

# Pure-Rotational H<sub>2</sub> Thermometry by Ultrabroadband Coherent Anti-Stokes Raman Spectroscopy

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Coherent anti-Stokes Raman spectroscopy (CARS) is a sensitive technique for probing highly luminous flames in combustion applications to determine temperatures and species concentrations. CARS thermometry has been demonstrated for the vibrational Q-branch and pure-rotational S-branch of several small molecules. Practical advantages of pure-rotational CARS, such as multi-species detection, reduction of coherent line mixing and collisional narrowing even at high pressures, and the potential for more precise thermometry, have motivated experimental and theoretical advances in S-branch CARS of nitrogen (N<sub>2</sub>), for example, which is a dominant species in air-fed combustion processes. Although hydrogen (H<sub>2</sub>) is of interest given its prevalence as a reactant and product in many gas-phase reactions, laser bandwidth limitations have precluded the extension of CARS thermometry to the H<sub>2</sub> S-branch. We demonstrate H<sub>2</sub> thermometry using hybrid femtosecond/picosecond pure-rotational CARS, in which a broadband pump/Stokes pulse enables simultaneous excitation of the set of H<sub>2</sub> S-branch transitions populated at flame temperatures over the spectral region 0–2200 cm<sup>-1</sup>. We present a pure-rotational H<sub>2</sub> CARS spectral model for data fitting and compare extracted temperatures to those from simultaneously collected N<sub>2</sub> spectra in two systems of study: a heated flow and a diffusion flame on a Wolfhard-Parker slot burner. From 300–650 K in the heated flow, the H<sub>2</sub> and N<sub>2</sub> CARS extracted temperatures are, on average, within 2% of the set temperature. For flame measurements, the fitted H<sub>2</sub> and N<sub>2</sub> temperatures are, on average, within 5% of each other from 300–1600K. Our results confirm the viability of pure-rotational H<sub>2</sub> CARS thermometry for probing combustion reactions.

## I. INTRODUCTION

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Coherent anti-Stokes Raman spectroscopy (CARS) remains an important tool for characterizing combustion environments after four decades of CARS thermometry research and application.<sup>1</sup> The coherent nature of the CARS technique affords a dramatic increase of CARS signals over the corresponding spontaneous Raman signals both in absolute strength and in intensity relative to collected background interference. One such interference results from probing the Raman signal in the presence of particulates, which are prevalent in highly luminous soot-forming flames associated with rich combustion conditions. In these environments, the coherent signal of CARS is particularly useful.<sup>2</sup> Although hydrogen ( $H_2$ ) is generally produced under the rich conditions of applied combustion, nitrogen ( $N_2$ ) CARS thermometry has been far more extensively developed.  $N_2$  is a dominant species in air-fed combustion reactions, and its linewidth, line-broadening, and other spectroscopic parameters are better known than those of  $H_2$ . Thus, the non-combustible  $N_2$  species has been widely used as an indirect probe to report on the spatially and temporally resolved temperature of the combustion reaction. The expansion of the lesser used  $H_2$  CARS methods is desirable, as they provide direct and versatile probing of  $H_2$ , which is a common reactant and product of combustion reactions in nitrogen-rich and nitrogen-free combustion environments.

Practical advantages exist for choosing  $H_2$  in CARS thermometry experiments.  $H_2$  is well-suited for vibrational CARS (VCARS) thermometry given the following characteristics: simplicity of  $H_2$  Q-branch spectra with relatively widely spaced and sparse transitions even at flame temperatures, the spectral separation of  $v = 0 \rightarrow v' = 1$  from  $v = 1 \rightarrow v' = 2$  transitions, strength of signal from a large Raman cross section distributed over only a few rotational transitions, and minimal coherent line mixing and collisional narrowing effects at low and moderate pressures.<sup>1</sup> Although the characterization of  $H_2$  linewidths is limited in the literature, initial and more recent

studies have provided essential molecular parameters for developing the spectral model for H<sub>2</sub> temperature fitting.<sup>3-6</sup> Time-resolved CARS measurements have been used to determine H<sub>2</sub> linewidths and broadening coefficients to further models for temperature assignment of H<sub>2</sub> CARS spectra.<sup>7-9</sup> Alternately, H<sub>2</sub> VCARS thermometry has been performed by frequency-domain fitting of the temperature-dependent relative spectral intensities of the well-separated rotational transitions within the first vibrationally excited band.<sup>10-12</sup> Specifically, H<sub>2</sub> VCARS experiments to determine flame temperatures, and in some cases H<sub>2</sub> concentration, have been performed in an ethylene-air Bunsen flame at atmospheric pressure,<sup>13</sup> in the exhaust of a CH<sub>4</sub>/air flame under pressure conditions,<sup>14</sup> and in a graphite tube furnace.<sup>15</sup> Shirley, et al. compared H<sub>2</sub> and O<sub>2</sub> VCARS thermometry accuracies across a diffusion flame from a Wolfhard-Parker burner.<sup>1, 16</sup> Later, Hancock et al. used a hydrogen/air flame from a Hencken burner to compare the relative thermometric accuracies of H<sub>2</sub> and N<sub>2</sub> VCARS.<sup>17</sup> Similarly, determining the thermometric accuracy of H<sub>2</sub> pure-rotational CARS (RCARS), with respect to more established N<sub>2</sub> CARS measurements is a goal of this work; however, the key difference is our development and use of an ultrabroadband CARS approach to enable H<sub>2</sub> RCARS thermometry.

Performing RCARS thermometry has garnered interest because of potential advantages over VCARS thermometry. One benefit is that the Raman rotational transitions are, in general, much more widely spaced compared to ro-vibrational transitions, which simplifies data analysis and increases the pressure of the onset of line mixing and collisional narrowing.<sup>1</sup> Another advantage of RCARS is that the rotational Raman spectra of many molecules fall within the same narrow frequency range, and multi-species detection has been demonstrated by several research groups.<sup>18-20</sup> The rotational Raman spectrum of H<sub>2</sub> is an exception to this rule, as the transitions extend much further in frequency. This large spectral range of H<sub>2</sub> rotational lines populated at flame

temperatures has previously prevented the detection of all lines necessary for temperature determination in one signal field. In the only previously published H<sub>2</sub> RCARS thermometry experiments, to our knowledge, Clauss and coworkers employed dual Stokes CARS to compare the simultaneously but separately collected signals from two S-branch transitions.<sup>21 22</sup> Our group has demonstrated a two-beam femtosecond/picosecond (fs/ps) hybrid CARS technique<sup>23-24</sup> capable of collecting the ultrabroadband H<sub>2</sub> S-branch spectrum. Equipped with this broadband technique, we investigate whether the extreme shift of the H<sub>2</sub> S-branch population with temperature could make H<sub>2</sub> more sensitive than other small molecules for RCARS thermometry across a wide range of temperatures.

