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Theoretical Formulation of an Ambient Stress State in Isotropic Elastic Media

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Theoretical Formulation of an Ambient Stress State in Isotropic Elastic Media

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Abstract

Due to the weight of overburden and tectonic forces, the solid earth is subject to an ambient stress state. This stress state is quasi-static in that it is generally in a state of equilibrium. Typically, seismology assumes this ambient stress field has a negligible effect on wave propagation. However, two basic theories have been put forward to describe the effects of ambient stress on wave propagation. Dahlen and Tromp (2002) expound a theory based on perturbation analysis that largely supports the traditional seismological view that ambient stress is negligible for wave propagation. The second theory, espoused by Korneev and Glubokovskikh (2013) and supported by some experimental work, states that perturbation analysis is inappropriate since the elastic modulus is very sensitive to the ambient stress states. This brief report reformulates the equations given by Korneev and Glubokovskikh (2013) into a more compact form that makes it amenable to statement in terms of a pre-stress form of Hooke's Law. Furthermore, this report demonstrates the symmetries of the pre-stress modulus tensor and discusses the reciprocity relationship implied by the symmetry conditions.

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NOMENCLATURE

Abbreviation	Definition
KG	Korneev and Glubokovskikh
DT	Dahlen and Tromp

1. INTRODUCTION

The rocks within the solid earth are subject to ambient stress fields due to the weight of overburden and tectonic forces. It is commonly assumed in seismology that the effects of this ambient stress state are negligible for typical seismological observations. However, ambient stress does to at least some extent affect wave propagation and two main theories have been advanced to describe these effects. These two theories, Dahlen and Tromp (2002) and Korneev and Glubokovskikh (2013), view the problem from two different aspects and come to two very different conclusions. Dahlen and Tromp take a linearized expansion theory approach to derive their pre-stress equations, while Korneev and Glubokovskikh make the case that expansion theory is inappropriate since the higher order terms relating the change in the elastic modulus tensor to a change in ambient stress state are actually larger in magnitude than the elastic modulus tensor itself. They retain the higher order terms, which in turn retains second order products of strains, making for a more complex theory.

In this report, the theory developed by Korneev and Glubokovskikh is expressed in simpler terms that makes it more amenable for future forward modeling in the presence of ambient stress and perhaps inversion of ambient stress from seismic data. The symmetries of the pre-stress elastic modulus tensor from their theory are derived and shown to match those of the standard elastic modulus tensor and on that basis an argument that the same reciprocity relationships apply in a pre-stressed media as in the assumed stress-less state.

2. AMBIENT STRESS EQUATIONS

2.1. Korneev and Glubokovskikh Equations

One of the two formulations of seismic wave propagation in the presence of static (or ambient or tectonic) stress is given in Korneev and Glubokovskikh (2013) (hereafter referred to KG). This is a nonlinear formulation as opposed to Dalen and Tromp (2002) (hereafter referred to DT), which assume linearity throughout. Much of KG's basic derivation is repeated here for completeness.

The total displacement (static plus transient) and total stress in the presence of ambient stress in an isotropic elastic media is related via (KG Equation 2, revised to clarify summation convention and verified in Gurbatov et al., 2012):

$$\begin{aligned}
\sigma_{ik} = {} & \lambda \frac{\partial u_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \\
& \left(\mu + \frac{A}{4}\right)\left(\frac{\partial u_s}{\partial x_i}\frac{\partial u_s}{\partial x_k} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_s}{\partial x_k}\frac{\partial u_i}{\partial x_s}\right) \\
& \frac{(B+\lambda)}{2}\left(\frac{\partial u_s}{\partial x_j}\frac{\partial u_s}{\partial x_j}\delta_{ik} + 2\frac{\partial u_i}{\partial x_k}\frac{\partial u_s}{\partial x_s}\right) + \frac{A}{4}\frac{\partial u_k}{\partial x_s}\frac{\partial u_s}{\partial x_i} \\
& \frac{B}{2}\left(\frac{\partial u_s}{\partial x_j}\frac{\partial u_j}{\partial x_s}\delta_{ik} + 2\frac{\partial u_k}{\partial x_i}\frac{\partial u_s}{\partial x_s}\right) + C\frac{\partial u_s}{\partial x_s}\frac{\partial u_s}{\partial x_s}\delta_{ik}
\end{aligned} \tag{1}$$

