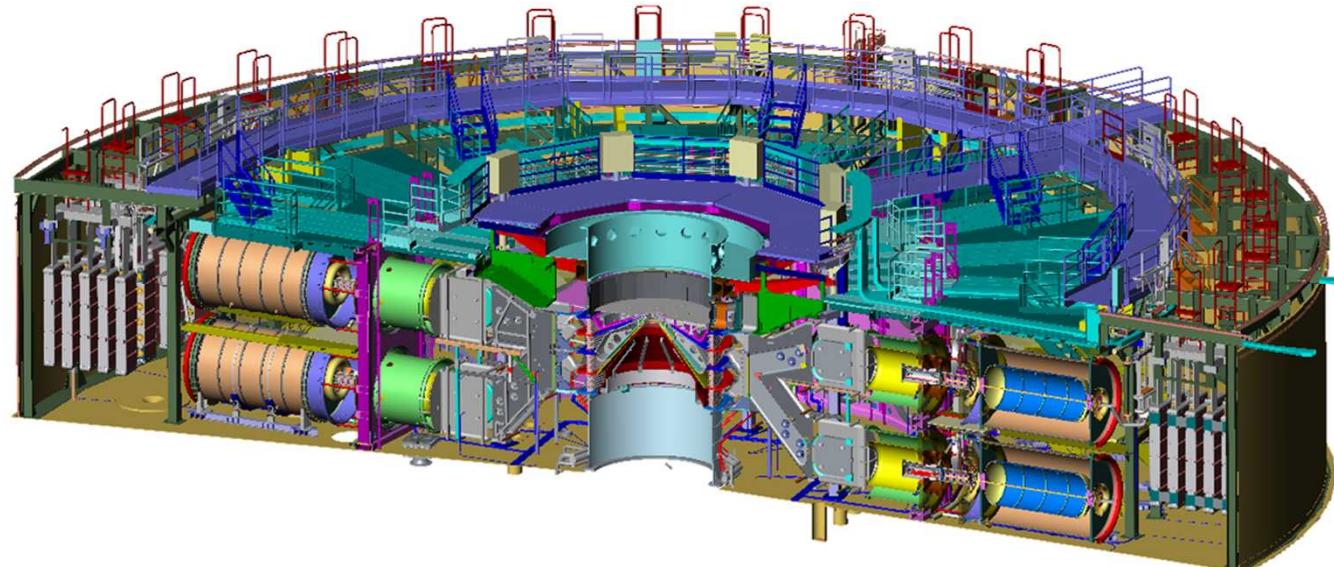
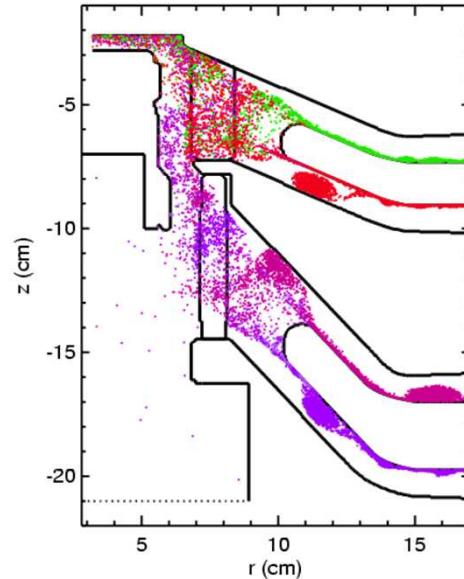


Quicksilver and EMPHASIS: Physics Overview for Modeling Pulsed Power

July 18, 2017



# Physics Overview for Modeling Pulsed Power Using Sandia's codes Quicksilver and EMPHASIS

Peggy Christenson  
7/18/2017



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# Acknowledgements



- Other Quicksilver/EMPHASIS/EMPIRE developers:
  - Tim Pointon, Keith Cartwright, Becky Coats, Matthew Bettencourt, Chris Moore, Edward Phillips, Richard Kramer, and Ed Love
- Thanks to George Laity for up-to-date Z images

# Overview

- What physics is required to model pulsed power flow from Marx bank to load?
  - Relativistic, electromagnetic, kinetic plasma, radiation and neutral transport
  - Volumetric plasma chemistry (collisions between: electrons, ions, neutrals, photons)
    - ionization, excitation, momentum transfer (charge-charge, charge-neutral)
  - Surface physics (neutral, ion and electron emission from surfaces)
    - field emission, thermal emission, desorption, secondary particle emission
- Requirements for credible and practical simulations
  - Particle emission models (space-charge-limited, plasma and/or neutral injection)
  - External circuits for power sources and complicated loads
  - Dynamic particle load-balancing (for efficient parallelization)
  - Implicit algorithms with  $\omega_p \Delta t > 1$ ,  $\omega_c \Delta t > 1$ ,  $\Delta x > \lambda_d$
  - Hybrid fluid/kinetic plasma model
  - High fidelity geometric representation (e.g. body fitted boundaries)

# Modeling capabilities (past)

- **QUICKSILVER:**
  - Structured-mesh particle-in-cell code
    - Parallel with dynamic load balancing
  - Explicit finite-difference EM-PIC (3-D, 2-D)
  - Coupling to transmission line and load models (boundary conditions)
  - Space-charge limited, thermal and secondary particle emission
  - Monte Carlo Collisions (charge-neutral with prescribed neutral background)
  - Coulomb collisions
  - Coupling to ITS for radiation transport
- 30 year history of Pulsed Power applications:
  - Z machine power flow
  - HERMES (and coupling to ITS for rad-transport)
  - Ursa Minor (next-gen radiographic driver)
  - Ion Diodes in the past (PBFA II, SABRE)

# Modeling capabilities (present)

- EMPHASIS™ (ElectroMagnetics PHysics AnalySIS):
  - Unstructured finite-element mesh particle-in-cell code
    - Parallel with dynamic load balancing
  - Implicit finite-element EM-PIC (3-D)
  - Coupling to Xyce for external circuit (transmission line and load models)
  - Space-charge limited, thermal, field, and secondary particle emission
  - Monte Carlo Collisions (charge-neutral with prescribed neutral background)
  - Coupling to ITS for radiation transport
  - Drift-diffusion model
    - PIC model coupled to rate equations and conductivity,  $\sigma(E)$ , and mobility,  $\mu(E)$
- Recent applications
  - Optimization of design of dynamic material load geometries on Z
  - Tapered MITL redesigns (e.g. HERMES)

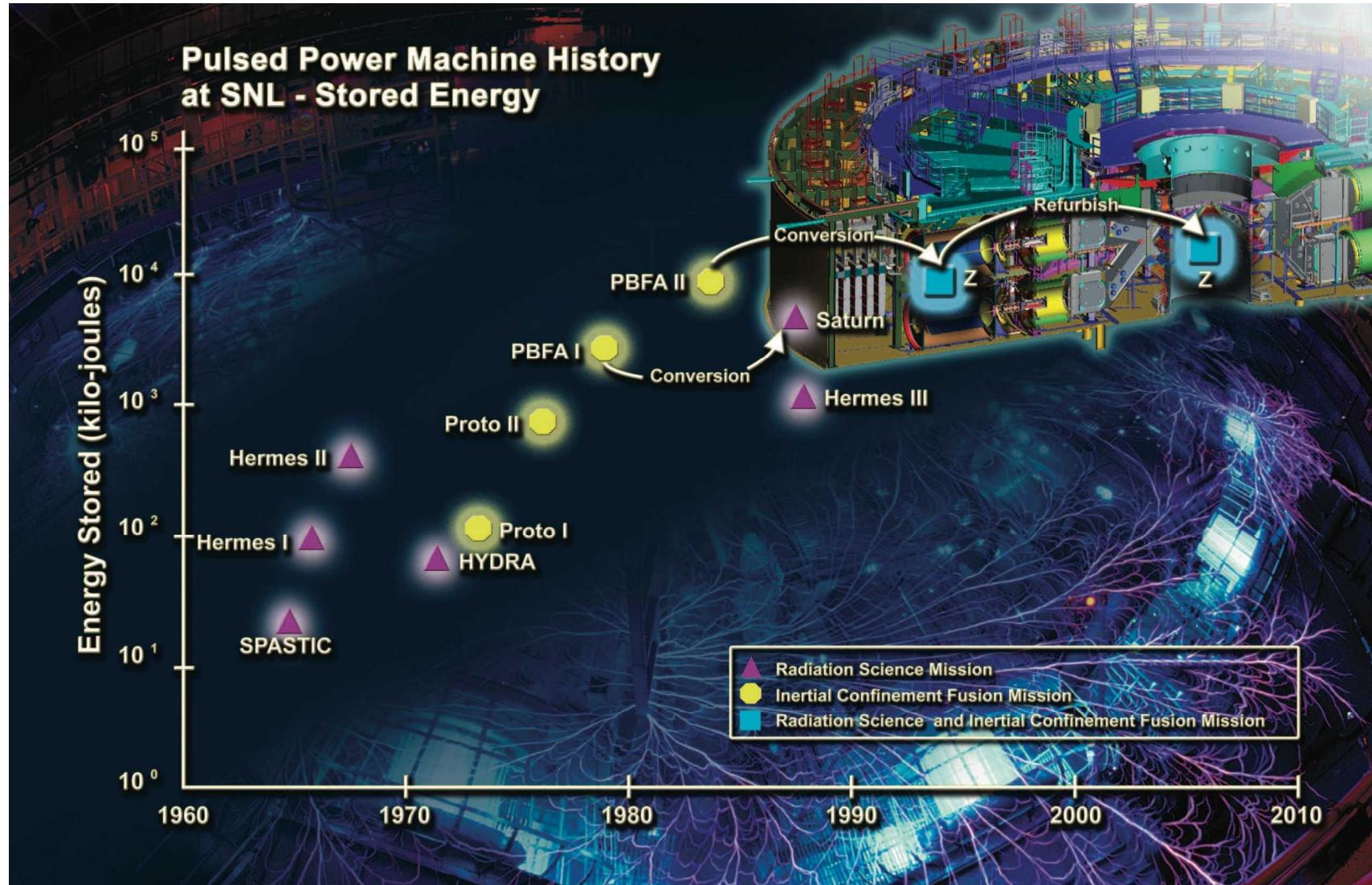
# Current Modeling Deficiencies of EMPHASIS

