SAND2017-9111PE

Artificial Viscosity and Solution Node Location Effects on Shock Capturing in High-Order, Entropy Stable, Finite Element Methods

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Contents

Motivation

2^{nd} order TVD finite volume schemes are great at what they're great at:

- e.g. robust capturing of strong shocks

But: Increasing interest in LES etc.

- High-order methods are superior for problems dominated by small smooth structures

Also: Important to understand error sources

- Difference with experiment from numerics or physics?
- High-order methods naturally extend to error analysis

To utilize high-order methods in hypersonics, **robust shock capturing needed**

Objective

Can we improve:

1. Regularization excessively diffusing discontinuities over many elements.
2. Regularization excessively dissipating flow features e.g. turbulent structures.
3. Sufficient regularization for discontinuities near element corners.

Contents

Numerical Method

Solve:

$$\mathbf{u}_{,t} + \nabla \cdot (\mathbf{f} - [c] \nabla \mathbf{u}) = 0 \quad (1)$$

Using **entropy stable**, spectral collocation methods based on summation-by-parts framework[1, 2, 3]:

$$\mathbf{u}_{,t} + \mathcal{P}^{-1} \left[\Delta \bar{\mathbf{f}} - \mathcal{D} [\hat{c}] \, \mathbf{\Theta} \right] = \mathcal{P}^{-1} \mathbf{g}^{int,q} \quad (2a)$$

$$\mathbf{\Theta} - \mathcal{D} \mathbf{w} = \mathcal{P}^{-1} \mathbf{g}^{int,\theta} \quad (2b)$$

Equivalency with nodal DG, **but** has a stronger entropy stability statement.

Two 'flavors':

- **Node-Centered**: Use Legendre-Gauss-Lobatto points for collocation (sub-optimal quadrature, evaluation of properties on boundaries, ...)
- **Cell-Centered**: Use Legendre-Gauss points (others possible) for collocation (optimal quadrature, solution variables evaluated on element interiors, ...)

Artificial Viscosity

Testing 3 artificial viscosity methods:

- **Shakib Based**: Based on the work of Shakib et al[4], uses a disconituity operator added a Peclet number based limit (not in Shakib's work)
- **Jump Base**:Based on the work of Barter and Darmofal[5], targets regions with large pressure jumps using a non linear switch to give a viscosity based on Peclet number
- **Entropy Viscosity**: Based on the work of Guermond et al[6], targets regions where entropy is produced (entropy equation residual and entropy flux jump).

Regularize Artificial viscosity field (Max.-Linear):

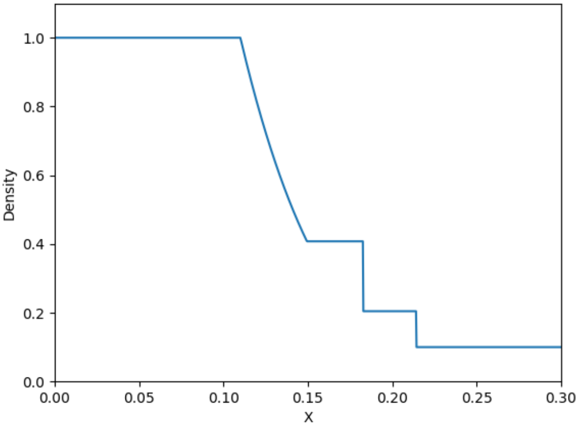
1. Nodal artificial viscosity is the max from adjacent elements
2. Nodal artificial viscosities are linearly interpolated to the elements

