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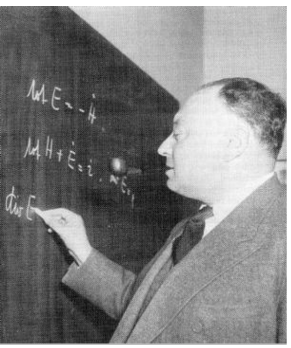

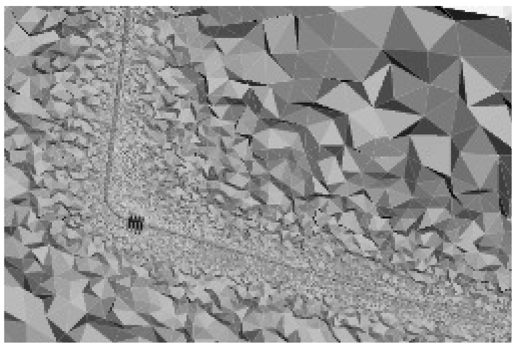
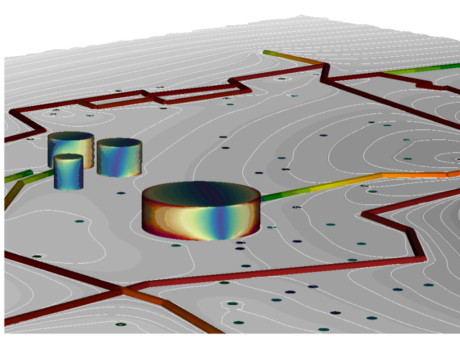

Electromagnetic modeling of geology cluttered with infrastructure and other thin conductors: a finite element method for hierarchical model parameters on volumes, faces and edges of an unstructured grid

Chester J Weiss

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The pain of volumetric discretization

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Example problem: discretization of steel casing in an oil well

0.2 m outer diameter, 0.025 m wall thickness, electrical conductivity 5e6 S/m

regular tet with edge length 0.025 m occupies a volume $(0.025\ \text{m})^3 / (6\sqrt{2}) = 1.84\text{e-}6\ \text{m}^3$

1 km of casing requires 7.4e6 tets

Over a 1 km^3 Earth model discretized at, say 10 m, 7.4/(7.4 + 8.5)*100% = 46.5% of the tets are devoted to 0.0000014% of the mesh volume.

This is **computationally explosive**, especially for realistic oilfield settings where there are 10s of km of steel casing + surface pipelines + storage tanks + electric cable + …

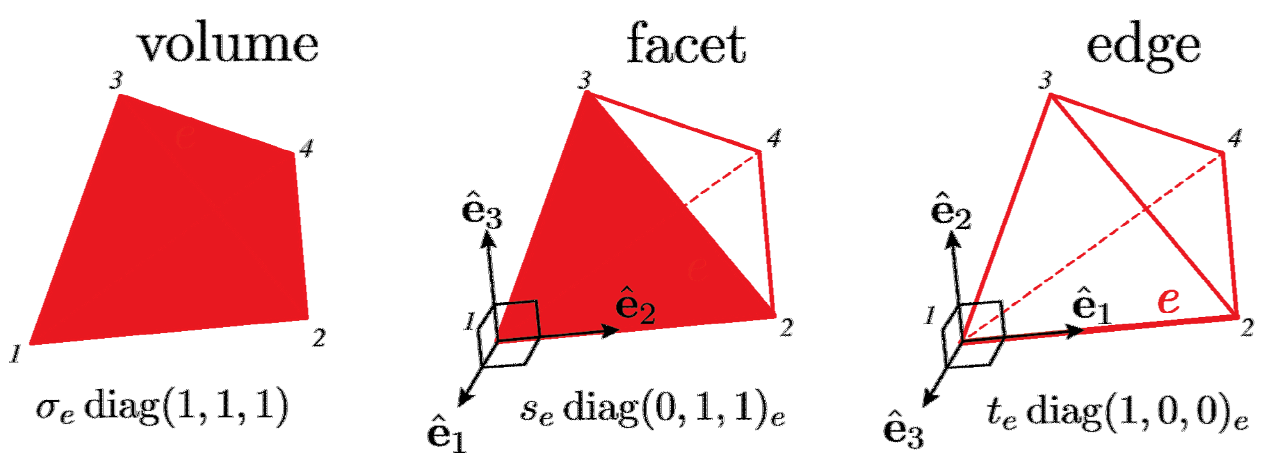
Typical approaches to the problem are

- specialized algorithms for parallel compute architectures (Commer et al., 2015, Hoversten et al., 2015, Um et al., 2015)
- Discretization of slightly “fatter” casing, whose large size reduces the element count with an acceptable reduction in accuracy (Haber et al., 2016; Weiss et al., 2016).

A New Hope

Hanging the material properties on the tets, faces and edges of the unstructured tetrahedral mesh allows for thin conductors to be economically represented by facets and edges, rather than 100s of millions of tiny tets.

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$



Hierarchical basis functions for material properties

This hierarchy of material distributions is made possible by using rank-2 tensor basis functions – an extension of the early work in 2D anisotropy by Weiss and Newman (Geophysics, 2002, 2003)

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

$$\boldsymbol{\psi}_e^V(\boldsymbol{x}) = \mathrm{diag}(1,1,1) \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{volume } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^E(\boldsymbol{x}) = \mathrm{diag}(1,0,0)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{edge } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^F(\boldsymbol{x}) = \mathrm{diag}(0,1,1)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{facet } e \\ 0 & \text{otherwise} \end{cases}$$

The tensor representation keeps the material properties local to the edges and facets in the Finite Element weak formulation / bilinear form.

Assembly and solution of the linear system

Poisson Eq for electro/magnetostatics

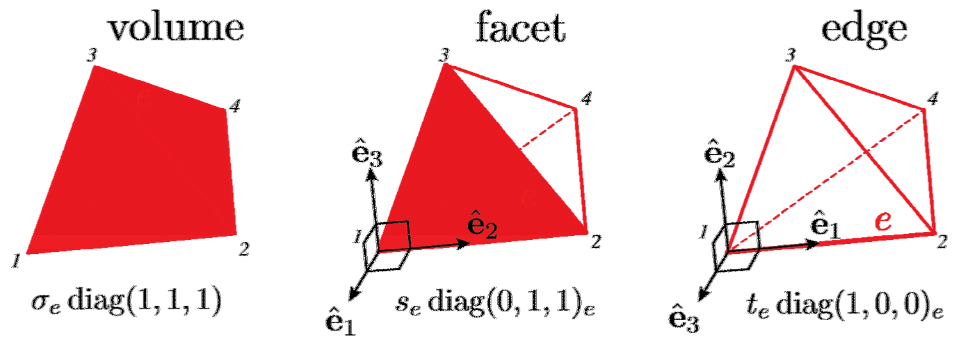
$$-\nabla \cdot (\boldsymbol{\sigma} \cdot \nabla u) = f \qquad \int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) \ dx^3 = \int_\Omega v f \ dx^3$$

Sparse anisotropic conductivity collapses 3D gradients to 2D and 1D gradients…

$$\boldsymbol{\sigma} = \mathrm{diag}(0, \sigma, \sigma) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_{23} v \cdot \nabla_{23} u$$

$$\boldsymbol{\sigma} = \mathrm{diag}(\sigma, 0, 0) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_1 v \cdot \nabla_1 u$$

…thus ensuring that the facet and edge material properties are local and not distributed over the tetrahedral volume.



Assembly and solution of the linear system

Variational formulation: $$\int_{\Omega} \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) \; dx^3 = \int_{\Omega} v f \; dx^3$$

Hierarchical model:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

3D inner products collapse to 2D and 1D inner products

$$\int_{\Omega} \nabla v \cdot \left[\sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) \right] \nabla u \; dx^3 = \sum_{e=1}^{N_V} \sigma_e \int_{V_e} \nabla v \cdot \nabla u \; dx^3 \; = \sum_{e=1}^{N_V} \sigma_e \, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^4 \mathbf{u}_e$$

$$\int_{\Omega} \nabla v \cdot \left[\sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) \right] \nabla u \; dx^3 = \sum_{e=1}^{N_F} s_e \int_{F_e} \nabla_{23} v \cdot \nabla_{23} u \; dx^2 = \sum_{e=1}^{N_F} s_e \, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^3 \mathbf{u}_e$$

$$\int_{\Omega} \nabla v \cdot \left[\sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x}) \right] \nabla u \; dx^3 = \sum_{e=1}^{N_E} t_e \int_{E_e} \nabla_1 v \cdot \nabla_1 u \; dx = \sum_{e=1}^{N_E} t_e \, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^2 \mathbf{u}_e$$

Global stiffness matrix is a sum of 3D, 2D and 1D element stiffness matrices.

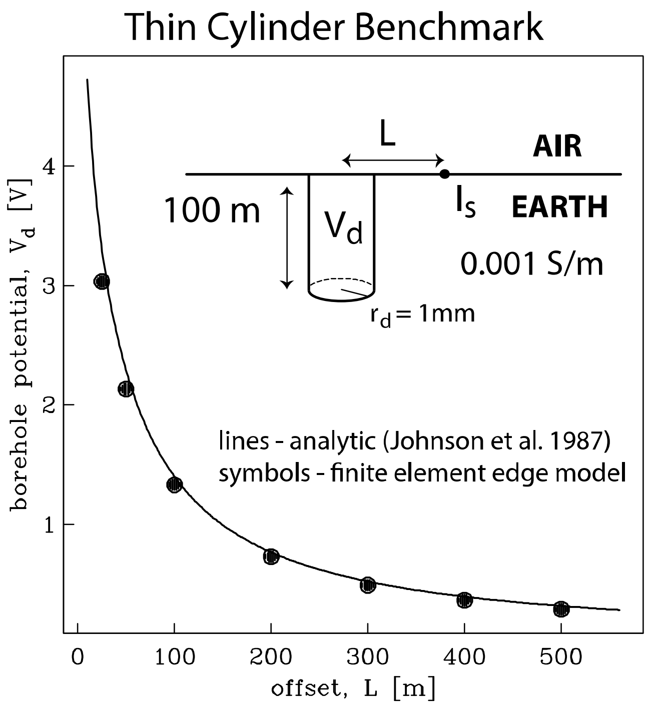
$$\mathbf{K}\mathbf{u} = \mathbf{b}$$

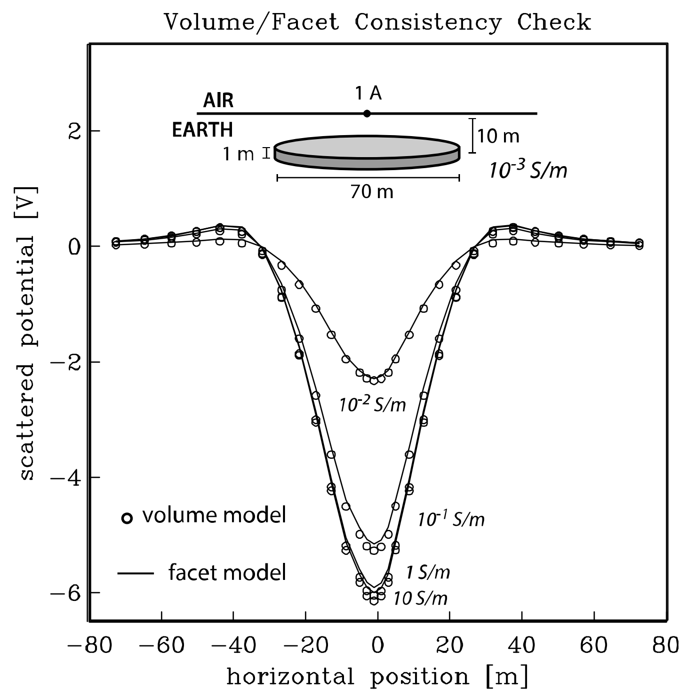
$$\mathbf{K} = \sum_{e=1}^{N_V} \sigma_e \mathbf{K}_e^4 + \sum_{e=1}^{N_F} s_e \mathbf{K}_e^3 + \sum_{e=1}^{N_E} t_e \mathbf{K}_e^2$$

Solve iteratively with Jacobi scaled conjugate gradients and on-the-fly matrix assembly (Weiss, 2001)

Benchmarking with analytics

Benchmarking and internal consistency checks show that for thin conductors, the facet/edge representation achieves acceptable accuracy over a range of geometries and material properties.

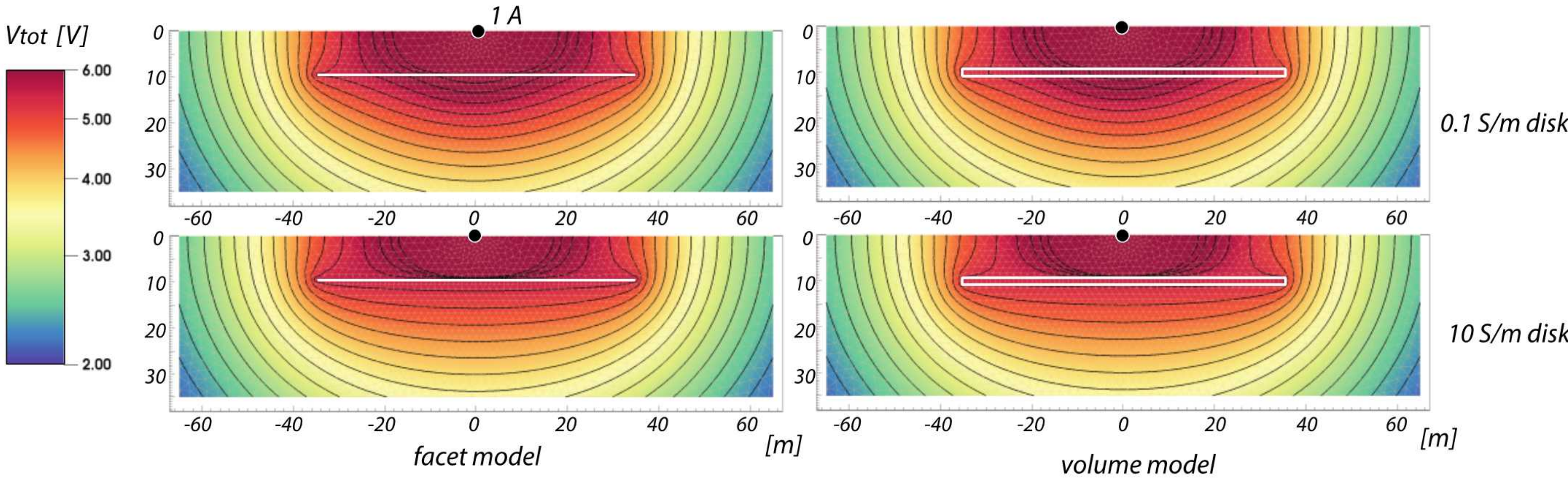




Volume/facet consistency test

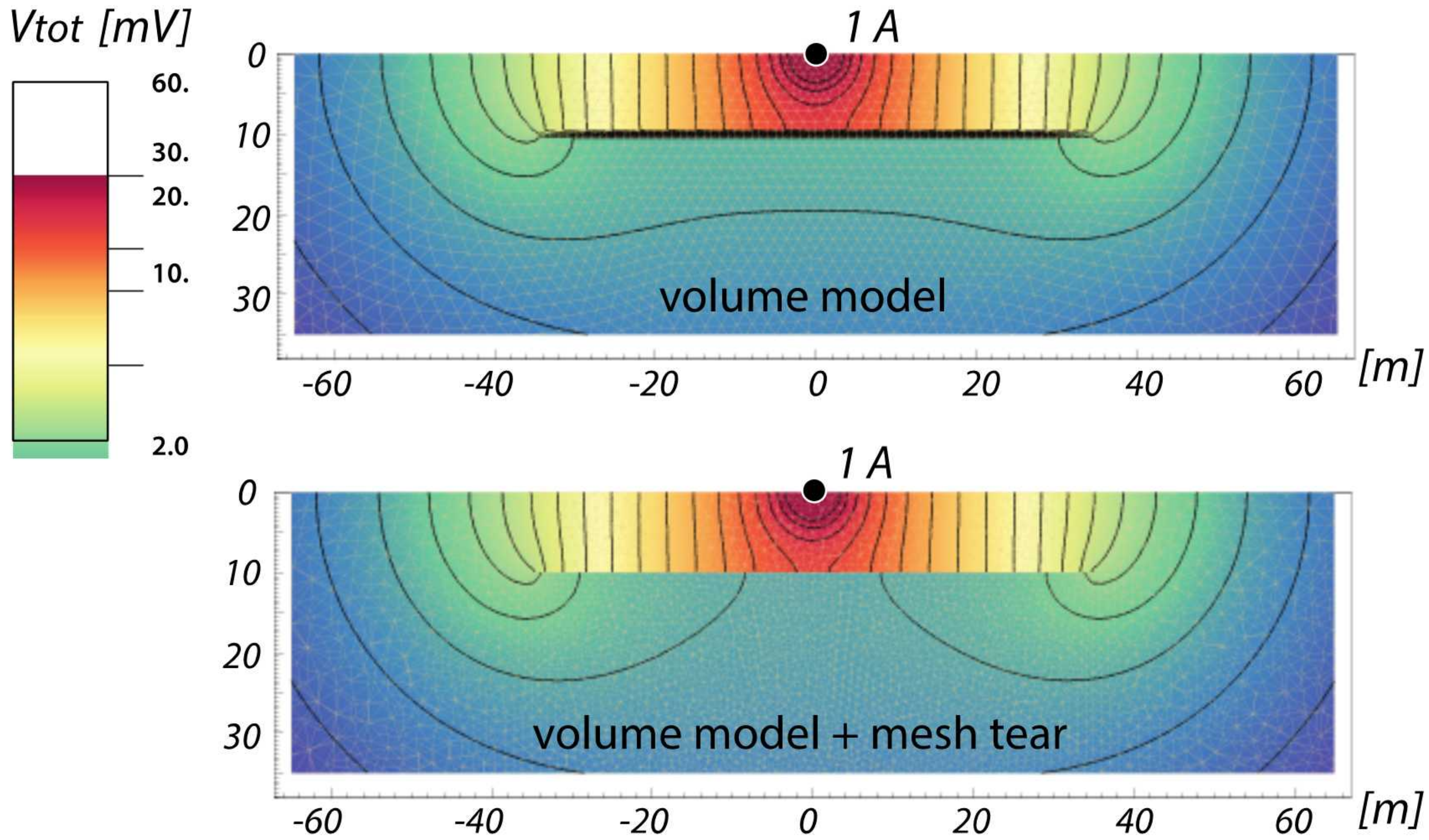
Visual inspection of thin disk results for facet elements (left) and many small tetrahedral elements (right).

