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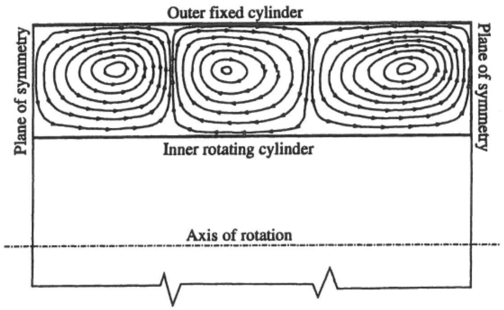

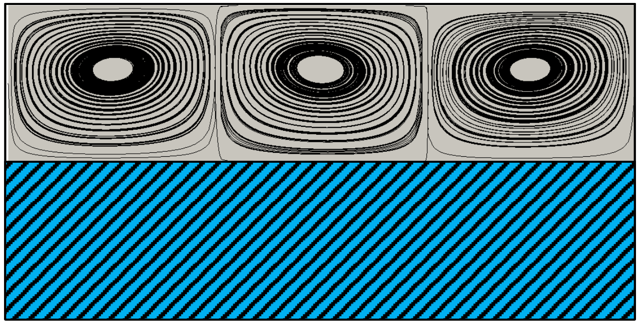

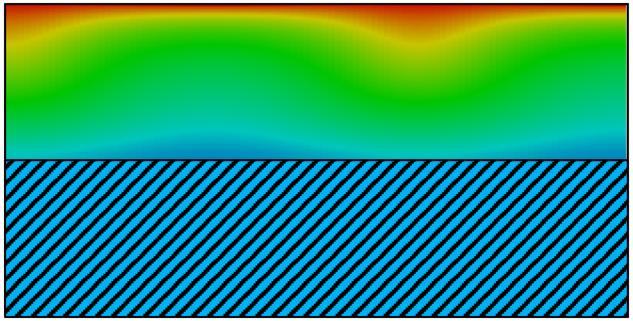

DSMC Simulations of Taylor-Couette Flow Instabilities

T.P. Koehler, M.A. Gallis, J.R. Torczynski, S.J. Plimpton

Sandia National Laboratories

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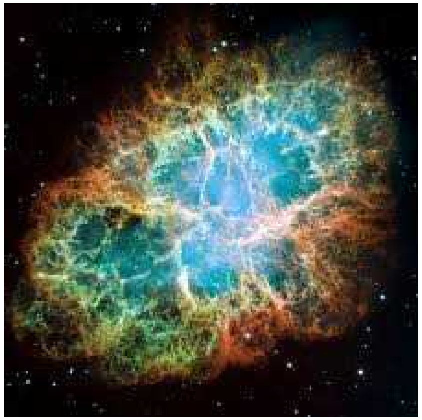
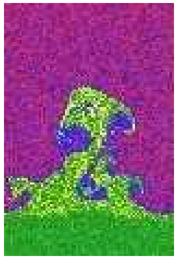
Nature and Hydrodynamic Instabilities



Richtmyer-Meshkov



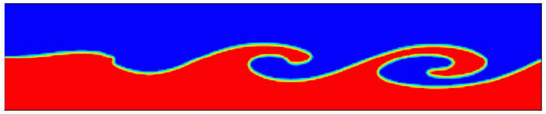
Hydrodynamic instabilities are of great scientific interest and engineering importance

Infinitesimal disturbances amplify spontaneously and ultimately dominate the flows.

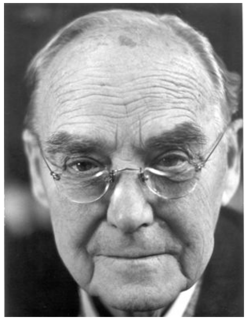
The hydrodynamic description assumes that changes in the fluid occur slowly so that the system can be considered in a state of local thermodynamic equilibrium.

If not, the fluid behavior deviates from the predictions of hydrodynamics, as molecular relaxation affects transport: diffusivity, viscosity, thermal conductivity.

