SAND2017-9990PE

Persistent Homology and Geology

Jason Heath, Sandia National Laboratories

For BYU's Applied Topology Seminar

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U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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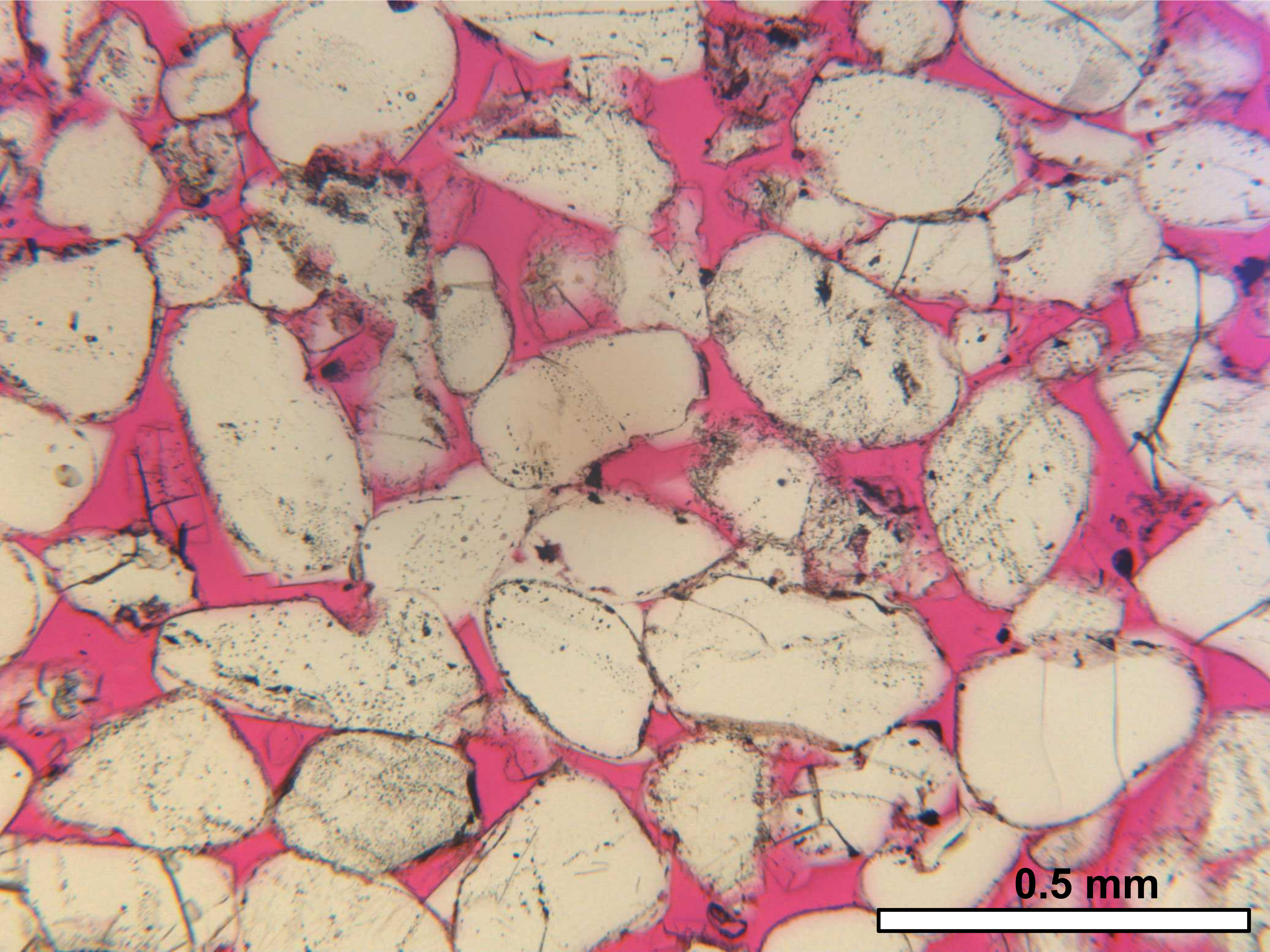
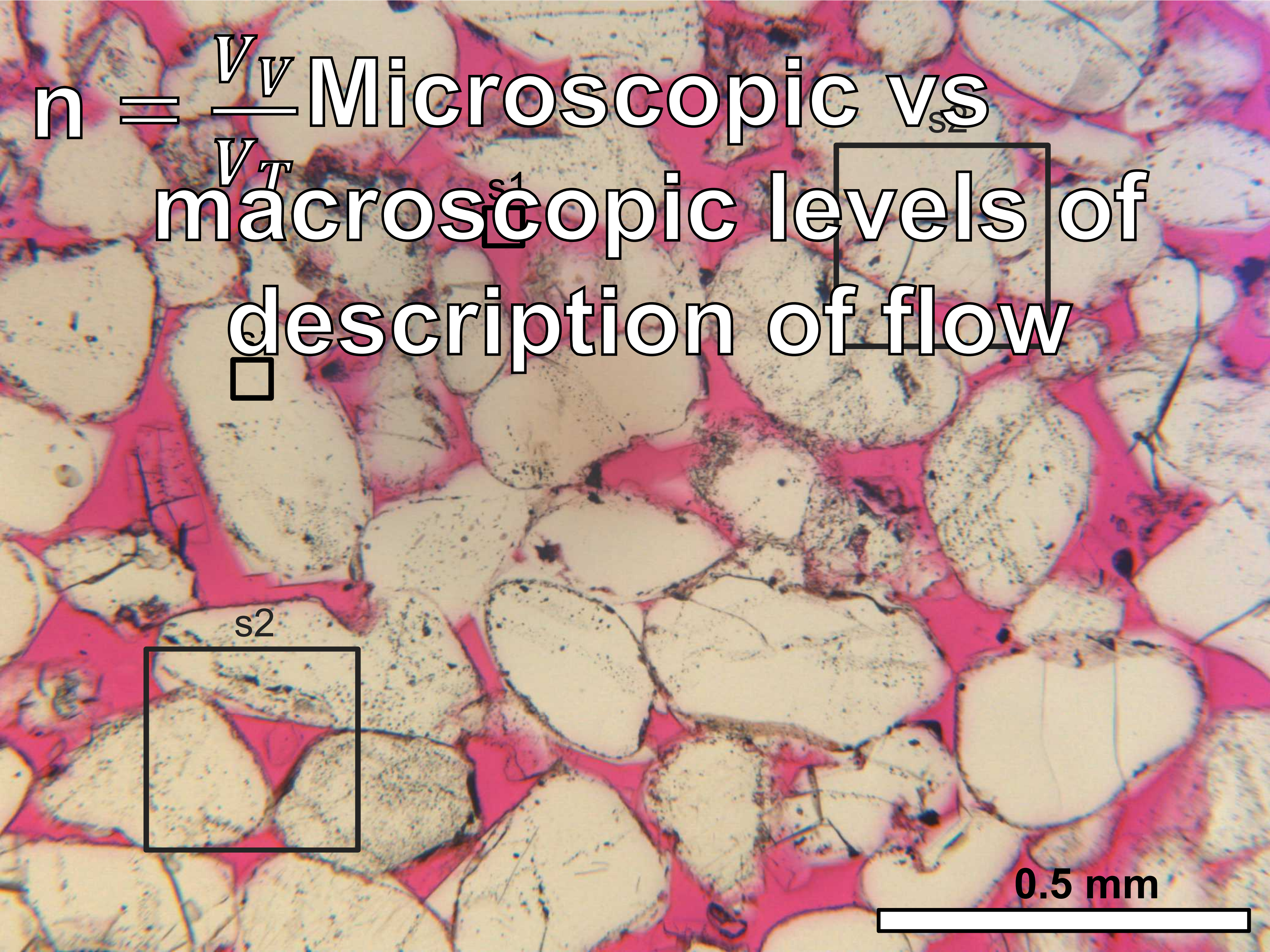
Outline

1. Connectivity and Geology
2. Persistent homology applications
   - Microstructure of Rocks
   - Seismic signals
3. Future direction and collaborations