Shown is a cross section of electric potential through the disk and surrounding geology for a weak conductor (top) and strong conductor (bottow). Background conductivity is 0.001 S/m.



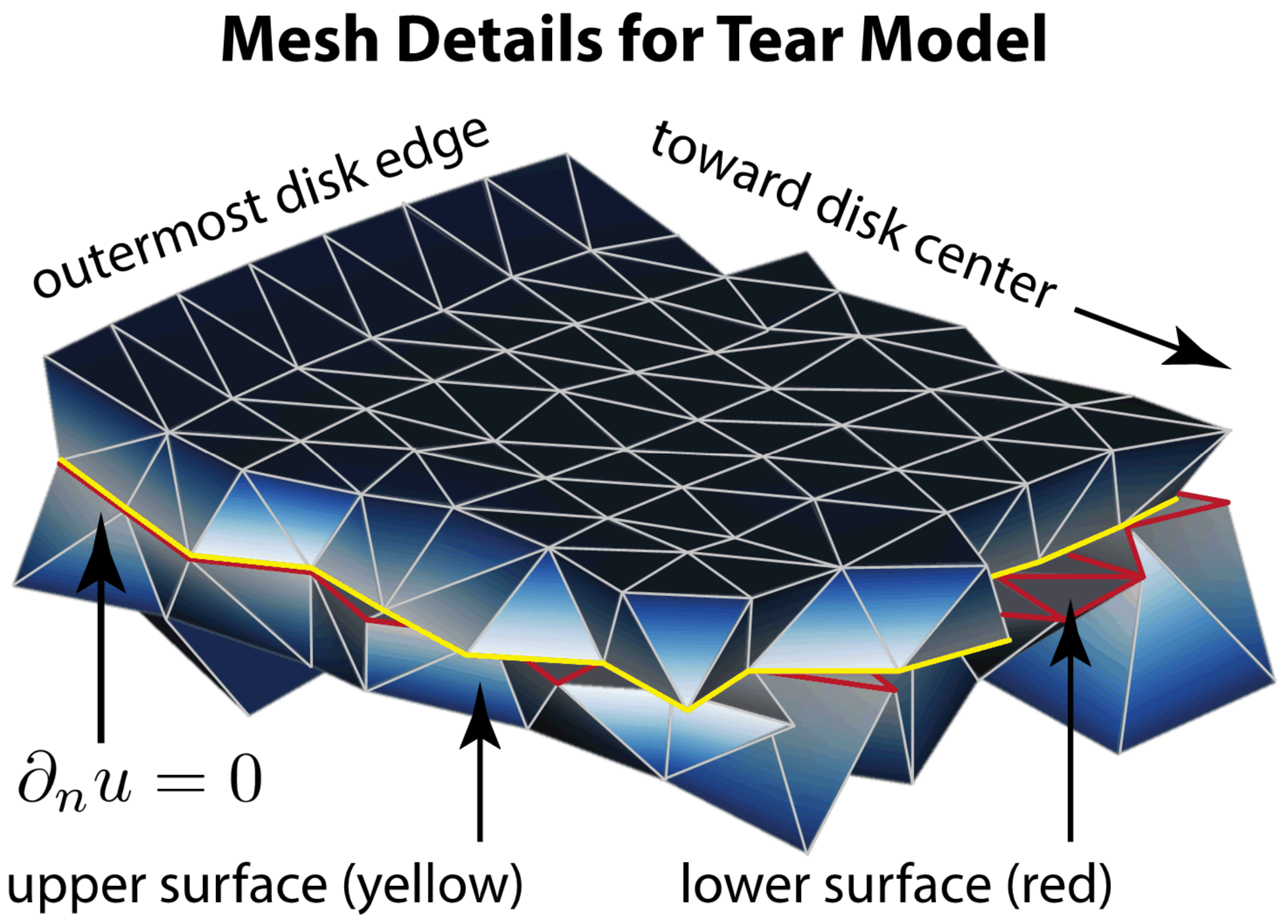
What about thin resistive elements?

Because of the continuity condition, thin resistors require a little more intervention. Specifically, we require that the infinitely thin resistor be multi-valued on the disk surface, with either side of the surface subject to a Neumann boundary condition. This is called a “tear” in the FE literature.



Mesh modification for perfect resistors

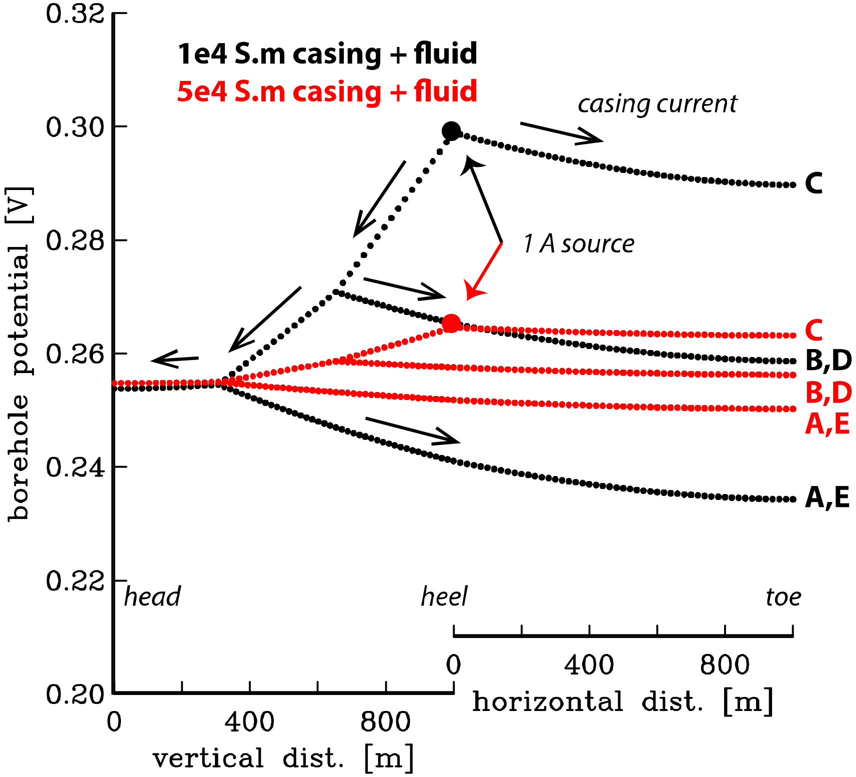
The ”tear” representing the thin resistor is doubly discretized, with one set of nodes corresponding to tets on one side of the tear, and second set for tets on the other side. Still, the surface is infinitely thin and we avoid extreme discretization of a thin, but finite thickness “slab” filled with millions of tiny tets.

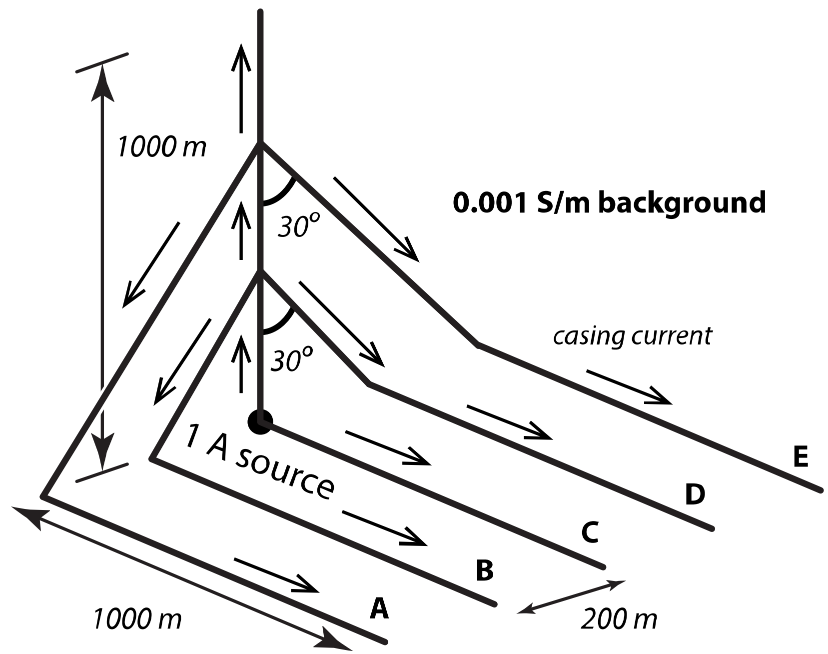


With the mathematical framework in place along with favorable benchmarking results, we're now emboldened to investigate oilfield problems.

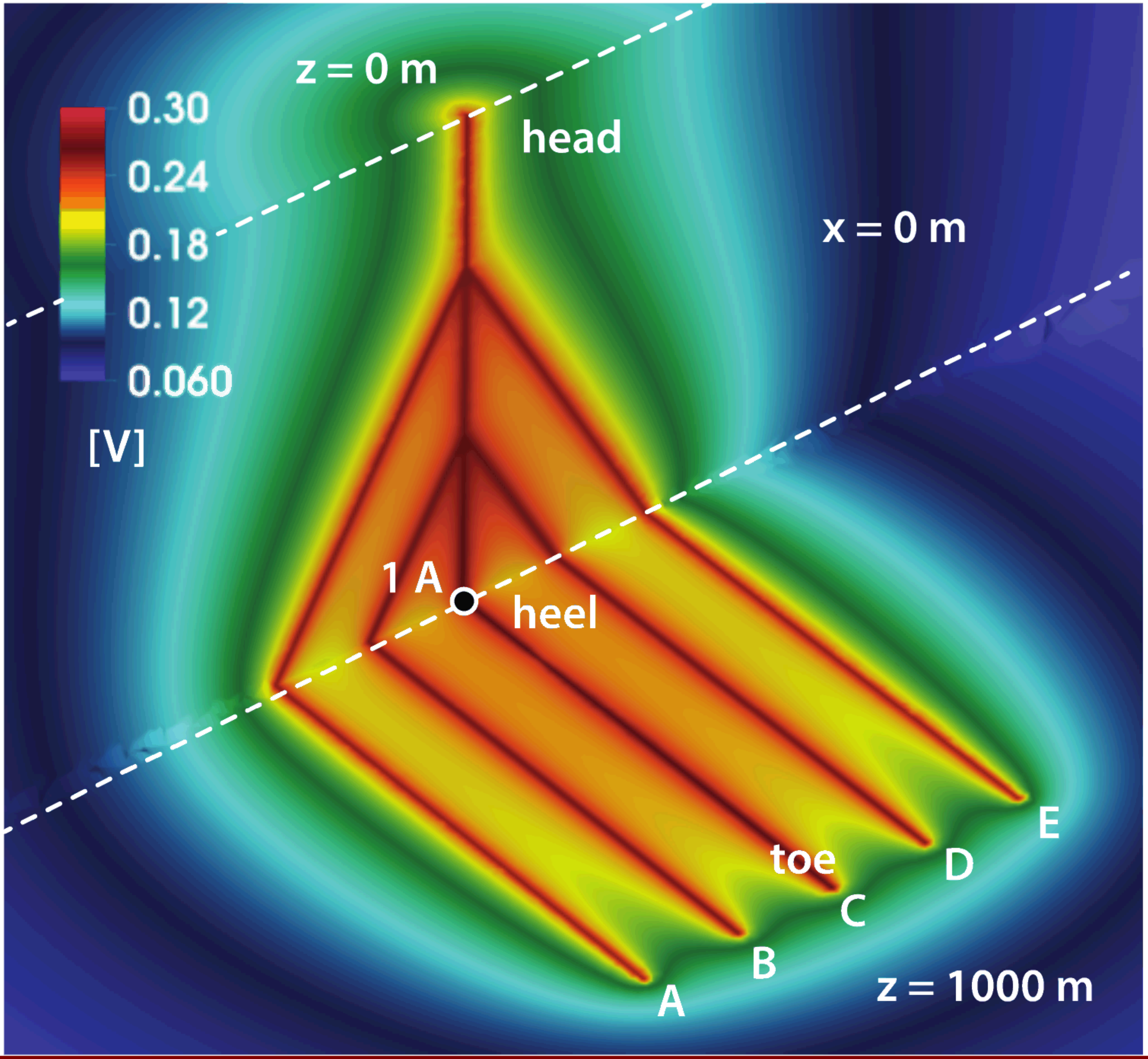
EXAMPLE PROBLEM: an idealized multi-lateral (edge elements for casing)

What is the effect of casing conductivity on the casing potential?





3D rendering of the multi-lateral system

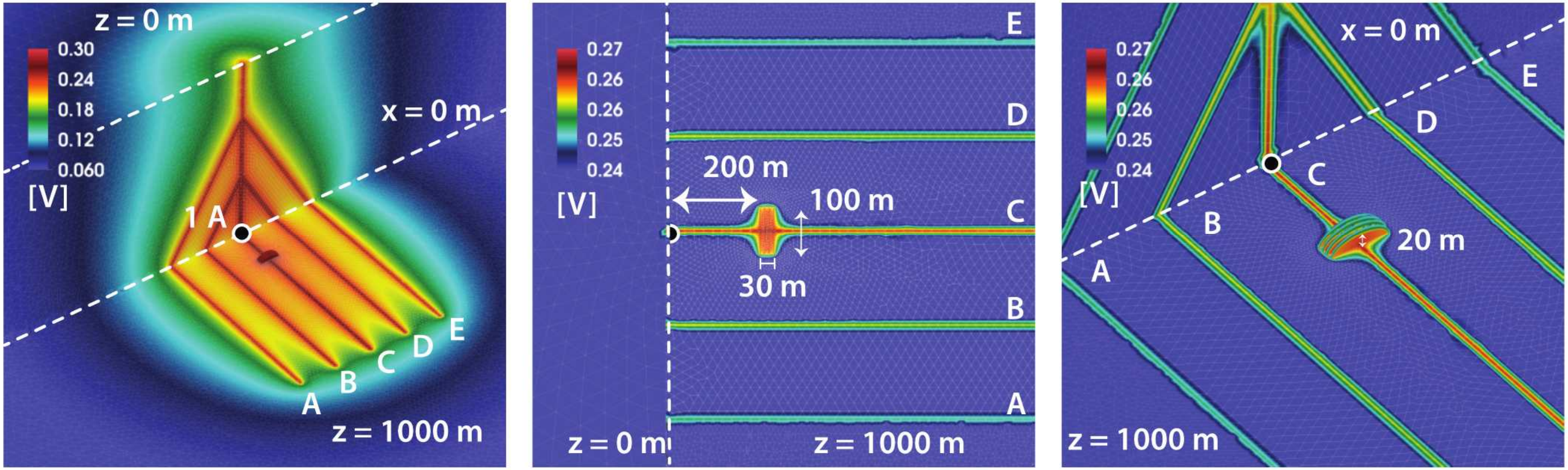


With the mathematical framework in place along with favorable benchmarking results, we're now emboldened to investigate oilfield problems.