Kelvin-Helmholtz

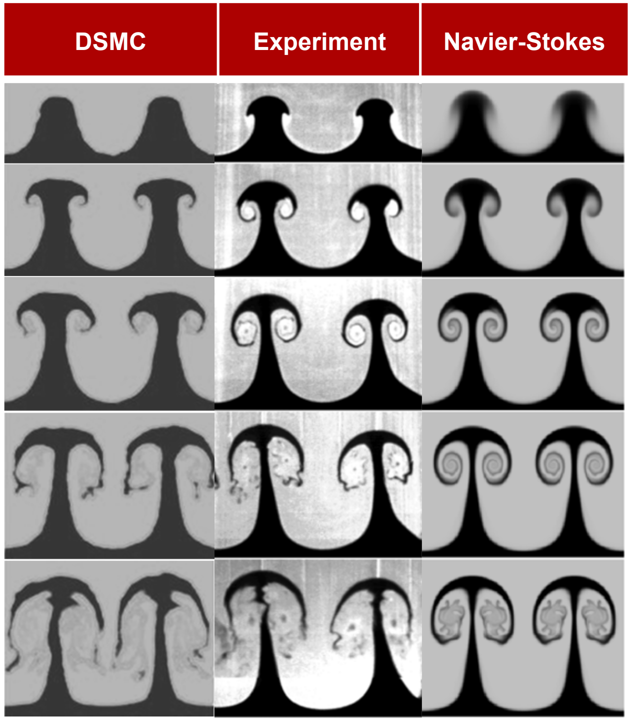


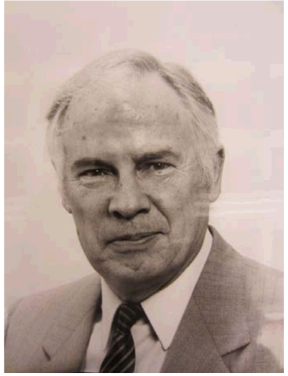
DSMC for Hydrodynamic Instabilities?



Sir Geoffrey Ingram Taylor

"A great many attempts have been made to discover some mathematical representation of fluid instability, but so far they have been unsuccessful in every case."





Graeme Bird

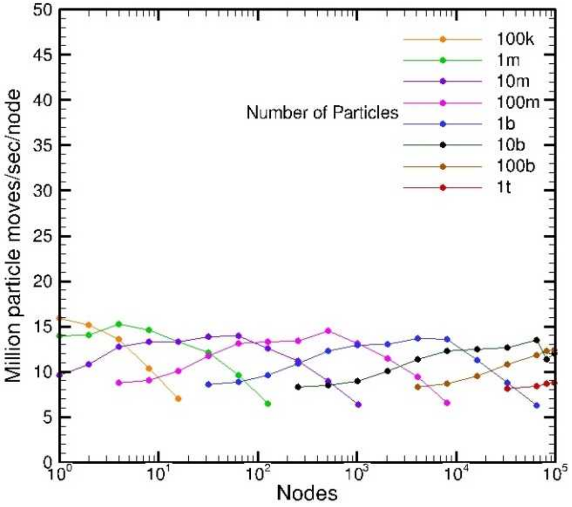
"it has now become clear that DSMC solutions can reach beyond the scope of … conventional mathematical models and that the range of applicability of the method is greater than first thought."

SPARTA: an Exascale DSMC Code

Sandia National Laboratories

SPARTA = Stochastic PArallel Rarefied-gas Time-accurate Analyzer

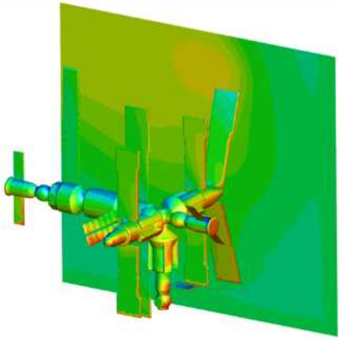
Scaling



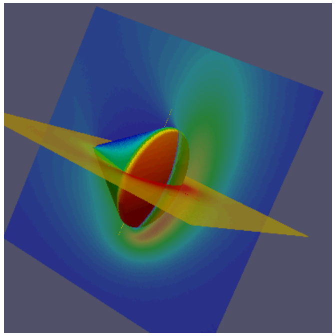
General features

- 1D, 2D, 2D-axisymmetric or 3D, serial or parallel
- Cartesian, hierarchical grid
  - Oct-tree (up to 16 levels in 64-bit cell ID)
  - Multilevel, general NxMxL instead of 2x2x2
- Triangulated surfaces cut/split the grid cells
  - 3D via Schwartzentruber algorithm
  - 2D via Weiler/Atherton algorithm
  - Formulated so can use as kernel in 3D algorithm
- C++, but really object-oriented C
  - Exascale-capable (scales to 1.6 Million cores, GPUs, Threading)
  - The code has been extensively verified and validated.
  - Includes advanced collision/chemistry models, boundary conditions, etc.