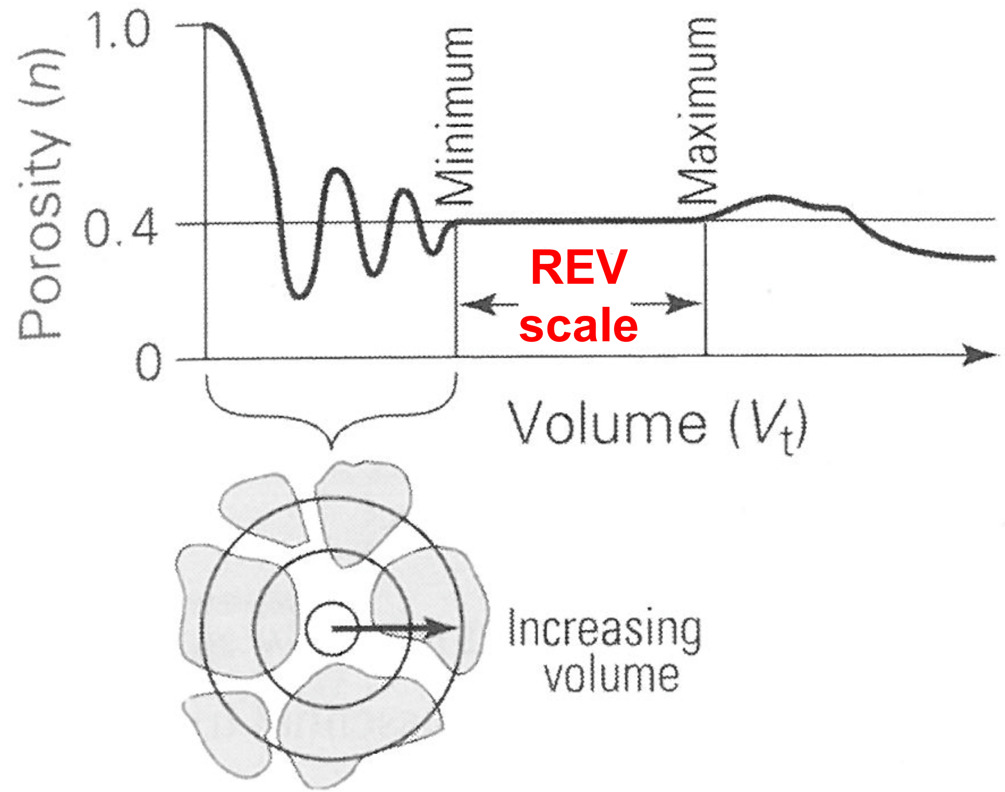

0.5 mm

$$n = \frac{V_V}{V_T}$$

Microscopic vs macroscopic levels of description of flow



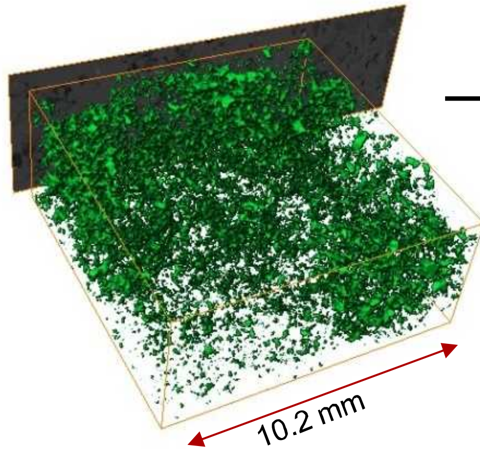
Representative elementary volume

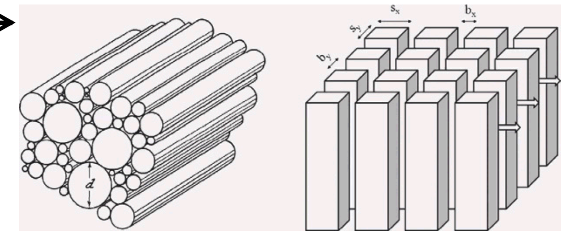


We may now define a porous medium as a portion of space occupied by a number of phases, at least one of which is a solid, for which an REV can be found (Bear, 1972).

Pore-scale representation

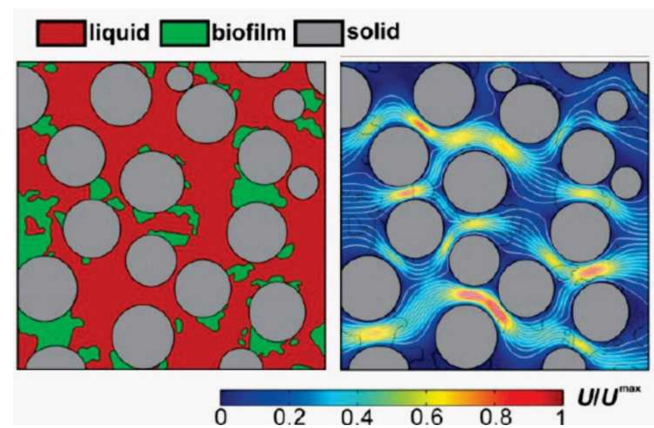
Geometric simplification





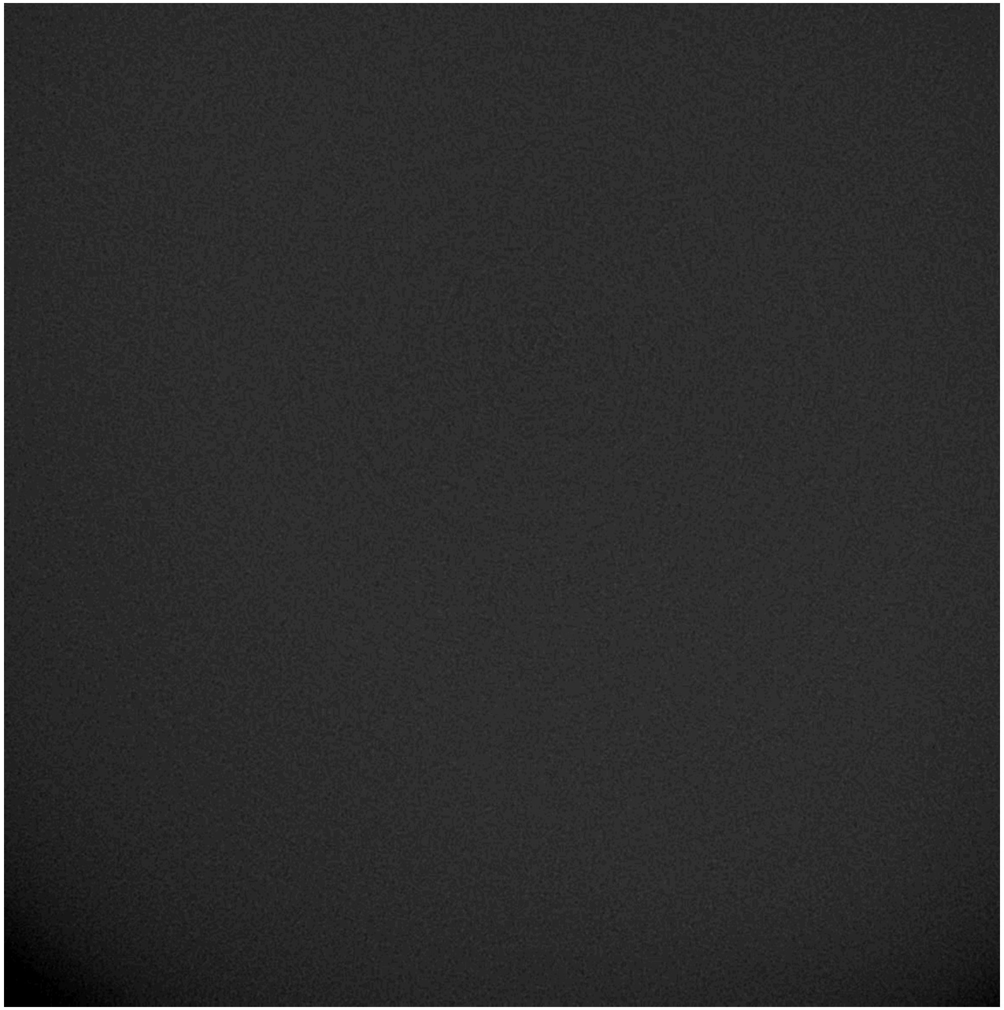
(from Chen et al., 2013)

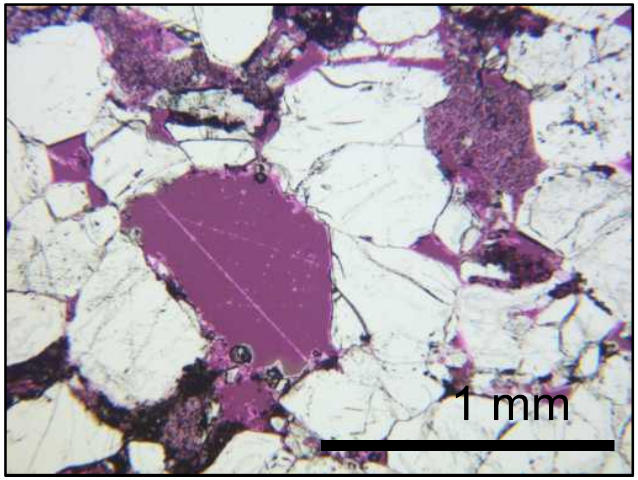
Keep geometric complexity, but intractable at large scales