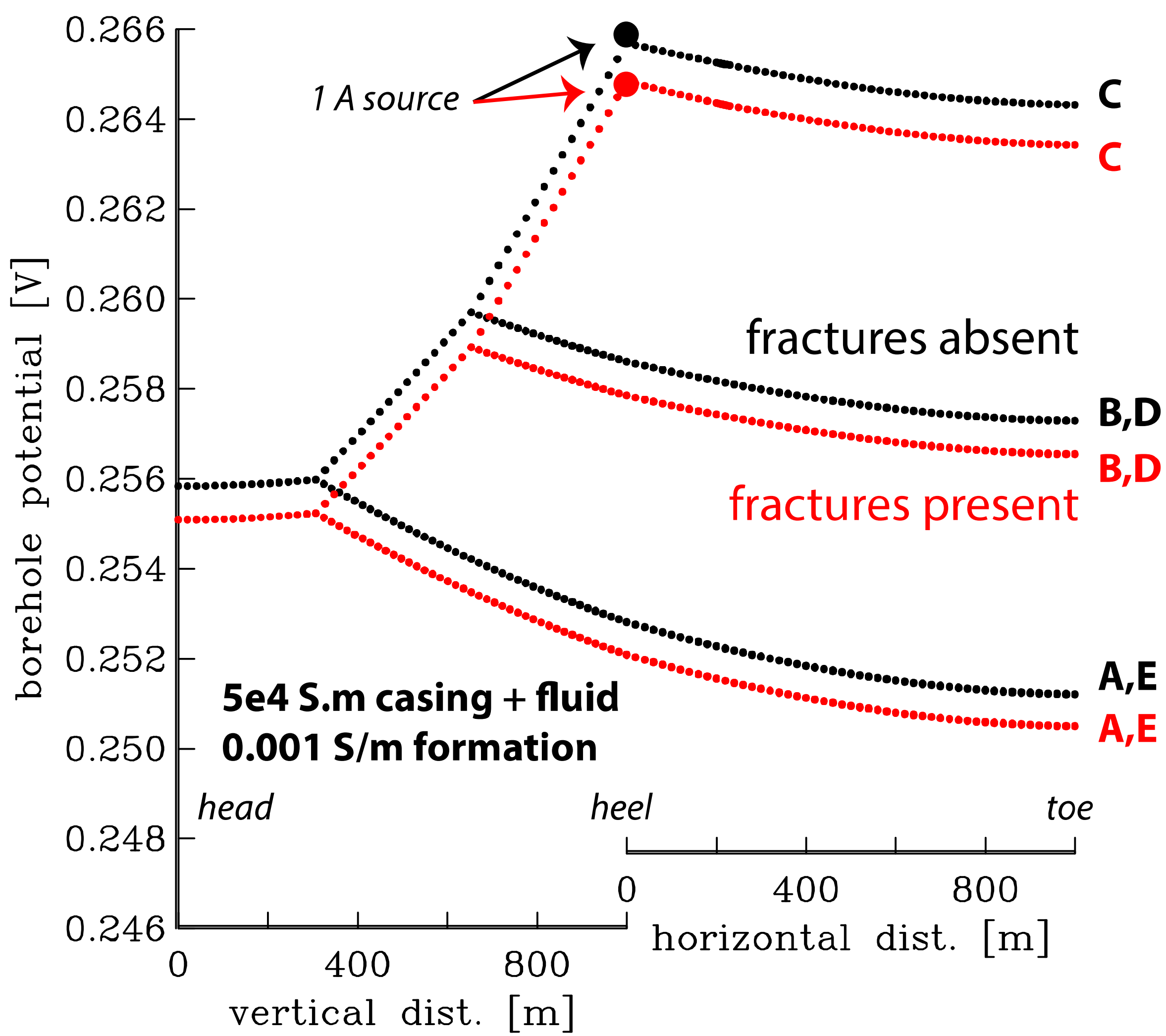
EXAMPLE PROBLEM: an idealized multi-lateral, now with a fracture.

What effect does a conductive fracture have on the casing system?

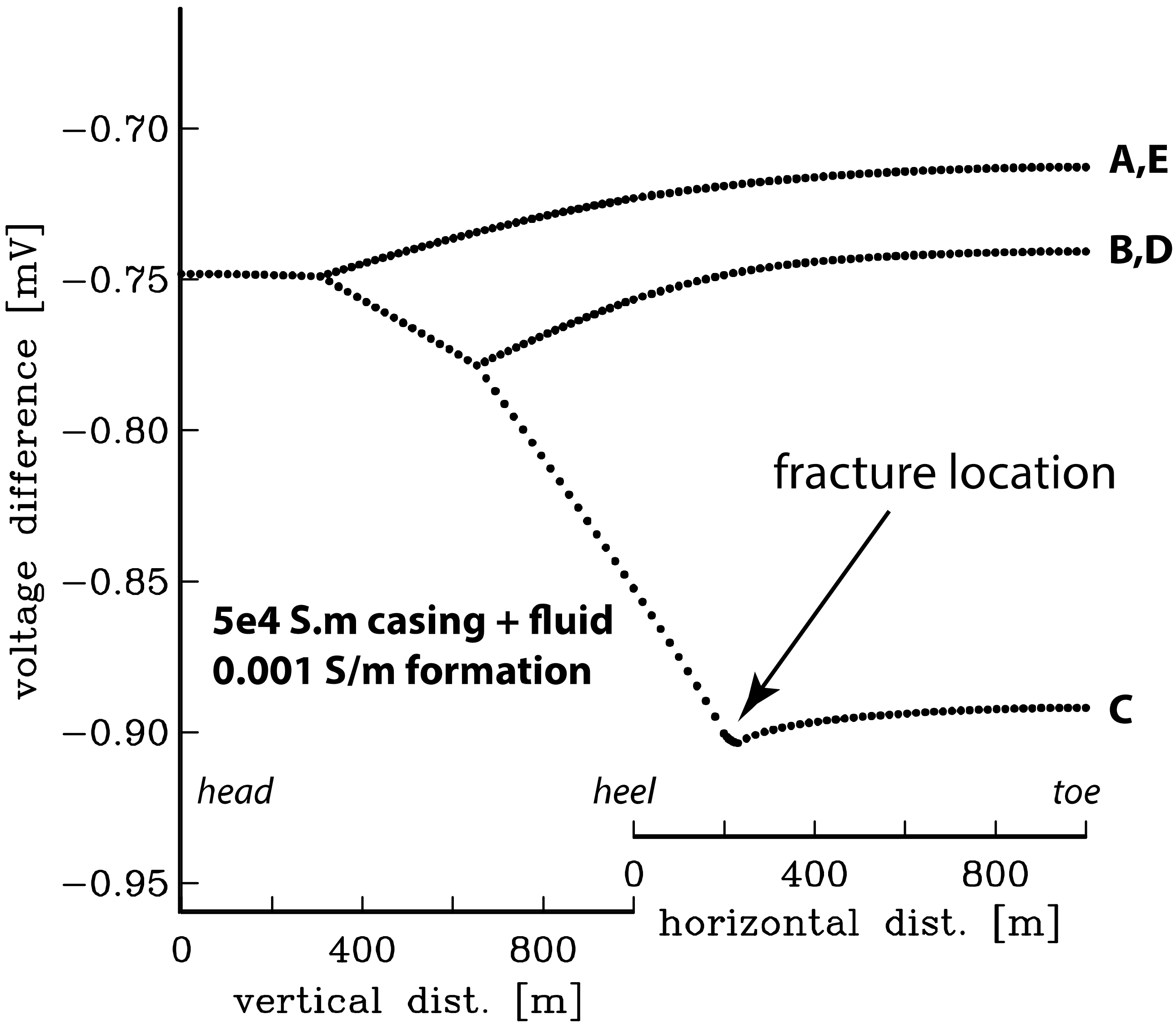
casing: edge elements, fracture: facet elements



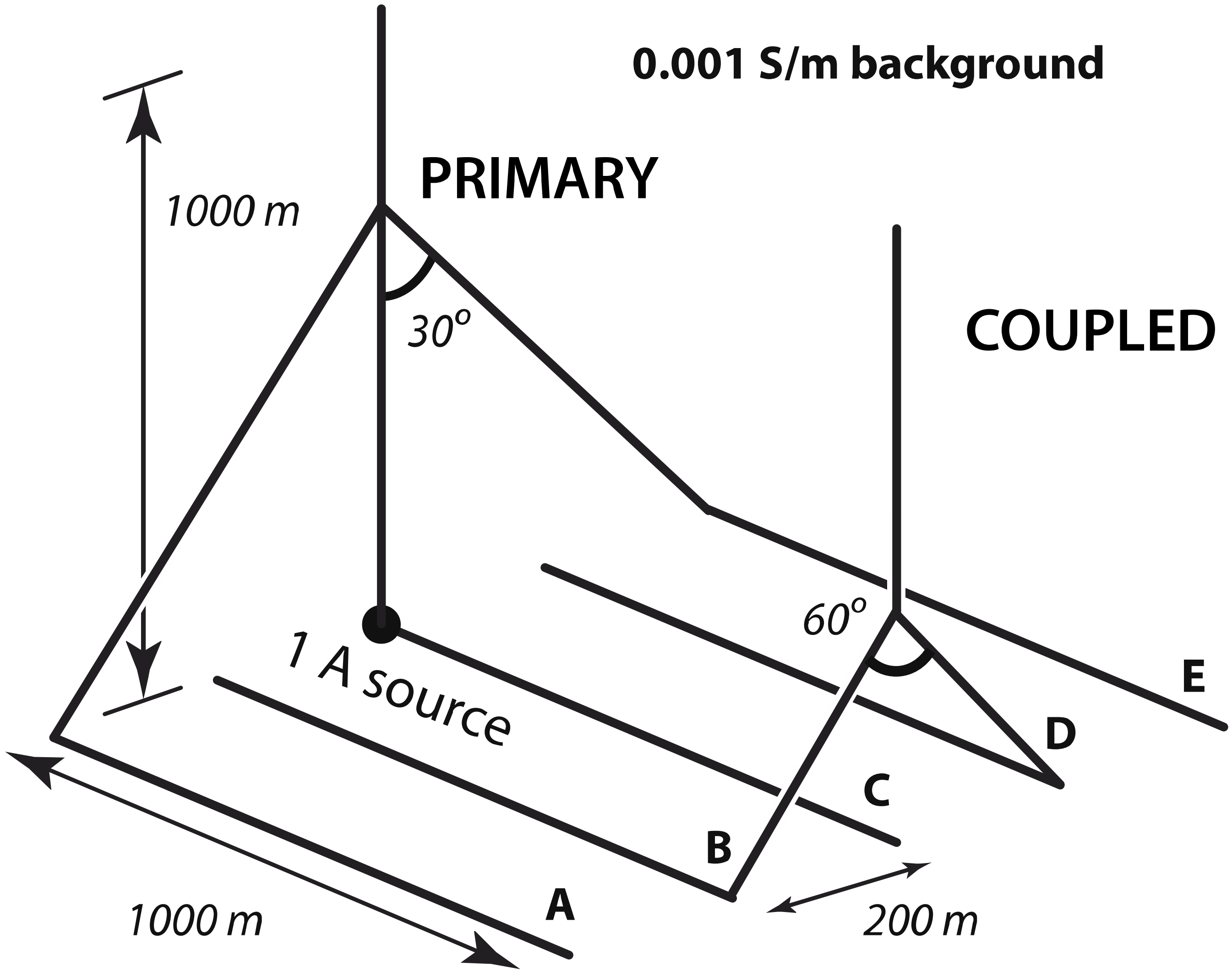
4D time-lapse response of casing response with conductive fractures introduced into the system



4D time-lapse differences of casing potential due to introduced fractures.



What happens when some of the horizontals aren't in direct electrical contact? I.e. passive coupling between all the steel in the ground.



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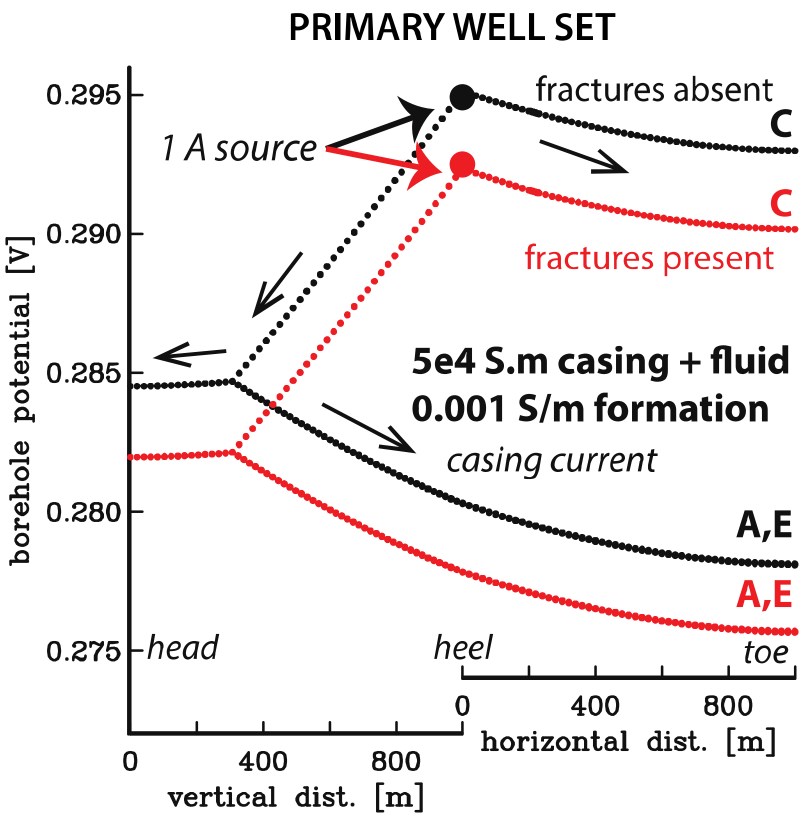
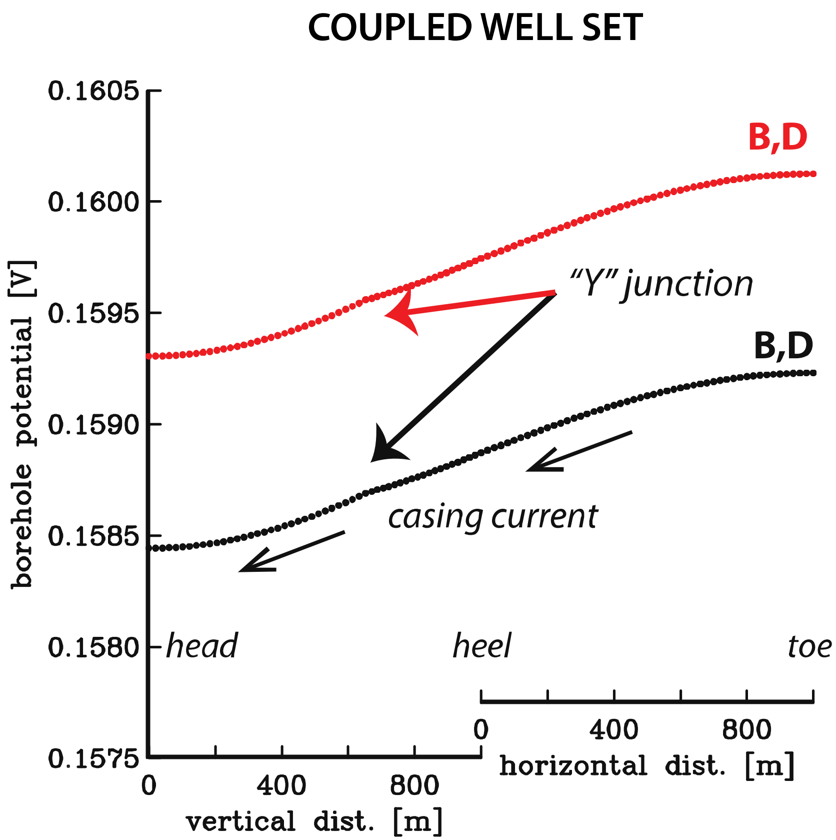
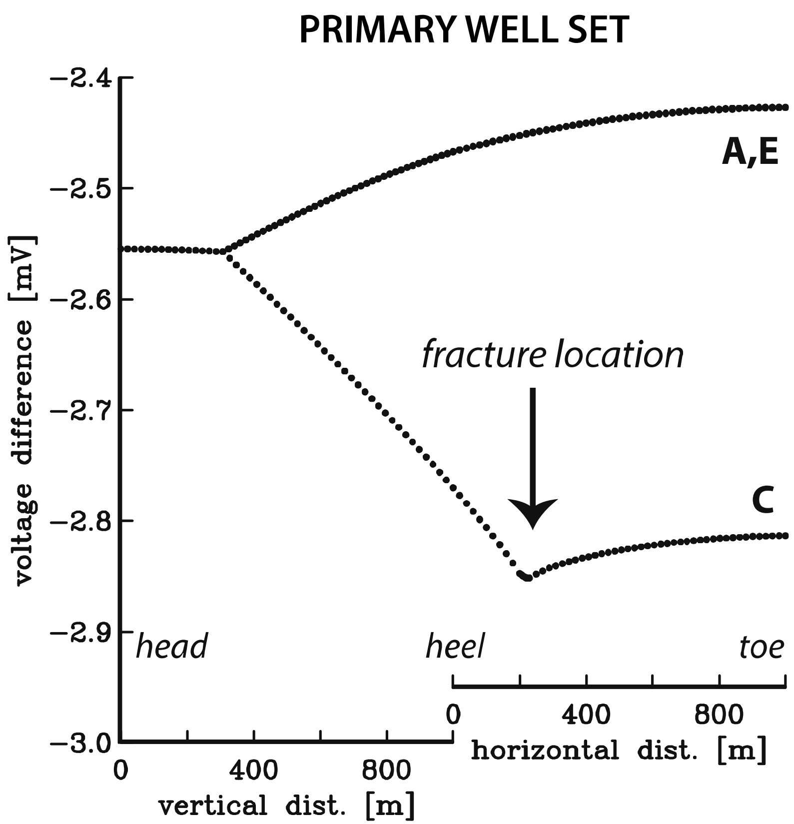
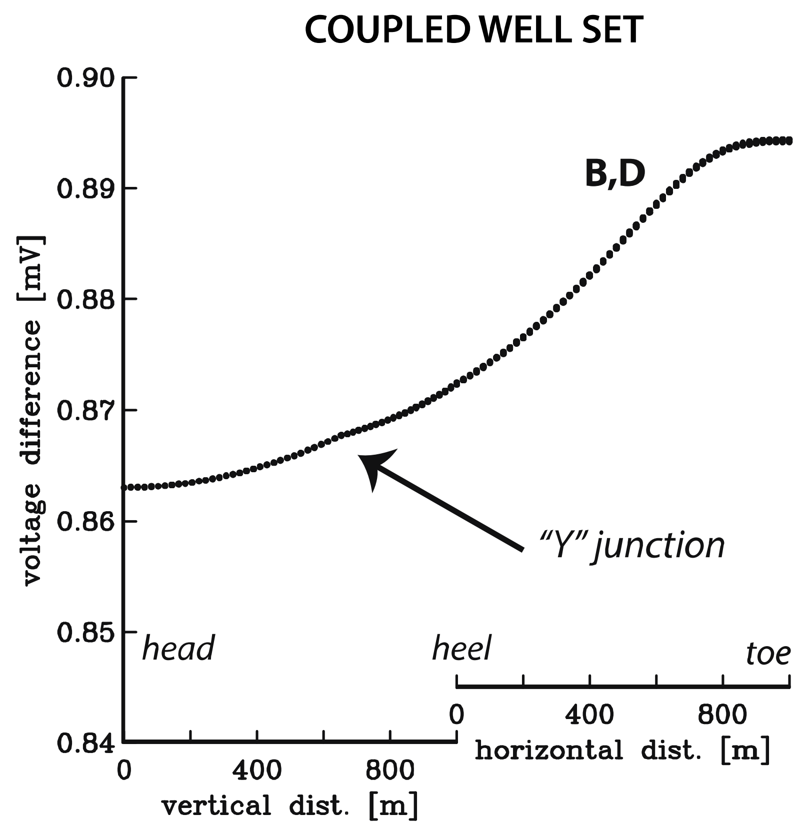