Linear artificial viscosity field that is continuous across elements

Contents

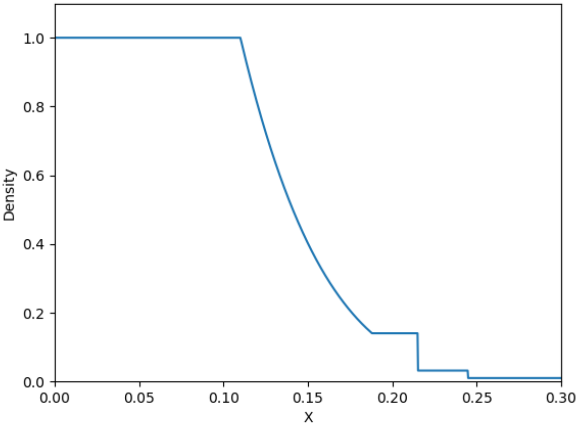
Density and Pressure Ratio of 10

Very similar to Sod's shock tube problem



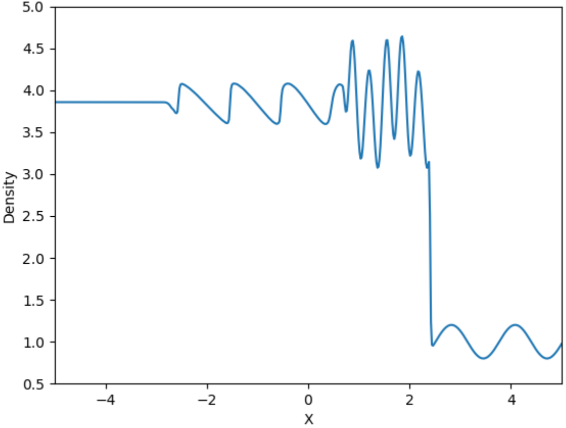
Density and Pressure Ratio of 100

Sonic point in expansion fan can cause issues



Shu-Osher Problem

Tests whether artificial viscosity preserves flow structures



Contents

Lifted vs. Interior Gradients in Residual Operator

Residual operator:

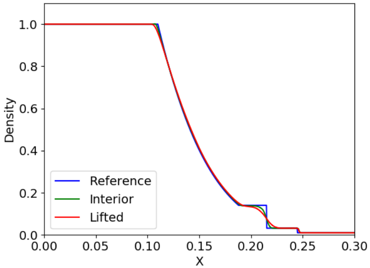
$$\mathcal{L}u = f_{,v}\, v_{,x} - f_{,x} \tag{3}$$

Two options for treating derivative terms:

- **Lifted:** Include jump penalties
  - Penalized discontinuties more
  - Larger effective stencil
- **Interior:** Use interior derivatives
  - Smaller non-zero artificial viscosity region
  - Smaller artificial viscosity

Riemann Problem Ratio 100

P3, 128 Elements, Node-Centered, Shakib-Based Viscosity



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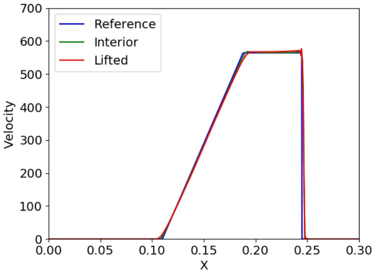
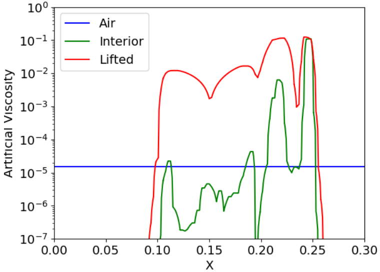


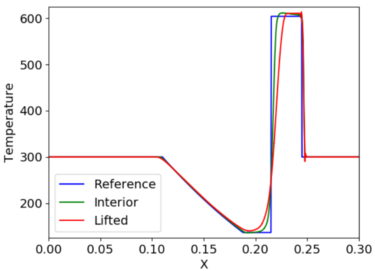
(b) Velocity

Riemann Problem Ratio 100

P3, 128 Elements, Node-Centered, Shakib-Based Viscosity



(c) Artificial Viscosity



(d) Temperature

Contents

Riemann Problem Ratio 10

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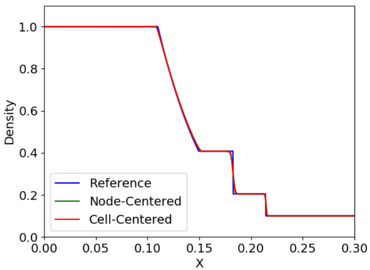

