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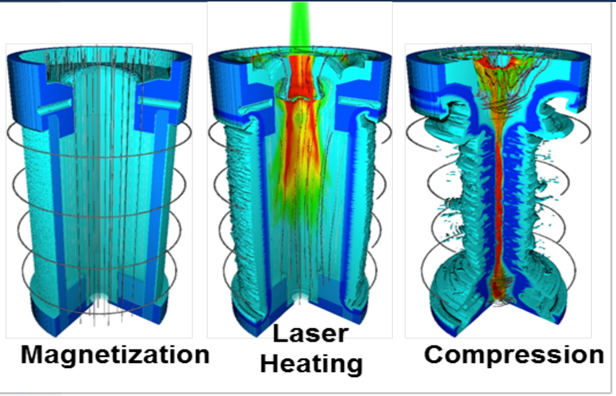

Bayesian analysis techniques for interpreting MagLIF data

Pat Knapp, Michael Glinsky, Matthew Evans, Matt Gomez, Stephanie Hansen, and Kelly Hahn

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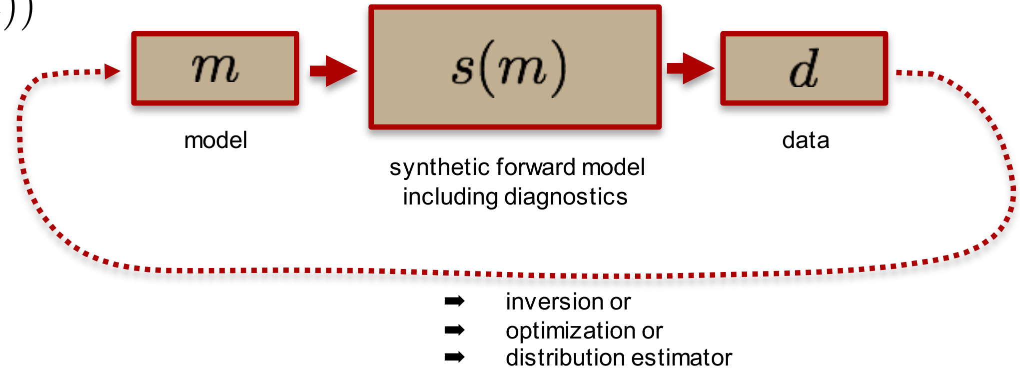
The problem…

- We have a set of data from each MagLIF experiment that needs to be used to determine the fundamental hotspot parameters (P, T, mix, etc.)
- Few of these parameters can be uniquely determined with a single diagnostic
- Pressure is impossible to measure directly
- We desire an approach to integrate multiple diagnostics simultaneously to constrain all of the interesting stagnation parameters

Using Bayesian statistics, we can efficiently explore the parameter space and determine the pdf's of the model parameters and their correlations

$$P(m|d) \propto P(m)P(d|m) = P(m)P(d - s(m))$$

$$P(d - s(m)) \propto \exp\left(-\frac{(d - s(m))^2}{2\sigma_d^2}\right)$$



The goal is to estimate the probability distribution of the model parameters, ***m***, given the measured data, ***d***.

Bayesian data analysis sees widespread use in a variety of applications

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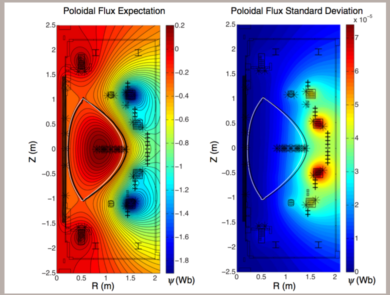
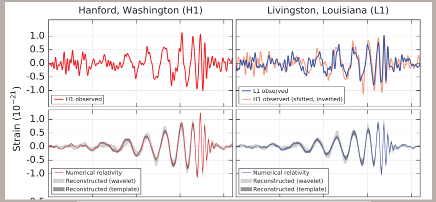
tokamak plasma profile estimation at installations such as JET and MAST



Svensson et al., Plasma Phys. Control. Fusion **50** 085002 (2008)
von Nessi et al., J. Phys. **A46** 185501 (2013)

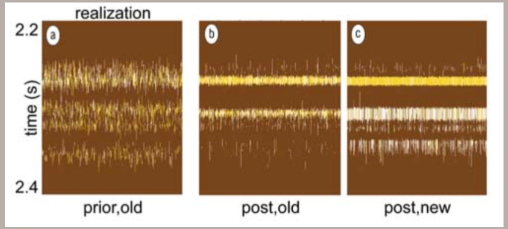
LIGO binary black hole merger analysis



Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

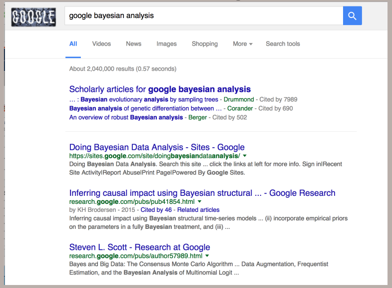
Veitch et al., Phys. Rev. D **91** 042003 (2015)

integrated seismic data, E&M data, and well log data analysis for petroleum exploration



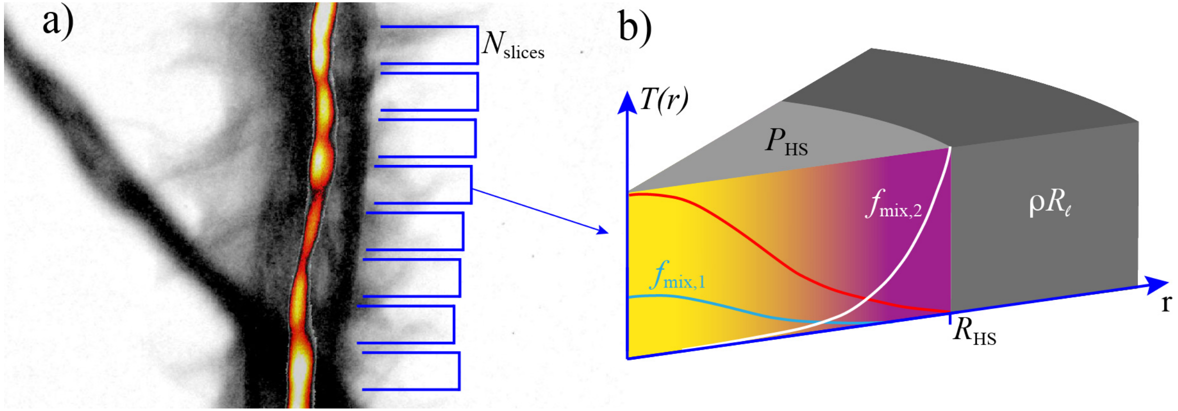
Gunning and Glinsky, Computers & Geosciences **30** 619 (2004)

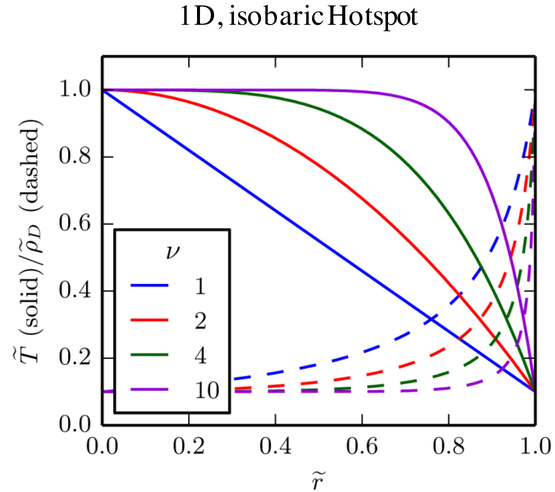
"big data" analysis such as the Google search engine



This technique, as applied to parameter estimation in HED systems, is in its infancy

We have developed a forward model that allows direct, quantitative comparison of the data with synthetic diagnostics





$$\{T_{\mathrm{i}}\} = \{T_{\mathrm{e}}\}$$
$$\{\tau_\nu^\ell\}$$
$$\{P_{HS}\}$$
$$\{f_{\mathrm{mix}}\}$$
$$\{Z_{\mathrm{mix}}\}$$
$$\{R_{\mathrm{HS}}\}$$

$$\overline{BR}$$
$$\tau_{\mathrm{burn}}$$

Assumptions:

- Each slice has its own independent parameters characterizing a 1D, isobaric hot spot surrounded by a liner
- Ideal gas EOS: $P_{\mathrm{HS}} = (1 + \langle Z \rangle) n_{\mathrm{i}} k_B T$
- All elements have same burn duration
- Electron and ion temperatures are equal
- Mix fraction is radially uniform and contaminant emission is dominated by bremsstrahlung radiation

Using this model, we can model existing MagLIF diagnostics

X-ray Emission:

$$Y_\nu = A_{f-f}\sum_{n=1}^{N} e^{-\tau_\nu^\ell}\tau_{\rm burn}2VP_{\rm HS}^2\int_0^1 \tilde{r}d\tilde{r}\frac{g_{\rm FF}\langle Z\rangle}{(1+\langle Z\rangle)^2}\sum_i f_i\tilde{j}_i\frac{e^{-h\nu/T}}{T^{5/2}}$$

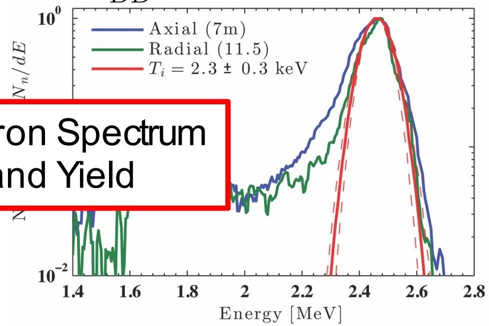
$$\tilde{j}_i \equiv \frac{j_i}{j_{\rm D}} = Z_i^2 + \frac{A_{f-b}}{A_{f-f}}\frac{Z_i^4}{T}e^{RyZ_i^2/T}$$

