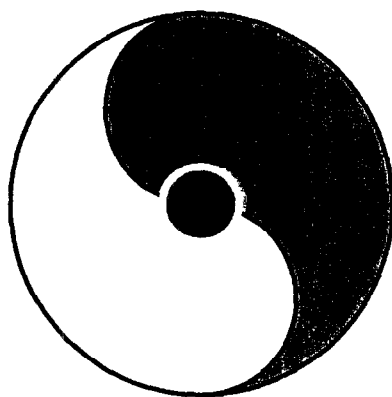


RIKEN Winter School

STRUCTURE OF HADRONS

—INTRODUCTION TO QCD HARD PROCESSES—

December 9–12, 1998

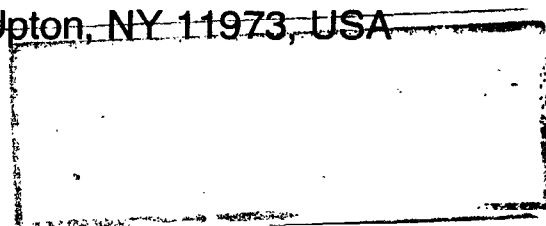


Organizers

Naohito Saito, Toshi-Aki Shibata and Koichi Yazaki

RIKEN BNL Research Center

Building 510, Brookhaven National Laboratory, Upton, NY 11973, USA



Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the “Rikagaku Kenkyusho” (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

*This work done under the auspices of U.S.D.O.E. contract no. DE-AC02-98CH10886.

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Introduction

The 9th Riken Winter School on "Structure of Hadrons - Introduction to QCD Hard Processes" was held from December 9th through 12th at Shimoda Tokyu Hotel, Shimoda, Shizuoka, Japan. The school was sponsored by the RIKEN BNL Research Center and the RIKEN Accelerator Research Facility. The purpose of the school was to offer young researchers an opportunity to learn the exciting physics associated with the structure of hadrons probed by QCD hard processes. The subjects discussed in the school cover theoretical and experimental aspects of the hard processes including polarization phenomena and the nucleon structure functions, starting from the basics of the perturbative QCD.

We had 3 main courses each consisting of 4 one-hour lectures, 3 tutorial sessions including Q&A and exercises on the contents of the lectures, 2 evening sessions for discussions on interesting and important subjects and 4 short lectures on specific topics. These were given by 3 main lecturers and 4 tutors. The list of them together with the titles of their lectures are given below.

Main Lecturers

R.Jaffe (MIT) "Spin in Hard QCD Processes"

J.Kodaira (Hiroshima) "Perturbative QCD and Introduction to Structure Functions"

K.Rith (Erlangen) "Quark-Gluon Structure of the Nucleon"

Tutors

N.Hayashi (Riken) "RHIC Spin Program"

Y.Koike (Niigata) "Twist-3 Effects in Hard Processes"

S.Kumano (Saga) "Polarized Proton-Deuteron Drell-Yan Processes"

K.Tanaka (Juntendo) "Hard Exclusive Processes and Vector Meson Wave Functions in QCD"

24 students attended the school and actively participated in the program. The 3 main lecturers gave excellent courses which were both pedagogical and inspiring. They also attended the tutorial and evening sessions and were kind enough to respond to any questions asked either during the sessions or in smaller circles.

The 4 tutors played very important roles by organizing the tutorial sessions, making exercise problems, helping students in understanding the lectures and giving interesting lectures on their own works.

At the end of the school, we asked the students to write their opinions about the school. The responses were all positive, and the students were grateful to the lecturers and the tutors. Many of them seem to have felt that 4 days were a little too short for fully understanding the broad field of QCD hard processes. They also wanted to have this kind of school regularly, every year if possible. We are thus considering the possibility of organizing such a school at least every two years.

We are grateful to Prof.T.D.Lee and Prof.M.Ishihara for their approval, which enabled us to organize this school. We also thank the lecturers, the tutors and the students for making the school fruitful. Special thanks are due to two secretaries, Ms.Yoko Kishino and Ms.Toshiko Nakamura, who did most of the administrative works and took care of drinks and snacks during the coffee breaks and the evening sessions. The hotel is located at a beautiful spot in Izu peninsula and we were lucky about the weather. We thank

Shimoda Tokyu Hotel for their excellent service and comfortable atmosphere.

Naohito Saito, Toshi-Aki Shibata and Koichi Yazaki

Introduction to Perturbative QCD and Structure Functions

Lectures Presented at the 9th RIKEN Winter School,

Shimoda, December 8-12, 1998

Jiro Kodaira

Department of Physics, Hiroshima University

Higashi-Hiroshima, 739-8526, Japan

Abstract

In this lecture I give a pedagogical introduction to the Perturbative QCD to understand the short-distance dynamics of the strong interaction. Starting with fundamental concepts such as the color degree of freedom of QCD, non-abelian gauge field theory, renormalization group equation etc., I explain a basic idea of the perturbative QCD and apply this idea to the e^+e^- processes and the structure functions. The notion of mass singularity and the necessity of its factorization is discussed in some detail.

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INTRODUCTION
TO
PERTURBATIVE QCD
AND
STRUCTURE FUNCTIONS

J.Kodaira (Hiroshima)

December 9-12 '98
at RIKEN WINTER SCHOOL

1. PROLOGUE

- Colored Quark Model
- Quantization
- QCD as Gauge Theory
- Parton Model

2. PERTURBATIVE QCD

- Renormalization
- Basic Idea
- Asymptotic Freedom

3. INFRARED SAFE OBSERVABLES

- KLN Theorem
- Jet
- e^+e^- Process

4. DEEP INELASTIC SCATTERING

- Factorization
 - Parton Distribution Function and OPE
-

1. PROLOGUE

♠ Colored Quark Model

1. History

Sakata Model $\rightarrow$ $SU_f(3)$ Classification
Eightfoldway of Hadrons

$$\underline{3} = (p, n, \Lambda)$$

QUARK Model by Gell-Mann and Zweig

$$\underline{3} = (u, d, s)$$

Quark Model tells us:

Meson $\leftarrow$ 2-quark Bound State

$$(\underline{3} \times \underline{3}^* = \underline{1} + \underline{8})$$

Baryon $\leftarrow$ 3-quark Bound State

$$(\underline{3} \times \underline{3} \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10})$$

e.g.

$$\pi^+ = u\bar{d} \quad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad \pi^- = \bar{u}d \quad \dots$$

$$p = uud \quad n = udd \quad \Sigma = uus \quad \Omega^- = s_{\uparrow}s_{\uparrow}s_{\uparrow} \quad \dots$$

2. Statistics

QUARKs are Fermions $\leftrightarrow$ Pauli Principle

e.g. for $\Omega^- \rightarrow$ Not S -wave

but S -wave from Exp. (*e.g.* Magnetic Moment)

QUARK Should have
an Additional Quantum Number



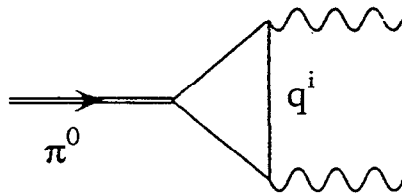
COLOR

For $SU_c(3)$

$$\epsilon_{ijk} q^i q^j q^k$$

is Antisymmetric !!

3. $\pi^0 \rightarrow 2\gamma$



if q is Colorless,

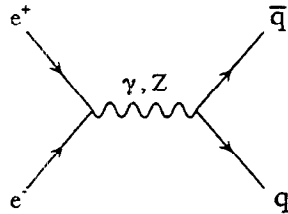
$$\text{Exp.} = 9 \text{ (Theor.)}_{\text{Colorless}}$$

if q is Colored,

$$\text{Theor.} = 3^2 \text{ (Theor.)}_{\text{Colorless}} = \text{Exp.}$$

4. R - ratio

$$R(s) \equiv \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-}$$



if q is Colorless, $R_{\text{Theor.}} = \sum_i Q_i^2$

if q is Colored, $R_{\text{Theor.}} = 3 \sum_i Q_i^2 = R_{\text{Exp.}}$

5. Append.

QUARKs are Confined



Colorless State is only Observable

6. Colored Quark

	$\leftarrow$	$\text{SU}_c(3)$	$\rightarrow$	
1	u d	u d	u d	$\uparrow \text{SU}_w(2)$
2	s c	s c	s c	$\downarrow \text{SU}_w(2)$
3	b t	b t	b t	$\downarrow \text{SU}_w(2)$

♠ QCD as Gauge Theory

Color $\leftrightarrow$ $SU_c(3)$
 $\rightarrow$ $SU_c(3)$ Non – Abelian Gauge Theory

Non-Abelian Gauge Field Theory

Start with Free Dirac Theory

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i \not{\partial} - m)\psi$$

For Matter (quark) field ψ ,

$$\psi \rightarrow \psi' = U\psi = e^{i\theta^a(x)T^a}\psi .$$

where T^a is Representation of Group ($SU(3)$)

$$[T^a, T^b] = if^{abc}T^c ,$$

Introduce Gauge Field A_μ^a

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ig T^a A_\mu^a ,$$

and Define its Transformation such that

$$D_\mu \psi \rightarrow (D_\mu \psi)' = U D_\mu \psi ,$$

$$\begin{aligned} (D_\mu \psi)' &= (\partial_\mu - ig T^a A_\mu^a) \psi' \\ &= U(\partial_\mu + U^{-1} \partial_\mu U - ig U^{-1} T^a A_\mu^a U) \psi , \end{aligned}$$

so

$$T^a A_\mu^a = U T^a A_\mu^a U^{-1} - \frac{i}{g} (\partial_\mu U) U^{-1} ,$$

With these Fields, we get Non-Abelian Gauge Field Theory

$$\mathcal{L} = \bar{\psi}(i \not{D} - m)\psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} ,$$

with

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c ,$$

♠ Quantization

1. General Strategy

PERTURBATION THEORY

- Take Interaction Picture

$$\mathcal{L}(x) \equiv \mathcal{L}_0(x) + \mathcal{L}_I(x)$$

- Quantize Free Part $\mathcal{L}_0$

2. Naïve (Canonical Quantization)

$$\mathcal{L}(x) = \mathcal{L}(\phi(x), \partial_\mu \phi(x))$$

$$\text{Canonical Variable} = \phi(x)$$

$$\text{Canonical Momentum} = \pi(x) \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)}$$

Commutation Relation:

$$\begin{aligned} [\phi(x), \pi(y)]_{x^0=y^0}^\pm &= i\delta(\vec{x} - \vec{y}), \\ [\phi(x), \phi(y)]_{x^0=y^0}^\pm &= [\pi(x), \pi(y)]_{x^0=y^0}^\pm = 0 \end{aligned}$$

3. Complexity for Gauge Field

- A_μ vs Physical Degree of Freedom (= 2)

Gauge Fixing

- Non-Abelian

Faddeev-Popov Ghost

Effective Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= \mathcal{L} + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{gc}} \\
\mathcal{L}_{\text{g.f.}} &= -\frac{1}{2\alpha}(f^a(A))^2 = -\frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2 \\
\mathcal{L}_{\text{gc}} &= -\bar{c}_a \frac{\partial f^a}{\partial A_\mu^b} D_\mu^{bd} c_d = \partial^\mu \bar{c}_a D_\mu^{bd} c_d
\end{aligned}$$

4. S-Matrix, Feynman Rule e.t.c.

- Dyson's S-Matrix

$$\begin{aligned}
|t = +\infty\rangle &\equiv S|t = -\infty\rangle \\
S &= T\left(i \int_{-\infty}^{\infty} d^4x \mathcal{L}_I(x)\right)
\end{aligned}$$

- Unitarity

$$S_{fi} = \langle f|S|i\rangle \equiv \delta_{fi} + i(2\pi)^4 \delta^4(p_f - p_i) T_{fi}$$

$$\text{Insert } SS^\dagger = S^\dagger S = 1$$

$$T_{fi} - T_{fi}^\dagger = i(2\pi)^4 \sum_n \delta^4(p_f - p_n) T_{fn} T_{ni}^\dagger$$

When $f = i$

$$2\text{Im}T_{ii} = (2\pi)^4 \sum_n \delta^4(p_i - p_n) |T_{in}|^2$$

- Cross section for $p_1 + p_2 \rightarrow k_1 + k_2 + \dots + k_n$

Define

$$M_{fi} \equiv \prod_l (2\pi)^{3/2} \sqrt{2p_l^0} \prod_j (2\pi)^{3/2} \sqrt{2k_j^0} T_{fi}$$

with

$$s_{ij}^\pm = s - (m_i \pm m_j)^2, \quad s = (p_1 + p_2)^2$$

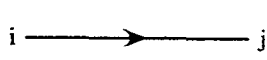
$$d\sigma = \frac{1}{2\sqrt{s_{12}^+ s_{12}^-}} (2\pi)^4 \delta^4\left(\sum_j k_j - p_1 - p_2\right) |M_{fi}|^2 \prod_j \frac{d^3k_j}{(2\pi)^3 2k_j^0}$$

- Feynman Rule

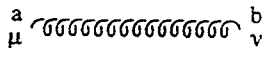
Field ϕ can be Expanded into Plane Waves in Int. Picture

$\Downarrow$

Coefficients : Creation and Annihilation Op.



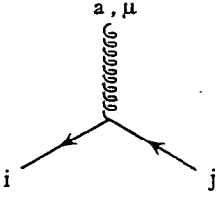
$$\frac{i}{\not{k} - m} \delta_{ij}$$



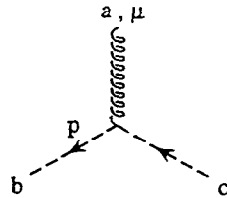
$$\frac{-i}{k^2} \delta^{ab} \left(g_{\mu\nu} - (1 - \alpha) \frac{k_\mu k_\nu}{k^2} \right)$$



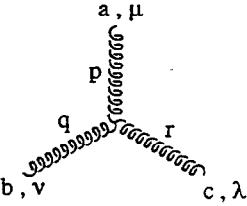
$$\frac{-i}{k^2} \delta^{ab}$$



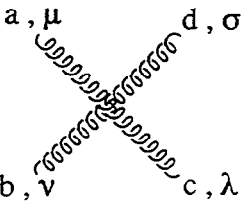
$$i g \gamma^\mu T_{ij}^a$$



$$g f^{abc} p^\mu$$



$$g f^{abc} \{ g_{\mu\nu} (p - q)_\lambda + g_{\nu\lambda} (q - r)_\mu + g_{\lambda\mu} (r - p)_\nu \}$$



$$\begin{aligned} & - i g^2 f^{abe} f^{cde} (g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}) \\ & - i g^2 f^{ace} f^{bde} (g_{\mu\nu} g_{\lambda\sigma} - g_{\mu\sigma} g_{\nu\lambda}) \\ & - i g^2 f^{ade} f^{bce} (g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\nu} g_{\sigma\lambda}) \end{aligned}$$

External Line

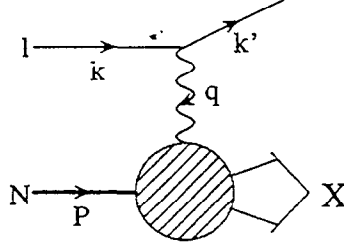
$$\leftrightarrow \bar{u}(p), u(p), \bar{v}(p), v(p), \epsilon_\mu(k)$$

♠ Parton Model

1. Deep Inelastic Scattering

Lorentz Invariants:

$$\begin{aligned} Q^2 &= -q^2 = -(k - k')^2 \\ P \cdot q &= M\nu \\ x &= \frac{1}{\omega} = \frac{Q^2}{2P \cdot q} \end{aligned}$$



Cross section:

$$k'_0 \frac{d\sigma}{d^3k'} = \frac{\pi}{4k \cdot P} e^4 \sum_X |M_{fi}|^2 \delta^4(P_X - P - q)$$

where

$$M_{fi} \equiv \langle eX | M | eN \rangle = \langle k' | j_\mu(0) | k, s \rangle \langle X | J_\nu(0) | PS \rangle D^{\mu\nu}(q)$$

Leptonic tensor:

$$L_{\mu\nu} = \frac{1}{2} \sum_{s'} \langle k, s | j_\mu(0) | k', s' \rangle \langle k', s' | j_\nu(0) | k, s \rangle$$

Hadronic tensor:

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2\pi} \sum_X \langle PS | J_\mu(0) | X \rangle \langle X | J_\nu(0) | PS \rangle (2\pi)^4 \delta^4(P_X - P - q) \\ &= \frac{1}{2\pi} \int d^4x e^{-iq \cdot z} \langle PS | J_\mu(0) J_\nu(z) | PS \rangle \\ &= \frac{1}{2\pi} \int d^4x e^{-iq \cdot z} \langle PS | [J_\mu(0), J_\nu(z)] | PS \rangle \end{aligned}$$

so

$$k'_0 \frac{d\sigma}{d^3k'} = \frac{1}{k \cdot P} \left(\frac{e^2}{4\pi Q^2} \right)^2 L^{\mu\nu} W_{\mu\nu}$$

In General

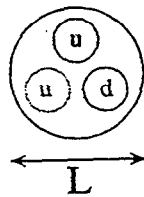
$$W_{\mu\nu} = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1 + \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \frac{2}{P \cdot q} F_2 \\ + 2i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ S^\sigma \frac{1}{P \cdot q} g_1 + (P \cdot q S^\sigma - \dot{q} \cdot S P^\sigma) \frac{1}{(P \cdot q)^2} g_2 \right\}$$

with

$$P \cdot S = 0 \quad , \quad S^2 = -M^2$$

2. Bjorken Scaling

How to see "Constituents = Quarks" Directly ?



Hit by "Light" which has
Wave Length $\ll L$
(Short W.L $\leftrightarrow$ High Energy)

Bjorken Limit:

$$x : \text{fixed} \quad Q^2, P \cdot q \rightarrow \infty$$

Bjorken Scaling:

Structure Functions F_i , (g_i)
depend on Two Invariants

$$F_i = F_i(x, Q^2) = F_i(P \cdot q, Q^2) \\ \rightarrow F_i(x)$$

3. Parton Model

Nucleon is Assembly of
Free Quarks (Partons) at High (Deep) Energy

Note: At High Energy (Short Distance), We will also See
Quantum Effects

So Not Only 3 Quarks but Many

Parton Model (Impulse Approximation)

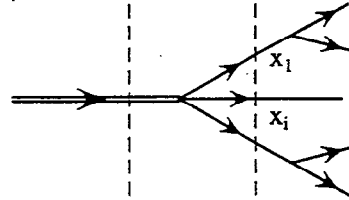


Interaction Time $\tau \sim \frac{1}{q^0} \ll$ Life Time of Partons T

Old Fashioned P.T. in Infinite Momentum Frame

$T^{-1} \sim$ Energy Denominator

$$\sum_i E_i - E_P = \frac{\sum_i [(x_i P)^2 + p_{i\perp}^2 + \mu_i^2]}{-\sqrt{P^2 + M^2}}$$



with $\sum_i x_i = 1$

Take $P \rightarrow \infty$

$$\sum_i E_i - E_P \simeq \sum_i \left[|x_i| P + \frac{p_{i\perp}^2 + \mu_i^2}{2|x_i|P} \right] - P - \frac{M^2}{2P}$$

When all $x_i > 0$, T becomes Max

$$T \sim \frac{2P}{\sum (p_{i\perp}^2 + \mu_i^2) / x_i - M^2} \sim \frac{2P}{M^2} \quad \text{when } p_{\perp} \sim \mu \sim 0$$

Take a Lepton-Nucleon CM frame

$$q = \left(\frac{2P \cdot q - Q^2}{4P}, q_{\perp}, -\frac{2P \cdot q + Q^2}{4P} \right)$$

Then

$$\tau \sim \frac{1}{q^0} = \frac{4P}{2P \cdot q - Q^2}$$

So,

$$\tau \ll T \Rightarrow P \cdot q(1-x) \gg M^2$$

Structure Function in Parton Model

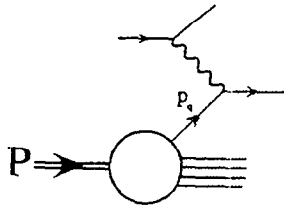
Introduce “Parton Distribution Function”

$$q(\xi)d\xi, \quad p_q = \xi P$$

Cross Section:

$$\sigma(l + N \rightarrow l' + X) = \sum_q \int \sigma_{\text{parton}}(l + p_q \rightarrow l' + X) q(\xi) d\xi$$

with $q = u, \bar{u}, d, \bar{d}, \dots$



Scaling:

$$\sigma_{\text{parton}} \propto \delta((\xi P + q)^2 - (\mu_{\text{parton}}^2 \sim 0))$$

$\Downarrow$

$$x = \xi$$

Structure Function:

For Helicity state of Nucleon, introduce

$$q_{\uparrow}(\xi) \quad \text{and} \quad q_{\downarrow}(\xi)$$

$$q(\xi) \equiv q_{\uparrow}(\xi) + q_{\downarrow}(\xi)$$

$$\delta q(\xi) \equiv q_{\uparrow}(\xi) - q_{\downarrow}(\xi)$$

Finally

$$F_1(x) = \frac{1}{x} F_2(x) = \sum_q e_q^2 q(x)$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \delta q(x)$$

2. PERTURBATIVE QCD

♣ Renormalization

1. Lagrangian and Physics

Two Interrogation about “Lagrangian”

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_0(i \not{D}_0 - m_0)\psi_0 - \frac{1}{4}F_{0\mu\nu}^a F_0^{a\mu\nu} + \dots \\ &= \bar{\psi}_0(i \not{\partial} - m_0)\psi_0 + g_0 \bar{\psi}_0 A_0^a T^a \psi_0 - \frac{1}{4}F_{0\mu\nu}^a F_0^{a\mu\nu} + \dots\end{aligned}$$

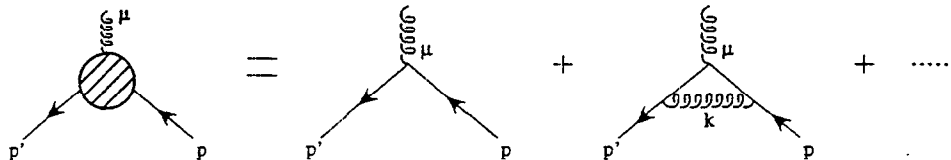
- Normalization of $\mathcal{L}$? Wrong with $2\mathcal{L}$?
- What are m_0 and g_0 ?