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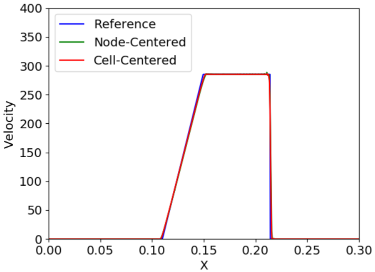

(b) Velocity

Riemann Problem Ratio 10

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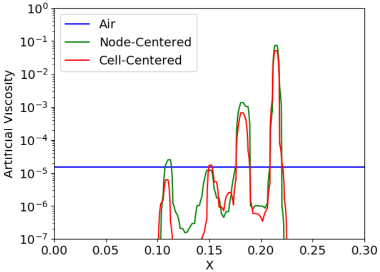

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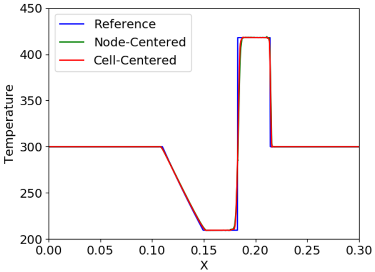

(d) Temperature

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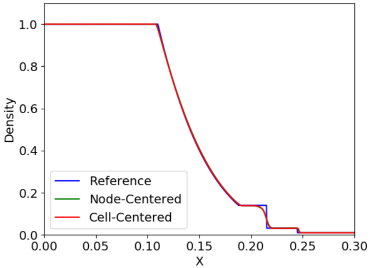

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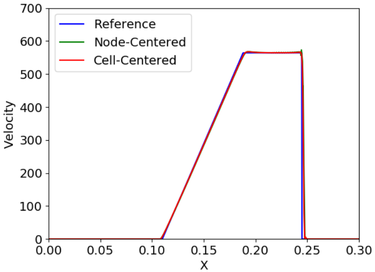

(b) Velocity

Riemann Problem Ratio 100

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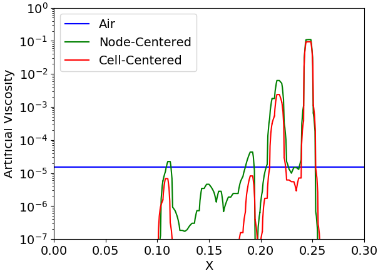

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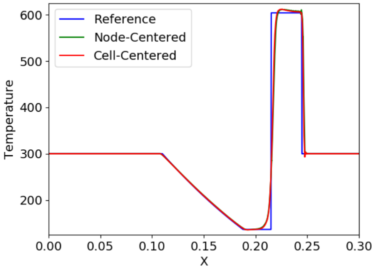
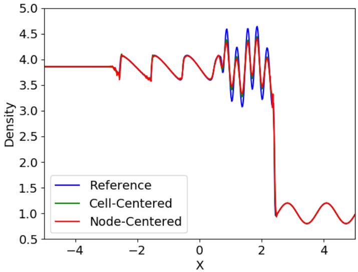


(d) Temperature

Shu Osher Problem

P3, 128 Elements, Shakib-Based Viscosity



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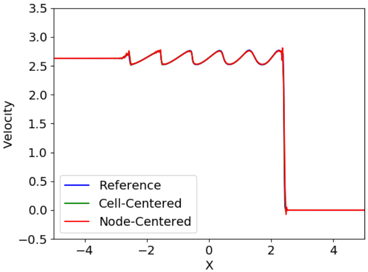

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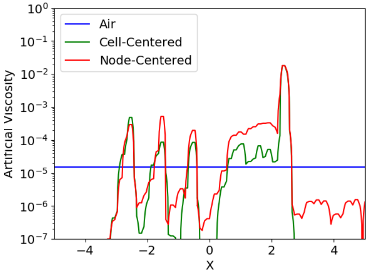

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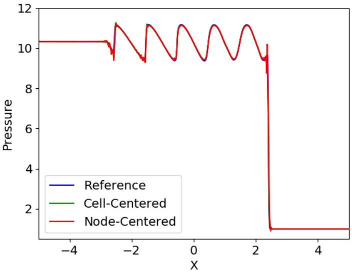
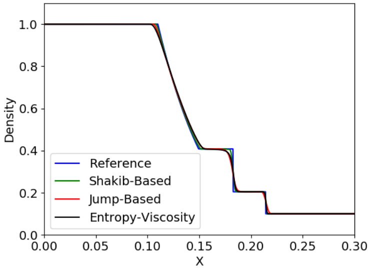


