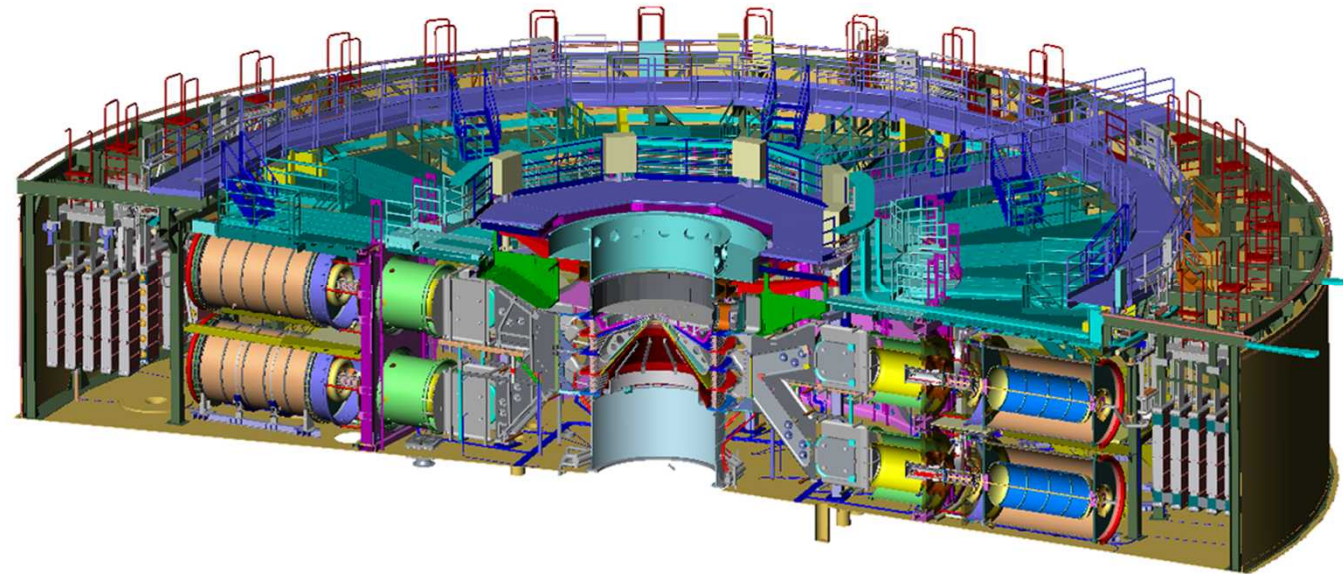
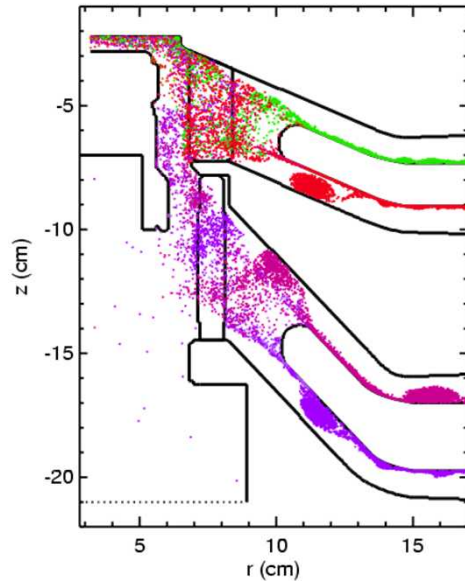


Modeling of the pulsed power system for the Z machine

Modeling of the pulsed power system for the Z machine



Physics Overview for Modeling Pulsed Power Using Sandia's codes Quicksilver and EMPHASIS

Peggy Christenson

7/18/2017

Acknowledgements

- Other Quicksilver/EMPHASIS/EMPIRE developers:
 - Tim Pointon, Keith Cartwright, Becky Coats, Matthew Bettencourt, Chris Moore, Edward Phillips, Richard Kramer, and Ed Love
- Thanks to George Laity for up-to-date Z images

- What physics is required to model pulsed power flow from Marx bank to load?
 - Relativistic, electromagnetic, kinetic plasma, radiation and neutral transport
 - Volumetric plasma chemistry (collisions between: electrons, ions, neutrals, photons)
 - ionization, excitation, momentum transfer (charge-charge, charge-neutral)
 - Surface physics (neutral, ion and electron emission from surfaces)
 - field emission, thermal emission, desorption, secondary particle emission
- Requirements for credible and practical simulations
 - Particle emission models (space-charge-limited, plasma and/or neutral injection)
 - External circuits for power sources and complicated loads
 - Dynamic particle load-balancing (for efficient parallelization)
 - Implicit algorithms with $\omega_p \Delta t > 1$, $\omega_c \Delta t > 1$, $\Delta x > \lambda_d$
 - Hybrid fluid/kinetic plasma model
 - High fidelity geometric representation (e.g. body fitted boundaries)

Modeling capabilities (past)

- QUICKSILVER:
 - Structured-mesh particle-in-cell code
 - Parallel with dynamic load balancing
 - Explicit finite-difference EM-PIC (3-D, 2-D)
 - Coupling to transmission line and load models (boundary conditions)
 - Space-charge limited, thermal and secondary particle emission
 - Monte Carlo Collisions (charge-neutral with prescribed neutral background)
 - Coulomb collisions
 - Coupling to ITS for radiation transport
- 30 year history of Pulsed Power applications:
 - Z machine power flow
 - HERMES (and coupling to ITS for rad-transport)
 - Ursa Minor (next-gen radiographic driver)
 - Ion Diodes in the past (PBFA II, SABRE)

Modeling capabilities (present)

- EMPHASIS™ (ElectroMagnetics PHysics AnalySIS):
 - Unstructured finite-element mesh particle-in-cell code
 - Parallel with dynamic load balancing
 - Implicit finite-element EM-PIC (3-D)
 - Coupling to Xyce for external circuit (transmission line and load models)
 - Space-charge limited, thermal, field, and secondary particle emission
 - Monte Carlo Collisions (charge-neutral with prescribed neutral background)
 - Coupling to ITS for radiation transport
 - Drift-diffusion model
 - PIC model coupled to rate equations and conductivity, $\sigma(E)$, and mobility, $\mu(E)$
- Recent applications
 - Optimization of design of dynamic material load geometries on Z
 - Tapered MITL redesigns (e.g. HERMES)

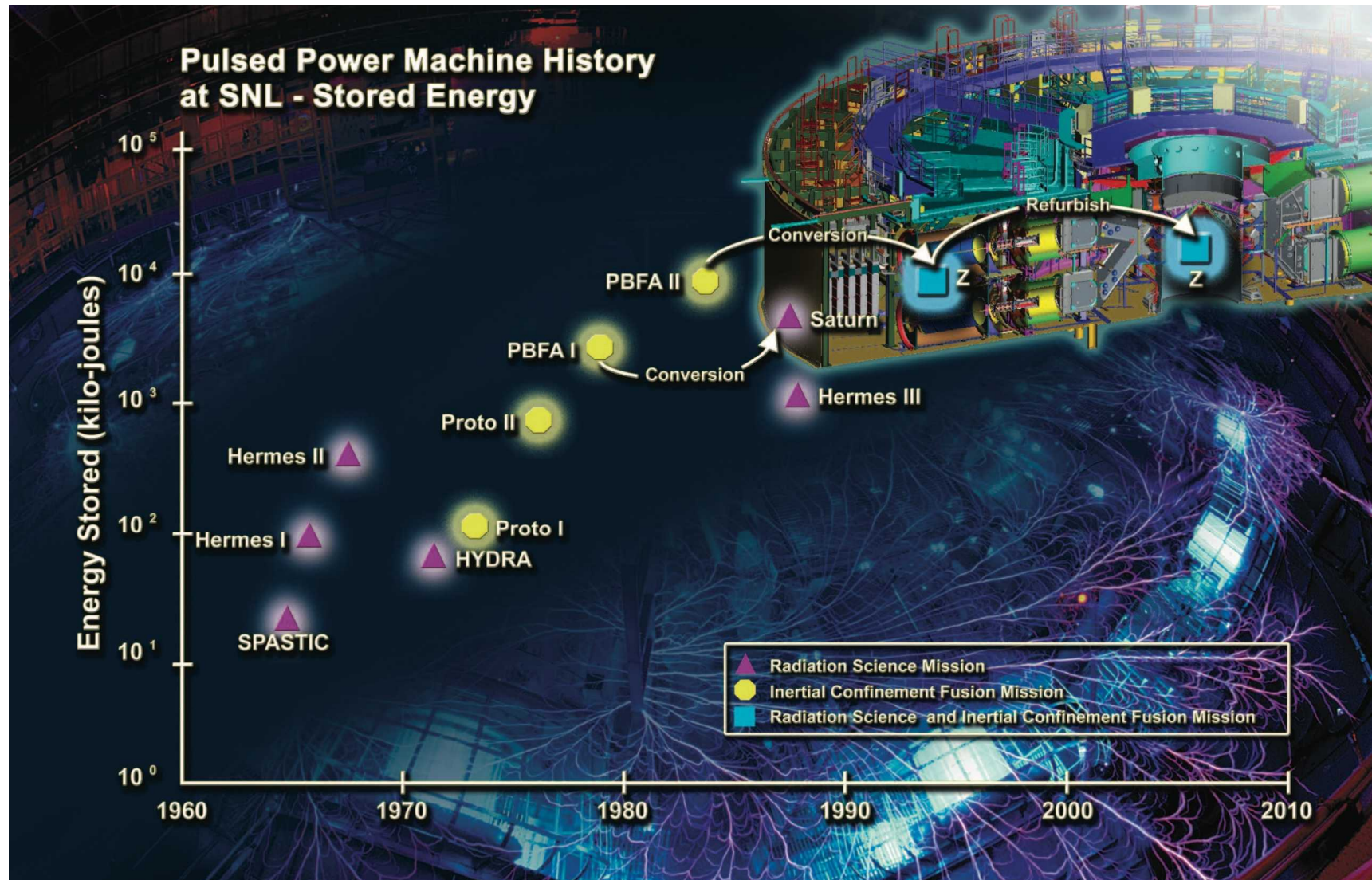
Current Modeling Deficiencies of EMPHASIS

