Performance Tradeoffs of Spectrum Sensing and Target State Estimation and Fusion

Qiang Liu
Computational Sciences and Engineering Division
Oak Ridge National Laboratory
Oak Ridge, TN 37831
Email: liuq1@ornl.gov

***Abstract*—We consider a scenario where a cognitive radio sensor performs spectrum sensing and target tracking alternately. Both tasks may impose certain performance guarantee requirements that necessitate considerations of how to schedule these tasks over time. In addition, multiple such cognitive radio sensors can perform collaborative spectrum sensing and target tracking, where a fusion center serves to collect the sensor estimates and their covariances to generate fused target state estimates. We study the performance tradeoffs of these two tasks from variable scheduling intervals. The effect of sensing errors on the target tracking performance due to induced loss from perceived channel unavailability is investigated and conclusions are drawn based on the observed performance tradeoffs.**

I. Introduction

There have been a plethora of research efforts in the areas of cognitive radio and dynamic spectrum sensing, allocation, sharing, and aggregation over the past decade [1]. The main motivation behind them are the inefficiencies of the current static channel allocation and usage, where a large chunk of spectrum remains under-utilized at any given moment. The channel usage is typically evolving spatially and temporally, facilitating cognitive radio (CR) devices to opportunistically sense and discover spectrum holes, and then share and aggregate these channels for their secondary use. Based on the dynamics of the licensed users, a database-based approach can be pursued for channels with well-behaved licensed usage and the secondary users can simply access the channel within the known OFF time of the licensees [4]. On the other hand, many channels exhibit highly dynamic and unpredictable behavior by the licensees, thereby necessitating cognitive radios to sense such channels more proactively in discovering the available spectrum opportunities, or “white spaces” that are otherwise unexplored.

Normally, the purpose of spectrum access by cognitive radios is for their own data transmission. We consider a specific case of secondary transmissions in this work, where a network of CRs periodically collect information on one or more dynamic targets and generate state estimates (e.g., the position and velocity) to be communicated to a fusion center. The fusion center then combines them to generate global state estimates [2]. The periodic nature of this task means that target tracking can be interleaved with the spectrum sensing task, with the most time being spent on the latter to guarantee channel availability and minimal interference to the user(s) licensed to that channel. During periods of unavailability (e.g., due to the presence of the licensed user), the CR will not communicate the generated estimates to the fusion center as scheduled, which can be equivalently regarded as information loss at the latter that can lead to degraded tracking performance.

We study the performance tradeoffs of scheduling spectrum sensing and target tracking in an interleaving manner, highlighting the major advantages and disadvantages of scheduling the two tasks at varying frequencies. We show that spectrum sensing decisions of an unavailable channel lead to *induced* communication loss for the tracking task, when a sensor suspends sending its target state estimates to the fusion center. While shorter scheduling intervals benefit the tracking task due to more frequent updates, it can degrade the overall spectrum sensing performance, which may become unacceptable to the quality-of-service (QoS) of licensed users. On the other hand, longer intervals can improve spectrum sensing accuracy due to more samples, but also lead to overall higher estimation errors at the fusion center. In addition, we also show that erroneous spectrum sensing decisions, i.e., false alarms and missed detections, can affect the tracking performance in more subtle ways. Since fusion inherently involves contributions from multiple CR sensors, we also explore the extent to which collaborative spectrum sensing with multiple CRs can benefit the overall sensing and tracking performance. We use an illustrative example to demonstrate joint spectrum sensing and target tracking performance as well as their performance tradeoffs.

The rest of this paper is organized as follows. In Section II, we present the system model and also review spectrum sensing and state estimation fundamentals. In Section III, we describe the interleaving sensing-tracking scheduling, exploring the performance tradeoffs of spectrum sensing and target tracking with one CR, and then extend the discussions to multiple CRs with collaborative or non-collaborative sensing and multi-sensor fusion. Section IV presents numerical case studies before the paper concludes in Section V.

II. System Model and Detection/Estimation Fundamentals

We consider a system in which the signal strengths of licensed users are weak, i.e., with low signal-to-noise ratios

(SNRs). This necessitates longer sensing times by the cognitive radios in general to meet the system sensing requirements. In addition, due to the periodic nature of sensors sending their state estimates to the fusion center (based on a certain schedule), we assume that such short messages, once started, can be successfully delivered to the fusion center and used for fusion, although such transmissions can incur interference to the licensed users. Therefore, we aim to reduce the number of erroneous channel sensing/detection decisions that may lead to secondary use while the licensed users are present.