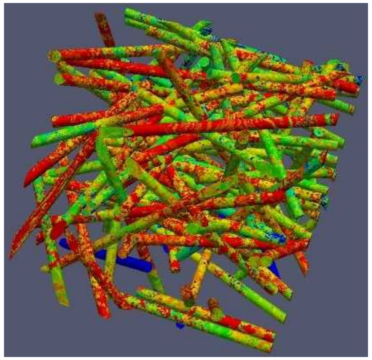
Satellites



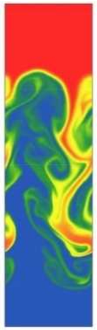
Re-entry



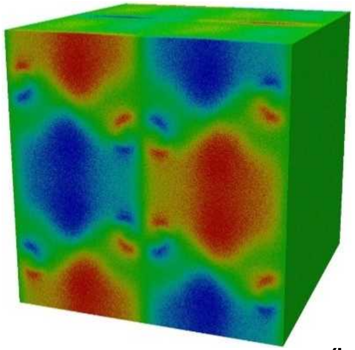
Porous Media



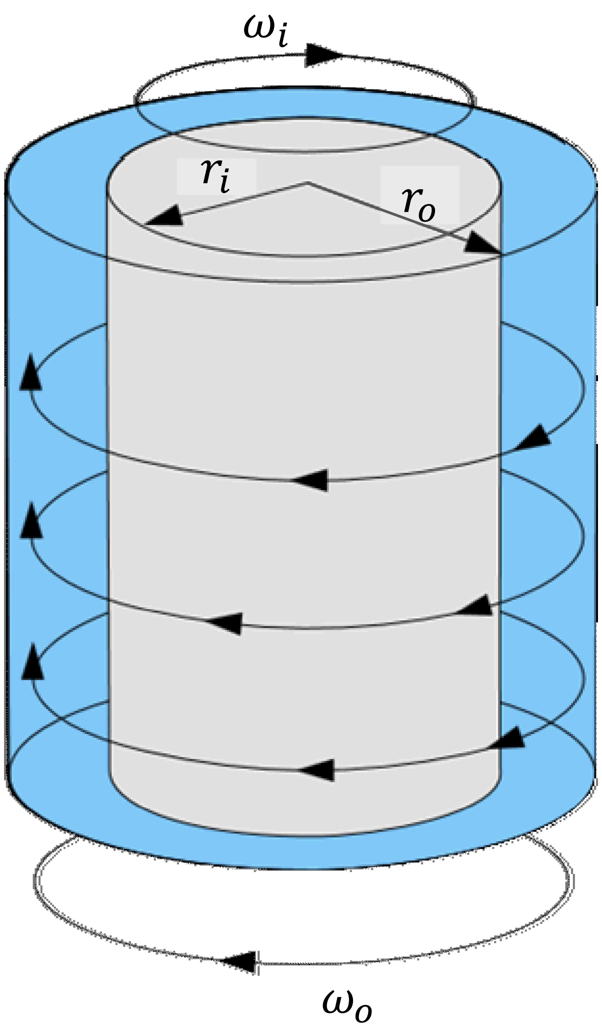
Instabilities



Turbulence



Taylor Instability of Couette Flow



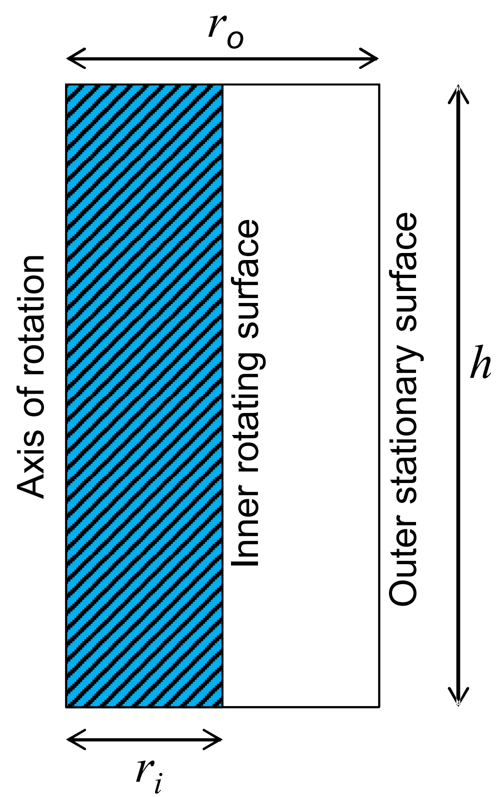
- Classical flow of viscous fluid in the annulus of two concentric cylinders, where one or both are rotating
- Couette flow: stable, linear velocity profile across annulus
- Taylor-Couette flow: inertial forces exceed viscous damping of perturbations leading to development of vortices
  - Taylor Number:

$$Ta = \frac{4\rho^2\omega^2 r_i^4}{\mu^2\left[1-\left({r_i}/{r_o}\right)^2\right]^2}$$

  - Inner Reynolds Number: $Re_i = r_i(r_o - r_i)\omega_i/\nu$
  - Outer Reynolds Number: $Re_o = r_o(r_o - r_i)\omega_o/\nu$

Bird's Simulations of Taylor-Couette Flow

- From G.A. Bird, *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, 1994, § 15.4, pg. 378
- Domain Definition:
  - Concentric cylinders with $r_o = 2r_i$ and $h = 2r_o$
    - Here: r_i = 0.5 m and r_o = 1.0 m
  - Boundary conditions:
    - Cylinder walls are diffusely reflective
    - Top/bottom of domain specularly reflective
  - Initial conditions:
    - Stationary and uniform gas with density such that $\lambda = (r_o - r_i)/50$ → Kn = 0.02
    - At t = 0 s, inner cylinder rotates with $\omega = 3c'_m$
      - For argon: ω = 2071 rad/s



- Taylor Number: $Ta = \frac{4\rho^2\omega^2 r_i^4}{\mu^2\left[1-\left(r_1/r_2\right)^2\right]^2} = 521{,}600$
  - Exceeds critical value of 33,110 → flow instabilities/vortices