- Radiation transport is decoupled from the plasma
 - Time independent radiation transport simulation modulated in time
- Full multi-fluid PIC hybrid
- Evolution of neutral and excited species
- Implicit algorithms with $\omega_p \Delta t > 1$, $\omega_c \Delta t > 1$, $\Delta x > \lambda_d$

Modeling capabilities (future)

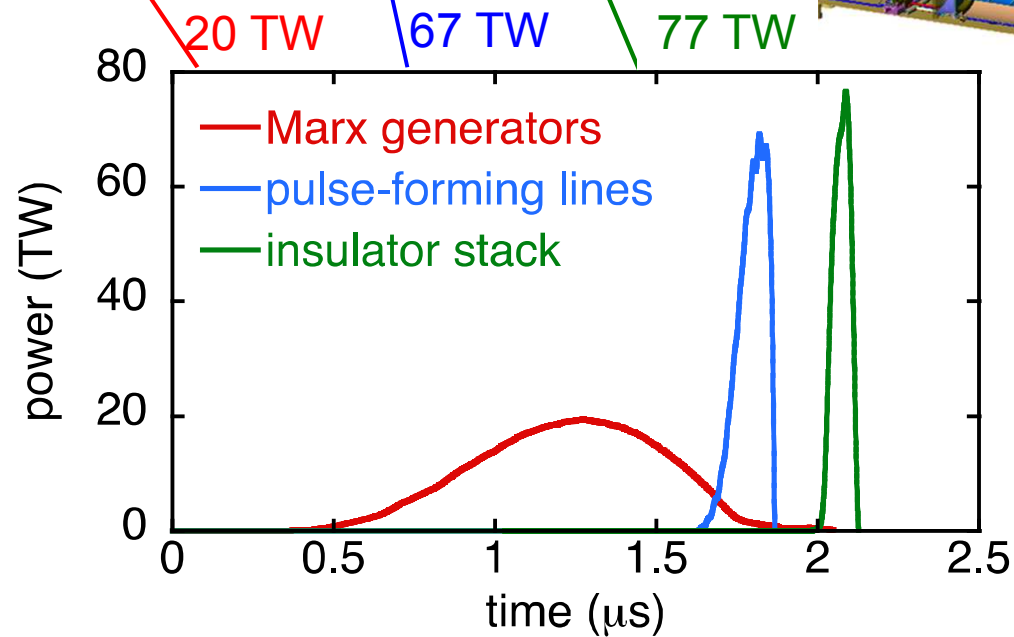
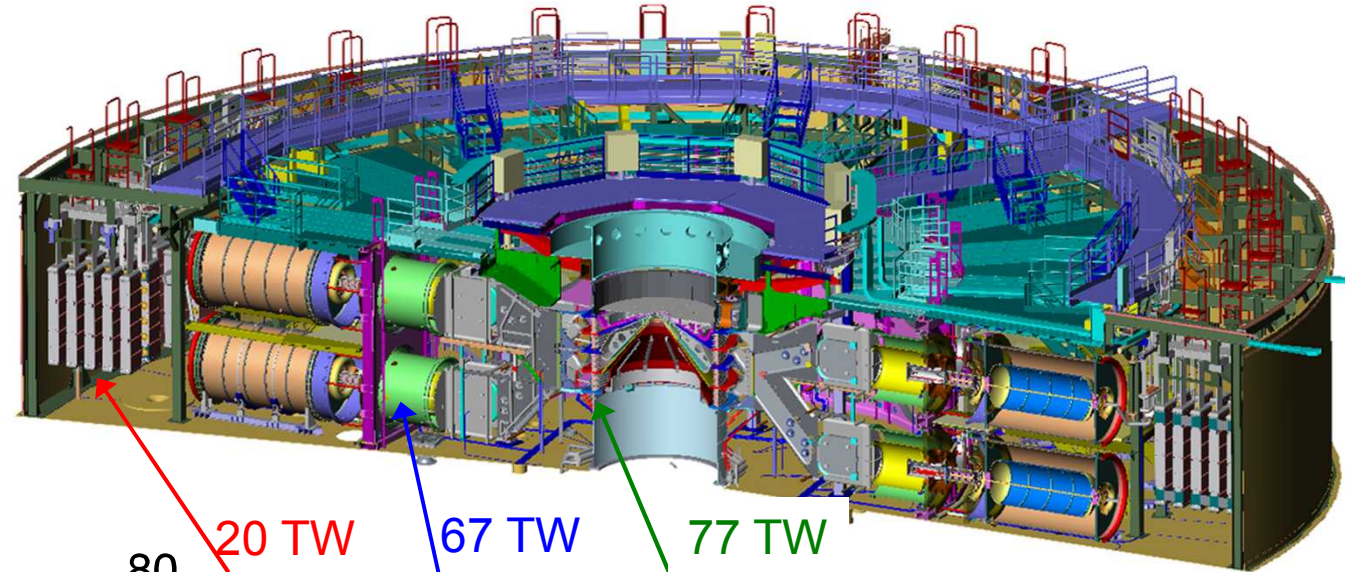
- EMPIRE: ElectroMagnetic Plasma with Integrated Radiation Effects
 - Implicit finite-element EM-PIC (3-D)
 - Hybrid structured/unstructured in-situ meshing
 - Built to operate on next generation platforms
 - Parallel with dynamic load balancing
 - Coupling to Xyce for external circuit (transmission line and load models)
 - Comprehensive emission models
 - Soon to have space-charge limited, thermal beam particle emission
 - Direct Simulation Monte Carlo (DSMC)
 - self-consistent evolution charged, neutral, and excited species
 - Monte Carlo Collisions (MCC)
 - charge-neutral with prescribed neutral background
 - Coupling to ITS for radiation transport
 - Will be replaced with a self consistent time accurate integrated radiation transport
 - Hybrid with full multi-fluid model
- Recent applications
 - Two stream instability
 - Landau damping
 - Linear plasma waves