Neutron Emission:

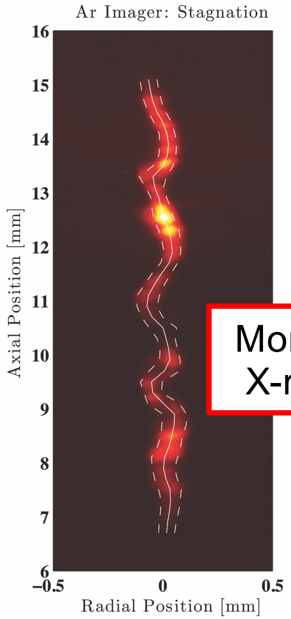
$$\frac{dN_{\rm DD}}{dE} = \sum_{n=1}^{N}\frac{P_{\rm HS}^2 V\tau_{\rm burn}}{(1+\sum_i f_i Z_i)^2}\int_0^1 \tilde{r}d\tilde{r}\frac{\langle\sigma v\rangle_{\rm DD}}{T_{\rm i}^2}I_o(E)$$

$$^*I_o(E) = e^{\frac{-2\bar{E}}{\sigma^2}(\sqrt{E}-\sqrt{\bar{E}})^2}$$

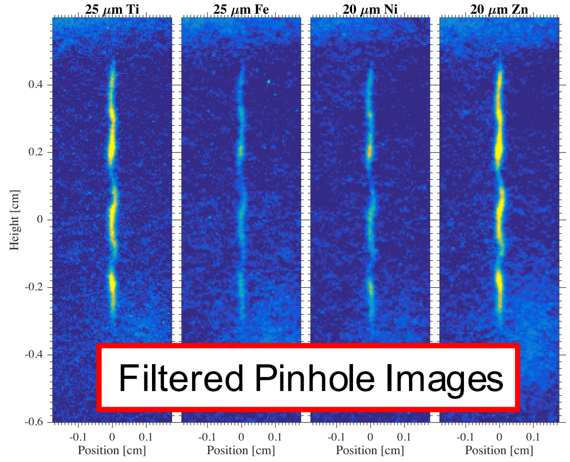
$$Y_{\rm DD} = 3.2\times10^{12}\pm20\%$$



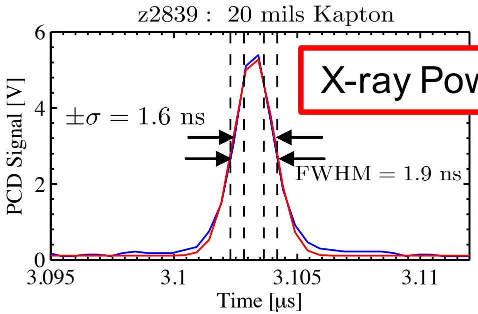
Neutron Spectrum and Yield



Monochromatic X-ray Imaging



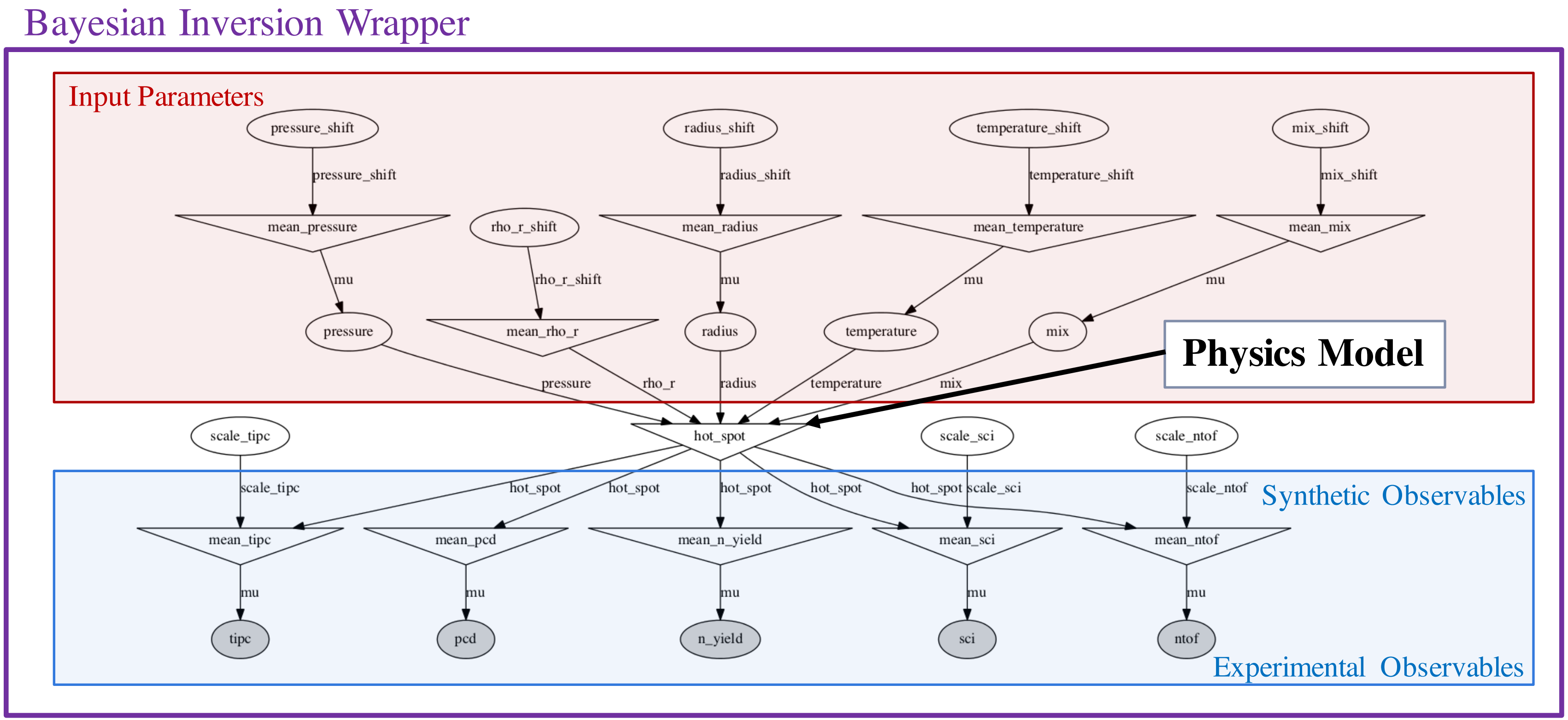
Filtered Pinhole Images



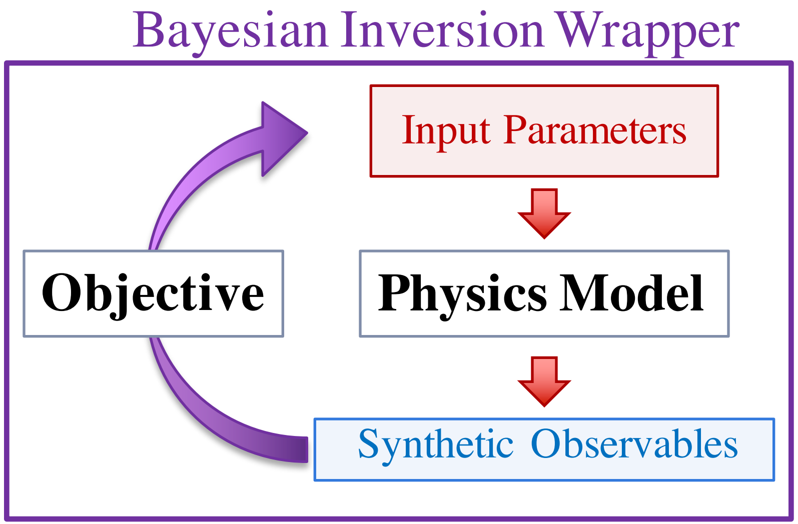
X-ray Power

$$Y_\gamma = 10\ \mathrm{J}\pm20\%$$

The inversion model can be represented as graph



The methodology is completely general and very modular

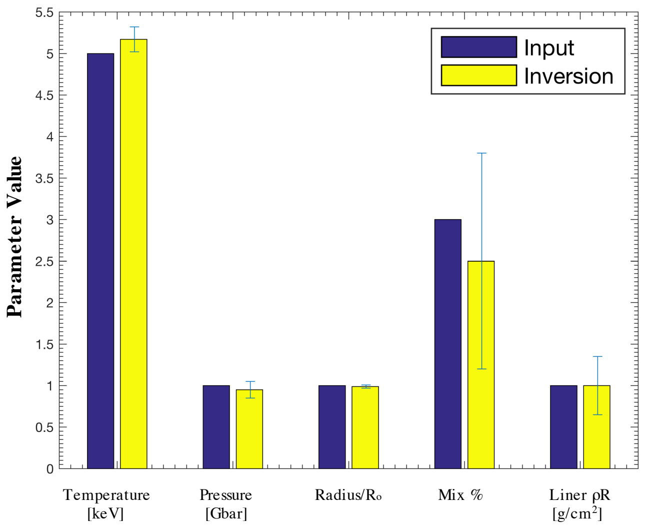


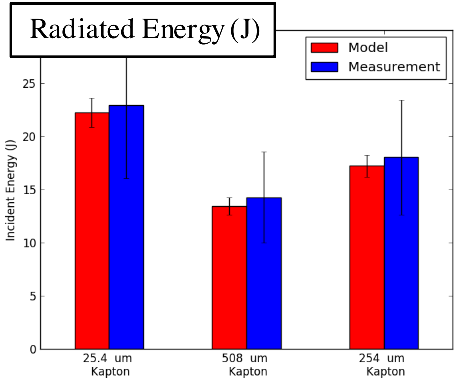
- The physics model and the synthetic observables (e.g. diagnostics) can be written to suit your application
- Everything is implemented in python with an architecture supporting this modular philosophy
  - The model is a class
  - Each of the diagnostics are classes that accept an instance of the model (e.g. a *hotspot* object) as an input
  - New diagnostics can be added trivially if no new physics is required

