Scalable Newton-Krylov-AMG Solution Methods for Implicit / IMEX Continuum Plasma Physics Models

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Abstract

We have developed ***scalable block preconditioners for implicit / IMEX plasma physics models with physics compatible discretizations. These include resistive [1] and extended MHD [2], and multifluid electromagnetic (EM) plasmas [3,4].*** The key is ***effective approximate Schur complement operators that encode the critical cross-coupling / fast time-scales*** of the physics.

Motivation (e.g. multifluid plasmas)

Multifluid EM plasma models are being evaluated for simulation of moderately-dense to dense / collisional systems in support of DOE magnetic and inertial confinement fusion (MCF, ICF). The models can be used as continuum approximations and as moment–based accelerators for multiscale continuum/kinetic models.

$$\frac{\partial \rho_a}{\partial t} + \nabla\cdot(\rho_a \mathbf{u}_a) = \mathcal{R}_{\rho_a}$$

$$\frac{\partial(\rho_a \mathbf{u}_a)}{\partial t} + \nabla\cdot(\rho_a \mathbf{u}_a \otimes \mathbf{u}_a + p_a I + \Pi_a) - q_a n_a (\mathbf{E} + \mathbf{u}_a \times \mathbf{B}) = \mathcal{R}_{\rho_a u_a}$$

$$\frac{\partial \mathscr{E}_a}{\partial t} + \nabla\cdot((\mathscr{E}_a + p_a)\mathbf{u}_a + \Pi_a \cdot \mathbf{u}_a + \mathbf{h}_a) - q_a n_a \mathbf{u}_a \cdot \mathbf{E} + Q_a^{src} = \mathcal{R}_{\mathscr{E}_a}$$

$$\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \frac{1}{\mu_0}\mathbf{B} + \mathbf{J} = \mathbf{0}; \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = \mathbf{0}; \quad \mathbf{J} = \sum_k q_k n_k \mathbf{u}_k$$

Block preconditioners must account for the relative time-scales of wave physics (hydrodynamics and EM), plasma and cyclotron frequencies, collisional transport and disparate spatial discretizations (nodal, edge, face, etc.).

Approach

Our block preconditioners **employ multilevel methods tailored to different discretization types** for scalability. The hydrodynamics variables (**F** - nodal) are partitioned, from electric (**E** - edge) and magnetic (**B** - face) fields and an approximate block LU factorization is designed.

$$\begin{bmatrix} \mathbf{Q}_B & \mathbf{K}_E^B & 0 \\ \mathbf{K}_B^E & \mathbf{Q}_E & \mathbf{Q}_F^E \\ \mathbf{Q}_B^F & \mathbf{Q}_E^F & \mathbf{D}_F \end{bmatrix} \begin{bmatrix} \mathbf{B} \\ \mathbf{E} \\ \mathbf{F} \end{bmatrix}; \quad \begin{bmatrix} \mathbf{Q}_B & \mathbf{K}_E^B & 0 \\ 0 & \hat{\mathcal{D}}_E & \mathbf{Q}_F^E \\ 0 & 0 & \hat{\mathbf{S}}_F \end{bmatrix}$$

$$\mathbf{S}_F = \mathbf{D}_F - \mathcal{K}_E^F \mathcal{D}_E^{-1} \mathcal{Q}_F^E \; ; \; \mathcal{D}^E = \mathbf{Q}_E - \mathbf{K}_B^E \bar{\mathbf{Q}}_B^{-1} \mathbf{K}_E^B$$

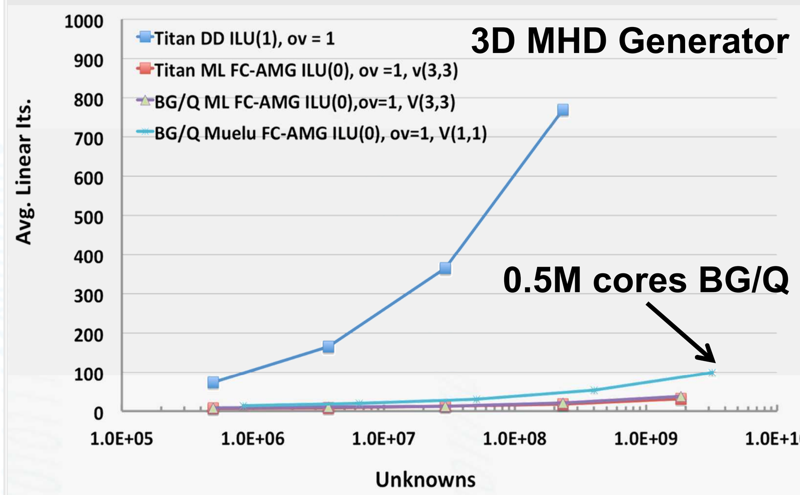
CFD – H(grad) AMG   E field – H(curl) AMG

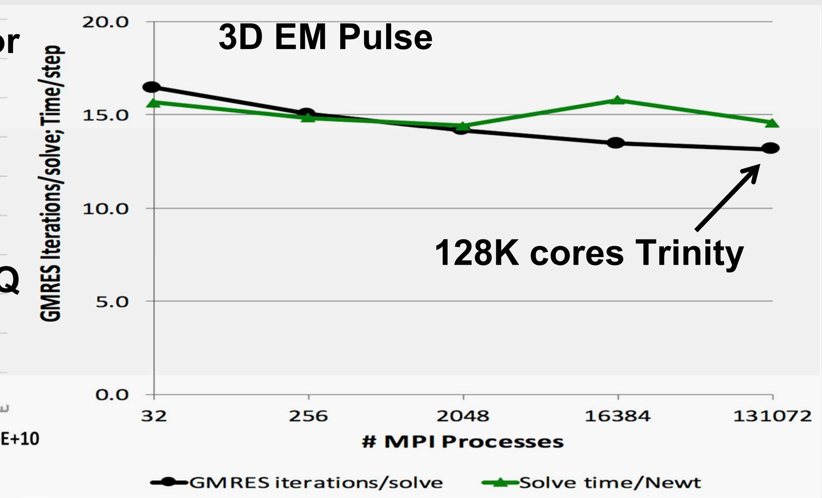
The **E** field Schur complement is close to singular for large values of CFL_c we use the grad-div stabilization for this weak form dominated by the curl-curl operator. Preliminary results have demonstrated **scalability following slow time-scales of interest and ability to solve in the long time-scale MHD limit.**