Pulsed Power at SNL

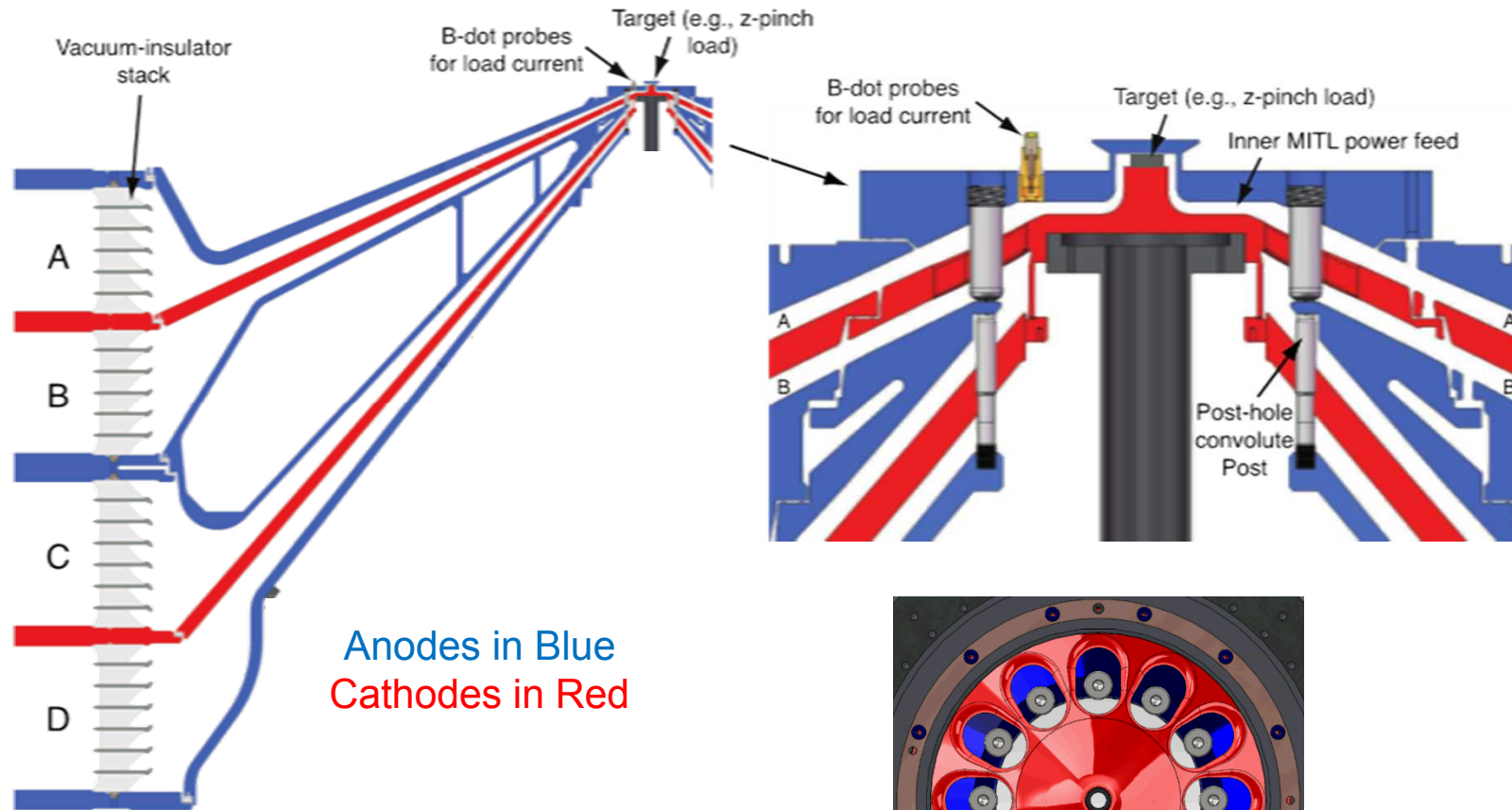


The Z machine Pulsed Power technology

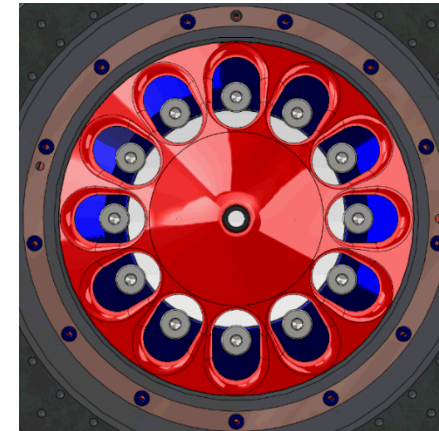
- Marx Generators
- Intermediate storage capacitors
- Laser-triggered gas switches
- Pulse-forming lines
- Self-breaking water switches
- Water convolute
- Vacuum-insulator stack
- Vacuum section



Vacuum Section



- 4 magnetically insulated transmission lines
- Coupled in parallel with a double post-hole convolute to the inner MITL

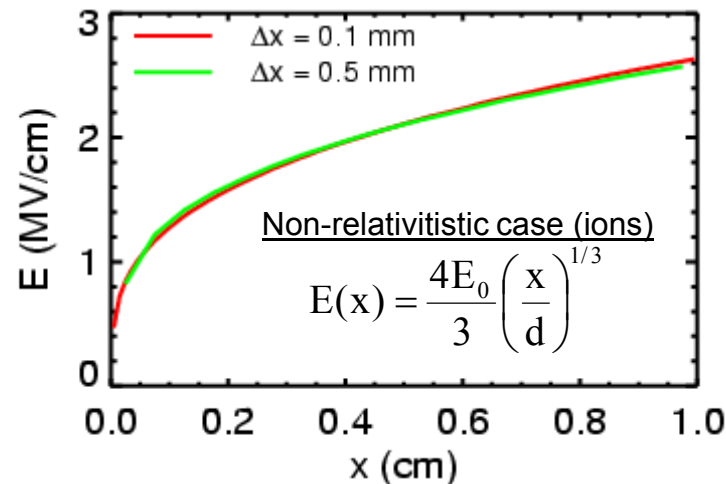


Space-charge-limited electron emission

- Electron emission from the cathode when $E > \sim 250$ kV/cm
- Electron current reduces E_{normal} at surface, and increases until J_e forces $E_n \rightarrow 0$ -- “space-charge-limited” emission
- Analytic solution for 1-D planar diode:
 - Gap d , voltage V , particle charge q , mass m
 - Child-Langmuir current: $J_{CL} = \frac{4\epsilon_0}{9} \left(\frac{2q}{m} \right)^{1/2} \frac{V^{3/2}}{d^2}$ (non-relativistic)

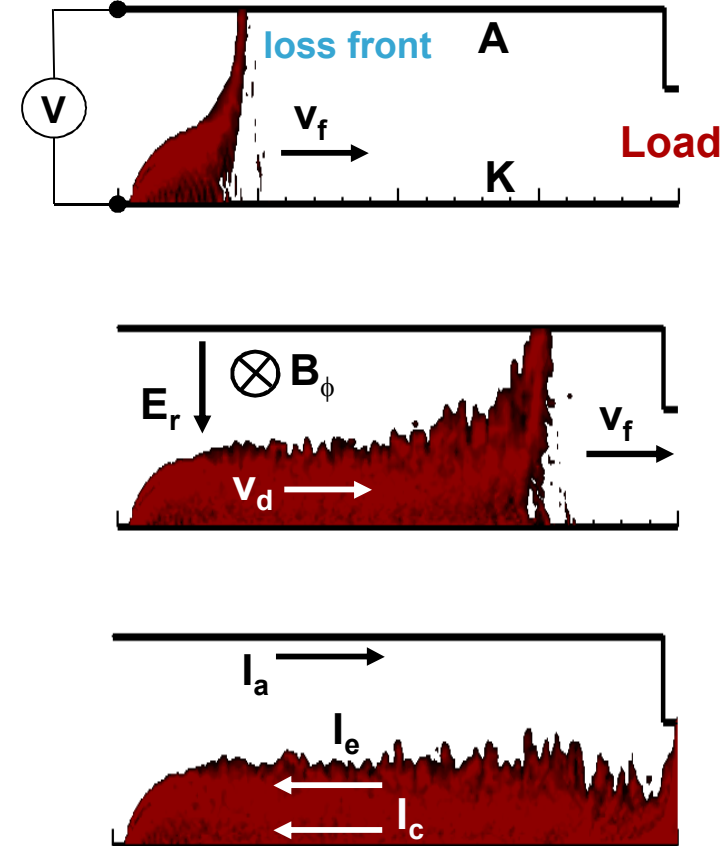
- Modeling challenge:
very steep E-field profile at
emission surface

- Ion diode: $d = 1$ cm,
 $V = 2$ MV; $E_0 = V/d$

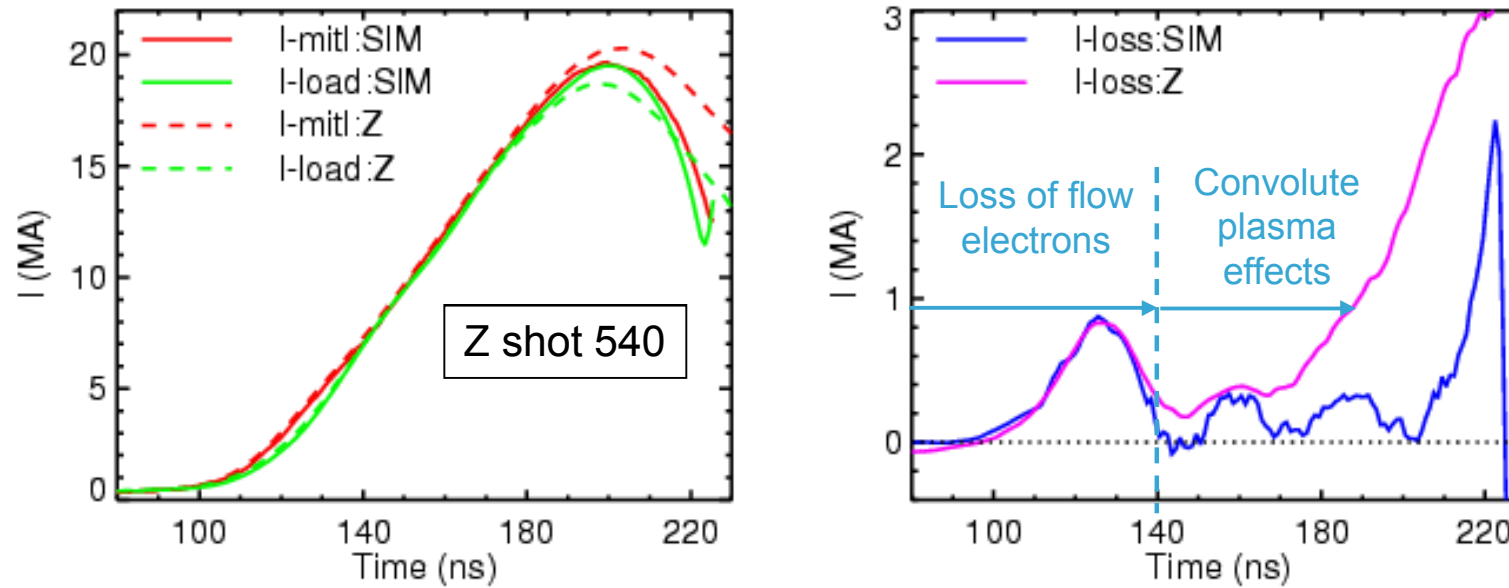


MITLs enable efficient power delivery

- Illustrated with coaxial MITL (courtesy of Steve Rosenthal)
- When emission first starts, electrons go straight across the A-K gap – current loss
- Behind the “loss front”, B-field from the current flow can stop electrons from reaching anode – “magnetic insulation”
- Useable current delivered to the load is the cathode current $I_c = I_a - I_e$, where I_e is the current flowing in the electron sheath (typically dumped as a loss elsewhere)
- MITL design is a tradeoff between:
 - Large gap: better insulation, lower flow
 - Small gap: lower inductance

