Preconditioning Strategies for Physics-Conforming Implicit Electromagnetics and Plasma Simulations

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Abstract

We have developed **a new block preconditioning strategy for a compatible mixed finite element discretization of the Maxwell equations**. This method has been designed to perform well when large time-steps are taken with respect to the speed of light. We have demonstrated the scalability, robustness, and efficiency of this approach on electromagnetics problems involving multiple time-scales. We have also incorporated the solver as a component within preconditioners for multi-fluid continuum plasma models.

Motivation

$$\varepsilon \frac{\partial E}{\partial t} - \nabla \times \frac{1}{\mu} B = -J$$
$$\nabla \cdot (\varepsilon E) = \rho_c$$
$$\frac{\partial B}{\partial t} + \nabla \times E = 0$$
$$\nabla \cdot B = 0$$

Electromagnetic phenomena governed by the Maxwell equations involve speed of light ($c = \sqrt{\frac{1}{\varepsilon\mu}}$) time-scales, requiring $CFL_c = c\frac{\Delta t}{\Delta x}$ less than one for explicit time integration. When other physics are coupled to electromagnetics, such as in plasma simulations, **time-scales of interest may be much slower than the speed of light.** Implicit time integration can efficient model relevant physics, but requires **scalable linear solvers that are robust to large time-steps.**

Approach

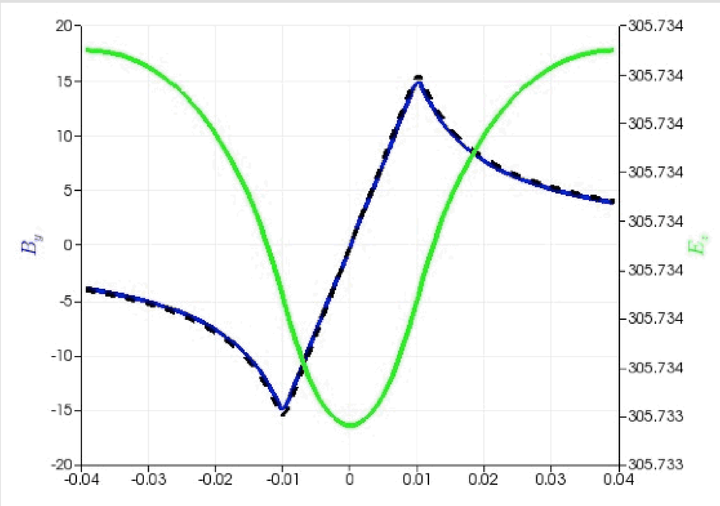
$$\underbrace{\begin{pmatrix} Q_B & K \\ -\hat{K}^t & Q_E \end{pmatrix}}\begin{pmatrix} B \\ E \end{pmatrix} = R$$
$$= \begin{pmatrix} I & 0 \\ -\hat{K}^t Q_B^{-1} & I \end{pmatrix}\begin{pmatrix} Q_B & K \\ 0 & S_E \end{pmatrix}$$
$$S_E = Q_E + \hat{K}^t Q_B^{-1} K$$

Employing and edge-face discretization for E and B implicitly enforces $\nabla \cdot B = 0$, and results in block-structured linear systems. Block preconditioners **employ multilevel methods tailored to different discretization types** for scalability. The Schur complement S_E is close to singular for large values of CFL_c. Based on properties of the discretization, we have been able to augment S_E to complete its kernel in a consistent way. This allows traditional multilevel methods to apply, such that our preconditioner **requires only block manipulation and application of traditional AMG to symmetric positive-definite operators.**