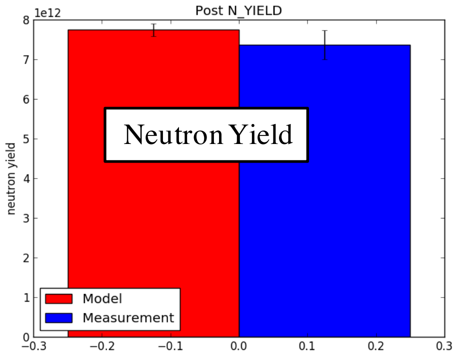
A validation effort is underway to convince ourselves that the model, diagnostics, and inversion are accurately implemented

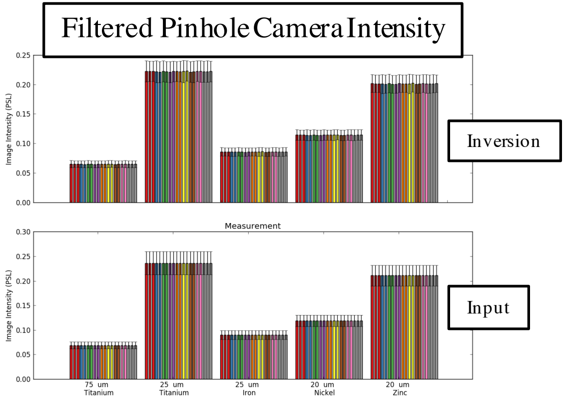
- Formulating a series of tests where synthetic data is generated using the model to test
  - Accuracy and stability of solution
  - Correlations between parameters
- Simple uniform plasma
- Simple variations in single parameters
- Variations in multiple parameters

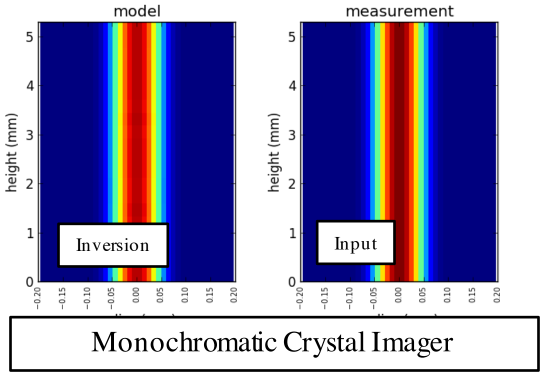
Initial tests with uniform plasma column are promising





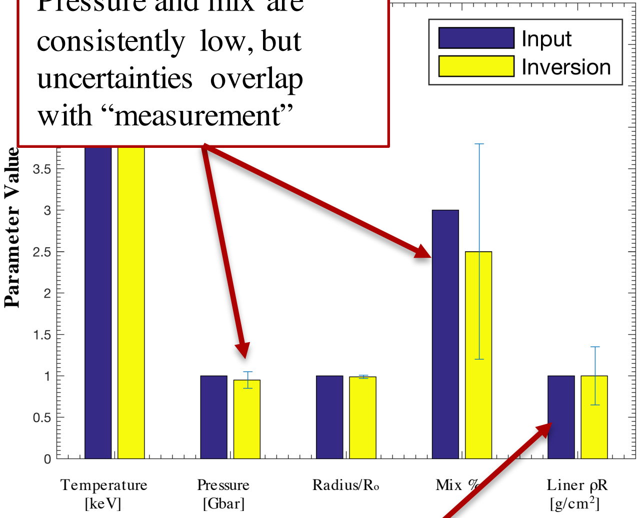






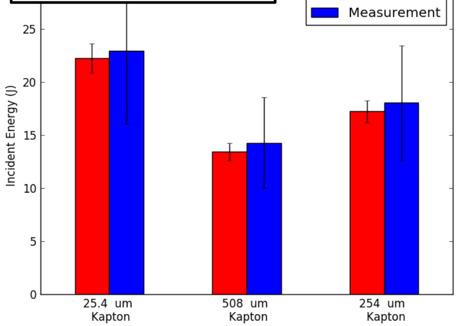
Initial tests with uniform plasma column are promising

Pressure and mix are consistently low, but uncertainties overlap with "measurement"

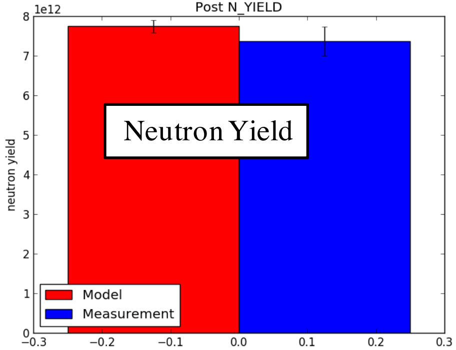


ρR is not being determined

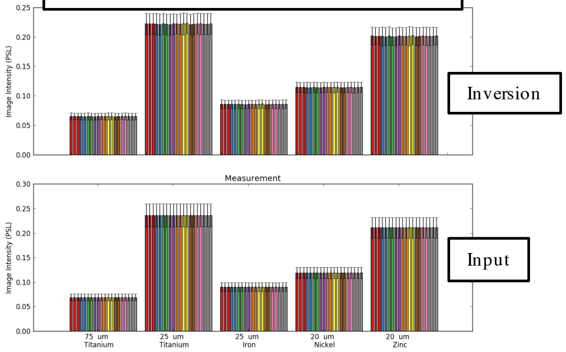
Radiated Energy (J)

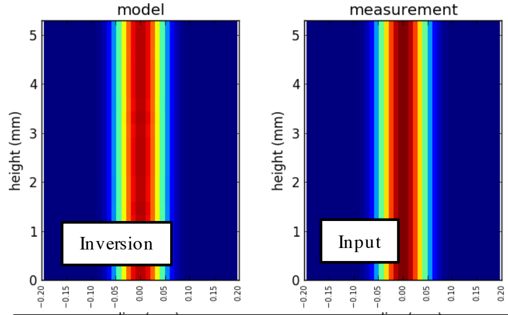


Neutron Yield



Filtered Pinhole Camera Intensity

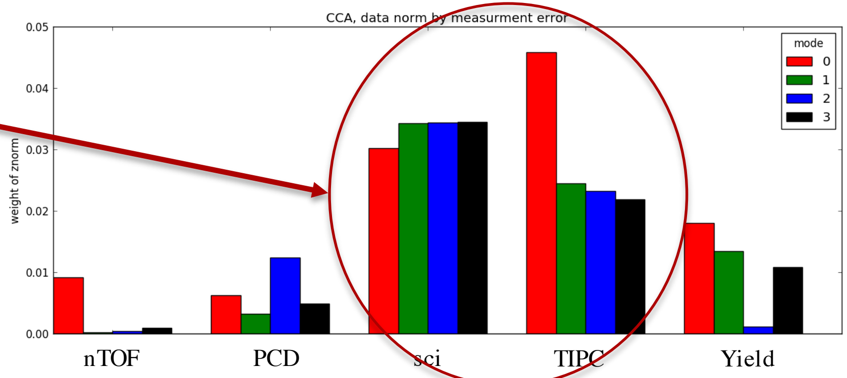


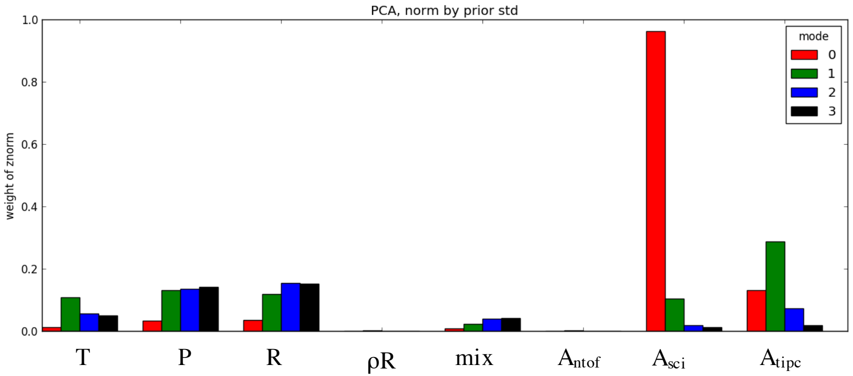


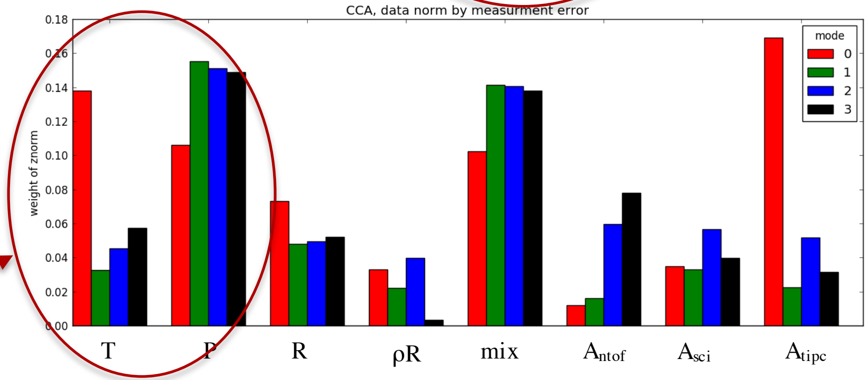
Monochromatic Crystal Imager

Correlation and principal components analysis shows which data and parameters and diagnostics contribute