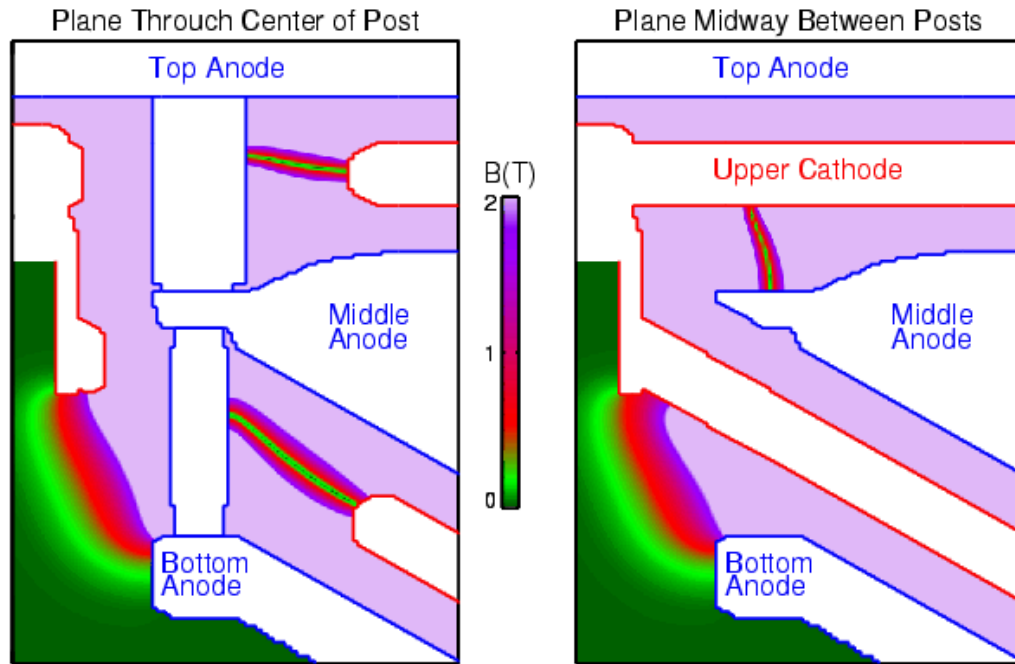


Quicksilver can accurately predict early-time losses

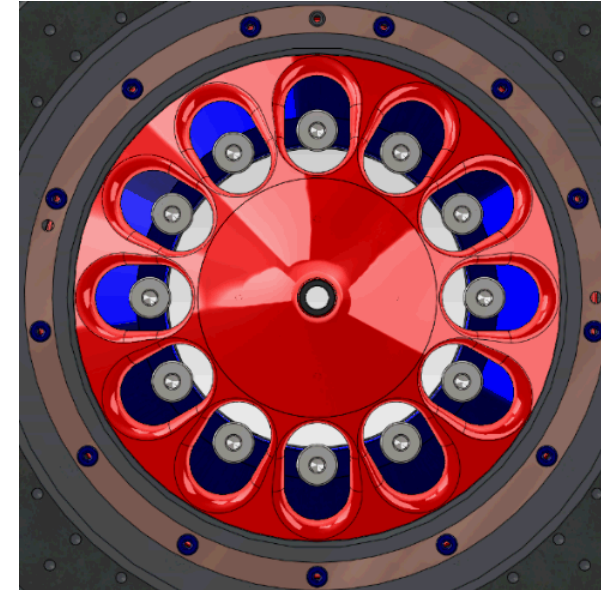


- Simulations with SCL emission on all cathode surface
- Early-time loss: MITL flow electrons lost to the anode ("loss front")
- Extra late-time loss is due to expanding electrode plasmas in the convolute
 - Experimental results in M.R. Gomez, *et al.*, Phys Rev ST-AB **20**, 010401 (2017)
 - Simulations with LSP [D.V. Rose, *et al.*, Phys Rev ST-AB **11**, 060401 (2008)].

Magnetic nulls in the convolute

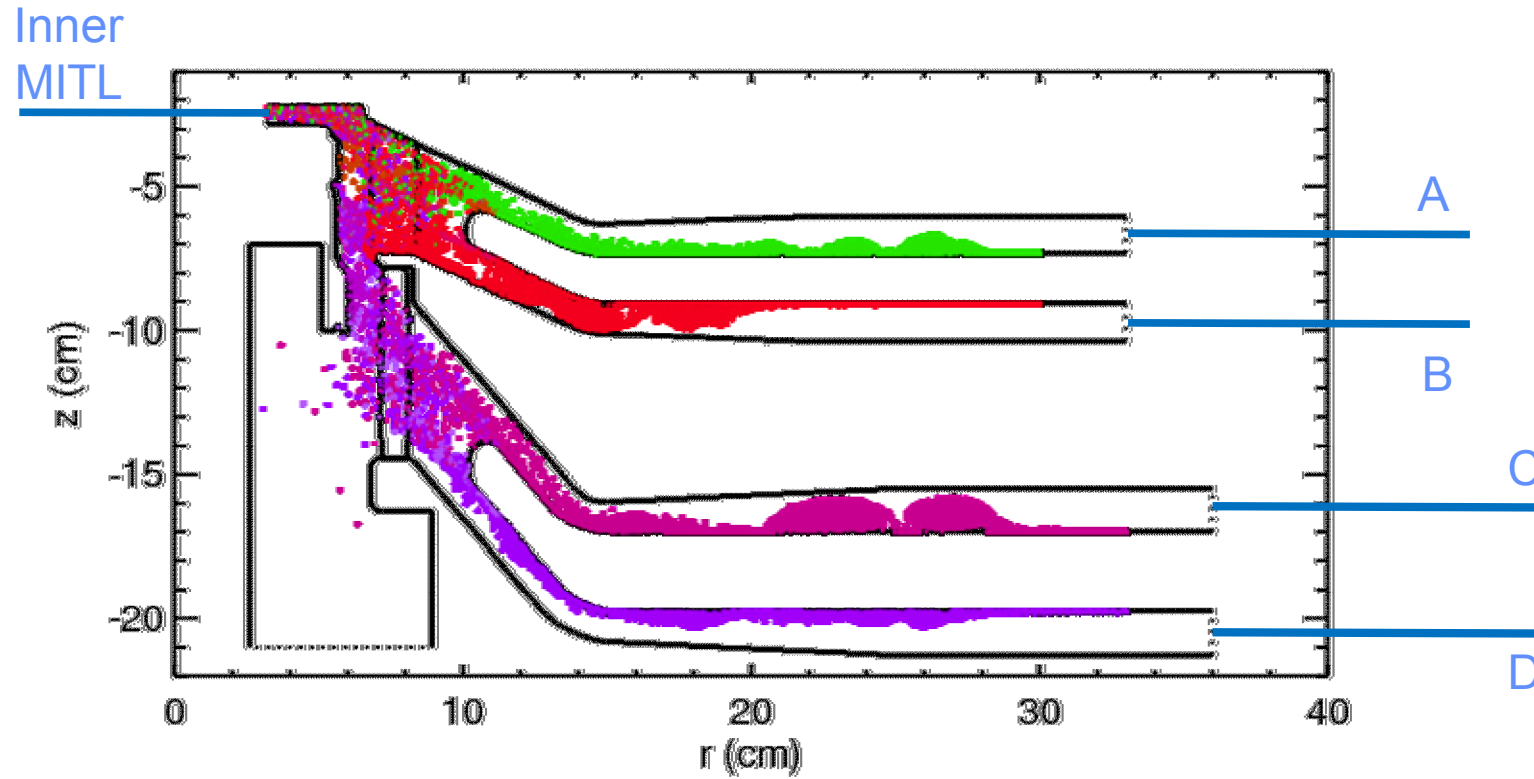


Convolute top view



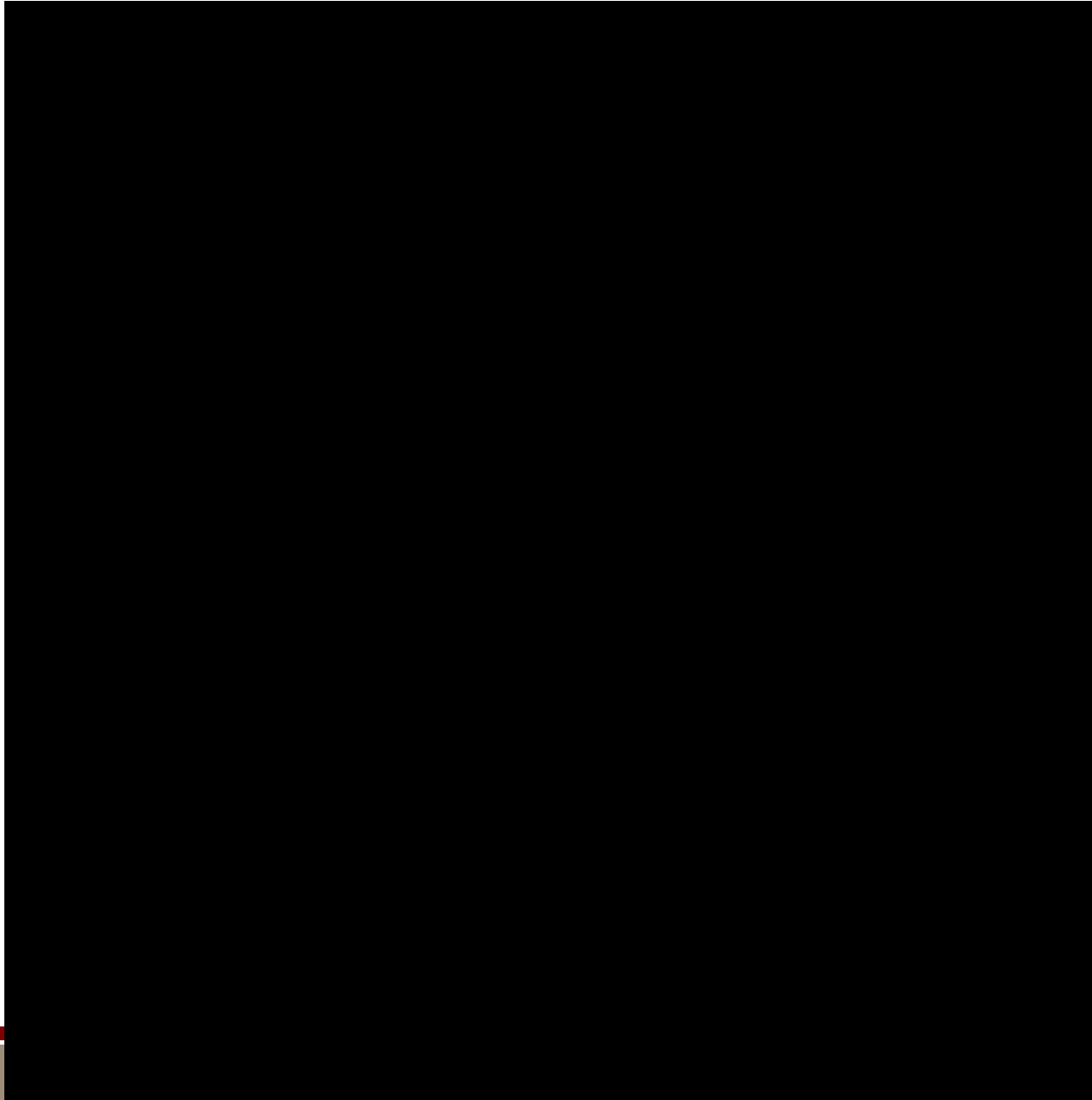
- Electrons flow into the convolute from the feed MITLs
 - Complicated by non-uniform impedance of feed MITLs, $Z \sim d/r$
- There will be electron losses at the nulls: “loss of magnetic insulation”
 - How much? How fast does the anode surface heat?

