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Time Domain Parameter Estimation in X-Ray Phase Contrast Imaging

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Summary

We show the advantage of reconstructing X-ray phase contrast imaging data in the time domain rather than the more common Fourier domain. Reconstruction in the time domain provides an opportunity to investigate the reliability of image products and the errors that arise in the data as a result of system noise, spurious signals, or imperfect grating stepping. The estimators used for the signal variables are more robust to sources of error than in Fourier domain processing. Furthermore, time domain analysis enables access to image products that can aid in other aspects of the image processing such as phase unwrapping.

Image Acquisition and Signal Model

In grating-based x-ray phase contrast imaging (XPCI), data are collected by making two sets of measurements:

1- a reference set with only the gratings in the beam path
2- a measurement set with the sample (and gratings)

The generic signal model for the data is give by:

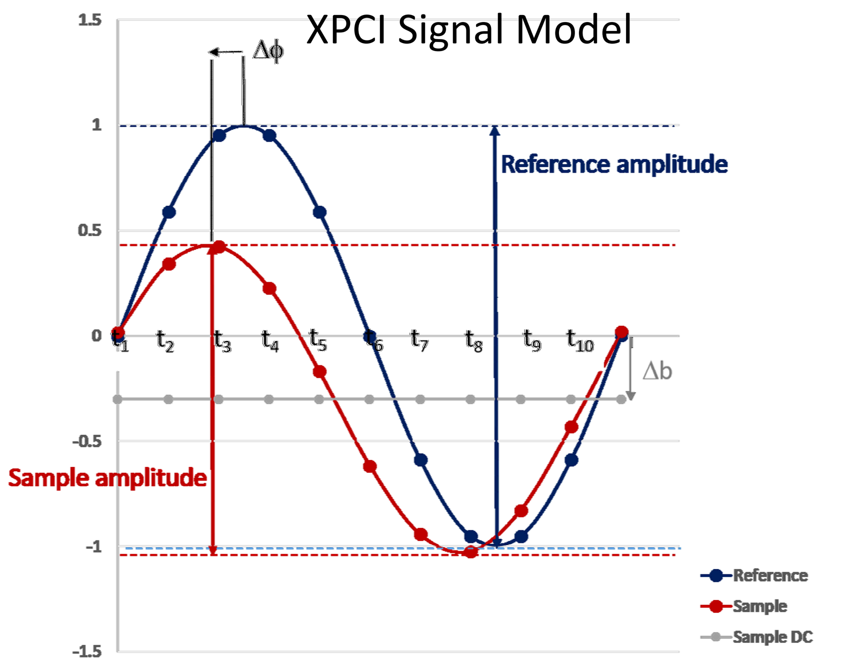
$$\mathrm{d}(n,m,t_l) = A_{nm}\cos(\omega t_l + \phi_{nm}) + b_{nm}$$

n, m : spatial indices of the detector $N \times M$ pixels
t_l : temporal variable of data acquisition
A : sinusoidal amplitude (for each spatial location)
ϕ : phase shift (for each spatial location)
b : bias (for each spatial location)

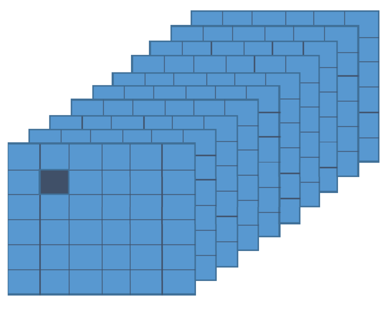
Change in the fringe pattern between the reference and sample enable characterization of the sample:

A : how much X-ray is scattered by the sample
ϕ : how much the phase is altered by the sample
b : absorption by the sample

The periodic nature of the signal makes Fourier methods a common approach for image reconstruction [1-3]. However, the batch processing intrinsic to Fourier methods make no allowance for spurious, or otherwise corrupted, measured values.