- Radiation transport is decoupled from the plasma
  - Time independent radiation transport simulation modulated in time
- Full multi-fluid PIC hybrid
- Evolution of neutral and excited species
- Implicit algorithms with  $\omega_p \Delta t > 1$ ,  $\omega_c \Delta t > 1$ ,  $\Delta x > \lambda_d$

# Modeling capabilities (future)

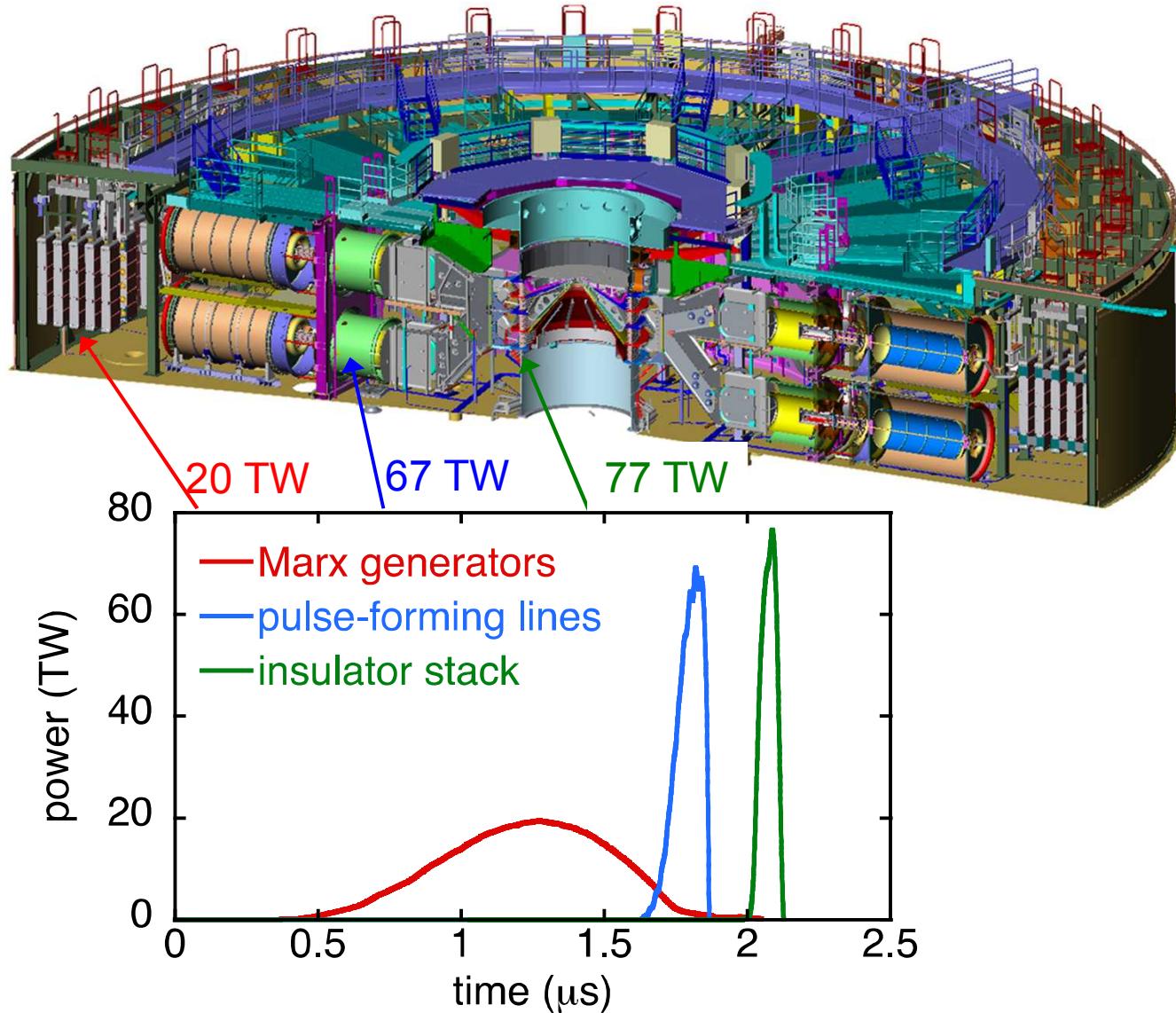
- EMPIRE: ElectroMagnetic Plasma with Integrated Radiation Effects
  - Implicit finite-element EM-PIC (3-D)
  - Hybrid structured/unstructured in-situ meshing
  - Built to operate on next generation platforms
  - Parallel with dynamic load balancing
  - Coupling to Xyce for external circuit (transmission line and load models)
  - Comprehensive emission models
    - Soon to have space-charge limited, thermal beam particle emission
  - Direct Simulation Monte Carlo (DSMC)
    - self-consistent evolution charged, neutral, and excited species
  - Monte Carlo Collisions (MCC)
    - charge-neutral with prescribed neutral background
  - Coupling to ITS for radiation transport
    - Will be replaced with a self consistent time accurate integrated radiation transport
  - Hybrid with full multi-fluid model
- Recent applications
  - Two stream instability
  - Laudau damping
  - Linear plasma waves

# Pulsed Power at SNL

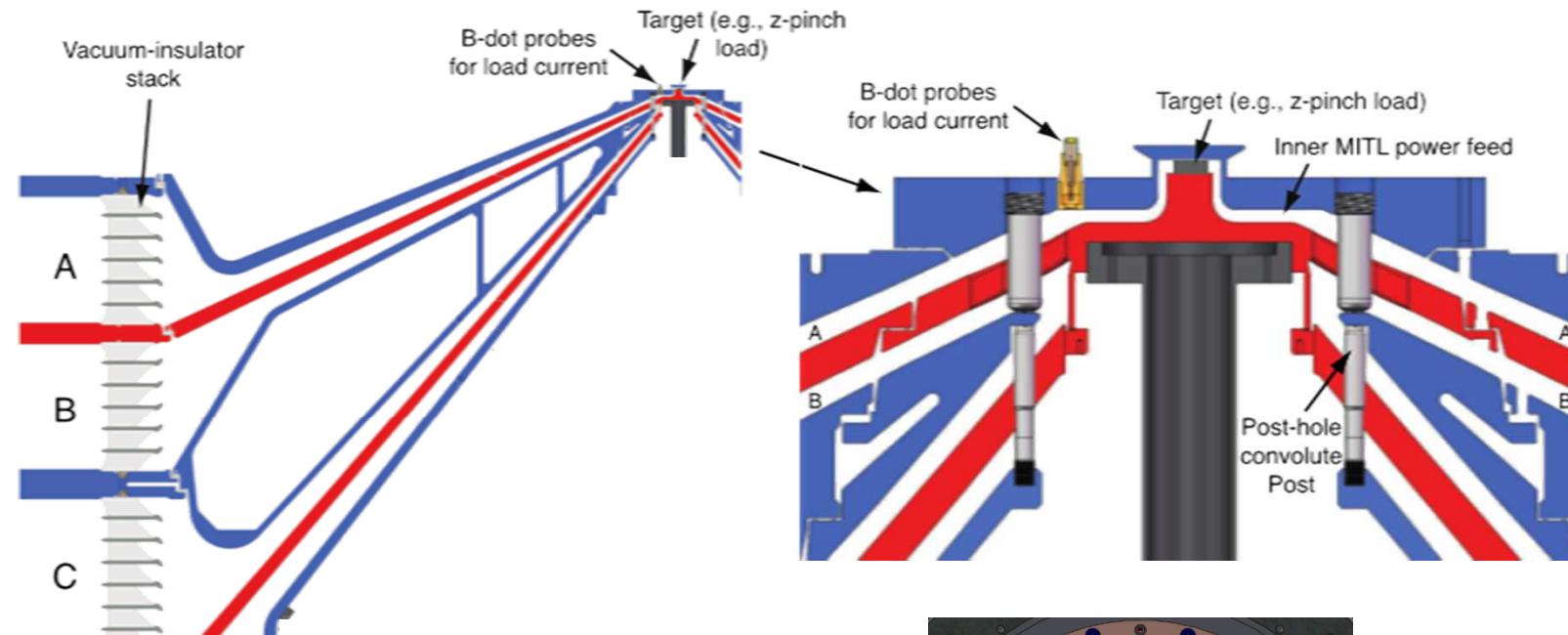


# The Z machine Pulsed Power technology

- Marx Generators
- Intermediate storage capacitors
- Laser-triggered gas switches
- Pulse-forming lines
- Self-breaking water switches
- Water convolute
- Vacuum-insulator stack
- Vacuum section