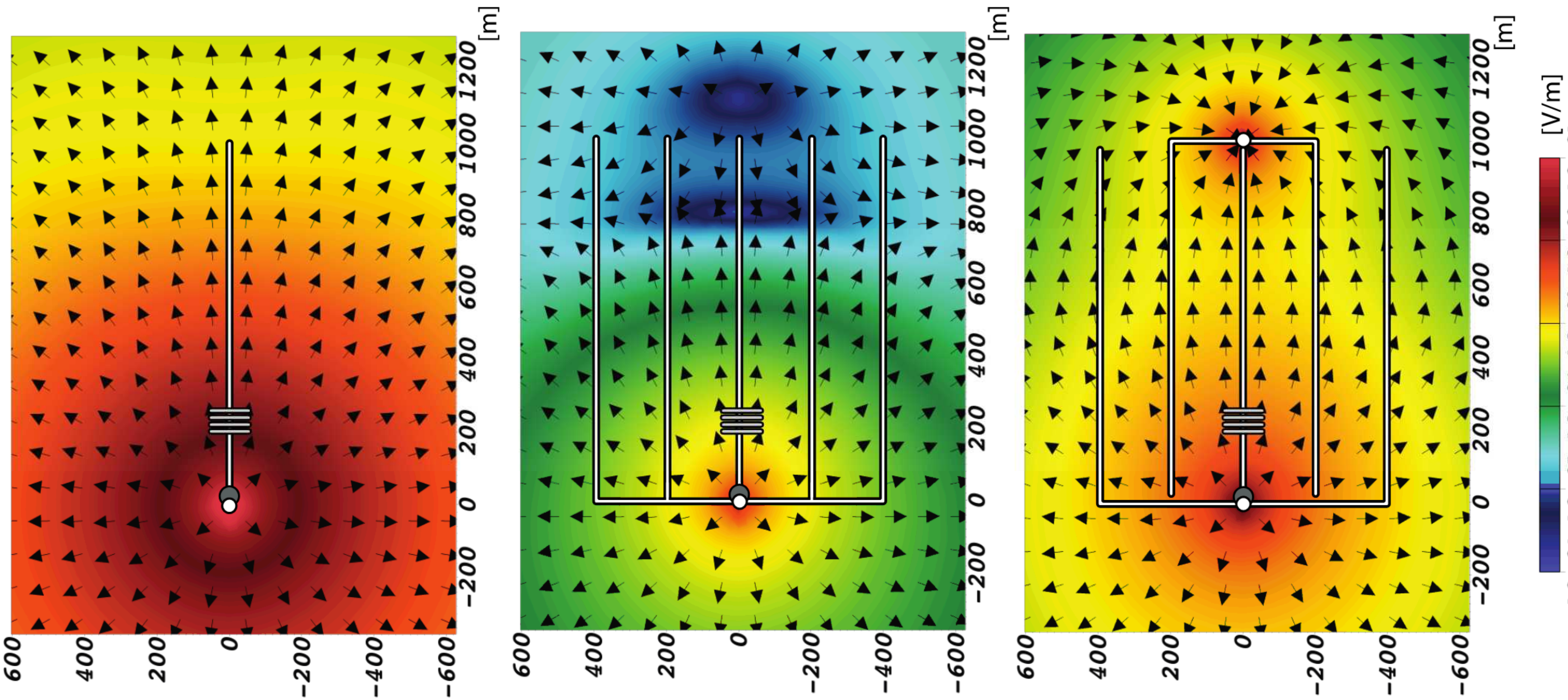


4D differences of casing potential for passively coupled casings.



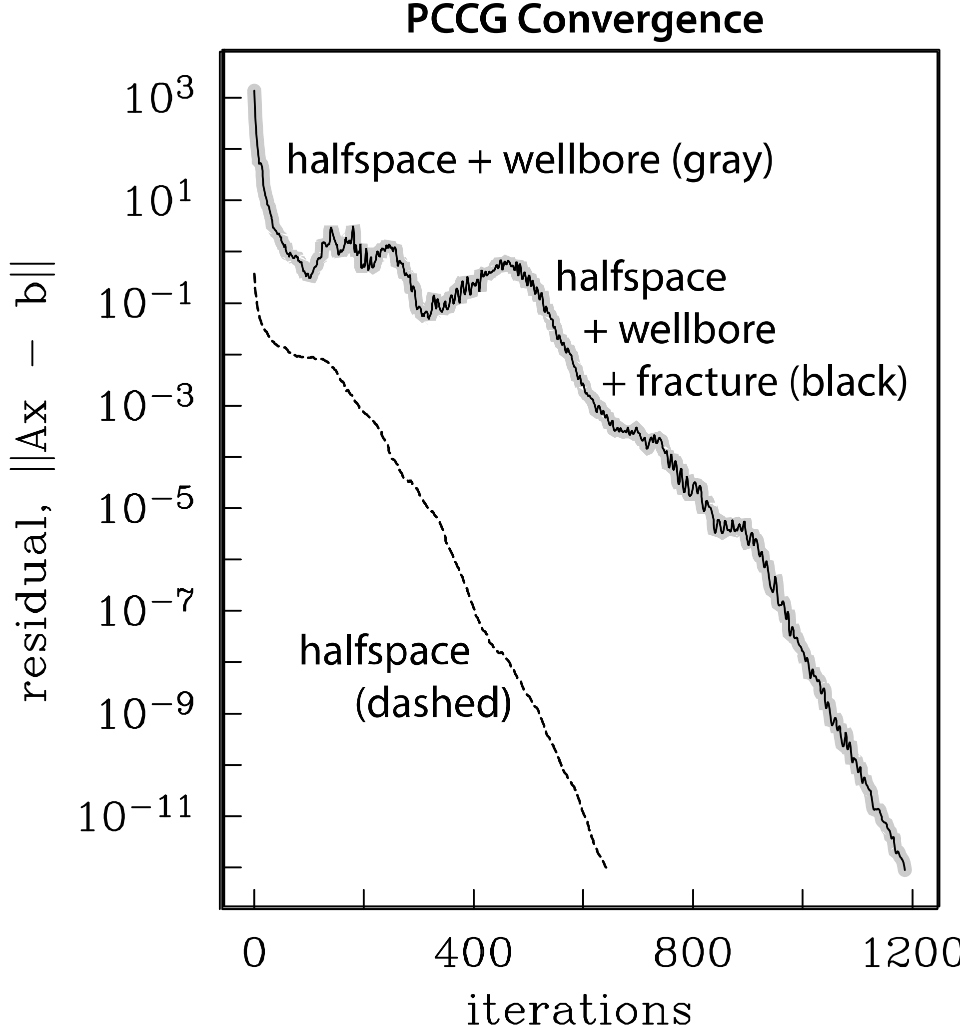


What does the time-lapse surface electric field look like for different casing configurations? Better get all the steel in your model!



Effect of hierarchical elements on PCCG convergence

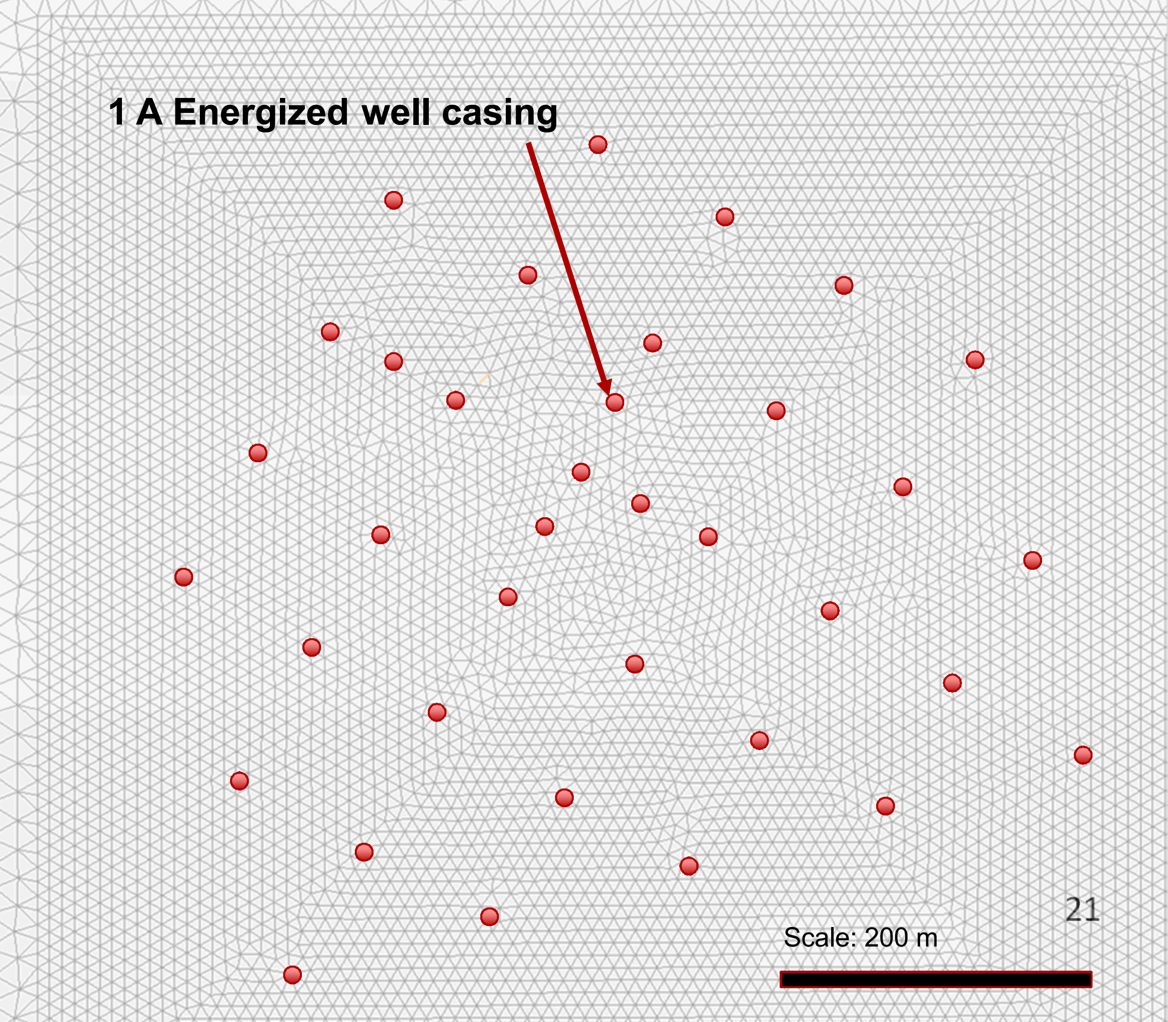
Adding steel to the model is known to increase the runtime. But still… the runtimes are on the order of a few minutes, rather than hours.



Example 1: Shallow, heavy oil reservoir.