A. Energy Detection

Energy detection is arguably the simplest yet often fairly effective method used by cognitive radios, where the energy of the ambient signal (i.e., sum of squares of discrete samples) is compared against a threshold, above which a decision is made that the presumed licensed user is actively using the channel. With a large number of samples, regardless of the underlying distribution of the licensed user signal, the energy sample is approximately normally distributed, thereby facilitating closed-from derivations of the detection performance. More specifically, using the Neyman-Pearson Lemma [5], suppose the false alarm (a licensed user is deemed present when it is actually not) rate is P_{FA}, then the probability of missed detection (a licensed user is present but deemed otherwise by the CR) can be derived as

$$P_{MD} = 1 - Q\left(\frac{Q^{-1}(P_{FA}) - \gamma\sqrt{M}}{\sqrt{1+2\gamma}}\right), \tag{1}$$

in which M is the total number of samples, γ denotes the SNR, and $Q(\cdot)$ is the tail distribution function of the standard normal distribution $\mathcal{N}(0,1)$ and $Q^{-1}(\cdot)$ its inverse. It is easy to show that with a fixed P_{FA}, sometimes known as the constant false alarm rate (CFAR) detector [8], the missed detection rate P_{MD} decreases monotonically with increasing SNR γ or increasing sample count M. In other words, with a certain fixed sampling rate, e.g., Nyquist rate for regular sensing or sub-Nyquist rate for compressed sensing [9], more samples can be used to improve the overall sensing accuracy performance in terms of reduced P_{MD}.

B. Linear Estimation

The goal of a state estimator is to extract the state information $\mathbf{x}$ from a measurement $\mathbf{y}$ corrupted by noise; this is done by sequentially running a filter that outputs the state estimate $\hat{\mathbf{x}}$ and its associated error covariance matrix $\mathbf{P}$. We consider a 2D ground target tracking scenario where the target state evolution is described by the discretized continuous white-noise acceleration (CWNA) model, a linear, nearly straight-line and nearly constant-velocity model [2]. The discrete-time target state equation is given by $\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{w}_k$, where (dropping the time index k), $\mathbf{x} = [\xi \quad \dot{\xi} \quad \eta \quad \dot{\eta}]^T$ is a vector consisting of the position and velocity components in both coordinates. The transition matrix $\mathbf{F}$ is given by

$$\mathbf{F} = \begin{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \end{bmatrix}, \tag{2}$$

where T is the sampling/estimation interval[1]. The covariance of the discrete-time process noise $\mathbf{w}_k$ is

$$\mathbf{Q} = \begin{bmatrix} \tilde{q}_\xi \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \tilde{q}_\eta \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \end{bmatrix}. \tag{3}$$

where $\tilde{q}_\xi$ and $\tilde{q}_\eta$ are respectively the power spectral densities (PSDs) of the underlying continuous-time white stochastic process along the axes.

The Kalman filter (KF) is the best known linear estimator and the equations describing its evolution can be found in [10]. When the sensor measurement model is also linear (e.g., with direct position measurements), then the KF conditions are met and the CR sensors can generate their state estimates of the target $\hat{\mathbf{x}}$ along with the error covariances $\mathbf{P} = \mathbb{E}[(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^T]$ based on KF evolutions. The KF estimator will reach its steady state sometime after initialization, regardless of the initial state, so that the Kalman gain becomes constant. The steady-state error covariances can be found by solving the discrete algebraic Riccati equation (DARE) with specified CWNA system model matrices and also sensor measurement model matrices [3]; alternatively, the steady-state errors can be easily found via numerical simulation. We focus on steady-state error performances in this paper.

III. Interleaved Spectrum Sensing and Target Tracking

We consider interleaved sensing and tracking scheduling, focusing on the effect of induced loss from sensing decisions on the overall tracking performance. Suppose that each estimation interval T consists of two parts, (i) the time dedicated to spectrum sensing $T_S = M/f_s$ in which f_s is the sampling frequency, and (ii) a small fixed amount of time $T_I << T_S$ for the CR sensor to process measurements to generate the target state estimate (and associated error covariance) for that time epoch and send the message to the fusion center. We primarily consider the effect of varying T_S, and in turn T, on both spectrum sensing and target tracking performances. We first investigate the performance tradeoff where there is only one CR sensor, and then extend the discussions to multi-sensor scenarios.