Bird’s DSMC2A Simulations (1994)

- Projection of streamlines on plane normal to rotation
- Assumes steady flow and averages results over the 21st through 30th revolutions of inner cylinder
- Observes development of three counter-rotating vortices through an unsteady process

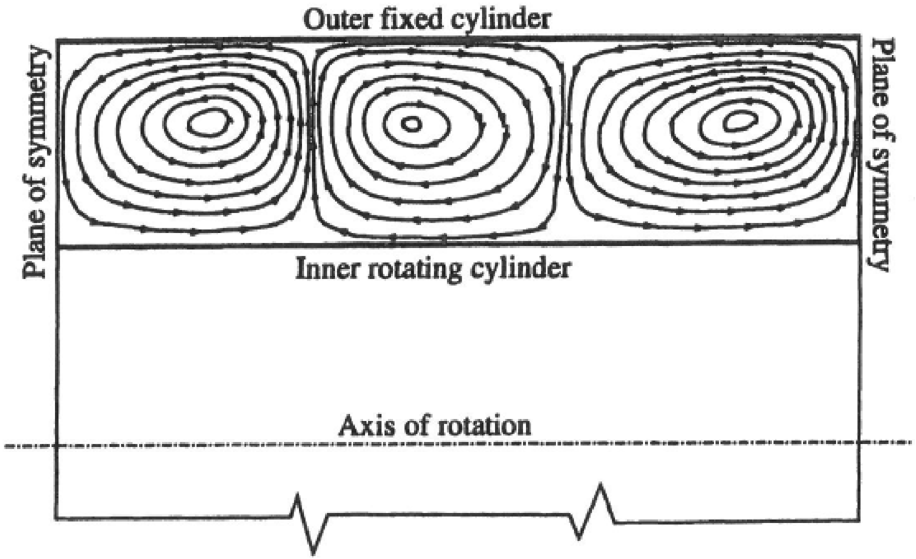
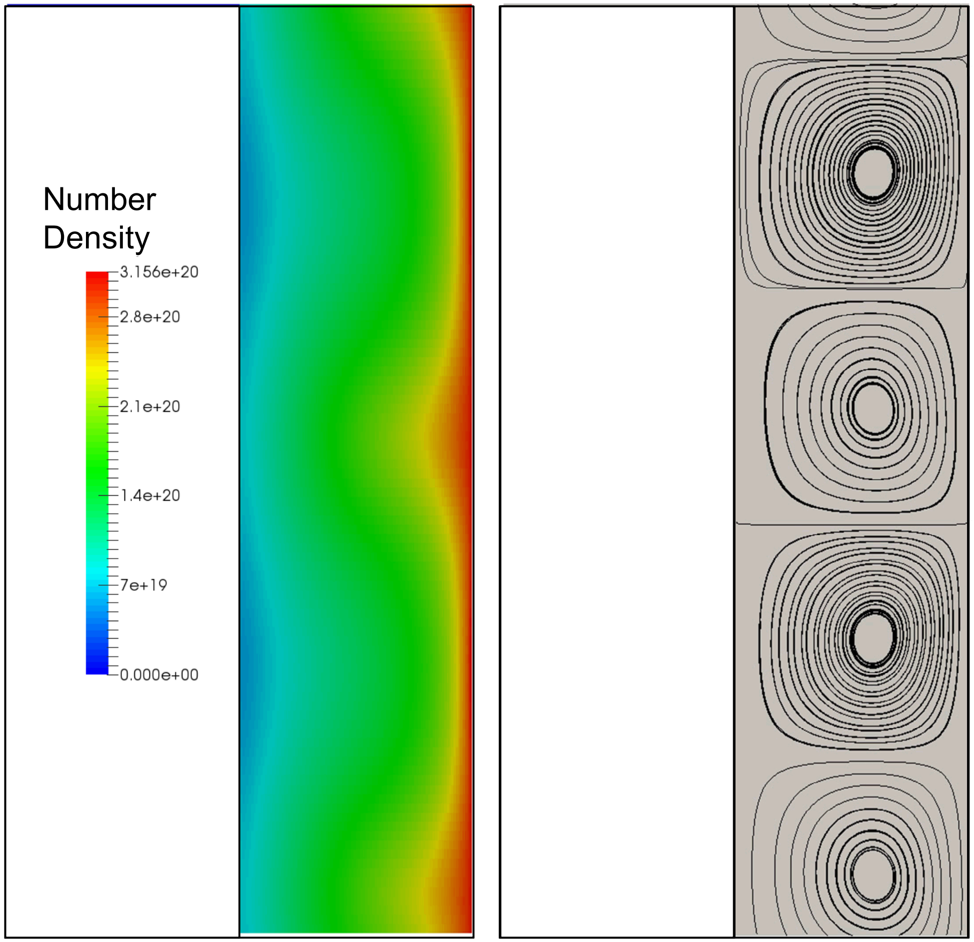