10 m node spacing, 16 km of casing, 36 vertical wells: 1628 edges
10 m node spacing on air/earth interface over oilfield
1.4M tets, 238k nodes, 10 x 10 x 5 km domain

Casing model:
20 cm OD
2.5 cm wall thickness
5e6 S/m conductivity
t_e = 5e4 S.m



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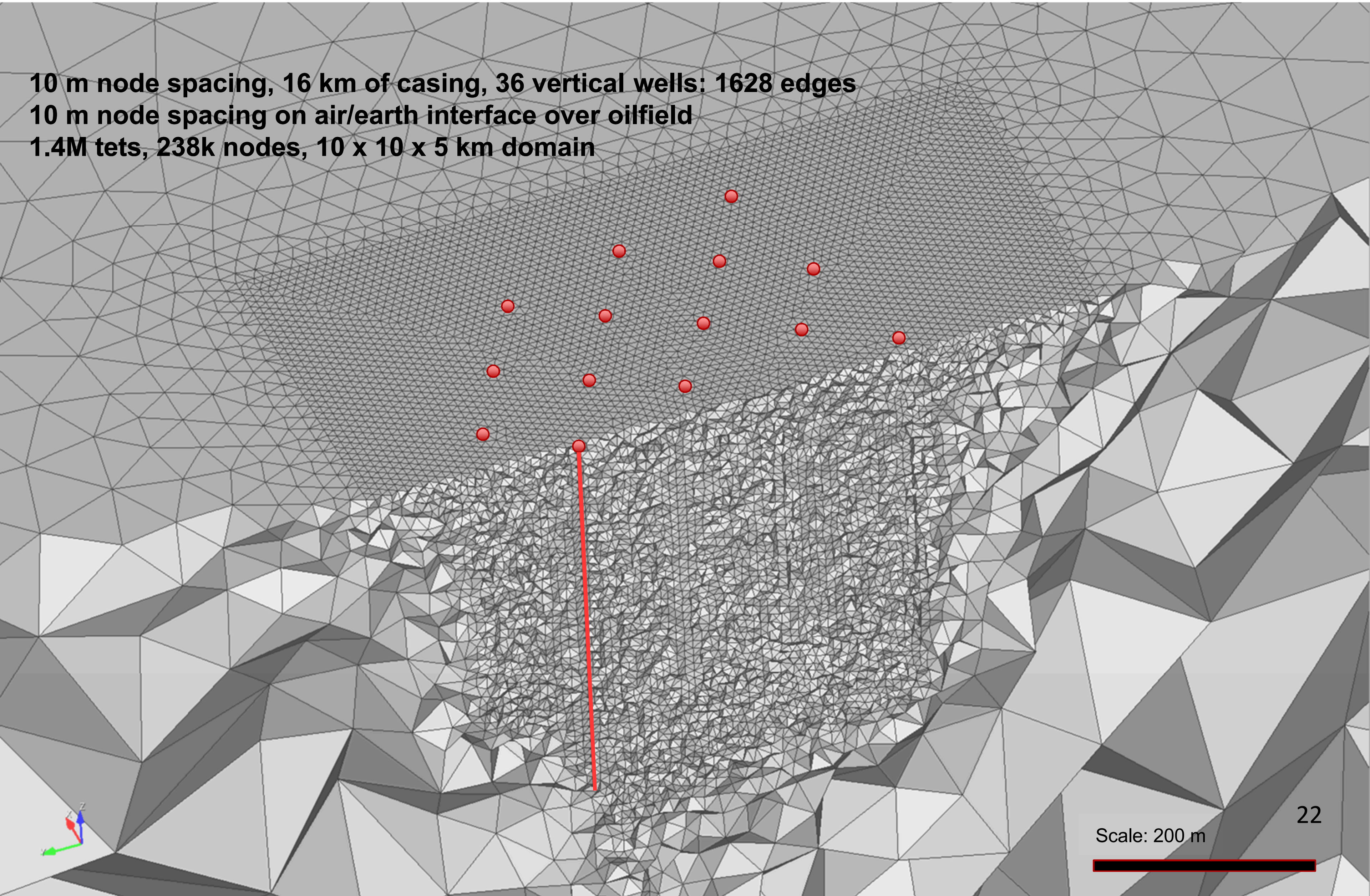

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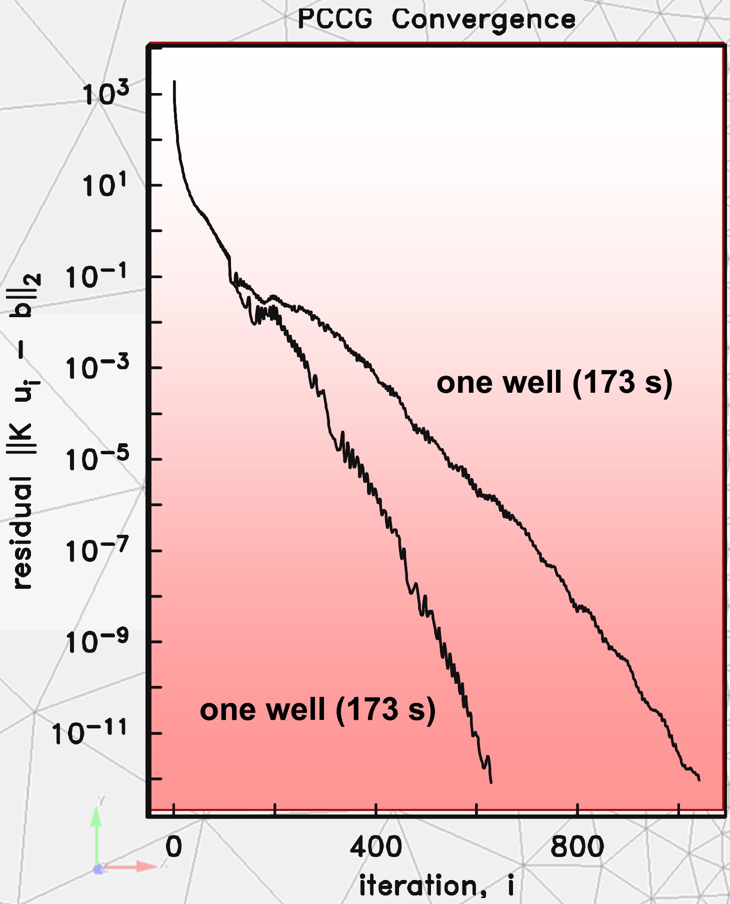
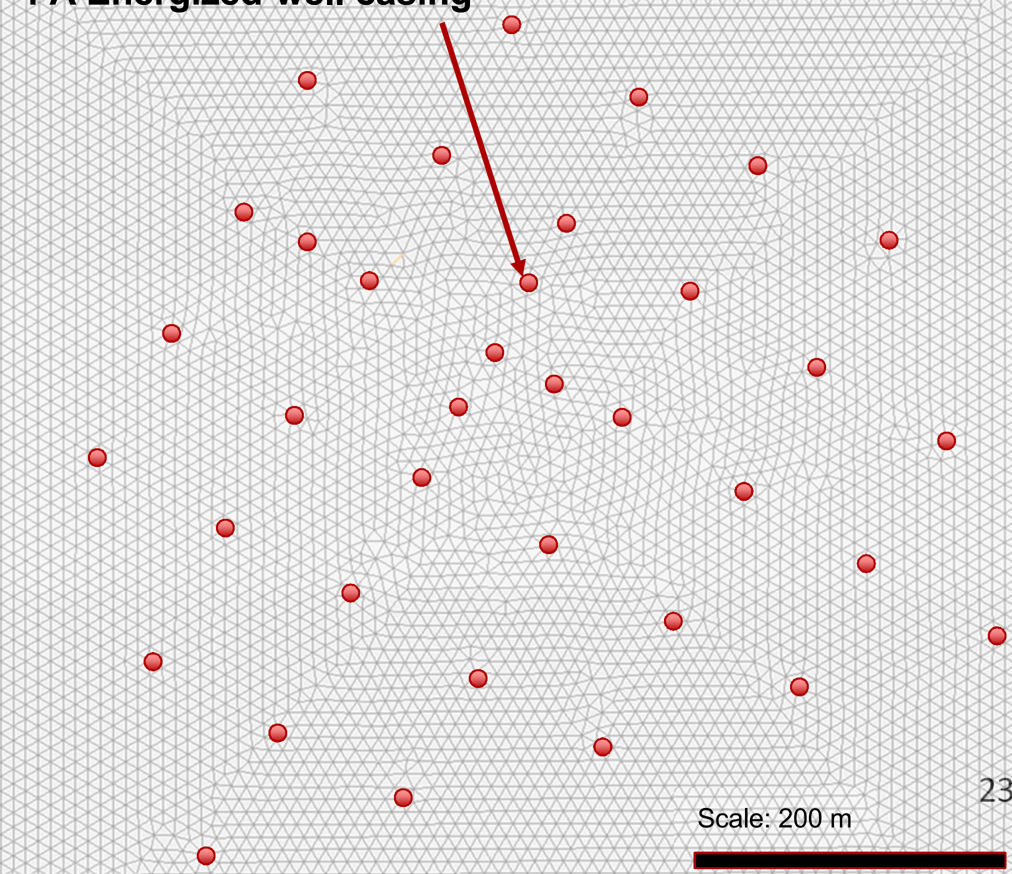
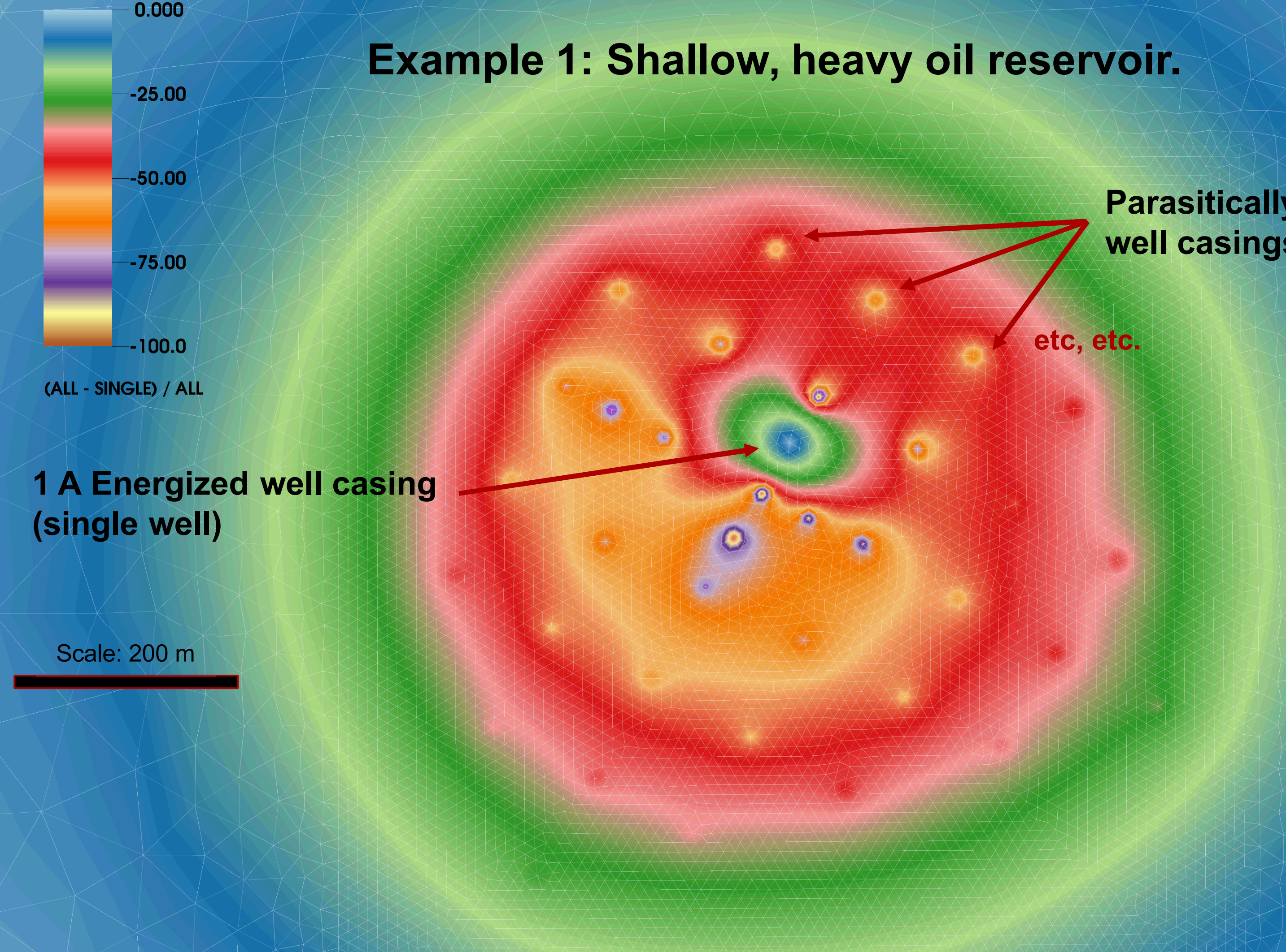
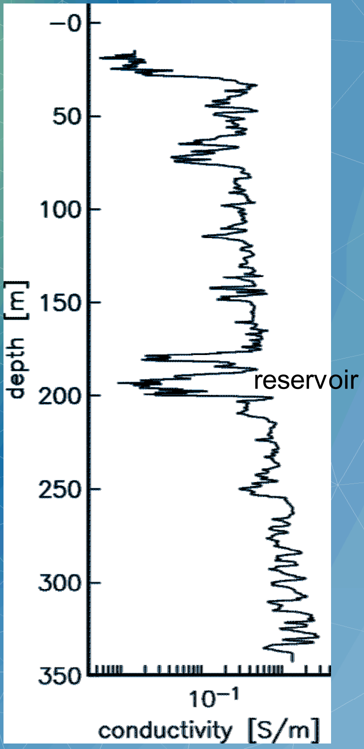




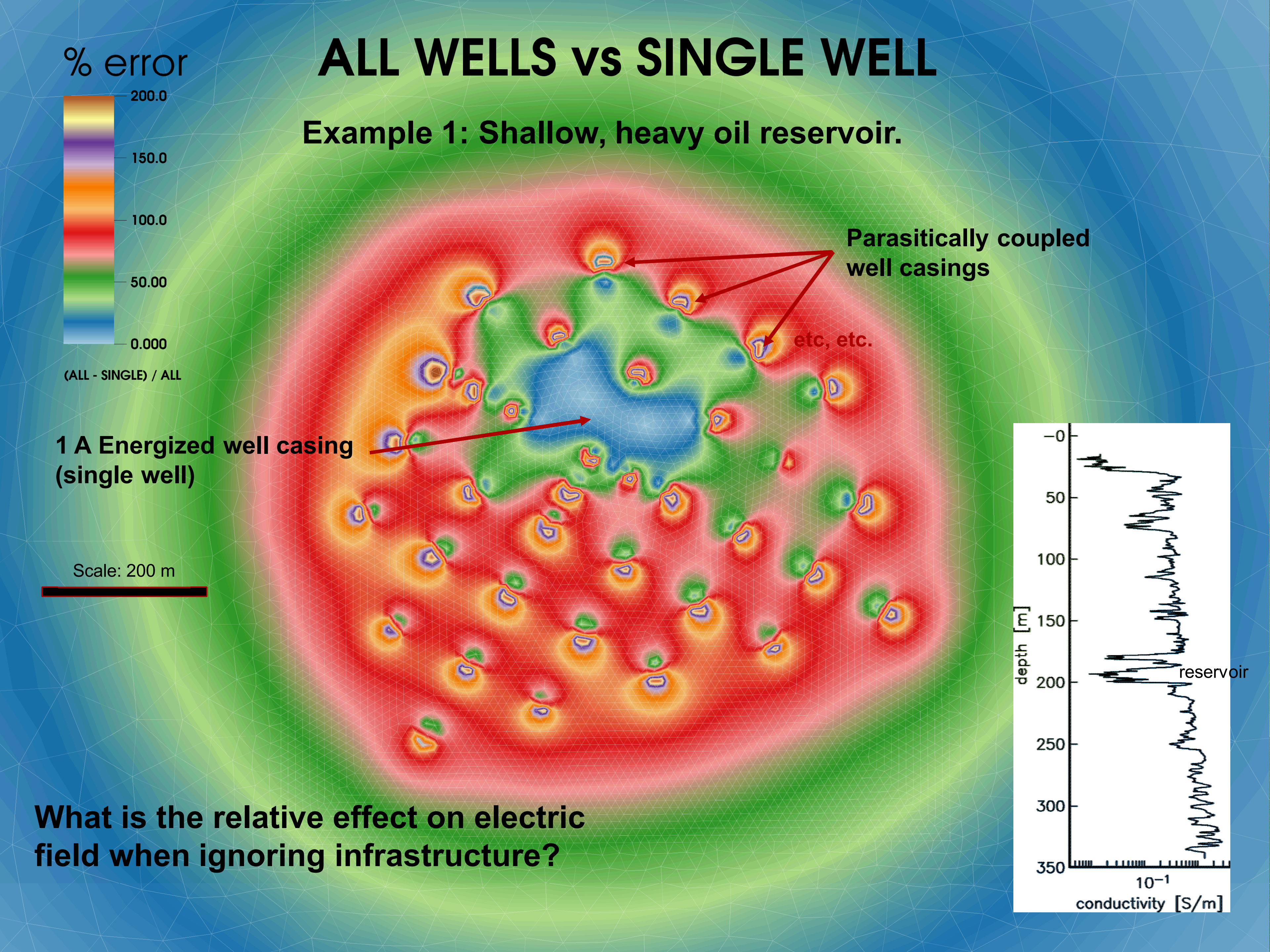
ALL WELLS vs SINGLE WELL

Example 1: Shallow, heavy oil reservoir.