TIPC and crystal imager dominate the inversion

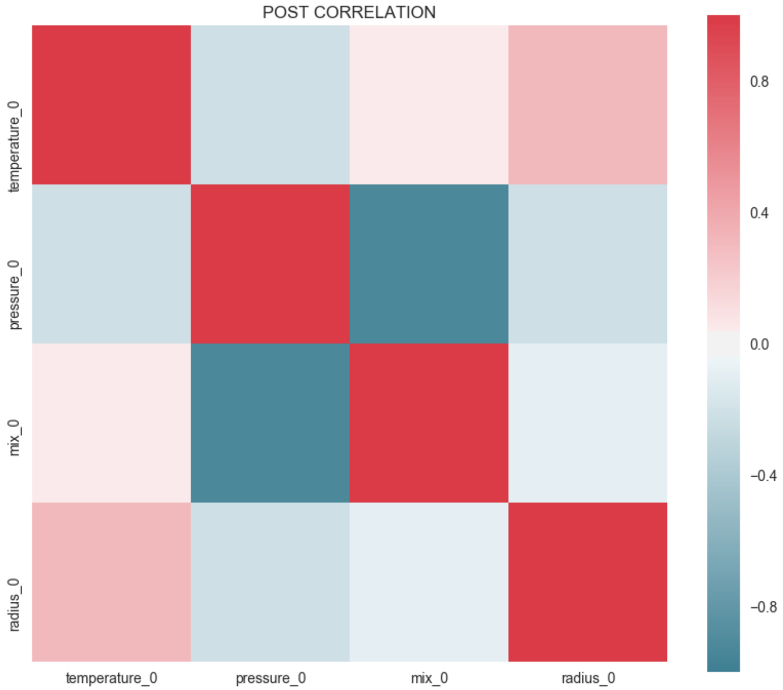


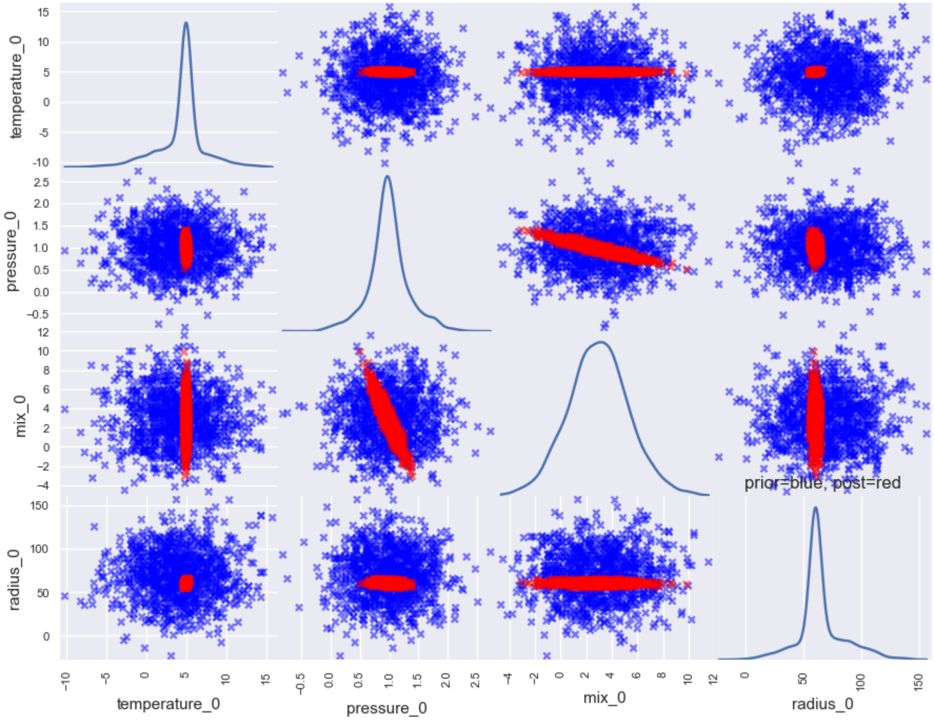




Temperature, mix and pressure are most strongly determined

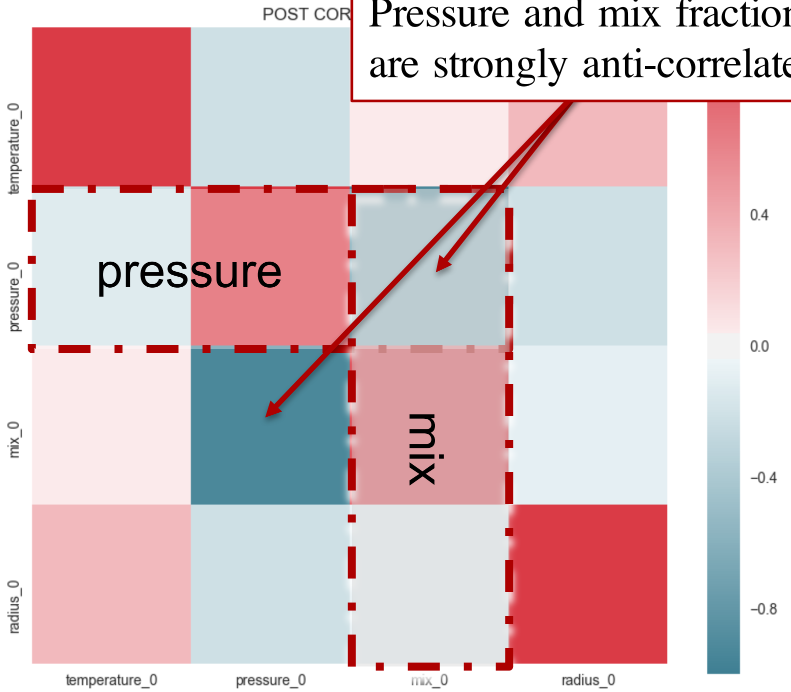
Visualizing the statistics gives insight into the physics of the system

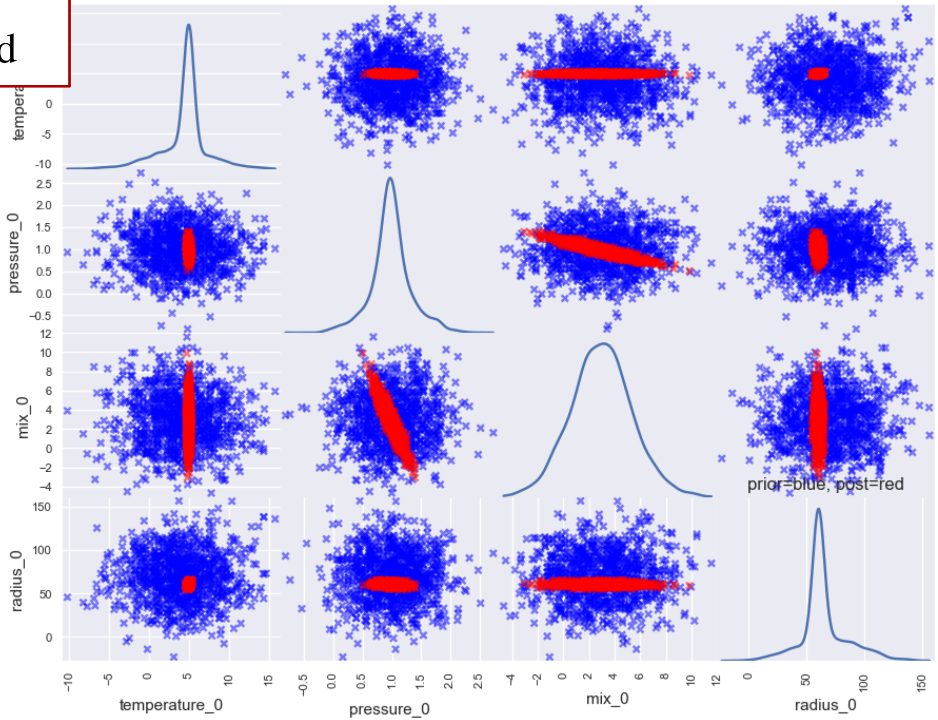




Visualizing the statistics gives insight into the physics of the system

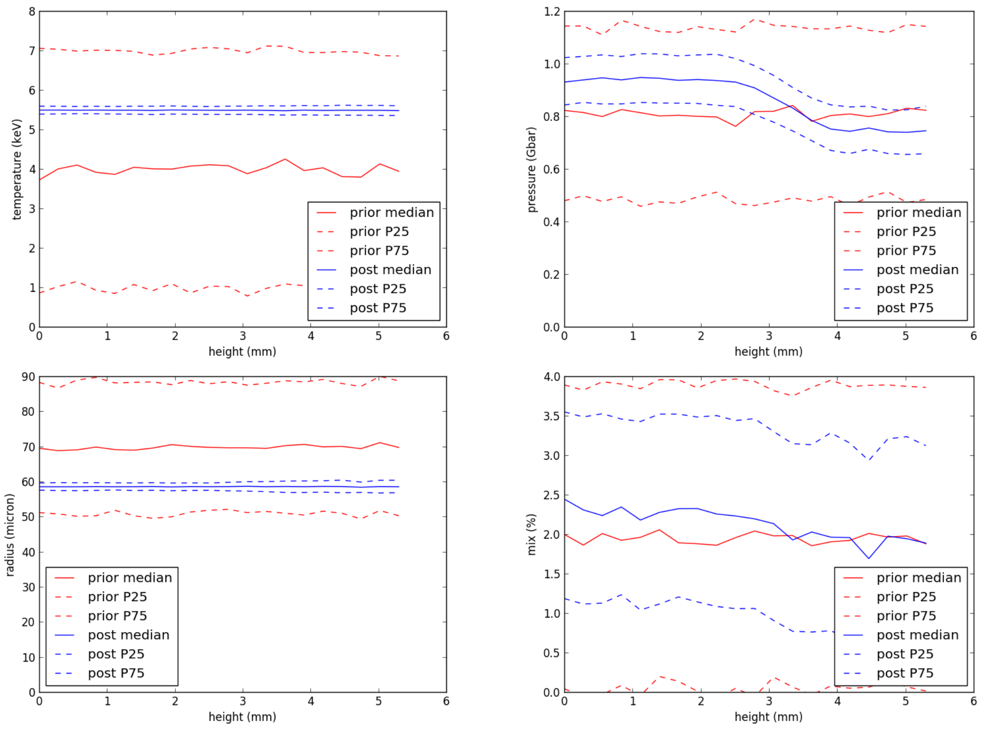
Pressure and mix fraction are strongly anti-correlated