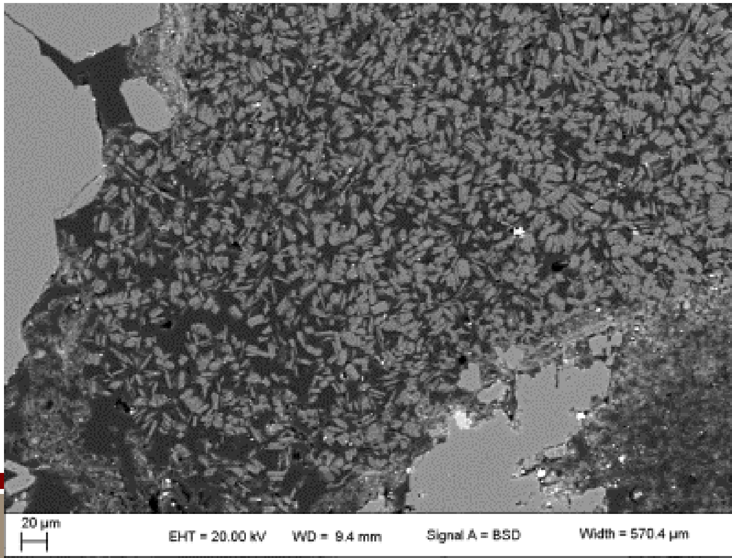
Need for consistency in treatment of pore structure for: poroelasticity, plastic deformation, fracture propagation, single and multiphase fluid flow, dynamic elastic wave propagation

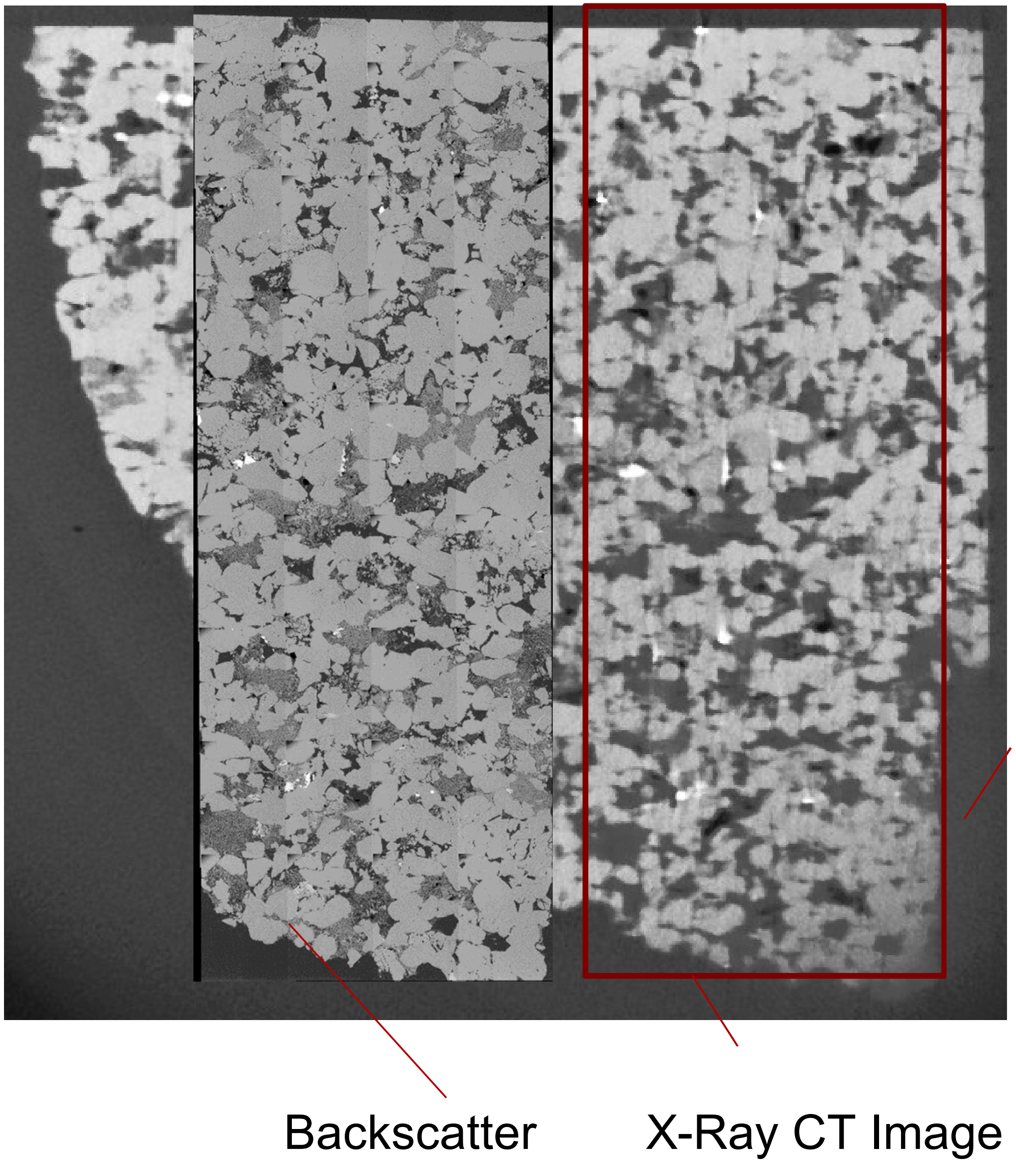
Now for persistent homology and rocks

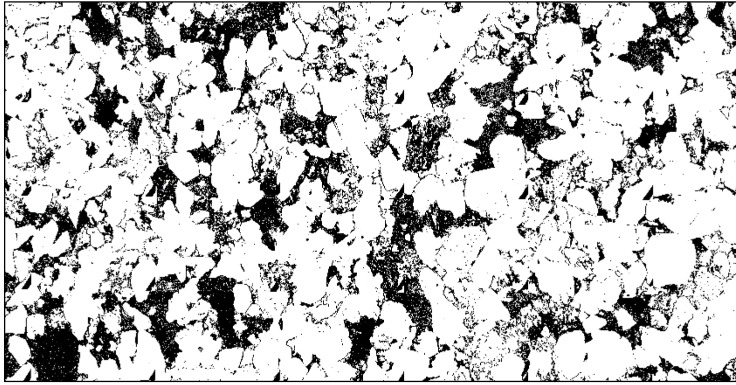




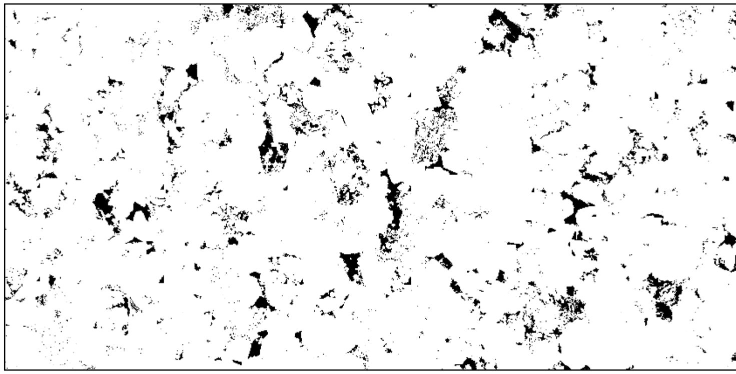
What is the core-scale connectivity of macropores and clay-associated micropores?