- Example of a 10 image stack
- Images are acquired at locations of filled circles, marked t_1 through t_{10}
- ω and t_l are such that one full cycle of the sinusoid is captured during image acquisition
- No image is acquired at the final, empty circle
- Reference and Sample images are taken at the same location(s), e.g. t_3 for the sample is acquired at the same transverse location as image x_3 for the reference
- The sine wave is sampled at every pixel location on the detector



Fourier Domain Processing

The forward discrete-time Fourier transform (DFT) is computed on the signal $x(n)$ as:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{j\left(\frac{2\pi}{N}\right)nk}$$

The first Fourier coefficient ($k = 0$) is:

$$X(k=0) = \sum_{n=0}^{N-1} x(n)$$

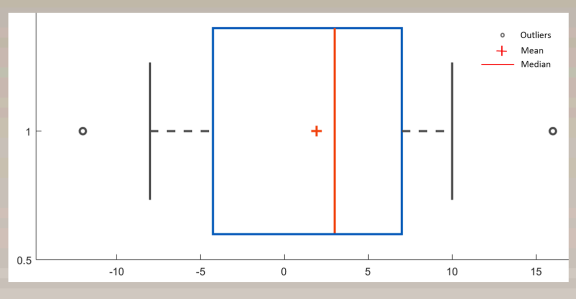
Estimation of DC offset: absorption

Computing the sample mean of $x(n)$ gives the same value as the first coefficient of the DFT (up to a deterministic scaling).
Sample-wise estimate of b with respect to t_l for spatial position (n, m):

$$\bar{b}(t_l) = d(n,m,t_l)$$

The sample-mean estimate of b (up to a deterministic scaling) is:

$$\hat{b}_\mu = \frac{1}{L}\sum_{t_l=0}^{L-1}\bar{b}(t_l)$$



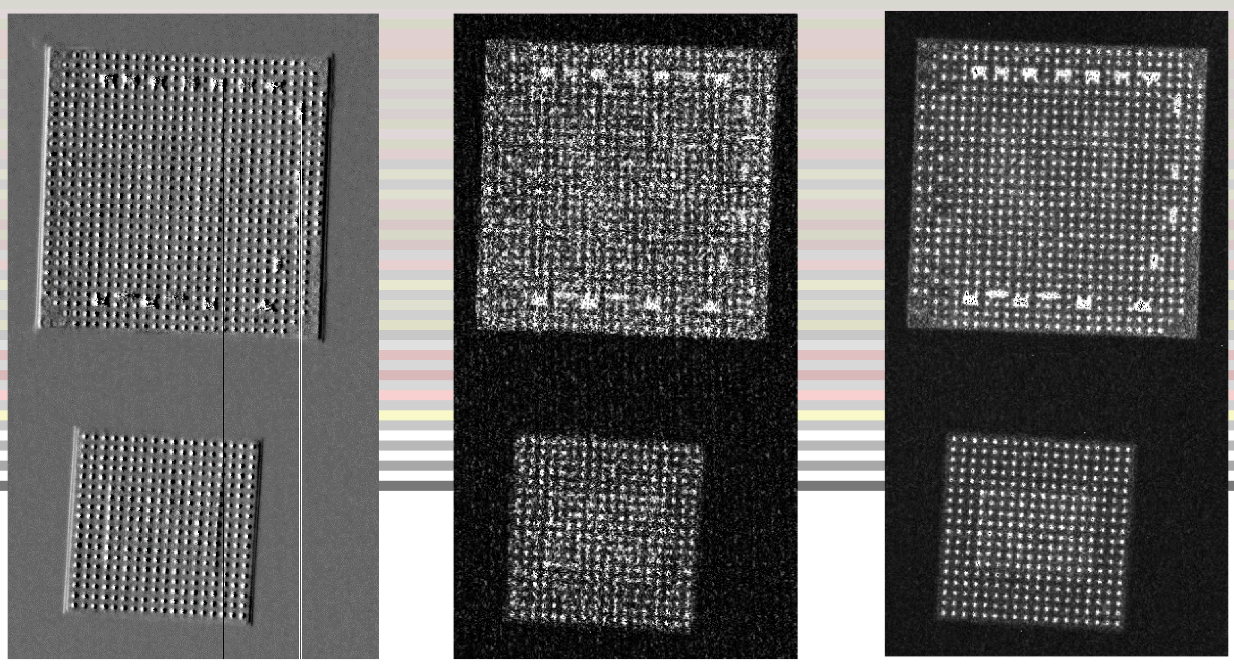
- Trimmed mean:

$$\hat{b}_{\mu_{trim}} = \frac{1}{t'_{l,high} - t'_{l,low} + 1}\sum_{t'_l = t'_{l,low}}^{t'_{l,high}} Sort\{\bar{b}(t_l)\}(t'_l)$$