Both were Fixed at Classical (Tree) Level ₀
but

m and g are Parameters Determined from Exp.

Exp. Value $\neq$ Tree Level

e.g.



$$ig_0 \Gamma^\mu(p, p') T_{ij}^a = ig_0 \gamma^\mu T_{ij}^a + ig_0^3 \gamma^\mu f(p, p') T_{ij}^a + \dots$$

2. Renormalization

One MUST Define Coupling Constant g

Corresponding to an Experiment, at $p^2 = p'^2 = \mu^2$

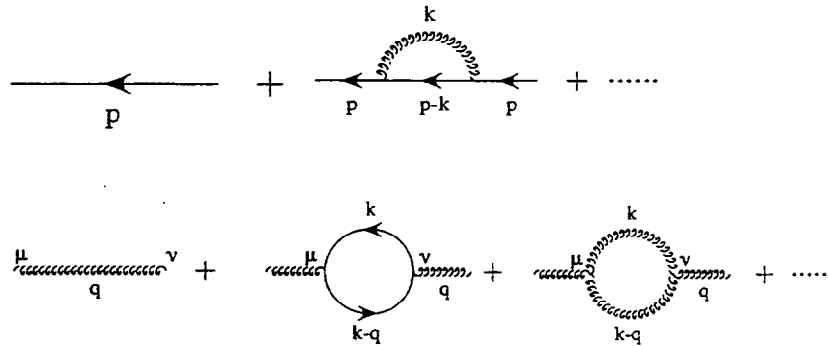
$$g(\mu) \equiv g_0 + g_0^3 f(p^2 = p'^2 = \mu^2) + \dots$$

$\Downarrow$

$$g_0 = Z_g(\mu)g(\mu)$$

“Physical” Coupling Const. $g(\mu)$
Depend on (arbitrary) Scale μ

The Same for m and Normalization



For Quark (Gluon) :

Residue and (Pole Position) of Propagator
Change from Tree Level

$\Downarrow$

Renormalization

$$\begin{aligned}\psi_0 &= Z_2^{1/2} \psi, \quad A_0^{a\mu} = Z_3^{1/2} A^{a\mu} \\ g_0 &= Z_g g, \quad m_0 = Z_m m, \quad \dots\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= Z_2 \bar{\psi}(i \not{\partial} - Z_m m) \psi + Z_g Z_2 Z_3^{1/2} g \bar{\psi} \not{A}^a T^a \psi \\ &\quad - \frac{Z_3}{4} F_{\mu\nu}^a F^{a\mu\nu} + \dots \\ &= \bar{\psi}(i \not{\partial} - m) \psi + g \bar{\psi} \not{A}^a T^a \psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \\ &\quad + (Z_2 - 1) \bar{\psi}(i \not{\partial} - m) \psi + Z_2(1 - Z_m) m \bar{\psi} \psi \\ &\quad + (Z_g Z_2 Z_3^{1/2} - 1) g \bar{\psi} \not{A}^a T^a \psi - \frac{1}{4} (Z_3 - 1) F_{\mu\nu}^a F^{a\mu\nu} + \dots\end{aligned}$$

Z_i should be Determined by some Condition

||

Renormalization Condition

Z_i depend on $\mu =$ Renormalization Scale

n.b. Z_i are in General Ultraviolet Divergent

3. Renormalization Group Equation

Physical Quantity *e.g.* Cross Section

CAN NOT Depend on μ

$$\mu \frac{d}{d\mu} \sigma(p, \mu) = 0$$

♣ Asymptotic Freedom

1. Anti-Screening

- μ Dependence of $g(\mu)$

Remember

$$g_0 = Z_g(\mu)g(\mu)$$

so

$$\mu \frac{d}{d\mu} g_0 = \left(\mu \frac{d}{d\mu} Z_g(\mu) \right) g(\mu) + Z_g(\mu) \mu \frac{d}{d\mu} g(\mu) = 0$$

i.e.

$$\mu \frac{d}{d\mu} g(\mu) = -g(\mu) \mu \frac{d}{d\mu} \ln Z_g(\mu) \equiv \beta(g)$$

- Asymptotic Freedom

For QCD

$$\begin{aligned} \beta(g) &= -\beta_0 g^3 - \beta_1 g^5 - \dots \\ &= -\frac{g^3}{(4\pi)^2} \frac{1}{3} (33 - 2n_f) - \mathcal{O}(g^5) \end{aligned}$$

This Means

when $\mu \rightarrow \text{Large}$, $g(\mu) \rightarrow \text{Small}$

if $\beta_0 > 0$ i.e. $33 - 2n_f > 0$

- Physics

QED v.s. QCD

Photon Does NOT have Charge

Gluon DOES have Charge

2. QCD Coupling Constant

$$\mu \frac{d}{d\mu} g(\mu) = \beta(g) = -\beta_0 g^3 - \beta_1 g^5 - \dots$$

- 1-loop:

$$\frac{1}{2} \ln \frac{Q^2}{\mu^2} = \frac{1}{2\beta_0} \left[\frac{1}{g^2(Q^2)} - \frac{1}{g^2(\mu^2)} \right]$$

Now Introduce μ Independent Parameter Λ_1

$$\Lambda_1 = \mu \exp \left(-\frac{1}{2\beta_0 g^2(\mu^2)} \right)$$

Then

$$g^2(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_1^2)}$$

- 2-loop:

Now

$$\Lambda_2 = \mu \exp \left(-\frac{1}{2\beta_0 g^2(\mu^2)} \right) (\beta_0 g^2(\mu^2))^{-\beta_1/2\beta_0^2}$$

Then

$$g^2(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_2^2)} - \frac{\beta_1 \ln(\ln(Q^2/\Lambda_2^2))}{\beta_0^3 \ln^2(Q^2/\Lambda_2^2)} + \dots$$

- QCD Parameter

Λ is the Only Parameter of QCD

except Quark Mass

R.G.E. $\rightarrow \mu$ and $g(\mu)$ are not Independent

3. Scheme Dependence

Explicit Value of Parameter $g, m, \dots$
Depend on Renormalization Condition
||
Called Scheme Dependence

- Definition of Λ

There are some arbitrariness in defining Z_g

$$g_0 = Z_g g = Z'_g g'$$

Relation between g and g'

$$g^2 = (Z'_g/Z_g)^2 g'^2 \equiv z_g^2 g'^2 = (1 + z_2 g'^2 + z_4 g'^4 + \dots) g'^2$$

This means QCD Λ Depend on Scheme

1-loop Example:

$$\begin{aligned} g^2(Q^2) &= \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} = g'^2(Q^2)(1 + z_2 g'^2(Q^2) + \dots) \\ &= \frac{1}{\beta_0 \ln(Q^2/\Lambda'^2)} \left[1 + \frac{z_2}{\beta_0 \ln(Q^2/\Lambda'^2)} \right] \end{aligned}$$

$$i.e. \quad \Lambda = \Lambda' e^{z_2/(2\beta_0)}$$

Commonly Used Scheme:

MS : Z_i has only $1/\epsilon$ in $D = 4 - 2\epsilon$

$\overline{\text{MS}}$: Z_i has $1/\epsilon - \gamma_E + \ln 4\pi$ in $D = 4 - 2\epsilon$

MOM : Z_i in Momentum Subtraction Scheme

♣ Basic Idea

1. General Strategy

In QCD When $\mu \rightarrow \infty$, $g(\mu) \rightarrow 0$

What will Happen ?
if we apply Perturbative Expansion
from the Beginning
to Quark and Gluon System

In Real World, Quark and Gluon are Confined

↑

Relation to Real Observable will be considered Later

Consider "Observable" in Quark and Gluon System

$$\mu \frac{d}{d\mu} \sigma \left(\frac{Q^2}{\mu^2}, \frac{p^2}{\mu^2}, \dots \right) = \left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \sigma = 0$$

Q = Typical Mom. , p = Mom. of External Particle

μ = Renormalization Scale

Neglect Mass Effects for Simplicity

Formal Answer:

$$\sigma \left(\frac{Q^2}{\mu^2}, \frac{p^2}{\mu^2}, \dots, g(\mu^2) \right) = \sigma \left(1, \frac{p^2}{Q^2}, \dots, g(Q^2) \right)$$

p^2 Should be Considered as

$$p^2 = m_{\text{quark}}^2 \simeq (\text{hadron scale})^2 \simeq 0 ,$$

- Scheme Dependence in Observables

Observable like Cross section
Should NOT depend on Scheme

$$\mathcal{S}(g) = \mathcal{S}'(g')$$

Expand in Perturbation Series

$$\begin{aligned}\mathcal{S}(g) &= s_0 + s_2 g^2 + s_4 g^4 + \dots \\ \mathcal{S}'(g') &= s'_0 + s'_2 g'^2 + s'_4 g'^4 + \dots\end{aligned}$$

Rewrite $\mathcal{S}(g)$ using Relation g and g'

$$\mathcal{S}(g) = s_0 + s_2 g'^2 + (s_4 + z_2 s_2) g'^4 + \dots$$

This Means

$$s_0 = s'_0 \quad , \quad s_2 = s'_2$$

s_{2n} ($n \geq 2$) Depend on Scheme
and

$$[\mathcal{S}'(g')]_{2n} - [\mathcal{S}(g)]_{2n} = \mathcal{O}(g^{2n+2})$$

$\Downarrow$

Perturbative Prediction for Observable
Depend on Scheme !!

Now, Perturbatively Expand when $Q^2 \sim \text{Large}$

$$\sigma \left(1, \frac{p^2}{Q^2}, g(Q^2) \right) = 1 + ag^2(Q^2) \ln \left(\frac{Q^2}{p^2} \right) + bg^2(Q^2) + \mathcal{O}(g^4)$$

Problem: Secon Term, if

$$\ln \left(\frac{Q^2}{p^2} \right) \gg 1 \quad \text{then} \quad g^2(Q^2) \ln \left(\frac{Q^2}{p^2} \right) \geq 1$$

Perturbative Expansion Breaks Down

Singularity in $p^2 = m^2 \rightarrow 0$ is Called

Mass Singularity

Related to Long Distance (Low Momentum) Behavior
of QCD

Two Cases to Overcome this Problem

- Factorization

$$\begin{aligned} \sigma &= 1 + ag^2 \ln \left(\frac{Q^2}{p^2} \right) + \dots \\ &= \left[1 + ag^2 \ln \left(\frac{Q_0^2}{p^2} \right) + \dots \right] \left[1 + ag^2 \ln \left(\frac{Q^2}{Q_0^2} \right) + \dots \right] \end{aligned}$$

- Singularity Free Case

$$a = 0$$

2. Sketch of Factorization

Consider DIS

Start with Parton Model (Parton Ansatz) $q^0(z)$

$$F(x, Q^2) = \int_x^1 \frac{dz}{z} F^0\left(\frac{x}{z}\right) q^0(z)$$

F^0 is Parton Structure Func.

$$F^0(\xi) = \delta(\xi - 1) + a(\xi)g^2 \ln\left(\frac{Q^2}{p^2}\right) + \dots ,$$

Take Moment (for Simplicity)

$$\begin{aligned} \int dx x^{n-1} F(x, Q^2) &= \int dz z^{n-1} q^0(z) \int dy y^{n-1} F^0(y, Q^2) \\ &\equiv M_n \left[1 + a_n g^2 \ln\left(\frac{Q^2}{p^2}\right) + \dots \right] \end{aligned}$$

with

$$M_n = \int dz z^{n-1} q^0(z) \quad , \quad a_n = \int dy y^{n-1} a(y) \quad ,$$

$\Downarrow$

$$\int dx x^{n-1} F(x, Q^2) = M_n(Q_0^2) \left[1 + a_n g^2 \ln\left(\frac{Q^2}{Q_0^2}\right) + \dots \right]$$

where

$$M_n(Q_0^2) \equiv M_n \left[1 + a_n g^2 \ln\left(\frac{Q_0^2}{p^2}\right) + \dots \right]$$

3. Interpretation and Comment

- Interpretation

- 1st Case

- Factorized Second Term Should
be a Prediction

- 2nd Case

- NOT Need Long Distance Information
So, σ itself Can be a Prediction

- Comment

- Two (Technically Different) Approach to

$$\text{Large Log} \quad \ln \left(\frac{Q^2}{p^2} \right)$$

- 1st Approach

$$p^2 \rightarrow 0 \quad \text{with} \quad Q^2 = \text{fixed}$$

$$\Downarrow$$

Factorization of Mass Singularity

- 2nd Approach

$$Q^2 \rightarrow \infty \quad \text{with} \quad p^2 = \text{fixed}$$

$$\Downarrow$$

Renormalization of U.V.Singularity

$$\Downarrow$$

Operator Product Expansion

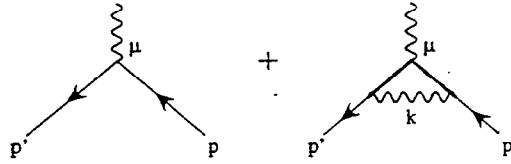
3. INFRARED SAFE OBSERVABLES

♠ KLN Theorem

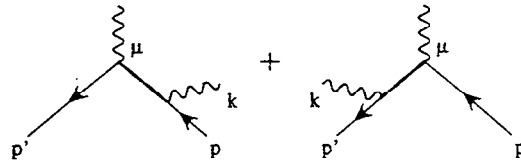
1. BN (Bloch-Nordsieck) Theorem

Ultraviolet Sing. $\leftrightarrow$ “Unphysical” Renormalization
 Infrared Sing. $\leftrightarrow$ “Physical” Massless Particle

e.g. for QED at 1-Loop



$$\sim e^2 \left\{ 2 \left(\ln \frac{-q^2}{m^2} - 1 \right) \ln \frac{\lambda^2}{m^2} - \ln^2 \frac{-q^2}{m^2} + 3 \ln \frac{-q^2}{m^2} + \dots \right\}$$



$$\sim e^2 \left\{ 2 \left(\ln \frac{-q^2}{m^2} - 1 \right) \ln \frac{4\omega_{\max}^2}{\lambda^2} - \ln^2 \frac{-q^2}{m^2} + 2 \ln \frac{-q^2}{m^2} + \dots \right\}$$

λ = Photon Mass , $\omega_{\max}$ = Max Energy of Photon

When $k \rightarrow 0$, Propagator — becomes “On-Shell”

Infrared Sing. Cancel in the Sum
 $\updownarrow$
 Zero Mom. Emission = No Emission
 $\Downarrow$
 Should Sum Degenerate States in Energy

2. KLN (Kinoshita-Lee-Nauenberg) Theorem

When $m \rightarrow 0$, Singularity $\ln m^2$ Remains !!

$\downarrow$
 Mass Singularity (in Narrow Sense)

$\uparrow$
 Massless Particle CAN DECAY
 into TWO On-Shell Particles

$$p \rightarrow k + l \quad \text{with} \quad k \neq l \neq 0 \quad \vec{k}/\vec{l}$$

Infrared Sing. + Mass Singularity (in Narrow Sense)

$\parallel$
 Mass Sing.

QED vs QCD

- $\alpha_{\text{QED}} \sim 1/137$ vs $\alpha_{\text{QCD}} \sim 1/10$
- Gluons Interact themselves

KLN Theorem

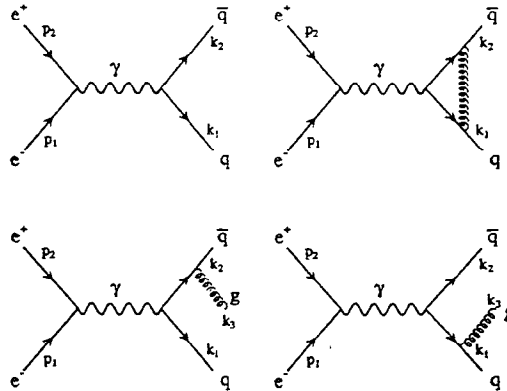
Mass Singularity Cancel when All Degenerate States in Energy are Summed over Initial and Final States

♠ e^+e^- Process

1. R - ratio

$$R(s) \equiv \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-}$$

- No QCD in the Initial State
- Total Cross Section
i.e. Sum up All States in the Final States
- No Mass Singularity



Tree Level

$$\sigma_0 = \frac{4\pi\alpha^2}{3s} N_c \sum_i Q_i^2$$

One-Loop Level

$$\sigma = \sigma_0 \left(1 + \frac{\alpha_s}{\pi}\right)$$

i.e.

$$R = N_c \sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi}\right)$$

2. Scheme Dependence and How to Determine Λ

$$R = N_c \sum_i Q_i^2 \left[1 + \frac{\alpha_s(Q^2)}{\pi} + B \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 + \dots \right]$$

B has been calculated in MS scheme

$$B = \frac{C_F}{4} \left[\frac{123}{8} N_c \frac{3}{8} C_F - \frac{11}{4} f + (11N_c - 2f) \left\{ -\zeta(3) + \frac{1}{4} (\ln 4\pi - \gamma_E) \right\} \right]$$

where

$$\zeta(3) = 1.202\dots, \quad \gamma_E = 0.5772\dots$$

Remove $\ln 4\pi - \gamma_E$ in $\overline{\text{MS}}$ scheme

When $N_c = 3$

$$B = \begin{cases} 7.36 - 0.44f & (\text{MS}) \\ 1.99 - 0.12f & (\overline{\text{MS}}) \end{cases}$$

Determination of Λ

Suppose One Determines Λ from Exp. at LO

$$R = N_c \sum_i Q_i^2 \left(1 + \frac{\alpha_s(Q^2/\Lambda_{\text{Exp.}}^2)}{\pi} \right)$$

Scheme Dependence Tells

$$\Lambda_B = \Lambda_{\text{Exp.}} \exp \left(-\frac{B}{8\pi^2\beta_0} \right),$$

e.g.

$$\Lambda_{\text{MS}} = 0.260\Lambda_{\text{Exp.}}, \quad \Lambda_{\overline{\text{MS}}} = 0.696\Lambda_{\text{Exp.}}$$

$\Downarrow$

Need NLO Results to Determine Λ !!

