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A nonhydrostatic model for atmospheric motion in ACME-HOMME

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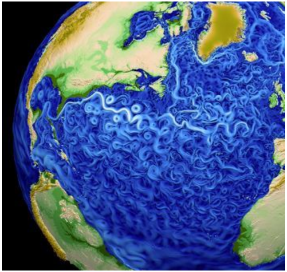
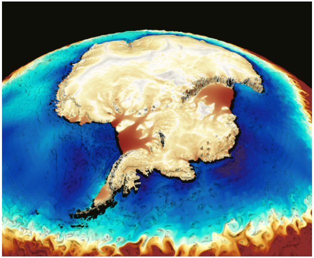
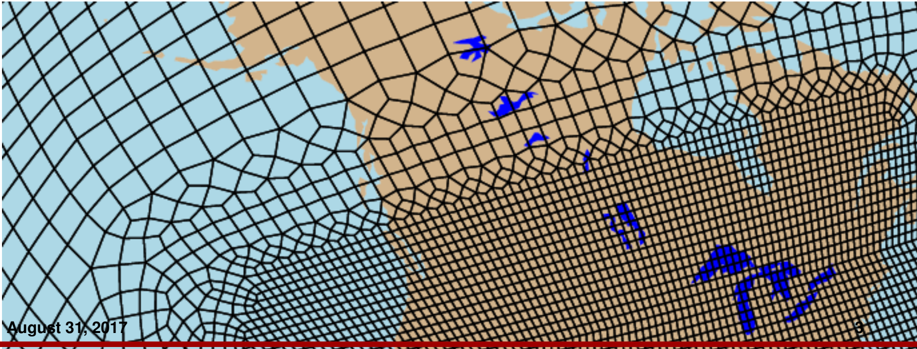
Introduction

- ACME is an earth system model intended for scientific research and also regional prediction (temperature extremes, precipitation, etc).
- Focus is on the water cycle, biogeomchemistry, and the cryosphere.
- Goal: produce a high-resolution earth system model running at 5 SYPD on DOE next-gen computers.
- ACME-HOMME is the atmospheric component of the ACME project.

Why non-hydrostatic?

- The ACME model is the first Earth system model with variable resolution in all components.
- Variable resolution allows for regional refinement down into the nonhydrostatic regime.
- Allows for accurate regional climate modeling and cloud resolving simulations for process studies.



Modeling choices

- Nonhydrostatic models must have w as a prognostic variable $\mathrm{D}w/\mathrm{D}t \neq 0$.
- We also add geopotential $\phi = gz$ as a prognostic variable.
- We use a scaled potential temperature density $\Theta = c_p^* \frac{\partial \pi}{\partial s} \theta$ rather than the real temperature T since $\mathrm{D}\theta/\mathrm{D}t \equiv 0$.
- Exner pressure: $T = \theta \left(\frac{p}{p_0}\right)^{\kappa} = \theta \Pi$
- Supports hydrostatic or nonhydrostatic simluation runs.
- Uses HOMME operators (2D mimetic-grad, curl, etc)
- This results in a formulation quite similar to that in (Laprise (1992)) and used in the GEM4.1 nonhydrostatic model.

Theta-NH model

$$
\begin{cases}
\dfrac{\partial \mathbf{u}}{\partial t} + (\nabla_s \times \mathbf{u} + 2\Omega) \times \mathbf{u} + \dfrac{1}{2}\nabla_s \mathbf{u}^2 + \dot{s}\dfrac{\partial \mathbf{u}}{\partial s} + \\
c_p^* \theta(\nabla_s \Pi - \dfrac{\partial \Pi}{\partial \kappa}\nabla_s \kappa) + \left(\dfrac{\partial p}{\partial s} / \dfrac{\partial \pi}{\partial s}\right) \nabla_s \phi = 0 \\
\frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla_s w + \dot{s}\frac{\partial w}{\partial s} + g - g\left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s}\right) = 0 \\
\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla_s \phi + \dot{s}\frac{\partial \phi}{\partial s} - g w = 0 \\
\frac{\partial \Theta}{\partial t} + \nabla_s \cdot (\Theta \mathbf{u}) + \frac{\partial}{\partial s}(\Theta \dot{s}) = 0 \\
\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial s}\right) + \nabla_s \cdot (\pi_s \mathbf{u}) + \frac{\partial}{\partial s}(\pi_s \dot{s}) = 0 \\
\rho = -\frac{\partial \pi}{\partial s} / \frac{\partial \phi}{\partial s}
\end{cases}
$$

Hamiltonian structure

The energy conserved (in the continuum) is given by

$$H = \int_A \int_s \underbrace{\frac{1}{2}\pi_s \mathbf{u}^2 + \frac{1}{2}\pi_s w^2}_{\text{kinetic energy}} + \underbrace{\Theta\Pi + \phi_s p + p_{\text{top}}\phi_{\text{top}}}_{\text{internal energy}} + \underbrace{\pi_s \phi}_{\text{potential energy}} \quad dA\ ds$$

In a mass or pressure based vertical coordinate s in non-hydrostatic mode this energy is actually the Hamiltonian and we can express

$$\dot{q} = J(q)\frac{\delta H}{\delta q}, \quad q = (u^T, w, \phi, \Theta, \pi_s)^T$$

where J is skew-symmetric operator. Skew-symmetry (but not conservation) is lost in hydrostastic mode.