- Median estimate of mean:

$$\hat{b}_M = Median\{\bar{b}(t_l)\}$$

- The uncertainty of the estimate can be quantified by finding the standard deviation, with respect to t_l, which gives an indication of the spread of the estimate of b.



Left: Fourier domain processed differential phase image product: streaks are columns lost from unsuccessful phase unwrapping with a 1D algorithm.
Center and Right: Standard deviation of the differential phase estimates. An image product accessible by time-domain processing, the image represents regions where there is a large standard deviation amongst the differential phase estimates. The standard deviation image can be thresholded to create either a weighted or binary mask for phase unwrapping algorithms. Center: Threshold = 0.75. Right: residual linear term of differential phase estimate, threshold = 0.05.

Estimation of phase

A sample-wise estimate of $\Phi = \omega t_l + \phi$ with respect to t_l for spatial position (n, m):

$$\bar{\Phi}_{t_l} = \angle d_C(n,m,t_l)$$

- $\bar{\Phi}(t_l)$ represents a line with an offset
- $\bar{\Phi}_{t_l}$ may have a 2π discontinuity due to phase wrapping, must unwrap before computing statistics, redefine $\bar{\Phi}_{t_l}$:

$$\bar{\Phi}_{t_l} = \mathrm{Unwrap}\{\angle d_C(n,m,t_l)\}.$$

For creating a dP image it isn't necessary to extract ω and ϕ from $\bar{\Phi}$, since only $\Delta\phi = \Phi_1 - \Phi_0 = \phi_1 - \phi_0$ is needed

For the dP image, $\Delta\hat{\phi}$ is estimated by computing the statistical measures as follows:

- Mean:

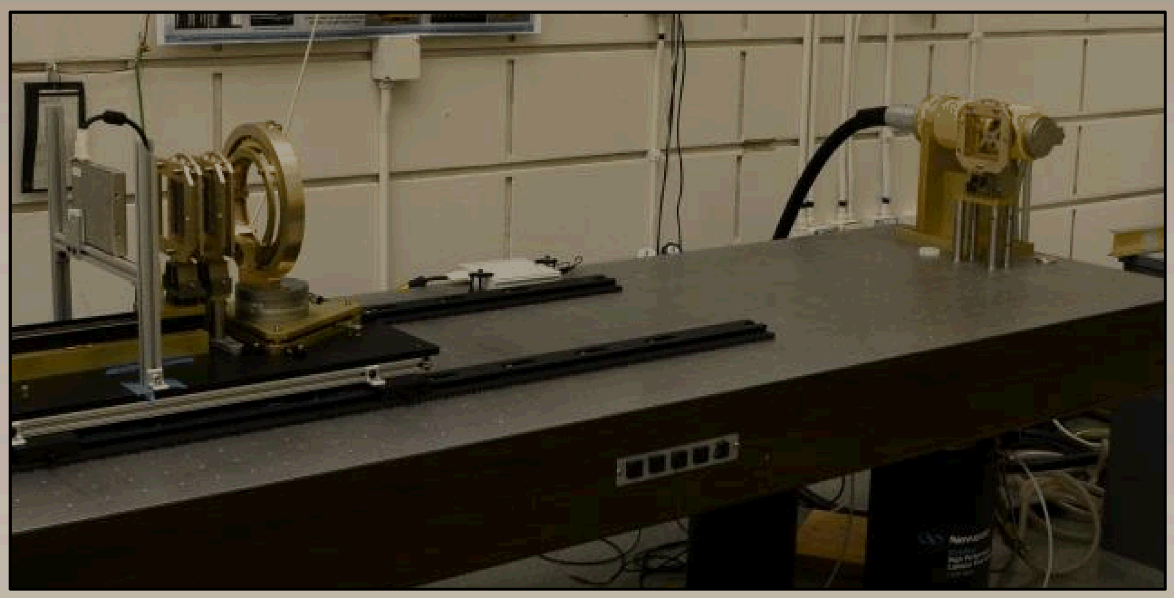
$$\Delta\hat{\phi}_\mu = \frac{1}{L}\sum_{t_l=0}^{L-1}\left(\bar{\Phi}_1(t_l) - \bar{\Phi}_0(t_l)\right)$$