A. Induced Communication Loss

Suppose the occupancy state of the channel of interest can be statistically modeled as a continuous-time alternating renewal process (ARP) between the ON and OFF states for its

[1]This interval is in reference to the tracking task, i.e., the interval at which a sensor sends its state estimates to the fusion center.

licensed user. The duration of ON and OFF times, T_{ON} and T_{OFF}, are respectively i.i.d. positive random variables, and the long-term proportion of time when the channel is busy can be expressed as [7]

$$\Pr\{ON\} = \frac{\mathbb{E}[T_{ON}]}{\mathbb{E}[T_{ON}] + \mathbb{E}[T_{OFF}]}. \quad (4)$$

The CR sensor will only use the channel to send target state estimates to the fusion center once the channel is deemed clear of active traffic from licensed user(s); in other words, whenever the sensor decides the channel to be busy, it refrains from transmitting messages, resulting in unavailable sensor data at the fusion center. In contrast to data loss due to adverse communication link conditions (such as channel fading, blockage, etc.), here the communication loss is *induced* due to presumed licensed user presence.

Had the CR sensor been able to make perfectly accurate sensing decisions, then the induced loss rate is simply $\Pr\{ON\}$, the proportion of time that the channel is actually busy. In reality, accounting for the erroneous sensing decisions including false alarms and missed detections, we can express the induced loss rate as

$$\begin{aligned} P_L &\triangleq \Pr\{\text{induced loss}\} \\ &= \Pr\{ON\}(1 - P_{MD}) + \Pr\{OFF\}P_{FA}. \end{aligned} \quad (5)$$

From Eq. (5), we can see that loss of sensor messages at the fusion center can be induced in two ways: (i) when the channel is busy, a correct decision (i.e., non-missed detection) will prevent the CR sensor to send its data to the fusion center; (ii) when licensed user traffic is absent, an erroneous decision (i.e., false alarm) will still lead the CR sensor to refrain from any data communication to the fusion center until the channel is deemed available again. Therefore, both P_{FA} and P_{MD} can affect the effective loss rate P_L, whereupon the fusion center applies its own predicted value in lieu of sensor estimates when the latter become unavailable due to induced loss.

B. Performance Tradeoffs with One CR Sensor

We discuss the qualitative sensing versus tracking performance tradeoffs with one CR sensor and induced loss. Two important facts on the steady-state tracking errors are in order here: First, in most tracking scenarios, steady-state estimation errors increase monotonically with the estimation interval T, due to the cumulative effect of the process noise over time, as shown in Eq. (3); second, with the same steady-state error performance, increasing loss would result in an increasing number of prediction steps by the fusion center and in turn higher tracking errors. Now, from Eq. (1), if we fix P_{FA} while using fewer samples M (and in turn, a smaller T_S), then P_{MD} increases and the overall sensing performance becomes worse; however, from Eq. (5), the loss rate P_L is reduced. The combined effect of smaller T and reduced P_L would lead to improved tracking performance overall, at the cost of degraded sensing performance.

The quantitative effect of induced loss on steady-state tracking errors with one CR sensor can be evaluated approximately by probabilistically combining error covariances resulting from finite-step predictions, although this approach will become less accurate with increasing loss rates. Alternatively, numerical simulation can be again used for a more accurate performance evaluation, which we will use in Section IV.

C. Performance Tradeoffs with Multiple CR Sensors

To improve both spectrum sensing and target tracking performance, we consider the effect of invoking multiple CR sensors for collaborative/non-collaborative sensing and tracking. For collaborative sensing, we use decision fusion rules to combine their individual sensing decisions, whereas for multi-sensor fusion, the fusion center combines sensor estimates to generate global fused estimates using closed-form rules.

1) Collaborative Sensing: Suppose there are a total of N sensors that jointly and synchronously sense the channel of interest. Sensing decisions are fused at the end of each sensing interval T_S and a joint decision is made that will lead to the same actions taken by all the sensors on data communication to the fusion center. In this case, a small overhead in gathering and processing of individual sensing decisions, and in disseminating the fused sensing decisions is entailed. The fusion center either receives nothing or data sent from all the sensors.

We consider a general l-out-of-N decision fusion rule where the channel is deemed unavailable only if at least $l = 1, 2, 3, ...N$ users have decided the channel as busy [6]. The false-alarm and missed-detection probabilities can be derived as follows:

$$P_{FA}^{collab} = \sum_{j=l}^{N} \binom{N}{j} P_{FA}^{j} \big(1 - P_{FA}\big)^{N-j}, \quad (6)$$

$$P_{MD}^{collab} = \sum_{j=0}^{l-1} \binom{N}{j} P_{MD}^{N-j} \big(1 - P_{MD}\big)^{j}, \quad (7)$$

where we have assumed that the sensors have the same sensing error probabilities P_{FA} and P_{MD}.