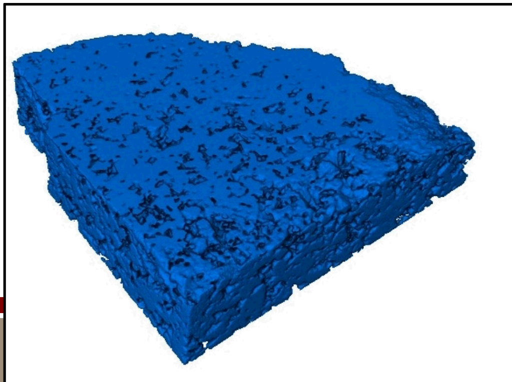




Clay-filled pores, 22.4% porosity

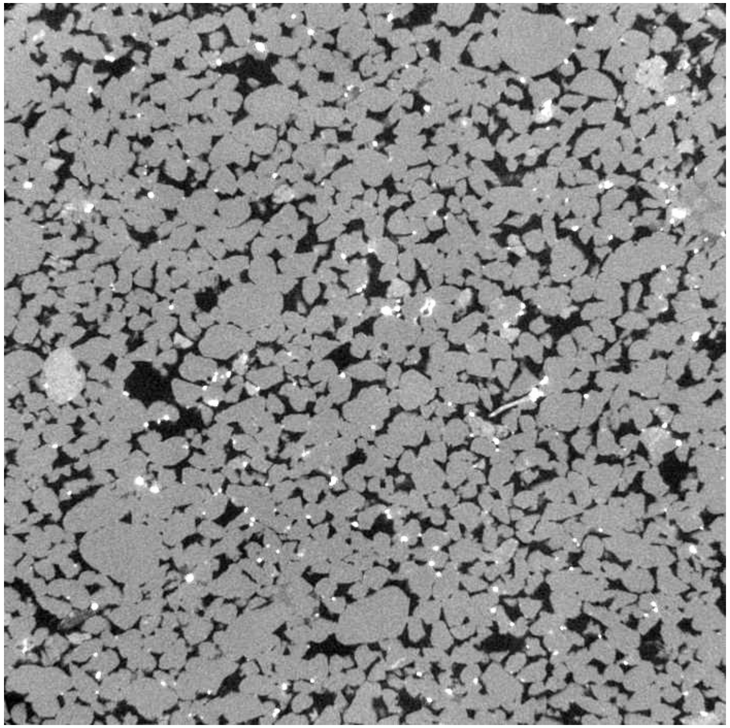


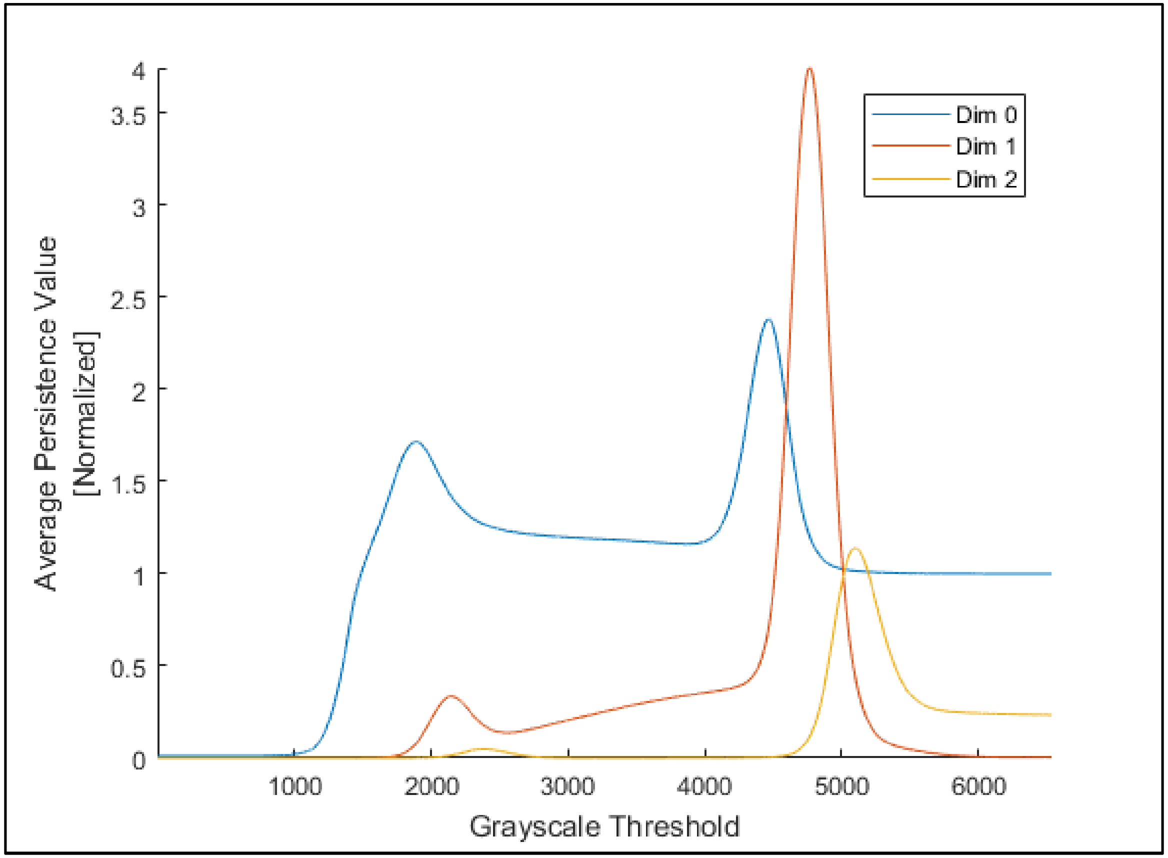
Macro-pores, 5.5 % porosity



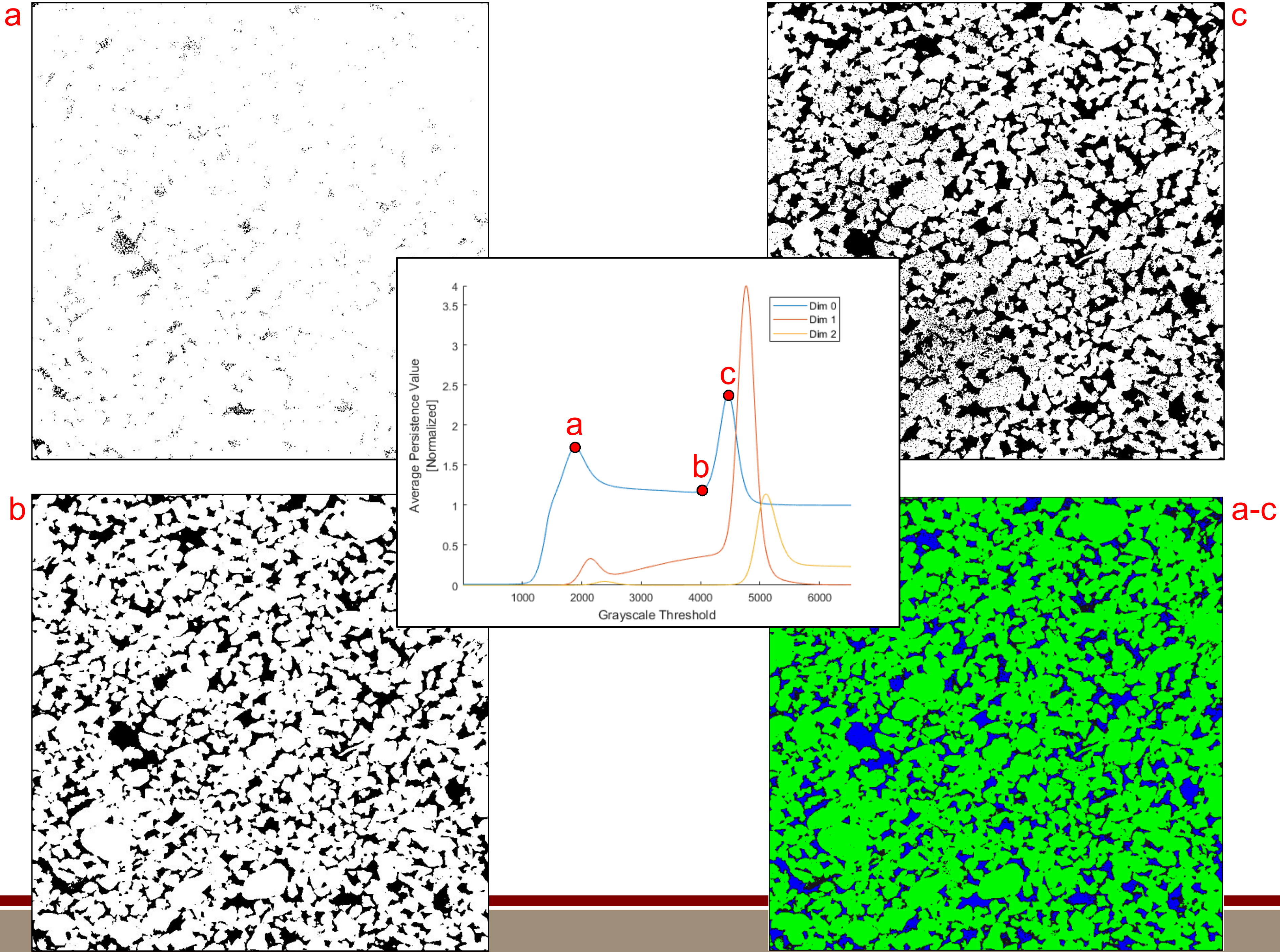
Average Persistence Value Curve