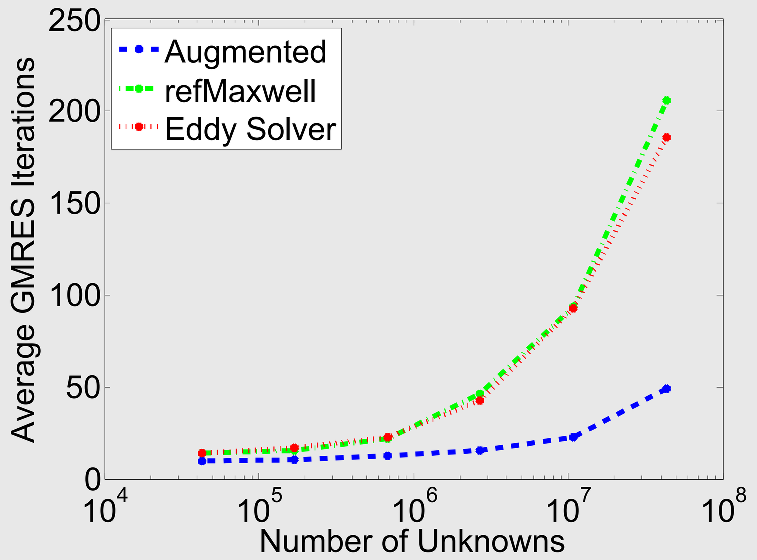
Results

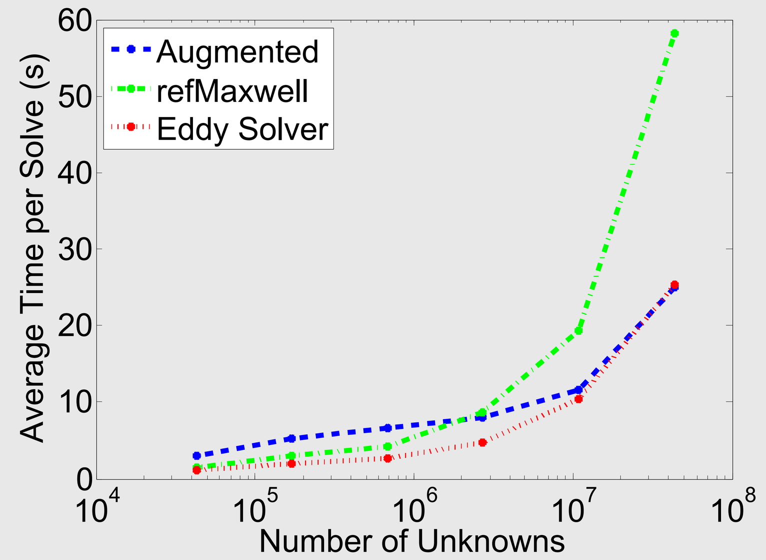
Wire in vacuum - current defined by Ohm's law ($J = \sigma E$)

We have recovered the long-time MHD limit of this problem without requiring a conductivity floor in the void.

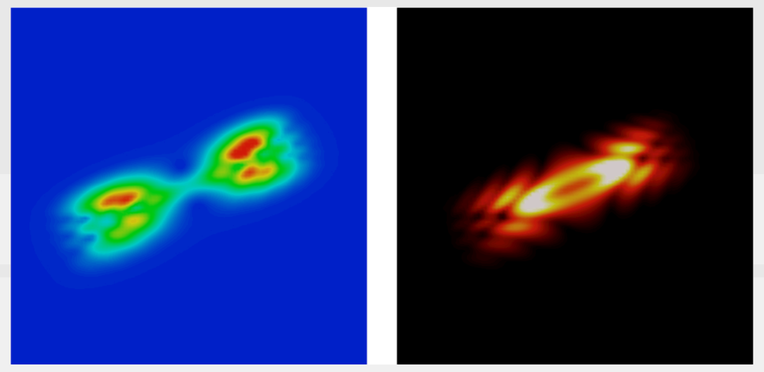


We have demonstrated **algorithmic scalability with large CFL_c** (up to 6,400 here), comparing well against state-of-the-art edge-based AMG routines applied to S_E.





Electron-ion plasma driven by an external current



Our approach to long time-scale electromagnetics has been **incorporated into a block preconditioner for multi-fluid plasma systems**. This preconditioner segregates the electromagnetics from the fluid variables and uses our augmented approach to solve for E and B.

Preliminary results have demonstrated **scalable performance while following slow time-scale of interest.** Performance can be improved by tuning sub-solve settings based on knowledge about physical time-scales.

Procs	DOFs	Avg Its	Time (s)
1	2.6e4	16.26	0.7833
4	1.0e5	16.45	1.031
16	4.0e5	17.95	2.141
64	1.3e6	27.74	3.923
256	6.3e6	32.68	5.078

Time-scale	CFL max
Light Wave	2.0e+1
Advection (e)	1.3e-2
Advection (i)	1.3e-5
Plasma Freq (e)	2.6e+1
Plasma Freq (i)	8.1e-1
Cyclotron Freq (e)	1.3e+1
Cyclotron Freq (i)	1.3e-2

Conclusions and Future Work

Our block preconditioning approach to the Maxwell equations, based on augmenting the Schur complement so that traditional multilevel methods apply, shows promising results on long time-scale problems of interest. This approach is actively used to enable the scalable electromagnetics solves in a DOE-funded particle-in-cell plasma application code at Sandia.

Preliminary incorporation of this approach into a coupled continuum plasma solver is promising. We plan to continue improving the coupled plasma solver for use with continuum and hybrid plasma application codes at Sandia. This will enable the effective application of implicit and IMEX time-integrators to plasma applications of interest to the DOE.

Areas in which we can help

Research involving plasma simulations and other applications including electromagnetics coupled to slower time-scale physics can benefit from this work. We provide a means to achieve implicit representation of fast EM physics in a scalable way while requiring only access to traditional multigrid technologies. New ASCR/OFES SciDAC and OFES plasma efforts would strongly benefit from collaboration on these topics.

Areas in which we need help

The preconditioner generally relies on expensive ILU type smoothers, and could benefit from research on lighter weight yet effective smoothers, such as Krylov smoothing approaches.

References

[1] Phillips, Shadid, and Cyr, Scalabale Preconditioners for Structure Preserving Discretizations of Maxwell Equations in First Order Form, Submitted to *SISC*, 2017.

[2] Phillips, Shadid, Cyr, and Pawlowski, Fast Linear Solvers for Multifluid Continuum Plasma Simulations, *NECDC* extended abstract/poster, 2016.

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