♠ Jet

1. Sterman-Weinberg Jet

Jet: Correlated Hadron's Beam in some Direction

Jet Definition by Sterman-Weinberg

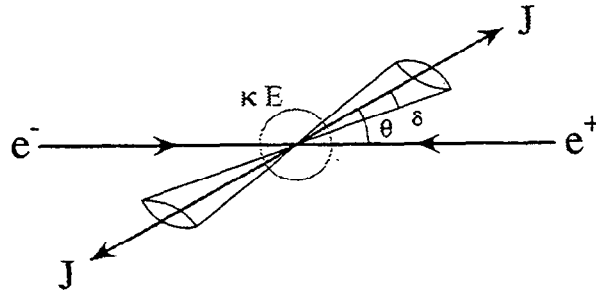


Direct Application of KLN Theorem

Definition of 2-Jet Production:

$$\sigma(\sqrt{s}, \Omega, \kappa, \delta)$$

Cross Section for Energy $\sqrt{s} \geq E \geq (1 - \kappa)\sqrt{s}$
Emitted into the Cone with Angle δ



Tree Level

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4s} N_c \sum_i Q_i^2 (1 + \cos^2 \theta)$$

One-Loop Level

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha_s}{\pi} C_F \left(4 \ln \delta \ln(2\kappa) + 3 \ln \delta + \frac{\pi^2}{3} - \frac{5}{2} \right) \right]$$

2. Thrust and Sphericity

Other Physical Quantities which are Free from M.S.

$$T = 2 \max \left(\frac{\sum_i' p_{||}^i}{\sum_i |\mathbf{p}^i|} \right)$$
$$S = \left(\frac{4}{\pi} \right)^2 \min \left(\frac{\sum_i |\mathbf{p}_T^i|}{\sum_i |\mathbf{p}^i|} \right)^2$$

where

$\mathbf{p}^i$ is momentum of i

Meaning of Sum:

Fix some Axis and take Sum over Produced Hadrons

↓

Move Axis such that $T(S)$ becomes Max (Min)

↓

This Axis is Called “Jet Axis”

S, T take

$$1 \geq T \geq \frac{1}{2} \quad , \quad 0 \leq S \leq 1 \quad ,$$

where

l.h.Limit = 2 – Jet Event , r.h.Limit = Spherical Event

- T, S DOES NOT Discriminate Degenerate States
- Mass Singularity Free

4. DEEP INELASTIC SCATTERING

♣ Factorization

1. Strategy and Scenario

$$\begin{aligned}
 F(x, Q^2) &= \int_x^1 \frac{dz}{z} F^0\left(\frac{x}{z}; (Q^2, p^2)\right) q^0(z; (p^2)) \\
 &\rightarrow \int_x^1 \frac{dz}{z} C\left(\frac{x}{z}; \alpha_s(\mu^2), \frac{Q^2}{\mu^2}\right) q(z, \mu^2)
 \end{aligned}$$

with C and q being Finite and $\mu^2 \gg \Lambda_{\text{QCD}}^2$

What we Should (Want to) Show

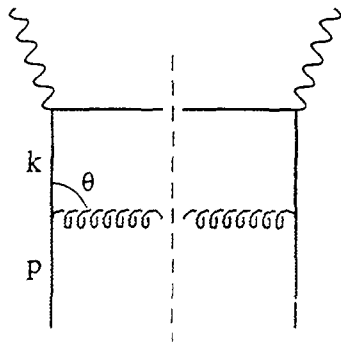
$$F^0 = C \otimes \Gamma, \quad q = \Gamma \otimes q^0$$

$\otimes$ = Complete Sum

Mass Singularity is Factorized into Γ

2. Preliminaries

• F^0 at 1-Loop Example



Mass Sing. occurs when

$$p^2 \rightarrow 0$$

$\Downarrow$

$$\theta \rightarrow 0 \Rightarrow k^2 \rightarrow 0$$

- Technical Points

Light-Cone Parametrization

$$p^\mu \equiv p(1, \mathbf{0}_\perp, 1), \quad n^\mu \equiv (1/2p)(1, \mathbf{0}_\perp, -1)$$

Sudakov Parametrization for k

$$k^\mu = zp^\mu + (k^2 + \mathbf{k}_\perp^2)/2x \, n^\mu + k_\perp^\mu \equiv k_\parallel^\mu + \dots$$

Regularization of M.S. (Dimensional)

$$D = 4 + 2\epsilon \quad , \quad g^2 \rightarrow g^2 \mu^{-2\epsilon}$$

$$\ln(Q^2/p^2) \rightarrow 1/\epsilon + \ln(Q^2/\mu^2) \quad \text{when } p^2 \rightarrow 0$$

Integration Measure

$$d^D k = \frac{dz}{2z} dk^2 d^{D-2} \mathbf{k}_\perp = \frac{dz}{4z} dk^2 d\Omega_{2+2\epsilon} dk_\perp^2 (k_\perp^2)^\epsilon$$

- Axial Gauge and Important Facts

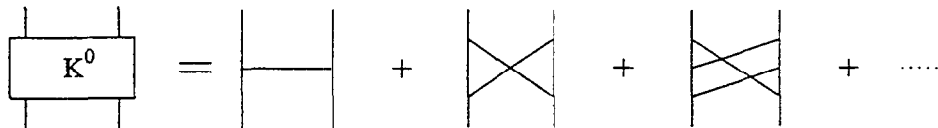
Axial gauge

$$D_{\mu\nu}(l) = i/l^2 \quad [-g_{\mu\nu} + (l_\mu n_\nu + l_\nu n_\mu)/n \cdot l]$$

Facts without Proof

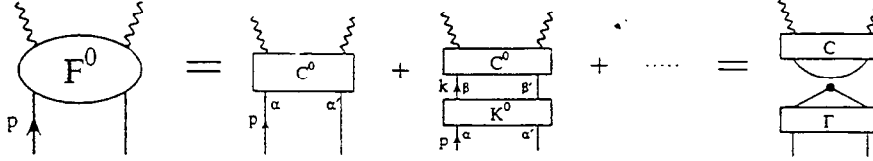
2-P-I Amplitudes are Finite
when External Mom. are Kept

2PI = Un-Separated when t-channel 2 Lines are Cut



3. Factorization

- Generalized Ladder in Axial Gauge



$$\begin{aligned}
 F^0 &= \left[C^0 (1 + K^0 + K^0 K^0 + \dots) \right]_{\alpha\alpha'} u^\alpha(p) \bar{u}^{\alpha'}(p) Z_F \\
 &= \left[C^0 \frac{1}{1 - K^0} \not{p} \right] Z_F
 \end{aligned}$$

- Suppress $\otimes$ ($= d^D k$, Sum over Spinor indices, Color)
- K^0 (C^0) Contains Upper lines (Propagators)
- $A_{\alpha\alpha'}^{**} \not{p}^{\alpha\alpha'} \equiv A \not{p}$, $\not{p}^{\alpha\alpha'} A_{\alpha\alpha'}^{**} \equiv [\not{p} A$

- Projection of Mass Singularity

We MUST Decouple both
in Spin Indices and k Integral
to Factorize M.S. (Pole in ϵ)

$$\mathcal{P} = \mathcal{P}^s \otimes \mathcal{P}^\epsilon$$

Consider *e.g.* the Term

$$\begin{aligned}
 K^0 \not{p} &= \frac{1}{(k^2)^2} (\text{dim. 1 func.}) \\
 &= \frac{1}{(k^2)^2} (A \not{k}_\parallel + B \not{k}_\perp + C \not{k}^2 + \dots)
 \end{aligned}$$

with $A, \dots$ being Dimensionless $A = A(k^2/\mu^2, x, \epsilon), \dots$

Note: $[n] = M^{-1}$ and $k_\parallel = xp$

Integration over k

$$\int d^D k \, K^0 \not{k} = \int \frac{dz}{4z} d\Omega_{2+2\epsilon} \\ \times \int_0^{\mathcal{O}(-Q^2)} dk^2 \int_0^{-(1-z)k^2} dk_\perp^2 \left(\frac{k_\perp^2}{\mu^2} \right)^\epsilon \frac{1}{(k^2)^2} (A \not{k}_\parallel + \dots)$$

- Upper Limit of k^2 Can be Any Scale
Since M.S. Comes from $k^2 \sim 0$
- Upper Limit of $k_\perp^2 \leftarrow (p-k)^2 > 0$

Only A Term and Region $k^2 \sim 0$ in Integral
Produces Mass Singularity

$\Downarrow$

$$\mathcal{P} K^0 \not{k} = \mathcal{P}^s \otimes \mathcal{P}^\epsilon K^0 \not{k} \\ = \int \frac{dz}{2z} \not{k} \Big|_{k^2=0} \int \frac{dk^2 d^{2+2\epsilon} \mathbf{k}_\perp}{(2\pi)^D} \left[\frac{\not{n}}{4k \cdot n} K^0 \not{k} \right] \\ = \int \frac{dz}{z} \not{k} \Big|_{k^2=0} z \int \frac{d^D k}{(2\pi)^D} \delta(z - k \cdot n) \left[\frac{\not{n}}{4k \cdot n} K^0 \not{k} \right]$$

Extension to Two Kernels A, B Connected by k

$$A \mathcal{P} B = A \mathcal{P}^s \otimes \mathcal{P}^\epsilon B \\ = A \not{k} \Big|_{k^2=0} \mathcal{P}^\epsilon \left[\frac{\not{n}}{4k \cdot n} B \right]$$

- $k^2 = 0 \rightarrow k = zp$
- $\mathcal{P}^\epsilon \left[\frac{\not{n}}{4k \cdot n} B \right]$ Extracts Poles in ϵ i.e. M.S.
- Def. of $\mathcal{P}^\epsilon$ is NOT Unique

Poles + *** $\rightarrow$ Factorization Scheme Dep.

- Factorization

$$M \equiv C^0(1 + K^0 + K^0 K^0 + \dots)$$

♡ Proof is Iterative

First Factorize Last Kernel

$$\begin{aligned} M &= C^0 \left[1 + \sum_{i=1}^{\infty} (K^0)^{i-1} \mathcal{P} K^0 + \sum_{i=1}^{\infty} (K^0)^{i-1} (1 - \mathcal{P}) K^0 \right] \\ &= C^0 \left[1 + \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P}) K^0 \right] + M \mathcal{P} K^0 \end{aligned}$$

So

$$M(1 - \mathcal{P} K^0) = C^0 \left[1 + \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P}) K^0 \right]$$

Next Factorize $K^0(1 - \mathcal{P})K^0$ in r.h.s.

$$\begin{aligned} &M(1 - \mathcal{P} K^0 - \mathcal{P}(K^0(1 - \mathcal{P})K^0)) \\ &= C^0 \left[1 + (1 - \mathcal{P})K^0 \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P})(K^0(1 - \mathcal{P})K^0) \right] \end{aligned}$$

Finally

$$M(1 - \mathcal{P} K) = C^0 \frac{1}{1 - (1 - \mathcal{P})K^0}$$

where

$$K \equiv K^0 \frac{1}{1 - (1 - \mathcal{P})K^0}$$

or

$$M = \left(C^0 \frac{1}{1 - (1 - \mathcal{P})K^0} \right) \left(\frac{1}{1 - \mathcal{P} K} \right)$$

♡ Final Result

$$\begin{aligned}
F^0(x, Q^2, (\alpha_s)_{\text{bare}}, 1/\epsilon) &= [M \not{p}] Z_F \\
&= \left[\left(C^0 \frac{1}{1 - (1 - \mathcal{P})K^0} \right) \not{p} \right. \\
&\quad \left. + \left(C^0 \frac{1}{1 - (1 - \mathcal{P})K^0} \right) \mathcal{P} K \frac{1}{1 - \mathcal{P}K} \not{p} \right] Z_F \\
&= \int \frac{dz}{z} \left[\left(C^0 \frac{1}{1 - (1 - \mathcal{P})K^0} \right) \not{k} \right]_{k^2=0} \\
&\quad \times Z_F \left\{ \delta(1 - z) \right. \\
&\quad \left. + \int \frac{d^D k}{(2\pi)^D} \delta(z - k \cdot n) z \left[\frac{\not{n}}{4 k \cdot n} K \frac{1}{1 - \mathcal{P}K} \not{p} \right] \right\} \\
&= \int \frac{dz}{z} C \left(\frac{x}{z}; \alpha_s(\mu^2), \frac{Q^2}{\mu^2} \right) \Gamma \left(z, \alpha_s, \frac{1}{\epsilon} \right)
\end{aligned}$$

4. Renormalization Group Equation

- Q^2 (μ^2) Independence of Γ

Possible Q^2 Dependence

$\Uparrow$

$$\int_0^{\mathcal{O}(-Q^2)} dk^2$$

Note: $\mathcal{P}$ acts on K

$$K = K^0 [1 + (1 - \mathcal{P})K^0 + (1 - \mathcal{P})(K^0(1 - \mathcal{P})K^0 + \dots)]$$

$\Downarrow$

Only Last Integral is Divergent

So with Finite Function Φ

$$\begin{aligned} & \Gamma\left(z, \alpha_s, \frac{1}{\epsilon}\right) \\ &= Z_F \left[\delta(1-z) + \int_0^{\mathcal{O}(-Q^2)} \frac{dk^2}{k^2} \Phi(k^2/\mu^2, z, \epsilon=0) \right] \\ &= Z_F \left[\delta(1-z) + \frac{1}{\epsilon} \Gamma_1(z, \alpha_s) + \frac{1}{\epsilon^2} \Gamma_2(z, \alpha_s) + \dots \right] \end{aligned}$$

By Differentiating Both Sides,

$$\frac{\partial}{\partial Q^2} \Gamma_i(z, \alpha_s) = 0$$

Namely

$$\frac{\partial}{\partial Q^2} \Gamma\left(z, \alpha_s, \frac{1}{\epsilon}\right) = 0 = \left[\frac{\partial}{\partial \mu^2} \Gamma\left(z, \alpha_s, \frac{1}{\epsilon}\right) \right]$$

- Renormalization Group Equation

F^0 (Bare) Physical Quantity
CAN NOT Depends on μ

$$\mu \frac{d}{d\mu} F^0 = \left(\mu \frac{\partial}{\partial \mu} + \beta(g, \epsilon) \frac{\partial}{\partial g} \right) F^0 = 0$$

Take Moment of F^0 for Simplicity

$$F^0\left(N, Q^2, (\alpha_s)_{\text{bare}}, \frac{1}{\epsilon}\right) = C\left(N, \frac{Q^2}{\mu^2}, \alpha_s\right) \Gamma\left(N, \alpha_s, \frac{1}{\epsilon}\right)$$

with

$$f(N) \equiv \int_0^1 dx x^{N-1} f(x)$$

Renormalization Group Equation

$$0 = \mu \frac{d}{d\mu} \ln F^0(N) = \mu \frac{d}{d\mu} \ln C(N) + \mu \frac{d}{d\mu} \ln \Gamma(N)$$

i.e.

$$\left[\mu \frac{d}{d\mu} - \gamma(N, \alpha_s) \right] C \left(N, \frac{Q^2}{\mu^2}, \alpha_s \right) = 0$$

with

$$\begin{aligned} C \left(N, \frac{Q^2}{\mu^2}, \alpha_s \right) \\ = C \left(N, 1, \alpha_s(Q^2) \right) \exp \left[- \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right] \end{aligned}$$

where

$$\gamma(N, \alpha) = -\mu \frac{d}{d\mu} \ln \Gamma(N) = -\beta(g, \epsilon) \frac{\partial}{\partial g} \ln \Gamma(N, \alpha_s, 1/\epsilon)$$

and

$$\Gamma \left(N, \alpha_s, \frac{1}{\epsilon} \right) = \exp \left[- \int_0^{\alpha_s(\mu^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right]$$

Note that μ can be Any Value

5. Final Step and DGLAP Equation

- Dressed Parton Distribution Function

For the Hadron Target, we prepare
“Bare” Distribution Function inside the Hadron

$$q^0 \left(z, \frac{1}{\epsilon} \right) = z \int \frac{d^D p}{(2\pi)^D} \delta \left(z - \frac{p \cdot n}{P \cdot n} \right) \left[\frac{\not{n}}{4 p \cdot n} H \not{p} \right]$$

Define “Dressed” Parton Distribution Function
“Renormalized” at μ^2

$$q(N, \mu^2) = \exp \left[- \int_0^{\alpha_s(\mu^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right] q^0 \left(N, \frac{1}{\epsilon} \right)$$

So

$$\begin{aligned} F(N, Q^2) &= F^0 \left(N, Q^2, (\alpha_s)_{\text{bare}}, \frac{1}{\epsilon} \right) q^0(N) \\ &= C \left(N, \frac{Q^2}{\mu^2}, \alpha_s \right) q(N, \mu^2) \\ &= C(N, 1, \alpha_s(Q^2)) q(N, Q^2) \end{aligned}$$

- DGLAP Equation

Q^2 Evolution of Dressed Parton
Distribution Function

$$Q \frac{d}{dQ} q(N, Q^2) = -\gamma(N, \alpha_s(Q^2)) q(N, Q^2)$$

In x Space

$$Q \frac{d}{dQ} q(x, Q^2) = \int_x^1 \frac{dz}{z} P \left(\frac{x}{z}, \alpha_s(Q^2) \right) q(z, Q^2)$$

with

$$\int_0^1 dx x^{N-1} P(x, \alpha_s(Q^2)) = -\gamma(N, \alpha_s(Q^2))$$

This is DGLAP Equation !!

♣ Parton Distribution Function and OPE

“Modern Def.” of Parton Distribution Function
Parton Model and Twist and OPE

1. Some Notation and Light Cone Dominance Bjorken Limit in Hadronic Tensor

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4x e^{-iq \cdot z} \langle PS | [J_\mu(0), J_\nu(z)] | PS \rangle$$

Only Region $0 \leq z^2 \leq 1/Q^2$ is Important
when $Q^2 \rightarrow \infty$, $x = Q^2/2(P \cdot q)$: fixed

Take Frame $\vec{P} // \vec{q} // \hat{e}_z$ and Introduce

$$p^\mu \equiv \frac{\mathcal{P}}{\sqrt{2}}(1, 0, 0, 1), \quad n^\mu \equiv \frac{1}{\sqrt{2}\mathcal{P}}(1, 0, 0, -1)$$

with $p^2 = n^2 = 0$, $p \cdot n = 1$

$$P^\mu = p^\mu + \frac{M^2}{2}n^\mu \simeq p^\mu$$

$$\lim_{\text{Bj}} q^\mu \simeq \left(P \cdot q + \frac{1}{2}M^2x \right) n^\mu - xp^\mu$$

Writing $z^\mu = \eta p^\mu + \lambda n^\mu + z_\perp^\mu$

$$\lim_{\text{Bj}} q \cdot z \simeq \eta P \cdot q - \lambda x$$

Riemann-Lebesgue Theorem says

$$|q \cdot z| \lesssim 1 \rightarrow |\eta| \lesssim 1/(P \cdot q), \quad |\lambda| \lesssim 1/x$$

Namely with Causality

$$0 \leq z^2 = 2\eta\lambda - z_\perp^2 \leq 2|\eta\lambda| \leq \frac{\text{const}}{Q^2}$$

2. Parton Distribution Function

- “Free Field Theory”

Wick Theorem

$$\begin{aligned} [J_\mu(0), J_\nu(z)] \\ = -(\partial^\alpha \Delta(z)) [S_{\mu\alpha\nu\sigma} \mathcal{O}_V^\sigma(0, z) - i\epsilon_{\mu\alpha\nu\sigma} \mathcal{O}_A^\sigma(0, z)] \end{aligned}$$

with

$$\begin{aligned} J_\mu &= \bar{\psi} \gamma_\mu \psi, \quad \Delta(z) = -(1/2\pi) \varepsilon(z^0) \delta(z^2) \\ S_{\mu\alpha\nu\sigma} &\equiv g_{\mu\alpha} g_{\nu\sigma} - g_{\mu\nu} g_{\alpha\sigma} + g_{\mu\sigma} g_{\nu\alpha} \end{aligned}$$

where

$$\begin{aligned} \mathcal{O}_V^\sigma(0, z) &\equiv \bar{\psi}(0) \gamma^\sigma \psi(z) - \bar{\psi}(z) \gamma^\sigma \psi(0) \\ \mathcal{O}_A^\sigma(0, z) &\equiv \bar{\psi}(0) \gamma^\sigma \gamma^5 \psi(z) + \bar{\psi}(z) \gamma^\sigma \gamma^5 \psi(0) \end{aligned}$$

Now

$$\begin{aligned} \eta \sim z_\perp \sim 0 \\ \Downarrow \\ \langle PS | \mathcal{O}_{V,A}^\sigma(0, z) | PS \rangle \simeq \langle PS | \mathcal{O}_{V,A}^\sigma(0, \lambda n) | PS \rangle \end{aligned}$$

e.g.

$$\begin{aligned} \langle PS | \bar{\psi}(0) \gamma^\sigma \psi(\lambda n) | PS \rangle &= 2p^\sigma \hat{q}(\lambda) + 2n^\sigma M^2 \hat{f}_4(\lambda) \\ &\simeq 2p^\sigma \hat{q}(\lambda) \end{aligned}$$

Insert $W_{\mu\nu}$ with $\Delta(z)$ and

$$\hat{q}(\lambda) = \int d\xi e^{-i\lambda\xi} q(\xi) = \int d\xi e^{-i\xi p \cdot z} q(\xi)$$

One gets

$$F_1(x) = (1/x) F_2(x) = q(x) - q(-x)$$

with

$$q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \not{x} \psi(\lambda n) | PS \rangle$$

$$-q(-x) \longleftrightarrow \text{Anti-Quark}$$

- “Full Theory”

QCD Interaction

$\Downarrow$

$$\Delta(z) \rightarrow \Delta(z)[0, z]$$

$$q(x) \rightarrow \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0)[0, \lambda n] \not{x} \psi(\lambda n) | PS \rangle$$

where

$$[y, z] = P \exp \left(ig \int_0^1 dt (y - z)_\mu A^\mu(ty + (1-t)z) \right)$$

New $q(x)$ is Gauge Invariant !!

