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Numerical Investigation of Coupling for Structural Acoustics

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Computational Solid Mechanics & Structural Dynamics

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Coupled structural-acoustics

- What is the noise radiated from a structure?
- How does a structure respond/transmit acoustic waves?
- Computational methods: FE for structure, FE or BE for acoustic domain
- Fully coupled methods
  - FE acoustics: dynamic system matrices no longer symmetric, can get very poorly conditioned systems
  - BE acoustics: dense matrices for acoustics
- Loosely coupled methods:
  - Coupling occurs through loading each domain
  - Allows use of optimized solvers
  - Downside: reduced accuracy?

Governing equations

- Structure: linearized elastodynamics for displacement *u*

$$\rho_s \ddot{\mathbf{u}} - \nabla \cdot \mathbf{T}_{\mathrm{R}} = \rho_s \mathbf{b}$$

- Acoustics: linearized Euler equations for velocity potential

$$\frac{1}{c_a^2}\ddot{\psi} - \nabla^2 \psi = 0, \quad p = -\rho_a \dot{\psi}$$

- Coupling condition: continuity of velocities and tractions

$$\mathbf{v}_s = \mathbf{v}_a = \nabla \psi$$

$$\mathbf{t}_s = \mathbf{t}_a = -p\,\mathbf{n}$$

Semi-discrete form

- Standard FEM method is used to discretize
- Resulting semi-discrete system is

$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} C_s & C_{sa} \\ C_{as} & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \psi \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ f_a \end{bmatrix}$$

- Matrices C_{sa} and C_{as} enforce the coupling conditions on the interface
  - Acoustic pressures are applied to the structure as tractions
  - Structural velocities are applied to the acoustic domain
- Newmark beta time integration

$$\begin{aligned} U^{n+1} &= U^n + (\Delta t)\dot{U}^n + \frac{(\Delta t)^2}{2}\left\{(1-2\beta)\ddot{U}^n + 2\beta\ddot{U}^{n+1}\right\} \\ \dot{U}^{n+1} &= \dot{U}^n + (\Delta t)\left\{(1-\gamma)\ddot{U}^n + \gamma\ddot{U}^{n+1}\right\} \end{aligned}$$

Partitioning the system

- Algebraic partitioning: integrate, partition, predict (IPP)
  - Time integrate the coupled system
  - Partition the fields
  - Predict field values that appear as loads
  - Use for coupling same physics, controls
- Differential partitioning: partition, predict, integrate (PPI)
  - Partition the block matrix system according to the physics
  - Predict fields values for coupling
  - Apply time integrator to the resulting system
  - Natural choice for multiphysics coupling
- We chose to use differential partitioning!

Partitioned system

- Rewrite the matrix system with coupling terms on the RHS

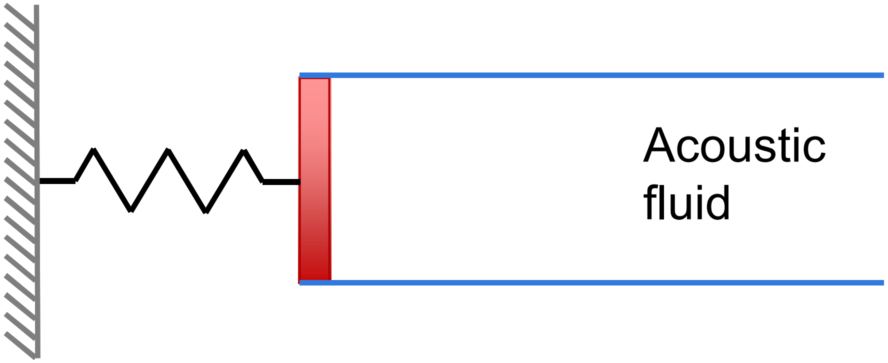
$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \psi \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ f_a \end{bmatrix} - \begin{bmatrix} 0 & C_{sa} \\ C_{as} & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\psi} \end{bmatrix}$$

- Structure velocity and acoustic pressure on RHS must be predicted
  - Predictions not entirely independent of solution algorithm!
- Predict both: solve whole system in parallel
- Predict one: system is solved in serial
- Correctors: correct the load on one partition after the other has been solved

Interfield coupling algorithms

- Parallel
  - Begin timestep
  - Exchange acoustic pressure, structural velocities
  - Acoustic and structural both solve
  - End timestep
- Serial
  - Begin timestep
  - Predict structure velocities, apply to acoustic domain
  - Solve acoustic domain
  - “Correct” acoustic pressures, apply to structure
  - Solve structure
  - End timestep