Starting with a clean sandstone



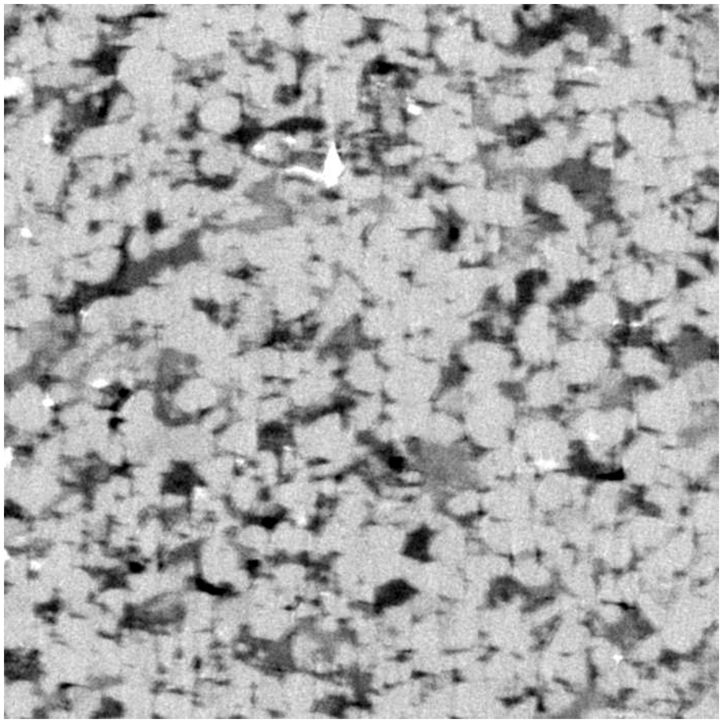


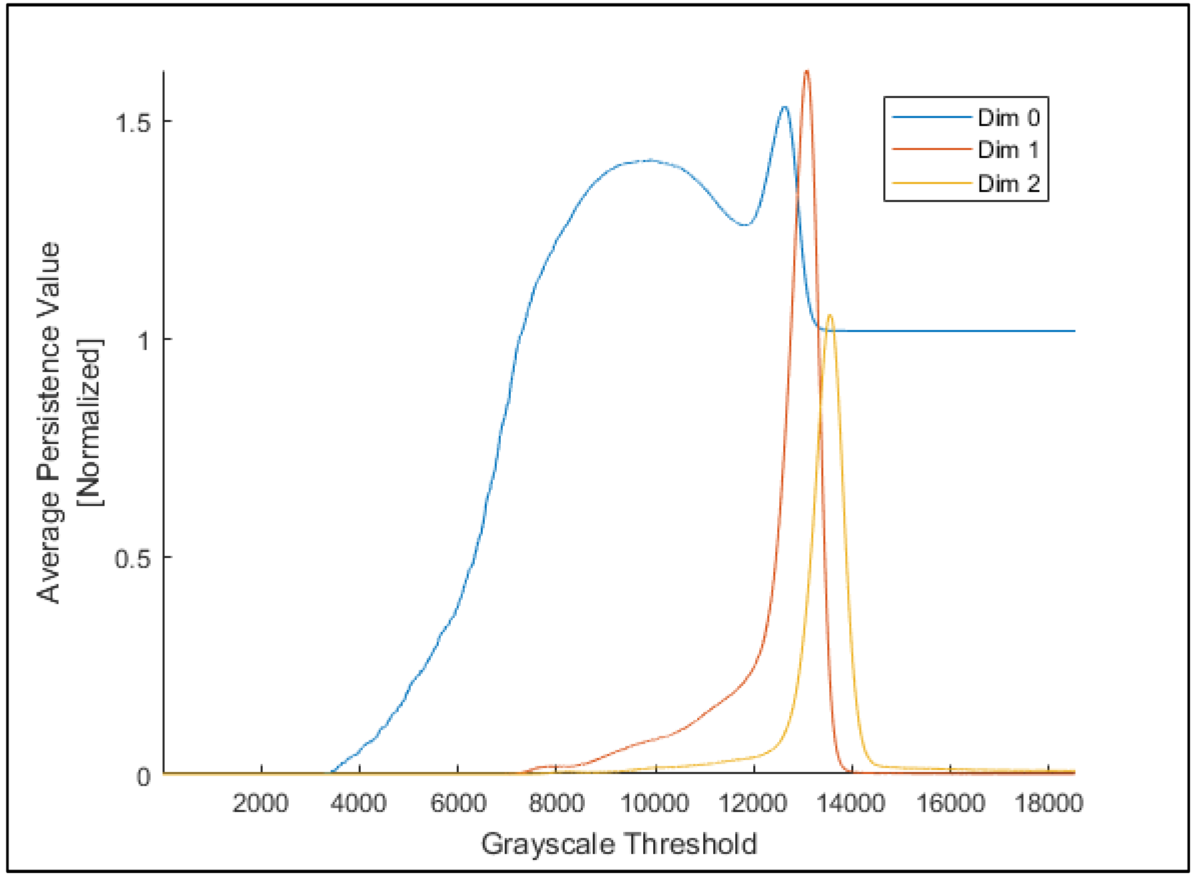
Histogram of grayscale values


a
b
c
a-c
Dim 0
Dim 1
Dim 2
Average Persistence Value [Normalized]
Grayscale Threshold
0
0.5
1
1.5
2
2.5
3
3.5
4
1000
2000
3000
4000
5000
6000