where σ_{ik} is the stress (function of space and time), u_i is total displacement (function of space and time), λ, μ, A, B, and C are unstressed elastic constants for nonlinear materials (functions of space), δ_{ik} is the kronecker delta function, and subscripts s and j are summation indices. For known materials A, B, and C (the second order elastic constants) are much larger in magnitude than λ, μ (Korneev and Glubokovskikh, 2013); thus, we can make the following approximation (KG Equation 3):

$$\begin{aligned}
\sigma_{ik} = {} & \lambda \frac{\partial u_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \\
& \frac{A}{4}\left(\frac{\partial u_s}{\partial x_i}\frac{\partial u_s}{\partial x_k} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_s}{\partial x_k}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_s}{\partial x_i}\right) \\
& \frac{B}{2}\left(\frac{\partial u_s}{\partial x_j}\frac{\partial u_s}{\partial x_j}\delta_{ik} + 2\frac{\partial u_i}{\partial x_k}\frac{\partial u_s}{\partial x_s} + \frac{\partial u_s}{\partial x_j}\frac{\partial u_j}{\partial x_s}\delta_{ik} + 2\frac{\partial u_k}{\partial x_i}\frac{\partial u_s}{\partial x_s}\right) \\
& + C\frac{\partial u_s}{\partial x_s}\frac{\partial u_s}{\partial x_s}\delta_{ik}
\end{aligned} \tag{2}$$

The equations of motion are the same as usual:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k} + f_i \tag{3}$$

where ρ is density (a function of space), t is time, and f_i is a component of a body force vector.

We can decompose the total displacement and stress into ambient and transient terms:

$$\begin{aligned} \mathbf{u} &\approx \mathbf{U} + \mathbf{w} \\ \sigma_{ik} &\approx \bar{\sigma}_{ik} + \tilde{\sigma}_{ik} \end{aligned} \tag{4}$$

where $\mathbf{U}$ is the static displacement, $\bar{\sigma}_{ik}$ is the static stress (both assumed here to be independent of time), $\mathbf{w}$ is the transient displacement, and $\tilde{\sigma}_{ik}$ is the transient stress.

Ignoring all transient terms, we find (KG Equation 7):

$$\begin{aligned} \bar{\sigma}_{ik} = {} & \lambda \frac{\partial U_s}{\partial x_s} \delta_{ik} + \mu \left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i} \right) \\ & \frac{A}{4} \left(\frac{\partial U_s}{\partial x_i} \frac{\partial U_s}{\partial x_k} + \frac{\partial U_k}{\partial x_s} \frac{\partial U_i}{\partial x_s} + \frac{\partial U_s}{\partial x_k} \frac{\partial U_i}{\partial x_s} + \frac{\partial U_k}{\partial x_s} \frac{\partial U_s}{\partial x_i} \right) \\ & \frac{B}{2} \left(\frac{\partial U_s}{\partial x_j} \frac{\partial U_s}{\partial x_j} \delta_{ik} + 2 \frac{\partial U_i}{\partial x_k} \frac{\partial U_s}{\partial x_s} + \frac{\partial U_s}{\partial x_j} \frac{\partial U_j}{\partial x_s} \delta_{ik} + 2 \frac{\partial U_k}{\partial x_i} \frac{\partial U_s}{\partial x_s} \right) \\ & + C \frac{\partial U_s}{\partial x_s} \frac{\partial U_s}{\partial x_s} \delta_{ik} \end{aligned} \tag{5}$$

The above equation relates static (ambient) stress to static displacements.