Image from G.A. Bird, Molecular Gas Dynamics and the Direct Simulation of Gas Flows, 1994, §15.4, pg. 379

SPARTA Simulations:
2D Axisymmetric with Periodic BCs

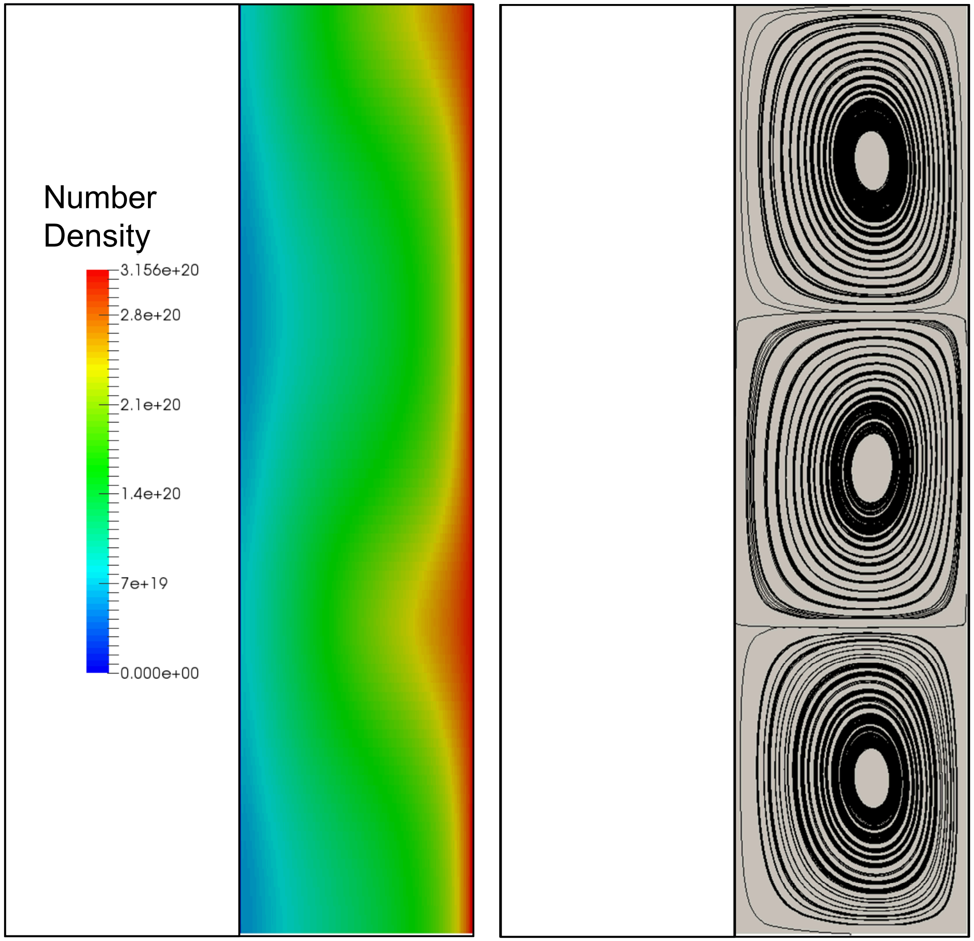
- Periodic conditions on upper and lower boundaries
- Steady counter-rotating vortex flow overlapping domain results
  - Not identical to Bird's results: BC
- Vortices aid mixing → number-density gradient across annulus is much less than base Couette flow, but still an order of magnitude