Verification: 1d acoustic piston



***Goal**: test loosely coupled algorithms to assess temporal accuracy*

Structure displacement

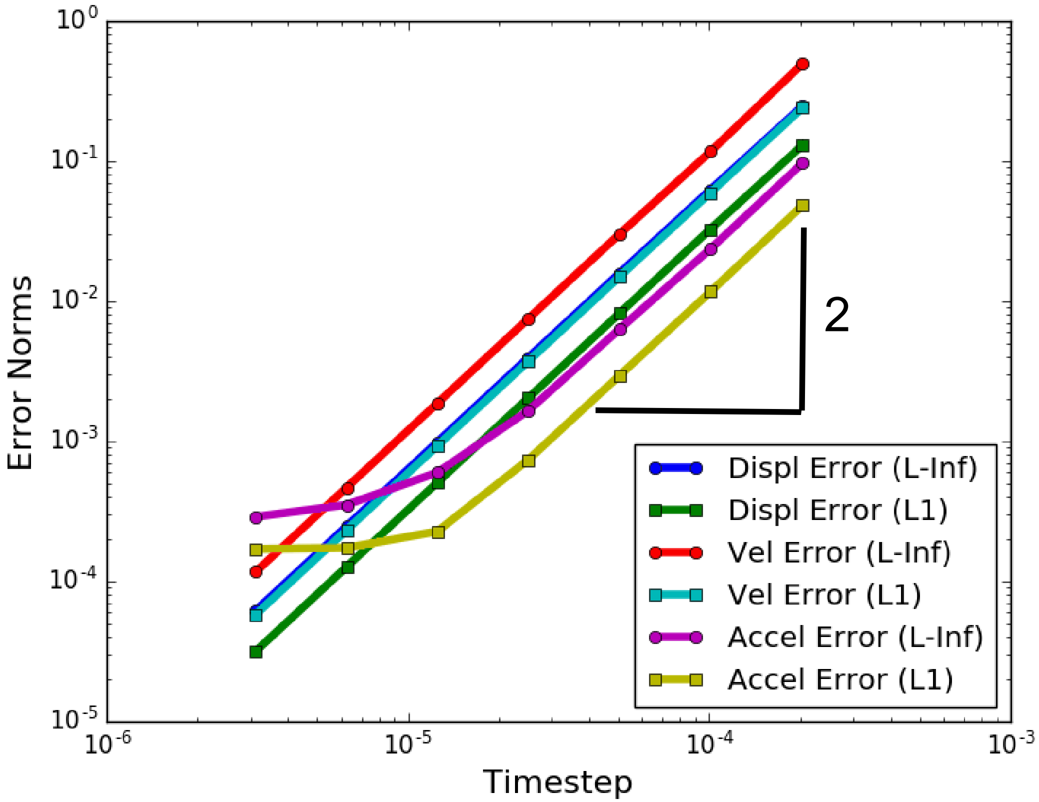
$$u_s(t) = \mathrm{e}^{-dt}\,(a\cos\omega t + b\sin\omega t) + \nu(t-\beta)$$

Fluid solution

$$v_a(t) = \dot{u}_s(t - x/c_a)H(t - x/c_a)$$
$$p_a(t) = p_\infty + \rho_a c_a v_a(t)$$