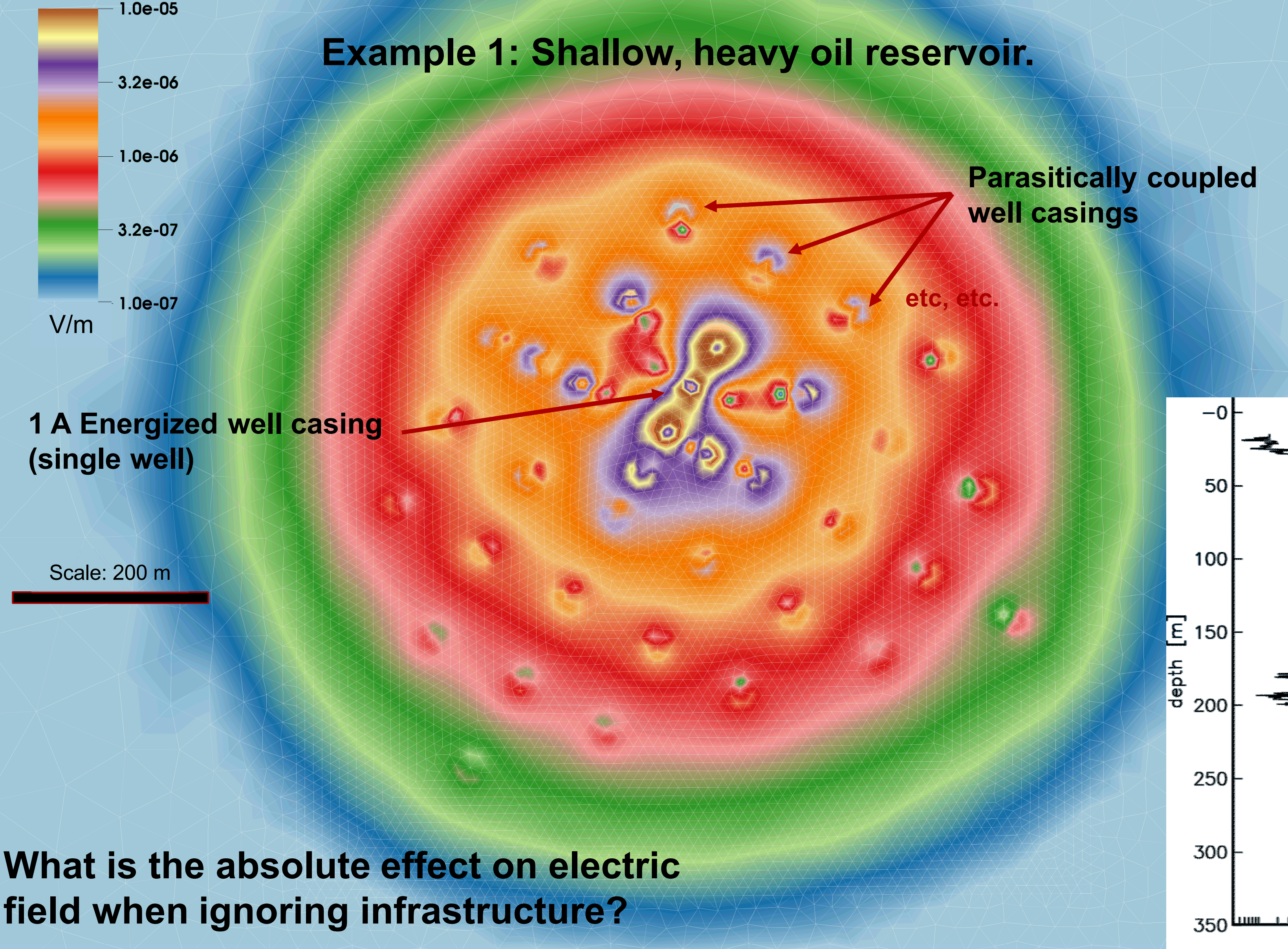
What is the relative effect on electrostatic potential when ignoring infrastructure?

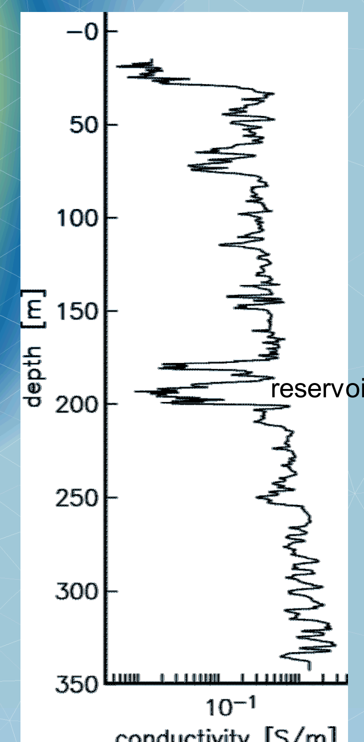


ALL WELLS vs SINGLE WELL
Example 1: Shallow, heavy oil reservoir.
% error
200.0
150.0
100.0
50.00
0.000
(ALL - SINGLE) / ALL
Parasitically coupled well casings
etc, etc.
1 A Energized well casing (single well)
Scale: 200 m
What is the relative effect on electric field when ignoring infrastructure?
depth [m]
0
50
100
150
200
250
300
350
reservoir
10^−1
conductivity [S/m]

ALL WELLS vs SINGLE WELL

Example 1: Shallow, heavy oil reservoir.

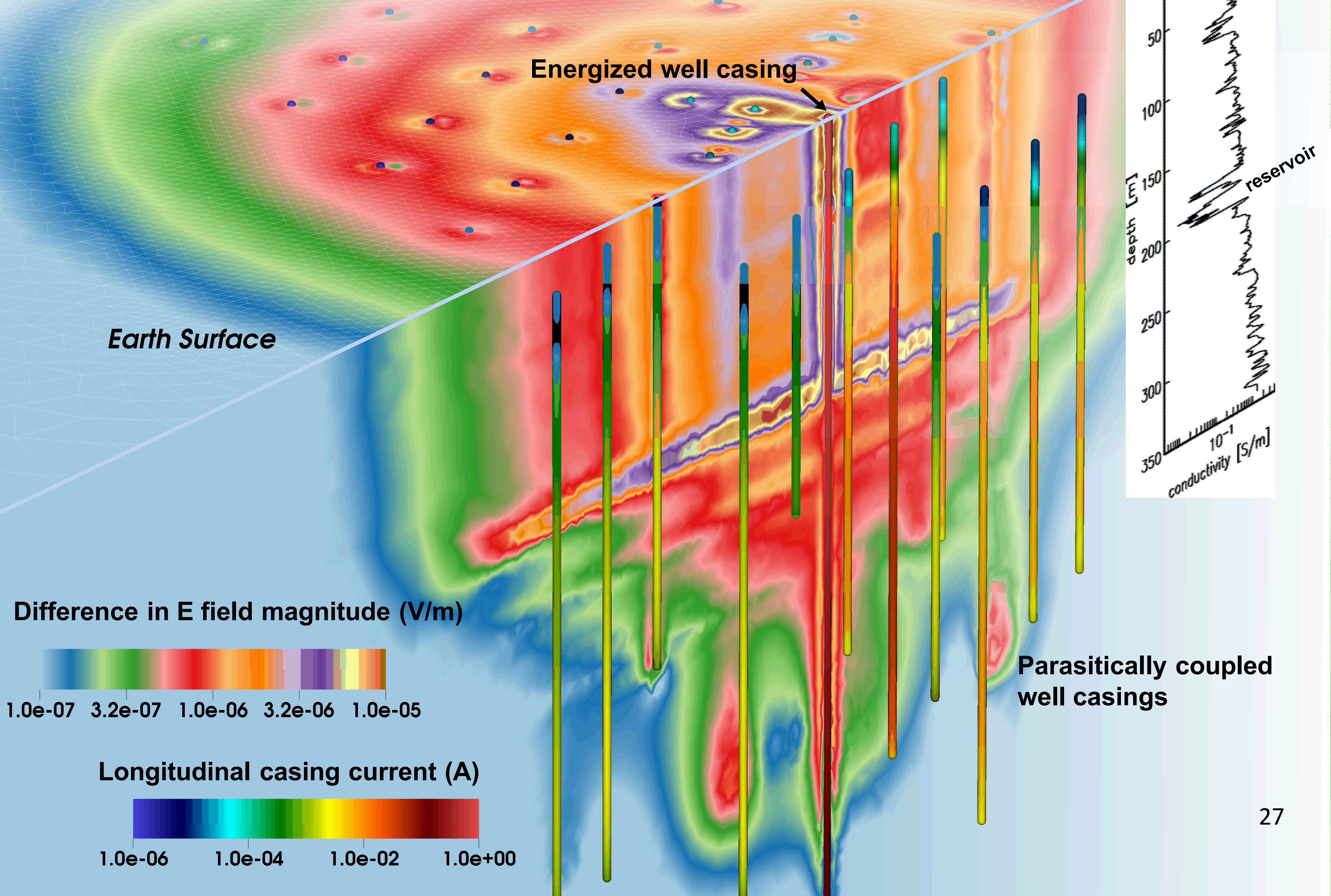




What is the absolute effect on electric field when ignoring infrastructure?

Effect of full clutter model on Electric fields

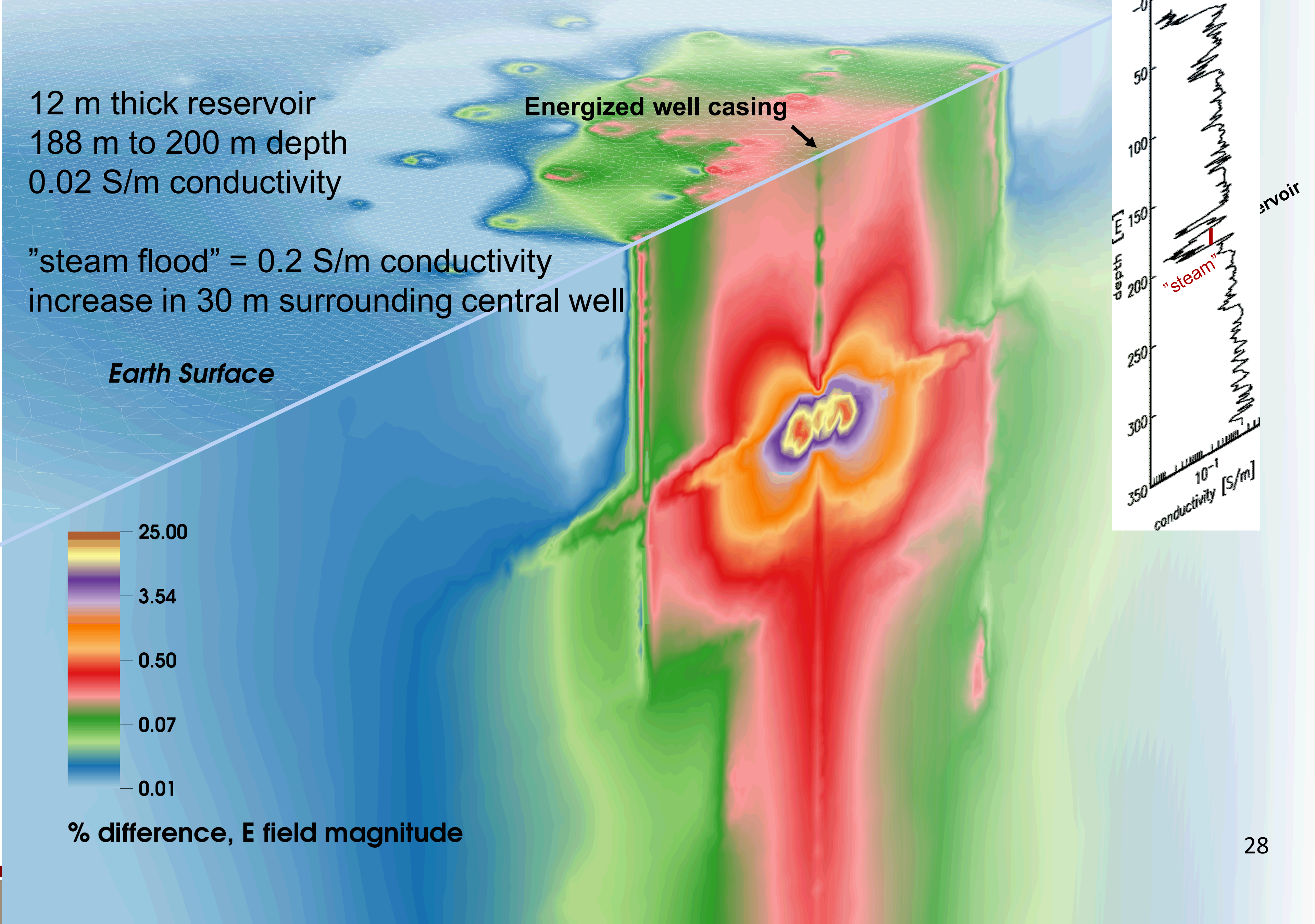
(E_all_wells – E_single_well)



Full clutter + steam injection: Electric fields

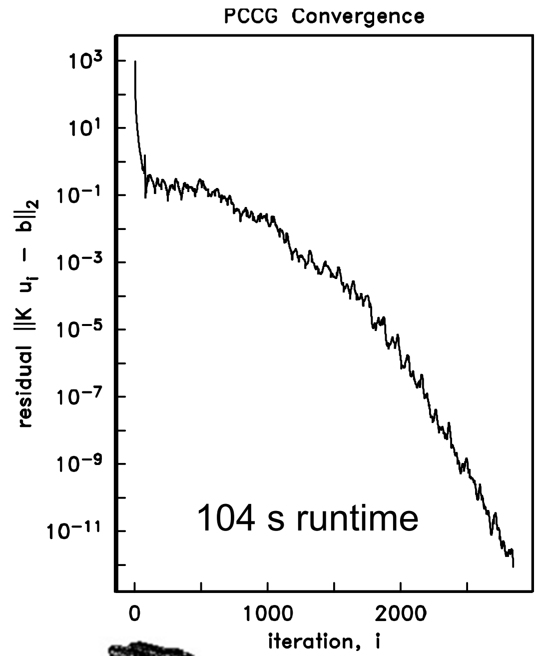
12 m thick reservoir
188 m to 200 m depth
0.02 S/m conductivity

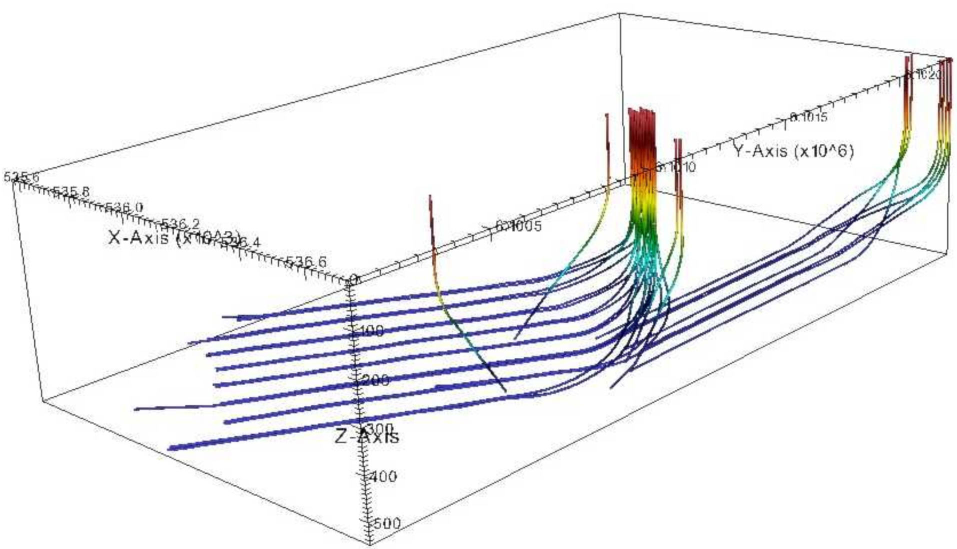
”steam flood” = 0.2 S/m conductivity
increase in 30 m surrounding central well