- Trimmed Mean:

$$\hat{\phi}_{\mu_{trim}} = \frac{1}{t'_{l,high} - t'_{l,low} + 1}\sum_{t'_l = t'_{l,low}}^{t'_{l,high}} Sort\{\bar{\Phi}_1(t_l) - \bar{\Phi}_0(t_l)\}(t'_l)$$

- Median:

$$\hat{\phi}_M = Median\{\bar{\Phi}_1(t_l) - \bar{\Phi}_0(t_l)\}$$



Estimation of amplitude: scattering

If the model was a complex sinusoid (assuming zero mean):

$$d_C(n,m,t_l) = A_{nm}e^{i(\omega t_l + \phi_{nm})}$$
$$= A_{nm}\cos(\omega t_l + \phi_{nm}) + iA_{nm}sim(\omega t_l + \phi_{nm})$$

A : magnitude of the complex-valued samples
$\Phi = \omega t_l + \phi$: extracted from the argument of each sample

- Sine function is $L/4$ samples out of phase with the cosine function
- Generate a pseudo-complex-valued sinusoid by cyclically rotating t_l by $L/4$ samples (denote this operation by $([t_l])_{L/4}$)

Zero-mean (remove $\hat{b}$) pseudo-complex-valued signal model is given by:

$$d_C(n,m,t_l) = \left(d(n,m,t_l) - \hat{b}\right) + i\left(d(n,m,([t_l])_{L/4} - \hat{b}\right)$$

The sample-wise estimate of A with respect to t_l becomes:

$$\bar{A}(t_l) = |d_C(n,m,t_l)|$$

Similar to before, alternative estimates of A can be computed :

- Sample mean

$$\hat{A}_\mu = \frac{1}{L}\sum_{t_l=0}^{L-1}\bar{A}(t_l)$$

- Trimmed mean

$$\hat{A}_{\mu_{trim}} = \frac{1}{t'_{l,high} - t'_{l,low} + 1}\sum_{t'_l = t'_{l,low}}^{t'_{l,high}} Sort\{\bar{A}(t_l)\}(t'_l)$$

- Median

$$\hat{A}_M = Median\{\bar{A}(t_l)\}$$

Benefits of time-domain processing

- The same image products can be generated with time-domain processing as with Fourier techniques
- The signal model variables, A, ϕ, and b, are truly statistical measures and can be estimated by different estimators
- Choice of estimator (eg mean, trimmed mean, or median estimators) enables more robustness to outliers or spurious data
- Additional statistics (e.g. standard deviation) can be computed for a measure of reliability of the estimates and associated image products
- The additional statistical measures can be used to assist further image processing
  - For example, a by-product of forming time-domain based dP images is the residual linear term. The linear phase term cancels unless the signal strength is low. Then the signal model does not hold and the linear phase term degenerates to system noise in the detector. This leaves a non-trivial residual linear term in the differential phase estimate, which can be used to form an image. A mask can also be generated from this image to help mask out problematic regions for phase unwrapping algorithms.

References

[1] Harasse, S., et. al. (2012) In: *International Workshop on X-ray and Neutron Phase Imaging with Gratings*, Tokyo, Japan 2012. AIP Conference Proceedings, p. 155.
[2] Bevins, N. B., et. al. (2012) In: *International Workshop on X-ray and Neutron Phase Imaging with Gratings*, Tokyo, Japan 2012. AIP Conference Proceedings, p. 169-174.
[3] Bech, M., et. al., *Physics in Medicine and Biology*, 55(18), 5529–39 (2010).

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