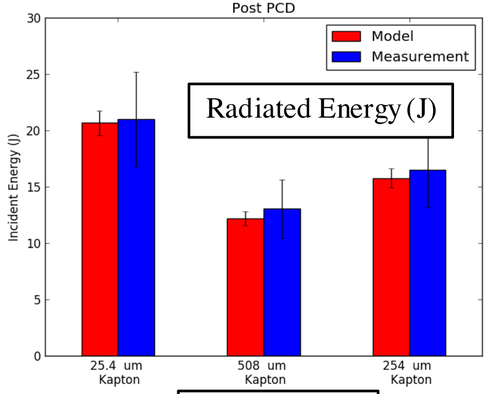
Very mild correlations between other stagnation parameters

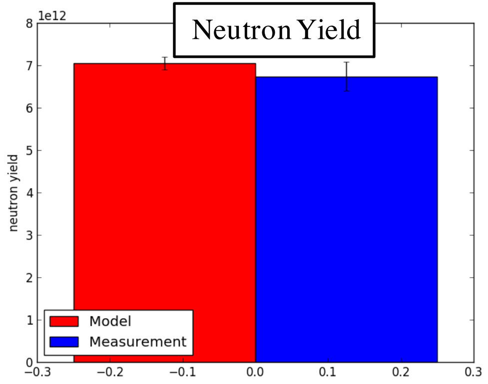
Test with an axial pressure gradient is able recover the input with reasonable accuracy

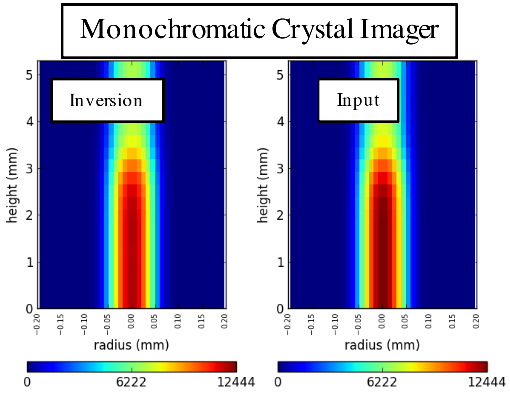


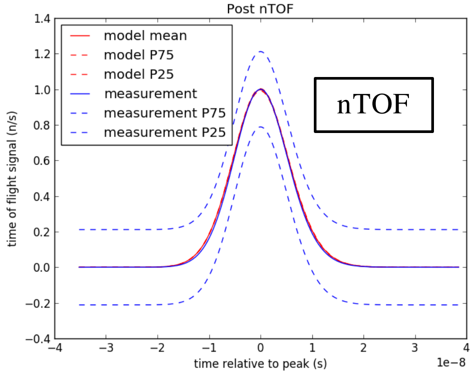
- The gradual step from 1.0 Gbar to 0.7 Gbar is closely recovered
- There is a slight slope to the mix profile is still low, but true value is within the uncertainties

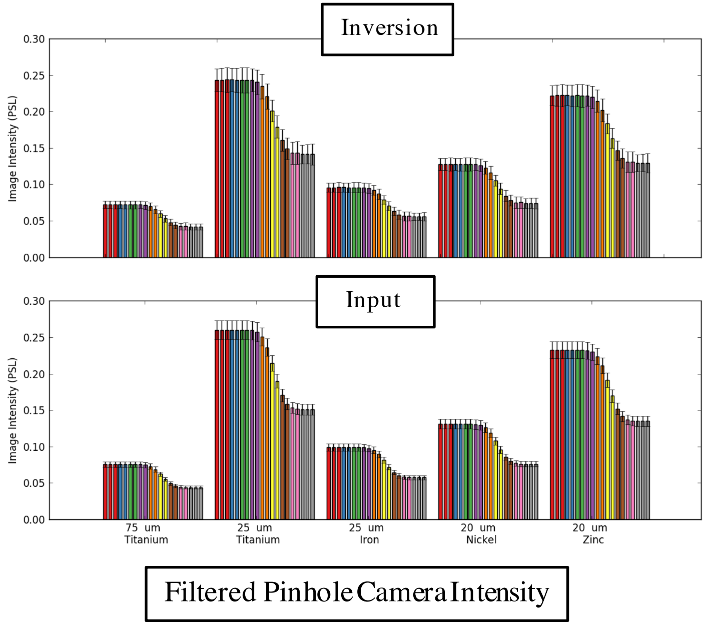
In both cases the diagnostic agreement is *excellent*





