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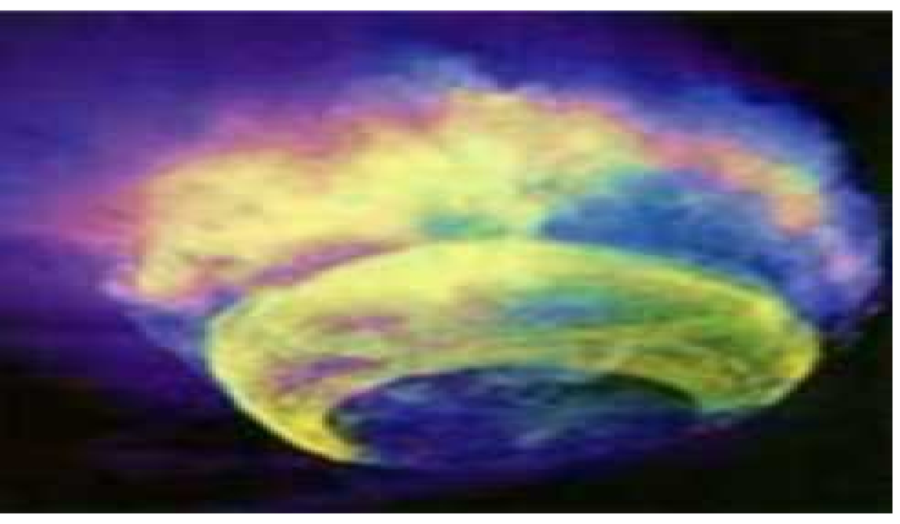

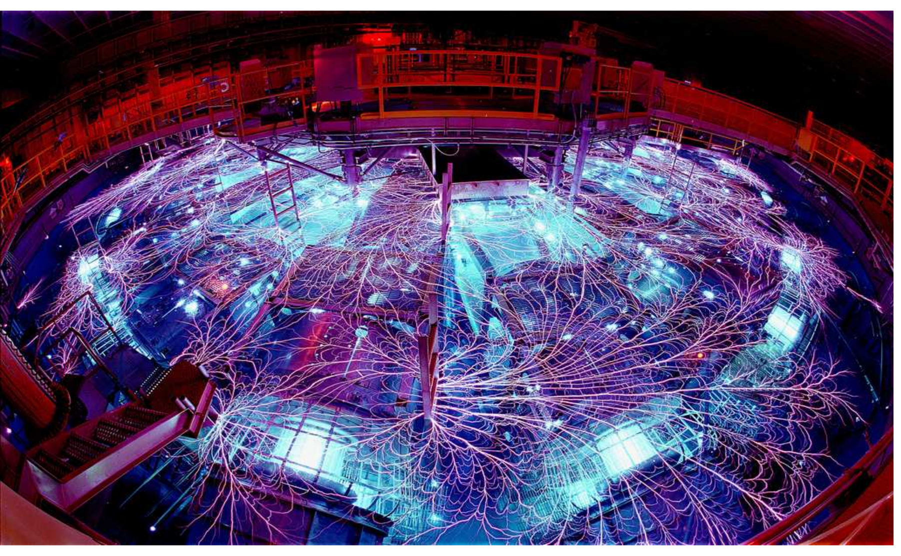

Stabilization of a Compatible Finite Element Plasma Discretization

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Problem Description

- We use a 5-moment multi-fluid plasma model (simplified to cold and collisionless).
- Compatible edge-face finite elements are used for the Maxwell equations
    - $\nabla \cdot \mathbf{B} = 0$ strongly
    - $\nabla \cdot (\varepsilon \mathbf{E}) = \rho_c$ weakly
- For simplicity, equal order nodal finite elements are used for all fluid DOFs.
- Application of interest
    - Electron plasma with ion background
    - Strong sources moving at the speed of light are applied to density and momentum.
    - An implicit time integrator resolves the EM time-scales.
    - Discretization of the large spatial scale of the problem makes the EM wave time-scale slow compared to the plasma frequency.

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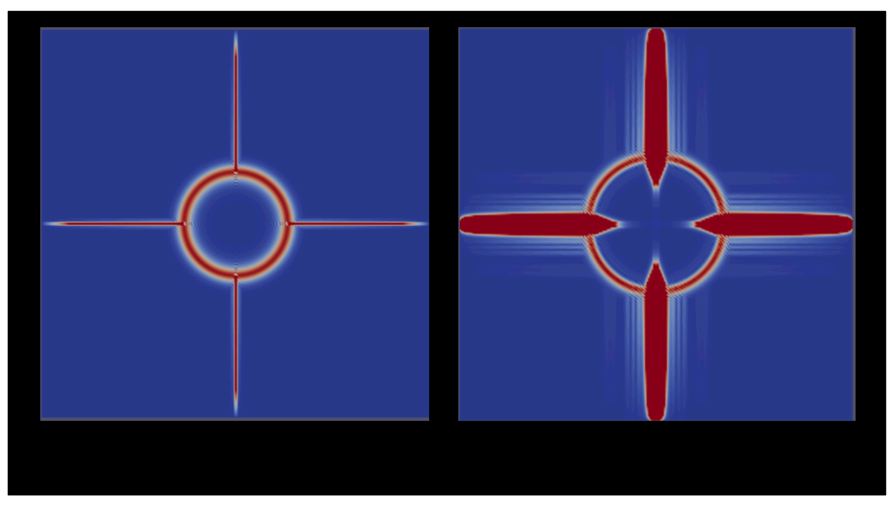
Edge-Node Instability