Time-accurate Quicksilver simulations of Z MITL/Convolute

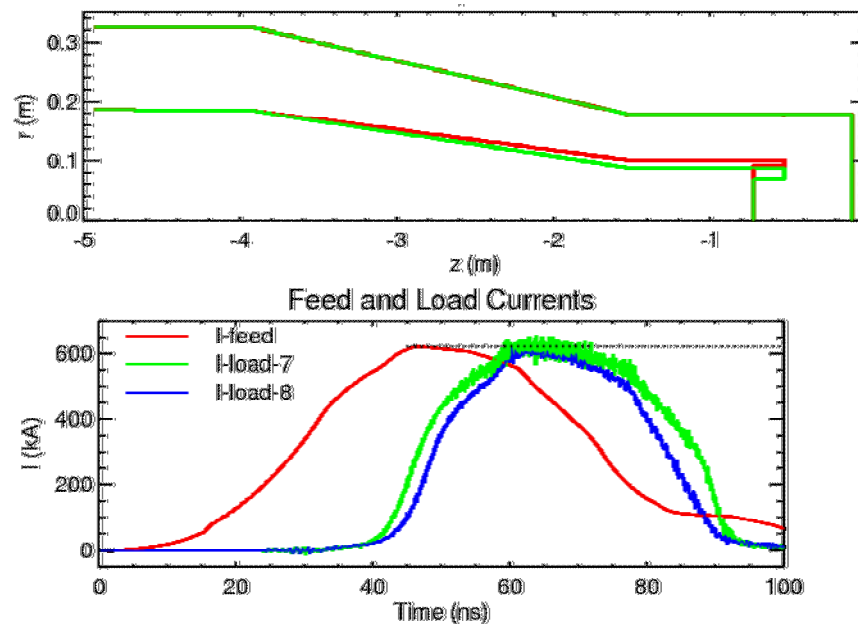


- Only possible with extensive use of 1-D transmission lines
- Reduces the 3-D system size – only emit electrons out to $r \sim 30$ cm
- Time-accurate source for all four feed MITLs (out to $r \sim 3$ m)
- Inner MITL connected to a 1-D TL terminated with a realistic load

Quicksilver MITL Movie

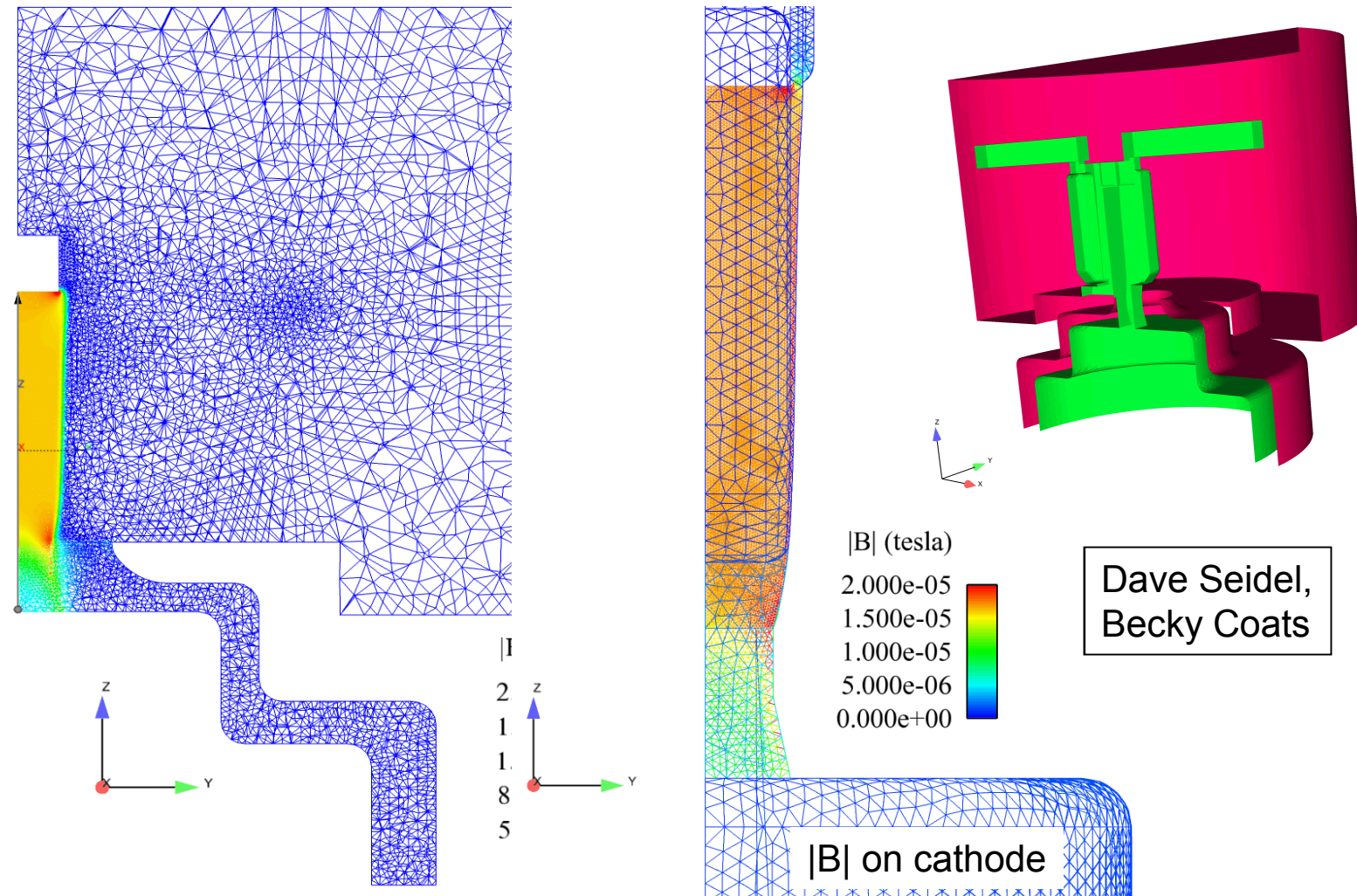


EMPHASIS could help HERMES MITL redesign



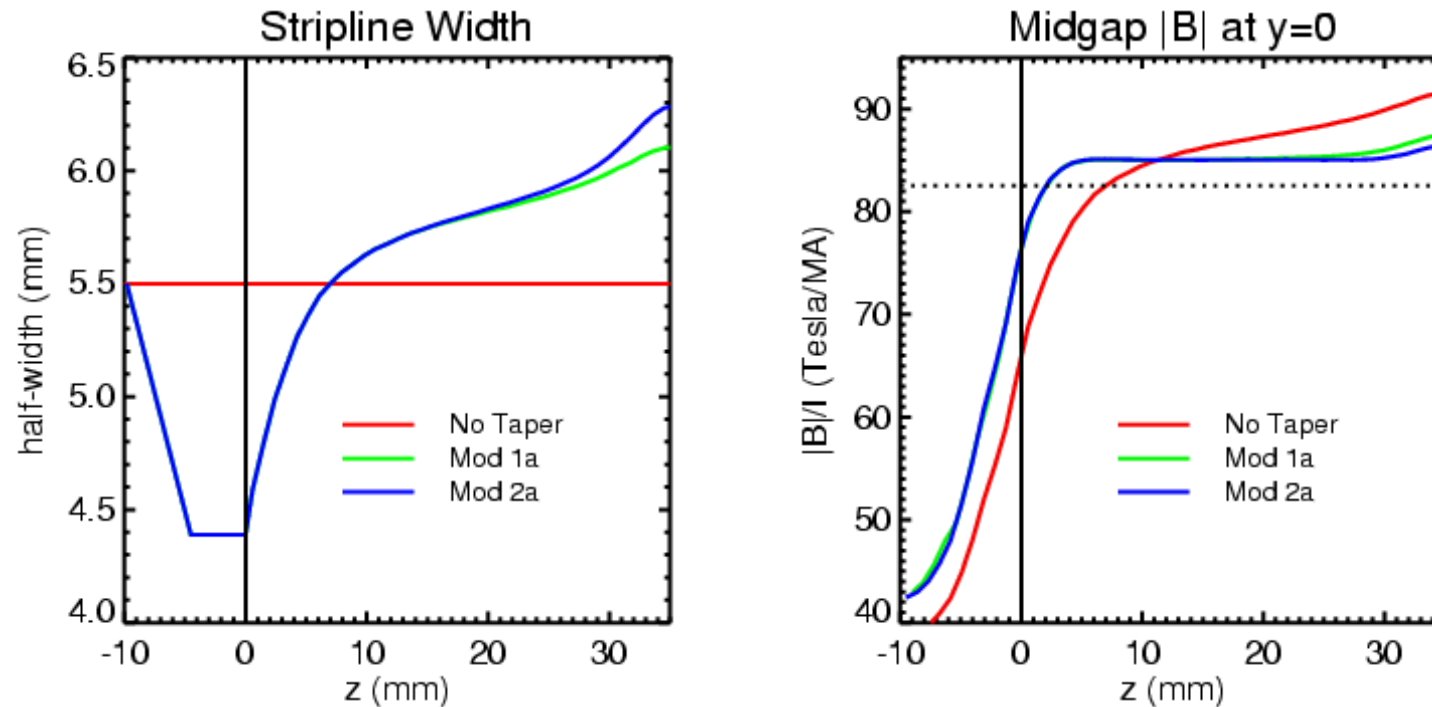
- Extended MITL with constant impedance taper using Quicksilver:
 - Anode: 25.5" \rightarrow 14"; cathode: 14.5" \rightarrow 8" diameter
- Reducing downstream cathode diameter to 7" is even better
 - Better mechanical design: larger A-K gap reduces alignment thresholds
 - Actually reduces current loss to zero
 - ... BUT Quicksilver has a numerical instability caused by the stair-stepped cathode