$\Delta(z)$ and $q(z)$ Develop Mass Singularity

$$\Delta(z)[0, z] \leftrightarrow F^0(z, Q^2, 1/\epsilon)$$

$$q(y) \leftrightarrow q^0(y, 1/\epsilon)$$

We Need Factorization (Renormalization)

$$\Delta(z) [0, z] \Rightarrow C(y, Q^2/\mu^2, \alpha_s)$$

$$q(y) \Rightarrow q(y, \mu^2) = \Gamma(y/\xi, \alpha_s, 1/\epsilon)$$

$$\otimes q^0(\xi, 1/\epsilon)$$

Similar for $g_1(x)$ and $g_2(x)$

3. Operator Product Expansion

Light Cone Dominance

$\Downarrow$

Systematic Expansion in x^2 of Current Product

$\Downarrow$

Operator Product Expansion (OPE)

Current (Operator) Product can be Expanded as

$$J(x)J(0) \sim \sum_{i,N} C_{i,N}(x^2) x^{\mu_1} x^{\mu_2} \dots x^{\mu_N} O_{\mu_1 \mu_2 \dots \mu_N}^{i,N}(0)$$

$C_{i,N}$: c-Number Function (Wilson's Coefficient Function)

Short-Distance Expansion:

What Happens when $x^\mu \rightarrow 0$?

$$C_i(x) \sim x^{d_O - d_A - d_B} \quad (d : \text{scale dim.})$$

Operators with Small Dimension Dominate

Light-cone Expansion:

What Happens when $x^2 \rightarrow 0$ *but* $x^\mu \neq 0$?

$$C_{i,N}(x^2) \sim (x^2)^{(d_O^N - N - d_A - d_B)/2}$$

Operators with Low Twist Dominate

$$\tau_N \equiv d_O^N - N$$

Caveat (short distance vs. light cone):

$$\text{OPE is valid when} \quad x^\mu \sim 0 \Leftrightarrow |q^2| \geq P \cdot q$$

$$\text{Deep inelastic region} \quad x^2 \sim 0 \Leftrightarrow |q^2| \leq P \cdot q$$

Moment Sum Rule: (Lorentz Structure Neglected)

Consider "Forward Virtual Compton Scatt"

$$T = i \int dx e^{iq \cdot x} \langle P | T J(x) J(0) | P \rangle$$

Valid in $P \cdot q / |q^2| \leq 1$

Relation to structure functions;

- Optical theorem $W = 1/\pi \text{Im}T$
- Crossing symmetry $W(q, P) = -W(-q, P)$

Calculation Goes as Follows

Take Fourier Transform

$$i \int d^4x e^{iq \cdot x} T J(x) J(0) \sim \sum_{i,N} C_{i,N}(Q^2) \left(\frac{2}{Q^2} \right)^N q_{\mu_1} \cdots q_{\mu_N} O_{i,N}^{\mu_1 \cdots \mu_N}$$

Matrix element of O

$$\langle P | O_{i,N}^{\mu_1 \cdots \mu_N}(0) | P \rangle \equiv 2 A^{i,N} P^{\mu_1} \cdots P^{\mu_N}$$

so

$$T(P \cdot q, q^2) = 2 \sum_{i,N} \omega^N A^{i,N} C_{i,N}(Q^2), \quad \omega = \frac{2P \cdot q}{Q^2} = \frac{1}{x}$$

Structure Functions in Physical Region $\omega \geq 1$ ($x \leq 1$)

$$\begin{aligned} \frac{1}{2\pi i} \oint d\omega \omega^{-N-1} T &= \quad \quad \quad \text{Diagram: A circle with a horizontal line passing through its center. The line has arrows pointing outwards from the circle. The center is labeled with a small 'c' and a prime symbol. The line is labeled with a small 'c' at both ends.} \\ &= \sum_i A^{i,N} C_{i,N}(Q^2) = \int_0^1 dx x^{N-1} W(x, Q^2) \end{aligned}$$

4. Relation to OPE

OPE

Lowest Twist Operator

$$O^{\mu_1 \cdots \mu_N} = i^{N-1} \bar{\psi} \gamma^{\mu_1} D^{\mu_2} \cdots D^{\mu_N} \psi$$

Moment Sum Rule

$$\int_0^1 dx x^{N-1} F(x, Q^2) = C \left(N, \frac{Q^2}{\mu^2}, \alpha_s \right) A^N(\mu^2)$$

where

$$\langle P | O^{\mu_1 \cdots \mu_N} | P \rangle = 2 A^N(\mu^2) P^{\mu_1} \cdots P^{\mu_N}$$

Namely

$$A^N(\mu^2) = \frac{1}{2} n_{\mu_1} \cdots n_{\mu_N} \langle P | O^{\mu_1 \cdots \mu_N} | P \rangle$$

Note $P \cdot n = 1$

On the Other Hand

Moment of (Renormalized) $q(x, \mu^2)$

$$\begin{aligned} q(N, \mu^2) &= \int dx x^{N-1} q(x, \mu^2) \\ &= \frac{1}{4\pi} \int d\lambda \left\{ \left(-i \frac{\partial}{\partial \lambda} \right)^{N-1} 2\pi \delta(\lambda) \right\} \\ &\quad \times \langle P | \bar{\psi}(0) [0, \lambda n] \not{n} \psi(\lambda n) | P \rangle \\ &= \frac{1}{2} \langle P | \bar{\psi}(0) \not{(in \cdot D)}^{N-1} \psi(0) | P \rangle = A^N(\mu^2) \end{aligned}$$

Equivalent to OPE !!

Spin in Hard QCD Processes: An Introduction

Lectures Presented at the 9th RIKEN Winter School,

Shimoda, December 8-12, 1998

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Abstract

The foundations and several applications of inclusive QCD spin physics are presented in four lectures. First, I discuss the properties of relativistic spin- $\frac{1}{2}$ and spin-1 particles and the tensor density describing angular momentum in QCD. Next I introduce the light cone description of parton distribution and fragmentation functions for quarks and gluons in a helicity basis. I discuss the classification of spin dependent effects with respect to helicity, chirality, transversity and twist. Two applications are discussed in some detail: the properties of the transverse spin structure function $g_2(x, Q^2)$, and the parton description of spin and orbital angular momentum.

Typeset using REVTeX

SPIN IN HARD QCD PROCESSES
AN INTRODUCTION

R L JAFFE
SHIMODA
DECEMBER 1998

- * THANK YOU TO RIKEN,
PROF YAZAKI, SHIBATA
DR SAITO
- * I HOPE YOU ALL HAVE OPPORTUNITY TO VISIT
RIKEN-BNL RESEARCH CENTER AT BROOKHAVEN
- * THANKS TO PROF. KODAIRA FOR HIS LECTURES
FORGIVE REPETITION, EXCURSIONS BEYOND.
WANT TO GIVE INTRODUCTION BUT ALSO INSPIRE
WITH POWERFUL RESULTS
- * CHANGES TO HELP WAKEFULNESS

• BJORKEN SUM RULE

$$\int_0^1 dx g_1^{p-n}(x, Q^2) = \frac{1}{6} \frac{g_A}{g_V} \left\{ 1 - \frac{\alpha_s}{\pi} - \frac{43}{12} \left(\frac{\alpha_s}{\pi} \right)^2 - 20.25 \left(\frac{\alpha_s}{\pi} \right)^3 \right\}$$

$$+ \frac{M^2}{Q^2} \int_0^1 dx x^2 \left\{ \frac{2}{9} g_1^{p-n}(x, Q^2) + \frac{1}{6} g_2^{p-n}(x, Q^2) \right\}$$

$$- \frac{4}{27} \frac{1}{Q^2} F^{u-d}(Q^2)$$

• MANY WORKERS

$$2F^{\mu}(Q^2)S^{\alpha} \equiv \langle PS | g \bar{u} \tilde{F}^{\alpha\lambda} \gamma_{\lambda} u | PS \rangle$$

- PQCD
- TARGET MASS
- HIGHER TWIST

- LOWEST NON-VANISHING MOMENT OF $g_2(x, Q^2)$ • E. SHURYAK
A. VAINSTEIN

$$\int_0^1 dx x^2 g_2(x, Q^2) = -\frac{2}{3} \int_0^1 dx x^2 g_1(x, Q^2) + \frac{1}{6} d_2(Q^2)$$

$$d_2(Q^2) [2 S_{\sigma} P_{\mu} P_{\nu} - S_{\mu} P_{\sigma} P_{\nu} - S_{\nu} P_{\mu} P_{\sigma}]$$

$$= \frac{3}{8} \langle PS | g \bar{q} (F_{\sigma\mu} \gamma_{\nu} + F_{\sigma\nu} \gamma_{\mu}) q | PS \rangle$$

Q^2

A SLAC, HERMES

CEBAF(?) PROJECT • 2. 14

SHIMODA DEC '98

SPIN IN HARD QCD PROCESSES

I. ANGULAR MOMENTUM $\neq$ RELATIVITY - INTRODUCTION

SPIN-0

SPIN-1/2

SPIN-1

II. KINEMATICS OF SPIN IN DEEP INELASTIC PROCESSES

DEEP INELASTIC SCATTERING

DISTRIBUTION $\neq$ FRAGMENTATION FUNCTIONS

CHIRALITY, TWIST, TRANSVERSITY ...

III. APPLICATIONS / EXAMPLES

SUM RULES (BRIEF)

TRANSVERSITY

g_2

PARTON DISTR. OF ORBITAL ANGULAR MOMENTUM

THANKS TO

ANEESH MANOHAR

XIANGDONG JI

GARY GOLDSTEIN

SERGEI BASHINSKY

I. ANGULAR MOMENTUM IN RELATIVISTIC QM $\neq$ QFT

A. DIRAC PARTICLES - SPIN $1/2$

MAIN REFERENCE

RLJ $\neq$ A. MANOHAR

N.P. 8337 (1990)

P 509

- CONSERVATION LAWS
- STRUCTURE OF DIRAC WAVE FUNCTIONS
- FREE DIRAC SPINORS
- SPIN, HELICITY, CHIRALITY...
- NOETHER'S THEOREM
- ANGULAR MOMENTUM TENSOR DENSITY

B. GAUGE FIELDS

- RECAP ALA SPIN $1/2$
- FREE FIELD POLARIZATION STATES
- ANGULAR MOMENTUM TENSOR DENSITY
- GAUGE DEPENDENCE

C. QCD

- ANGULAR MOMENTUM TENSOR DENSITY (LAST TIME!)
- GAUGE $\neq$ INTERACTION ISSUES

A. SPIN-1/2

- DIRAC EQUATION $(i\not{\partial} - m)\Psi = 0 \Rightarrow (\not{\partial} - m)u = 0$

- $\Psi \neq u$ are four component, DIRAC SPINORS, but Dirac equation describes only 2 degrees of freedom for each $E \neq \vec{p}$

$$\Lambda_{\pm} = \frac{1}{2m} (m \pm \not{p})$$

- $\Lambda_{\pm}(p)$ are projection operators $\Lambda_{\pm}\Lambda_{\mp} = 0$
They project out a 2-dim. subspace. So for each $E \neq \vec{p}$, $\exists$ only 2 modes (spin) for relativistic Dirac particle
 $\Lambda_{\pm}^2 = \Lambda_{\pm}$
 $\Lambda_{+} + \Lambda_{-} = 1$

- Spin-1/2 $\neq$ Polarization

Any state of spin-1/2 particle is described by 2-component spinor in its rest frame:

$$u_j \quad j=1,2 \quad u^\dagger u = 1$$

Spin density matrix $M_{ij} \equiv u_i u_j^*$

$$1. M^\dagger = M \quad 2. \text{tr } M = 1 \quad 3. M^2 = M$$

Mixed states: spinor formalism difficult
 M formalism excellent

Most general polarization state for spin $-1/2$

$$M \text{ with } \underline{M^\dagger = M} \quad \underline{\text{tr } M = 1}$$

Parameterize by $M = \frac{1}{2} (1 + P \vec{\sigma} \cdot \hat{S})$

$$\left. \begin{array}{l} P = \text{polarization} \\ \hat{S} = \text{direction of spin} \end{array} \right\} \text{All you need to know @ spin half.}$$

Pure state? $M^2 = M \Rightarrow P=1 \quad M = \frac{1}{2} (1 + \vec{\sigma} \cdot \hat{S})$

Suggests ANY PURE SPIN STATE FOR SPIN HALF IS A (POSITIVE EIGENVALUE) EIGENSTATE OF $\vec{\sigma} \cdot \hat{S}$ FOR SOME $\hat{S}$

PROOF: Let $\hat{S} \equiv U^\dagger \vec{\sigma} U$

Then $\hat{S} \cdot \vec{\sigma} U = U$ left as exercise

So any dirac particle is an eigenstate of spin in some direction. This is not true for spin $> 1/2$ and simple misconceptions arise.

• SYMMETRIES OF DIRAC EQ.

(in gluon or general backgrd.)

★ $\vec{J}$ $H = \vec{\alpha} \cdot \vec{p} + \beta m + V$

($V = U \ln \beta$, ETC.)

$\vec{L} = \vec{r} \times \vec{p} \quad \vec{S} = \frac{1}{2} \vec{\sigma}$

$\vec{J} = \vec{L} + \vec{S} \quad \vec{p} = -i \vec{\nabla}$

$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$ TEMPORARY

$\vec{\sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$ DIRAC MATRICES

$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

• $[H, \vec{L}] = -i \vec{\alpha} \times \vec{p} \neq 0$

• $[H, \vec{S}] = i \vec{\alpha} \times \vec{p} \neq 0$

• $[H, \vec{J}] = 0$

SO FOR DIRAC MOTION IN A CENTRAL FIELD (THE NUCLEON IS AT LEAST THIS COMPLICATED) ONLY

$\vec{J}^2 \implies j(j+1)$

$J_3 \implies m_j$ ARE GOOD Q.N.

$\vec{L} \neq \vec{S}$ are not conserved. They mix in Dirac energy eigenstates

★ π PARITY $\pi \equiv P\beta$ WHERE $P|\vec{x}\rangle = |- \vec{x}\rangle$
IS NAIVE PARITY

$[\pi, H] = 0$ for parity
invariant interactions

• Symmetries implemented on states

Standard γ -matrix basis $\gamma^0 = \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \quad \Pi \psi(\vec{r}) = \begin{pmatrix} +\phi(-\vec{r}) \\ -\chi(-\vec{r}) \end{pmatrix} \quad \text{ γ did it}$$

$$Y_{lm}(-\hat{r}) = (-1)^l Y_{lm}(\hat{r})$$

So ϕ and χ differ in l - by 1 unit $l_\phi = l_\chi \pm 1$

Example (important)

$$\left. \begin{array}{l} \bullet \text{ Typically lowest energy state} \\ \bullet \text{ Reduces to } 1S_{1/2} \text{ in NR limit} \\ \bullet \text{ Upper components - } l=0 \\ \text{Lower components - } l=1 \end{array} \right\} \psi_m = \begin{pmatrix} \overset{\text{s-wave}}{f(r)} \\ i\vec{\sigma} \cdot \hat{r} \overset{\text{p-wave}}{g(r)} \end{pmatrix} u_m$$

$$\bullet \langle S_3 \rangle = \frac{1}{2} \int_0^\infty r^2 dr (f^2 - \frac{1}{3} g^2)$$

$$\langle L_3 \rangle = \frac{2}{3} \int_0^\infty r^2 dr g^2$$

$$\text{NB } \langle S_3 \rangle + \langle L_3 \rangle = \frac{1}{2} \quad \left(\int_0^\infty r^2 dr (f^2 + g^2) = 1 \right)$$

Moral $\Rightarrow$ Orbital and spin angular momentum cannot be separated in a relativistic theory.

$\rightarrow$ Even the lowest energy Dirac states carry $\langle L_3 \rangle$ in a relativistic Q.

• Free Dirac Spinors

$$u(p, s) \quad v(p, s)$$

(set $v(p, s)$ aside)

$$\left. \begin{aligned} (\not{p} - m) u(p, s) &= 0 \\ (m + \gamma_5 \not{s}) u(p, s) &= 0 \end{aligned} \right\} \text{CONVENTIONAL}$$

• in rest frame $\vec{\sigma} \cdot \hat{s} u(p, s) = u(p, s)$
 $s^\mu = m(0, \hat{s})$

OR

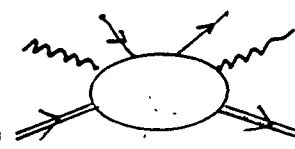
$$\begin{aligned} \bar{u}(p, s) \gamma^\mu u(p, s) &= 2p^\mu \\ \bar{u}(p, s) \gamma^\mu \gamma_5 u(p, s) &= 2s^\mu \end{aligned}$$

$$p^\mu p_\mu = m^2 \quad s^\mu s_\mu = -m^2 \quad p^\mu s_\mu = 0$$

p^μ is a 4-vector s^μ is an axial 4-vector

Note: Consider an amplitude with a fermion scattering forward -

Bilinear in $\bar{u}(p, s) \quad u(p, s)$



$\not{u}(p, s)$

$\Rightarrow$ AT MOST LINEAR IN s^μ

• Bases

$$\vec{\sigma} = \begin{pmatrix} \sigma^1 & \sigma^3 \\ 0 & \sigma^1 \end{pmatrix}$$

* HELICITY $\lambda = \frac{1}{2} \vec{\sigma} \cdot \hat{p}$

$$[\lambda, H] = 0 \quad [\lambda, \vec{P}] = 0 \quad \lambda u(p, \pm) = \pm \frac{1}{2} u(p, \pm)$$

* CHIRALITY

γ_5

$$[\gamma_5, H] = 0 \text{ IFF } m = 0$$

Of course we can use a chiral basis whether or not $[\gamma_5, H] = 0$.

$$\gamma_5 u_R(p) = u_R(p)$$

$$\gamma_5 u_L(p) = -u_L(p)$$

* TRANSVERSITY

RHJ & X. Ji, NP B375 (1992)
P 527.

X TRANSVERSE SPIN DOES NOT COMMUTE WITH H :

$$\text{EG. } [\sigma_1, H] = [\sigma_1, \vec{\alpha} \cdot \vec{p}] = [\sigma_1, \alpha_3] p_3 = -i\alpha_2 p_3$$

SO A DIRAC PARTICLE WITH ENERGY $E \neq$ MOMENTUM $\vec{p}$
CANNOT BE LABELLED BY TRANSVERSE SPIN.

* HOWEVER $\vec{\gamma}_\perp \gamma_5$ DOES COMMUTE WITH $H = \vec{\alpha}^3 p^3 + \beta m$

$$[\gamma_1 \gamma_5, \alpha_3 p_3] = 0 \text{ SO THIS "TRANSVERSITY" IS A GOOD Q.N.}$$

$$\text{LET } \hat{S} = m \hat{e}_\perp$$

$$\frac{1}{2}(m \pm \gamma_5 \not{S}) \Rightarrow \frac{1}{2}m (1 \pm \gamma_5 \gamma^\perp)$$

$$\gamma_5 \gamma^\perp u_\perp(p) = u_\perp(p)$$

$$\gamma_5 \gamma^\perp u_T(p) = -u_T(p)$$

$$\gamma^\perp = \gamma^1 \text{ or } \gamma^2$$

RELATIONS

- For $\gamma^+ = \gamma^1$

$$u_{\perp}(p) = \frac{1}{\sqrt{2}} (u(p,+) + u(p,-))$$

$$u_{\top}(p) = \frac{1}{\sqrt{2}} (u(p,+) - u(p,-))$$

So relation between transversity & helicity is analogous to σ_1 and σ_3 in NR Q.

- Of course chirality & helicity coincide for $m=0$.
- Angular Momentum in Q.F.Q. (Spin-0 & $\frac{1}{2}$)
- Conservation of 4-momentum + Noether's Thm $\Rightarrow T_{\mu\nu}$

$$\partial^\mu T_{\mu\nu} = 0, \quad T_{\mu\nu} = T_{\nu\mu}$$

$$P^\nu \equiv \int d^3x T^{0\nu} \quad \frac{d}{dt} P^\nu = \int d^3x (-\partial_i T^{i\nu}) = 0$$

- Lorentz invariance + Noether's Thm $\Rightarrow M_{\mu\nu\lambda}$

$$M_{\mu\nu\lambda} = -M_{\mu\lambda\nu}$$

$M_{\mu ij}$ ROTATIONS

$M_{\mu 0k}$ BOOSTS

"Simple" construction

$$M_{\mu\nu\lambda} \equiv x_\nu T_{\mu\lambda} - x_\lambda T_{\mu\nu}$$

(SEE RLJ &
MANOHAR)

$$\begin{aligned}\partial^\mu M_{\mu\nu\lambda} &= \delta^\mu_\nu T_{\mu\lambda} - \delta^\mu_\lambda T_{\mu\nu} \\ &\quad + x_\nu \partial^\mu T_{\mu\lambda} - x_\lambda \partial^\mu T_{\mu\nu} \\ &= 0 \quad \text{IF } T_{\mu\nu} \text{ IS SYMMETRIC \& CONSERVED}\end{aligned}$$

$$\int d^3x M^0_{ij} = \epsilon_{ijk} J_k \quad \text{ROTATIONS}$$

$$\int d^3x M^0_{0\alpha} = B_\alpha \quad \text{BOOSTS}$$

NOTE: $M_{\mu\nu\lambda}$ obtained from this construction and from Noether's theorem are slightly different

$$M_{\mu\nu\lambda}^{(T)} = M_{\mu\nu\lambda}^{(N)} + \partial^\beta B_{[\mu,\beta]\nu\lambda}$$

SEE RLJ \&
MANOHAR
NP B337 (1990)

↑ "superpotential" does not
contribute to charges

What matters is what's observable!

- Scalar field ϕ

$$M^{\mu\nu\lambda} = \partial^\mu \phi (x^\nu \partial^\lambda - x^\lambda \partial^\nu) \phi - (g^{\mu\lambda} x^\nu - g^{\mu\nu} x^\lambda) \mathcal{L}$$

Interpret? $J^k = \int d^3x \ i \dot{\phi} (\vec{x} \times \vec{\nabla})^k \phi$

Canonical! $L^k = \int d^3x \sum_{\alpha} i \pi^{\alpha}(x) \left(\vec{x} \times \frac{\vec{\nabla}}{i} \right)^k \Phi^{\alpha}(x)$

So $\pi^k (\vec{x} \times \vec{\nabla}) \phi^k$ is "convective" or "orbital" angular momentum.