RCARS thermometry is generally regarded as more accurate than its vibrational counterpart at lower temperatures because of the absence of a significantly populated hot vibrational band. However, once temperatures are high enough, a significant fractional population exists in the first vibrational excited state, and VCARS is regarded as the more accurate technique. For the N<sub>2</sub> molecule, this temperature threshold is around ~1200 K, at which point a Boltzmann analysis shows ~6% of the vibrational population resides in the  $v = 1$  initial state. The H<sub>2</sub> molecule has a larger vibrational transition energy, and a temperature of ~2140 K is required to achieve the same 6% population of  $v = 1$ . Thus, the accuracy advantage of RCARS over VCARS for H<sub>2</sub> thermometry may conceivably exist at even higher temperatures than for N<sub>2</sub> thermometry.

Several advantages of H<sub>2</sub> RCARS thermometry motivate the development and validation of a H<sub>2</sub> RCARS model. With the multiplexed nature of the signal generated in ultrabroadband CARS, robust global fits to the available data across multiple species and molecular manifolds will improve both accuracy and precision of temperature and species concentration measurements. Isolation of the pure-rotational H<sub>2</sub> CARS signal creates an opportunity for detailed studies of



nonequilibrium, or nonthermalized, molecular energy distributions, for which the equipartition theorem is not valid. The wide frequency spacing of the pure rotational states provides a unique test case for exponential gap collisional energy transfer models. Therefore, a time-domain RCARS model for  $\text{H}_2$  that can be incorporated into global fits of two-beam ultrabroadband CARS data must be developed and validated to exploit these potential advantages and new directions.

In this paper, we detail the development of RCARS  $\text{H}_2$  thermometry, including the experimental setup and methodology, and the theoretical models that we have adapted for fitting of temperature-dependent  $\text{H}_2$  S-branch CARS spectra. The resulting temperature profiles from the S-branch spectra of  $\text{H}_2$  are compared to the profiles evaluated from simultaneously collected  $\text{N}_2$  RCARS spectra to provide an assessment of the accuracy and precision of the  $\text{H}_2$  RCARS thermometry. For this assessment, we use both a heated flowing mixture of  $\text{H}_2$  and  $\text{N}_2$ , and a  $\text{N}_2$ -seeded  $\text{H}_2$ -fueled diffusion flame from a Wolfhard-Parker slot burner.

## II. EXPERIMENT

### A. Optical setup

Femtosecond and picosecond amplified laser systems generate the pulses used in these hybrid CARS experiments. The experimental layout is similar to that presented previously,<sup>25</sup> and it is depicted in Fig. 1(a). The mode-locked oscillators seeding the two laser systems are phase locked to the same external 100 MHz radio frequency source, which enables precise electronic timing between the femtosecond and picosecond pulses with low jitter ( $<1$  ps). The pump/Stokes beam originates from the output of a commercial (KM Laboratories Wyvern 1000) Ti:Sapphire regenerative amplifier operating at 1 kHz (780 nm, 45 fs). A 2 mJ portion of amplified 780 nm light is focused into a 310- $\mu\text{m}$  diameter hollow-core fiber that is 1 m in length (Kaleidoscope,

Femtolasers) and housed in a pressure vessel at  $\sim 350$  Torr Ar. The incoming pulses are frequency broadened via self-phase modulation, and the  $\sim 0.8$  mJ pulses exiting the fiber are compressed to  $\sim 7$  fs duration (0.6 mJ/pulse) after a series of reflections from mirrors that impart negative chirp to the pulse. A cylindrical concave silver mirror with 400 mm focal length focuses the horizontally polarized pump/Stokes beam to a vertical sheet. The picosecond probe pulses originate from a high-energy seeded picosecond Nd:YAG regenerative amplifier operating at 20 Hz ( $\sim 24$  mJ/pulse, 65 ps). A half-wave plate rotates the probe polarization to match that of the pump/Stokes beam, and a cylindrical plano-convex lens with 300 mm focal length focuses the beam to a vertical sheet. The two beams intersect at their common horizontal focus; a shallow crossing angle ( $5^\circ$ ) relaxes the phase-matching conditions for detecting a large range of Raman shifts simultaneously. This  $5^\circ$  intersection of beams with focused diameters of 50  $\mu\text{m}$  full width and half maximum (FWHM) yields an interaction length of  $\sim 1$  mm. In this work, the sample (labeled S in Fig. 1(a)) is either a  $\text{N}_2$ -seeded  $\text{H}_2$  diffusion flame or a heated flow mixture of  $\text{N}_2$  and  $\text{H}_2$ , as discussed in Section II.B. The temporal delay,  $\tau$ , of the probe is electronically set to 160 ps to suppress the nonresonant signal fully while still probing relatively early to help balance the detection of  $\text{N}_2$  and  $\text{H}_2$  with large differences in dephasing times. We determine  $\tau$  and the probe temporal duration of 65 ps via intensity cross-correlation of the nonresonant spectral portion (integration of  $< 2900\text{ cm}^{-1}$  Raman shifts) of a methane signal.

The imaging and detection scheme (Fig. 1(a)) first involves the isolation of the CARS signal ( $> 40\text{ cm}^{-1}$  Raman shifts) from the probe after transmission through an angle-tuned short-wave-pass filter (Semrock, SP01-561RU-25). The CARS signal generated at the sample vertical sheet interaction region is relay imaged  $\sim 1:1$  onto the entrance slit of a 0.550 m Czerny-Turner spectrometer (Horiba iHR550, 600 or 2400 grooves/mm grating), and spectrally dispersed onto a

400×1600 pixel (16  $\mu\text{m}$  pixel size), water-cooled ( $-90^\circ\text{C}$ ), back-illuminated CCD array (Andor DU971N-BV). We apply a  $2\times 2$  binning in the CCD software, resulting in an array of  $200\times 800$  effective pixels with size 32  $\mu\text{m}$ . The collection of each single-shot CCD frame (30 ms exposure time) is triggered by a probe pulse; however, the collection rate of the full frame is electronically limited beyond the 20 Hz laser repetition rate to 6 Hz. A shutter (UniBlitz) at the entrance of the spectrometer is synchronized to the CCD collection to block all additional laser shots.