(d) Pressure

Contents

Riemann Problem Ratio 10

P3, 128 Elements, Cell-Centered



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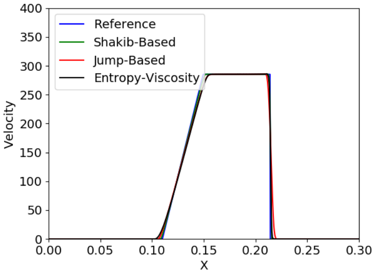
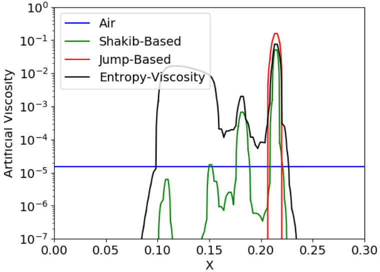


(b) Velocity

Riemann Problem Ratio 10

P3, 128 Elements, Cell-Centered



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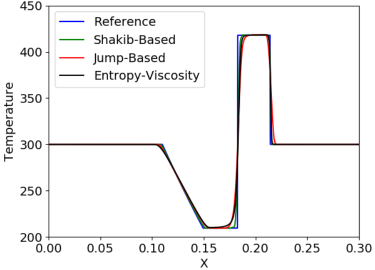
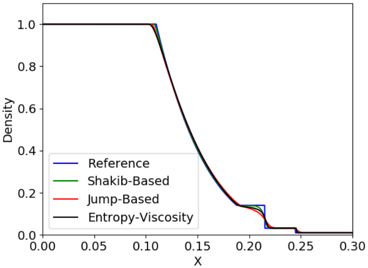


(d) Temperature

Riemann Problem Ratio 100

P3, 128 Elements, Cell-Centered



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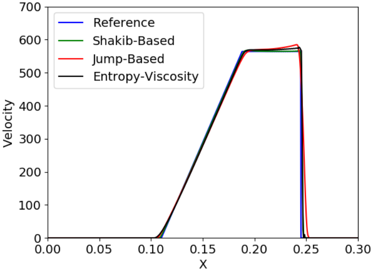
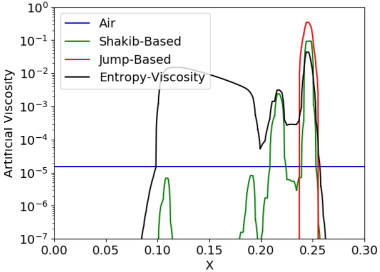


(b) Velocity

Riemann Problem Ratio 10

P3, 128 Elements, Cell-Centered



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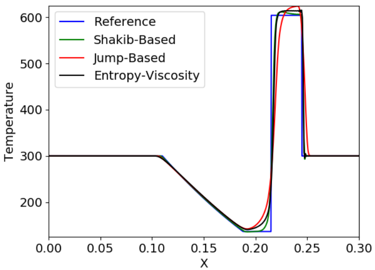
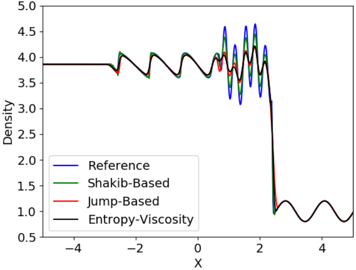


(d) Temperature

Shu Osher Problem

P3, 128 Elements, Cell-Centered



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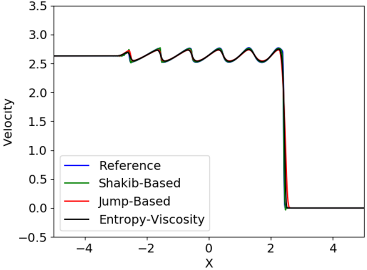