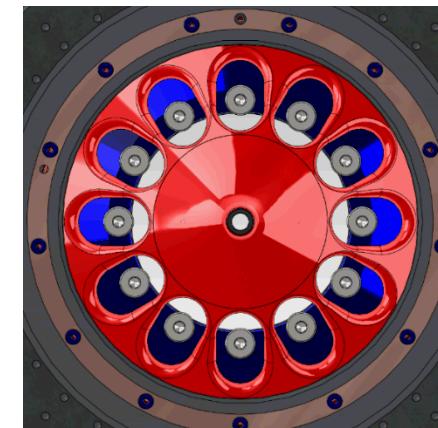


# Vacuum Section



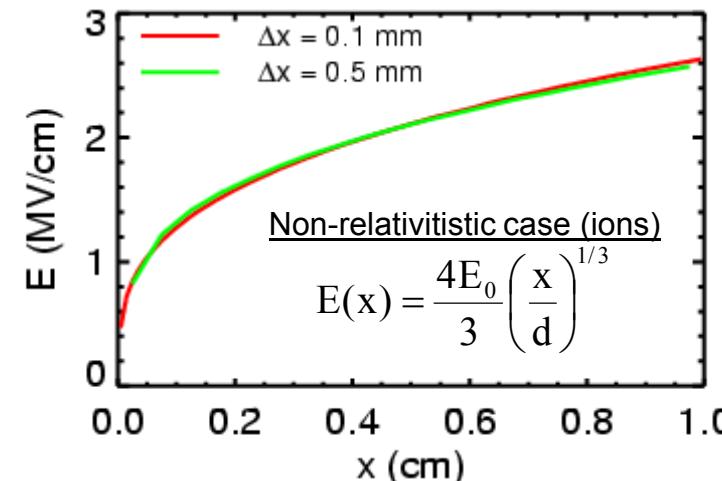
Anodes in Blue  
Cathodes in Red

- 4 magnetically insulated transmission lines
- Coupled in parallel with a double post-hole convolute to the inner MITL



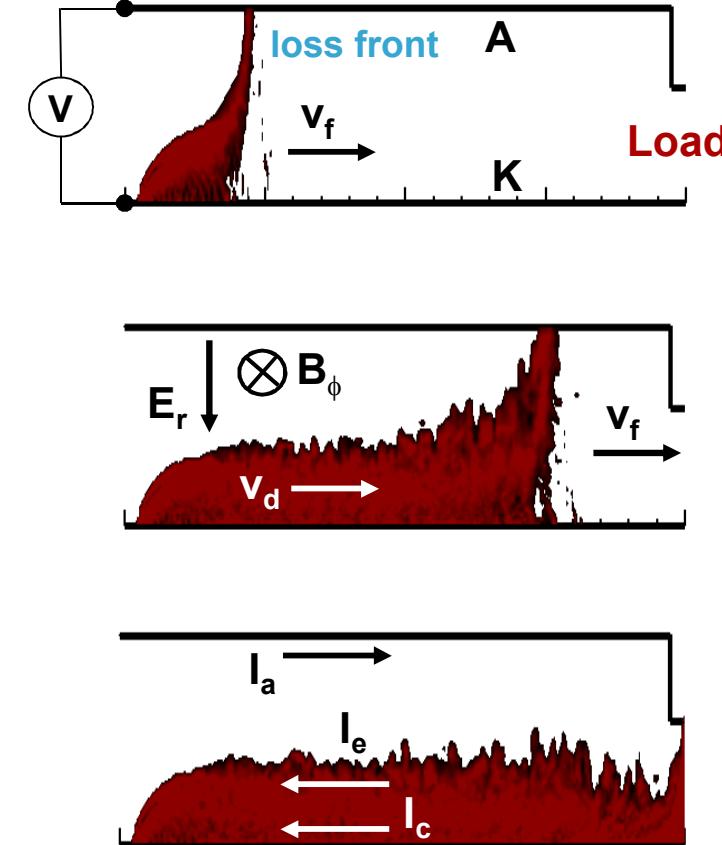
# Space-charge-limited electron emission

- Electron emission from the cathode when  $E > \sim 250 \text{ kV/cm}$
- Electron current reduces  $E_{normal}$  at surface, and increases until  $J_e$  forces  $E_n \rightarrow 0$  -- “space-charge-limited” emission
- Analytic solution for 1-D planar diode:
  - Gap  $d$ , voltage  $V$ , particle charge  $q$ , mass  $m$
  - Child-Langmuir current:  $J_{CL} = \frac{4\epsilon_0}{9} \left( \frac{2q}{m} \right)^{1/2} \frac{V^{3/2}}{d^2}$  (non-relativistic)
- Modeling challenge:  
very steep E-field profile at  
emission surface
  - Ion diode:  $d = 1 \text{ cm}$ ,  
 $V = 2 \text{ MV}$ ;  $E_0 = V/d$

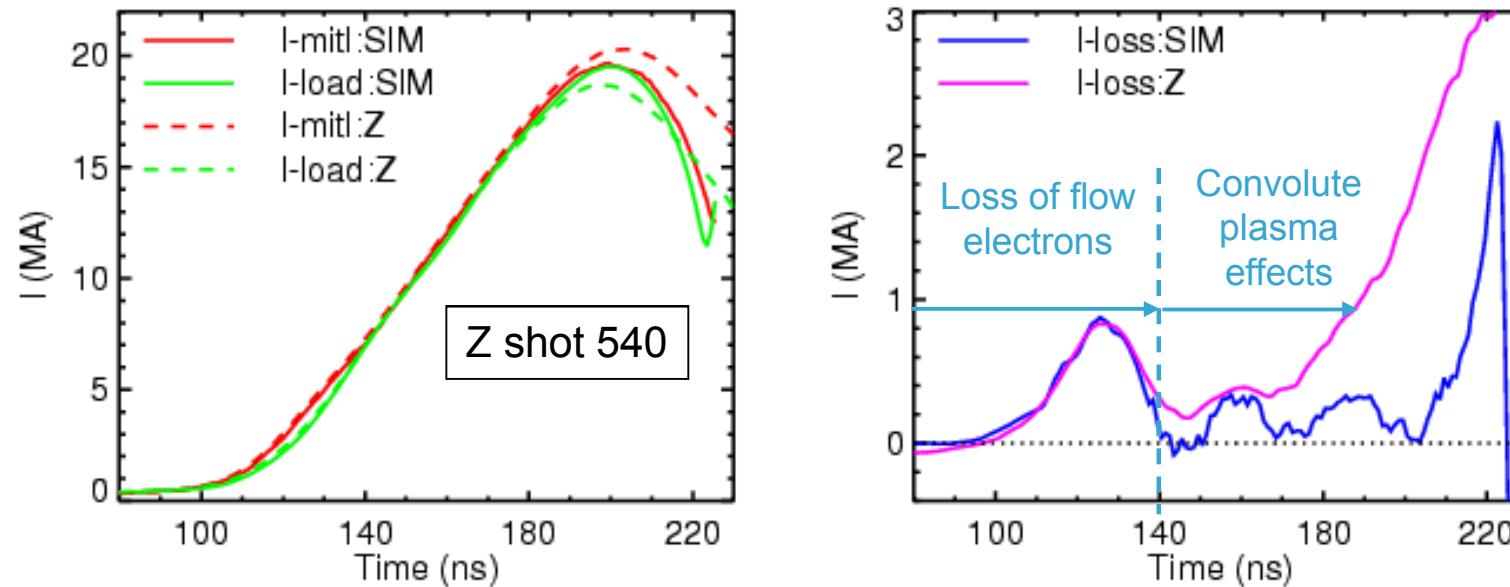


# MITLs enable efficient power delivery

- Illustrated with coaxial MITL (courtesy of Steve Rosenthal)
- When emission first starts, electrons go straight across the A-K gap – current loss
- Behind the “loss front”, B-field from the current flow can stop electrons from reaching anode – “magnetic insulation”
- Useable current delivered to the load is the cathode current  $I_c = I_a - I_e$ , where  $I_e$  is the current flowing in the electron sheath (typically dumped as a loss elsewhere)
- MITL design is a tradeoff between:
  - Large gap: better insulation, lower flow
  - Small gap: lower inductance



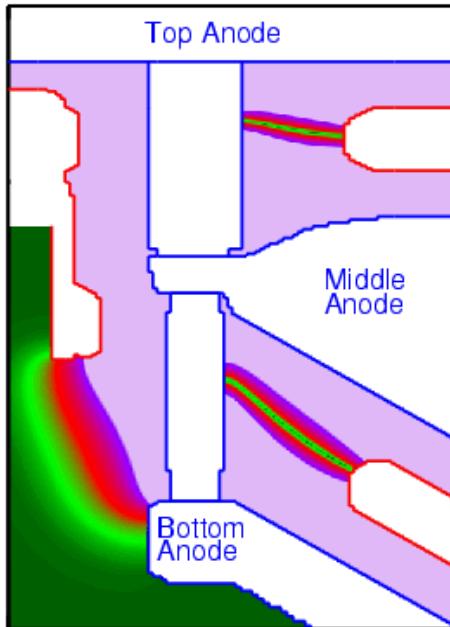
# Quicksilver can accurately predict early-time losses



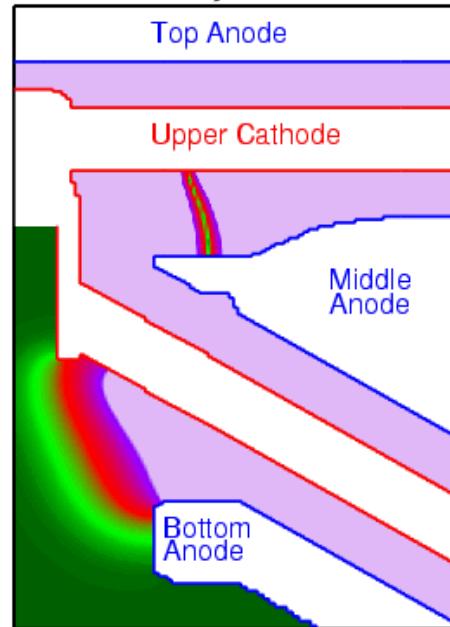
- Simulations with SCL emission on all cathode surface
- Early-time loss: MITL flow electrons lost to the anode ("loss front")
- Extra late-time loss is due to expanding electrode plasmas in the convolute
  - Experimental results in M.R. Gomez, *et al.*, Phys Rev ST-AB **20**, 010401 (2017)
  - Simulations with LSP [D.V. Rose, *et al.*, Phys Rev ST-AB **11**, 060401 (2008)].