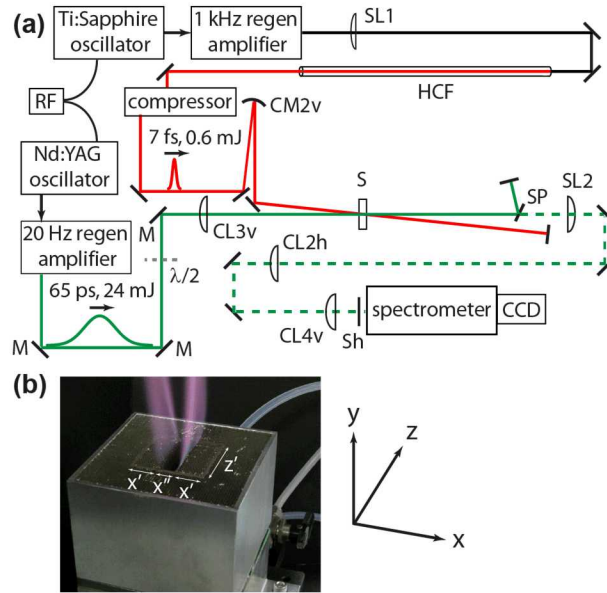


FIG. 1. Two-beam fs/ps CARS experiment. (a) The output of a 1 kHz Ti:Sapphire regenerative amplifier is frequency broadened in a hollow-core fiber (HCF) and compressed with chirped mirrors; each visible/near-IR pulse provides pump and Stokes field interactions. Probe pulses (532 nm) originate from a Nd:YAG regenerative amplifier and transmit through a half-wave plate ( $\lambda/2$ ). Polarization-matched beams are focused to intersecting vertical sheets at the sample (S). The CARS signal transmits through a short-pass filter (SP) and is 1:1 relay imaged to the spectrometer and CCD detector. Labels for spherical lenses (SL) and cylindrical lenses (CL) and mirrors (CM) include focal lengths (1,  $f=1\text{ m}$ ; 2,  $f=400\text{ mm}$ ; 3,  $f=300\text{ mm}$ ; 4,  $f=75\text{ mm}$ ); CL and CM labels also indicate the focusing axis of symmetry (v, vertical; h, horizontal). Sh, shutter; M, dielectric mirror; silver mirrors are unlabeled. (b) Laser beams propagate along  $z$  above the Wolfhard-Parker burner with labeled slot dimensions of  $x'=16\text{ mm}$ ,  $x''=9\text{ mm}$ , and  $z'=41\text{ mm}$ .

## B. CARS measurements

In this work, we demonstrate  $\text{H}_2$  RCARS thermometry in a heated flow and across a diffusion flame. The first sample is a mixture of  $\text{H}_2$  and  $\text{N}_2$  flowing at 1 L/min and 4 L/min, respectively,

that is gradually heated up to  $\sim 800$  K, as measured by a thermocouple across the flow exit. The second sample is a laminar, nearly two-dimensional flame produced at a fuel-air interface of a Wolfhard-Parker slot burner.<sup>26</sup> Figure 1(b) depicts the burner and its experimental orientation in x (lateral), y (vertical), and z (beam propagation) dimensions. The proportions of three adjacent slots are taken from Ref. <sup>27</sup> and are as follows:  $9 \times 41$  mm ( $x'' \times z'$ , Fig. 1(b)) center fuel (here,  $H_2$  seeded with  $N_2$ ) slot and one  $16 \times 41$  mm ( $x' \times z'$ , Fig. 1(b)) slot of flowing air on each side of the center slot. Ideally, the gases from each side of a common boundary (here, each  $z' = 41$  mm edge) rise vertically, diffusing into each other in a laminar flow.<sup>26</sup> The following procedures ensure a steady flame front and a laminar flow: flowing  $N_2$  through an outer shroud that increases each of the x and z burner dimensions to 76.2 mm (also  $y = 76.2$  mm), filling this outer shroud and the air slots with fine sintered bronze honeycomb mesh that is flush with the burner surface, and placing a steel mesh chimney (not shown in Fig. 1(b)) above the burner. The flow rates are 21 L/min for  $H_2$  and 4 L/min for  $N_2$  in the fuel slot and 50 L/min air for the two air slots.

For either sample, the 1D CARS imaging system produces RCARS spectra for both  $N_2$  and  $H_2$ , where each CCD row reports on a unique spatial (and thus temperature profile) position in y above the flow tube exit or burner surface. The beam intersection is roughly centered on the stationary flow tube in the x and z dimensions. The exact intersection position for the heated flow is chosen to minimize spatial averaging effects (SAE)<sup>27-28</sup> along the z-dimension of the probe volume, which corresponds to recovered temperatures in close agreement to those reported by the thermocouple. Orienting the Wolfhard-Parker burner such that the probe volume is parallel, or along the 2D flame front has been shown to result in a temperature measurement with little influence from SAE.<sup>27</sup> Because of the steepness of the flame front, our probe region does not span the vertical range necessary to sample the complete reactants-to-products cross section of the



flame, so we translate the burner in the x dimension and measure the temperature at a constant height. The vertical spatial intensity maxima of the pump/Stokes and probe beams are both located  $\sim 3.5$  mm above the flow tube exit or burner surface. This region of strongest signal is vertically centered on the CCD. An average of 10 (5 after binning) vertical pixels, or  $160\text{ }\mu\text{m}$  of the image, defines the temperature profiles presented here.