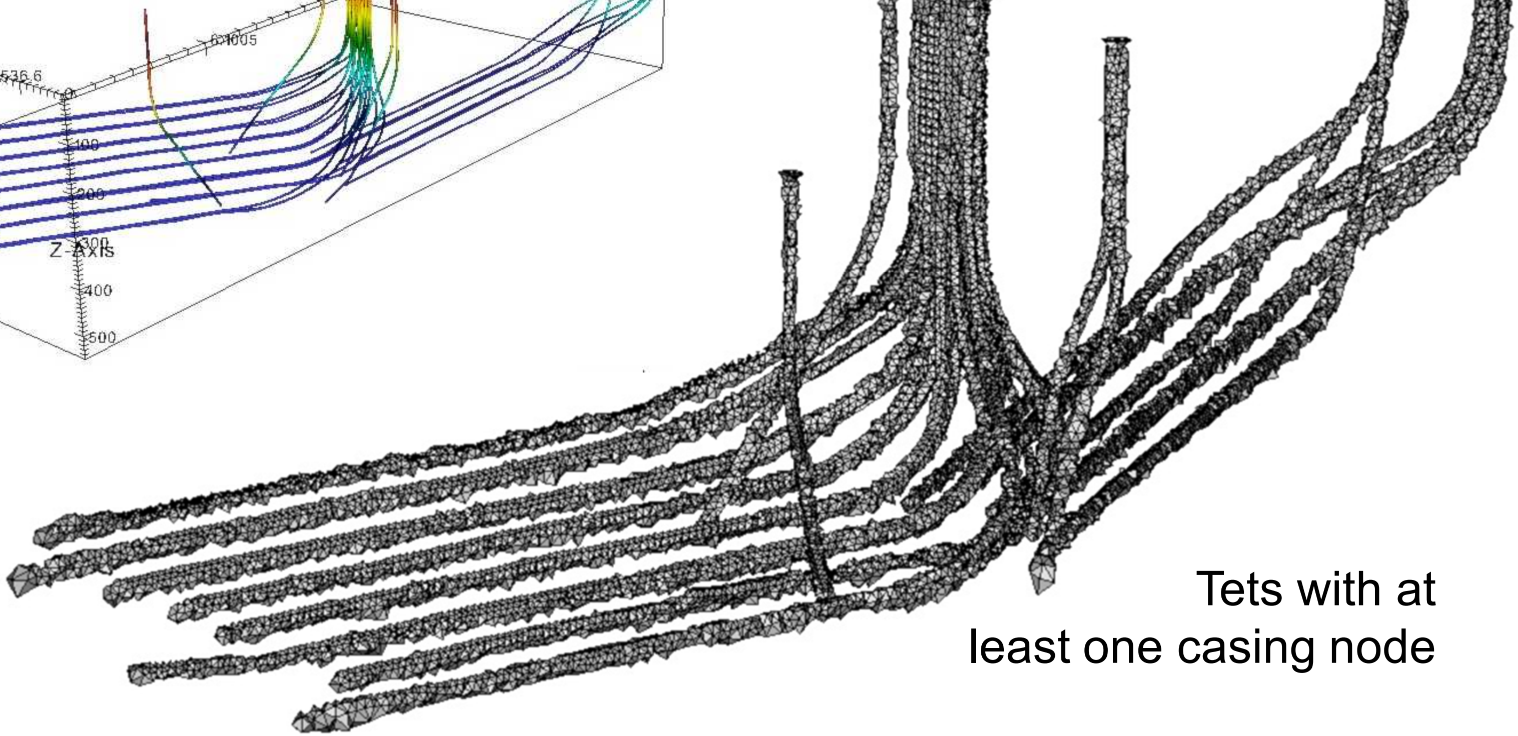


Example 2: SAGD multilateral

20 m node spacing, 45 km of casing, 31 wells: 2313 edges
50 m node spacing on air/earth interface over oilfield
332k tets, 60k nodes, 10 x 10 x 5 km domain

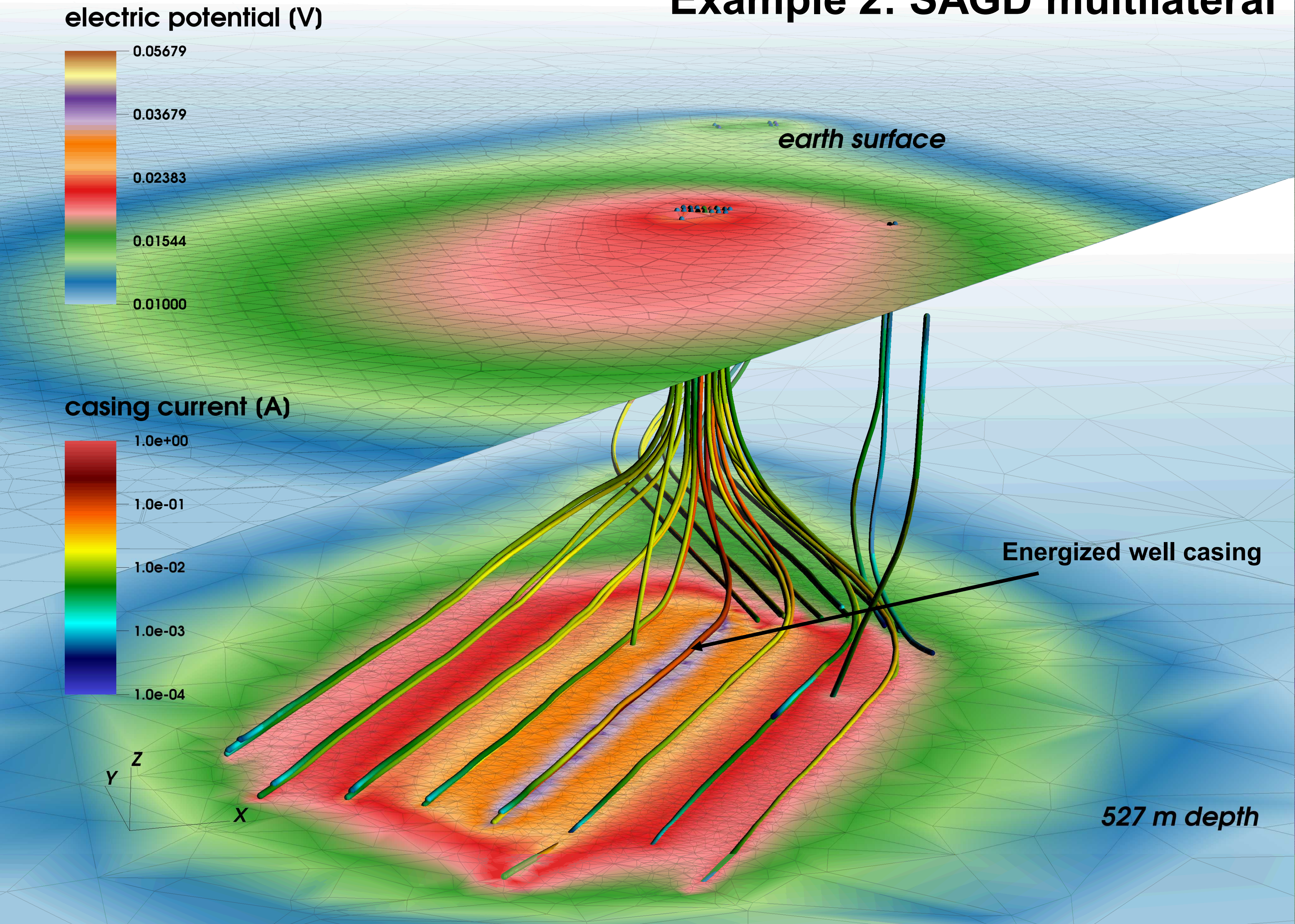




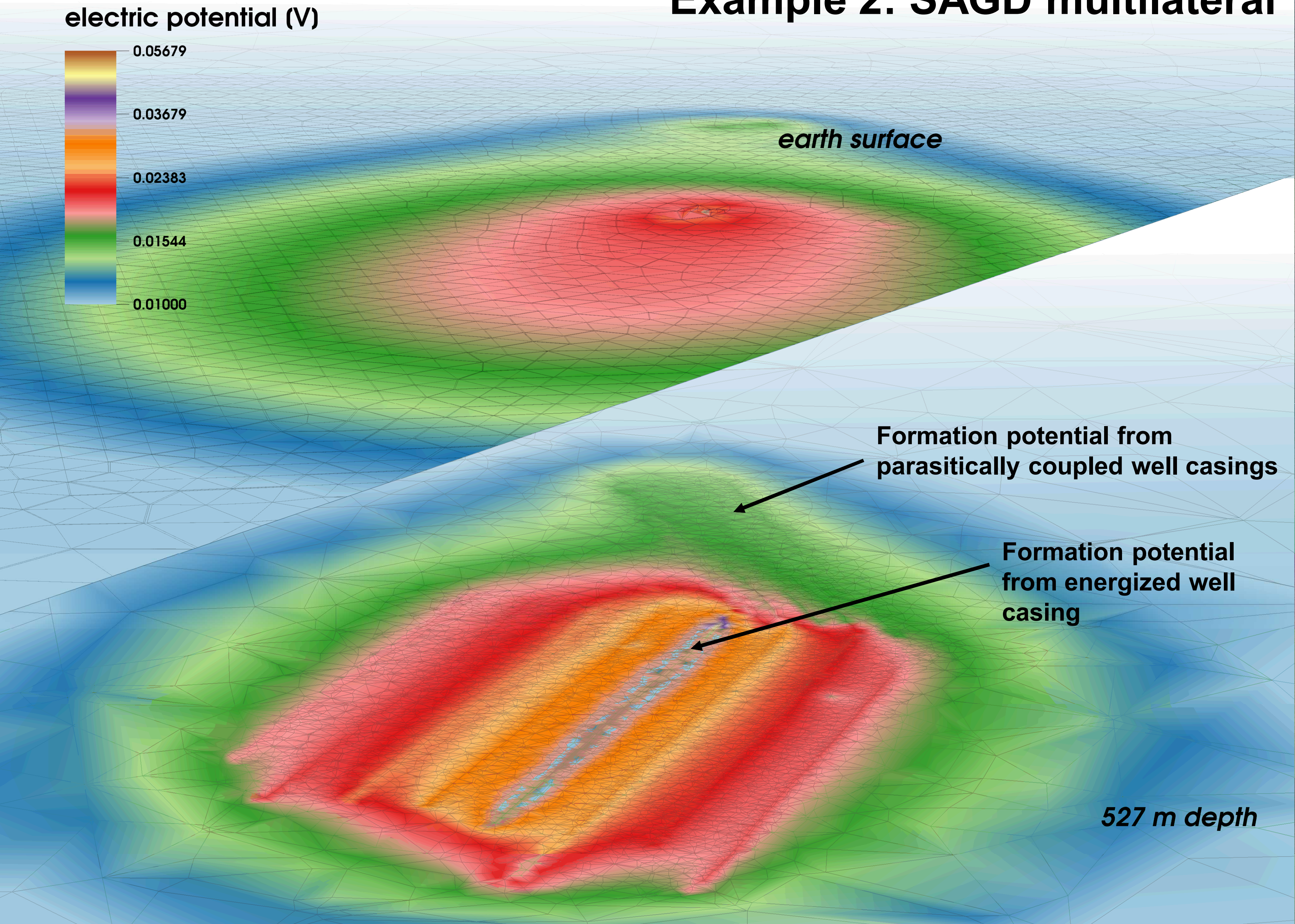


Tets with at
least one casing node

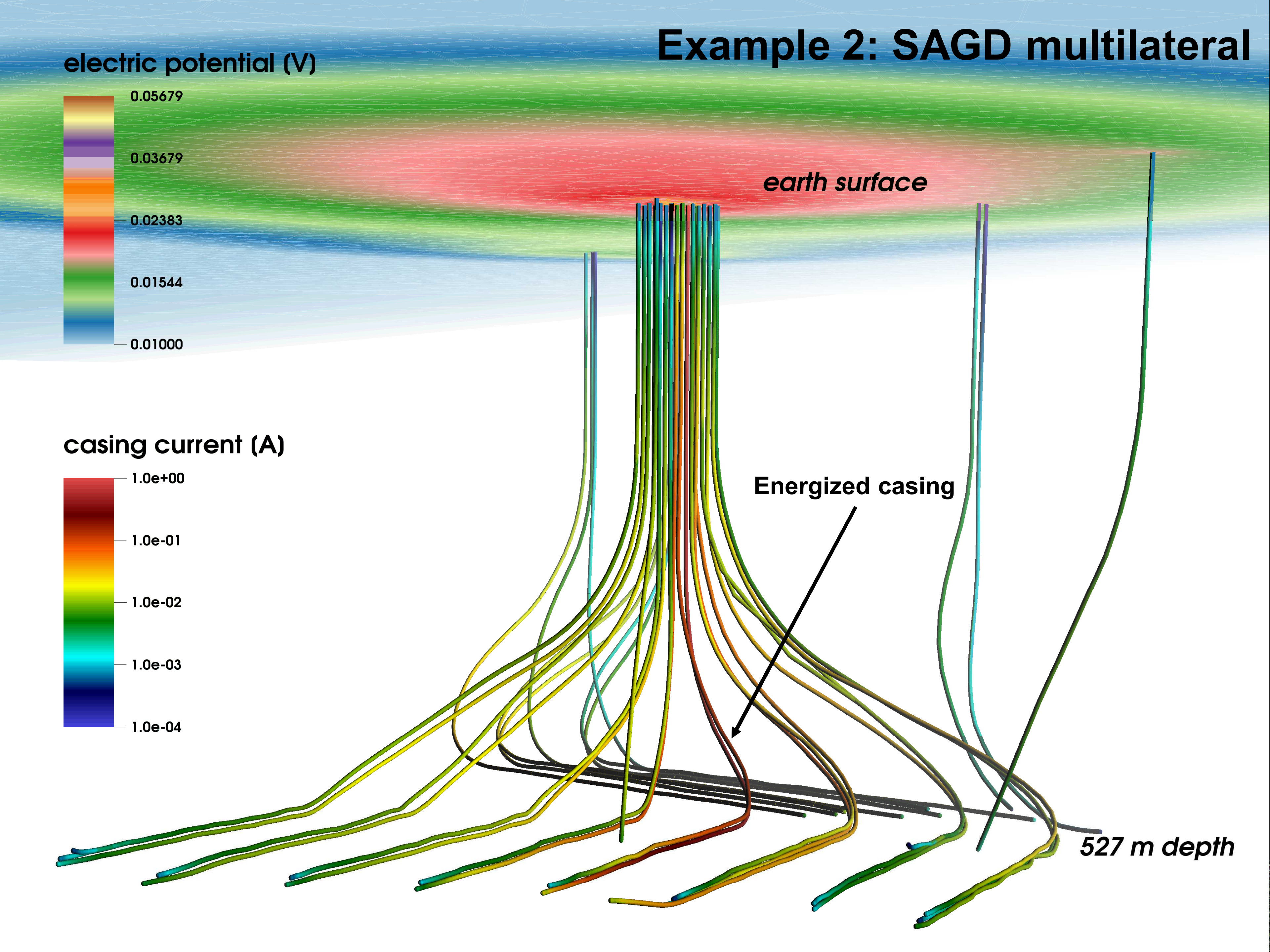
Example 2: SAGD multilateral



Example 2: SAGD multilateral



Example 2: SAGD multilateral



Example 3: Casing + surface infrastructure

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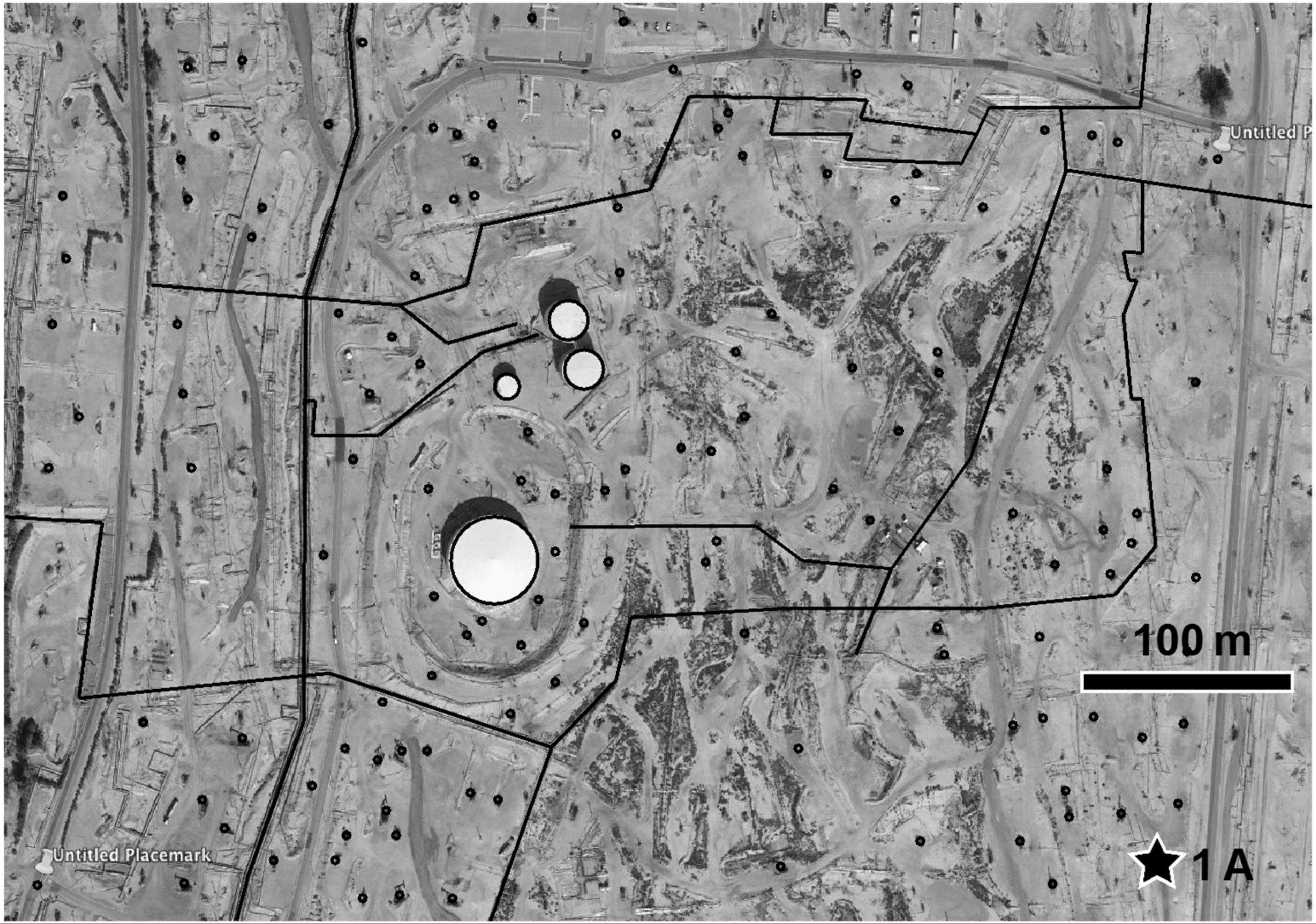
Kern River Formation Site