- Unphysical behavior occurs along the axes when this discretization is applied to the above problem.
- The effect was reproduced in a simplified formulation where only the momentum to E-field coupling was represented, suggesting that the **instability is due to the incompatibility of nodal and edge elements in capturing the plasma oscillation coupling.**

- The fully discrete plasma oscillation system contains only mass matrix operators.

$$\underbrace{\begin{pmatrix} \frac{1}{\Delta t} Q_n & -\omega_p Q_e^n \\ \omega_p Q_n^e & \frac{1}{\Delta t} Q_e \end{pmatrix}}_{} \begin{pmatrix} \rho \mathbf{u} \\ \mathbf{E} \end{pmatrix} = \begin{pmatrix} R_{\rho \mathbf{u}} \\ R_{\mathbf{E}} \end{pmatrix}$$

$$= \begin{pmatrix} I & -\Delta t \omega_p Q_e^n Q_e^{-1} \\ 0 & I \end{pmatrix} \begin{pmatrix} S_n & 0 \\ \omega_p Q_n^e & \frac{1}{\Delta t} Q_e \end{pmatrix}$$

- The Schur complement S_n discretely captures the momentum to E coupling in one operator.

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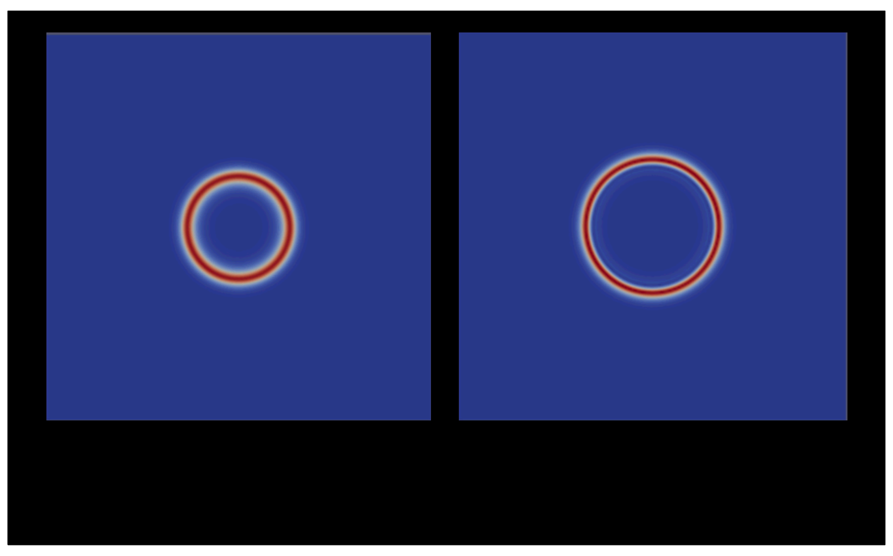
- But a continuous analysis of the PDEs followed by discretization results in a mass operator $\left(\frac{1}{\Delta t} + \Delta t \omega_p^2\right) Q_n$
- It can be shown that $Q_e^n Q_e^{-1} Q_n^e \neq Q_n$ and, hence, that **the Schur complement is an incompatible representation of plasma oscillation effects**.

Stabilization Scheme

- We stabilize the system by adding a new DOF, $\hat{\rho \mathbf{u}}$, to correct the Schur complement to be consistent.

$$\begin{pmatrix} Q_e & -Q_n^e & 0 \\ -\Delta t \omega_p^2 Q_e^n & \left(\frac{1}{\Delta t} + \Delta t \omega_p^2\right) Q_n & -\omega_p Q_e^n \\ 0 & \omega_p Q_n^e & \frac{1}{\Delta t} Q_e \end{pmatrix} \begin{pmatrix} \hat{\rho \mathbf{u}} \\ \rho \mathbf{u} \\ \mathbf{E} \end{pmatrix} = \begin{pmatrix} 0 \\ R_{\rho \mathbf{u}} \\ R_{\mathbf{E}} \end{pmatrix}$$



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- $\hat{\rho \mathbf{u}}$ is an edge projection of $\rho \mathbf{u}$ and converges to $\rho \mathbf{u}$ as the mesh is refined.
- The method extends directly to multi-fluid cases.
- Future work includes elimination of extra constraint, impact of additional spatial dissipation, and evaluation on more challenging problems.

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