Performing accurate  $\text{H}_2$  thermometry, especially at high temperatures, requires overcoming laser stability and signal-to-noise challenges associated with broadband pulses and spectra. For averaged results, each CARS spectrum is the average of 500 frames for the flame measurements and 200 frames for the heated flow measurements. This averaging provides an adequate CARS signal-to-noise ratio given the spectral and intensity instabilities in the broadband pump/Stokes pulses required to excite  $\text{H}_2$  rotational Raman transitions from  $\sim 350$  to  $\sim 2150\text{ cm}^{-1}$  simultaneously. Additionally, averaged results may be compared between the two thermometry methods ( $\text{H}_2$  and  $\text{N}_2$ ) for an assessment of accuracy, even though burner fluctuations have been observed and will reduce the single-shot precision. Shot-to-shot fluctuations in integrated intensity or energy at specific frequencies within the ultrabroadband pulse spectrum do not directly equate to precision errors in the CARS spectrum and temperature evaluation. Spectrally uniform intensity fluctuations should not affect the relative CARS peak areas or fitted temperatures. Also, the amplitude of each CARS signal peak depends on the pump/Stokes impulsive excitation, which includes contributions from all frequency pairs (likely with uncorrelated noise fluctuations) with differences equal to the S-branch transition. Therefore, precise relative peak amplitudes and evaluated temperatures may occur with spectral noise, and signals with large Raman shifts (fewer pump/Stokes combination frequencies) should be more sensitive to noise. Each data set includes two additional spectra, averaged over the corresponding number of frames as the CARS signal data: a background

spectrum recorded without pump/Stokes illumination, and a nonresonant spectrum generated in a flow of methane at  $\tau = 0$ . Background-subtracted CARS spectra are divided by background-subtracted nonresonant spectra to account for the spectral sensitivity of the experiment (pulse bandwidth, phase-matching, optical coatings, and spectral responsivity of grating and detector) before fitting to model CARS spectra (Section II.C). The wide frequency spacing of H<sub>2</sub> rotational peaks can generate CARS spectra at low temperatures with very few populated transitions or at high temperatures with populated transitions at large Raman shifts where the pump/Stokes bandwidth limits the excitation profile. The nonresonant third-order susceptibility of methane is relatively high (more than 2.5 times that of Ar, N<sub>2</sub>, or O<sub>2</sub>),<sup>29</sup> which enables us to collect nonresonant spectra with the signal levels across the H<sub>2</sub> frequency window necessary for H<sub>2</sub> temperature measurements.

### **C. Rotational CARS modeling**

The focus of this work is on H<sub>2</sub> RCARS, but we also present N<sub>2</sub> RCARS results to assess and validate H<sub>2</sub> thermometry. The bulk of this section outlines the modeling and fitting procedures for H<sub>2</sub> spectra, and differences in these procedures for CARS of N<sub>2</sub> or H<sub>2</sub>/N<sub>2</sub> mixtures are addressed at the end of the section. The time-domain CARS signal calculations presented here are similar to previous methods.<sup>18,30</sup> While the experimental probe pulse electric field is included explicitly in this model, the pump and Stokes interaction pulse is assumed to be transform limited and a delta function in time. Therefore, we divide the experimental CARS spectra by a nonresonant spectrum (Section II.B.), which serves as a spectral responsivity function, to account for the experimental deviation from infinite bandwidth of Raman state preparation in the model. In these experiments, the probe follows the pump/Stokes pulse at a delay,  $\tau$ , that is large enough to assure no pulse



overlap or nonresonant signal contributions. The time-domain CARS signal simplifies to the time-dependent Raman-resonant signal with polarizability, calculated as

$$\chi(t) = \sum_{\Delta J=+2} W_{JJ'} \exp[(i\omega_{JJ'} - 2\Gamma_J)t]. \quad (1)$$

In Eq. (1),  $\chi(t)$  is the sum of contributions from Stokes transitions from states  $J$  to  $J' = J + 2$ , where  $J$  is the angular-momentum quantum number. These transitions occur at positive  $\omega_{JJ'}$  Raman frequencies, which are calculated from differences between rotational energies,  $F(J + 2) - F(J)$ , where

$$F(J) = B_{v=0}J(J+1) - D_{v=0}J^2(J+1)^2 + H_{v=0}J^3(J+1)^3, \quad (2)$$

with ground vibrational state rotational constants from Ref. <sup>31</sup>. The  $J$ -dependent Raman linewidths,  $\Gamma_J$ , in Eq. (1) model the collisional dephasing and are given by the Lorentzian half-width at half maximum (HWHM) of the Raman transition. Following a treatment similar to those of Refs. 3-4, 7, we express the Raman linewidths as

$$\Gamma_J = 2\pi D_0(T) \nu_{JJ'}^2 / c\rho + \gamma_J(T)\rho, \quad (3)$$

where  $D_0 = 0.01176T^{0.8314}$  cm<sup>2</sup>amagat/s is the temperature-dependent optical diffusion coefficient,<sup>3</sup>  $\nu_{JJ'}$  are the Raman transition frequencies in cm<sup>-1</sup> ( $\nu_{JJ'} = \omega_{JJ'} / 2\pi c$ ),  $c$  is the speed of light,  $\rho$  is the density, and  $\gamma_J(T) = \tilde{\gamma}_J T + \gamma_0$  is the temperature-dependent collisional broadening coefficient in units of cm<sup>-1</sup>amagat<sup>-1</sup> with  $\tilde{\gamma}_J$  and  $\gamma_0$  values taken from Table 1 of Ref. 4. Removing the  $J$ -dependence of  $\gamma_J$  results in 1% lower evaluated temperatures, on average, over a 340–800 K range. Note that the wavenumber units of  $\Gamma_J$  from Eq. (3) must be converted to radial frequency units for Eq. (1). Finally, the  $W_{JJ'}$  terms in Eq. (1) are weighting functions that, for treatment of an individual species, are simply

$$W_{JJ'} = (N_{J'} - N_J) b_{JJ'} F_J, \quad (4)$$

where  $N_J$  ( $N_{J'}$ ) are the temperature-dependent rotational Boltzmann population distribution fractions for the  $J$  ( $J'$ ) state,  $b_{JJ'}$  are the Placzek-Teller coefficients, and  $F_J$  are the Herman-Wallis factors. For the Stokes transitions ( $J' = J + 2$ ) in the experiments presented here, the Placzek-Teller coefficients are given by

$$b_{J,J+2} = \frac{3(J+1)(J+2)}{2(2J+3)(2J+1)}. \quad (5)$$

The Herman-Wallis factors, which account for vibration-rotation interaction, are as follows:<sup>32-34</sup>

$$F_J = \left[ 1 + \frac{\eta^2}{\chi} (J^2 + 3J + 3) \right]^2, \quad (6)$$

where the variable  $\chi$  is defined as the ratio of anisotropic polarizability expansion coefficients<sup>32-33</sup> and is distinct from  $\chi(t)$  in Eq. (1). In Eq. (6),  $\chi = 0.38$  for the H<sub>2</sub> S-branch,<sup>33</sup> and  $\eta = 2B_e / \omega_e$  includes the rotational constant ( $B_e = 60.8 \text{ cm}^{-1}$ ) and vibrational frequency ( $\omega_e = 4401 \text{ cm}^{-1}$ ) of H<sub>2</sub>.<sup>35</sup> For the H<sub>2</sub> S-branch transitions in this work, the  $F_J$  factors increase from 1.01 to 1.50 for transitions out of initial states  $J = 0$  to  $J = 9$ , respectively. The rigid rotor approximation ( $F_J = 1$  for all  $J$  transitions) results in falsely high evaluated temperatures; the error increases with temperature, e.g., 1% near room temperature to 4% near 800 K.