EMPHASIS Simulation of ICE stripline geometry



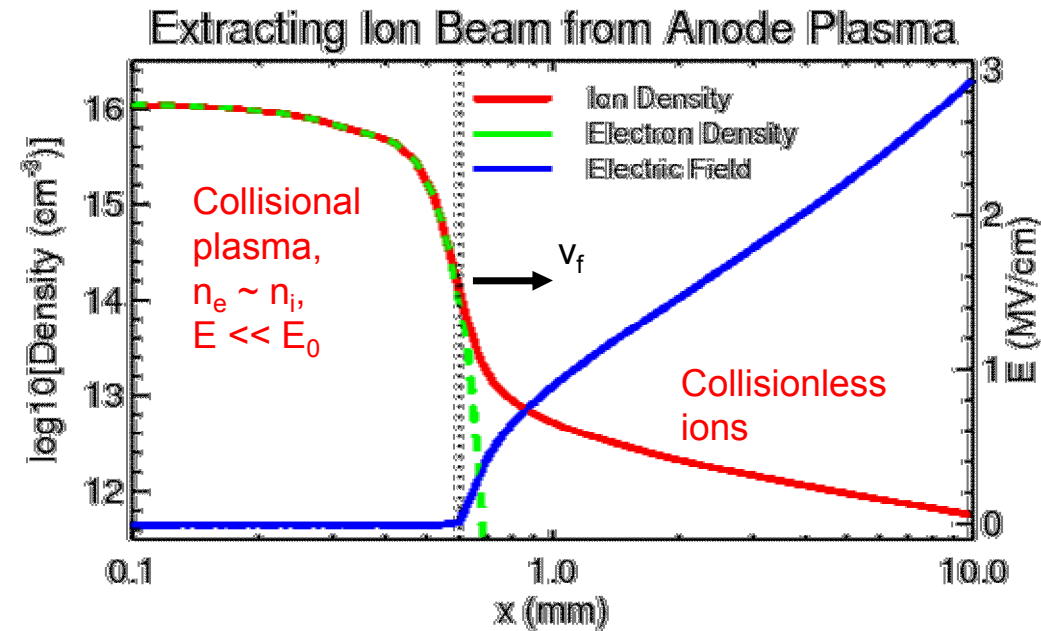
Goal: Improve B-field uniformity on the cathode ($< 1\%$ variation)

Optimize axial uniformity of $|B|$ in stripline



- Cut “notch” in width of the stripline below the sample area
- Vary width as a function of z above the notch
- Uniformity of $B(z)$ profile over the samples is greatly improved

Electrode plasmas



1D multi-fluid
simulation of an
ion diode

$V = 2$ MV
 $d_0 = 1$ cm
 $E_0 = 2$ MV/cm

- Typical plasma parameters: $n = 10^{16} - 10^{18}$ cm⁻³; $T = 1 - 10$ eV
 - Debye length $\lambda_{De} \sim 100$ nm; Plasma frequency $\omega_{pe} \sim 10^{13}$ s⁻¹
- Expansion velocity of plasma front, $v_f \sim 1 - 20$ cm/ μ s
- Dynamic effective anode-cathode gap: $d_{eff}(t) = d_0 - v_f t$
 - A decreasing A-K gap is almost always bad

Modeling electrode plasmas is very difficult

- Clearly, must operate at $\Delta x / \lambda_{De} \gg 1$
 - Possible with energy-conserving PIC: stable, but no guarantee of accuracy
- Large E-fields introduce a new, severe constraint: $eE\Delta x / T_e < \sim 1$
 - A significant fraction of the plasma electrons and ions must be able to cross a cell against the E-field to sustain expansion
 - For $d = 1$ cm, $V = 2$ MV, $T_e = 1$ eV, need $\Delta x \sim 1$ μ m
- Cathode plasma expansion is harder to model than anode plasma
 - Less drag from Coulomb collisions as electrons accelerate ($v_{ei} \sim 1/v^3$)
- Only path forward seems to be a hybrid fluid-PIC model
 - Fluids for dense plasma with plasma-front tracking scheme
 - Creation of PIC particles at the front

Summary

- Quicksilver has had success modeling many pulsed power systems
 - PBFA II, SABRE, Z, Ursa Minor, HERMES, and many more
- Key enabling capabilities
 - Massively parallel with dynamic load balancing
 - Particle emission models
 - External circuits (1-D transmission lines or Xyce)
- Unstructured mesh capabilities of EMPHASIS and EMPIRE can address key geometry issues not possible with legacy structured EM-PIC
 - Work needed on emission models and external circuits (Xyce)
- Development of advanced plasma emission models are necessary to simulate late-time current loss on Z (and impedance collapse in other devices)
 - A very difficult problem

Backup slides

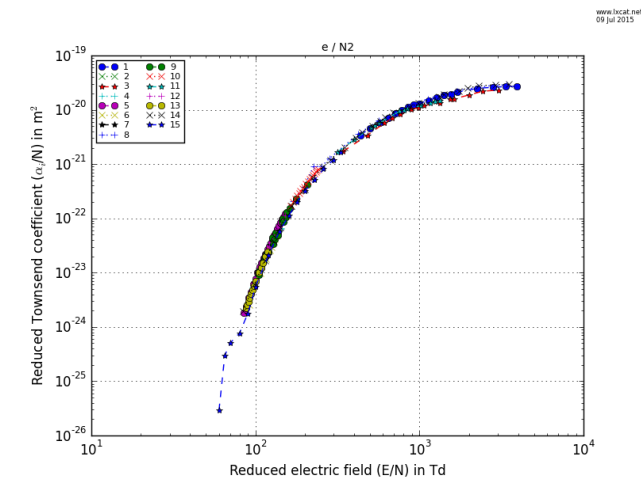
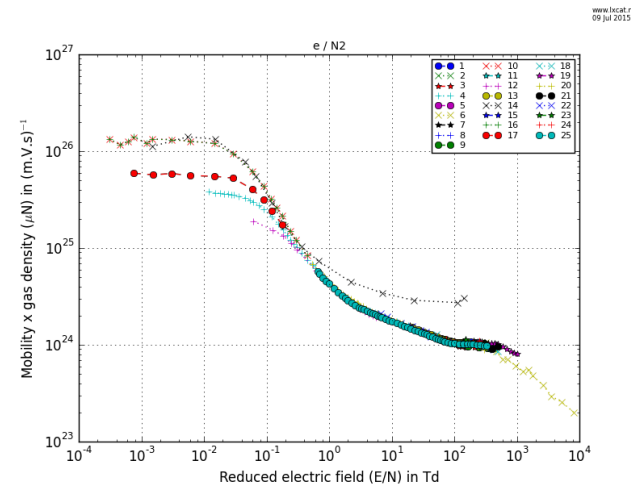
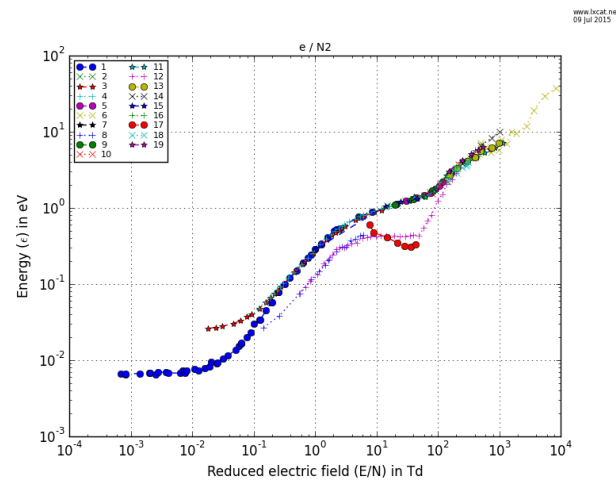
Surface Plasma Generation Models

| Emission Model | Incident Particle | Secondary Particle | Thermal Dependence |
|--|-------------------|--------------------|--------------------|
| Desorption (Hertz-Knudsen-Langmuir)(Sanford et.al.) | None | Neutral | Yes |
| Thermo-Field Emission (Murphy & Good) | None | Electron | Yes |
| Space Charge Limited Emission (limit of thermo-field emission for high voltage) (Shiffler) | None | Electron | No |
| Secondary Emission (Lin and Joy) | Electron | Electron | No |
| Secondary Emission (Raizer 1991) | Ion | Electron | No |
| Associative Desorption (Lieberman and Lichtenberg) | Ion/Neutral | Ion/Neutral | Yes |
| Sputtering (Langley) | Ion | Ion/Neutral | Weak |
| Stimulated ion emission | Electron | Ion | Weak |