If we assume that for wave propagation that the transient displacements are small relative to the static displacements, it allows us to neglect terms in the square of $\mathbf{w}$. The transient stress-displacement equations, derived by substituting Equation 4 into Equation 2 and ignoring terms involving the square of $\mathbf{w}$ (KG Equation 9):

$$
\begin{aligned}
\tilde{\sigma}_{ik} &= \lambda \frac{\partial w_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right) \\
&+ \frac{A}{4}\left(\frac{\partial U_s}{\partial x_i}\frac{\partial w_s}{\partial x_k} + \frac{\partial w_s}{\partial x_i}\frac{\partial U_s}{\partial x_k} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_i}{\partial x_s}\right. \\
&\left. + \frac{\partial U_s}{\partial x_k}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_s}{\partial x_k}\frac{\partial U_i}{\partial x_s} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_s}{\partial x_i} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_s}{\partial x_i}\right) \\
&+ B\left(\frac{\partial U_s}{\partial x_j}\frac{\partial w_s}{\partial x_j}\delta_{ik} + \frac{\partial U_i}{\partial x_k}\frac{\partial w_s}{\partial x_s} + \frac{\partial w_i}{\partial x_k}\frac{\partial U_s}{\partial x_s} + \frac{\partial U_s}{\partial x_j}\frac{\partial w_j}{\partial x_s}\delta_{ik}\right. \\
&\left. + \frac{\partial U_k}{\partial x_i}\frac{\partial w_s}{\partial x_s} + \frac{\partial w_k}{\partial x_i}\frac{\partial U_s}{\partial x_s}\right) + 2C\frac{\partial U_j}{\partial x_j}\frac{\partial w_s}{\partial x_s}\delta_{ik}
\end{aligned} \tag{6}
$$

Equations 5 and 6 form the basis from which we will derive the remaining equations.

2.2. Reformulated Equations

In seismology we typically have information related to the in-situ elastic constants via seismic characterizations, instead of the unstressed constants used in the above equations. Therefore, the first transformation is to back out the implied in-situ λ and μ, that we will call λ' and μ'. To find these in-situ values, we will need to group terms that match the forms of the unstressed λ and μ terms found in Equation 6. Thus, for λ we search for terms of the form $\frac{\partial w_s}{\partial x_s}\delta_{ik}$. For μ we search for terms of the form $\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right)$. Doing this, we find

$$
\begin{aligned}
\lambda' &= \lambda + 2C\frac{\partial U_j}{\partial x_j} \\
\mu' &= \mu + B\frac{\partial U_j}{\partial x_j}
\end{aligned} \tag{7}
$$

We can then substitute λ' and μ' into Equation 6 to slightly simplify it:

$$\tilde{\sigma}_{ik} = \lambda' \frac{\partial w_s}{\partial x_s}\delta_{ik} + \mu'\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right)$$
$$+\frac{A}{4}\left(\frac{\partial U_s}{\partial x_i}\frac{\partial w_s}{\partial x_k} + \frac{\partial w_s}{\partial x_i}\frac{\partial U_s}{\partial x_k} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_i}{\partial x_s}\right.$$
$$\left.+\frac{\partial U_s}{\partial x_k}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_s}{\partial x_k}\frac{\partial U_i}{\partial x_s} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_s}{\partial x_i} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_s}{\partial x_i}\right) \tag{8}$$
$$+B\left(\frac{\partial U_s}{\partial x_j}\frac{\partial w_s}{\partial x_j}\delta_{ik} + \frac{\partial U_i}{\partial x_k}\frac{\partial w_s}{\partial x_s} + \frac{\partial U_s}{\partial x_j}\frac{\partial w_j}{\partial x_s}\delta_{ik} + \frac{\partial U_k}{\partial x_i}\frac{\partial w_s}{\partial x_s}\right)$$