# Magnetic nulls in the convolute

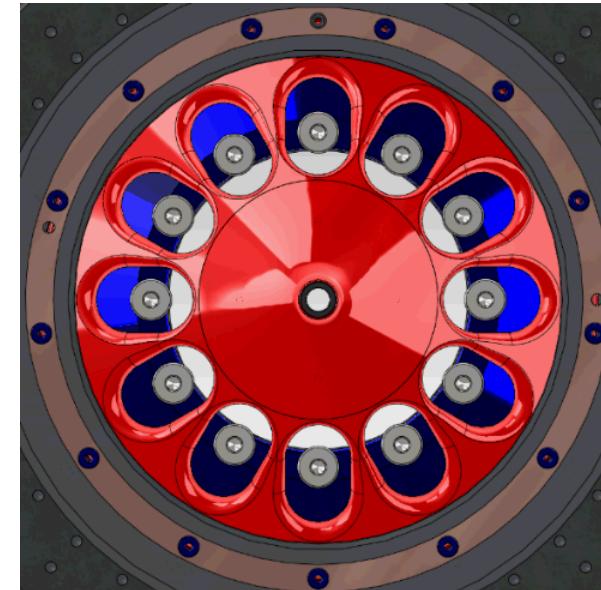
Plane Through Center of Post



Plane Midway Between Posts

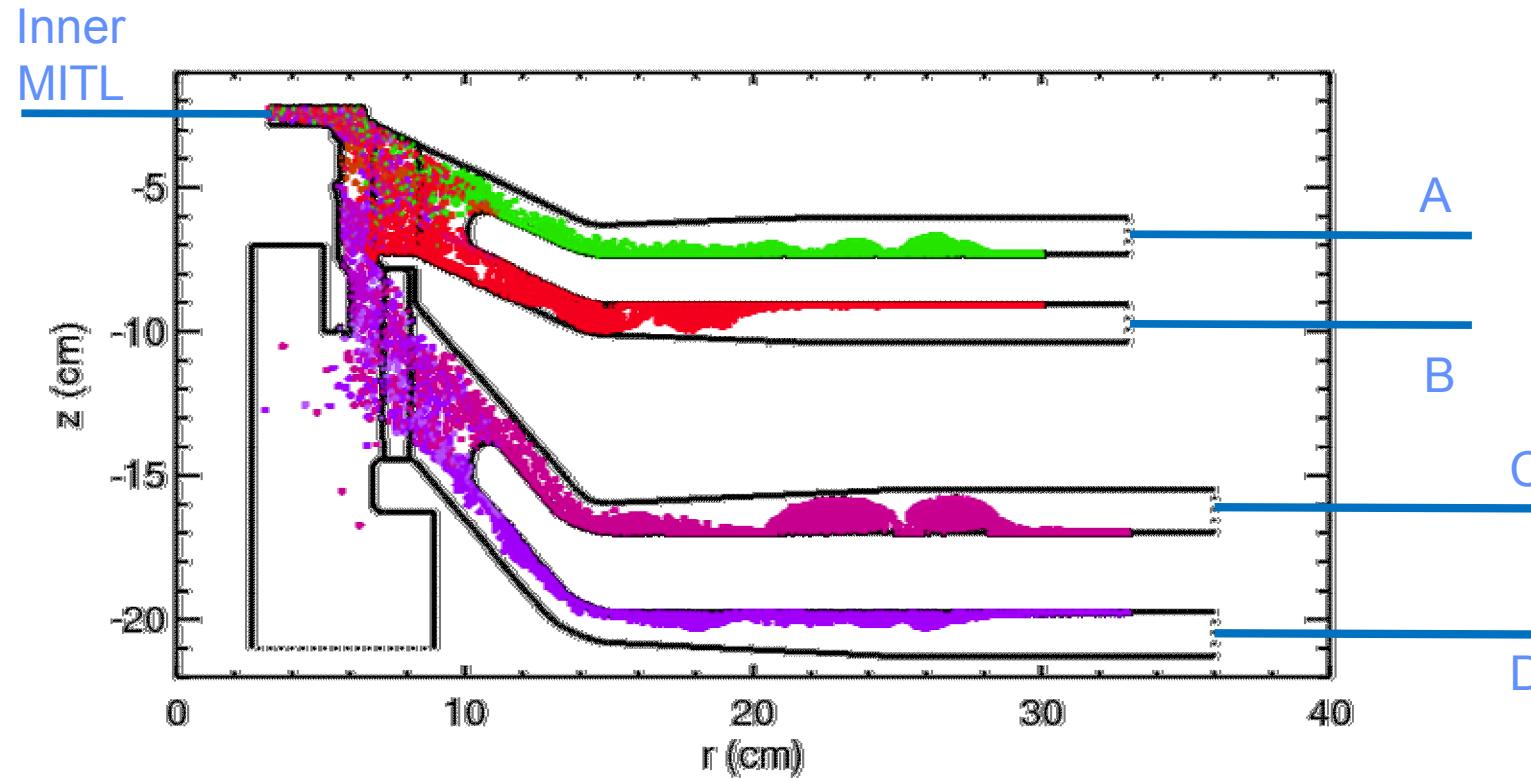


Convolute top view



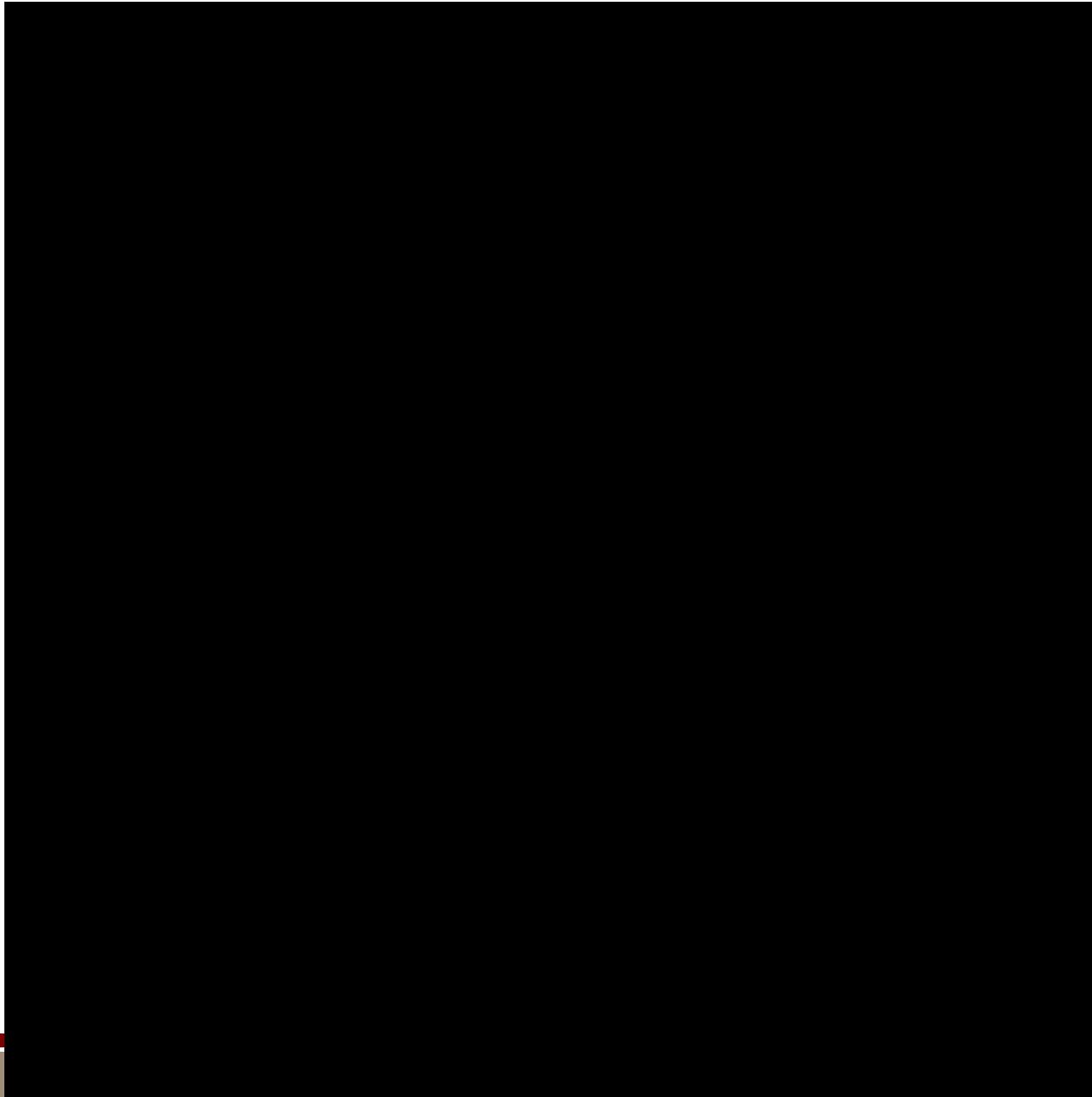
- Electrons flow into the convolute from the feed MITLs
  - Complicated by non-uniform impedance of feed MITLs,  $Z \sim d/r$
- There will be electron losses at the nulls: “loss of magnetic insulation”
  - How much? How fast does the anode surface heat?