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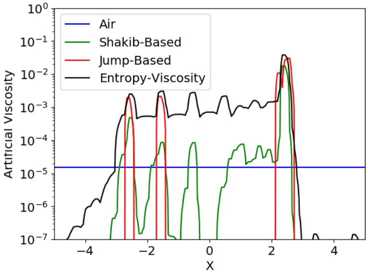
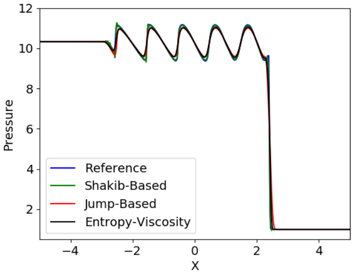


(c) Artificial Viscosity



(d) Pressure

Contents

Regularization

Max. element Peclet numbers are in approx. range 1-10

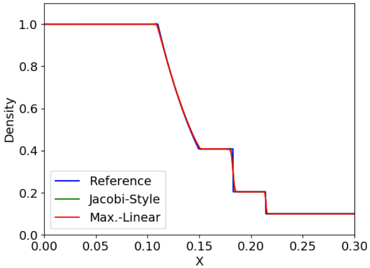
- Rule of thumb: Peclet number ≤ 2 for no oscillations in Burgers Equation

Proposal: The magnitude of the artificial viscosity is fine, but it is distributed over too large region
Improvement: Pick a new regularization

- Jacobi style smoothing with additional step to ensure continuity of artificial viscosity over interfaces
- NB: Smoothing does not preserve maximum in artificial viscosity

Riemann Problem Ratio 10

P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



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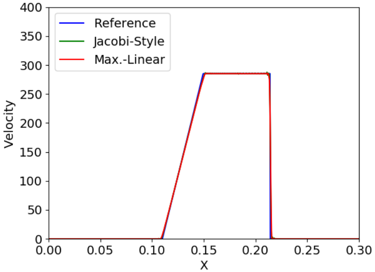
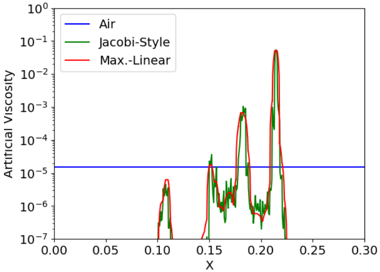


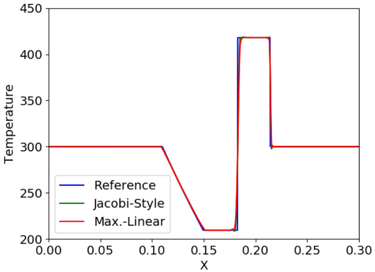
(b) Velocity

Riemann Problem Ratio 10

P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



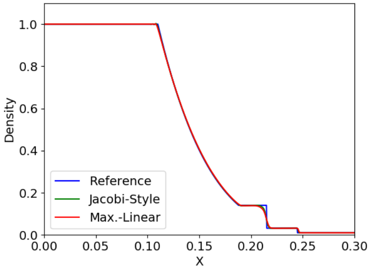
(c) Artificial Viscosity



(d) Temperature

Riemann Problem Ratio 100

P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



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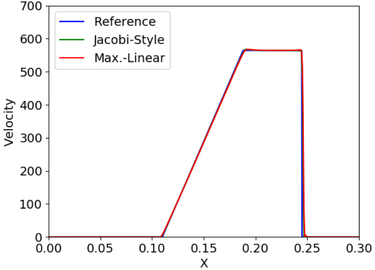

(b) Velocity

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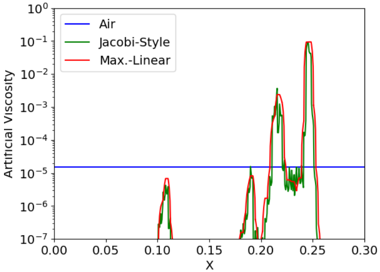
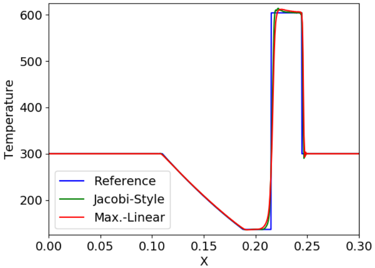
P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