Streamline projections averaged over one revolution shown at 2 fps

SPARTA Simulations: 2D Axisymmetric with Specular BCs

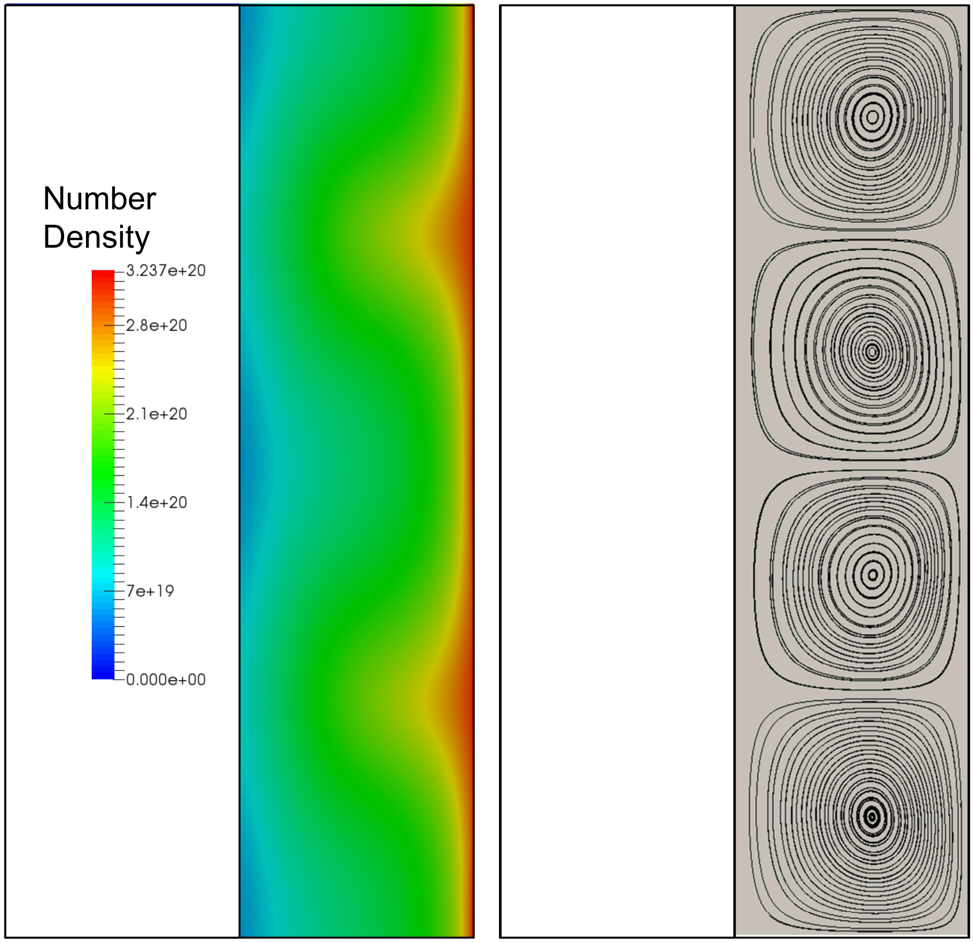
- Specularly reflecting upper and lower surface
- Three counter-rotating vortices, as seen by Bird
- Number-density gradient still exceeds an order of magnitude across annulus
  - Mean free path? Implications on grid resolution?



Streamline projections averaged over one revolution shown after 30 revolutions

SPARTA Simulations:
2D Axisymmetric with Specular BCs (fine grid)

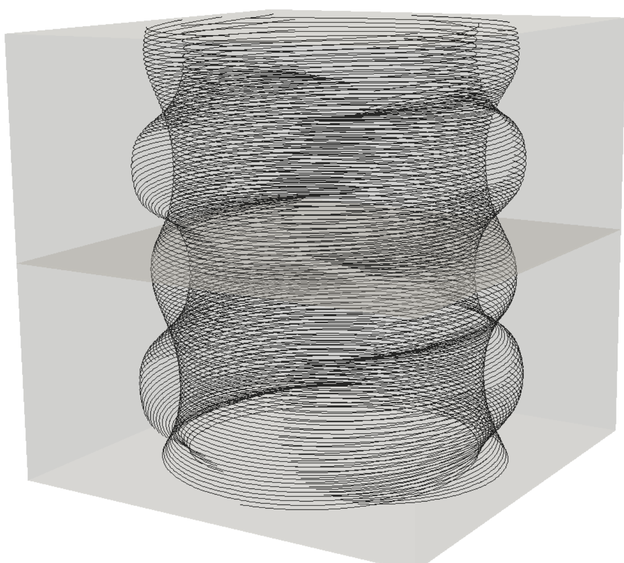
- Grid Study:
  - Bird's grid: 200x400 cells
  - Fine grid: 1000x2000 cells
- Four counter-rotating vortices instead of three as seen previously
- Number density increases slightly