# Time-accurate Quicksilver simulations of Z MITL/Convolute

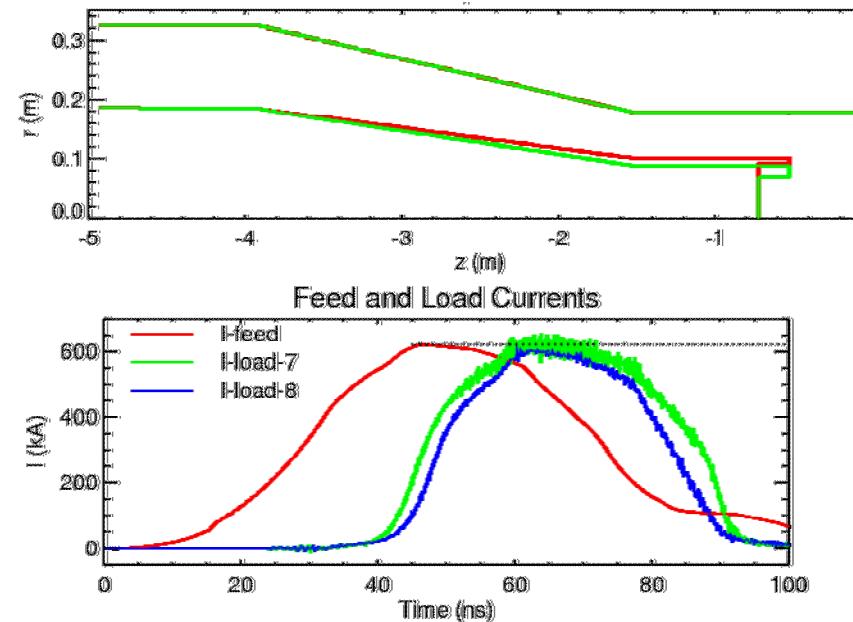


- Only possible with extensive use of 1-D transmission lines
- Reduces the 3-D system size – only emit electrons out to  $r \sim 30$  cm
- Time-accurate source for all four feed MITLs (out to  $r \sim 3$  m)
- Inner MITL connected to a 1-D TL terminated with a realistic load

# Quicksilver MITL Movie

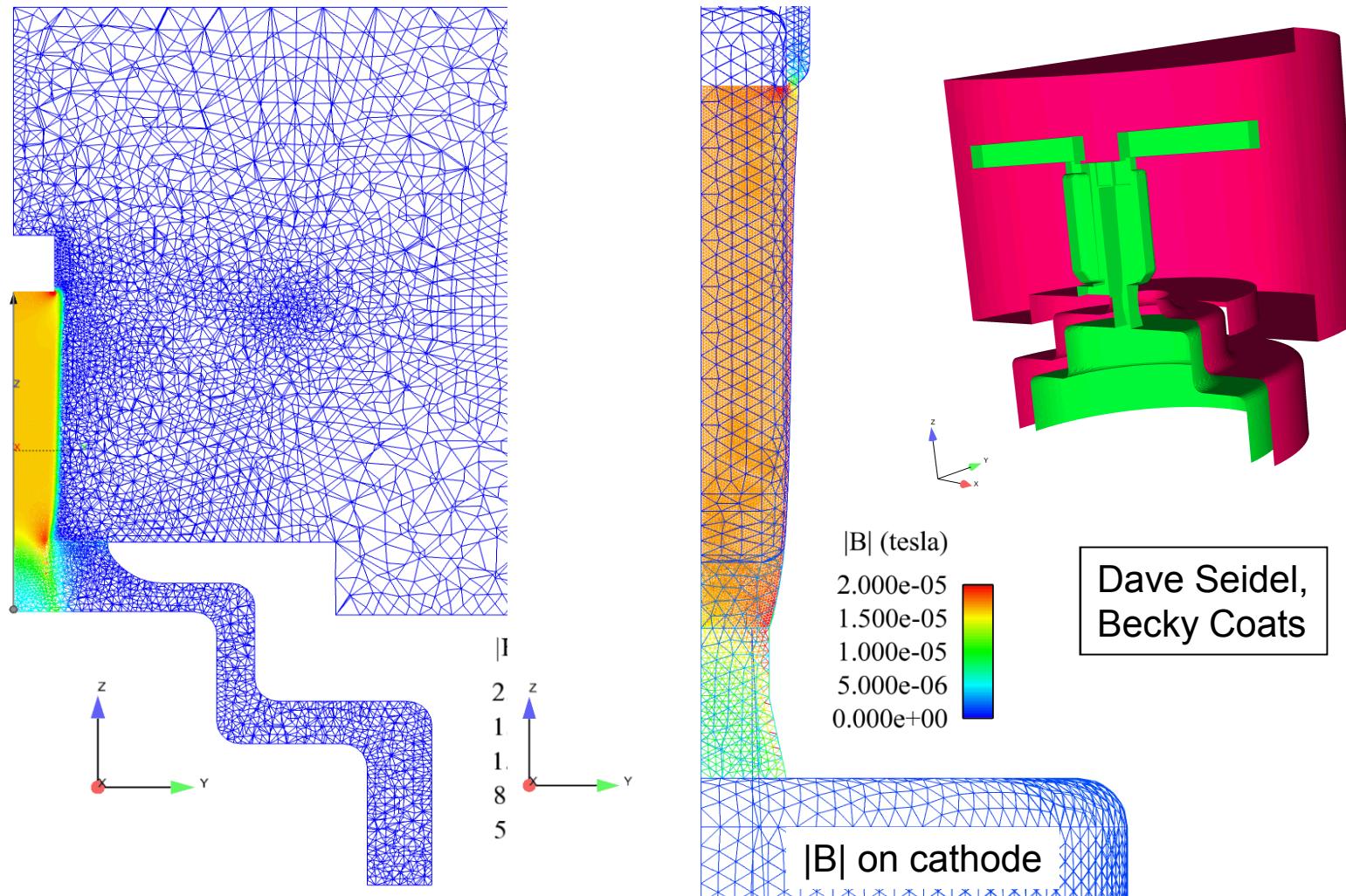


# EMPHASIS could help HERMES MITL redesign

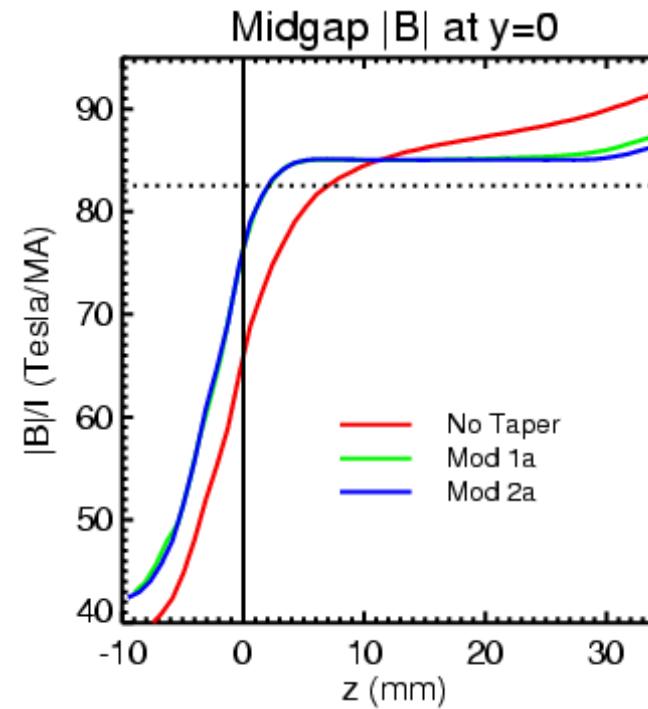
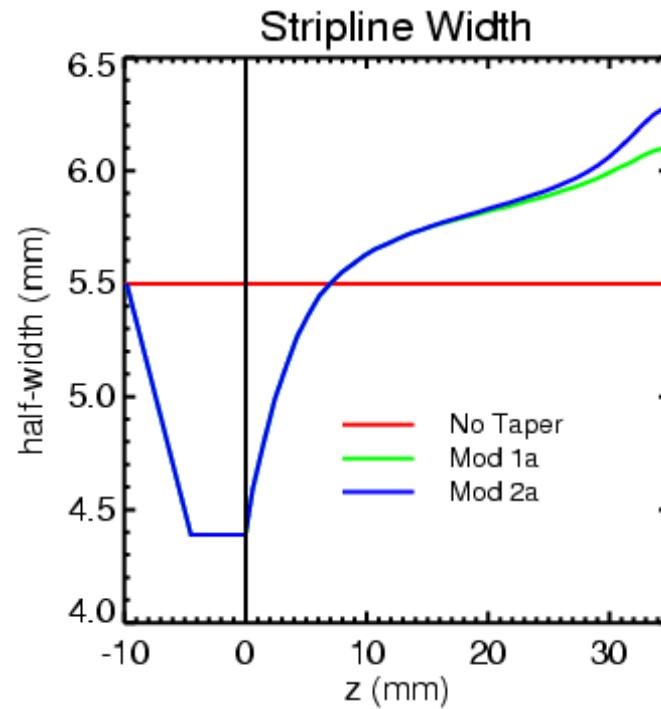