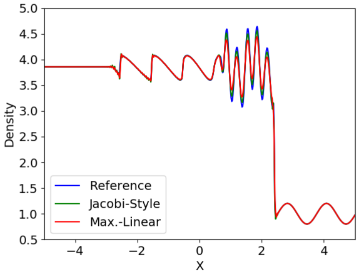
(c) Artificial Viscosity



(d) Temperature

Shu Osher Problem

P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



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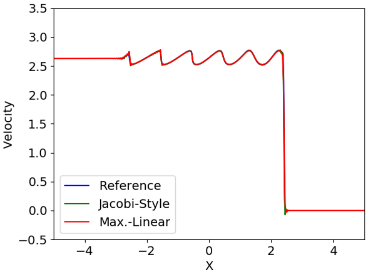
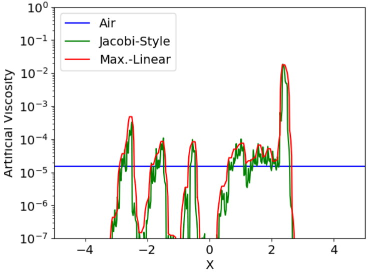


(b) Velocity

Shu Osher Problem

P3, 128 Elements, Cell-Centered, Shakib-Based Viscosity



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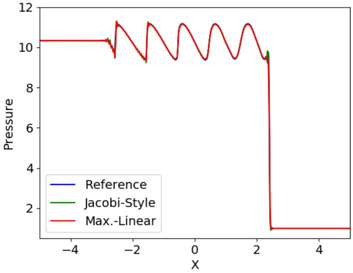
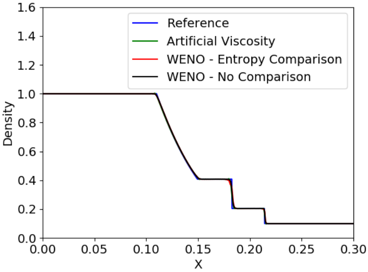


(d) Pressure

Contents

Riemann Problem Ratio 10

WENO: P4, 512DOFs
SSSCE: P3, 128Elements



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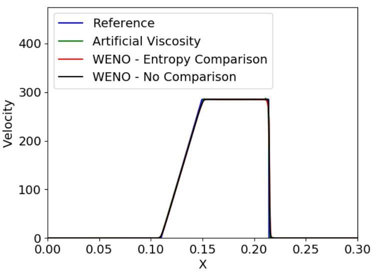
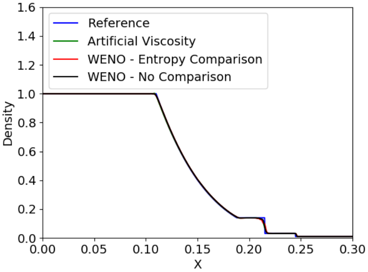


(b) Velocity

Riemann Problem Ratio 100

WENO: P4, 512DOFs
SSSCE: P3, 128Elements



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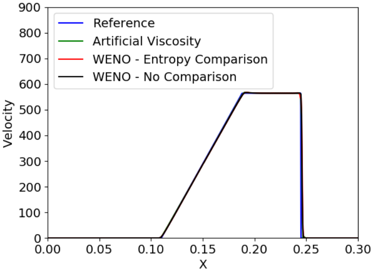

(b) Velocity

Shu Osher Problem

WENO: P4, 512DOFs
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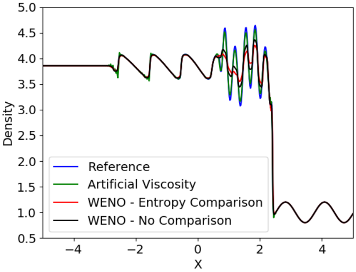