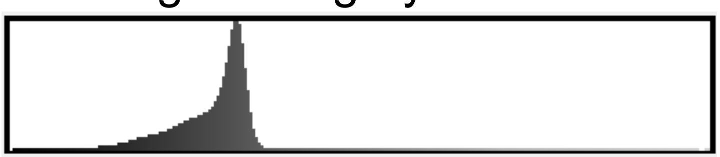
Average Persistence Value Curve

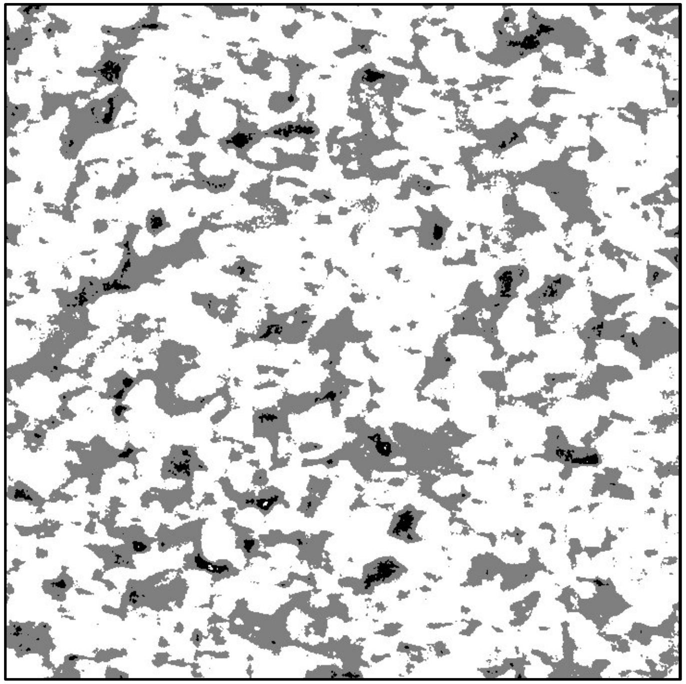
More difficult sample





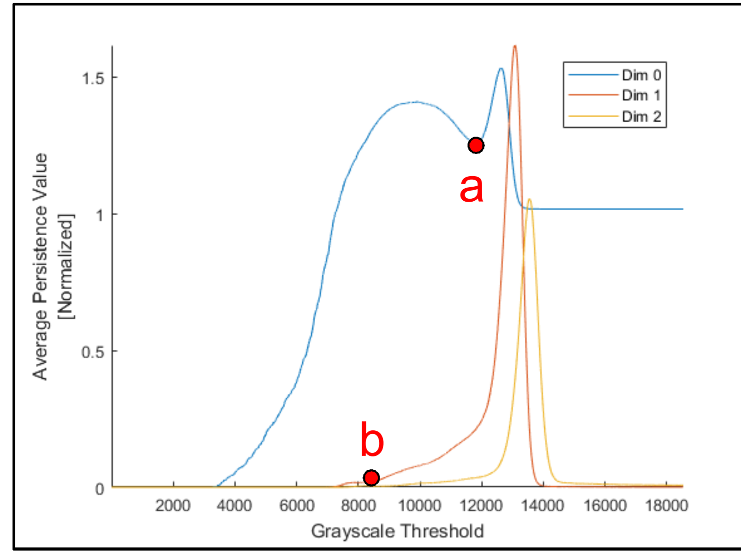
Histogram of grayscale values



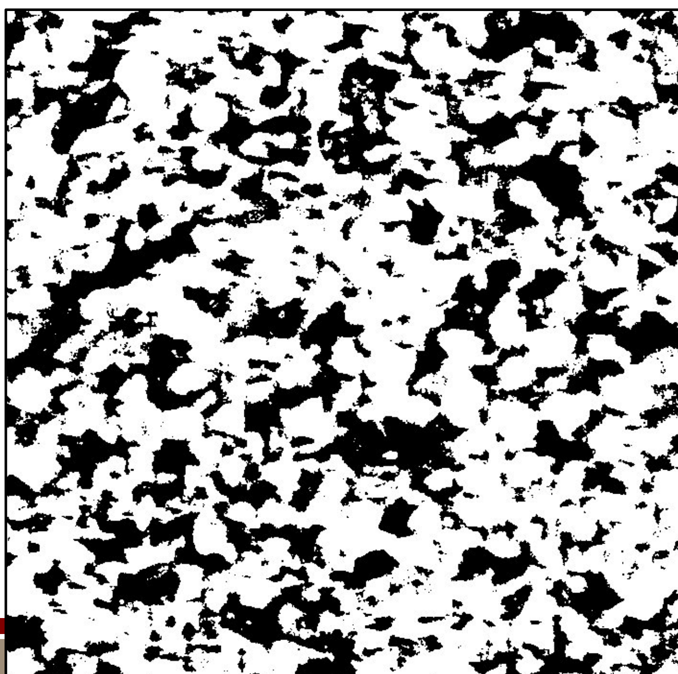


clay-grain interface

Direct sum of pics a and b



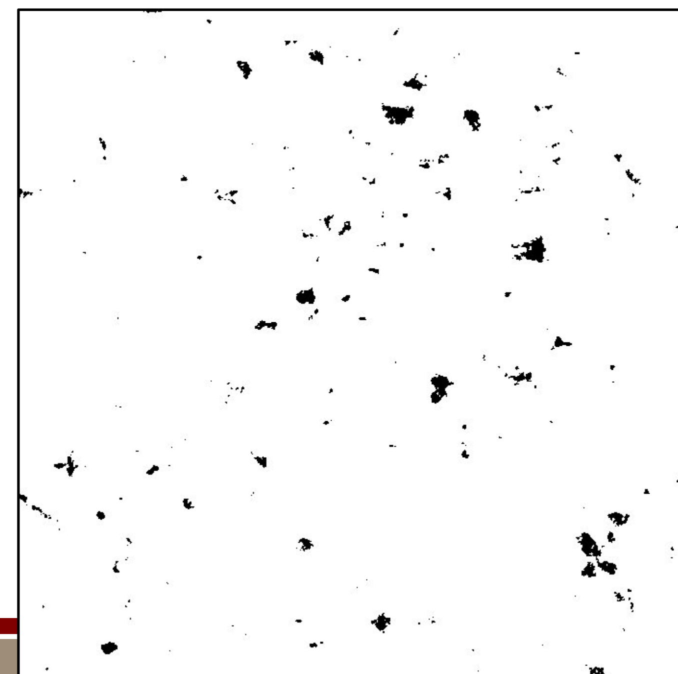
a



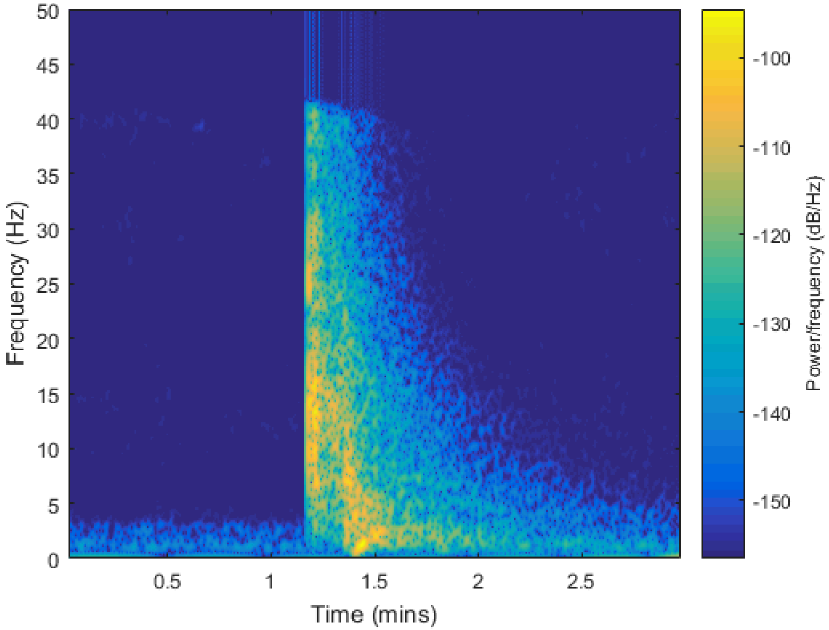
clay and pore

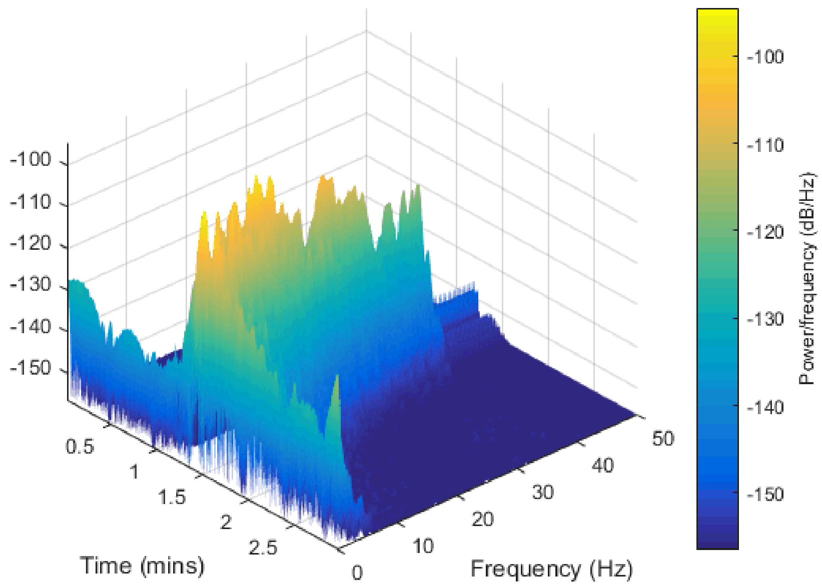
b

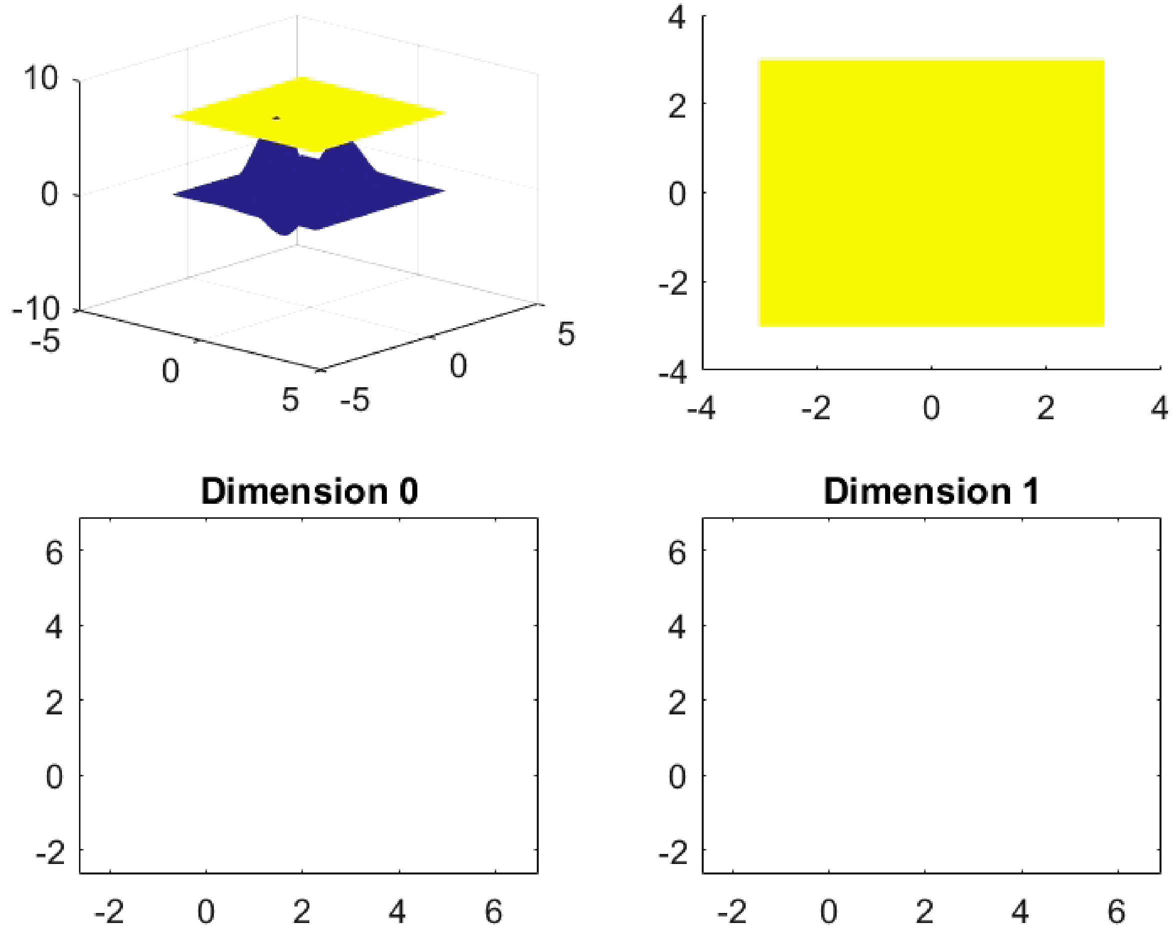
pore

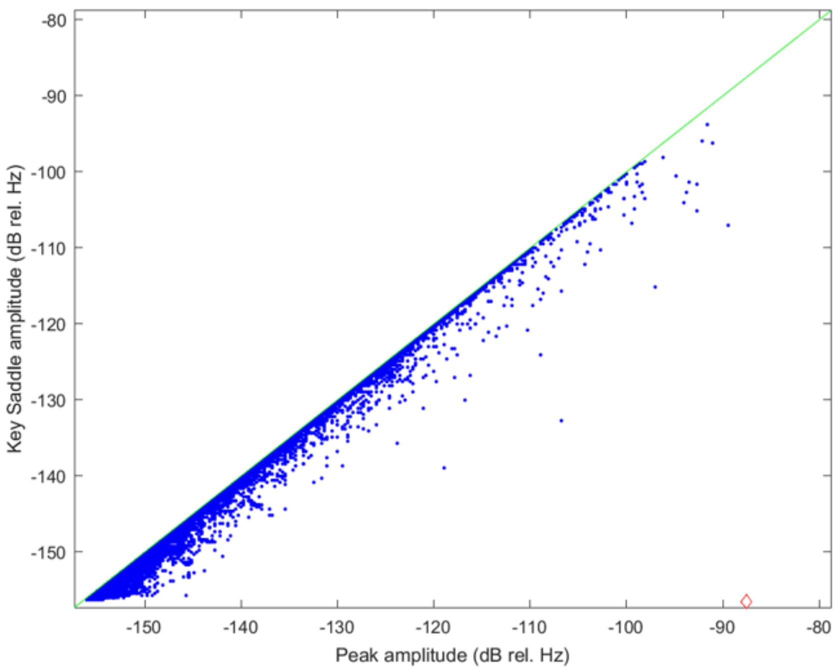


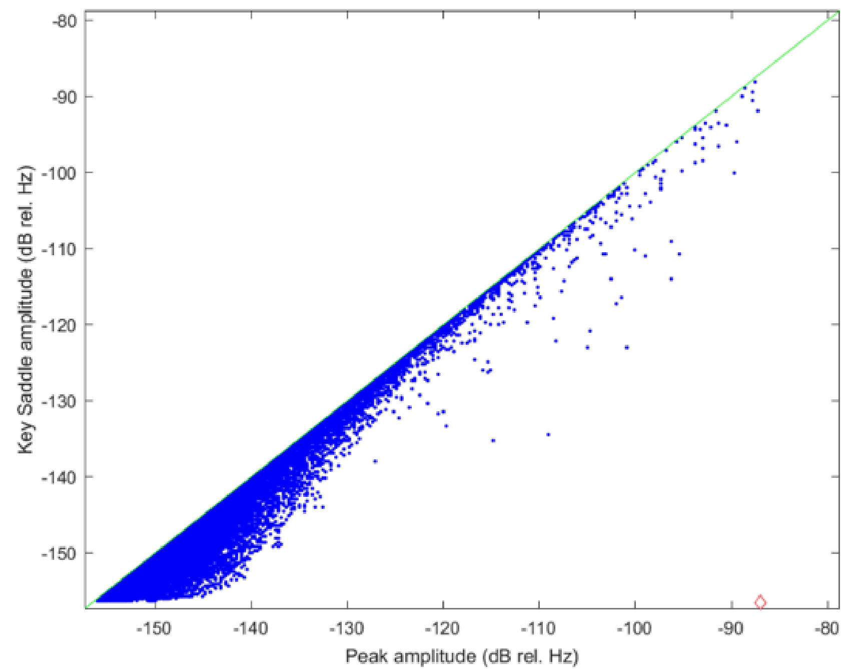
Persistent homology and seismic data