It is necessary to couple the desorption and thermo-field emission to a thermal solve on the electrodes to accurately model surface plasma generation

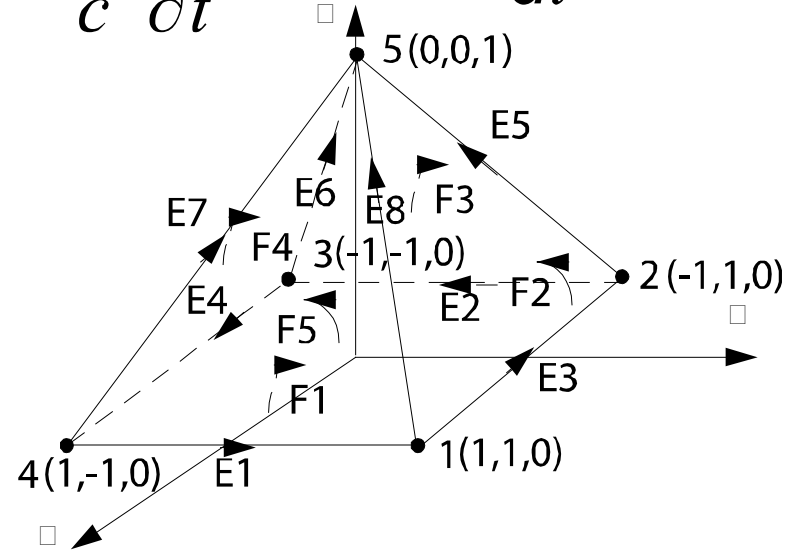
Modeling

- EMPHASIS EM/PIC
 - EM field solve
- Conductivity Model
 - Mobility/conductivity as a function of **electric field** and density
 - One can write the time update as one big matrix,
 - Use Ohms law for the current



Second Order EM Formulation

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \bar{E} \right) + \frac{\epsilon_r}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} + \mu_0 \sigma \frac{d \bar{E}}{dt} + \mu_0 \bar{E} \frac{d \sigma}{dt} = -\mu_0 \frac{\partial \bar{J}}{\partial t}$$



$$\left[T \right] \frac{d^2 E}{dt^2} + \left[B \right] \frac{d E}{dt} + \left[S \right] E + F = 0$$

Newmark time integration

$$[T] \frac{E^{n+1} - 2E^n + E^{n-1}}{\Delta t^2} + [B] \frac{E^{n+1} - E^{n-1}}{2\Delta t} + [S](\alpha_1 E^{n+1} + \alpha_2 E^n + \alpha_3 E^{n-1}) + F = 0$$

Second order accuracy

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Unconditional stable

$$\alpha_2^2 = 4\alpha_1\alpha_3$$

Energy conserving (symplectic)

$$\alpha_1 = \alpha_3$$

Leads to

$$\alpha_1 = 1/4, \alpha_2 = 1/2, \text{ and } \alpha_3 = 1/4$$

Godfrey update

$$E^{n+1} - 2E^n + E^{n-1} + 4\Delta t \frac{([B] + (2\alpha_1 + \alpha_2)\Delta t[S])E^n + (-[B] + (1 - (2\alpha_1 + \alpha_2))\Delta t[S])E^{n-1} + \Delta t F)}{4[T] + 2\Delta t[B] + 4\alpha_1\Delta t^2[S]} = 0.$$

[B] and [S] are updated as the conductivity changes

Fill time is on the order of the solve time

Rate Equations

$$\frac{\partial n}{\partial t} + \nabla (n\mathbf{u}) = G - L$$

Secondary Electron Rates

$$\frac{\partial N_e}{\partial t} = S_e + (\nu_i(E) - \nu_a) N_e - \alpha_{ei}(E) N_e N_+ \quad (5.1)$$

N_e is the electron density

N_+ is the positive ion density

N_- is the negative ion density

S_e is the photoelectrons created by the primary electrons

ν_i is the rate of production of the secondary electrons by collisions between secondary electrons

ν_a is the attachment rate of the electrons to neutral air atoms

α_{ei} is the recombination between electrons and positive ions

Ion Rate equations

Positive Ion Rate

$$\frac{\partial N_+}{\partial t} = S_e + \nu_i(E)N_e - \alpha_{ei}(E)N_eN_+ - \alpha_{ii}N_+N_-$$

α_{ei} is the recombination between negative and positive ions

Negative Ion Rate

$$\frac{\partial N_-}{\partial t} = \nu_a N_e - \alpha_{ii}N_+N_-$$

Conductivity

$$\sigma = \mu_e N_e + \mu_+ N_+ + \mu_- N_-$$

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \bar{E} \right) + \frac{\epsilon_r}{c^2} \frac{\partial^2}{\partial t^2} \bar{E} + \mu_0 \sigma \frac{d}{dt} \bar{E} + \mu_0 \bar{E} \frac{d\sigma}{dt} = -\mu_0 \frac{\partial}{\partial t} \bar{J}$$

$$mn \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = qn (\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla \cdot \Pi + f|_c$$

$$\frac{\partial}{\partial t} \equiv 0, \mathbf{B} = 0, (\mathbf{u} \cdot \nabla) \mathbf{u} \text{ is zero } \nabla \cdot \Pi = \nabla p, \text{ isothermal relation } p = nkT$$

$$f|_c = - \sum_{\beta} mn \nu_{m\beta} (\mathbf{u} - \mathbf{u}_{\beta}) - m\mathbf{u} (G - L)$$

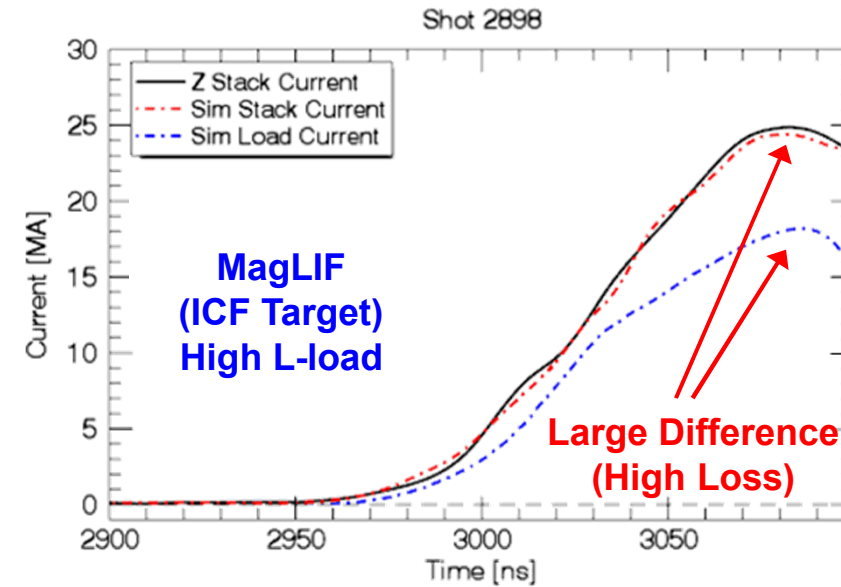
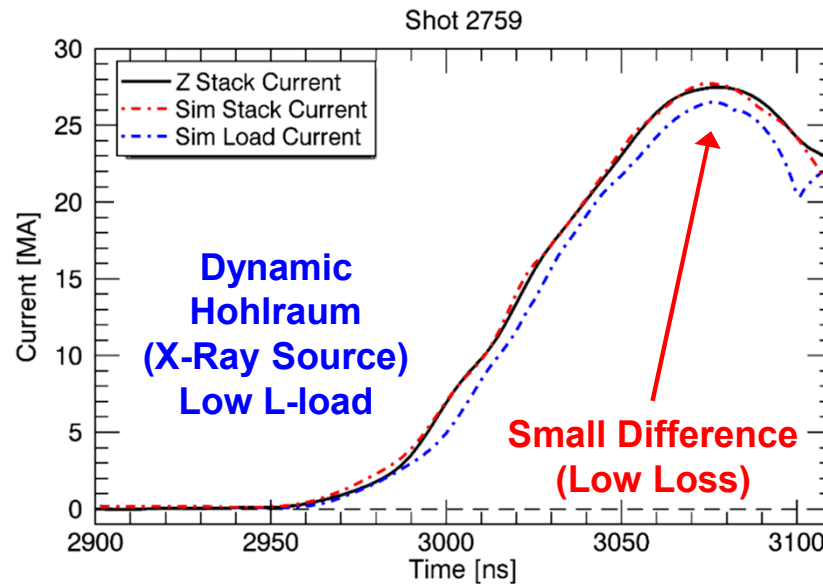
$$f|_c \approx -nm\mathbf{u}\nu_m$$

$$0 = qn\mathbf{E} - \nabla p - mn\nu_m\mathbf{u}$$

$$\mathbf{J} = \frac{q^2 n}{m\nu_m} \mathbf{E} - \frac{q}{m\nu_m} \nabla p$$

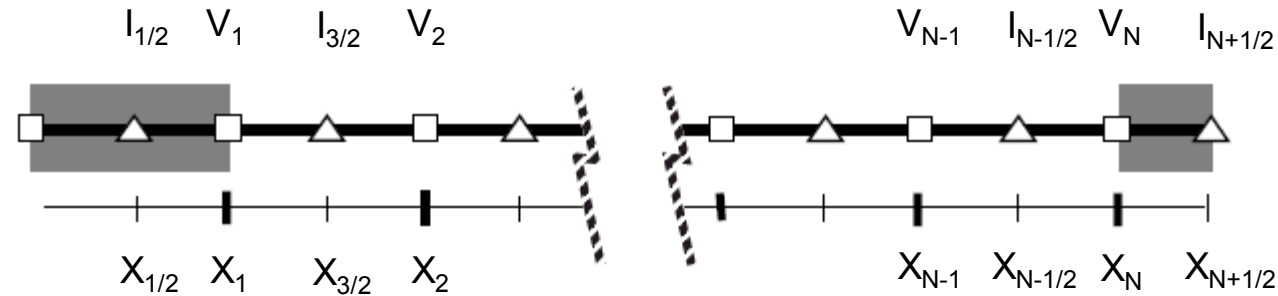
$$\mathbf{J} = qn\mu_m \mathbf{E} - \mu_m \nabla p$$