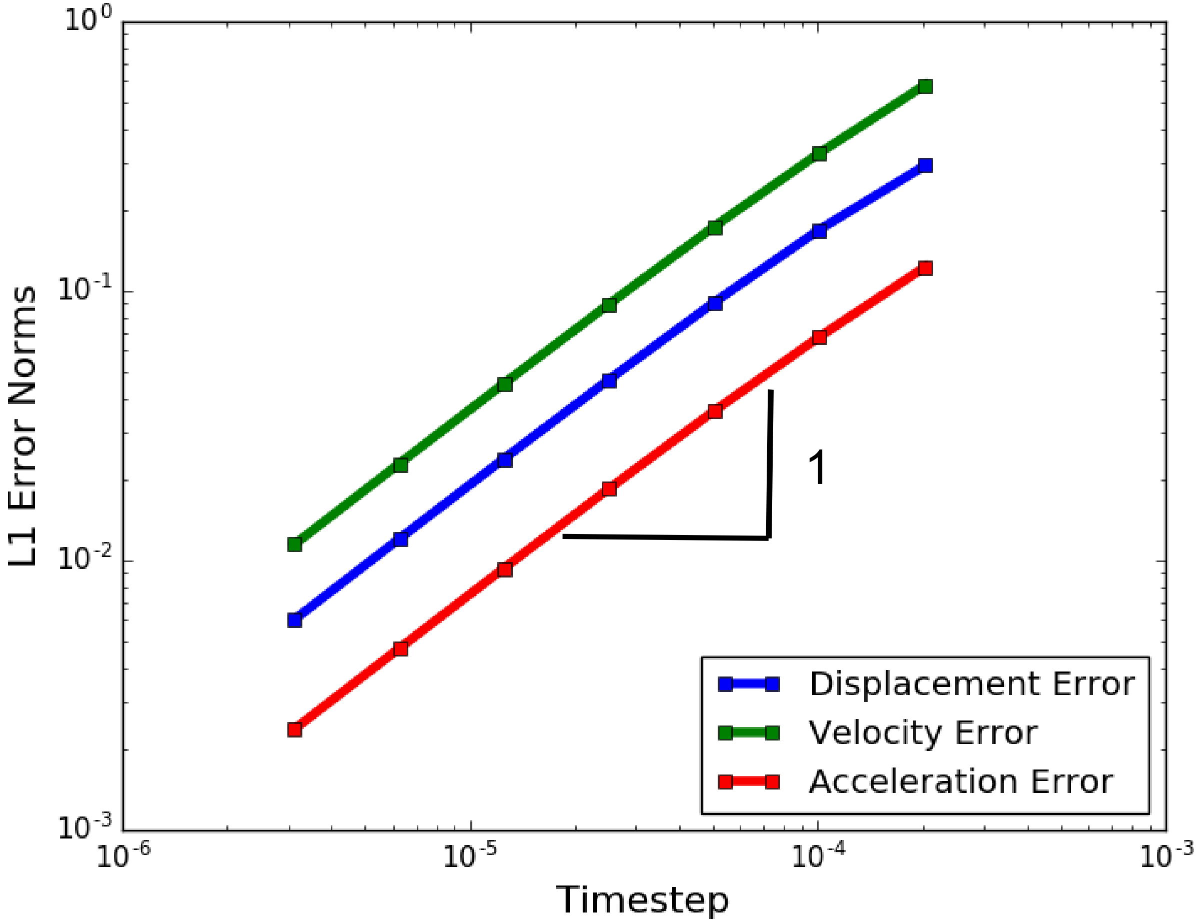
Baseline: tightly coupled solution

- Coupled structural-acoustics are solved as a single linear system
  - Preconditioner GDSW is *necessary* for efficiency
  - Second order time accurate