Discrete energy non-conservation

- Homme operators are defined in 2 dimensions and our discretization gives us horizontal and vertical discrete integration by parts.
- Can show: energy is conserved if

$$\frac{\partial \pi}{\partial s} w \frac{\partial w}{\partial t} - \frac{w^2}{2} \nabla \cdot \left(\frac{\partial \pi}{\partial s} \mathbf{u} \right) + \left(g - g \left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s} \right) \right) w = 0$$

Compare to: $\frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla_s w + \dot{s} \frac{\partial w}{\partial s} + g - g \left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s} \right) = 0.$

- Using 3D vector invariant form, discrete integration by parts is sufficient for energy conservation Dubos, T. and Tort, M. (2014).

Theta-NH model (Hydrostatic mode)

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Hydrostatic approximation implies $\left(\frac{\partial p}{\partial s}/\frac{\partial \pi}{\partial s}\right) \equiv 1$

$$\begin{cases} \frac{\partial w}{\partial t} + \mathbf{u}\cdot\nabla_s w + \dot{s}\frac{\partial w}{\partial s} + g - g\left(\frac{\partial p}{\partial s}/\frac{\partial \pi}{\partial s}\right) = 0 \\ \frac{\partial \phi}{\partial t} + \mathbf{u}\cdot\nabla_s \phi + \dot{s}\frac{\partial \phi}{\partial s} - gw = 0 \end{cases}$$

$$dw/dt \equiv 0, \quad d\phi/dt \equiv g\cdot \text{constant}$$

Number of non-trivial prognostic equations decreases by two and discrete energy is conserved.

Energy diagnostics

- Due to dissipation and vertical remap energy will never be conserved exactly by the model.

$$\frac{1}{2}w^2\nabla\cdot\left(\mathbf{u}\frac{\partial\pi}{\partial s}\right)+\frac{\partial\pi}{\partial s}w\mathbf{u}\cdot\nabla w\neq 0$$

- To prevent spurious growth/decay the energy gained/lost is tracked and added back as heat.
- All errors including time-truncation become internal energy in the model.

Acoustic wave time-step restriction

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$$\begin{cases} \frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla_s w + \dot{s} \frac{\partial w}{\partial s} + g - g \left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s} \right) = 0 \\ \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla_s \phi + \dot{s} \frac{\partial \phi}{\partial s} - g w = 0 \end{cases}$$

- For the JW Baroclinic test case (Jablonowski, C. and Williamson, D. (2006)) the maximum usable step size drops from around $\Delta t = 600$ seconds in the hydrostatic model to $\Delta t = 2$ seconds in the non-hydrostatic model.
- The time-step restriction is a result of the non-hydrostatic system supporting vertically propagating acoustic waves and the vertical scale being much smaller than the horizontal scale.

HEVI splitting

Horizontally explicit vertically implicit splitting:

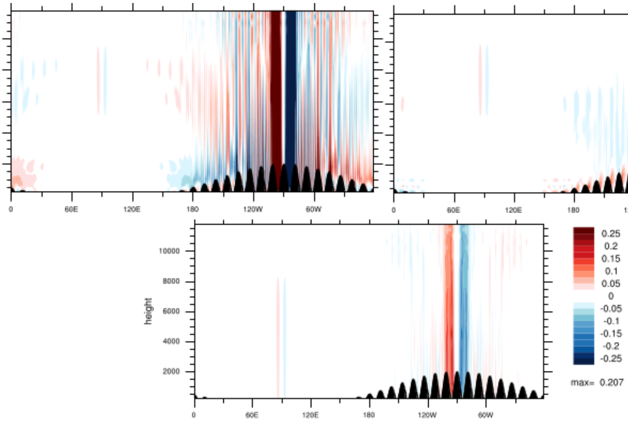
$$
\begin{cases}
\frac{\partial \mathbf{u}}{\partial t} + (\nabla_s \times \mathbf{u} + 2\Omega) \times \mathbf{u} + \frac{1}{2}\nabla_s \mathbf{u}^2 + \dot{s}\frac{\partial \mathbf{u}}{\partial s} + \\
c_p^* \theta(\nabla_s \Pi - \frac{\partial \Pi}{\partial \kappa}\nabla_s \kappa) + \left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s}\right) \nabla_s \phi = 0 \\
\frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla_s w + \dot{s}\frac{\partial w}{\partial s} + g - g\left(\frac{\partial p}{\partial s} / \frac{\partial \pi}{\partial s}\right) = 0 \\
\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla_s \phi + \dot{s}\frac{\partial \phi}{\partial s} - g w = 0 \\
\frac{\partial \Theta}{\partial t} + \nabla_s \cdot (\Theta \mathbf{u}) + \frac{\partial}{\partial s}(\Theta \dot{s}) = 0 \\
\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial s}\right) + \nabla_s \cdot (\pi_s \mathbf{u}) + \frac{\partial}{\partial s}(\pi_s \dot{s}) = 0 \\
\rho = -\frac{\partial \pi}{\partial s} / \frac{\partial \phi}{\partial s}
\end{cases}
$$