Z is modeled with the Bertha circuit code



- POC: Brian Hutssel
- Empirical and physics-based models for all components
 - PIC simulations provide guidance for the vacuum section
- Can simulate entire pulse ($\sim 3 \mu\text{s}$) in $< \sim 1$ minute

Quicksilver 1-D transmission lines

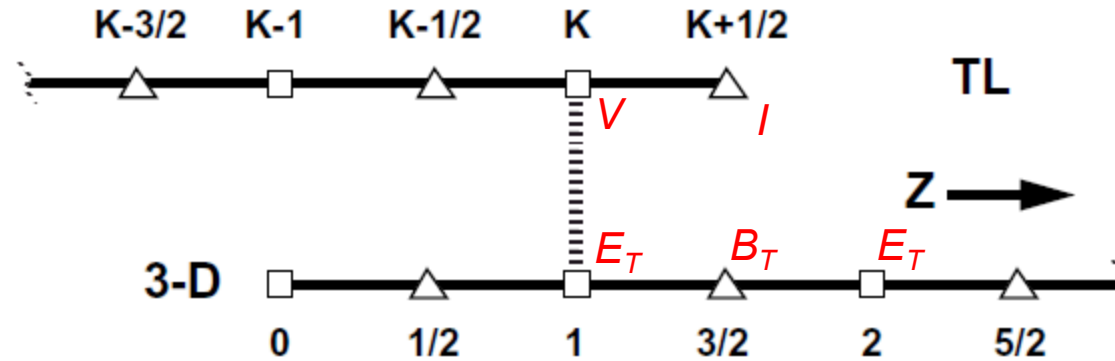


- Telegrapher's equations: $\frac{\partial V}{\partial t} = -\frac{1}{C_l} \frac{\partial I}{\partial x}$; and $\frac{\partial I}{\partial t} = -\frac{1}{L_l} \frac{\partial V}{\partial x}$
- Discretize in space and time: $V_i^n = V(x_i, n\Delta t)$; $I_{i+1/2}^{n-1/2} = I(x_{i+1/2}, (n-1/2)\Delta t)$
- $C_l(x_{i+1/2})$ and $L_l(x_{i+1/2})$ constant in each "full grid" cell: $[x_i, x_{i+1})$
- Leapfrog time advance:

$$V_i^n = V_i^{n-1} - \gamma_i (I_{i+1/2}^{n-1/2} - I_{i-1/2}^{n-1/2}); \text{ where } \gamma_i = \frac{2\Delta t}{\Delta x_{i-1/2} [C_l(x_{i-1/2}) + C_l(x_{i+1/2})]}$$

$$I_{i+1/2}^{n+1/2} = I_{i+1/2}^{n-1/2} - \lambda_i (V_{i+1}^n - V_i^n); \text{ where } \lambda_i = \frac{\Delta t}{\Delta x_i L_l(x_{i+1/2})}$$

Connect 1-D TLs to 3-D system at “TLports”



Connection of a TL to a z-normal port (lower z boundary)

- A *TLport* is a rectangular region on a boundary face of a 3-D block, with two conductors separated by a vacuum gap
 - Parallel plates, coaxial lines, more complicated cross-sections
- No interpolation needed in time or normal ordinate (z here) to match:
 - Transverse $\mathbf{E}_T = (E_x, E_y)$ with V at boundary plane
 - Transverse $\mathbf{B}_T = (B_x, B_y)$ with I half a cell inside

TEM voltage and current in the 3-D system

- Solve a 2-D Poisson solution for TEM E-field in the port plane:

- $\nabla_T^2 \phi = 0$, with BCs $\phi = 0$ on cathode, and $\phi = V_0$ (1 Volt) on anode

- “Poisson E-field”: $E_p = -\nabla \phi$

- QS computes E_p for arbitrary port geometry at $t = 0$

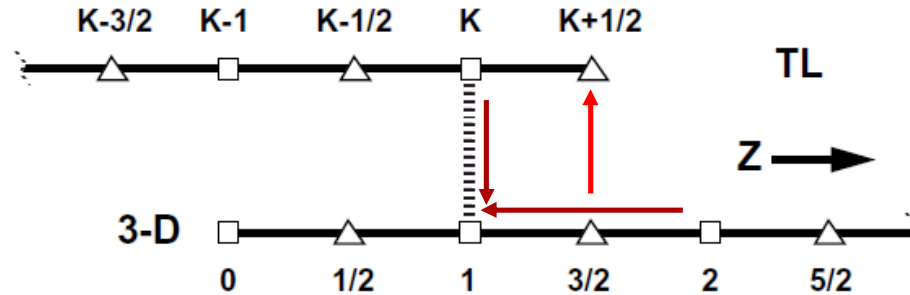
- E_p is orthogonal to any non-TEM mode profile E_{NT} :
$$\int E_p \bullet E_{NT} dA = 0$$

- **Filtering out the non-TEM fields is crucial to stability**

- TEM voltage:
$$V = \frac{\epsilon}{C_l V_0^2} \int E_p \bullet E_T dA, \text{ where } C_l = \frac{\epsilon}{V_0^2} \int E_p^2 dA$$

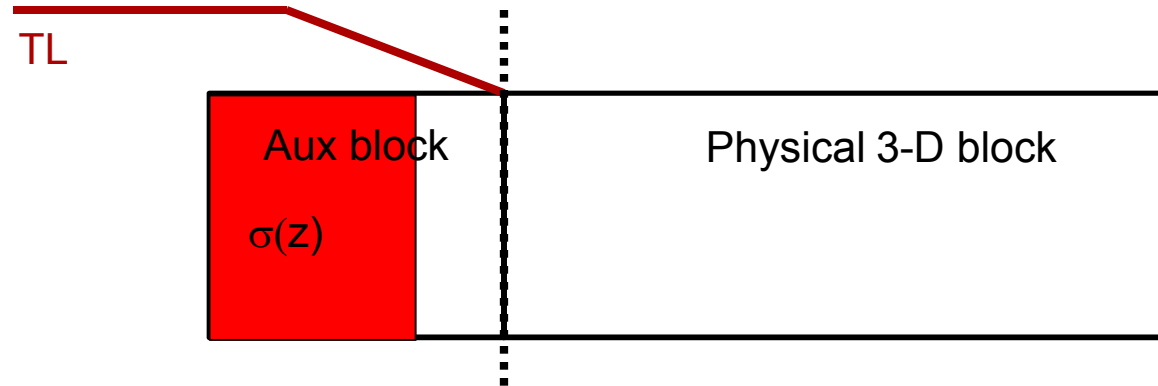
- TEM current:
$$I = \frac{1}{\mu V_0} \int \hat{e}_z \times E_p \bullet B_T dA$$

Basic filtered TLport algorithm



- Non-TEM transverse field at $k = 1, 2$ planes: $E_k^{*n} = E_{Tk}^n - (V_{3D,k}^n / V_0) E_P$
- In the port plane, set $E_T^n = (V_K^n / V_0) E_P + E_1^{*n}$
- Compute $E_1^{*n+1} = (1 - \delta) E_1^{*n} + \delta E_2^{*n}$; where $\delta = v_{ph} \Delta t / \Delta x$
 - Assumption: all non-TEM waves are travelling upstream with same v_{ph} (usually c)
- Use $B_{T,3/2}$ to set boundary value $I_{K+1/2}$ for V_K on next timestep
- Usually works fine for inlet power feeds

“Auxiliary block” TLport algorithm



- Filtered TLport algorithm failure modes:
 - Electrostatic fields from non-TEM structure too close to the port
 - Particles flowing through the boundary
- Solution: add auxiliary field block with conductivity to port boundary
 - For the field solver, this is a “parallel load” with the transmission line
 - TEM fields coupled to the transmission line V and I
 - non-TEM fields coupled to the aux-block
 - Particles drift ballistically into the aux-block (never come back)
 - Decay particle charge and absorb in conductivity region

Pulsed Power boundary conditions for TLs

- Thevenin equivalent source: $V_{oc}(t)$ and source impedance R_s
- Series RLC circuit for driving system with charged capacitors
 - Variable $R(t)$ acting as the switch (start high, then fall)
- T-junctions and multi-line junctions
- Time-dependent inductive loads (dynamic material shots on Z)
- Z-pinch loads:
 - Thin shell $r_w(t)$, height h , mass M , return current radius b

- Time-dependent inductance:
$$L(t) = L_h \ln\left(\frac{b}{r_w(t)}\right), \text{ where } L_h = \frac{\mu_0 h}{2\pi}$$

- Force on the wire array:
$$M\ddot{r}_w = -\frac{L_h I^2}{2r_w}$$

- Small set of non-linear ODEs to determine boundary V and I