Results:

Examples of scalable performance include:

H(grad) coupled system AMG for nodal FE resistive MHD, and H(curl) AMG for an edge / face FE Maxwell solver.





Block preconditioners for electron-ion EM plasma driven by an external current (fast EM waves, plasma /cyclotron freq.)

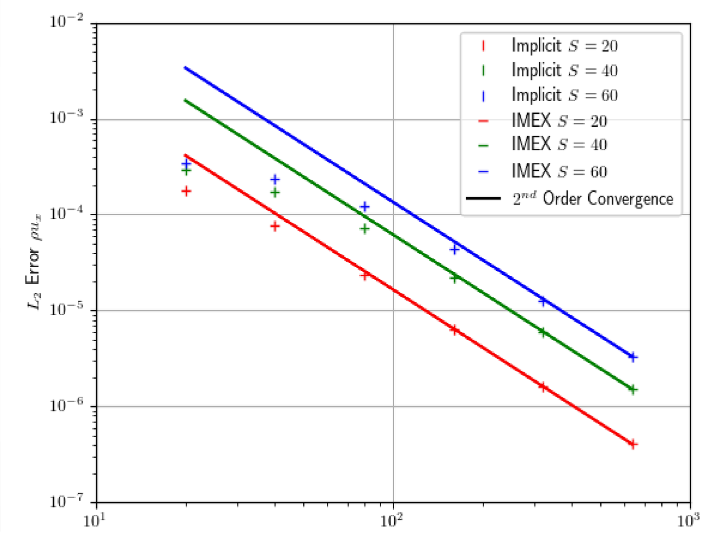
Procs	DOFs	Avg Its	Time (s)
1	2.6e4	16.26	0.7833
4	1.0e5	16.45	1.031
16	4.0e5	17.95	2.141
64	1.3e6	27.74	3.923
256	6.3e6	32.68	5.078

Implicit L-stable and IMEX SSP/L-stable time integration and block preconditioners enable solution of multifluid EM plasma model in the asymptotic resistive MHD limit.

Plasma Scales for $S = 60$

	Electrons	Ions
$\omega_p \Delta t$	$4 \cdot 10^7 - 1.3 \cdot 10^9$	$9.4 \cdot 10^5 - 3 \cdot 10^7$
$\omega_c \Delta t$	$1.7 \cdot 10^6 - 5.5 \cdot 10^7$	$9.4 \cdot 10^2 - 3 \cdot 10^4$
$\nu_{\alpha\beta} \Delta t$	$1.7 \cdot 10^{10} - 5.5 \cdot 10^{11}$	$9.4 \cdot 10^6 - 3 \cdot 10^8$
$v_s \frac{\Delta t}{\Delta x}$	$7 \cdot 10^{-3}$	$2 \cdot 10^{-4}$
$u \frac{\Delta t}{\Delta x}$	$2 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
$\frac{\mu}{\rho}\frac{\Delta t}{\Delta x^2}$	$0.4 - 12$	$0.01 - 0.3$
$c \frac{\Delta t}{\Delta x}$	167	

IMEX terms: implicit/explicit



Convergence / order of accuracy

Conclusions and Future Work

Our development of implicit / IMEX plasma physics formulations and block preconditioning approaches have demonstrated scalable solution for multiple time scale plasma systems. Effective Schur complement approximations (1) must preserve critical off-diagonal hyperbolic and strong source term coupling, and (2) decompose the coupled system into sub-systems to which optimal H(grad) and H(curl) AMG can be applied.

Preliminary incorporation of these type of approaches into R&D and production mulifluid continuum plasma solvers for DOE/SNL applications is promising. Currently pursuing demonstration and application for large-scale fusion applications for MCF (e.g. Tokamaks) and for ICF (e.g. magnetic implosions for Z-pinch).

Areas in which we can help

R&D involving continuum plasma models (resistive and extended MHD, and multifluid plasmas) both as continuum solvers and accelerators for multiscale continuum/kinetic models. New ASCR/OFES SciDAC and OFES plasma efforts, as well as NNSA plasma physics R&D would strongly benefit from collaboration on these topics.

Areas in which we need help

Continued development of efficient and scalable H(grad) / H(curl) AMG algorithms / software at extreme scale and R&D for these methods on next generation architectures. R&D on light-weight / robust smoothers for AMG that require less memory than DD-ILU.

References

[1] Shadid, Pawlowski, Cyr, Tuminaro, Chacon, Weber, Scalable Implicit Incompressible Resistive MHD with Stabilized FE and Fully-coupled Newton-Krylov-AMG CMAME 2016

[2] Chacon and Stanier, "A scalable, fully implicit algorithm for the reduced two-field low-β extended MHD model," JCP, 2016

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[4] Phillips, Shadid, Cyr, Scalabale Preconditioners for Structure Preserving Discretizations of Maxwell Equations in First Order Form, Submitted to SISC, 2017.

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