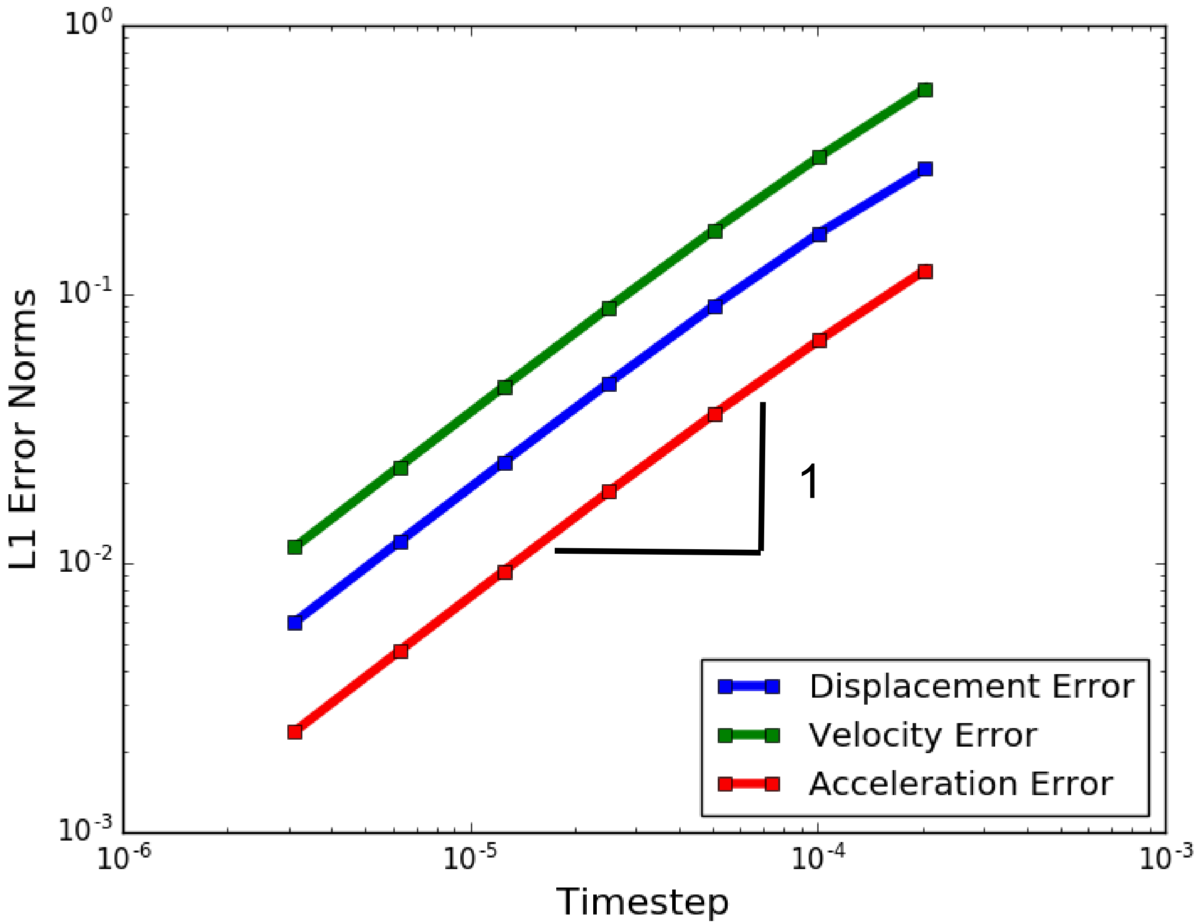
Interfield parallel solution

- Predict both the structure velocities and acoustic pressures
  - Use values from previous timestep



Interfield serial solution

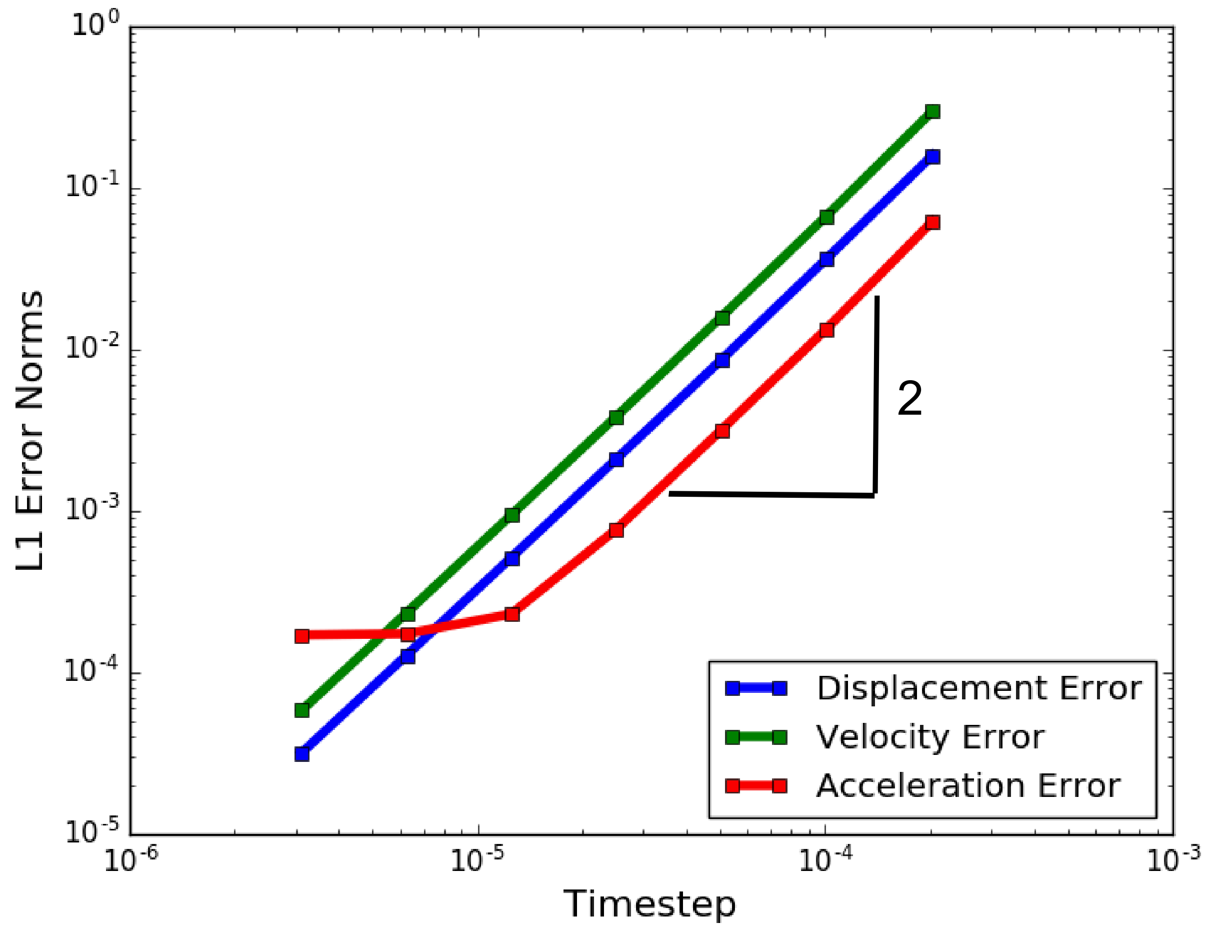
- Structural velocity predictor: $\mathbf{v}^P(t^{n+1}) = \mathbf{v}(t^n)$