- Dirac field ψ

[illegible]

* Note appearance of spin associated with axial vector operator.

or The fact that spin tensor density reduces to an axial vector is, in fact, a degeneracy associated with spin- $1/2$ fields and does not hold for higher spins.

B. SPIN 1 (AND GAUGE FIELDS)

- Many aspects $\Leftrightarrow$ spin $-1/2$

- * $\vec{L} \neq \vec{S}$ are not separately conserved - only $\vec{J}$
- * Stationary states in external field mix various l values to obtain eigenstates of $\vec{J}^2 \neq$ parity
- * Three propagating degrees of freedom (helicity $+1, 0, -1$) for each $E \neq \vec{p}$. Gauge invariance $3 \Rightarrow 2$. No helicity 0 for gluons

- Polarization in rest frame (needs specialization for gauge particles)

ϵ_k $k=1,2,3$ POLARIZATION VECTOR

* M 3×3 POLARIZATION DENSITY MATRIX

$$\underline{M^\dagger = M} \quad \underline{\text{tr } M = 1} \quad \underline{M^2 = M} \quad (\text{for pure state})$$

$\vec{\Sigma}$: 3×3 spin matrices $(\Sigma^k)_{ij} = -i \epsilon_{kij}$

To parameterize M requires 8 3×3 hermitian matrices (cf. Gell Mann's $SU(3)$)

$$\Theta_{ij} \equiv \{\Sigma_i, \Sigma_j\} - \frac{1}{2} \delta_{ij} \quad \text{Tr } \Theta = \Theta_{jj} = 0 \quad 5 \text{ INDEP. } \Theta_{ij}$$

$$\text{Then } M = \frac{1}{3} \mathbb{I} + \underbrace{P \hat{S} \cdot \vec{\Sigma}}_{\text{VECTOR POL.}} + \underbrace{Q_{ij} \Theta_{ij}}_{\text{TENSOR POL.}}$$

So polarization state of spin-1 particle contains both vector ($\hat{S}$) and tensor (Q) information.

* Spin direction $\hat{S} = \vec{E}^* \vec{\Sigma} \vec{E} \quad (\text{cf. } U^\dagger \vec{\sigma} U)$

$$S_k = \underline{E_i^* (-i \epsilon_{kij}) E_j} = \underline{-i \vec{E}^* \times \vec{E}}$$

However - an arbitrary $\vec{E}$ IS NOT an eigenstate of $\vec{\Sigma}$ along some direction $\hat{S}$. Can prove $\vec{E}$ is eigenstate iff $\vec{E} \cdot \vec{E} = 0$. [Then $(\vec{\Sigma} \cdot \hat{S}) \vec{E} = \vec{E}$]

* Relativistic generalization $\vec{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$

$$\vec{E} \Rightarrow E^\mu \text{ with } k_\mu E^\mu = 0 \quad (\text{still only 3 dof.})$$

• SCALAR: $E_\mu^* E^\mu$

• VECTOR POLARIZATION: $S_\mu = i \epsilon_{\mu\nu\alpha\beta} P^\nu E^{*\alpha} E^\beta$

• TENSOR POLARIZATION: $\gamma_{\mu\nu} = E_\mu^* E_\nu + E_\nu^* E_\mu - \frac{1}{2} g_{\mu\nu} E_\alpha^* E^\alpha$

SCALAR $\epsilon^* \cdot \epsilon$

VECTOR $S_\mu = i \epsilon_{\mu\nu\alpha\beta} P^\nu \epsilon^{*\alpha} \epsilon^\beta$

TENSOR $\eta_{\mu\nu} = \epsilon_\mu^* \epsilon_\nu + \epsilon_\nu^* \epsilon_\mu - \frac{1}{2} g_{\mu\nu} \epsilon_\alpha^* \epsilon^\alpha$

Spin physics of gluons is richer because they can carry tensor polarization, but poorer (than massive spin-one) because no helicity zero state exists.

- $M_{\mu\nu\lambda}$ for gauge fields

$$M_g^{\mu\nu\lambda} = -F^{\mu\alpha} (x^\nu \partial^\lambda - x^\lambda \partial^\nu) A_\alpha + F^{\mu\lambda} A^\nu + F^{\nu\lambda} A^\mu - \frac{1}{4} F^2 (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}) \quad (\text{ABELIAN})$$

↙ ORBITAL
↘ SPIN

- Spin part (no x^α , no ∂^α) is not related to a vector operator

(Note $F^{\mu\lambda} A^\nu + F^{\nu\lambda} A^\mu + \overline{F^{\lambda\nu} A^\mu} = \epsilon^{\mu\nu\lambda\sigma} K_\sigma$ but we don't have the third term.)

- $M_g^{\mu\nu\lambda}$ is gauge invariant, but separate spin and orbital pieces are not.

Relate to well-known expression -

$$J^k = \frac{1}{2} \epsilon^{klm} \int d^3x M^{0lm}(\vec{x})$$

$$\vec{J} = \int d^3x E^l (\vec{x} \times \vec{\nabla}) A^l + \int d^3x \vec{E} \times \vec{A}$$

$$\vec{J} = \int d^3x \vec{x} \times (\vec{E} \times \vec{B}) \quad \text{familiar}$$

• QCD

$$M^{\mu\nu\lambda} = \underbrace{\frac{i}{2} \bar{\psi} \gamma^\mu (x^\nu \partial^\lambda - x^\lambda \partial^\nu) \psi}_{\text{QUARK ORBITAL}} + \underbrace{\frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi}_{\text{QUARK SPIN}} \\ - 2 \underbrace{\text{Tr} \{ F^{\mu\alpha} (x^\nu \partial^\lambda - x^\lambda \partial^\nu) A_\alpha \}}_{\text{GLUON ORBITAL}} + 2 \underbrace{\text{Tr} \{ F^{\mu\lambda} A^\nu + F^{\nu\mu} A^\lambda \}}_{\text{GLUON SPIN}} \\ - \underbrace{\frac{1}{2} \text{Tr} F^2 (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu})}_{\text{BOOSTS}}$$

* Lack of gauge invariance clouds interpretation of the individual terms. On the other hand, no interaction dependent terms appear ($M^{\mu\nu\lambda}$ is quadratic in canonical variables)

• Ji suggested gauge invariant decomposition by $\partial_\alpha \rightarrow D_\alpha$ - addition terms in quark part cancel against gluon part -

X. Ji, PRL 73 (1997) 510.

$$\begin{aligned}
 M_{\text{QCD}}^{\mu\nu\lambda} = & \underbrace{\frac{i}{2} \bar{\Psi} \gamma^\mu (x^\nu D^\lambda - x^\lambda D^\nu) \Psi}_{\text{QUARK ORBITAL}} + \underbrace{\frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\Psi} \gamma_\sigma \gamma_5 \Psi}_{\text{QUARK SPIN}} \\
 & - 2 \underbrace{\text{Tr} \{ F^{\mu\alpha} (x^\nu D^\lambda - x^\lambda D^\nu) A_\alpha \}}_{\text{GLUON ORBITAL}} + 2 \underbrace{\text{Tr} \{ F^{\mu\lambda} A^\nu - F^{\nu\lambda} A^\mu \}}_{\text{GLUON SPIN}}
 \end{aligned}$$

* SUMMARY

- QUARK SPIN - Separately gauge invariant and directly measurable
- GLUON SPIN - not separately gauge invariant but a quantity is measurable that coincides with this in a definite gauge (!) (see last lecture)
- QUARK ORBIT - Separately gauge invariant but measurement unknown.
- GLUON ORBIT - not separately gauge invariant and measurement unknown.

SEE LATER LECTURE FOR MORE DISCUSSION

- Why is it hard to measure matrix elements of orbital angular momentum?

R.L.J. & MANOHAR

Analog with charge and magnetic moment

$$j^\mu = \bar{\Psi} \gamma^\mu \Psi \quad \langle PS | j^0 | PS \rangle = 2P^0 Q$$

$$\text{But } \langle PS | \vec{j} | PS \rangle = 0 \quad \text{-----}$$

Moment of $\vec{J}$ appears in off-forward matrix element

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$$\int d^4x e^{iq \cdot x} \langle P' S' | j_\mu(\vec{x}) | P S \rangle$$

$$= \bar{u}(P' S') \left\{ \gamma_\mu F_1(q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2M} F_2(q^2) \right\} u(P S)$$

Expand about $\vec{q} = 0$. Linear term on RHS

$$\text{RHS} \propto F_2(0) \vec{q} \times \vec{S} \propto \vec{q} \times \vec{S} \mu$$

$$\text{LHS} \propto \vec{q} \times \int d^3x \langle P S | \frac{1}{2} \vec{x} \times \vec{J} | P S \rangle$$

Lesson: To measure a coordinate space moment of an operator you must measure the operator matrix element at non-zero momentum transfer

Angular momenta are moments of $T^{\mu\nu}$ ($x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$). So one must measure $\langle P' S' | T^{\mu\nu} | P S \rangle$.

Hard QCD processes naturally give forward matrix elements — hard.

But see X. Ji PR D55 (1997) 7114.

II KINEMATICS OF SPIN IN DEEP INELASTIC PROCESSES

A. DEEP INELASTIC SCATTERING FROM A SPIN- $\frac{1}{2}$ TARGET

- STRUCTURE FUNCTIONS
- MEASUREMENT

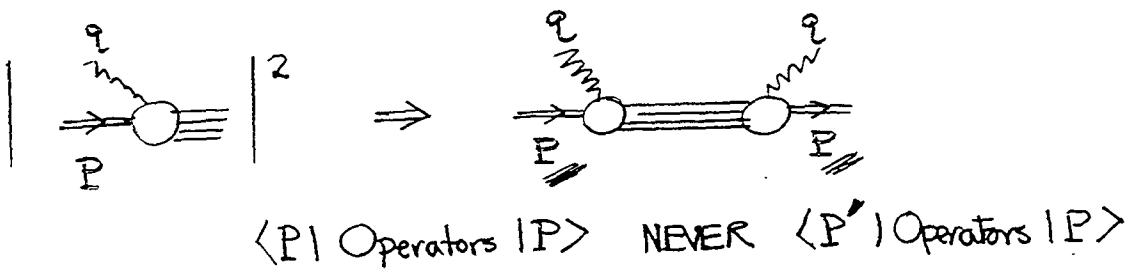
B. INTRODUCTION TO QUARK & GLUON DISTRIBUTION & FRAGMENTATION FUNCTIONS

- A GRAPHICAL GUIDE
- FROM GRAPHS TO EQUATIONS
- GENERALIZATIONS, DIFFICULTIES, F.A.Q.
- SUMMARY
- LEARNING FROM THE LIGHT CONE

C. HELICITY BASIS & DENSITY MATRIX NOTATION

DETAILS

- QCD hard processes $\Rightarrow$ forward matrix elements



- How magnetic moment $\vec{\mu} = \frac{\vec{x} \times \vec{J}}{2}$ can be measured?

$$\int d^4x e^{iq \cdot x} \langle P'S' | j_k(x) | PS \rangle = (2\pi)^4 \delta^4(P+q-P')$$

$$\bar{U}(P'S') \left[\gamma_k F_1(q^2) + \frac{i}{2M} \sigma_{k\nu} q^\nu F_2(q^2) \right] U(PS)$$

Take $\vec{q} \Rightarrow 0$ $q^0 \approx \vec{q}^2/2M$ $\gamma_k = \frac{(P+P')_k}{2M} + \frac{i \sigma_{k\nu} q^\nu}{2M}$

GORDON IDENTITY

LHS: $e^{i\vec{q} \cdot \vec{x}} = 1 + i\vec{q} \cdot \vec{x} + \dots$ NOTE: term linear in $\vec{x}$ is also linear in $\vec{q}$

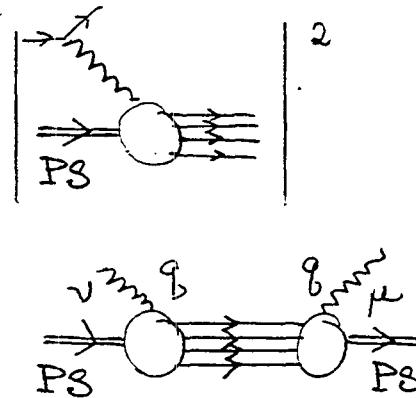
ALGEBRA (see e.g. JACKSON)

$$\int d^3x \frac{1}{2} \vec{x} \times \langle PS | \vec{J} | PS \rangle \propto \bar{U}(P'S) \vec{\sigma} U(PS) \frac{F_1(0) + F_2(0)}{2M}$$

REQUIRES $\vec{q} \neq 0$ EXPT.

A. DEEP INELASTIC SCATTERING FROM A SPIN-1/2 TARGET

$$\left. \begin{aligned} P \cdot q &= \nu = M v_{\text{exp}} \\ q^2 &= -Q^2 < 0 \\ x &= Q^2/2\nu \\ y &= \nu/ME = (E^* - E)/E \\ K &= 1 + 4M^2 x^2/Q^2 \\ [S] &= \text{mass} \end{aligned} \right\} \text{SCALING VARIABLES}$$



• Structure Functions

$$\begin{aligned} W_{\mu\nu}(q, P, S) &= \frac{1}{4\pi} \sum_x (2\pi)^4 \delta^4(q + P - x) \langle PS | J_\mu | x \rangle \langle x | J_\nu | PS \rangle \\ &= \int d^4\xi e^{iq \cdot \xi} \langle PS | J_\mu(\xi) J_\nu(0) | PS \rangle \\ &= \frac{1}{2\pi} \text{Im } T_{\mu\nu}(q, P, s) \end{aligned}$$

$$\bullet \quad q^\mu W_{\mu\nu} = 0 \quad \bullet \quad W_{\mu\nu}^* = W_{\nu\mu} \quad \bullet \quad \leq \text{linear in } S^\mu$$

$$\begin{aligned} W_{\mu\nu} &= -g_{\mu\nu} F_1(x, Q^2) + \frac{P_\mu P_\nu}{\nu} F_2(x, Q^2) \\ &+ \frac{i\epsilon_{\mu\nu\alpha\beta}}{\nu} q^\alpha S^\beta g_1(x, Q^2) \quad \leftarrow \text{LONGITUDINAL} \\ &+ \frac{i\epsilon_{\mu\nu\alpha\beta}}{\nu^2} (\nu S^\beta - q \cdot S P^\beta) q^\alpha g_2(x, Q^2) \end{aligned}$$

$g_1 \neq g_2$ are the playthings of $\vec{DIS}$

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Why are they called longitudinal & transverse ?

DEFINE IMPORTANT LIGHT-LIKE BASIS VECTORS

$$\sqrt{2} p^\mu \equiv p(1, 0, 0, 1) \quad -n^2 = p^2 = 0$$

$$\sqrt{2} n^\mu \equiv \frac{1}{p}(1, 0, 0, -1) \quad n \cdot p = 1$$

WORK IN FRAME WHERE NUCLEON & PHOTON
 DEFINE $\hat{z}$ -AXIS (Theorist's frame - changes event by event)

NUCLEON AT REST
 PHOTON ALONG $\hat{z}$ -AXIS

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu \quad q^\mu = (v + \frac{1}{2} M^2 x) n^\mu - x p^\mu$$

$$S^\mu = S \cdot n p^\mu + S \cdot p n^\mu + S_\perp^\mu \quad S \cdot p \ll S \cdot n$$

Rest frame $p = M\sqrt{2}$

$W_{\mu\nu}$ = spin independent terms

$$S \cdot p = 0$$

$$+ i \epsilon_{\mu\nu\lambda\sigma} (S \cdot n) n^\lambda p^\sigma g_1(x, Q^2)$$

$$+ i \epsilon_{\mu\nu\lambda\sigma} n^\lambda S_\perp^\sigma (g_1(x, Q^2) + g_2(x, Q^2))$$

So $g_1 \Leftrightarrow$ LONGITUDINAL

$$g_1 + g_2 \equiv g_T \Leftrightarrow \text{TRANSVERSE}$$

WHY THE LIGHT CONE?

Bjorken limit : $P \cdot q \rightarrow \infty$ $x = \frac{Q^2}{2P \cdot q}$ fixed
 $Q^2 \rightarrow \infty$

Rest frame :

$$P = (M, 0, 0, 0)$$

$$q = (\nu/M, 0, 0, -(\nu^2/M^2 - Q^2)^{1/2})$$

$$\lim_{Bj} q = (\nu/M, 0, 0, -\nu/M + M Q^2/2\nu) \quad q^\pm = \frac{1}{\sqrt{2}} (q^0 \pm q^3)$$

$$= (\nu/M, 0, 0, -\nu/M + Mx) \quad a \cdot b = a^+ b^- + \underbrace{a^-}_{-a_\perp} \underbrace{b^+}_{b_\perp}$$

$$\underline{q^+ = \frac{Mx}{\sqrt{2}}} \quad \underline{q^- = \sqrt{2} \frac{\nu}{M}} \quad \underline{P^+ = P^- = M/\sqrt{2}}$$

So Bj-limit is single component of $q^\mu \rightarrow \infty$ SIMPLE

- $q^- \rightarrow \infty \Rightarrow \xi^+ \rightarrow 0$ uncertainty principle

What coordinate is ξ^+ ?

It's a Compton scattering coord!



- $\xi^2 = 2\xi^+\xi^- - \xi_\perp^2 > 0$ (causality) $[J(\xi), J(0)]$

$$\xi^- \sim 1/q^+ \sim \sqrt{2}/Mx \quad \text{so} \quad \underline{\xi^2 \rightarrow 0 \text{ IF } x \not\rightarrow 0}$$

• Measurement

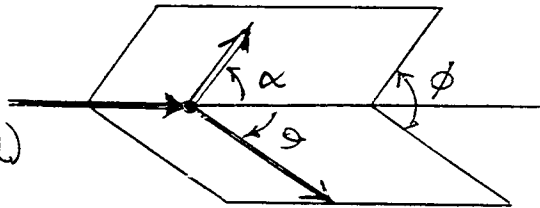
RLT Comments 14 (1990) 239.

Electron spin (helicity) asymmetry

$$d\sigma_R = d\bar{\sigma} + d\Delta\sigma$$

$$d\sigma_L = d\bar{\sigma} - d\Delta\sigma$$

$$d\bar{\sigma} \Rightarrow F_1 \neq F_2 \text{ (standard)}$$



$$\frac{d\Delta\sigma}{dx dy d\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \cos \alpha \left[\left(1 - \frac{y}{2} - \frac{y^2}{4}(K-1)\right) g_1 - \frac{y}{2}(K-1) g_2 \right] \right. \\ \left. - \sin \alpha \cos \phi \sqrt{(K-1) \left(1 - \frac{y}{2} - \frac{y^2}{4}(K-1)\right)} \left(\frac{y}{2} g_1 + g_2 \right) \right\}$$

NB $K-1 \sim \frac{1}{Q^2} \left(= \frac{4M^2 x^2}{Q^2} \right)^{1/2}$

- * Complication: electron beam is not parallel to virtual photon.
- * At any fixed polarization $\alpha \neq 90^\circ$, g_1 dominates and one measures $\approx (1-y/2) g_1(x, Q^2)$
- * Exactly at 90° g_1 term drops out and subdominant g_2 term is measured
- * g_2 term is multiplied by an extra $\frac{1}{\sqrt{Q^2}}$ ($\sim \sqrt{K-1}$) signalling non-leading twist.
- * This is the best & most important example where twist-3 dominates because twist-2 is kinematically suppressed.

B. INTRODUCTION TO QUARK & GLUON DISTRIBUTION & FRAGMENTATION FUNCTIONS

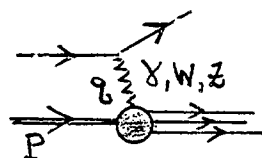
I. A GRAPHICAL GUIDE

Too much to do and too little time to start with first principles and develop everything mathematically. Instead motivate definitions pictorially.