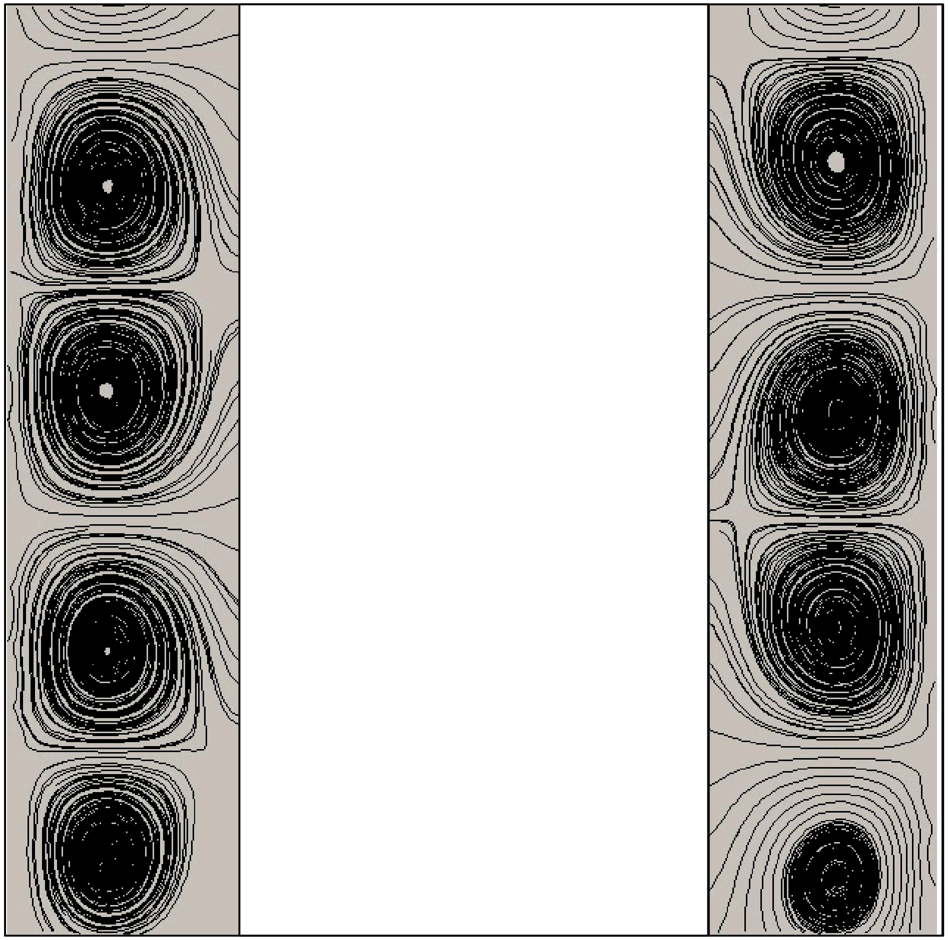


Streamline projections averaged over one revolution shown after 30 revolutions

SPARTA Simulations: 3D Axisymmetric with Periodic BCs

- Four counter-rotating vortices develop as before
- Vortex centers are not at equal heights on opposite sides of the slice
- Wavy 3D structure in streamlines is observed when full domain is shown

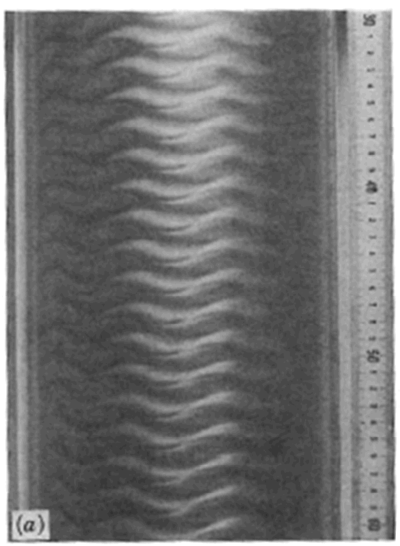




Streamline projections averaged over one revolution shown at 2 fps

Taylor-Couette Instability in Literature

- Inner Reynolds Number: $Re_i = r_i(r_o - r_i)\omega_i/\nu$
- Outer Reynolds Number: $Re_o = r_o(r_o - r_i)\omega_o/\nu$
- Here, $\omega_o = 0$ rad/s and $\nu = 2\ \mathrm{m^2/s}$
  - $Re_o = 0$ and $Re_i = 259$