- Extended MITL with constant impedance taper using Quicksilver:
  - Anode: 25.5"  $\rightarrow$  14"; cathode: 14.5"  $\rightarrow$  8" diameter
- Reducing downstream cathode diameter to 7" is even better
  - Better mechanical design: larger A-K gap reduces alignment thresholds
  - Actually reduces current loss to zero
    - ... BUT Quicksilver has a numerical instability caused by the stair-stepped cathode

# EMPHASIS Simulation of ICE stripline geometry

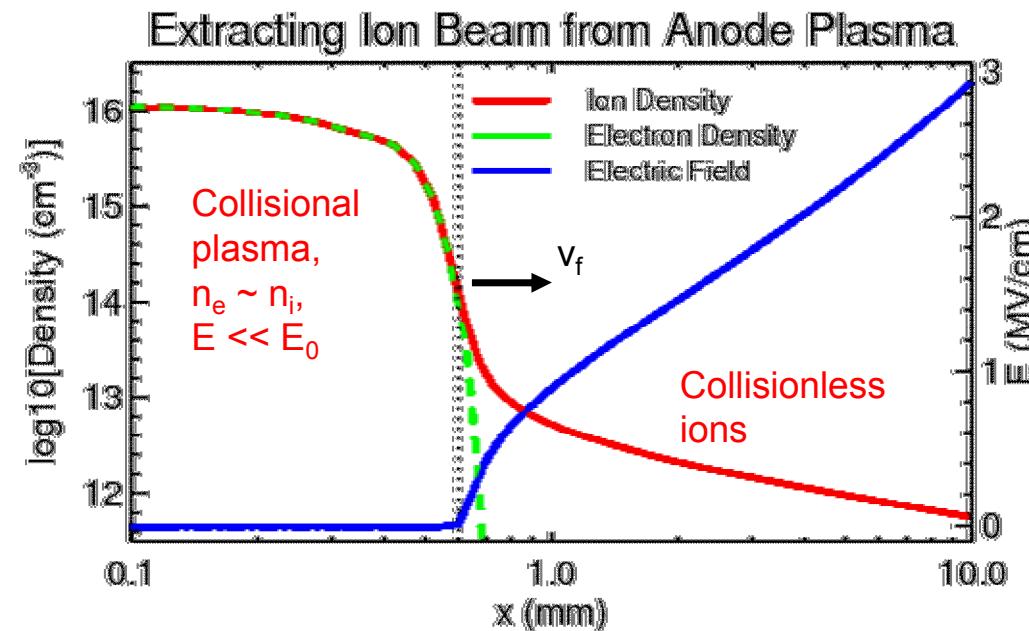


# Optimize axial uniformity of $|B|$ in stripline



- Cut “notch” in width of the stripline below the sample area
- Vary width as a function of  $z$  above the notch
- Uniformity of  $B(z)$  profile over the samples is greatly improved

# Electrode plasmas



1D multi-fluid simulation of an ion diode

$V = 2$  MV

$d_0 = 1$  cm

$E_0 = 2$  MV/cm

- Typical plasma parameters:  $n = 10^{16} - 10^{18}$  cm<sup>-3</sup>;  $T = 1 - 10$  eV
  - Debye length  $\lambda_{De} \sim 100$  nm; Plasma frequency  $\omega_{pe} \sim 10^{13}$  s<sup>-1</sup>
- Expansion velocity of plasma front,  $v_f \sim 1 - 20$  cm/μs
- Dynamic effective anode-cathode gap:  $d_{eff}(t) = d_0 - v_f t$ 
  - A decreasing A-K gap is almost always bad

# Modeling electrode plasmas is very difficult

- Clearly, must operate at  $\Delta x/\lambda_{De} \gg 1$ 
  - Possible with energy-conserving PIC: stable, but no guarantee of accuracy
- Large E-fields introduce a new, severe constraint:  $eE\Delta x/T_e < \sim 1$ 
  - A significant fraction of the plasma electrons and ions must be able to cross a cell against the E-field to sustain expansion
  - For  $d = 1 \text{ cm}$ ,  $V = 2 \text{ MV}$ ,  $T_e = 1 \text{ eV}$ , need  $\Delta x \sim 1 \mu\text{m}$
- Cathode plasma expansion is harder to model than anode plasma
  - Less drag from Coulomb collisions as electrons accelerate ( $v_{ei} \sim 1/v^3$ )
- Only path forward seems to be a hybrid fluid-PIC model
  - Fluids for dense plasma with plasma-front tracking scheme
  - Creation of PIC particles at the front

# Summary

- Quicksilver has had success modeling many pulsed power systems
  - PBFA II, SABRE, Z, Ursa Minor, HERMES, and many more
- Key enabling capabilities
  - Massively parallel with dynamic load balancing
  - Particle emission models
  - External circuits (1-D transmission lines or Xyce)
- Unstructured mesh capabilities of EMPHASIS and EMPIRE can address key geometry issues not possible with legacy structured EM-PIC
  - Work needed on emission models and external circuits (Xyce)
- Development of advanced plasma emission models are necessary to simulate late-time current loss on Z (and impedance collapse in other devices)
  - A very difficult problem

# Backup slides



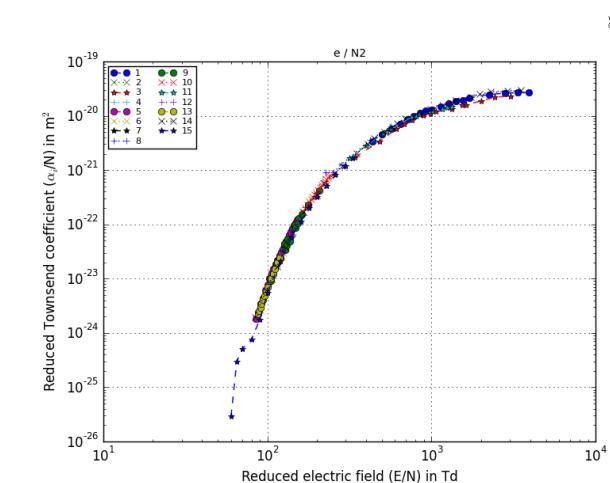
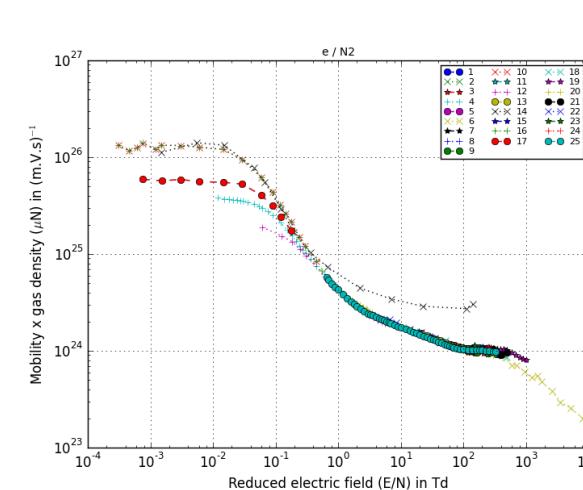
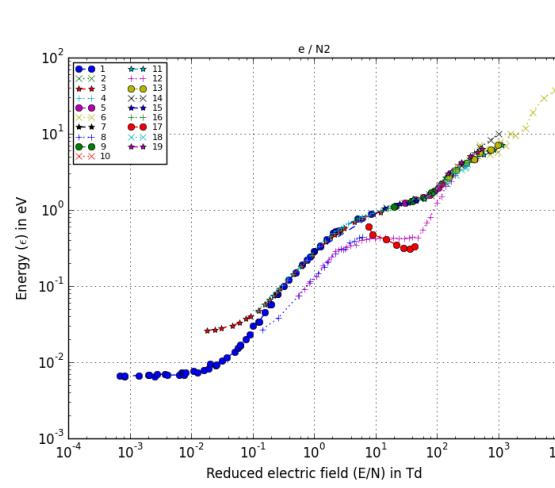
# Surface Plasma Generation Models

Emission Model	Incident Particle	Secondary Particle	Thermal Dependence
Desorption (Hertz-Knudsen-Langmuir)(Sanford et.al.)	None	Neutral	Yes
Thermo-Field Emission (Murphy & Good)	None	Electron	Yes
Space Charge Limited Emission (limit of thermo-field emission for high voltage) (Shiffler)	None	Electron	No
Secondary Emission (Lin and Joy)	Electron	Electron	No
Secondary Emission (Raizer 1991)	Ion	Electron	No
Associative Desorption (Lieberman and Lichtenberg)	Ion/Neutral	Ion/Neutral	Yes
Sputtering (Langley)	Ion	Ion/Neutral	Weak
Stimulated ion emission	Electron	Ion	Weak

It is necessary to couple the desorption and thermo-field emission to a thermal solve on the electrodes to accurately model surface plasma generation