Deep inelastic processes:

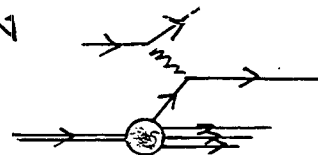
- large four-momentum transfer $Q^2 \gg \Lambda_{QCD}^2$
- unconstrained development

DEEP INELASTIC SCATTERING
 $lP \rightarrow l'X$

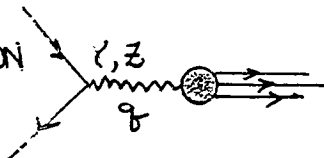


QCD/PARTON FACTORIZATION

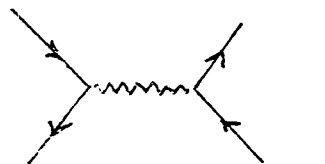
$$Q^2, P \cdot q \rightarrow \infty$$



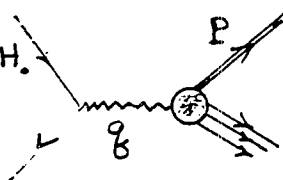
e^+e^- ANNIHILATION
 $e^+e^- \rightarrow X$



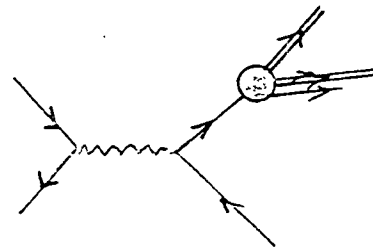
$$Q^2 \rightarrow \infty$$



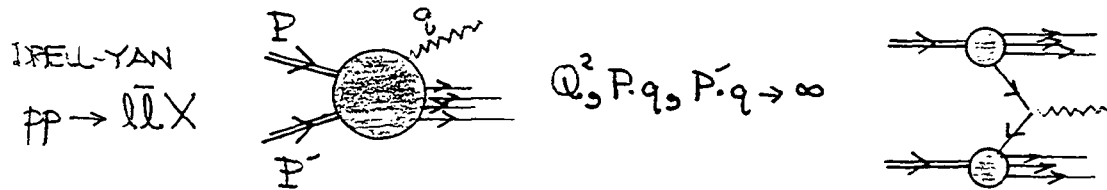
SINGLE PARTICLE INCLUSIVE ANNIH.
 $e^+e^- \rightarrow hX$



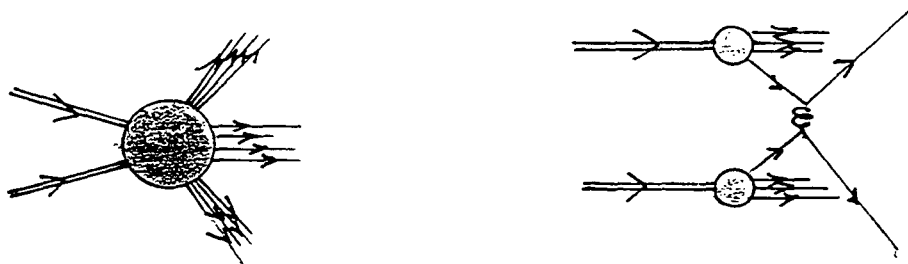
$$Q^2, P \cdot q \rightarrow \infty$$



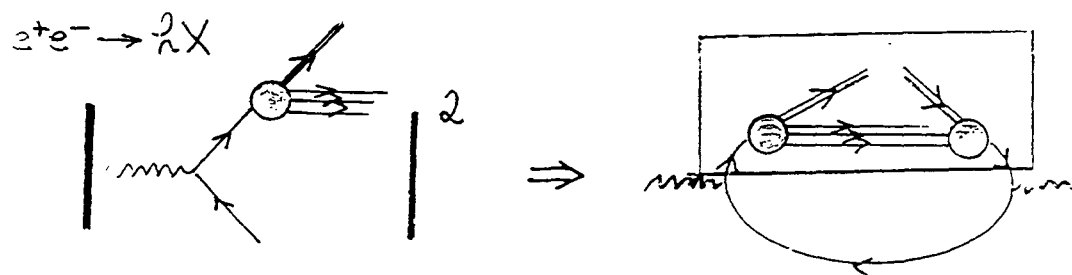
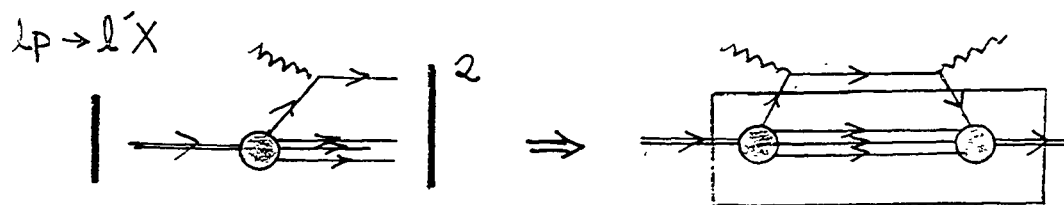
More processes (suppress lepton lines $\rightarrow$)



2 JET PRODUCTION
 $pp \rightarrow \text{jet} + \text{jet} + X$

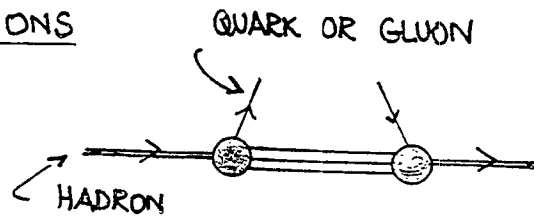


Experimenters measure cross sections

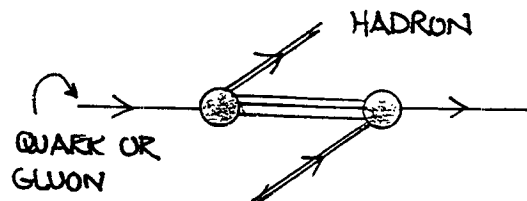


Basic building blocks

DISTRIBUTION FUNCTIONS

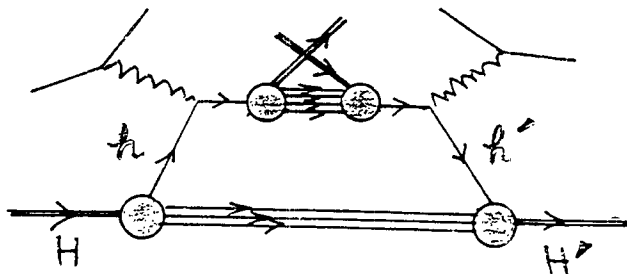


FRAGMENTATION FUNCTIONS



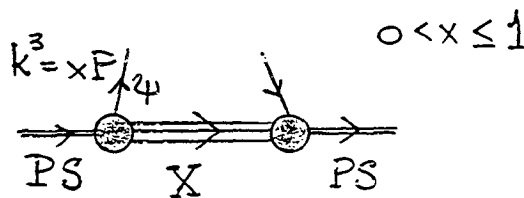
Cross sections for deep inelastic processes in QCD are composed of distribution & fragmentation functions $\otimes$ hard parton cross sections $\otimes$ QCD radiative corrections

Example SINGLE HADRON INCLUSIVE ELECTROPRODUCTION $ep \rightarrow ehX$



- BUILDING BLOCKS
- HELICITY DENSITY MATRIX FORMALISM

2. FROM GRAPHS TO EQUATIONS



RLT IN "RELATIVISTIC
DYNAMICS & QUARK
NUCLEAR PHYSICS"
(NILEY, 1988)

$$P(x, P, S, \eta) = \sum_X \delta(k^3 - xP^3) |\langle X | \bar{\eta} \psi | P, S \rangle|^2$$

SUM ON UNOBSERVED
HADRONS

↑
QUARK WITH
LONGITUDINAL
MOMENTUM
FRACTION x

↑
↑
DIRAC FIELD FOR QUARK
↑
↑
ARBITRARY SPINOR TO
ABSORB DIRAC INDICES
↔ REST OF DIAGRAM

NOTE:

- $P^2 = M^2$, $S^2 = -M^2$, $P \cdot S = 0$
- Formulated in INFINITE MOMENTUM FRAME
 $P = (\sqrt{P^2 + M^2}, 0, 0, P)$ $P \rightarrow \infty$
- Only $k^3 = xP^3 = xP$ of quark is observed
 $\sum_X$ includes other components of k^μ
- P is a probability, $P \geq 0$
 η (a four component Dirac spinor, selects quark spin state)

TOWARD A FRAME INDEPENDENT FORMULATION OF $\mathcal{F}$.

ASIDE ON LIGHT-CONE COORDINATES

$$a^\mu = (a^0, a^1, a^2, a^3) \text{ a lorentz 4-vector}$$

$$a^+ = \frac{1}{\sqrt{2}} (a^0 + a^3) \quad a^- = \frac{1}{\sqrt{2}} (a^0 - a^3)$$

$$a_\mu a^\mu = 2a^+a^- - \vec{a}_\perp^2 = (a^0)^2 - \vec{a}^2$$

$$a^\perp{}^\mu = (0, a^1, a^2, 0)$$

TARGET MOMENTUM $P^\mu \approx (p + \frac{M^2}{2p}, 0, 0, p)$

$$\begin{array}{l} P^+ \approx \sqrt{2} p \rightarrow \infty \\ P^- \approx \frac{M^2}{2\sqrt{2}p} \approx 0 \end{array}$$

P^μ has only one non-trivial component P^+

DEFINE TWO "UNIT" VECTORS

$$p^\mu = \frac{p}{\sqrt{2}} (1, 0, 0, 1)$$

$$p^2 = 0$$

$$n^\mu = \frac{1}{\sqrt{2}p} (1, 0, 0, -1)$$

$$n^2 = 0$$

$$p \cdot n = 1$$

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu$$

$$S^\mu = (S \cdot n) p^\mu + (S \cdot p) n^\mu + S_\perp^\mu$$

Delta function in $\frac{d^4k}{(2\pi)^4}$

$$k^3 = x P^3 \quad (P^3 = p \rightarrow \infty)$$

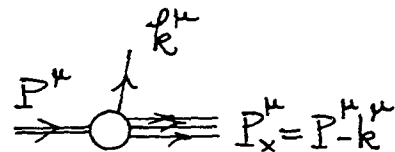
$$\Rightarrow k \cdot n = x P \cdot n = x$$

$\int$

$$f(x, P, S, \eta) = \sum_X \delta(k \cdot n - x) \langle PS | \bar{\psi} \eta | X \rangle \langle X | \bar{\eta} \psi | PS \rangle$$

$$\sum_X \delta(k \cdot n - x) = \sum_X \int \frac{d\lambda}{2\pi} e^{i\lambda x - i\lambda k \cdot n}$$

$$k \cdot n = (P - P_x) \cdot n$$



$$e^{i(P_x - P) \cdot n \lambda} \langle X | \bar{\eta} \psi | P, S \rangle = \langle X | \bar{\eta} \psi(\lambda n) | PS \rangle$$

Perform $\sum_X$

$$f(x, P, S, \eta) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi}(0) \eta, \bar{\eta} \psi(\lambda n) | P, S \rangle$$

QUARK DISTRIBUTION FUNCTION IS FOURIER TRANSFORM OF LIGHT-CONE CORRELATION FUNCTION

- Starting point for field theoretic analysis
- Frame independent! $p = M/\sqrt{2} \rightarrow$ rest frame
- Light cone physics controls distribution functions

I. STANDARD PARTON DISTRIBUTIONS

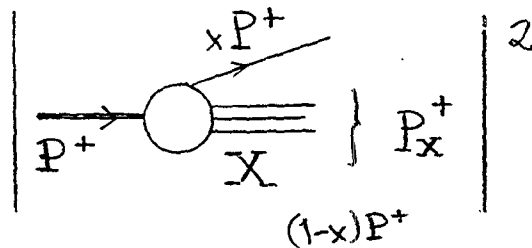
• A PARTON DISTRIBUTION

$$f(x) = \sum_X \delta(P_X^+ - (1-x)P^+) \left| \langle X | \psi_+(0) | P \rangle \right|^2$$

LIGHT CONE
MOMENTUM DISTR.

"GOOD" COMPONENT
OF QUARK FIELD

↑
TO BE EXPLAINED
SHORTLY



- PROBABILITY
- SELECTS OUT SPECIAL DIRECTION! $\sqrt{2}P^+ = P^0 + P^3$
DETERMINED BY EXPT.
- DISTRIBUTION IN $x = x_{Bj}$
- Q^2 DEPENDENCE (FROM QCD RADIATIVE CORRECTIONS)
IS SUPPRESSED.

• IN COORDINATE SPACE

$$f(x) = \frac{1}{2\pi} \int dx^- e^{iP^+x^-x} \langle P | \psi_+^+(0) \psi_+(x^-) | P \rangle$$

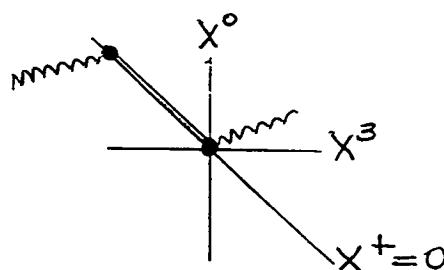
$x^+ = 0$
 $x_\perp = 0$

$$f(x) = \frac{1}{2\pi} \int dx^- e^{iP^+ x^- x} \langle P | \psi_+^\dagger(0) \psi_+(x^-) | P \rangle$$

• GAUGE INVARIANCE :

NOT MANIFEST. ACTUALLY (*) IS THE FORM
OF A MANIFESTLY G.I. EXPRESSION IN $A^+=0$ GAUGE

$$\psi_+^\dagger(0) \psi_+(x^-) \implies \psi_+^\dagger(0) \underbrace{\mathbb{P} \exp i \int_0^{x^-} dy^\mu A_\mu(y)}_{\text{WILSON LINE ON L.C.}} \psi_+(x^-)$$



• LORENTZ INVARIANCE

$$\psi_+^\dagger(0) \dots \psi_+(x^-) = \bar{\psi}(0) \gamma^+ \dots \psi(x^-)$$

$\gamma^+ dx^-$ ≠ ALL ELSE IS BOOST INVARIANT
ALONG x^3 DIRECTION

$$f(x) = \frac{1}{2\pi} \int dx^- e^{i(P^+ x^-)x} \langle P | \bar{\psi}(0) \gamma^+ \mathbb{P} e^{i \int_0^{x^-} A \cdot dy} \psi(x^-) | P \rangle$$

NB: Path dependence

• AND LOCAL OPERATORS

$$\int_{-1}^1 dx x^{n-1} f(x) = A_n$$

$$S \langle P | \bar{\psi} \gamma^{\mu_1} i D^{\mu_2} \dots i D^{\mu_n} \psi | P \rangle = S(P^{\mu_1} \dots P^{\mu_n}) A_n$$

$S = \text{SYMMETRIZE} \neq \text{REMOVE TRACES}$
 $(S(P^{\mu_1} P^{\mu_2}) = P^{\mu_1} P^{\mu_2} - g^{\mu_1 \mu_2} M^2/4)$

- OPERATORS ARE G.I., DEFINITE TWIST ($t=2$), WITH
 NEW DEFINED ANOMALOUS DIMENSIONS (HENCE Q^2 -
 EVOLUTION)

• SUMMARY: AN IDEAL PARTON DISTRIBUTION

- WEIGHTED LIGHT-ONE MOMENTUM PROBABILITY DISTRIBUTION
 (WEIGHTED BY SPIN, FLAVOR, TRANSVERSITY, ETC.)
- MANIFESTLY G.I. BI (OR MULTI-) LOCAL LIGHT-ONE
 FOURIER TRANSFORM
- TOWER OF LOCAL, GAUGE INVARIANT OPERATORS
 OF DEFINITE TWIST

• EXAMPLES OF IDEAL PARTON DISTRIBUTIONS

• MOMENTUM (p^+) DISTRIBUTION

$$q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \gamma \psi(\lambda n) | P \rangle$$

$$\phi = \frac{1}{\sqrt{2}} (p, 0_{\perp}, p)$$

• HELICITY DISTRIBUTION

$$\Delta q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_{11} | \bar{\psi}(0) \gamma_5 \psi(\lambda n) | PS_{11} \rangle$$

$$n = \frac{1}{\sqrt{2}} \left(\frac{1}{p}, 0_{\perp}, -\frac{1}{p} \right)$$

• TRANSVERSITY DISTRIBUTION

$$S q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_{\perp} | \bar{\psi}(0) [\not{x}_{\perp}, \gamma] \gamma_5 \psi(\lambda n) | PS_{\perp} \rangle$$

• GLUON MOMENTUM DISTRIBUTION

$$G(x) = \int \frac{d\lambda}{4\pi x} e^{i\lambda x} \langle P | n_{\mu} F^{\mu\alpha}(0) n_{\nu} F^{\nu}_{\alpha}(\lambda n) | P \rangle$$

• NOT SO IDEAL PARTON DISTRIBUTIONS

• HIGHER TWIST DISTRIBUTIONS

• GLUON SPIN DISTRIBUTION

• TRANSVERSE MOMENTUM $\neq$ ANGULAR MOMENTUM DISTRIBUTIONS

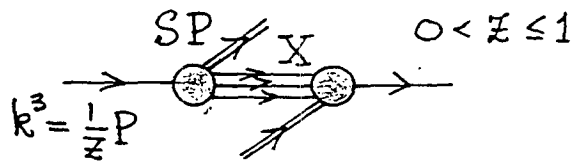
III GENERALIZATIONS, DIFFICULTIES & FAQ

- Generalization to fragmentation functions

JOLINS & SOPER NP B152, 109 (1982)

SLIT & JI PRL 71, 2547 (1993)

SI PRD 49, 114 (1992)



$$\hat{f}(z, P, S, \eta) = \sum_X \delta(k^3 - \frac{1}{z} P) \langle 0 | \bar{\eta} \psi | PS; X \rangle \langle PS; X | \bar{\psi} \eta | 0 \rangle$$

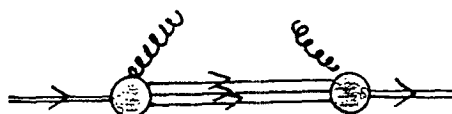
After similar algebra

$$\hat{f}(z, P, S, \eta) = \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \bar{\eta} \psi(0) | PS; X \rangle \langle PS; X | \bar{\psi}(\lambda n) \eta | 0 \rangle$$

FRAGMENTATION FUNCTION IS ALSO A L-C CORRELATION FUNCTION, BUT INHERENTLY MORE COMPLICATED BECAUSE $\sum_X$ CANNOT BE PERFORMED

- Generalization to gluons

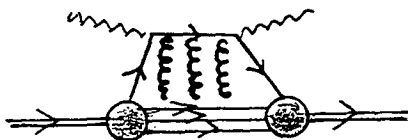
$$G(x, P, S, \eta) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \eta^{\mu\nu} G_{\mu\nu}(0) \eta^{\alpha\beta} G_{\alpha\beta}(\lambda n) | PS \rangle$$



- Gauge invariance

- $\bar{\Psi}(0) \eta \bar{\eta} \Psi(\lambda n)$ is not gauge invariant ?

- Such problem usually means a class of diagrams has been omitted -



- The "active" quark propagates in a gluon background field

$$\bar{\Psi}(0) \eta \bar{\eta} \Psi(\lambda n) \Rightarrow \bar{\Psi}(0) \eta P(e^{i \int_{\lambda}^0 d\beta n \cdot A(\beta n)}) \bar{\eta} \Psi(\lambda n)$$

$P()$ IS WILSON LINK FROM $\lambda n \rightarrow 0$
SUMS UP GLUON INSERTIONS

- Simplify with gauge choice

$$\underline{n \cdot A = 0} \quad \Leftrightarrow \quad \underline{\text{LIGHT-CONE GAUGE}}$$

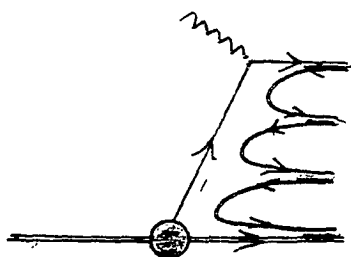
$P() \Rightarrow 1$ SO GAUGE INVARIANT RESULT REDUCES TO

$$\bar{\Psi}(0) \eta \bar{\eta} \Psi(\lambda n)$$

IN L-C GAUGE

- "But the intermediate state $|X\rangle$ is colored!"

Soft hadronization physics is assumed not to spoil dynamical predictions. What actually happens



Hadronization occurs at a much later time scale $\sim 1/\Lambda_{\text{QCD}}$ after hard scattering. "Lead brick effect"

- But $\bar{\Psi}(0)\eta\bar{\eta}\Psi(\lambda n)$ doesn't exist in QCD ?!

