Two-qudit deterministic optical quantum logic in a single photon

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Abstract: We demonstrate deterministic two-qudit gates using the time and frequency degrees of freedom of a single photon, showing the potential of our scheme for deterministic quantum computing in high-dimensional Hilbert spaces.

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Optical quantum gates have been demonstrated in multiple photonic degrees of freedom, such as polarization, orbital angular momentum, time, and frequency. Compared to other platforms, optical states have the advantages of close-to-no decoherence and suitability for long-distance communications. However, two-qubit gates suffer from being probabilistic with standard linear optics and photon counting [1]. To overcome this issue, encoding qubits in different degrees of freedom (DoFs) in a single photon has been demonstrated, where each DoF carries one qubit and the interactions between different qubits can be made deterministic [2,3]. Even though in these demonstrations two and three-qubit operations can be executed with unity success probability, each DoF contains only one qubit, and the number of DOFs in a photon is limited; thus, the size of the Hilbert space in which these deterministic transformations can occur is fairly moderate.

To overcome the issue of a limited Hilbert space, we take advantage of the high dimensionality present in two particular single-photon DoFs—namely, time and frequency, which are both compatible with fiber optical routing—to encode one *qudit* per DoF, where a qudit is a high-dimensional ($d > 2$) unit of quantum information. We consider multiple time bins and frequency bins; as long as the frequency spacing between different modes (Δf) and the time-bin spacing (Δt) are chosen such that they far exceed the Heisenberg limit (i.e., $\Delta f \Delta t \gg 1$), we are able to manipulate the time and frequency DoFs independently in a hyper-encoding fashion, using concepts developed in time-division and wavelength-division multiplexing, respectively. Since our single photons can potentially be generated in a superposition of many time and frequency bins, multiple qubits can be encoded in each DoF, making our proposed scheme a favorable platform for deterministic optical quantum computing algorithms on large Hilbert spaces.

First, we experimentally realize a generalized three-dimensional Pauli-X gate (cyclic shift) in the time domain as a building block for single-qudit operations, defined on time-bin eigenstates as $X|n\rangle = |n \oplus 1\rangle$. We prepare three time bin states $\{|0\rangle_t, |1\rangle_t, |2\rangle_t\}$—3 ns wide each and 6 ns apart from each other—by carving out a continuous wave laser with an intensity modulator, and manipulate their relative phases with a phase modulator. To perform the X operation, we separate the time bins $|0\rangle_t$ and $|1\rangle_t$ from time bin $|2\rangle_t$, and delay the first route by 3 bins (18 ns) using a balanced 1×2 Mach-Zehnder modulator (MZM) switch. After the path-dependent delay, another 1×2 MZM (operating in the reverse direction) can recombine the time bins in a deterministic fashion, although in this proof-of-principle experiment, we employ a 2×2 fiber coupler for probabilistic recombination, due to equipment availability. We synchronize a single-photon detector (SPD) and time interval analyzer with the generated time bins for state measurement. The transformation matrix of the measured X gate [Fig. 1(a)] yields a computational basis fidelity $\mathcal{F}_C$ of 0.996 ± 0.001. To show the phase coherence of our gate, we next prepare superposition states as input and interfere the transformed time bins after the gate with a cascade of 1-bin and 2-bin delay-stabilized unbalanced interferometers, and also stabilize the path lengths of the X gate itself. We apply a phase of 0, ϕ and 2ϕ to the time-bins $|0\rangle_t$, $|1\rangle_t$ and $|2\rangle_t$, respectively, and sweep ϕ from 0 to 2π, obtaining the interference pattern shown in Fig. 1(b). After subtraction of dark counts, we calculate a visibility of 0.94 ± 0.01, showing strong phase coherence between the time bins after the gate.

Next, we incorporate our high-performance X gate into a frequency network to realize deterministic two-qudit gates, where the frequency DoF acts as the control and the time DoF is the target qudit. For this demonstration, we utilize true single photons, heralded by detecting the partner photon of a frequency-bin entangled pair created through spontaneous four-wave mixing in an on-chip microresonator. In this case, the pump's time bins couple into a microring with a free spectral range of 380 GHz, generating a biphoton frequency comb (BFC).