Serial solution method

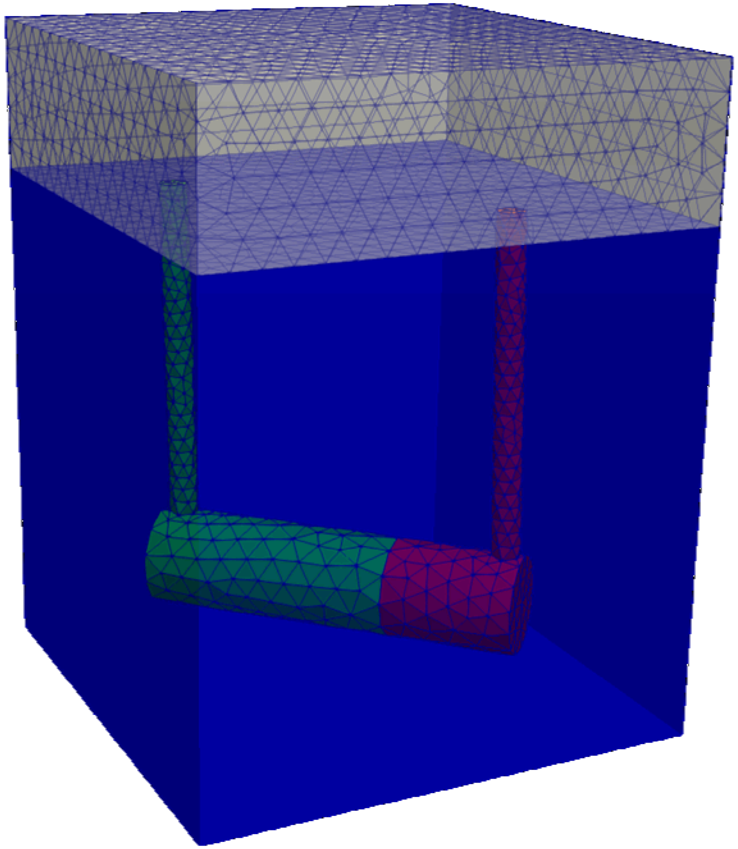
- Structural velocity predictor: (GSS)

$$\mathbf{v}^P(t^{n+1}) = \mathbf{v}(t^n) + \Delta t\dot{\mathbf{v}}(t^n) + \frac{1}{2}\Delta t\left(\dot{\mathbf{v}}(t^n) - \dot{\mathbf{v}}(t^{n-1})\right)$$



Performance test

- Underground tunnel: Acoustic domain is air above ground and filling the tunnel. Structure domain is the “stone” ground.



Performance test

- Acoustic domain
  - 20,833 linear tetrahedra
  - 4,455 DOFs
- Structural domain
  - 84,315 linear tetrahedra
  - 46,230 DOFs
- Loosely coupled:
  - 1 acoustic processor, 1 structural processor: 2 mins, 13s
  - 1 acoustic processor, 10 structural processors: 51s
- Tightly coupled:
  - 1 processor: 2mins, 40 s
  - 11 processors: 55s

Performance test

Pressure applied on the structure

Performance test

Summary

- Loosely-coupled structural-acoustics was accomplished through monolithic and partitioned approaches
- Both approaches achieve second order accuracy in time
- Partitioned allows flexibility for MPMD coupling with other software, e.g., Sierra/Solid Mechanics
- Predictor/corrector coupling has many options that will be explored in future work