Using the calculated Raman-resonant polarizability,  $\chi(t)$ , and probe experimental pulse parameters, we model the experimental H<sub>2</sub> CARS spectrum at several temperatures from 200 K to >2000 K to include in a library for fitting. Boltzmann fractions (Eq. (4)) dominate the intrinsic temperature dependence of  $\chi(t)$ , but linewidths (Eq. (3)) also contribute. For example, fixing  $\Gamma_J$  ( $J$ ) from Eq. (3) to constant values calculated at  $T=600$  K, the actual dependence of  $\Gamma_J$  on  $T$  results

in evaluated temperature errors up to 5% for a set of 300–800 K spectra when the linewidths are not properly calculated. The set of Stokes transitions out of initial states  $J = [0, 9]$  is included in the Eq. (1) summation of  $\chi(t)$  and is sufficient to describe the H<sub>2</sub> Raman spectrum for all temperatures measured in this work. The experimental probe pulse is assumed to be transform limited with electric field amplitude,  $E_{pr} = \sqrt{I_{pr}}$ , where the intensity cross-correlation of the femtosecond pump/Stokes and picosecond probe pulses is considered to be equal to  $I_{pr}$ . Given the carrier frequency,  $\omega_0$ , and delay,  $\tau$ , of the probe, we can express the CARS time-domain electric field as,<sup>18</sup>

$$E_{CARS}(t - \tau) = E_{pr}(t - \tau) \exp[i\omega_0(t - \tau)] \times \chi(t). \quad (7)$$

A Fourier transform with respect to  $t$  of Eq. (7) yields the complex-valued frequency-domain  $E_{CARS}$  field, and the square modulus of this field is the CARS spectrum,  $I_{CARS}(\omega + \omega_0)$ , which is shifted by  $\omega_0$  from  $I_{CARS}(\omega)$ . Modeled  $I_{CARS}(\omega)$  spectra at 300 and 2000 K illustrate the large spectral bandwidth and strong temperature dependence of the  $J = [0, 9]$  H<sub>2</sub> transitions (Fig. 2). Additionally, the characteristic 9:1 intensity alternation exists between odd and even transitions, which is related to the nuclear spin statistics of H<sub>2</sub>.

In this work, the fitting of an experimental CARS spectrum to the temperature library of model spectra involves the comparison of sets of  $J$ -indexed peak areas. In this context, an experimental CARS spectrum exists at each row of a CARS spectral image from the 2D array detector; each row spectrum can be fit to a model CARS spectrum (e.g., Fig. 2). Compared to alternate methods of fitting one complete spectrum with another,<sup>18</sup> the peak area fitting strategy has advantages of relaxing both the calibration requirement of the experimental detection axis and the consistent wavelength assignment of spectral peaks with asymmetric and varying pixel sensitivity functions

due to grating spectrometer coma and other aberrations.<sup>36-38</sup> A nonlinear least-squares algorithm is used to find the temperature library model spectrum that minimizes the residual between sets of experimental and model peak areas, which have been treated in the following manner: the set of experimental peak areas is normalized to its maximum; a set of model peak areas has the total area under peaks scaled to that under the set of normalized experimental peaks. Interpolation of the library temperatures yields a fit to the nearest 0.5 K.

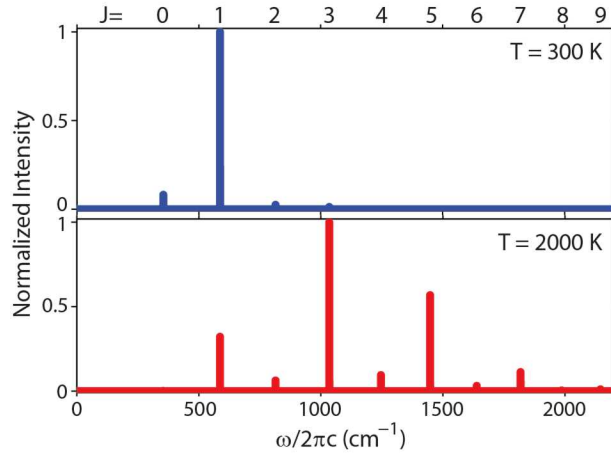


FIG. 2. The height-normalized set of pure-rotational H<sub>2</sub> CARS transitions out of initial states  $J = [0, 9]$  are simulated for two temperatures: 300 K (top panel, blue) and 2000 K (bottom panel, red).

The N<sub>2</sub> RCARS model and experimental fitting differ from the H<sub>2</sub> methods in the following aspects. To fit the temperatures measured in this work, the N<sub>2</sub> model includes the set of Stokes transitions out of initial states  $J = [0, 40]$  in Eq. (1). In our experiments, spectral filtering of the CARS signal from the probe limits fitting to detected lines  $J = [4, 40]$ ; furthermore the use of a 600 grooves/mm grating for simultaneous N<sub>2</sub> and H<sub>2</sub> collection reduces the spectral resolution and restricts fitting to the higher nuclear spin-weighted transitions out of even-numbered initial  $J$  states. Instead of calculating the  $\Gamma_J$  using Eq. (3), we take the temperature-dependent linewidths from those previously obtained from time-domain coherence measurements<sup>39</sup> and then interpolate to the spectral library temperatures. Additionally, the N<sub>2</sub> model incorporates Herman-Wallis factors that

differ from those of H<sub>2</sub>, and the specific coefficients and constants in the right-hand side of Eq. (6) are taken from Ref. <sup>40</sup> for N<sub>2</sub>. Finally, collisions between H<sub>2</sub> and N<sub>2</sub> molecules in mixtures affect the linewidth of each molecule. We modify our H<sub>2</sub> and N<sub>2</sub> CARS spectral models by incorporating species-specific, concentration-weighted linewidth sums in a method that has been shown to result in more accurate evaluations of temperature.<sup>4,41</sup> The N<sub>2</sub> thermometry model combines the N<sub>2</sub>-N<sub>2</sub> linewidths<sup>39</sup> and experimental N<sub>2</sub>-H<sub>2</sub> linewidths;<sup>41</sup> the H<sub>2</sub> model requires additional  $\tilde{\gamma}_J$  and  $\gamma_0$  values<sup>4</sup> to calculate  $\gamma_J^{\text{H}_2\text{-H}_2}(T)$  and  $\gamma_J^{\text{H}_2\text{-N}_2}(T)$ , which are then combined to solve Eq. (3) for  $\Gamma_J$ .

