock masses using numerical  
814 analysis methods: Effects of fracture geometry and loading direction. *J. Geophys. Res.* 106 (B11):  
815 26671-687.

816 [40] Pearce CJ, Thavalingam A, Liao Z, Bićanić N. Computational aspects of the discontinuous de-  
817 formation analysis framework for modelling concrete fracture. *Eng. Fract. Mech.* 2000; 65: 283–98.

818 [41] Rutqvist J, Leung C, Hoch A, Wang Y, Wang Z. Linked multicontinuum and crack tensor approach  
819 for modeling of coupled geomechanics, fluid flow and transport in fractured rock. *Journal of Rock*  
820 *Mechanics and Geotechnical Engineering.* 2013; 5: 18-31.

821 [42] Settari A, Sullivan RB, Walters DA, Wawrynek PA. 3-D analysis and prediction of microseismicity  
822 in fracturing by coupled geomechanical modeling. *Society of Petroleum Engineers* 2002. Paper  
823 75714. <http://www.onepetro.org/conference-paper/SPE-75714-MS>.

824 [43] Ji LJ, Settari A, Sullivan R. A novel hydraulic fracturing model fully coupled with geomechanics  
825 and reservoir stimulation. *Soc. Pet. Eng. J.* 2009; 14 (3): 423–30.

826 [44] Hou ZM, Zhou L, Kracke T. Modelling of seismic events induced by reservoir stimulation in an  
827 enhanced geothermal system and a suggestion to reduce the deformation energy release. *Rock*  
828 *Dynamics and Applications – State of the Art.* Taylor and Francis Group, London, United Kingdom  
829 (pp. 161–175), 2013.

830 [45] Wangen M. Finite element modeling of hydraulic fracturing in 3D. *Comput. Geosci.* 2013; 17 (4):  
831 647–59.

832 [46] Yarushina VM, Bercovici D, Oristaglio M. Rock deformation models and fluid leak-off in hydraulic  
833 fracturing. *Geophys. J. Int.* 2013; 194 (3): 1514–26.

834 [47] Itasca. *FLAC3D, Version 5.0. User's Manual.* Itasca Consulting Group, Minneapolis; 2012.

835 [48] Pruess K, Oldenburg C, Moridis G. *TOUGH2 user's guide, version 2.0,* Lawrence Berkeley Na-  
836 tional Laboratory Report, LBNL-43134, Berkeley, p.198; 2011.

837 [49] Figueiredo B, Tsang CF, Rutqvist J, Niemi A. A study of changes in deep fractured rock permea-  
838 bility due to coupled hydro-mechanical effects. *Int. J. Rock Mech. Min. Sci.* 2015; 79: 70-85.

839 [50] Cappa F, Rutqvist J. Modeling of coupled deformation and permeability evolution during fault re-  
840 activation induced by deep underground injection of CO<sub>2</sub>. *Int. J. Greenhouse Gas Control* 2011; 5:  
841 336-46.

842 [51] Attewell PB, Farmer IW. *Principles of engineering geology.* John Wiley & Sons Inc., New York;  
843 1976.

844 [52] Zhao J. Properties of rock materials. In: *Rock mechanics for civil engineering,*  
845 [www.epf.ch/en/ensei/Rock-Mechanics/ENS-080312-EN-Notes-Chapter-4.pdf](http://www.epf.ch/en/ensei/Rock-Mechanics/ENS-080312-EN-Notes-Chapter-4.pdf). 2008. 49 p.

846 [53] Chin LY, Raghavan R, Thomas LK. Fully coupled geomechanics and fluid flow analysis of wells  
847 with stress-dependent permeability. *Soc. Petrol. Eng.* 2000; 5: 32-45.

848 [54] Rinaldi A, Rutqvist J, Cappa F. Geomechanical effects of CO<sub>2</sub> leakage through fault zones during  
849 large-scale underground injection. *Int. J. Greenhouse Gas Control* 2014; 20: 117-31.

850 [55] Witherspoon PA, Wang JSY, Iwai K, Gale JE. Validity of cubic law for fluid flow in a deformable  
851 rock fracture. *Water Resour. Res.* 1980; 16: 1016-24.

852 [56] Mutlu O, Pollard D. On the patterns of wing crack along an outcrop scale flaw: a numerical mod-  
853 eling approach using complementarity. *J. Geophys. Res.* 2008; 113: B06403, doi:  
854 10.1029/2007JB005284.

855 [57] Pollard DD, Segall P. Theoretical displacements and stresses near fractures in rock: with applica-  
856 tions to faults, joints, veins, dikes, and solution surfaces. In: *Fracture mechanics of rock*, BK Atkin-  
857 son, editor. Academic Press Inc; 1987. pp. 277-349.