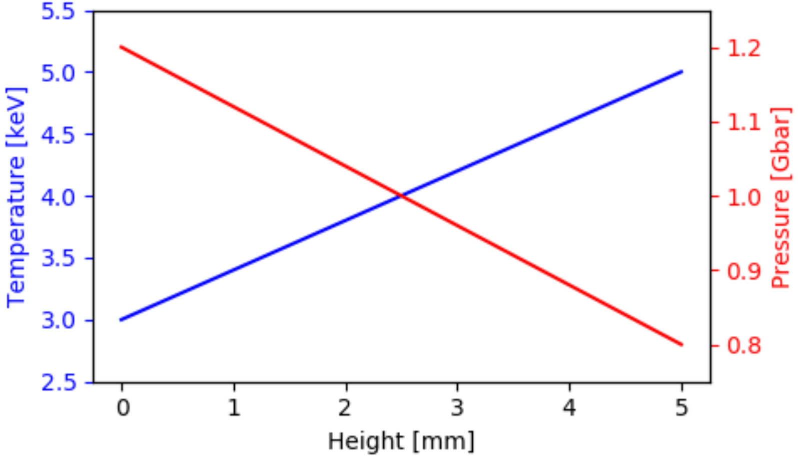


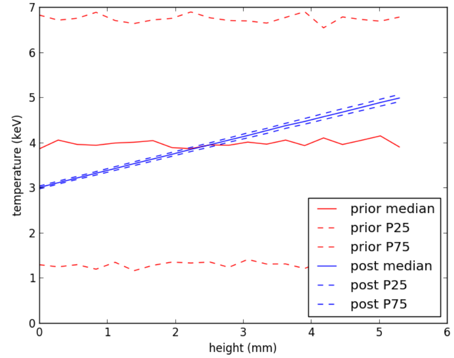


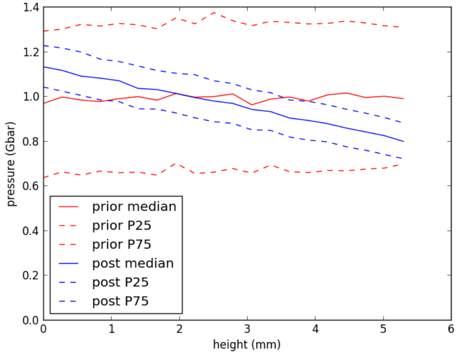


Lets make things more interesting: Combined Pressure and Temperature gradients

Input Temperature and Pressure Profiles

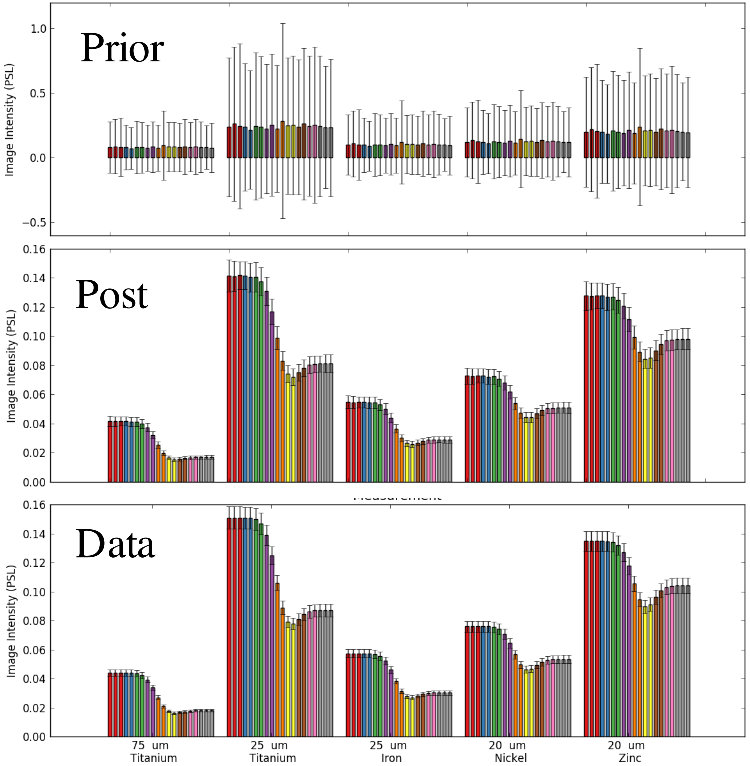


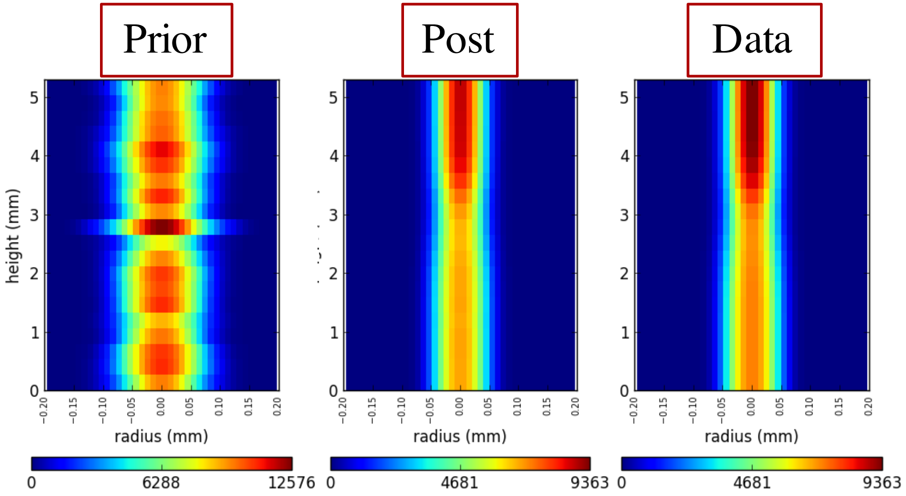




- Radius
  - Input: 70 μm
  - Output: 70 +/- 3 μm
- Mix
  - Input: 3%
  - Output: 3 +/- 1.2%
- Areal density still undetermined

Lets make things more interesting: Combined Pressure and Temperature gradients

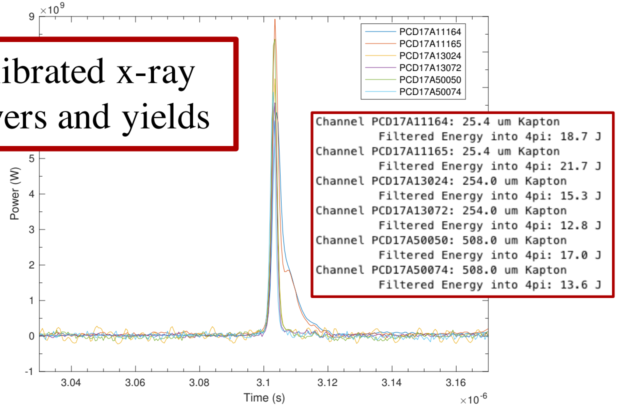




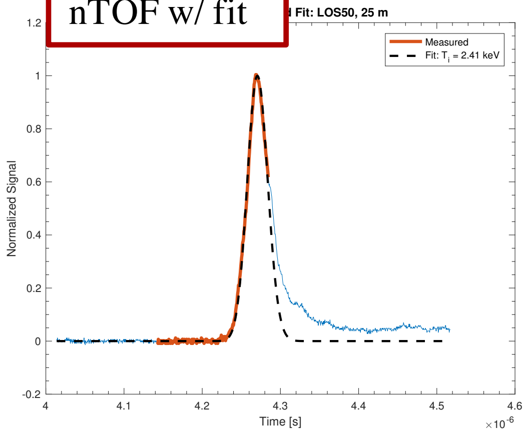
Diagnostic matches are excellent!