### III. RESULTS AND DISCUSSION

The temperature profiles presented below are for H<sub>2</sub> and N<sub>2</sub> CARS thermometry of a heated flow and diffusion flame. **For these demonstration experiments with repetition rates defined in Section II.A, averaged results are presented along with the corresponding single-shot precisions that help determine the potential for H<sub>2</sub> CARS measurements in transient turbulent combustion processes.** The temperature traces are obtained using the following procedure: the experimental spectrum from each CCD row is fit to the appropriate model (Section II.C.); the fitted temperatures are averaged over 5 (2x2 binned) vertical pixels (160  $\mu\text{m}$ ) in the peak signal region  $\sim 3.5$  mm above the surface of the flow or burner. This procedure is repeated for each thermocouple-measured temperature (heated flow, Section III.A.) or lateral position (Wolfhard-Parker slot burner, Section III.B.) to create temperature profiles for thermometry comparison.

#### A. Heated flow CARS thermometry

For the heated-flow mixture of H<sub>2</sub> and N<sub>2</sub>, RCARS spectra are collected with a 600 grooves/mm grating, which assures the detection of the complete set of H<sub>2</sub> S-branch transitions

while retaining the minimum resolution needed for N<sub>2</sub> S-branch temperature fitting. Extracted temperature profiles from fitting a single set of data are shown in Fig. 3(a) for H<sub>2</sub> (red trace) and N<sub>2</sub> (black dotted trace). The x-axes in Fig. 3(a,b) represent the set temperatures of the flow, as measured by a thermocouple (type K, 240  $\mu$ m diameter) centered at the flow exit. Since the actual temperature at the center of the flow exit may differ from the set temperature, we first assess the H<sub>2</sub> RCARS thermometric accuracy by comparison with the simultaneously collected N<sub>2</sub> RCARS data. The fs/ps RCARS technique has been well benchmarked for accuracy previously with N<sub>2</sub>.<sup>30, 42-44</sup> We calculate the magnitudes of H<sub>2</sub> and N<sub>2</sub> temperature differences (gray trace in Fig. 3(a), expressed as percentage of thermocouple-measured set temperature) and find that the H<sub>2</sub> and N<sub>2</sub> temperature profiles demonstrate good agreement. The average difference is 16.5 K (3.0%) over the full range (room temperature through  $\sim$ 800 K) with all differences  $<6.7\%$  (Fig. 3(a)). While good agreement is thus demonstrated, there is still room for improvement. In repeated day-to-day measurements, the low-temperature region from 290-350 K proves to be the most difficult, and the evaluated H<sub>2</sub> temperature can differ from the fitted N<sub>2</sub> temperature by as much as 20% near 300 K in an otherwise accurate data set; at and above 350 K, temperature evaluations are far more robust. The evaluated H<sub>2</sub> temperatures are quite sensitive to the exact excitation profile and its recovery, which reduces the reproducibility of accurate low-temperature measurements. As mentioned previously, the more drastic change of the H<sub>2</sub> versus N<sub>2</sub> S-branch spectra with temperature might imply that H<sub>2</sub> is a more sensitive molecule for thermometry. However, this benefit is counterbalanced by the increased influence of shot noise errors on temperature fitting for H<sub>2</sub>, which has only  $\sim 1/5$  of the number of populated transitions as N<sub>2</sub> at a given temperature in these experiments.



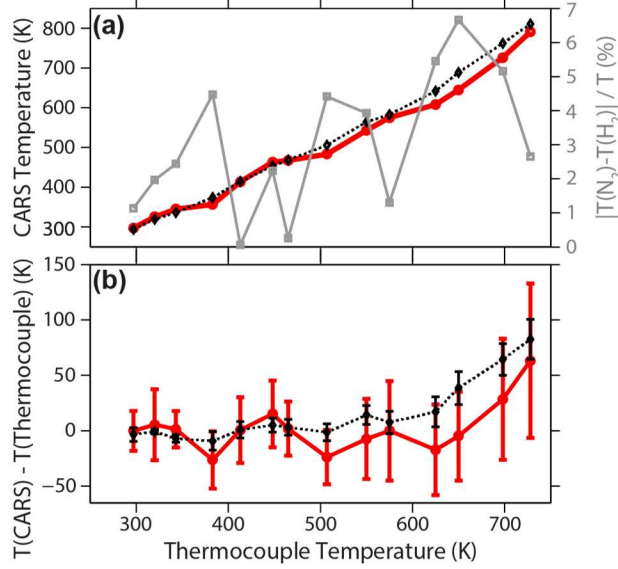


FIG. 3. Evaluated temperatures and statistics of heated flow CARS spectra taken at a series of thermocouple-measured set temperatures. (a) Temperatures extracted from pure-rotational CARS models are plotted for H<sub>2</sub> (red trace) and N<sub>2</sub> (dotted black trace) measured simultaneously (600 grooves/mm grating). Absolute values of the differences between H<sub>2</sub> and N<sub>2</sub> temperatures are plotted as percentages of thermocouple set temperatures (gray trace, right axis). (b) Differences between fitted temperatures and set temperatures are shown for H<sub>2</sub> (red trace) and N<sub>2</sub> (dotted black trace). The  $\pm\sigma$  error bars give the single-shot precision of H<sub>2</sub> and N<sub>2</sub> RCARS thermometry measurements.