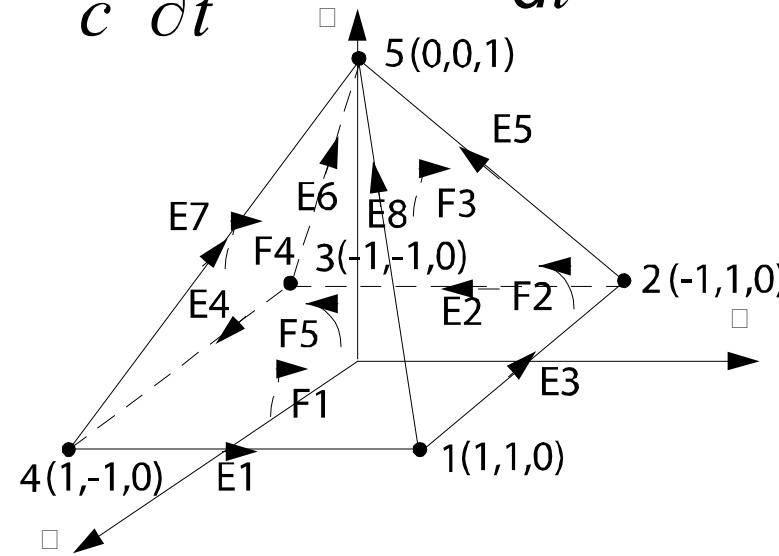
# Modeling

- EMPHASIS EM/ PIC
  - EM field solve
- Conductivity Model
  - Mobility/conductivity as a function of **electric field** and density
  - One can write the time update as one big matrix,
    - Use Ohms law for the current



# Second Order EM Formulation

$$\nabla \times \left( \frac{1}{\mu_r} \nabla \times \bar{E} \right) + \frac{\epsilon_r}{c^2} \frac{\partial^2}{\partial t^2} \bar{E} + \mu_0 \sigma \frac{d}{dt} \bar{E} + \mu_0 \bar{E} \frac{d\sigma}{dt} = -\mu_0 \frac{\partial}{\partial t} \bar{J}$$



$$[T] \frac{d^2 E}{dt^2} + [B] \frac{\square E}{\square t} + [S] E + F = 0$$

# Newmark time integration

$$[T] \frac{E^{n+1} - 2E^n + E^{n-1}}{\Delta t^2} + [B] \frac{E^{n+1} - E^{n-1}}{2\Delta t} + [S](\alpha_1 E^{n+1} + \alpha_2 E^n + \alpha_3 E^{n-1}) + F = 0$$

Second order accuracy

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Unconditional stable

$$\alpha_2^2 = 4\alpha_1\alpha_3$$

Energy conserving (symplectic)

$$\alpha_1 = \alpha_3$$

Leads to

$$\alpha_1 = 1/4, \alpha_2 = 1/2, \text{ and } \alpha_3 = 1/4$$

# Godfrey update

$$E^{n+1} - 2E^n + E^{n-1} + \\ 4\Delta t \frac{([B] + (2\alpha_1 + \alpha_2)\Delta t[S])E^n + (-[B] + (1 - (2\alpha_1 + \alpha_2))\Delta t[S])E^{n-1} + \Delta tF]}{4[T] + 2\Delta t[B] + 4\alpha_1\Delta t^2[S]} = 0.$$

[B] and [S] are updated as the conductivity changes

Fill time is on the order of the solve time

# Rate Equations

$$\frac{\partial \mathbf{n}}{\partial t} + \nabla (n \mathbf{u}) = G - L$$

Secondary Electron Rates

$$\frac{\partial N_e}{\partial t} = S_e + (\nu_i(E) - \nu_a) N_e - \alpha_{ei}(E) N_e N_+ \quad (5.1)$$

$N_e$  is the electron density

$N_+$  is the positive ion density

$N_-$  is the negative ion density

$S_e$  is the photoelectrons created by the primary electrons

$\nu_i$  is the rate of production of the secondary electrons by collisions between secondary electrons

$\nu_a$  is the attachment rate of the electrons to neutral air atoms

$\alpha_{ei}$  is the recombination between electrons and positive ions

# Ion Rate equations

Positive Ion Rate

$$\frac{\partial N_+}{\partial t} = S_e + \nu_i(E)N_e - \alpha_{ei}(E)N_eN_+ - \alpha_{ii}N_+N_-$$

$\alpha_{ei}$  is the recombination between negative and positive ions

Negative Ion Rate

$$\frac{\partial N_-}{\partial t} = \nu_a N_e - \alpha_{ii} N_+ N_-$$

# Conductivity

$$\sigma = \mu_e N_e + \mu_+ N_+ + \mu_- N_-$$

$$\nabla \times \left( \frac{1}{\mu_r} \nabla \times \bar{E} \right) + \frac{\epsilon_r}{c^2} \frac{\partial^2}{\partial t^2} \bar{E} + \mu_0 \sigma \frac{d}{dt} \bar{E} + \mu_0 \bar{E} \frac{d\sigma}{dt} = -\mu_0 \frac{\partial}{\partial t} \bar{J}$$

# Fluid to Drift-Diffusion

$$mn \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = qn (\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla \cdot \boldsymbol{\Pi} + f|_c$$

$\frac{\partial}{\partial t} \equiv 0$ ,  $\mathbf{B} = 0$ ,  $(\mathbf{u} \cdot \nabla) \mathbf{u}$  is zero  $\nabla \cdot \boldsymbol{\Pi} = \nabla p$ , isothermal relation  $p = nkT$

$$f|_c = - \sum_{\beta} mn \nu_{m\beta} (\mathbf{u} - \mathbf{u}_{\beta}) - m \mathbf{u} (G - L)$$

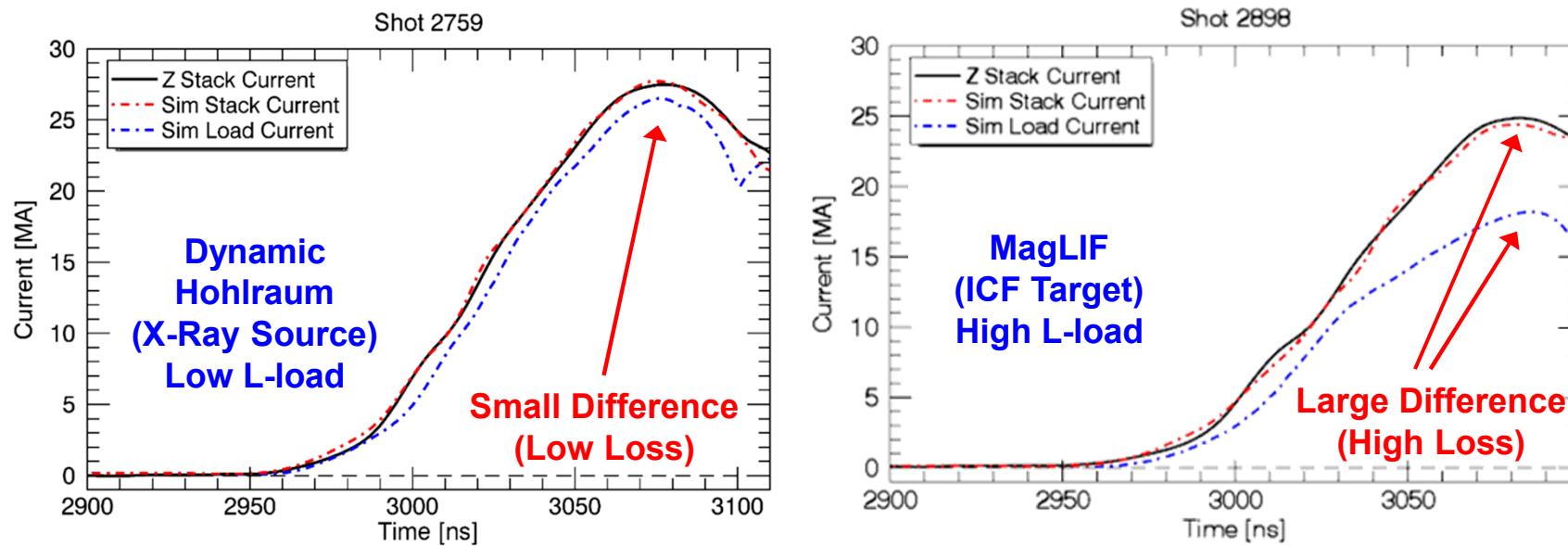
$$f|_c \approx -nm \mathbf{u} \nu_m$$

$$0 = qn \mathbf{E} - \nabla p - mn \nu_m \mathbf{u}$$

$$\mathbf{J} = \frac{q^2 n}{m \nu_m} \mathbf{E} - \frac{q}{m \nu_m} \nabla p$$

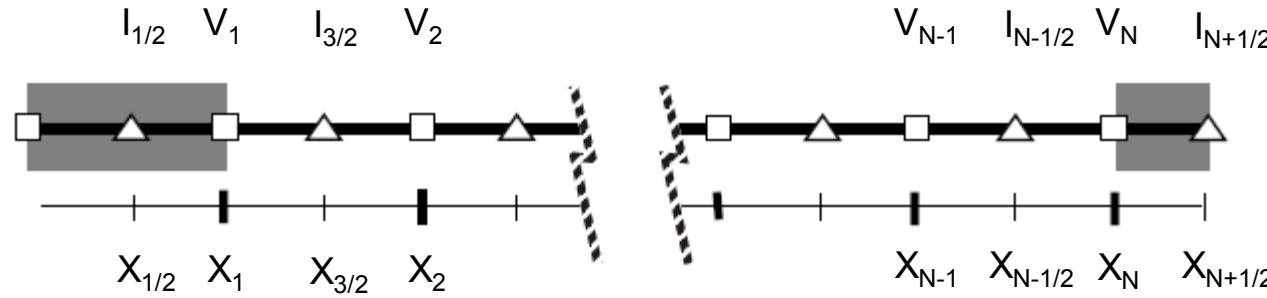
$$\mathbf{J} = qn \mu_m \mathbf{E} - \mu_m \nabla p$$