0.7 km^2 area + 122 wells + ~2 km surface pipes



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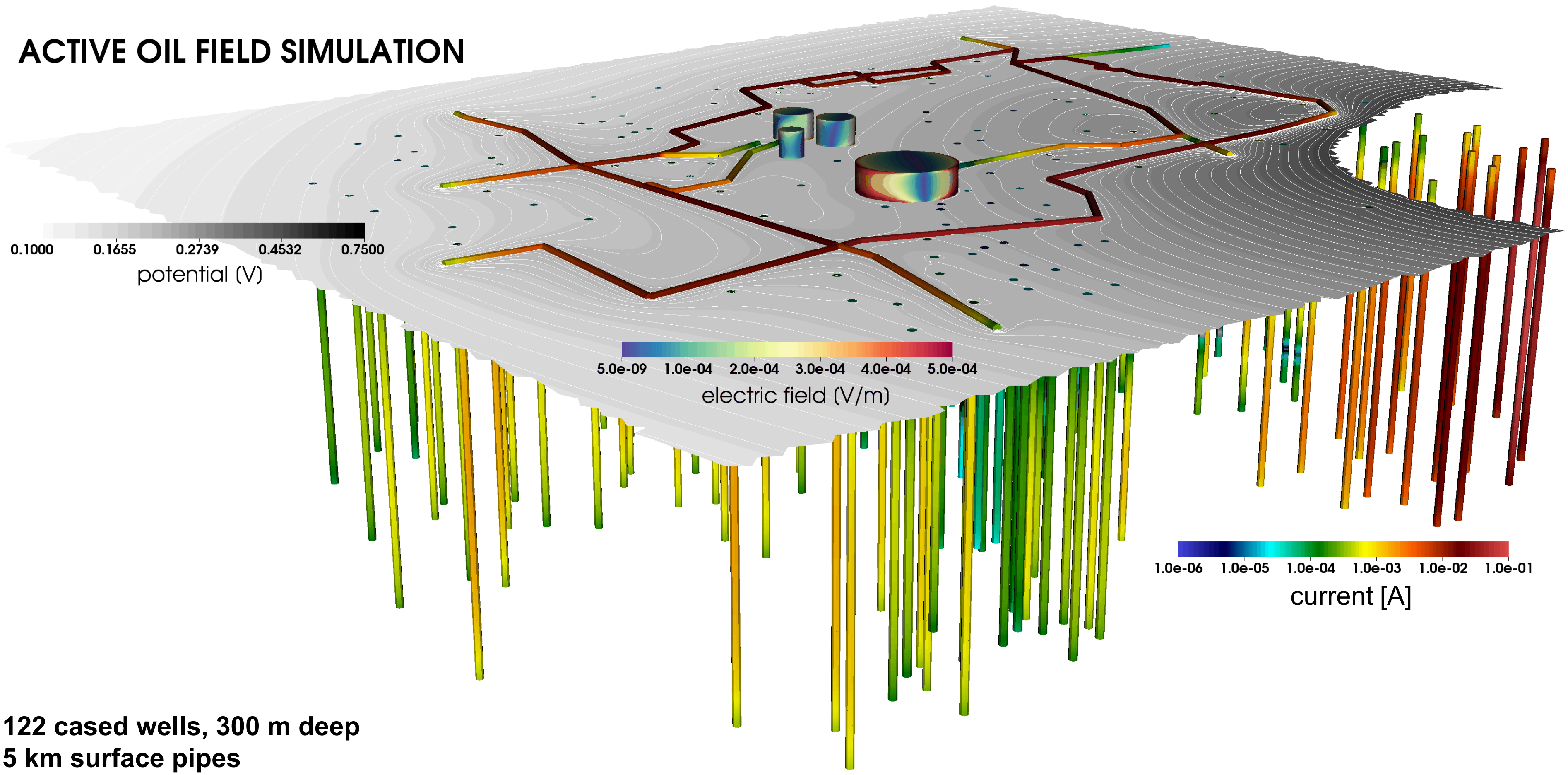
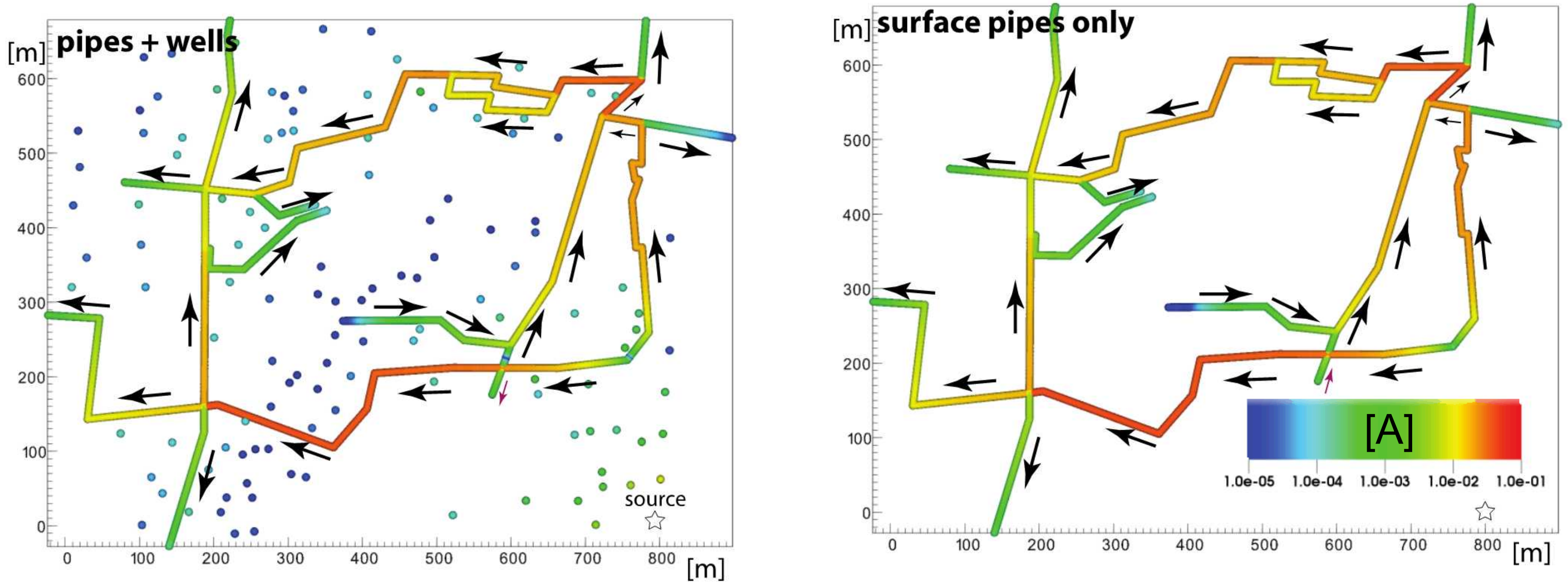


122 cased wells, 300 m deep
5 km surface pipes
~35 km pipeline/casing modeled at 10 m grid spacing: 3500 elements
Traditional FEM requires ~7e6 elements per km of pipline/casing.
HFEM decreases computational burden by ~4 orders of magnitude in this example (10 min vs 2 mo, estimated runtime)

Example 3: Casing + surface infrastructure

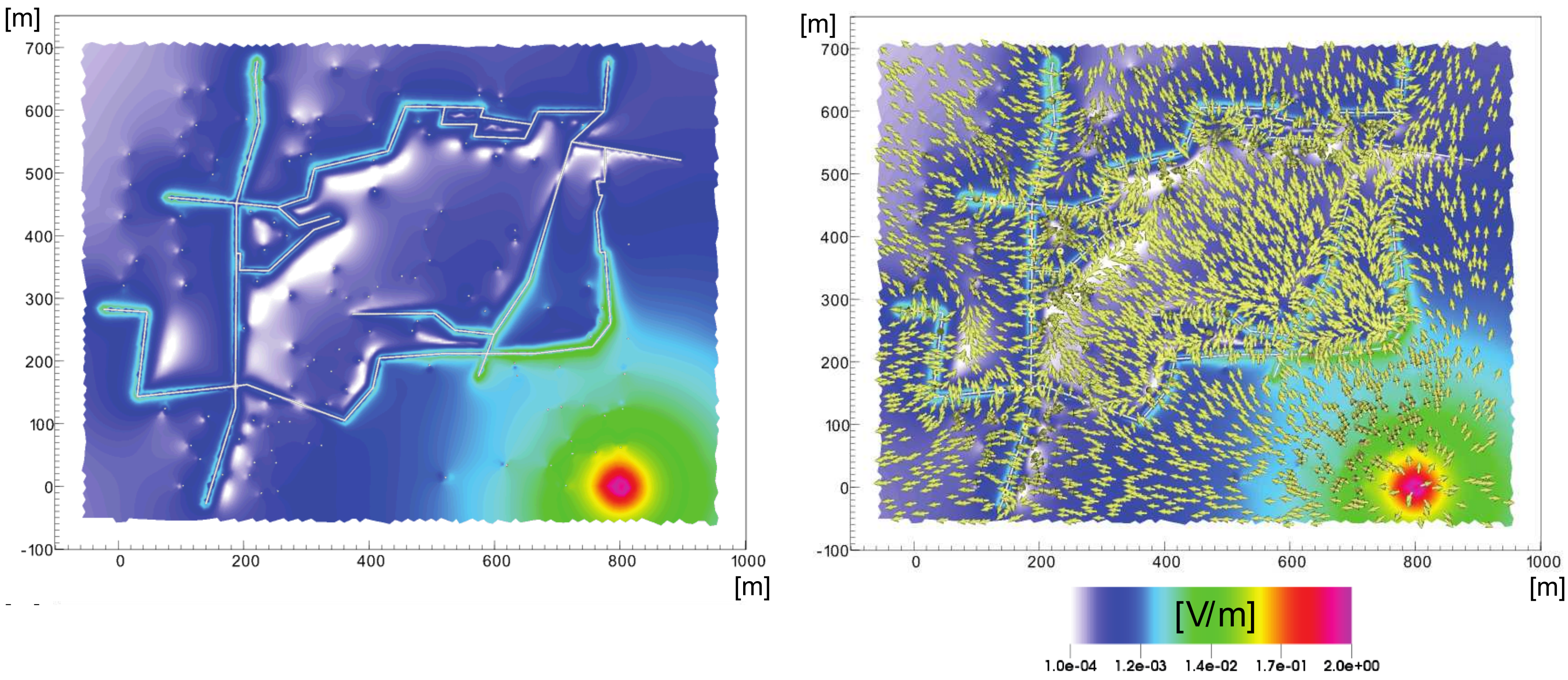
Longitudinal current (colors + arrows) for full casing/pipeline coupling (left) and pipeline alone (right).



Weiss and Wilson (2017)

Example 3: Casing + surface infrastructure

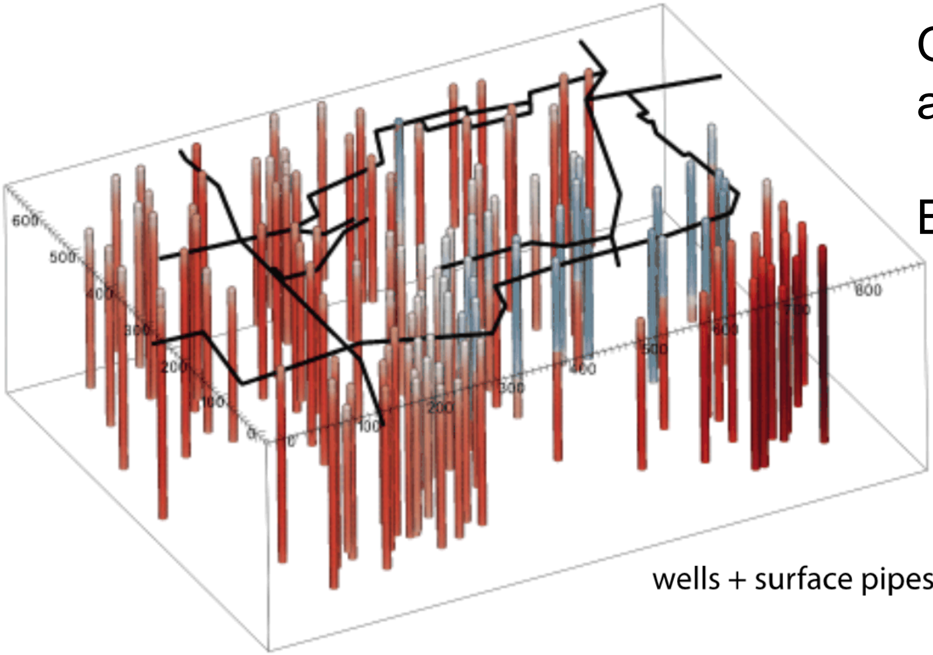
Surface electric field for fully coupled model. Amplitude only (left), amplitude + direction (right)



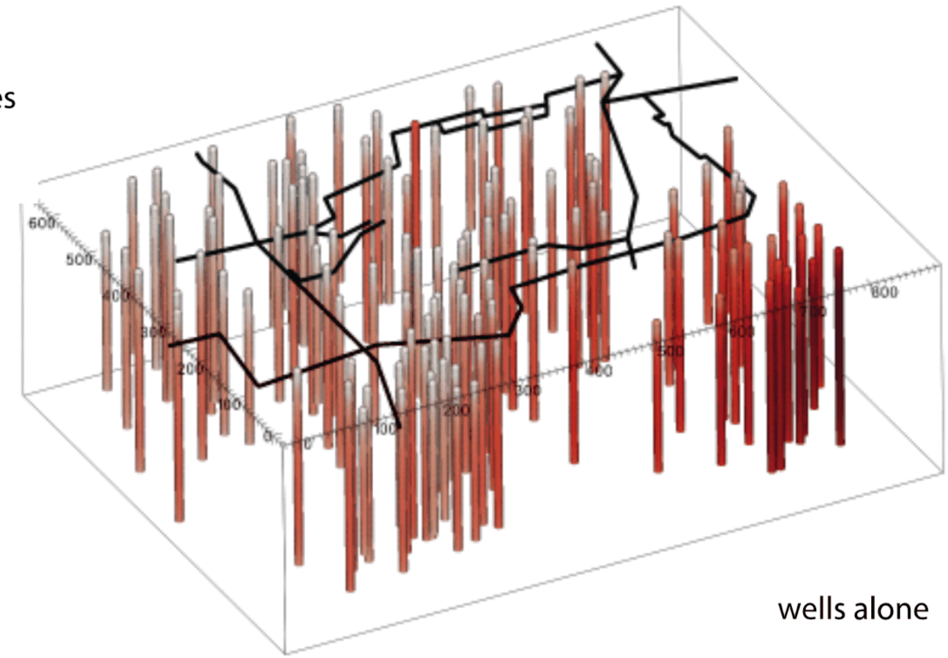
Kern River Formation Site

QUESTION: How does pipeline coupling affect casing current direction?

Blue, upward current; red, downward.



wells + surface pipes



wells alone

Conclusions and comments

Hierarchical material properties in finite element analysis offers a computationally economical way for modeling sharp, volumetrically insignificant regions, with elevated material property values (e.g. conductivity in electrostatics)

The reduction is computational burden over volumetric discretization can reach several orders of magnitude, thus leading to “real time” solutions and evaluation of problems previously believed intractable.

For the electrostatic problem, hierarchical FE solutions compare favorably with independent analytic solutions and are internally consistent with solutions from volume discretizations.

Hierarchical FE method has been applied to various “real world” oilfield examples with complex infrastructure. Although solution times are fast (10s of seconds to a minute or so), mesh generation and metadata management issues are more acute.

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