The H<sub>2</sub> and N<sub>2</sub> temperature profiles closely match the set temperature through  $\sim 650$  K, which is evident in temperature differential plots for each molecule (Fig. 3(b)). At temperatures in excess of 650 K, the thermocouple readout was systematically lower than both the N<sub>2</sub> and H<sub>2</sub> evaluated temperatures, in which case the comparison between the benchmark N<sub>2</sub> data provides the best assessment of the accuracy of the H<sub>2</sub> thermometry data. The average deviations of the N<sub>2</sub> temperature from the thermocouple are 18.2 K (2.9%) over the full temperature range and 9.0 K (1.7%) for set temperatures  $\leq 650$  K. The residuals for H<sub>2</sub> for the corresponding temperature ranges are similar: 14.0 K (2.5%) and 8.7 K (1.9%). In repeated averaged-shot experiments and among individual laser shots, we find that the reproducibility of the H<sub>2</sub> thermometry near room temperature is difficult, because with only two intense rotational transitions (Fig. 2, top panel), even subtle fluctuations in excitation efficiency significantly affect the evaluated temperature. One possible solution would be to collect the experimental and nonresonant spectra simultaneously,

and this is currently under development in our laboratory. For single-laser-shot implementation of RCARS  $\text{H}_2$  thermometry, such a referencing scheme could help improve the single-shot precision of evaluated temperatures to match that of RCARS  $\text{N}_2$  thermometry. For our current experiment, the standard deviations of 200 single-shot spectral temperature fits are given as red (black) error bars for  $\text{H}_2$  ( $\text{N}_2$ ) in Fig. 3(b) for each set temperature. The  $\text{N}_2$  thermometry yields smaller error bars on average ( $\sigma = 1.8\%$  of the mean fitted temperature), but  $\text{H}_2$  also provides fairly high precision with  $\sigma = 6.8\%$  of the mean. The use of temperature fits to quantify indirectly the laser pulse fluctuations is motivated by the impulsive excitation (see Section II.B) and signal-probe frequency nondegeneracy of hybrid CARS. One other quantifier of relevant experimental noise is the ratio of two dominant peak amplitudes within the  $\text{H}_2$  S-branch; this ratio has an average shot-to-shot standard deviation of 15%. Our aim in this work is to provide an assessment of the viability of  $\text{H}_2$  wideband S-branch thermometry, and improvements in the overall precision of single-shot measurements are planned by incorporating single-shot corrections for the excitation profile.

## B. Diffusion flame CARS thermometry

For the Wolfhard-Parker burner measurements, two sets of CARS spectra are collected from the  $\text{N}_2$ -seeded  $\text{H}_2$  diffusion flame that is translated laterally across the fixed probe volume. Like in the heated flow case, a dataset detected with the 600 grooves/mm grating can be used for simultaneous thermometry of the two species. Figure 4(a) is a spectrum from a  $\sim 1200$  K point in the flame that has been fitted for temperature using  $J = [0, 7]$   $\text{H}_2$  and  $J = [4, 40]$   $\text{N}_2$  transitions; magnification levels emphasize the disparity of signal intensity between  $\text{N}_2$  and  $\text{H}_2$  and across the  $\text{H}_2$  series due to a combination of transition strength, excitation efficiency (dashed trace, Fig. 4(a)), and density of molecules. Resulting temperature profiles discussed later in this section are truncated in flame regions where low signal-to-noise levels prevent temperature fitting of one

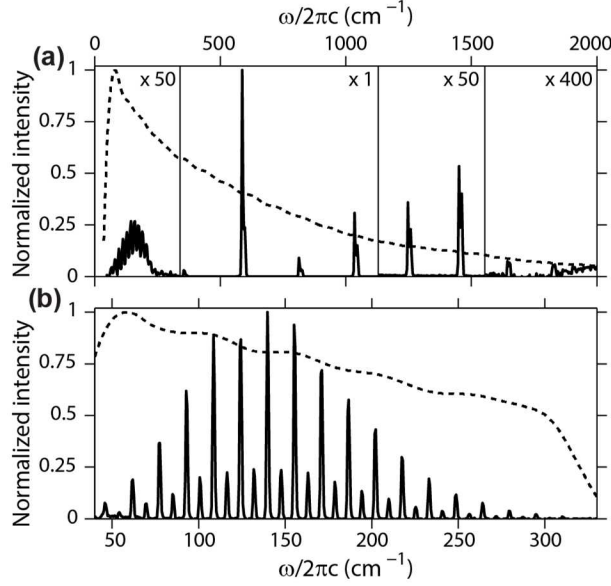


FIG. 4. Experimental RCARS spectra in diffusion flame ( $\sim 1200$  K), probed at  $y = 3.5$  mm above burner surface and  $x = 3.15$  mm from fuel slot center. (a) Lower resolution RCARS spectrum (solid lines) contains  $N_2$  rotational transitions below  $350\text{ cm}^{-1}$  and  $H_2$  transitions out of  $J = 0$  ( $\sim 354\text{ cm}^{-1}$ ) through  $J = 7$  ( $1818\text{ cm}^{-1}$ ) states. After overall height normalization, the labeled magnification is applied to each spectral region. (b) Higher resolution, height-normalized RCARS spectrum (solid lines) of  $N_2$  rotational transitions out of  $J = 4$  ( $\sim 44\text{ cm}^{-1}$ ) through  $J = 40$  ( $327\text{ cm}^{-1}$ ) states. (a,b) Height-normalized nonresonant methane spectra (dashed lines) are overlaid.

molecule. In this experiment, the detected  $H_2$  signal disappears at peak flame temperatures and also outside of the fuel slot where no  $H_2$  is present. Above the fuel slot and away from the air slot,  $N_2$ -seeding of the  $H_2$  fuel is necessary to detect the  $N_2$  S-branch spectrum. Even with seeding, the  $N_2$  signal is low on the fuel side of the flame (Fig. 4(a)). The region between flame walls produces CARS spectra with much stronger  $H_2$  than  $N_2$  transitions, because of the 21:4  $H_2$ : $N_2$  flow rate ratio, probe delay (160 ps), and broad pump/Stokes spectrum. The attenuation via neutral density filters necessary to avoid CCD saturation of  $H_2$  lines results in  $N_2$  lines that are too weak for temperature fitting. Sacrificing simultaneous species thermometry between flame walls and focusing only on the unattenuated  $N_2$  spectrum still poses the challenge of scattering within the spectrometer of the extremely strong, low-temperature, low-frequency  $H_2$  transitions into the CCD image of the  $N_2$  S-branch. High-resolution  $N_2$  spectra characterizing the complete burner surface are detected after a 2400 grooves/mm spectrometer grating and a spectral filter to eliminate  $H_2$  transitions. Resulting

spectrally filtered N<sub>2</sub> RCARS and nonresonant spectra from this configuration are shown in Fig. 4(b), where the N<sub>2</sub> spectrum of  $J = [4, 40]$  Raman transitions is taken at an identical point in the flame as in Fig. 4(a). Thus, this high resolution dataset provides both a check for flame and laser pulse stability over the  $\sim 1$  hour of data collection and validation of the reduced peak fitting procedure described in Section II.C. for N<sub>2</sub> RCARS thermometry using the lower resolution N<sub>2</sub> spectra.