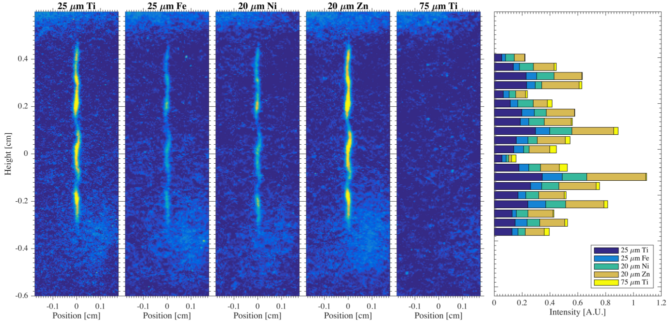
Example fully processed data for z2839

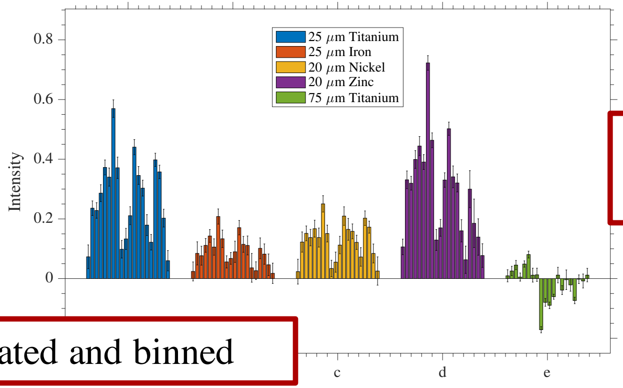
Calibrated x-ray powers and yields



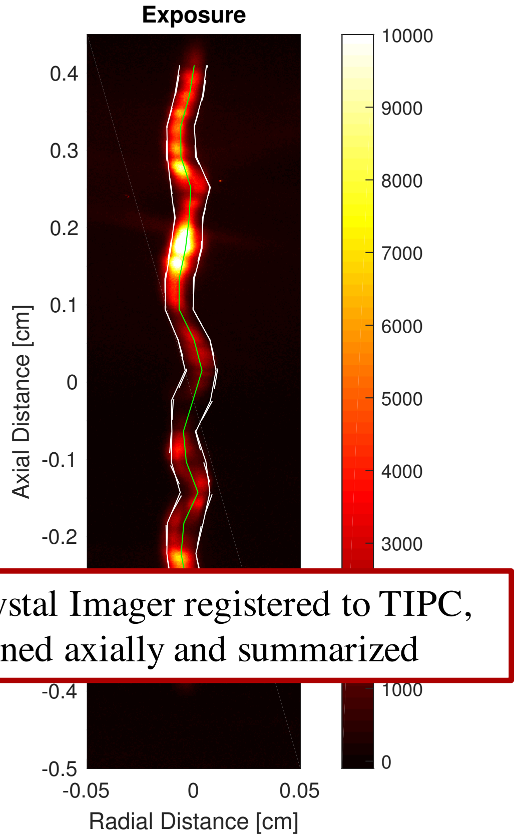
nTOF w/ fit





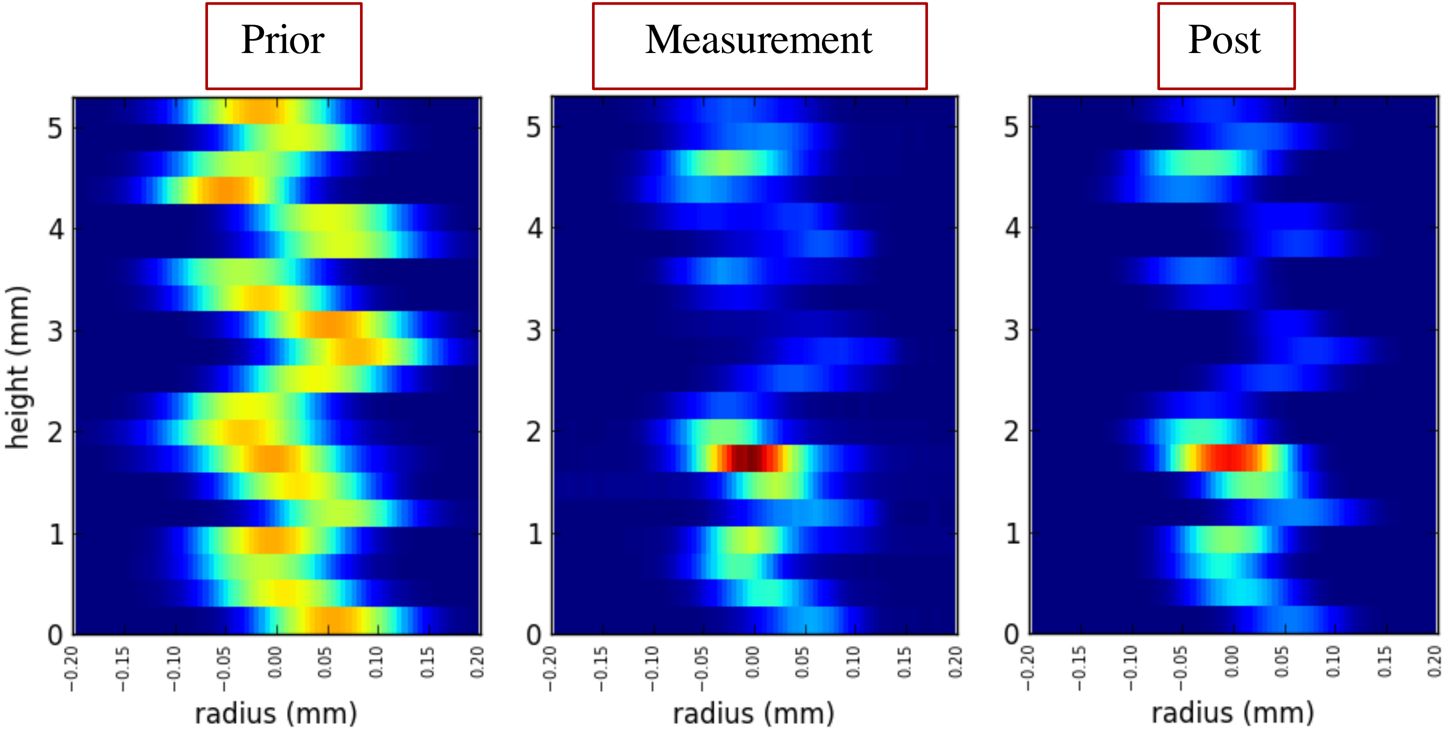


TIPC registered, calibrated and binned

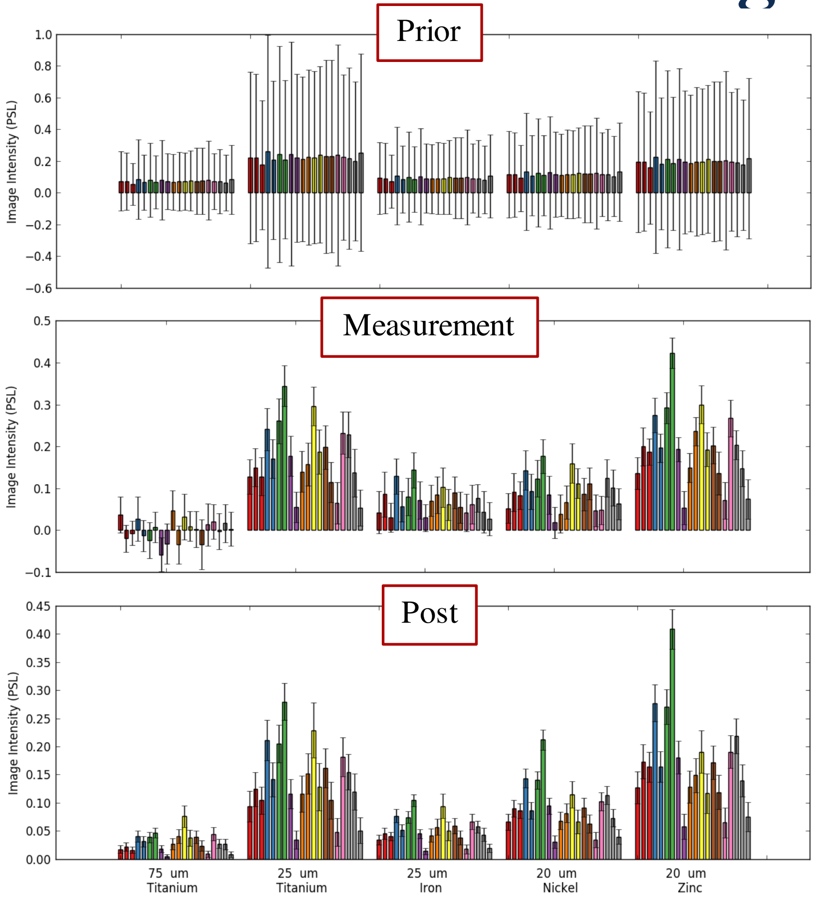


Crystal Imager registered to TIPC, binned axially and summarized

Data: Spherical Crystal Imager

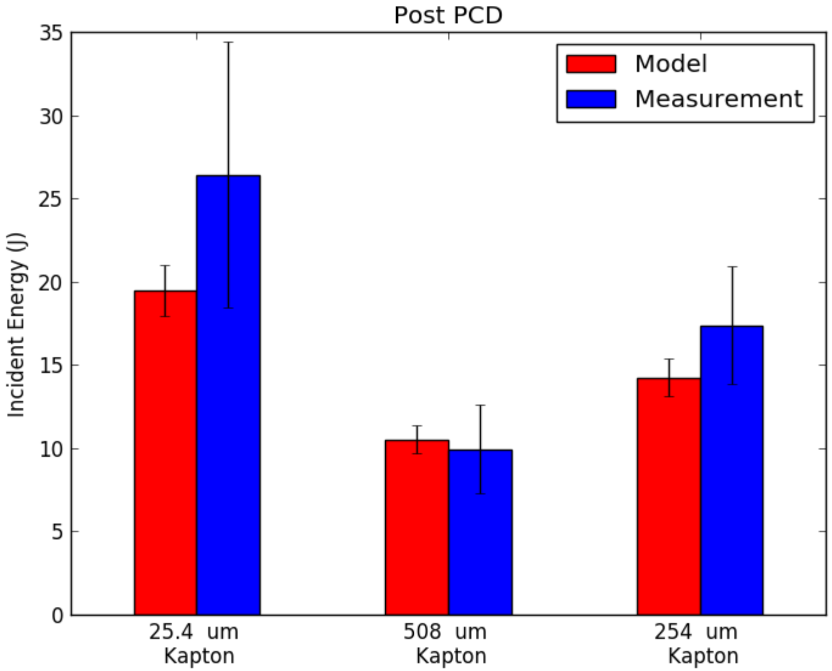


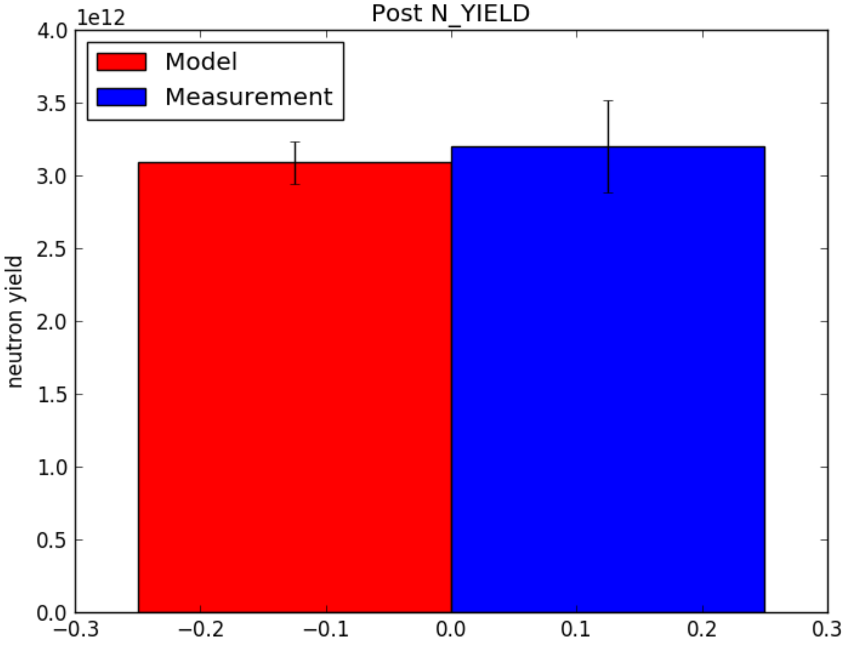
Data: Filtered Time integrated Pinhole Camera



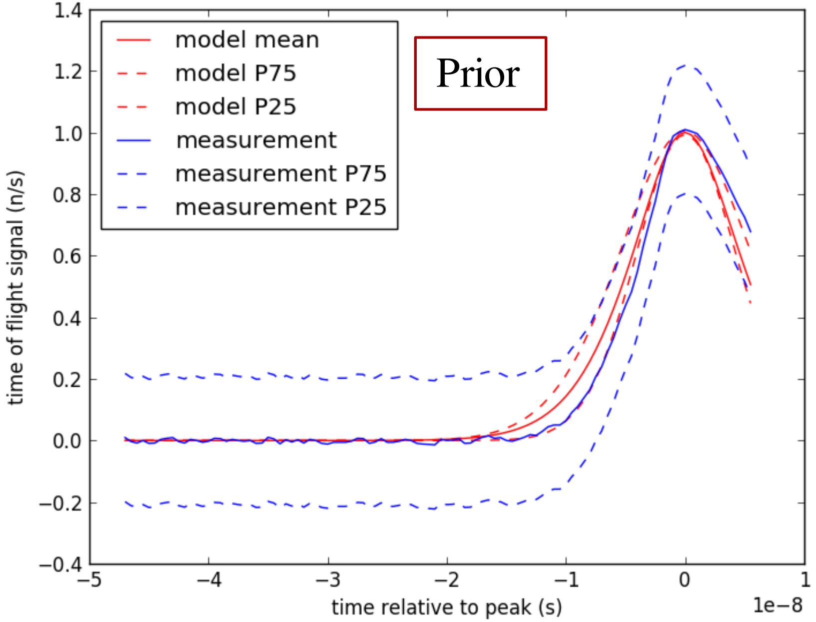
- TIPC values are matched everywhere within the uncertainties, except where the data is noise

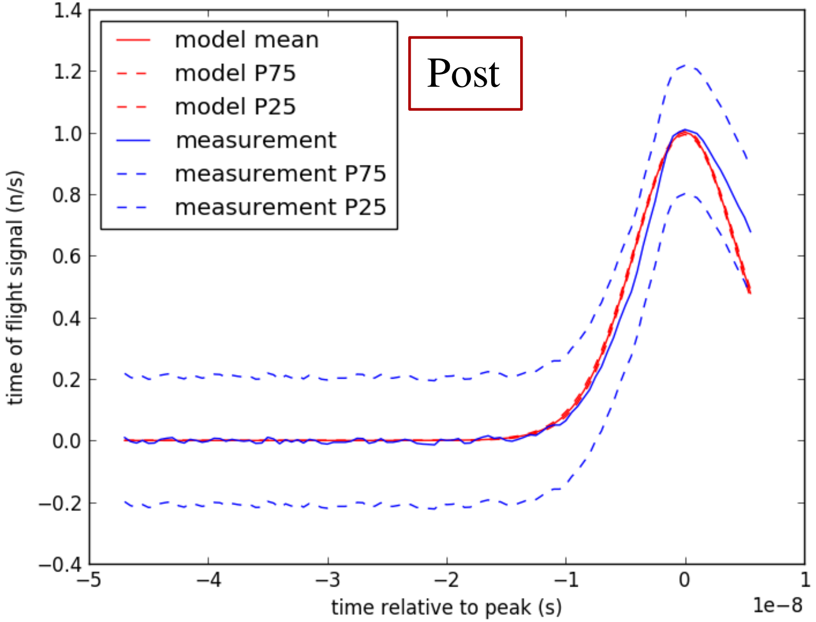
Data: X-ray and Neutron Yield





Data: Neutron time-of-flight





- The inversion produces a decent match to the nTOF, but with zero uncertainty
- Suggests that the modeled nTOF is completely determined by other diagnostics and is not contributing to the inversion