Dimension 0
Dimension 1


Key Saddle amplitude (dB rel. Hz)
Peak amplitude (dB rel. Hz)
-80
-90
-100
-110
-120
-130
-140
-150
-150
-140
-130
-120
-110
-100
-90
-80


Key Saddle amplitude (dB rel. Hz)
Peak amplitude (dB rel. Hz)
-80
-90
-100
-110
-120
-130
-140
-150
-150
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-120
-110
-100
-90
-80

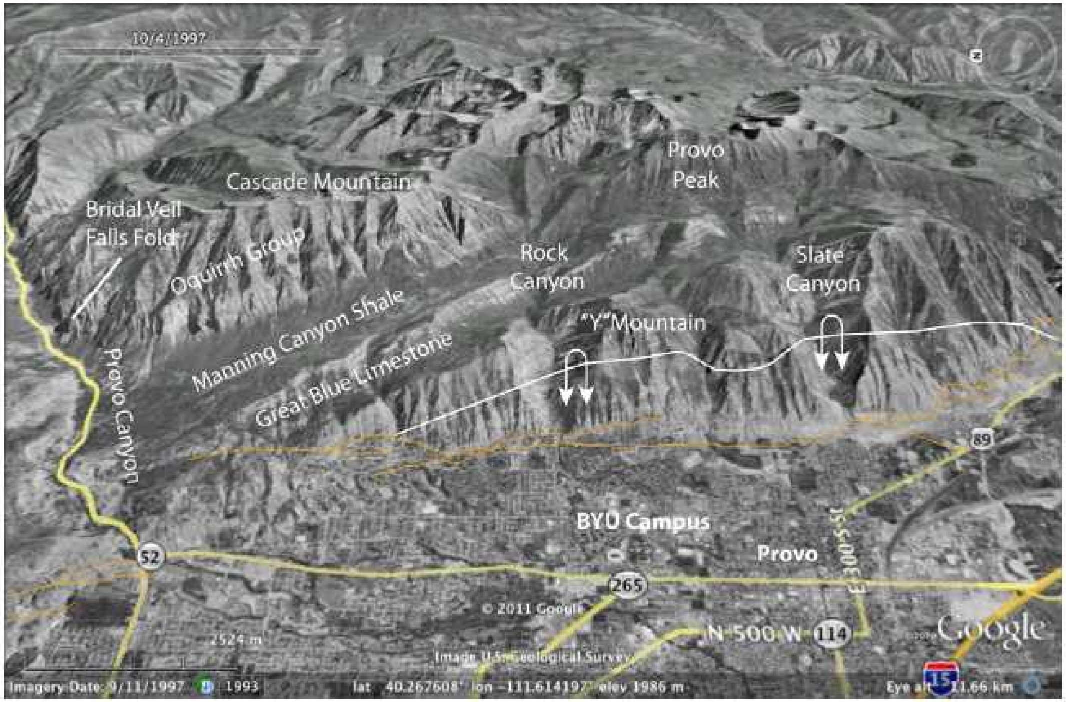
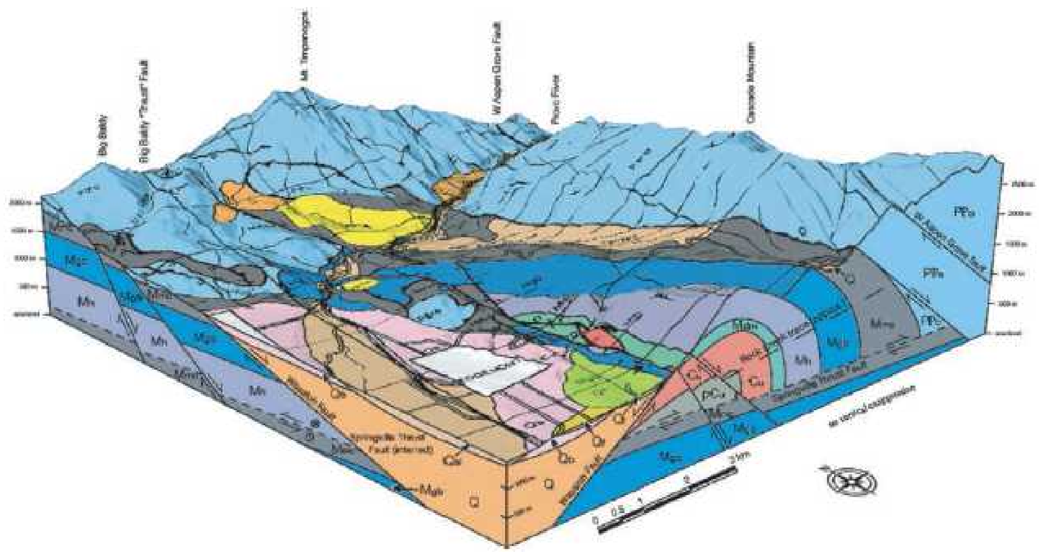


Figure 9. Location map of the "Y" Mountain anticline near Provo (oblique view looking east). Note plunge to the north and south, and strike valley of Manning Canyon Shale.