Wavy Taylor Vortices

Image from Koschmieder, 1979

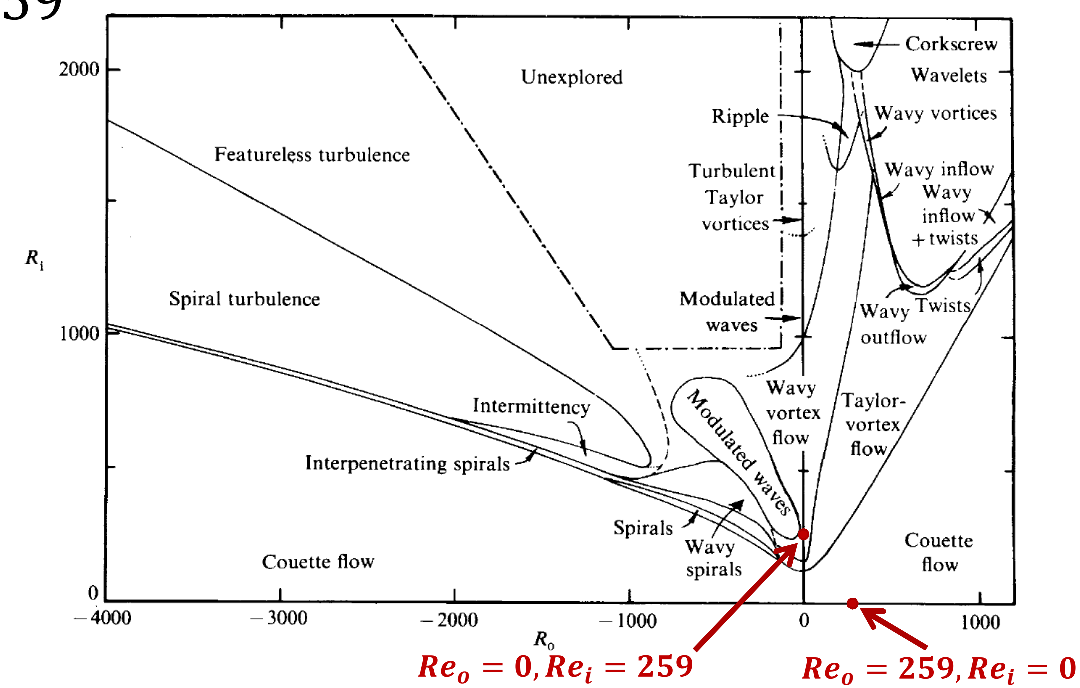
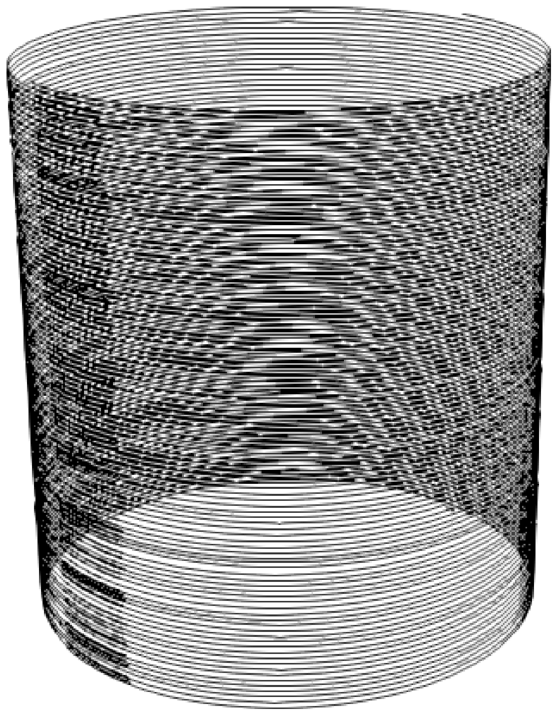
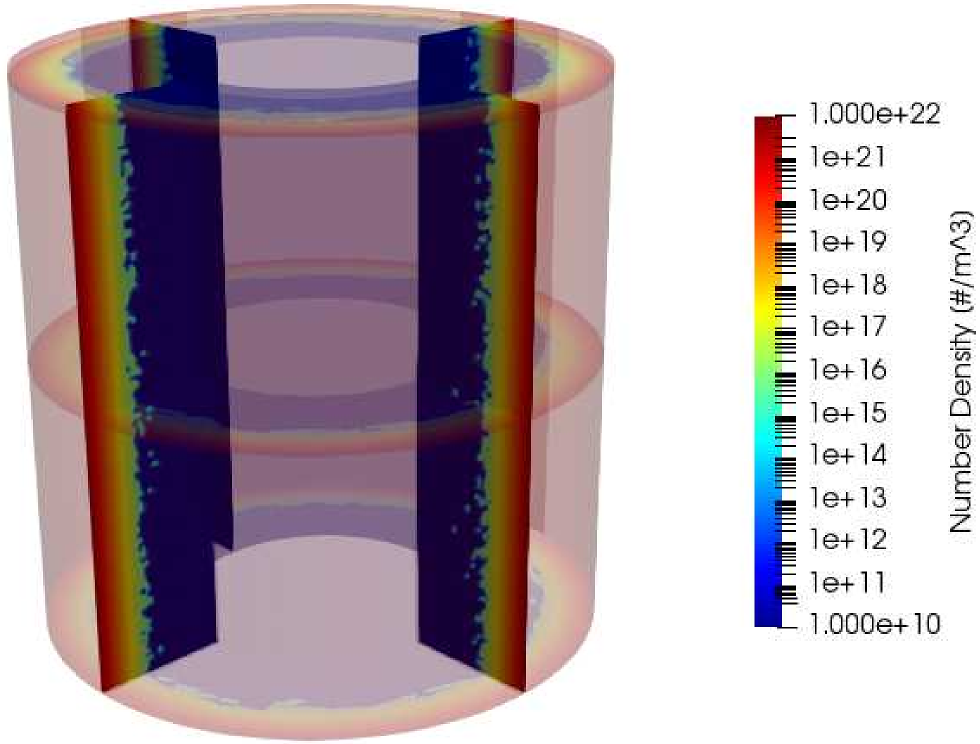


Image from Andereck et al., 1986

SPARTA Simulations: Stable Couette Flow

- When the inner cylinder is fixed and the outer cylinder rotates, SPARTA simulations reproduce stable Couette flow, as expected
  - Note: density difference across annulus is > 10 orders of magnitude





$\omega_o = 2071 \frac{\mathrm{rad}}{\mathrm{s}}, \omega_i = 0\ \mathrm{rad}/s$ and $\nu = 2\mathrm{m}^2/s \rightarrow Re_o = 259$ and $Re_i = 0$

DSMC for hydrodynamic instabilities? Yes, but be careful!

- SPARTA verification for hydrodynamic instabilities
  - Steady Taylor-Couette vortices as presented by Bird are accurately reproduced by SPARTA

- Additional considerations
  - Gradients across flow fields require additional grid refinement
  - Hydrodynamic instabilities are usually three-dimensional