Now define the static strain as:

$$E_{ik} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right) \tag{9}$$

Then Equation 8 becomes

$$\tilde{\sigma}_{ik} = \lambda' \frac{\partial w_s}{\partial x_s}\delta_{ik} + \mu'\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right)$$
$$+\frac{A}{4}\left(2E_{is}\frac{\partial w_s}{\partial x_k} + 2E_{ks}\frac{\partial w_s}{\partial x_i} + 2E_{ks}\frac{\partial w_i}{\partial x_s} + 2E_{is}\frac{\partial w_k}{\partial x_s}\right) \tag{10}$$
$$+B\left(E_{js}\left(\frac{\partial w_s}{\partial x_j} + \frac{\partial w_j}{\partial x_s}\right)\delta_{ik} + 2E_{ik}\frac{\partial w_s}{\partial x_s}\right)$$

Finally, using the standard definition of infinitesimal (transient) strain:

$$\epsilon_{ik} = \frac{1}{2}\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right) \tag{11}$$

we obtain

$$\tilde{\sigma}_{ik} = \lambda'\epsilon_{ss}\delta_{ik} + 2\mu'\epsilon_{ik}$$
$$+A\left(E_{is}\epsilon_{ks} + E_{ks}\epsilon_{is}\right) \tag{12}$$
$$+2B\left(E_{js}\epsilon_{js}\delta_{ik} + E_{ik}\epsilon_{ss}\right)$$

If one is attempting to estimate the ambient stress from inversion of seismic data, one possibility would be to retrieve estimates of the E_{ij} using Equation 12. However, since Equation 5 is written in terms of static displacement gradients, not static strains, the conversion to static stress is not obvious. Likewise, if one desires to perform a forward simulation of transient signals in the presence of ambient stress, we typically do not know the static strains or displacements directly since this would require knowledge of what the configuration of particles would be in the absence of any stress. In either case we need Equation 5 expressed in terms of static strains. To do this, we rewrite Equation 5 by using Equations 9 and 7, to obtain

$$\begin{aligned}\bar{\sigma}_{ik} &= \lambda' E_{ss}\delta_{ik} + 2\mu' E_{ik} \\ &\quad + A\left(E_{is}E_{ks}\right) \\ &\quad + B\left(E_{js}^2\delta_{ik}\right) - C\left(E_{ss}^2\delta_{ik}\right)\end{aligned} \tag{13}$$

This forms a set of 6 coupled nonlinear equations for E_{ij} and $\bar{\sigma}_{ik}$. In the case of inversion of seismic data for static strains in Equation 12, Equation 13 can be used directly to easily obtain static stresses implied by those strains. For the forward problem, since we could, in principle, obtain estimates of the static stress, we can use Equation 13 to find the static strains implied by those stresses. Equation 13 can be solved for the 6 independent E_{ij} using a numerical root solver. Both means of utilizing Equation 13 can be performed provided that λ', μ', A, B, C are known. λ' and μ' can usually be estimated based on cores, tomography, or other methods. The A, B, and C terms have been experimentally determined for some earth materials, but these are comparatively poorly known for many materials. One possibility is that inversions for these nonlinear material parameters could be accomplished using these equations.

2.3. Symmetries

Hooke's Law expresses the stress-strain relationship as (e.g., Aki and Richards, 2002)

$$\tilde{\sigma}_{ij} = C_{ijkl}\epsilon_{kl} \tag{14}$$

where C_{ijkl} is the elastic modulus tensor. We can write Equation 12 in the form of Equation 14, where C_{ijkl} is interpreted as the pre-stress modulus tensor. First, note that $\tilde{\sigma}_{ik}$ as defined in Equation 12 is symmetric since E_{ij} and ϵ_{ij} are both symmetric by Equations 9 and 11. This means there are at most 36 unique entries in C_{ijkl} since there are 6 unique stress components and 6 unique strain components. We can therefore fully write out Equation 12 and find the 36 components of the pre-stress modulus tensor for an isotropic elastic solid as:

$$C_{1111} = \lambda' + 2\mu' + 2AE_{11} + 4BE_{11}$$

$$C_{1112} = AE_{12} + 2BE_{12}$$

$$C_{1113} = AE_{13} + 2BE_{13}$$

$$C_{1122} = \lambda' + 2BE_{11} + 2BE_{22}$$

$$C_{1123} = 2BE_{23}$$

$$C_{1133} = \lambda' + 2BE_{11} + 2BE_{33}$$

$$C_{1211} = AE_{12} + 2BE_{12}$$

$$
\begin{aligned}
&C_{1212} = \mu' + \frac{1}{2}AE_{11} + \frac{1}{2}AE_{22} \\
&C_{1213} = \frac{1}{2}AE_{23} \\
&C_{1222} = AE_{12} + 2BE_{12} \\
&C_{1223} = \frac{1}{2}AE_{13} \\
&C_{1233} = 2BE_{12} \\
&C_{1311} = AE_{13} + 2BE_{13} \\
&C_{1312} = \frac{1}{2}AE_{23} \\
&C_{1313} = \mu' + \frac{1}{2}AE_{11} + \frac{1}{2}AE_{33} \\
&C_{1322} = 2BE_{13} \\
&C_{1323} = \frac{1}{2}AE_{12} \\
&C_{1333} = AE_{13} + 2BE_{13} \\
&C_{2211} = \lambda' + 2BE_{11} + 2BE_{22} \\
&C_{2212} = AE_{12} + 2BE_{12} \\
&C_{2213} = 2BE_{13} \\
&C_{2222} = \lambda' + 2\mu' + 2AE_{22} + 4BE_{22} \\
&C_{2223} = AE_{23} + 2BE_{23} \\
&C_{2233} = \lambda' + 2BE_{22} + 2BE_{33} \\
&C_{2311} = 2BE_{23} \\
&C_{2312} = \frac{1}{2}AE_{13} \\
&C_{2313} = \frac{1}{2}AE_{12} \\
&C_{2322} = AE_{23} + 2BE_{23} \\
&C_{2323} = \mu' + \frac{1}{2}AE_{22} + \frac{1}{2}AE_{33} \\
&C_{2333} = AE_{23} + 2BE_{23} \\
&C_{3311} = \lambda' + 2BE_{11} + 2BE_{33}
\end{aligned} \tag{15}
$$

$$C_{3312} = 2BE_{12}$$

$$C_{3313} = AE_{13} + 2BE_{13}$$

$$C_{3322} = \lambda' + 2BE_{22} + 2BE_{33}$$

$$C_{3323} = AE_{23} + 2BE_{23}$$

$$C_{3333} = \lambda' + 2\mu' + 2AE_{33} + 4BE_{33}$$

Not all 36 coefficients are unique. There are 14 terms that all others can be easily expressed with: λ', μ', AE_{ij}, and BE_{ij}, the latter two contributing 6 terms each. However, there are actually only 12 linearly independent coefficients. We can observe this if we express the stress-strain relationship in Equation 12 as

$$\begin{bmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ \tilde{\sigma}_{33} \\ \tilde{\sigma}_{23} \\ \tilde{\sigma}_{13} \\ \tilde{\sigma}_{12} \end{bmatrix} = \begin{bmatrix} a & b & c & d & e & f \\ b & g & h & i & j & f \\ c & h & k & i & e & l \\ d & i & i & \frac{1}{4}\left(g+k\right) - \frac{1}{2}h & \frac{1}{2}\left(f-l\right) & \frac{1}{2}\left(e-j\right) \\ e & j & e & \frac{1}{2}\left(f-l\right) & \frac{1}{4}\left(a+k\right) - \frac{1}{2}c & \frac{1}{2}\left(i-d\right) \\ f & f & l & \frac{1}{2}\left(e-j\right) & \frac{1}{2}\left(i-d\right) & \frac{1}{4}\left(a+g\right) - \frac{1}{2}b \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \end{bmatrix} \quad (16)$$

where

$$\begin{aligned} a &= C_{1111} \\ b &= C_{1122} \\ c &= C_{1133} \\ d &= C_{1123} \\ e &= C_{1113} \\ f &= C_{1112} \\ g &= C_{2222} \\ h &= C_{2233} \\ i &= C_{2223} \\ j &= C_{2213} \\ k &= C_{3333} \\ l &= C_{3312} \end{aligned} \quad (17)$$

The pre-stress modulus tensor displays the same symmetries, i.e., $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$, as the traditional elastic modulus tensor.