Estimation of stagnation parameter axial profiles

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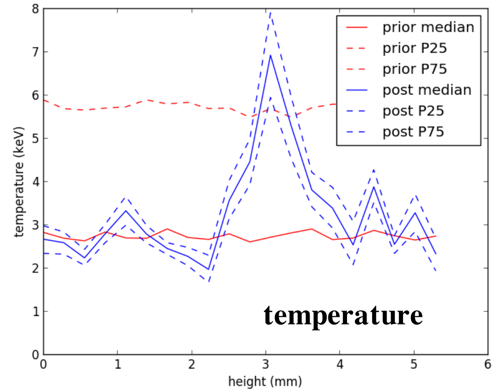
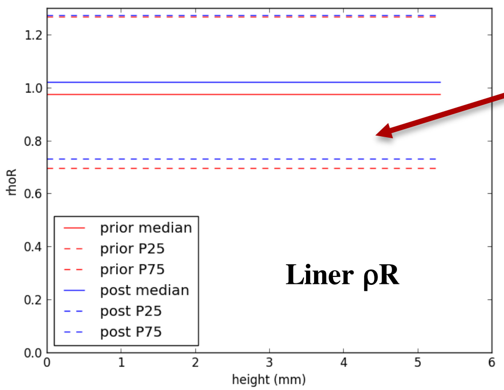
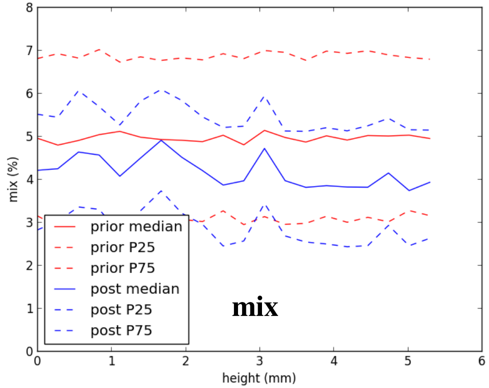
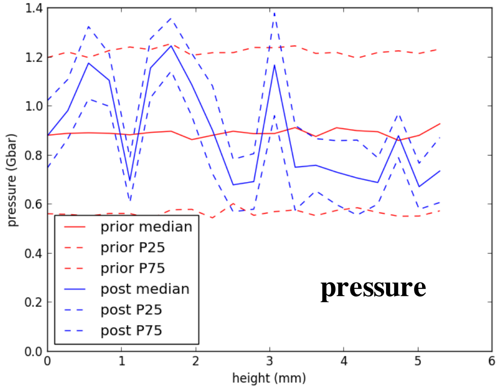
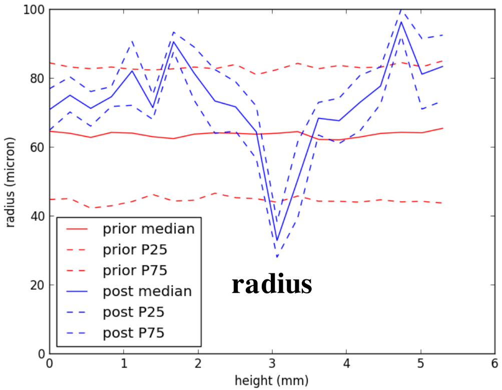




Real variation in ρR will affect the inference on other quantities



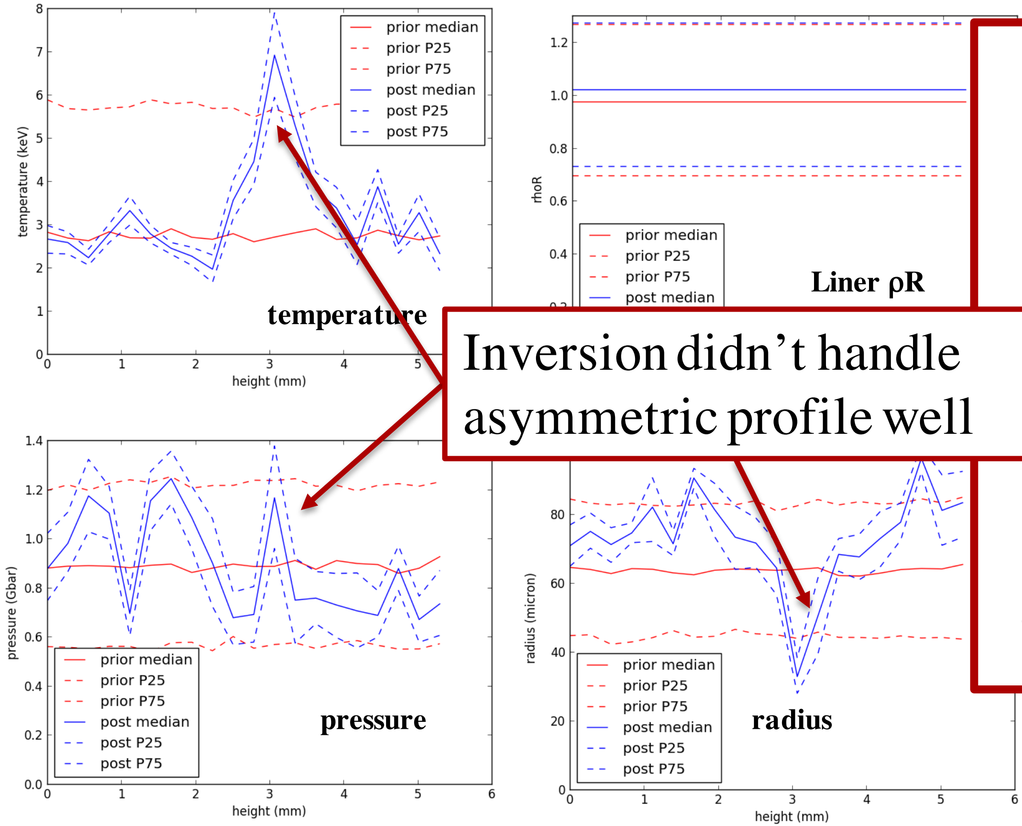




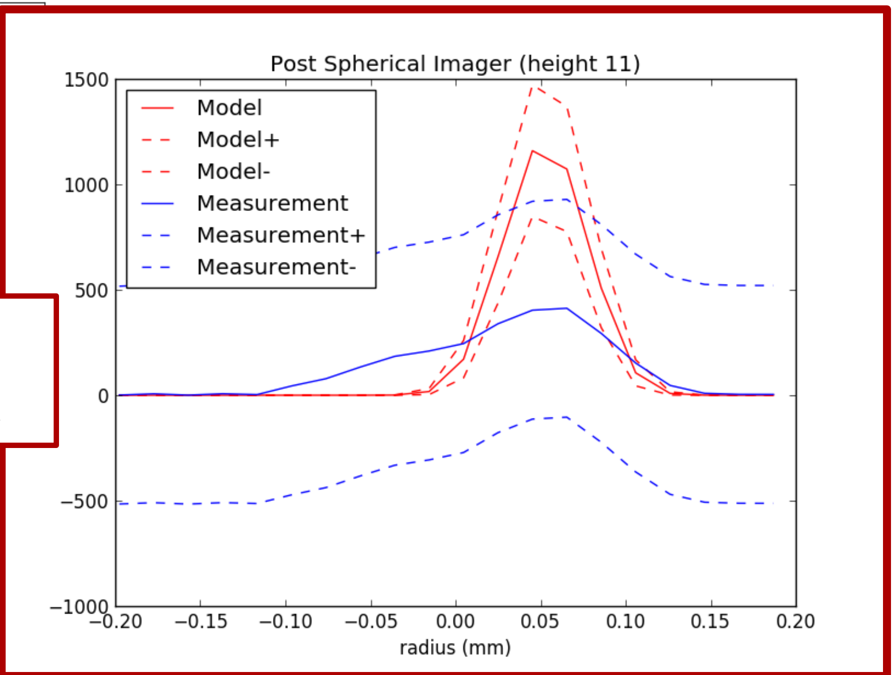
Inferred Pτ~1.7 atm-s

The lack of sensitivity to liner ρR will introduce a systematic bias in other quantities

Estimation of stagnation parameter axial profiles



Inversion didn’t handle asymmetric profile well



The lack of sensitivity to liner ρR will introduce a systematic bias in other quantities

Next Steps and Future Work

- Continue Validation effort
  - More synthetic tests with the hotspot model
  - Use synthetic diagnostics generated from a variety of GORGON simulations
- Improve liner areal density estimation
  - Using the full nTOF spectrum with downscatter
  - Optimize filtered x-ray diagnostics for areal density
  - Include XRS3 data in the inversion
- Improve mix determination
  - Optimize x-ray power measurements for mix
  - Synthetic tests using new diagnostics -> XRS3, neutron imaging
- Refine MCMC algorithm and implement "multi-shoot" approach to explore parameter space and look for multimodality
- Implement an iterative procedure to combat bias from the prior