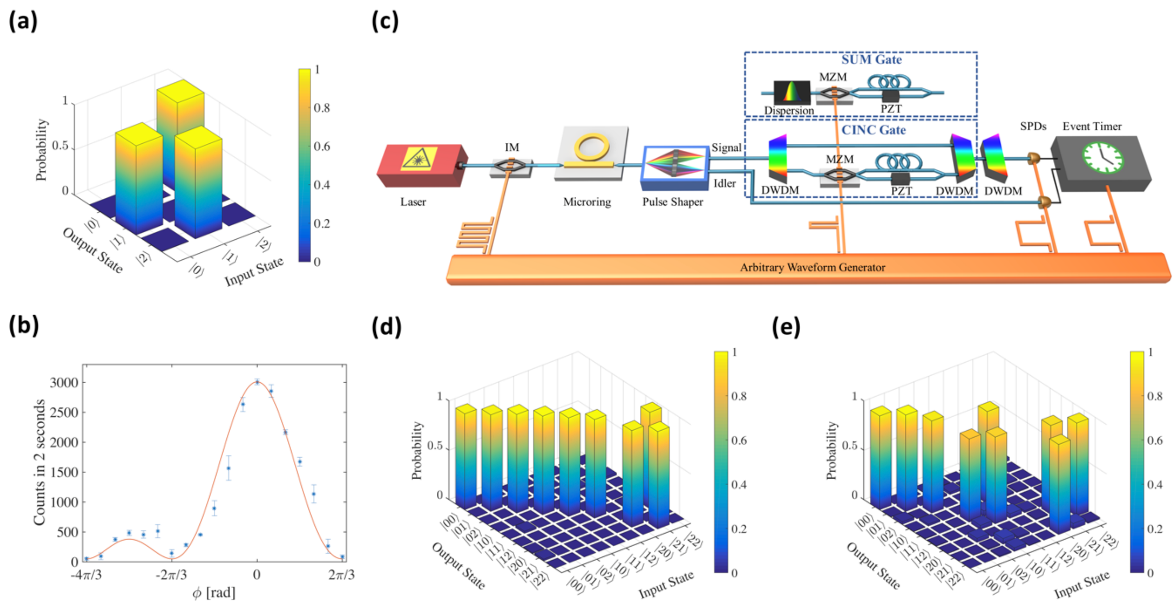

Fig. 1. (a) Experimental transformation matrix of the X gate. (**b)** Counts measured after overlapping all three output time bins, for a time-bin superposition state input into the X gate. The blue error bars are obtained from 5 measurements for each phase. **c.** Experimental setup for CINC and SUM gates. IM: intensity modulator. DWDM: dense wavelength division multiplexing filter. MZM: Mach-Zehnder modulator. SPD: single-photon detector. (**d)** and (**e)** Experimental transformation matrices of the CINC and SUM, respectively.

The time-bin and frequency-bin entanglement of such sources have been proven recently [4-7]. The signal and idler photons from the first three comb line pairs of the BFC are then selected with a pulse shaper [Fig. 1(c)]. In our setup, the idler photons are used as heralding photons, while the signal photons carry the two qudits in their time bins and frequency bins $\left\{|0\rangle_f, |1\rangle_f, |2\rangle_f\right\}$.

As our two-qudit gates, we demonstrate the controlled increment (CINC) and the modulo sum (SUM) gates. In a CINC operation, an X gate is applied to the time-bin qudit only when the frequency qudit is in the state $|2\rangle_f$. To carry out this operation, we separate the frequency state $|2\rangle_f$ from the other two frequencies with a dense wavelength division multiplexing (DWDM) filter and apply the time-bin X gate to the route that consists of the frequency state $|2\rangle_f$. These frequencies are then recombined utilizing another DWDM filter [Fig. 1(c)]. In the SUM operation, the control qudit is added to the target qudit, using a chirped fiber Bragg grating inducing a dispersion of -2 ns/nm on the photons, which imparts 6-ns (1-bin) and 12-ns (2-bin) delays for the temporal modes of $|1\rangle_f$ and $|2\rangle_f$, respectively, as required for the SUM operation. This dispersion module is then followed by a cyclic shift interferometer—which has the same structure as the time-bin X gate—to bring back the two time bins that are pushed out of the computational space ($|3\rangle_t$ and $|4\rangle_t$) and operate as the cyclic addition [Fig. 1(c)]. Another DWDM filter and single-photon detector are used at the output of the gate for state measurement. The transfer matrices of the CINC and SUM gates are shown in Figs. 1(d) and 1(e) by counting the coincidences between the two SPDs for different input and output states, with accidental-subtracted fidelities of $\mathcal{F}_C = 0.90 \pm 0.01$ and $\mathcal{F}_C = 0.92 \pm 0.01$, respectively.

In conclusion, we have demonstrated deterministic single- and two-qudit quantum gates using the time and frequency degrees of freedom of a single photon for encoding the qudits. Hyper-entangled time-frequency entangled states can be generated in integrated on-chip sources, and utilizing on-chip components such as pulse shapers, switches, intensity and phase modulators, can potentially lead to demonstration of this quantum gating scheme on an integrated circuit. As the time and frequency degrees of freedom can be extended to much higher dimensions, these examples of deterministic quantum gates have the potential to make time-frequency-encoded photons a valuable platform for practical quantum computing.

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