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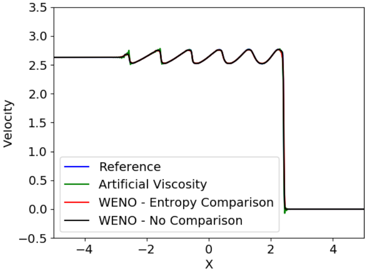
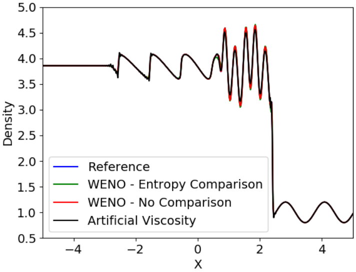


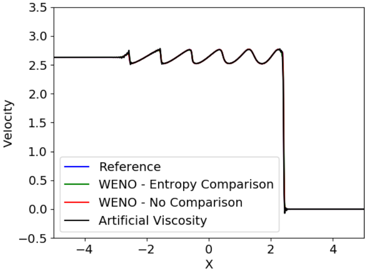
(b) Velocity

Shu Osher Problem

WENO: P6, 512DOFs
SSSCE: P3, 128Elements



(a) Density



(b) Velocity

Comparison to WENO

Cell-centered elements allow for a less aggresive artificial viscosity scheme

- Cell-centered requires less artificial viscosity
- Less artificial viscosity $\Rightarrow$ preserve flow features
- No oscillation around shock

Stronger Shocks

Is the artificial viscosity robust enough for stronger shocks?
Examine the Woodward Colella problem

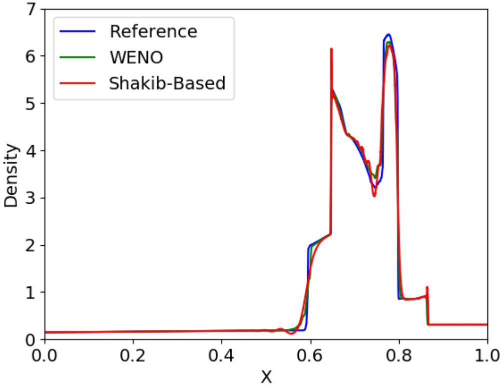
- Interacting blast waves with pressure ratios of 100 and 1000
- Initial discontinuities at $x = 0.1$ and $x = 0.9$

Table : Initial Conditions

	Left State	Middle State	Right State
Density	1.0	1.0	1.0
Velocity	0.0	0.0	0.0
Pressure	1000	0.01	100

Results

So far P1 with artificial viscosity and P4 WENO (1000DOFs)



Conclusion

Cell-centered methods out perform nodal methods

- Significantly less oscillatory primitive variable fields than nodal methods
- Allows less conservative artificial viscosity

Using Shakib-based artificial viscosity (least aggressive smoothing) gives best resolution of flow features

- Can be made more conservative (like Entropy Viscosity) with lifted gradients

Future Work

- Is artificial viscosity applied to finite difference method competitive with WENO?
- Investigate the effect of cell-centered finite difference for WENO
- What is the efficiency trade-off between higher and lower order methods with constant DOF count?

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Valentin Zingan, Jean-Luc Guermond, Jim Morel, and Bojan Popov.
Implementation of the entropy viscosity method with the discontinuous galerkin method.
Computer Methods in Applied Mechanics and Engineering, 253:479 – 490, 2013.

Backup Slides

Entropy Stability

Define *entropy*: $S\left(u\right)$

- Convex in solution variables
- For Euler/NS

$$S = -\frac{\rho R}{\gamma - 1} \ln p\rho^{-\gamma} \tag{4}$$

Entropy stability:

$$\frac{\partial S}{\partial t} + \nabla \cdot F \leq 0 \tag{5}$$

Interpretation:

- Bounded convex function of solution variables $\Rightarrow$ L_2-style stability
- Euler/NS: solutions obey 2^{nd} law of thermodynamics

Still require shock capturing

Shakib Based Artificial Viscosity

Based on the work of Shakib et al[4].

$$\hat{\mu} = \sqrt{\frac{(\mathcal{L}u)^T\, w_{,u}\, (\mathcal{L}u)}{w^T u + w_{,x} g_{ij} u_{,w} w_{,x}}}. \tag{6}$$