In particular, we have the following special cases:

- When $l = 1$, we have the *OR-rule*, where
$$P_{FA}^{OR} = 1 - (1 - P_{FA})^N, \text{ and } P_{MD}^{OR} = P_{MD}^N; \quad (8)$$
- When $l = N$, we have the *AND-rule*, where
$$P_{FA}^{AND} = P_{FA}^N, \text{ and } P_{MD}^{AND} = 1 - (1 - P_{MD})^N; \quad (9)$$
- When $l = \lceil \frac{N+1}{2} \rceil$, we have the *MAJORITY-rule*, and the resulting sensing error probabilities can be obtained from Eqs. (6) and (7).

2) Non-Collaborative Sensing: In non-collaborative sensing, each CR sensor performs its own sensing (still synchronized across the sensors), whose decisions will be used *by itself* regarding whether to send data to the fusion center. As such, both sensing and tracking tasks are performed independently, and the fusion center gathers available sensor data and interpolates the missing ones using prediction for those absent. The effect of such non-collaborative sensing on the overall accuracy metrics can be expressed as

$$P_{FA}^{non-collab} = 1 - (1 - P_{FA})^N, \tag{10}$$

$$P_{MD}^{non-collab} = 1 - (1 - P_{MD})^N. \tag{11}$$

3) Fusers: We consider two simple closed-form fusers. First, the simplest average fuser (AF) calculates the arithmetic mean of the sensor estimates as the fuser output:

$$\hat{\mathbf{x}}_k^{AF} = \frac{1}{N} \sum_{i=1}^{N} \hat{\mathbf{x}}_k^{(i)}, \tag{12}$$

in which the subscript k is the time index and the superscript (i) denotes Sensor i.

Alternatively, the simple track-to-track fuser (T2TF) is a convex combination of the sensor estimates as follows [2]:

$$\hat{\mathbf{x}}_k^{T2TF} = \left(\sum_{i=1}^{N} (\mathbf{P}_k^{(i)})^{-1} \right)^{-1} \sum_{i=1}^{N} \left((\mathbf{P}_k^{(i)})^{-1} \hat{\mathbf{x}}_k^{(i)} \right). \tag{13}$$

From the equation, the sensor error covariance matrices are needed for this fuser, along with their state estimates. It is well known that the common process noise results in correlation in the error cross-covariance across sensor estimates. However, it is generally difficult to derive the exact cross-covariances over time; as a result, one may assume that the cross-covariance is negligible in order to apply this simplified fuser, even though the result will be suboptimal.

4) Performance Tradeoffs: We can see from Eq. (8) that compared to single-CR sensing, the OR-rule increases the overall false-alarm rate and decreases the missed detection probability, which in turn will lead to increased P_L according to Eq. (5). In contrast, the AND-rule can be found to result in decreased P_L. In other cases, such as with non-collaborative sensing, both overall false-alarm and missed-detection probabilities will increase. Indeed, it becomes more difficult to qualitatively show the performance tradeoffs with various scheduling intervals in the context of multi-sensor joint spectrum sensing and target tracking. We use numerical examples in the next section to demonstrate such performance tradeoffs.

IV. Numerical Examples

A. Simulation Setup

Suppose five sensors are deployed to utilize a $100\,\mathrm{kHz}$ bandpass channel with an average signal-to-noise ratio of $SNR = -20\,\mathrm{dB}$. The long-term licensed channel usage is specified by the mean busy and idle times $\mathbb{E}[ON] = 10\,\mathrm{s}$ and $\mathbb{E}[OFF] = 40\,\mathrm{s}$ respectively; consequently, the channel is unavailable for about 20% of the time. Ambient signals are sampled at the Nyquist sampling rate, i.e., $100\,\mathrm{kbps}$. For the tracking task, suppose the target motion follows the CWNA model defined in Section II with noise PSDs along both axes $\tilde{q}_\xi = \tilde{q}_\eta = 0.02\,\mathrm{m}^2/\mathrm{s}^3$. The sensor position measurement noise standard deviations are $40\,\mathrm{m}$, $25\,\mathrm{m}$, $15\,\mathrm{m}$, $25\,\mathrm{m}$, and $18\,\mathrm{m}$ respectively. We investigate in the following the effect of adopting different values of estimation interval T on sensing and tracking performance.