DCMIP Test cases

- DCMIP (Dynamical Core Model Intercomparison Project) 2012 established new non-hydrostatic test cases for atmospheric dycores.
- If interested: check out the Test Case Document by Ullrich et al on the DCMIP project website.
- Compare 3 models: PREQX (old HOMME dycore), THETA-H (theta model in hydrostatic mode), and THETA-NH (theta model in non-hydrostatic mode.
- Make use of ”small planets” where non-hydrostatic effects are more apparent.

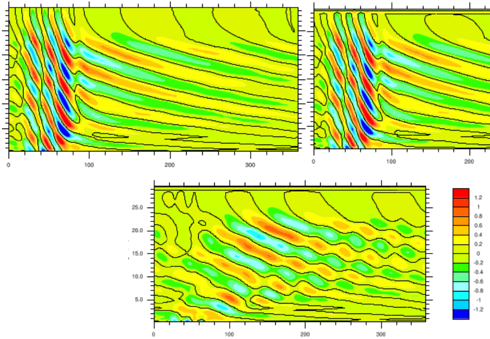
DCMIP 2-0

Figure: Atmosphere at rest, non-rotating plaet, Schaer-like mountain (Girard, C. et al. (2002)), horizontal velocity at the equator after 6 days with 1°, 30 vertical levels. PREQX (t. left), THETA-H (t. right), THETA-NA (bottom).



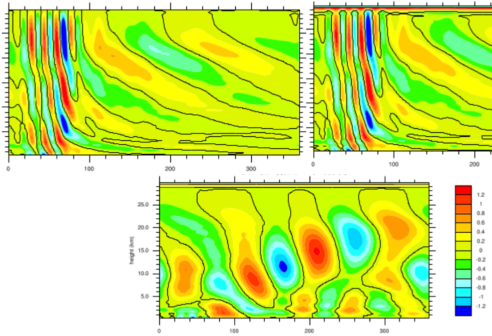
DCMIP 2-1

Figure: Temperature perturbation after 2 hours for flow over a Schaer-like mountain without wind shear at the equator, 1.5° horizontal grid, 60 vertical levels, small planet $\times$500. PREQX (t. left), THETA-H (t. right), THETA-NH (bottom)



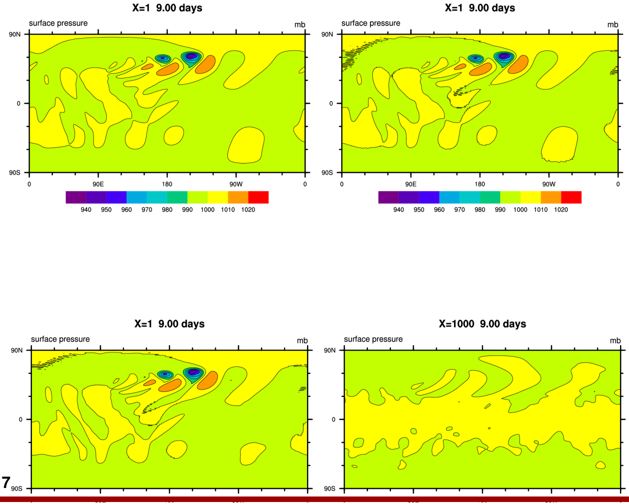
DCMIP 2-2

Figure: Temperature perturbation after 2 hours for flow over a Schaer-like mountain with wind shear at the equator, 1.5° horizontal grid, 60 vertical levels, small planet $x500$. PREQX (t. left), THETA-H (t. right), THETA-NH (bottom)



DCMIP 4 Baroclinic instability

Figure: Surface pressure at 1 degree. PREQX (t. left), THETA-H (t. right), THETA-NH (b. left), THETA-NH small planet x1000 (b. right)



Fin

Questions?

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