Backup slides

Processes for our macroscopic continuum-level description of flow & transport

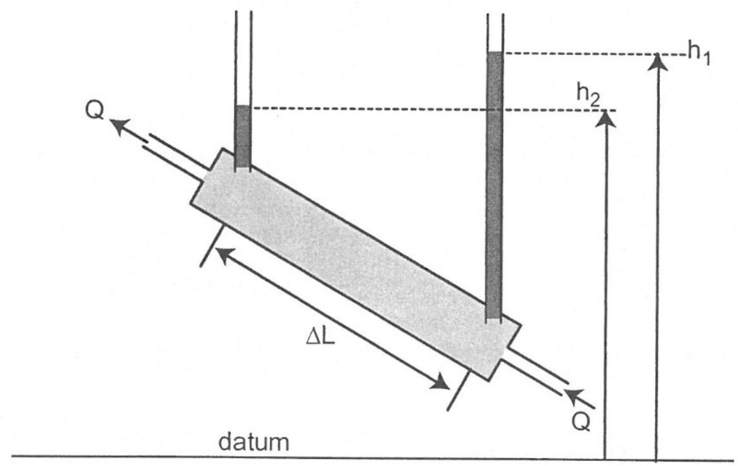
- Advection
- Hydrodynamic dispersion (molecular diffusion and mechanical dispersion)
- Chemical reactions (e.g., sorption, decay of a solute)

What is advection?
How is advection quantified?

$$K = \frac{k\rho g}{\mu}$$

$$Re < 10$$

Laminar flow in rocks



$$Q = -KA\frac{\Delta h}{\Delta L}$$

$$q = \frac{Q}{A}$$

$$v = \frac{q}{n_e}$$

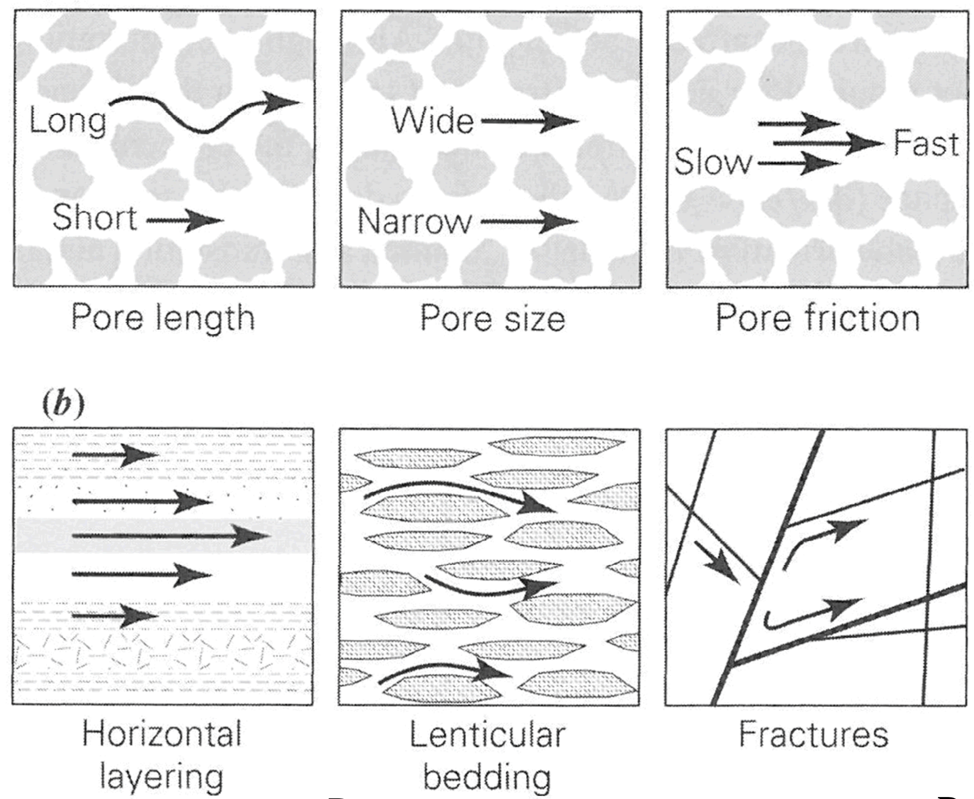
$$q_a = vC$$

$$\frac{\partial C}{\partial t} = -v_x \frac{\partial C}{\partial x}$$

1D advection equation

Mechanical dispersion - spreading

Heterogeneities at multiple scales create variance in the average linear velocity –
This results in an indirect transport process called mechanical dispersion
Mechanical dispersion acts to disperse chemical mass in groundwater, like molecular diffusion



Mechanical dispersion depends on

- Advection to operate
- Dominates at higher velocities
- Rate coefficient is generally treated as a second-order tensor

Combined effects of mechanical dispersion and molecular diffusion is called **<u>hydrodynamic dispersion</u>**

$$\overline{D_{ij}} = \alpha_{ijkl} \frac{v_k v_l}{|v|} + \frac{D_m}{n_e}$$

Assuming isotropic medium: $D_{xx} = D_L = \alpha_L v_x + \frac{D_m}{n_e}$ $\quad D_{yy} = D_T = \alpha_T v_x + \frac{D_m}{n_e}$

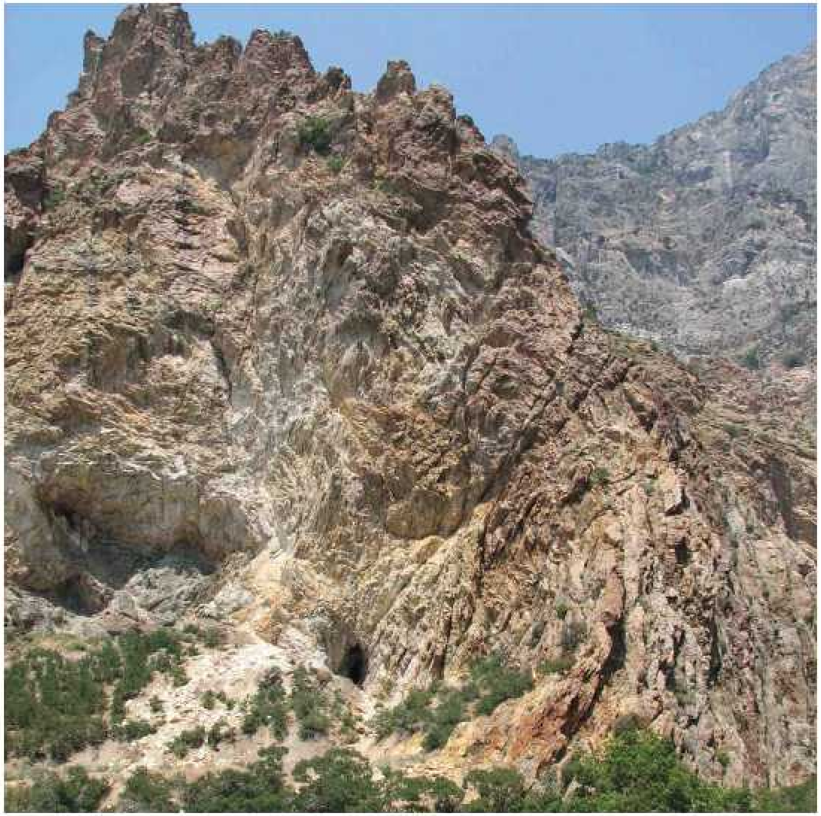


Figure 12. Hinge zone of the "Y" Mountain anticline in Rock Canyon looking east. Closely spaced bedding planes in the Tintic Quartzite change from near horizontal to vertical and overturned from left to right. In shade at far left are opposing thrust faults that bring tillite over quartzite.

Figure 13. Looking south along strike at the vertical to slightly overturned beds of the "Y" Mountain anticline. "Y" Mountain is right of center. Provo Peak (11,068 feet elevation) is on the far left. The strike valley of the Manning Canyon Shale is at the base of Provo Peak (see figure 9). Flexural slip along the weakest units moves structurally higher dip domains over lower ones, which accounts for the abrupt shallowing in dip across these horizons upsection (see explanation in Bridal Veil Fall fold).