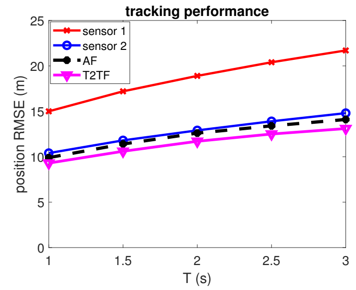


Fig. 1. Tracking performance (position estimate RMSE) with ideal lossless assumption and variable estimation interval T

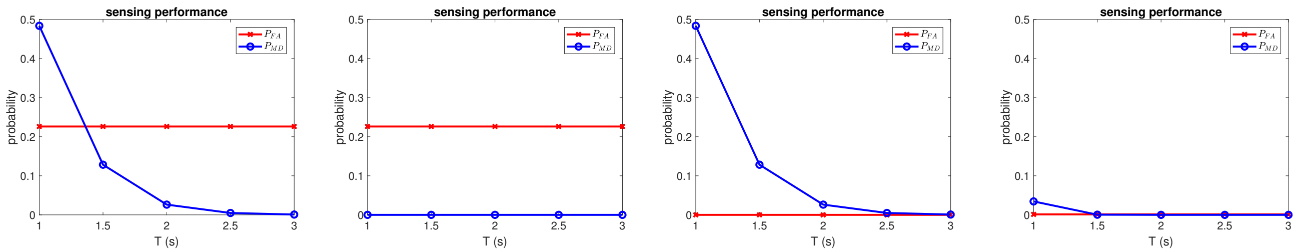
TABLE I
Performance with two sensors and non-collaborative spectrum sensing

Estimation interval T (in s)	1.0	1.5	2.0	2.5	3.0
Effective P_{FA}	0.10	0.10	0.10	0.10	0.10
Effective P_{MD}	0.23	0.05	0.01	0.00	0.00
Effective loss rate	0.22	0.23	0.24	0.24	0.24
Position RMSE (in m): AF	10.1	11.7	13.0	13.9	14.9
Position RMSE (in m): T2TF	9.4	10.9	12.1	13.0	13.8

B. Sensing and Tracking Performance

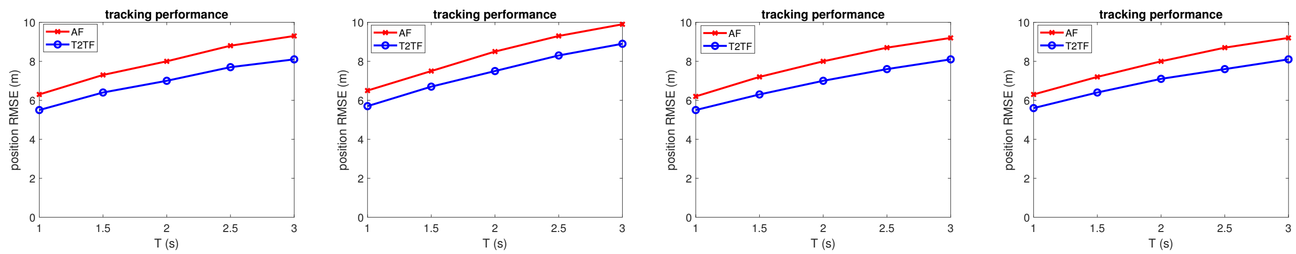
1) Lossless Performance: We first examine the performance using only two sensors with measurement standard deviations $40\,\mathrm{m}$ and $25\,\mathrm{m}$. Fig. 1 plots position root-mean-square error (RMSE) performance of both sensors and two fusers, namely, AF and T2TF, under the ideal lossless condition, with variable estimation intervals T from $1\,\mathrm{s}$ up to $3\,\mathrm{s}$. It can be seen that with increasing estimation interval T, the tracking error gradually increases as well, thanks to the cumulative effect of process noise that results in higher uncertainty with larger gaps between adjacent estimates. In addition, both fusers effectively improve the overall estimation performance, with the T2TF yielding better fused estimates compared to their AF counterparts.