The discontinuity operator is then limited:

$$\hat{\mu}_{max} = C_{\hat{\mu}} \frac{|u| + c}{h} \tag{7}$$

That is, the resulting artificial viscosity is defined by:

$$\mu = g_{ij} \max\{\hat{\mu}, \hat{\mu}_{max}\} \tag{8}$$

To avoid using the temporal term, the operator $\mathcal{L}u$ is defined as:

$$\mathcal{L}u = f_{,v}\, v_{,x} - f_{,x} \tag{9}$$

Jump Based Artificial Viscosity

Based on the work of Barter and Darmofal[5].
Targets regions where jumps in a property (pressure) are large

$$\mu_k = \frac{h\left(|u| + c\right)}{p} S_K \tag{10}$$

Where S_K is a nonlinear shock switch given by:

$$S_K\left(J\right) = \begin{cases} 0, & \text{if } \log_{10} J \leq \psi_0 - \Delta\psi \\ 1, & \text{if } \log_{10} J \geq \psi_0 + \Delta\psi \\ \frac{1}{2}\left(1 + \sin\frac{\pi\left(\log_{10} J - \psi_0\right)}{2\Delta\psi}\right), & \text{otherwise} \end{cases} \tag{11}$$

And J is the jump indicator, given by:

$$J = \frac{1}{|\partial\kappa|} \int_{\partial\kappa} \left| \frac{[\![p]\!]}{\{p\}} \right| \cdot \mathbf{n}\, ds \tag{12}$$

Entropy Viscosity

Based on the work of Guermond et al[6].
Targets regions where entropy is produced

$$\mu_k = \max\{C_{max} h_k \max_{\Delta\Omega}(|u| + c), C_e h_k^2 D_k\} \tag{13}$$

Where D_k is based on the entropy production in the element and at each interface,

$$D_k = \frac{\max\{\max_{\Delta\Omega}|R|, \max_{\Delta\Omega}|J|\}}{N} \tag{14}$$

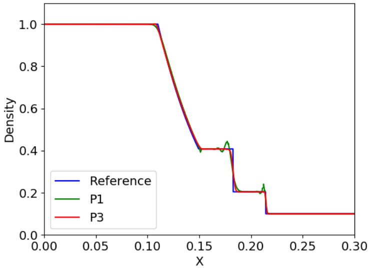
$$R = S_{,t} + \nabla \cdot uS \tag{15}$$

$$J = \frac{1}{h_k} \cdot [\![uS]\!] \tag{16}$$

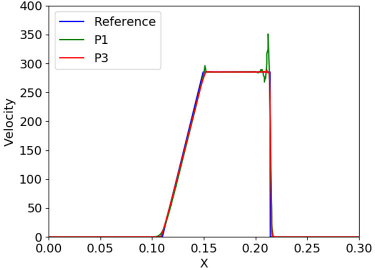
$$N = \max_{x\in\Omega}\left|S(u) - \frac{1}{|\Omega|}\int_{\Omega} S \, d\Omega\right| \tag{17}$$

Riemann Problem Ratio 10

P1 256Elements vs. P3 128 Elements (Same DOF Count)



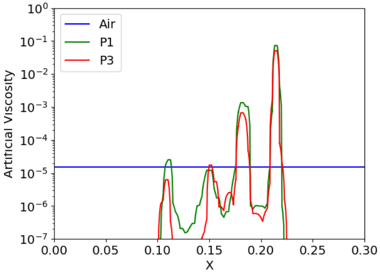
(a) Density



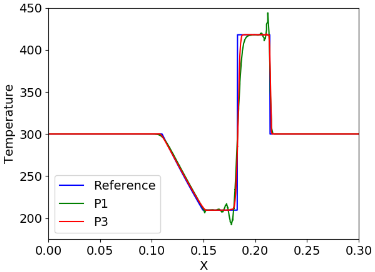
(b) Velocity

Riemann Problem Ratio 10

P1 256Elements vs. P3 128 Elements (Same DOF Count)



(c) Artificial Viscosity



(d) Temperature

Woodward Colella

So far P1 with artificial viscosity and P4 WENO (1000DOFs)