Based on the fact that the pre-stress modulus tensor has the same symmetries as the standard elastic modulus tensor, it is apparent that the reciprocity of source and receiver commonly assumed in seismology is valid. This follows from Betti's theorem (Equation 2.34 in Aki and Richards, 2002):

$$\begin{aligned}&\iiint_V \left(\mathbf{f}-\rho\ddot{\mathbf{u}}\right)\cdot\mathbf{v}\,d\mathrm{V}+\iint_S \mathbf{T}_{\mathbf{u}}(\mathbf{n})\cdot\mathbf{v}\,d\mathrm{S}\\ &\qquad=\iiint_V \left(\mathbf{g}-\rho\ddot{\mathbf{v}}\right)\cdot\mathbf{u}\,d\mathrm{V}+\iint_S \mathbf{T}_{\mathbf{v}}(\mathbf{n})\cdot\mathbf{u}\,d\mathrm{S}\end{aligned} \tag{18}$$

where **u** and **v** are displacements due to body forces **f** and **g**, respectively, and $\mathbf{T}_{\mathbf{u}}(\mathbf{n})$ and $\mathbf{T}_{\mathbf{v}}(\mathbf{n})$ are the tractions caused by displacements **u** and **v**, respectively, on surface normals **n**. This relationship is possible because of the symmetries of the elastic modulus tensor. Assuming that transient displacements are zero prior to some time and that boundary conditions are homogeneous and independent of time (pre-stress must vanish at the free surface of the earth just like transient stresses) allow us to derive the source-receiver reciprocity relations found in equations 2.38-2.40 in Aki and Richards (2002). These form the foundation of many assumptions made in processing and imaging seismic data and they are valid for KG theory as well.

3. CONCLUSIONS

We have reformulated the equations for linearized seismic wave propagation given by Korneev and Glubokovskikh (2013) into an easier to utilize form. This form is more compact, being written in terms of ambient strains. However, since ambient strains are generally inaccessible, the derived equations allow for the inversion of the ambient strains given ambient stresses, which are, in principle, measurable. This formulation enables a direct mapping between dynamic stresses and strains analogous to Hooke's Law. The resulting pre-stress modulus tensor demonstrates the same symmetries as the traditional elastic modulus tensor. This latter property implies that the same source-receiver reciprocity relationship applies in the presence of ambient stress as in the absence of ambient stress. The stress-strain relationship also shows that for a medium that is isotropic in the absence of ambient stress, the medium becomes anisotropic with 12 independent coefficients when in the presence of ambient stress.

REFERENCES

1. Dahlen, F.A. and J. Tromp, *Theoretical Global Seismology*, Princeton University Press, Princeton, NJ, 1998.
2. Gurbatov, S.N., O.V. Rudenko, A.I. Sachet, Types of Acoustic nonlinearities and Methods of Nonlinear Acoustic Diagnostics, in Waves and Structures in *Nonlinear Nondispersive Media*, Springer Berlin Heidelberg, doi: 10.1007/978-3-642-23617-4_8, pp 271-307, 2012.
3. Korneev, V., and S. Glubokovskikh, Seismic velocity changes caused by an overburden stress, *Geophys*., 78 (5), doi:10.1190/GEO2012-0380.1, WC25-WC31, 2013.
4. Aki, K., and P. Richards, *Quantitative Seismology, 2nd Edition*, University Science Books, Sausalito, CA, 2002.

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