2) Sensing vs. Tracking Performance with Two Sensors and Non-Collaborative Sensing: Table I lists key sensing and tracking performance metrics with variable estimation interval scheduling and non-collaborative sensing under the above two-sensor setting. Here each sensor is supposed to run a CFAR detector with a false alarm rate of 0.05. From the table, we see the effective missed detection rate is over 20% when $T = 1\,\mathrm{s}$, which could be too high for licensed user protection; however, setting T at higher values can quickly alleviate such sensing performance degradation while incurring only a slightly higher loss rate. The fused position estimate RMSEs



(a) Non-collaborative sensing (b) Collaborative sensing: OR-rule (c) Collaborative sensing: AND-rule (d) Collaborative sensing: MAJORITY-rule

Fig. 2. Sensing performance (false-alarm and missed-detection probabilities) with variable estimation interval T



(a) Non-collaborative sensing (b) Collaborative sensing: OR-rule (c) Collaborative sensing: AND-rule (d) Collaborative sensing: MAJORITY-rule

Fig. 3. Tracking performance (position estimate RMSE) with variable estimation interval T

here are somewhat higher than the ones shown in Fig. 1 because nearly a quarter of the total sensor estimates are missing at the fusion center. Overall, setting T in the range of $1.5\,\mathrm{s}$ to $2\,\mathrm{s}$ appears to achieve the best balance in both sensing and tracking performances.

3) Sensing vs. Tracking Performance with Five Sensors: Finally, we consider the case where all five sensors are used with both collaborative and non-collaborative sensing. Figs. 2 and 3 show the sensing and tracking performance respectively. Comparing results in Fig. 3a with those in Table I, one can see the overall tracking error performance improvement with five-sensor fused estimates. In addition, comparing across different sensing configurations, we observe that (i) the OR-rule yields slightly worse tracking performance compared to the other cases; (ii) the MAJORITY-rule yields the best sensing performance with the lowest false-alarm and missed-detection rates. The overall sensing and tracking performance is best balanced if we select T to be around $1.5\,\mathrm{s}$.

V. Conclusion

We investigated interleaving spectrum sensing and target tracking performance tradeoffs by considering the effect of estimation/fusion interval scheduling on their performances. Both non-collaborative sensing and collaborative sensing were explored, along with two types of fusers for combining target state estimates, in their effect on the overall induced loss. A numerical example was used to demonstrate how to select a configuration to achieve a reasonably good performance tradeoff among the alternatives. Future work may include more detailed modeling of other detection methods as well as sensing decision fusion and state estimate fusion rules. Also of future interest are methods that incorporate noise uncertainty and adaptively update the sensing/tracking interval based on evolving channel and target dynamics.

Acknowledgment

This work was funded in part by the Mathematics of Complex, Distributed, Interconnected Systems Program, Office of Advanced Computing Research, U.S. Department of Energy, and SensorNet Project of Office of Naval Research, and was performed at Oak Ridge National Laboratory managed by UT-Battelle, LLC for U.S. Department of Energy under Contract No. DE-AC05-00OR22725.

References

[1] I. F. Akyildiz, W. y. Lee, M. C. Vuran, and S. Mohanty. A survey on spectrum management in cognitive radio networks. *IEEE Communications Magazine*, 46(4):40–48, Apr. 2008.
[2] Y. Bar-Shalom, P. K. Willett, and X. Tian. *Tracking and Data Fusion: A Handbook of Algorithms*. YBS Publishing, 2011.
[3] Control System Toolbox: Solve discrete-time algebraic Riccati equations (DAREs). https://www.mathworks.com/help/control/ref/dare.html.
[4] R. Doost-Mohammady and K. R. Chowdhury. Design of spectrum database assisted cognitive radio vehicular networks. In *7th International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, pages 1–5, Jun. 2012.
[5] E. L. Lehmann and J. P. Romano. *Testing Statistical Hypotheses*. Springer, 2008.
[6] S. Maleki, S. P. Chepuri, and G. Leus. Energy and throughput efficient strategies for cooperative spectrum sensing in cognitive radios. In *IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications*, pages 71–75, San Francisco, CA, Jun. 2011.
[7] S. M. Ross. *Stochastic processes*. Wiley, 1996.
[8] L. L. Scharf and C. Demeure. *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*. Addison-Wesley, 1991.
[9] S. K. Sharma, E. Lagunas, S. Chatzinotas, and B. Ottersten. Application of compressive sensing in cognitive radio communications: A survey. *IEEE Communications Surveys Tutorials*, 18(3):1838–1860, thirdquarter 2016.
[10] D. Simon. *Optimal state estimation: Kalman, H [infinity] and nonlinear approaches*. Wiley-Interscience, 2006.