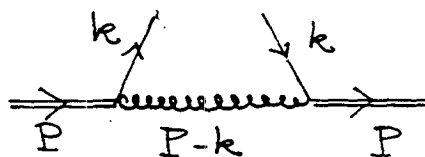
eg. CHENG & LI OR OTHER STANDARD TEXT

In the Σ_X is an integral over the other 3 components of the quark (or gluon) momentum. This integral generally diverges in renormalizable field theories

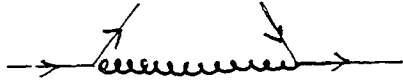
$$f(x, P, S, \eta) = \int d^4k \delta(k \cdot n - x) \sum_X' \langle PS | \bar{\Psi} \eta | X \rangle \langle X | \bar{\eta} \Psi | PS \rangle$$

Example

$$f \sim \int \frac{d^2k_{\perp}}{k_{\perp}^2} \sim \log \Lambda^2$$



Aside



$$f(x, \dots) \propto \int \frac{d^4 k}{(2\pi)^4} \bar{U}(PS) \gamma^\mu \frac{1}{\not{k}} \eta \bar{\eta} \frac{1}{\not{k}} \gamma_\mu U(PS) \delta(k \cdot n - x)$$

$$\otimes \delta((P-k)^2) + \text{gauge terms}$$

$$\text{Brief calculation} \Rightarrow \int \frac{dk_\perp^2}{k_\perp^2}$$

Remedy - In physical processes divergence is cut off by physical momentum transfer - eg. Q^2 in deep inelastic electron scattering

$$\int dk_\perp^2 \Rightarrow \int_{\frac{Q^2}{\lambda^2}}^{Q^2} dk_\perp^2$$

- $f(x, P, S, \eta)$ depends logarithmically on Q^2

$$\bar{\Psi}(0) \eta \bar{\eta} \Psi(\lambda n) \Rightarrow \bar{\Psi}(0) \eta \bar{\eta} \Psi(\underline{x}_\perp \approx \frac{1}{Q^2}, x^- = \lambda)$$

Not quite local on light-cone

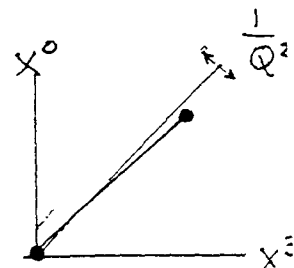
Light-cone correlator doesn't actually exist ($\underline{x}_\perp = 0$)

Problem is logarithmic and well-understood

- $\ln q^2 \iff$ QCD radiative corrections

$$f(x, PS, \eta) \rightarrow f(x, \ln Q^2, PS, \eta)$$

$$\bar{\Psi}(0) \eta \bar{\eta} \Psi(\lambda n) |_{\sim}$$



SUMMARY - DISTRIBUTION FUNCTIONS

QUARKS

$$q(x, Q^2) = \frac{1}{\sqrt{2}P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \psi_+^\dagger(0) \mathcal{P}(0, \lambda n) \psi_+(\lambda n) | P \rangle \Big|_{Q^2}$$

$$\Delta q(x, Q^2) = \frac{1}{\sqrt{2}P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_z | \psi_+^\dagger(0) \mathcal{P}(0, \lambda n) \gamma_5 \psi_+(\lambda n) | PS_z \rangle \Big|_{Q^2}$$

$$\delta q(x, Q^2) = \frac{1}{\sqrt{2}P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \psi_+^\dagger(0) \mathcal{P}(0, \lambda n) \gamma_\perp \gamma_5 \psi_+(\lambda n) | PS_\perp \rangle \Big|_{Q^2}^*$$

$q(x, Q^2)$, $\Delta q(x, Q^2) \neq \delta q(x, Q^2)$ describe all quark inclusive hard physics that survives at large- Q^2 (twist-2)

$$q(x, Q^2) \Leftrightarrow f_1(x, Q^2) \text{ SPIN AVERAGE}$$

$$\Delta q(x, Q^2) \Leftrightarrow g_1(x, Q^2) \text{ LONGITUDINAL SPIN}$$

$$\delta q(x, Q^2) \Leftrightarrow h_1(x, Q^2) \text{ TRANSVERSE SPIN}^*$$

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu$$

NOTE: NO g_2 - SUBDOMINANT
 $h_1(x, Q^2)$ DECOUPLES FROM
 INCLUSIVE D.I.S.

$$\mathcal{P}(0, \lambda n) = \mathbb{P} \left(\exp i \int_{\lambda n}^0 d\xi^\mu A_\mu(\xi) \right)$$

$$\psi_+ = \mathbb{P}_+ \psi \quad \mathbb{P}_\pm = \frac{1}{2} \gamma^\mp \gamma^\pm^* \quad \gamma^\pm = \frac{\gamma^0 \pm \gamma^3}{\sqrt{2}}$$

*

* GLUONS

$$G(x, Q^2) = -\frac{i}{\sqrt{2} x p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | F^{+\alpha}(0) \rho(0, \lambda n) F_{\alpha}^+(\lambda n) | P \rangle \Big|_{Q^2}$$

$$\Delta G(x, Q^2) = \frac{i}{\sqrt{2} x p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_z | F^{+\alpha}(0) \rho(0, \lambda n) \tilde{F}_{\alpha}^+(\lambda n) | PS_z \rangle \Big|_{Q^2}$$

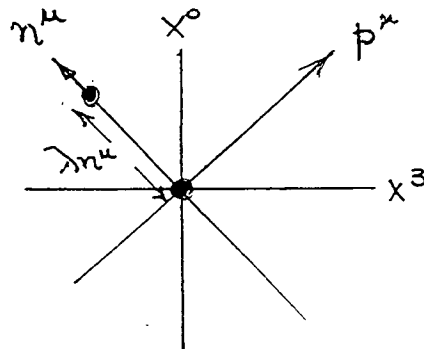
$G(x, Q^2) \Leftrightarrow$ SPIN AVERAGE GLUON DISTRIBUTION
 $\Delta G(x, Q^2) \Leftrightarrow$ LONGITUDINAL SPIN DISTRIBUTION *

• LEARNING FROM THE LIGHT CONE

All the quantities of interest appear as correlation functions at light-like coordinate separation

Suspect there is something fundamental to learn from formulating QCD on the light cone.

Large subject, long history.
 Small piece here.



* MORE ABOUT THIS LATER

* IF WE QUANTIZE DIRAC FIELD ON SURFACES OF CONSTANT ξ^+ , THEN CORRELATION FUNCTION WILL REDUCE TO A DISTRIBUTION OF FOURIER COMPONENTS OF INDEPENDENT DEGREES OF FREEDOM JUST AS EQUAL TIME CORR. FN. REDUCES TO A MOMENTUM DISTRIBUTION *

Formal basis for extension of parton model to spin dependent & higher twist effects

LIGHT CONE QUANTIZATION

$$\gamma^\pm \equiv \frac{1}{\sqrt{2}} (\gamma^0 \pm \gamma^3) \quad \underline{\gamma}^\perp = (\gamma^1, \gamma^2)$$

Useful projection operators

$$\underline{P}_\pm \equiv \frac{1}{2} \gamma^\mp \gamma^\pm$$

$$P_+ \psi \equiv \psi_+$$

$$P_- \psi \equiv \psi_-$$

$$P_+ P_- = P_- P_+ = 0$$

$$P_\pm^2 = P_\pm$$

$$P_+ + P_- = 1$$

$\psi_+ \neq \psi_-$ ARE TWO COMPONENT SPINOR FIELDS

$\psi_+ \Leftrightarrow$ "GOOD" COMPONENTS OF ψ

$\psi_- \Leftrightarrow$ "BAD" COMPONENTS OF ψ

NEW NOTATION	$\psi_+ \Rightarrow \phi$	$\psi_- \Rightarrow \chi$

Importance of $\psi_{\pm}$: At constant x^+ , x^+ is evolution variable
analogous to time in usual formulation

Using $P^{\pm}$, Dirac equation

$$\frac{\partial}{\partial x^+} \psi_+ = \dots$$

$$(i\not{\partial} - g\not{A})\psi = m\psi$$

$$i \frac{\partial}{\partial x^-} \psi_- = g \cancel{A_-} \psi_- + \frac{1}{2} [(i\partial_j - gA_j)\gamma^j + m] \gamma^0 \psi_+$$

GAUGE $n \cdot A = 0 \rightarrow n^- A_- = 0 \Rightarrow \underline{A_- = A^+ = 0}$

EQN. FOR ψ_- DOES NOT INVOLVE x^+ -EVOLUTION.

IT IS A CONSTRAINT

LIKEWISE FOR $A^- \equiv A_+$

$$i \frac{\partial}{\partial x^-} \psi_- = F[\underline{A_+}, \psi_+]$$

SO ψ_- IS NOT AN INDEPENDENT FIELD — IT IS A
FUNCTIONAL OF $\psi_+ \neq \underline{A_+}$

- ψ_+ AND $\underline{A_+}$ ARE INDEPENDENT CANONICAL VARIABLES
- ψ_- AND A_+ ARE DEPENDENT AND SHOULD
BE ELIMINATED FROM FORMULAS TO DISPLAY
THEIR PARTICLE CONTENT

ON TO SPIN & RELATED ISSUES

$\psi_{\pm}$ are still two component spinors. Decompose this spin space. Choose a suitable set of γ -matrices

$$\gamma^0 \equiv \rho_1 \sigma_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \quad \gamma^2 = i\sigma_2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}$$

$$\gamma^1 \equiv i\sigma_1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix} \quad \gamma^3 = -i\rho_2 \sigma_3 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \rho_3 \sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

Why? $P^+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $P^- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $\sigma^3 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$

So $\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$ Upper $\equiv$ good Lower $\equiv$ bad

NOTE: γ^1 , γ^2 , γ_5 $\neq$ σ_3 ALL COMMUTE w/ $P_{\pm}$

CHOICES ① DIAGONALIZE γ_5 (AND σ_3)
HELICITY OR CHIRALITY BASIS

② DIAGONALIZE γ_1 OR γ_2
TRANSVERSITY BASIS

① $\sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

$\gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$

$\psi = \begin{pmatrix} \phi_+ \\ \phi_- \\ \chi_+ \\ \chi_- \end{pmatrix}$ GOOD
± ARE
HELICITY
LABELS
BAD

2. ROTATE SO γ' IS DIAGONAL

$$Q_{\pm} = \frac{1}{2} (1 \pm \gamma^1 \gamma^5)$$

$$\psi = \begin{pmatrix} \phi_+ \\ \phi_- \\ \chi_+ \\ \chi_- \end{pmatrix} \quad \begin{matrix} \pm \text{ ARE} \\ \text{TRANSVERSITY} \\ \text{LABELS} \end{matrix}$$

$$Q_+ \phi_+ = \phi_+ \quad \text{POSITIVE TRANSVERSITY}$$

$$Q_- \phi_- = \phi_- \quad \text{NEGATIVE TRANSVERSITY}$$

Of course ϕ_+ and ϕ_- are linear combinations of $\phi_{\pm}$.
Also γ^2 could be equally well used to define transversity

SUMMARY

COMPONENT	HELICITY	CHIRALITY	EFFECTIVE TWIST*
ϕ_+	$+\frac{1}{2} \uparrow$	$+ \text{ R}$	1
ϕ_-	$-\frac{1}{2} \downarrow$	$- \text{ L}$	1
χ_+	$+\frac{1}{2} \uparrow$	$- \text{ L}$	2
χ_-	$-\frac{1}{2} \downarrow$	$+ \text{ R}$	2

* See subsequent discussion

$\uparrow \downarrow$ Alternative notation for helicity

L.R Alternative notation for chirality. 94

e43

WHY DO ϕ_+ AND χ_+ HAVE SAME HELICITY
BUT OPPOSITE CHIRALITY?

$$\left[\text{NOTE} \quad \sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \right]$$

ANSWER — $\chi_{\pm}$ ARE ACTUALLY QUARK-GLUON
COMPOSITE FIELDS

$$\chi_+ \propto (\phi \tilde{A}_+)_+$$

$(\phi \tilde{A}_+)_+$ IS QUARK + GLUON IN HELICITY $+\frac{1}{2}$
STATE:

$$\begin{array}{c} \uparrow \\ \oplus \\ \downarrow \end{array}$$

$$+1 - \frac{1}{2} = +\frac{1}{2}$$

SO χ_+ REALLY CONTAINS $\phi_- \Rightarrow$ ODD CHIRALITY.

(IMPORTANT FOR SORTING OUT TWIST)

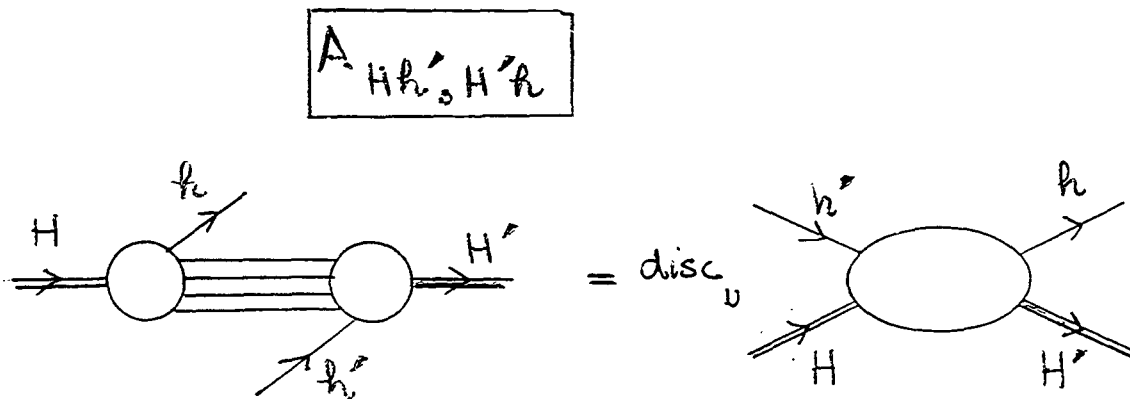
C. QUARK & GLUON DISTRIBUTION & FRAGMENTATION FUNCTIONS IN A HELICITY BASIS

$q, \Delta q, \delta q$ all seem to be described as helicity projections of quark-hadron scattering

- * quark $h = \pm 1/2$ "good" light-cone components
- * gluon $h = \pm 1$ "good" light-cone components
- * nucleon $H = \pm 1/2$

K. CHEN, G. GOLDSTEIN
RLJ, X. JI, Y.P. B 445 (1995)
380

RLJ, X. JIN & J. TANG
PHYS REV LETT 20 (1992) 1166
PHYS REV D57 (1998) 5920



• CONSERVATION OF J_3 : $H + h' = H' + h$

• PARITY $A_{Hh', H'h} = A_{-H-h', -H'-h}$

• T-INVARIANCE* $A_{Hh', H'h} = A_{H'h, Hh'}$

- QUARKS IN THE NUCLEON

$$A_{\frac{1}{2} \frac{1}{2}, \frac{1}{2} \frac{1}{2}} = q(x, Q^2) + \Delta q(x, Q^2)$$

$$A_{\frac{1}{2} -\frac{1}{2}, \frac{1}{2} -\frac{1}{2}} = q(x, Q^2) - \Delta q(x, Q^2)$$

$$A_{\frac{1}{2} -\frac{1}{2}, -\frac{1}{2} \frac{1}{2}} = \delta q(x, Q^2)$$

- GLUONS IN THE NUCLEON

$$A_{\frac{1}{2} 1, \frac{1}{2} 1} = G(x, Q^2) + \Delta G(x, Q^2)$$

$$A_{\frac{1}{2} -1, \frac{1}{2} -1} = G(x, Q^2) - \Delta G(x, Q^2)$$

- HELICITY DENSITY MATRIX FORMALISM

Gather non-vanishing helicity amplitude into a matrix notation

QUARK-NUCLEON:

$$A(x, Q^2) = q(x, Q^2) \mathbb{I} \otimes \mathbb{I} + \Delta q(x, Q^2) \sigma_3 \otimes \sigma_3 + \delta q(x, Q^2) (\sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2)$$

TOTAL
QUARK/
NUCLEON
SPIN
STRUCTURE
ATTWIST-2

GLUON-NUCLEON:

$$A(x, Q^2) = G(x, Q^2) \mathbb{I} \otimes \mathbb{I} + \Delta G(x, Q^2) \sigma_3 \otimes \Sigma_3$$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2 DIRAC COVARIANTS, DIMENSION & TWIST

Return to $\bar{\psi}_i \psi_j$ and construct $\bar{\psi} T \psi$ for $T=1, \gamma_5, \dots$

ASIDE ON DIMENSIONS

Dimension means "engineering dimension" or units in $[\text{mass}]^d$

Operator Θ , notation $[\Theta] = m^{d_\Theta}$

- Examples
- $S = \int d^4x \mathcal{L} \Rightarrow [\mathcal{L}] = m^4 \Rightarrow d_{\mathcal{L}} = 4$
 - $[\bar{\psi} \psi] = m^4 \Rightarrow d_{\psi} = 3/2$
 - $[G_{\mu\nu} G^{\mu\nu}] = m^4 \Rightarrow d_G = 2$
 - $d_p = 1 \quad d_n = -1$
 - $\langle P|P' \rangle = (2\pi)^3 2E \delta^3(\vec{P} - \vec{P}')$
 $\Rightarrow [\langle P|P' \rangle] = m^{-2}$

Dimensional consistency requires

$$\underline{[\langle PS | \bar{\psi}_i(0) \psi_j(\lambda n) | PS \rangle]} = m^1$$

↑
Dimensions can be made up from powers of p^μ , n^μ , S^μ and target mass.

POWER COUNTING (TWIST)

- Looking for largest contribution as momentum transfers $\rightarrow \infty$
- Hard parton cross sections are scale invariant (up to logs)

$$\frac{d\hat{\sigma}}{da db \dots} \sim \frac{1}{Q^d} \quad \text{where } d \text{ is determined by dimensional analysis}$$

No masses in denominator

- Any masses introduced in f or $\hat{f}$ will persist through to end of calculation and suppress contribution by $\Theta(m/Q)^S$
- Leading contribution (no m/Q suppression) is "TWIST-2"
Each numerator factor of m increases suppression and twist by 1.

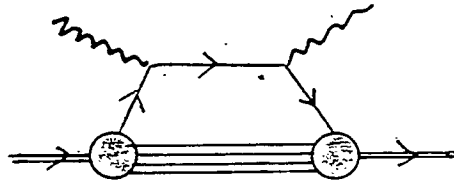
EXAMPLE $\Gamma \Rightarrow \gamma^\mu$

$$\begin{aligned} \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma^\mu \psi(\lambda n) | PS \rangle_{Q^2} \\ = 2 \left\{ \underbrace{f_1(x, Q^2)}_{\text{TWIST-2}} p^\mu + \underbrace{M^2 f_4(x, Q^2)}_{\text{TWIST-4}} n^\mu \right\} \end{aligned}$$

f_1 WILL EVENTUALLY CONTRIBUTE AT LEADING ORDER $\mathcal{O}(1)$.

f_4 WILL CONTRIBUTE AT ORDER $1/Q^2$.

DEMONSTRATION (D.I.S.)



$$W_{\mu\nu} = \frac{1}{4} \text{Tr}(\gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma) \frac{(q+xP)^\rho}{2P \cdot q} f^\sigma$$

$$f^\sigma = \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle P | \bar{\psi}(0) \gamma^\sigma \psi(\lambda n) | P \rangle = 2 (f_1 p^\sigma + M^2 f_4 n^\sigma)$$

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu$$

$$P^2 = M^2$$

$$P \cdot q \cong \nu$$

$$q^\mu \cong \nu n^\mu - x p^\mu$$

$$q^2 = -2x\nu = -Q^2$$

$$\frac{1}{4} \text{Tr} \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma = g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}$$

$$W_{\mu\nu} = -g_{\mu\nu} \frac{q \cdot f}{2q \cdot P} + \left\{ \frac{(q^\mu + xP^\mu) f^\nu}{2q \cdot P} + (\mu \leftrightarrow \nu) \right\}$$

$$= -g_{\mu\nu} f_1(x, Q^2) + \frac{q^\mu P^\nu + q^\nu P^\mu}{2q \cdot P} f_1(x, Q^2) + x \frac{P^\mu P^\nu}{q \cdot P} f_1$$

DOMINANT ↗

$$+ g_{\mu\nu} \frac{x M^2}{q \cdot P} f_4 + \frac{q^\mu q^\nu M^2}{(q \cdot P)^2} f_4 + \dots$$

↖ SUPPRESSED

DECOMPOSITION INTO DISTRIBUTION FUNCTIONS

RLT $\neq$ XLT
NFB 375, 527 (1992)

SPIN $\frac{1}{2}$ TARGETS (suppress $\log Q^2$ dependence)

$$\frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_\mu \psi(\lambda n) | PS \rangle = 2 \left\{ f_1^{(2)}(x) p_\mu + f_4^{(4)}(x) n_\mu M^2 \right\}$$

$$\frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_\mu \gamma_5 \psi(\lambda n) | PS \rangle = 2 \left\{ g_1^{(2)}(x) S \cdot n p_\mu + g_T^{(2)}(x) S_{\perp\mu} + g_3^{(4)}(x) M^2 S \cdot n n_\mu \right\}$$

$$\frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle PS | \bar{\psi}(0) \psi(\lambda n) | PS \rangle = 2 M e^{(3)}(x)$$

$$\frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle PS | \bar{\psi}(0) \sigma_{\mu\nu} i \gamma_5 \psi(\lambda n) | PS \rangle = 2 \left\{ h_1^{(2)}(x) (S_{\perp\mu} p_\nu - S_{\perp\nu} p_\mu) / M + h_L^{(2)}(x) M (p_\mu n_\nu - p_\nu n_\mu) S \cdot n + h_3^{(4)}(x) M (S_{\perp\mu} n_\nu - S_{\perp\nu} n_\mu) \right\}$$

NOTE: IN DIMENSIONAL ANALYSIS $S_\perp$ COUNTS AS POWER
OF MASS IN NUMERATOR

THREE LEADING TWIST DISTRIBUTION FUNCTIONS f_1, g, h_1

THREE SUBDOMINANT $\mathcal{O}(1/Q)$ DISTR. FUNCTIONS e, g_T, h_L

THREE TWIST-4, $\mathcal{O}(1/Q^2)$ DISTR. FUNCTIONS — INDISTINGUISHABLE
FROM MULTIPARTICLE CORRELATIONS ARISING ELSEWHERE

V LEADING TWIST DISTRIBUTION FUNCTIONS

- $f_1(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \gamma \psi(\lambda n) | P \rangle$
- $g_1(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_{||} | \bar{\psi}(0) \gamma \gamma_5 \psi(\lambda n) | PS_{||} \rangle$
- $h_1(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_{\perp} | \bar{\psi}(0) [\not{x}_{\perp} \gamma] \gamma_5 \psi(\lambda n) | PS_{\perp} \rangle$