In the absence of known temperatures at each flame position, we assess the performance of H<sub>2</sub> RCARS thermometry in a flame by comparison of H<sub>2</sub> and N<sub>2</sub> temperature profiles (Fig. 5). At each burner position, we calculate the residual temperature and percent error of the extracted H<sub>2</sub> or N<sub>2</sub> temperature (simultaneously collected spectra) with respect to the temperature extracted from the higher resolution N<sub>2</sub> spectra, which serves as the common reference. First, we determine the self-consistency of the N<sub>2</sub> temperature measurements between different datasets with the low and high resolution detection. Over the flame positions where the N<sub>2</sub> temperature profiles collected from low and high resolution gratings overlap, the average residual of N<sub>2</sub> temperatures (dotted black trace – blue trace, Fig. 5) is 34.7 K (4.0%), which reduces to 18.1 K (2.6%) with the removal of points at  $\sim 3.2$  and  $\sim 6.3$  mm locations within the steepest portions of the temperature profile. Possible explanations for the larger residuals at these locations include laser shot noise or positional reproducibility errors due to flame instability or translation stage motion. The overall agreement between these two N<sub>2</sub> data sets confirms the thermometry results from lower resolution N<sub>2</sub> spectra here and in the heated flow (Section III.A.) and justifies the comparison of H<sub>2</sub> and N<sub>2</sub> extracted temperatures from asynchronously collected spectra.



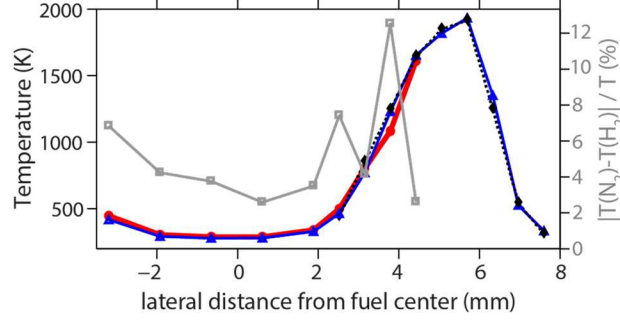


FIG. 5. Diffusion flame temperature profile spanning the burner fuel slot and one flame wall. CARS spectra are collected  $\sim 3.5$  mm above the burner surface at a series of lateral burner positions. Temperatures extracted from models are plotted for  $H_2$  (red trace) and  $N_2$  (dotted black trace) measured simultaneously (600 grooves/mm grating) and for  $N_2$  (blue trace) collected in higher resolution spectra (2400 grooves/mm grating). Differences between  $H_2$  and  $N_2$  temperatures are plotted as absolute value percentages of reference temperatures (gray trace, right axis).

The error of averaged  $H_2$  temperatures by comparison to the averaged  $N_2$  reference temperatures (red trace – blue trace, Fig. 5) serves to quantify the accuracy of our  $H_2$  S-branch thermometry approach, and is plotted in gray in Fig. 5 as an absolute value percentage of the reference  $N_2$  temperatures. Over the range of temperature profile overlap, the average  $H_2$  residual is 36.3 K (5.3%). Despite the similarity to the average  $N_2$  residual, the  $H_2$  and  $N_2$  residuals differ in magnitude and sign at common points, so the temperature profiles of simultaneously collected  $H_2$  and  $N_2$  spectra are not better matched. The standard deviation of extracted temperatures from a series of multiple-shot spectra is largest at the flame front positions, e.g., 3–4 mm from fuel center (Fig. 5). We can see from single-laser-shot assessments that the precision of the RCARS thermometry measurements is worse for the flame than for the previous heated flow both for  $N_2$  and  $H_2$  for all conditions. In the flame front, the precisions of  $H_2$  and  $N_2$  measurements are  $\sigma = 17\%$  and  $\sigma = 14\%$ , respectively. Therefore, the flame is operating with real fluctuations, and single-laser-shot statistical fluctuations are not a good representation of the precision of the technique itself.

## V. CONCLUSION

We have demonstrated the feasibility and accuracy of H<sub>2</sub> RCARS thermometry by using it to measure temperatures in a heated flow and a diffusion flame. For each system, we compare temperatures extracted from the collected broadband H<sub>2</sub> S-branch CARS spectra to those extracted with well-established methods from N<sub>2</sub> S-branch CARS spectra. The recovered H<sub>2</sub> and N<sub>2</sub> temperatures are, on average, within 3.0% of each other over a temperature range of 290–800 K in a heated flow and within 5.3% over 290–1610 K in a diffusion flame. Additionally, our assessment of H<sub>2</sub> and N<sub>2</sub> thermometry accuracy requires the comparison of measured temperatures to set temperatures of the heated flow. For thermocouple readings  $\leq 650$  K, the measured temperatures are, on average, within 2% of the set temperature for N<sub>2</sub> and H<sub>2</sub>. Finally, the H<sub>2</sub> temperature precisions over multiple averaged-shot spectra and for single-shot measurements are 3% and 6.8%, respectively. These results compare favorably with previous H<sub>2</sub> VCARS flame thermometry results of 4% repeated measurement precision, and 3% or even 5-10% error from N<sub>2</sub>, thermocouple, or calculated temperatures.<sup>16-17</sup> In our work, differences in H<sub>2</sub> and N<sub>2</sub> results at low temperatures can arise from limited H<sub>2</sub> transitions for fitting, while a significant improvement in reproducibility is obtained at temperatures over approximately 350 K. Further, the chaotic nature of the supercontinuum generation scheme yields pulse-to-pulse spectral fluctuations that degrade the single-shot precision, and current work in our lab is targeted at developing a shot-to-shot correction for the varying excitation profile. The ability to measure the complete H<sub>2</sub> rotational distribution will be important to studies of non-equilibrium molecular energy transfer, where it has been shown that small molecules, such as H<sub>2</sub>, may have persistent modal temperature differences during rapid energy release or relaxation of a gas ensemble including a larger collision partner, which is generally the case for H<sub>2</sub>.<sup>45</sup>



In summary, we have performed H<sub>2</sub> RCARS thermometry across a wide temperature range at atmospheric pressure. Our experimental results and spectral model indicate the viability of using S-branch H<sub>2</sub> CARS spectra to directly probe temperatures in combustion reactions that occur under rich conditions. The accuracy and versatility of H<sub>2</sub> RCARS thermometry motivate the continued development of this laser diagnostics tool to characterize high-temperature chemical processes.

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