# Z is modeled with the Bertha circuit code



- POC: Brian Hutsel
- Empirical and physics-based models for all components
  - PIC simulations provide guidance for the vacuum section
- Can simulate entire pulse ( $\sim 3 \mu\text{s}$ ) in  $< \sim 1$  minute

# Quicksilver 1-D transmission lines

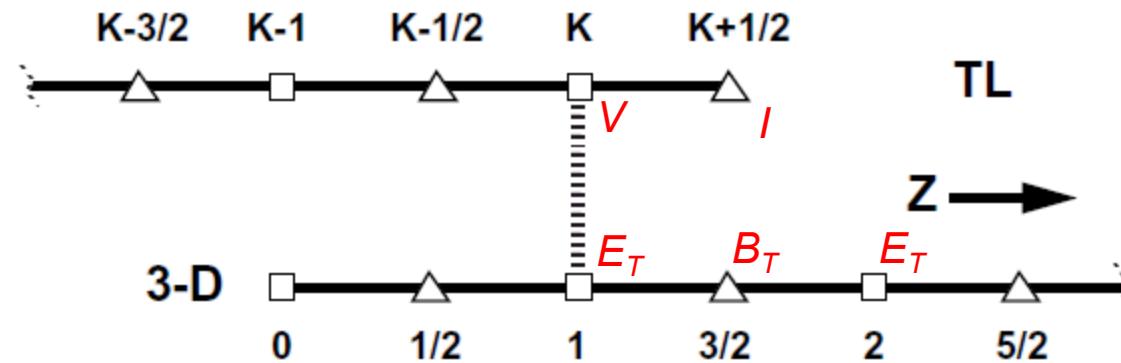


- Telegrapher's equations:  $\frac{\partial V}{\partial t} = -\frac{1}{C_l} \frac{\partial I}{\partial x}$ ; and  $\frac{\partial I}{\partial t} = -\frac{1}{L_l} \frac{\partial V}{\partial x}$
- Discretize in space and time:  $V_i^n = V(x_i, n\Delta t)$ ;  $I_{i+1/2}^{n-1/2} = I(x_{i+1/2}, (n-1/2)\Delta t)$
- $C_l(x_{i+1/2})$  and  $L_l(x_{i+1/2})$  constant in each “full grid” cell:  $[x_i, x_{i+1}]$
- Leapfrog time advance:

$$V_i^n = V_i^{n-1} - \gamma_i (I_{i+1/2}^{n-1/2} - I_{i-1/2}^{n-1/2}); \text{ where } \gamma_i = \frac{2\Delta t}{\Delta x_{i-1/2} [C_l(x_{i-1/2}) + C_l(x_{i+1/2})]}$$

$$I_{i+1/2}^{n+1/2} = I_{i+1/2}^{n-1/2} - \lambda_i (V_{i+1}^n - V_i^n); \text{ where } \lambda_i = \frac{\Delta t}{\Delta x_i L_l(x_{i+1/2})}$$

# Connect 1-D TLs to 3-D system at “TLports”



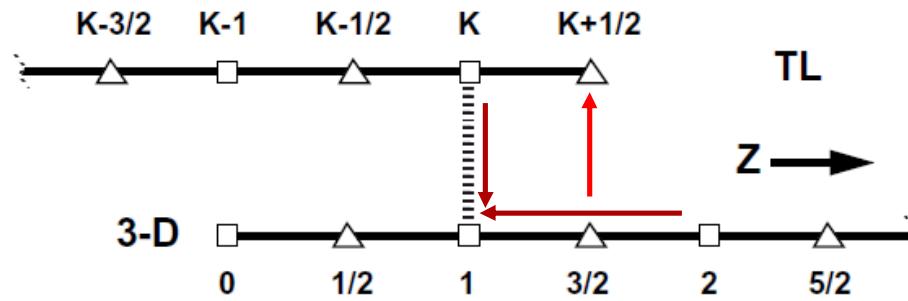
Connection of a TL to a z-normal port (lower z boundary)

- A *TLport* is a rectangular region on a boundary face of a 3-D block, with two conductors separated by a vacuum gap
  - Parallel plates, coaxial lines, more complicated cross-sections
- No interpolation needed in time or normal ordinate (z here) to match:
  - Transverse  $\mathbf{E}_T = (E_x, E_y)$  with  $V$  at boundary plane
  - Transverse  $\mathbf{B}_T = (B_x, B_y)$  with  $I$  half a cell inside

# TEM voltage and current in the 3-D system

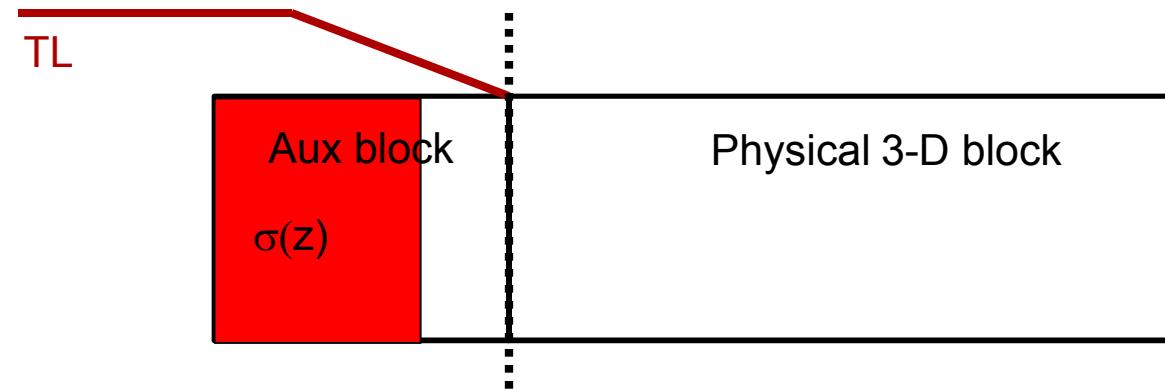
- Solve a 2-D Poisson solution for TEM E-field in the port plane:
  - $\nabla_T^2 \phi = 0$ , with BCs  $\phi = 0$  on cathode, and  $\phi = V_0$  (1 Volt) on anode
  - “Poisson E-field”:  $E_p = -\nabla \phi$
- QS computes  $E_p$  for arbitrary port geometry at  $t = 0$
- $E_p$  is orthogonal to any non-TEM mode profile  $E_{NT}$ :  $\int E_p \bullet E_{NT} dA = 0$
- **Filtering out the non-TEM fields is crucial to stability**
  - TEM voltage:  $V = \frac{\epsilon}{C_l V_0^2} \int E_p \bullet E_T dA$ , where  $C_l = \frac{\epsilon}{V_0^2} \int E_p^2 dA$
  - TEM current:  $I = \frac{1}{\mu V_0} \int \hat{e}_z \times E_p \bullet B_T dA$

# Basic filtered TLport algorithm



- Non-TEM transverse field at  $k = 1, 2$  planes:  $E_k^{*n} = E_{T_k}^n - (V_{3D,k}^n / V_0) E_P$
- In the port plane, set  $E_{T_p}^n = (V_K^n / V_0) E_P + E_1^{*n}$
- Compute  $E_1^{*n+1} = (1 - \delta) E_1^{*n} + \delta E_2^{*n}$ ; where  $\delta = v_{ph} \Delta t / \Delta x$ 
  - Assumption: all non-TEM waves are travelling upstream with same  $v_{ph}$  (usually  $c$ )
- Use  $B_{T,3/2}$  to set boundary value  $I_{K+1/2}$  for  $V_k$  on next timestep
- Usually works fine for inlet power feeds

# “Auxiliary block” TLport algorithm



- Filtered TLport algorithm failure modes:
  - Electrostatic fields from non-TEM structure too close to the port
  - Particles flowing through the boundary
- Solution: add auxiliary field block with conductivity to port boundary
  - For the field solver, this is a “parallel load” with the transmission line
    - TEM fields coupled to the transmission line  $V$  and  $I$
    - non-TEM fields coupled to the aux-block
  - Particles drift ballistically into the aux-block (never come back)
    - Decay particle charge and absorb in conductivity region

# Pulsed Power boundary conditions for TLs

- Thevenin equivalent source:  $V_{oc}(t)$  and source impedance  $R_s$
- Series RLC circuit for driving system with charged capacitors
  - Variable  $R(t)$  acting as the switch (start high, then fall)
- T-junctions and multi-line junctions
- Time-dependent inductive loads (dynamic material shots on Z)
- Z-pinch loads:
  - Thin shell  $r_w(t)$ , height  $h$ , mass  $M$ , return current radius  $b$
  - Time-dependent inductance: 
$$L(t) = L_h \ln\left(\frac{b}{r_w(t)}\right), \text{ where } L_h = \frac{\mu_0 h}{2\pi}$$
  - Force on the wire array: 
$$M\ddot{r}_w = -\frac{L_h I^2}{2r_w}$$
  - Small set of non-linear ODEs to determine boundary  $V$  and  $I$