Decompose in light-cone chirality basis

$$f_1(x) = \frac{1}{x} \int d^2 k_{\perp} \langle P | \phi_R^+(x p, \underline{k}_{\perp}) \phi_R(x p, \underline{k}_{\perp}) + \phi_L^+(x p, \underline{k}_{\perp}) \phi_L(x p, \underline{k}_{\perp}) | P \rangle$$

$$g_1(x) = \frac{1}{x} \int d^2 k_{\perp} \langle P \hat{e}_3 | \phi_R^+(x p, \underline{k}_{\perp}) \phi_R(x p, \underline{k}_{\perp}) - \phi_L^+(x p, \underline{k}_{\perp}) \phi_L(x p, \underline{k}_{\perp}) | P \hat{e}_3 \rangle$$

$$h_1(x) = \frac{2}{x} \text{Re} \int d^2 k_{\perp} \langle P \hat{e}_1 | \phi_L^+(x p, \underline{k}_{\perp}) \phi_R(x p, \underline{k}_{\perp}) | P \hat{e}_1 \rangle$$

or in light-cone transversity basis

$$g_1(x) = \frac{2}{x} \text{Re} \int d^2 k_{\perp} \langle P \hat{e}_3 | \phi_T^+(x p, \underline{k}_{\perp}) \phi_{\perp}(x p, \underline{k}_{\perp}) | P \hat{e}_3 \rangle$$

$$h_1(x) = \frac{1}{x} \int d^2 k_{\perp} \langle P \hat{e}_1 | \phi_T^+(x p, \underline{k}_{\perp}) \phi_T(x p, \underline{k}_{\perp})$$

$$- \phi_{\perp}^+(x p, \underline{k}_{\perp}) \phi_{\perp}(x p, \underline{k}_{\perp}) | P \hat{e}_1 \rangle$$

LIGHT-CONE	HELICITY	CHIRALITY
GOOD	+	+
GOOD	-	-
BAD	+	-
BAD	-	+

TWIST-TWO - (GOOD - GOOD)

SCALING

HELICITY AVERAGE

$$(\uparrow\uparrow \Rightarrow \uparrow\uparrow + \downarrow\downarrow \Rightarrow \downarrow\downarrow)$$

unpolarized target & beam

chiral even



HELICITY DIFFERENCE

$$(\uparrow\uparrow \Rightarrow \uparrow\uparrow - \downarrow\downarrow \Rightarrow \downarrow\downarrow)$$

longitudinal spin asymmetry

chiral even



HELICITY FLIP

$$(\uparrow\downarrow \Rightarrow \downarrow\uparrow)$$

transverse spin asymmetry

chiral odd



TWIST-3 EXAMPLE : $e(x)$

- SPIN AVERAGE
- CHIRAL ODD

WHICH DIRAC STRUCTURE ?

$$2Me(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \psi(\lambda n) | P \rangle$$

- CLEARLY SPIN-AVERAGE, CLEARLY CHIRAL-ODD
- TWIST-3 $\Theta(M/Q)$ BY DIMENSIONAL ANALYSIS
- LIGHT-ONE HELICITY ANALYSIS

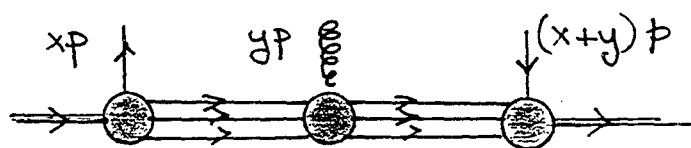
$$e(x) = \frac{1}{x} \int d^2 k_{\perp} \langle P | \phi_R^+(x p, k_{\perp}) \chi_R(x p, k_{\perp}) \\ + \chi_R^+(x p, k_{\perp}) \phi_R(x p, k_{\perp}) + (L \leftrightarrow R) | P \rangle$$

- REDUCTION TO INDEPENDENT FIELDS $\chi = F[\phi, A_{\perp}]$

$$e(x) = -\frac{1}{4Mx} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\phi}(0) i \not{x} \not{D}_{\perp}(\lambda n) \phi(\lambda n) | P \rangle + \text{H.C.}$$

- NO SIMPLE FOCK SPACE INTERPRETATION OF $\mathcal{P}(X)$

- GRAPHICALLY



- GENERALIZATIONS

- Each unit of twist adds independent fields

$$\phi^+ \phi \Leftrightarrow \text{TWIST 2} \quad \begin{array}{c} \uparrow \quad \downarrow \\ \text{---} \text{---} \end{array}$$

$$\phi^+ \chi \Leftrightarrow \text{TWIST 3} \quad \begin{array}{c} \uparrow \quad \varepsilon \quad \downarrow \\ \text{---} \text{---} \end{array}$$

$$\chi^+ \chi \Leftrightarrow \text{TWIST 4} \quad \begin{array}{c} \uparrow \quad \varepsilon \quad \varepsilon \quad \downarrow \\ \text{---} \text{---} \end{array}$$

- twist-3 is (barely) tractible, twist-4 is not yet.

III. SPIN DYNAMICS

SHORT EXAMPLES

A. SUM RULES

B. TRANSVERSE SPIN EFFECTS

C. $g_2 \neq \text{TWIST-3}$

D. A LOCAL OPERATOR FOR GLUON SPIN

A. SUM RULES

* GENERIC

$$f(x) = \frac{1}{\sqrt{2}} p^+ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \phi^\dagger(0) \Gamma \rho(0, \lambda n) \phi(\lambda n) | PS \rangle_{Q^2}$$

$$\begin{aligned} \int_{-\infty}^{\infty} dx f(x) x^n &= \dots \int dx \int \frac{d\lambda}{2\pi} x^n e^{i\lambda x} \dots \\ &= \dots \int dx \int \frac{d\lambda}{2\pi} \left(\frac{1}{i} \frac{\partial}{\partial \lambda} \right)^n e^{i\lambda x} \dots \\ &= \dots \int d\lambda \int \frac{dx}{2\pi} e^{i\lambda x} \left(i \frac{\partial}{\partial \lambda} \right)^n \dots \end{aligned}$$

$$\left(\int \frac{dx}{2\pi} e^{i\lambda x} = \delta(\lambda) \right)$$

$$= \dots \left(i \frac{\partial}{\partial \lambda} \right)^n \langle PS | \phi^\dagger(0) \Gamma \rho(0, \lambda n) \phi(\lambda n) | PS \rangle_{Q^2}$$

$$\int_{-\infty}^{\infty} dx x^n f(x, Q^2) = \dots \langle PS | \phi^\dagger(0) \Gamma \left(i D \cdot n \right)^n \phi(0) | PS \rangle_{Q^2}$$

$$\begin{aligned} \text{QUARKS} &\rightarrow f(x, Q^2) = -\bar{f}(-x, Q^2) \\ \text{GLUONS} &\rightarrow G(x, Q^2) = -G(-x, Q^2) \end{aligned} \quad \begin{aligned} &(\Delta q, \delta q \neq \Delta G \\ &\text{have various signs}) \end{aligned}$$

* EXAMPLES (STRIPPED OF QCD RADIATIVE CORRECTIONS)

• BJORKEN (1966) $\bar{\psi} \gamma^+ \gamma_5 \tau_3 \psi$

$$\int_0^1 dx [g_1^{ep}(x) - g_1^{en}(x)] = \frac{1}{6} \frac{g_A}{g_V} \propto \Delta u - \Delta d \quad \begin{array}{l} \text{ISOSPIN} \\ \text{QCD} \end{array}$$

• EWS-RJ

$$\int_0^1 dx \, g_1^{ep}(x) = \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \propto \langle \bar{\psi} \gamma^+ \gamma_5 \not{Q}^2 \psi \rangle$$

$$\propto \frac{1}{6} F + \frac{1}{18} D + \frac{1}{9} \Sigma \quad \begin{array}{l} \text{SU(3) SYMMETRY} \\ \text{QCD} \end{array}$$

$F \neq D \Leftrightarrow$ BARYON β -DECAY AXIAL CHARGES

$\Sigma \Leftrightarrow \langle \text{PSI} | \bar{\psi} \gamma^+ \gamma_5 \psi | \text{PS} \rangle = \text{SPIN FRACTION}$

• BURKHARDT-COTTINGHAM SR

$$\int_0^1 dx \, g_2(x) = 0$$

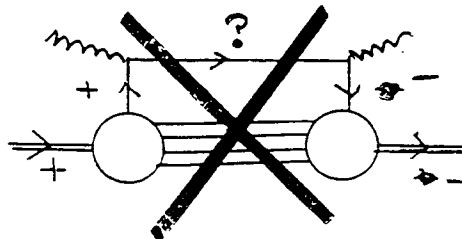
THERE IS NO OPERATOR
(SPIN-0, DIMENSION-3)
FOR R.H.S.

QCD
ABSENCE OF "J=0
FIXED POLE"

B. TRANSVERSE SPIN EFFECTS

$\approx$ S_q DECOUPLES FROM INCLUSIVE D.I.S.

$$S_q \propto A \begin{array}{cccc} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ & \uparrow & & \uparrow \\ & \text{QUARK} & & \\ & \text{HELICITY} & & \\ & \text{FLIP} & & \end{array}$$



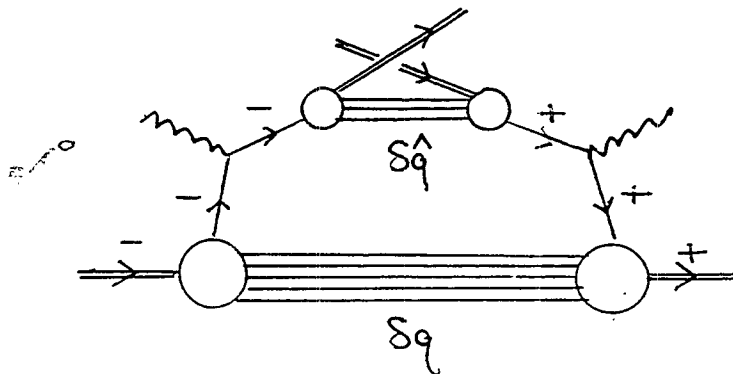
* S_q DOES NOT MIX WITH GLUONS

There is no twist-2 gluon helicity flip

$$\begin{array}{c} \text{X} \quad \frac{1}{2} \quad -1 \quad \longrightarrow \quad -\frac{1}{2} \quad +1 \\ \quad \uparrow \quad \uparrow \quad \quad \quad \uparrow \quad \uparrow \\ \quad \quad \Delta H = -1 \quad \quad \quad \Delta H = +2 \end{array}$$

* S_q DOMINATES TRANSVERSE SPIN EFFECTS IN HARD PROCESSES W/ TWO ACTIVE HADRONS

EX. $e\vec{P} \rightarrow eHx$



* $A_{\pi}/A_u \ll 1$ IN $\vec{p}\vec{p} \rightarrow 2 \text{ jets}$

- 2jet production is dominated by $gq \neq qq$ but there is no gluon transversity.

C. g_2 IS HIGHER TWIST & HAS NO SIMPLE PARTON INTERPRETATION

$$g_2 \propto \bar{\psi} \gamma^+ \gamma_5 \psi \rightarrow \psi_+^\dagger \gamma^+ \gamma_5 \psi_- + \psi_-^\dagger \gamma^+ \gamma_5 \psi_+$$

$$g_1 \propto \bar{\psi} \gamma^+ \gamma_5 \psi$$

BAD L.C. COMPONENTS
 $\Rightarrow$ HIGHER TWIST.

GLUONS WILL APPEAR:

$$\int_0^1 dx x^2 \bar{g}_2(x, Q^2) \propto \langle PS | \frac{1}{8} g \bar{\psi} \tilde{F}_{\sigma\mu} \gamma_\lambda \psi | PS \rangle$$

[SHURYAK-VAINSHTEYN S.R.]

- BAD NEWS: Nearly infinite number of papers claiming to "understand" g_2 in parton model are flawed (or at least very model dependent).
- GOOD NEWS: g_2 probes quark gluon correlations

D. A LOCAL OPERATOR FOR GLUON SPIN FRACTION

RLJ PHYS. LETT.

$$\Delta G(x, Q^2) = G_\uparrow(x, Q^2) - G_\downarrow(x, Q^2)$$

3365 (1996) 259

$$\int_0^1 dx \Delta G(x, Q^2) = \Delta G \quad \bullet \text{ WHAT IS RELATION TO GENERATOR OF GLUON SPIN ROTATIONS IN } M^{\mu\nu\lambda}?$$

- RHS SHOULD BE SPIN-1, DIM-3 BUT THERE IS NO LOCAL, GAUGE INVARIANT, $S=1, d=3$ GLUON OP.

I. WHY IS $g_2(x, Q^2)$ INTERESTING?

$g_2(x, Q^2)$ IS NOT THE QUARK TRANSVERSE SPIN DISTRIBUTION

NOTE: $g_1(x, Q^2)$ IS THE LONGITUDINAL SPIN DISTRIBUTION:

$$g_1(x) = \sum_a [q_a^\uparrow(x) - q_a^\downarrow(x)] e_a^2$$

WHERE $q_a^\uparrow$ ($q_a^\downarrow$) IS THE PROBABILITY TO FIND A QUARK w/ FLAVOR a , MOMENTUM FRACTION x $\neq$ HELICITY $\uparrow$ OR $\downarrow$ RELATIVE TO THE TARGET NUCLEON

I.1 USING OPERATOR PRODUCT EXPANSION $g_2(x, Q^2)$ CAN BE RELATED TO QUARK-GLUON OPERATORS:

$$\int_0^1 dx x^n g_1(x, Q^2) = \frac{1}{4} a_n(Q^2)$$

$$\int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{4} \frac{n}{n+1} (d_n(Q^2) - a_n(Q^2))$$

SHUR'AK & VAINSHTEIN

(EARLY WORK BY

AHMED & ROSS,

RATCLIFFE, WANDZURA

& WILCZEK)

$$\begin{aligned} \rightarrow g_2 &= \bar{g}_2 + g_2^{ww} \\ g_2^{ww} &= -g_1 + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \end{aligned}$$

$$\int_0^1 dx x^n \bar{g}_2(x, Q^2) = \frac{1}{4} \frac{n}{n+1} d_n(Q^2)$$

$$\langle PS | \Theta [\sigma_3 \{\mu_1\} \mu_2 \dots \mu_n] | PS \rangle \Big|_{Q^2} = \frac{d_n(Q^2)}{(n+1)} \left[(S_\sigma P_{\mu_1} - S_{\mu_1} P_\sigma) P_{\mu_2} \dots P_{\mu_n} \right. \\ \left. + (S_\sigma P_{\mu_2} - S_{\mu_2} P_\sigma) P_{\mu_3} \dots P_{\mu_n} + \dots \right]$$

- traces

$$\Theta [\sigma_3 \{\mu_1\} \mu_2 \dots \mu_n] = \mathcal{S}_n \left\{ \frac{g}{8} \sum_{l=0}^{n-2} i^{n-2} \bar{q} D_{\mu_1} \dots D_{\mu_l} \tilde{F}_{\mu_{l+1} \mu_{l+2}} D_{\mu_{l+3}} \dots D_{\mu_{n-1}} \gamma_{\mu_n} q \right. \\ \left. + \frac{g}{16} \sum_{l=0}^{n-3} i^{n-3} \bar{q} D_{\mu_1} \dots D_{\mu_l} (D_{\mu_{l+1}} \tilde{F}_{\mu_{l+2} \mu_{l+3}}) D_{\mu_{l+4}} \dots D_{\mu_{n-1}} \gamma_{\mu_n} \gamma_5 q \right. \\ \left. - \text{traces} \right\}$$

$$g \text{ (QCD COUPLING)} = 0 \Rightarrow \bar{g}_2 = 0$$

$\bar{g}_2$ IS INTERACTION DEPENDENT

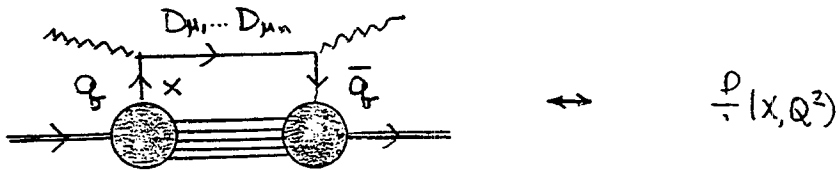
$$\int_0^1 dx \bar{g}_2(x, Q^2) = 0 \quad \text{B.C.S.R.}$$

* SIMPLEST NON-TRIVIAL CASE — $n=2$

$$\Theta [\sigma_3 \{\mu\} \nu] = \frac{g}{8} \bar{q} (\tilde{F}_{\mu\nu} \gamma_\nu + \tilde{F}_{\nu\mu} \gamma_\mu) q$$

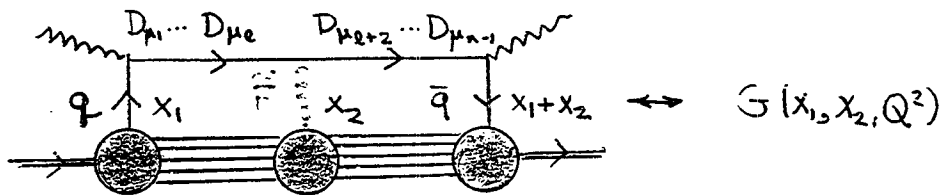
2.2 PARTON STRUCTURE

GENERIC TWIST-TWO STRUCTURE FUNCTION



GENERIC TWIST-THREE STRUCTURE FUNCTION

RLT 1983



THEN $g_T(x)$ IS OBTAINED FROM $G(x_1, x_2)$

$$g_T(x) = \frac{1}{2x} \int dy \{ G(x, y) - G(y, x) + \tilde{G}(x, y) - \tilde{G}(y, x) \}$$

$G(x, y)$ IS A "NATURAL" LIGHT-CONE DISTRIBUTION FUNCTION

$g_T(x)$ HAS INFORMATION INTEGRATED OUT (COMPRESSED)

NOTE: THE PROCESS $G(x_1, x_2) \Rightarrow g_T(x)$ IS NOT REVERSIBLE

III.3 THEORETICAL INTERLUDE:

9.

• EXPRESSIONS FOR $G(x,y)$ & $\tilde{G}(x,y)$

X.11 & RLJ

KINEMATICS: LIGHTCONE VARIABLES $p^\mu \neq n^\mu$

$$p^2 = n^2 = 0$$

$$p \cdot n = 1$$

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu$$

LIGHT CONE GAUGE $n \cdot A = 0$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x + i\mu(y-x)} \langle PS | \bar{q}(0) i D^\alpha(\mu n) \not{n} q(\lambda n) | PS \rangle$$

$$= 2i \epsilon^{\alpha\beta\mu\nu} n_\beta S_\mu p_\nu G(x,y) + \dots$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x + i\mu(y-x)} \langle PS | \bar{q}(0) i D^\alpha(\mu n) \not{n} \gamma_5 q(\lambda n) | PS \rangle$$

$$= 2S^\alpha \tilde{G}(x,y) + \dots$$

where ... denote other tensor structures of higher twist (> 3)

SUPPORT: ALWAYS "PARTONIC"

RLJ 1983

$$\left. \begin{array}{l} -1 < x < 1 \\ -1 < y < 1 \\ -1 < y-x < 1 \end{array} \right\} \text{Analysis shows these regions have} \\ \text{parton interpretations}$$

- AN EQUIVALENT -- AND DECEPTIVE -- EXPRESSION FOR $g_2(x, Q^2)$

40.

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda, n) | PS \rangle \Big|_{Q^2} = 2 \left\{ g_1(x, Q^2) p^\mu S \cdot n + g_T(x, Q^2) S_\perp^\mu + g_3(x, Q^2) S \cdot n n^\mu \right\}$$

Looks simple: g_1 and g_T both related to quark distribution alone.

But: $\bar{q} \gamma^+ \gamma_5 q \rightarrow q_+^+ \gamma_5 q_+$ is independent light cone distribution
whereas

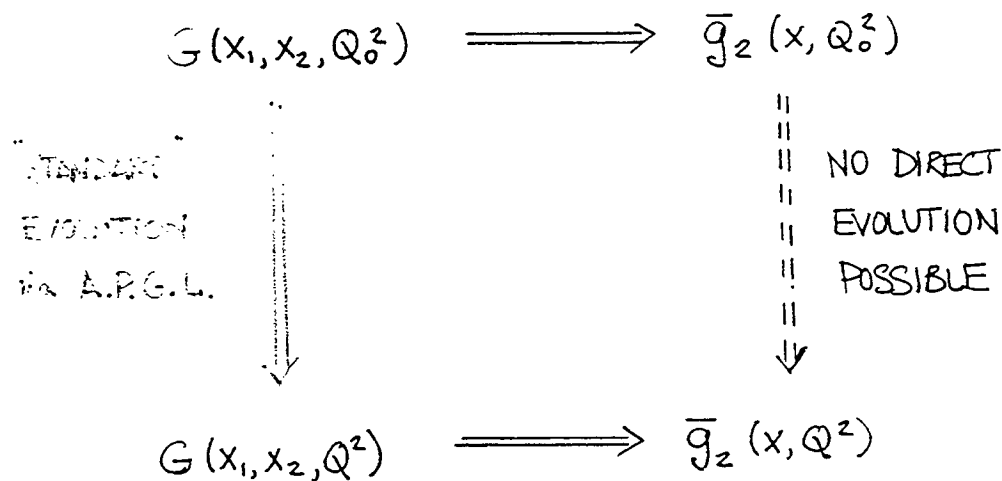
$\bar{q} \gamma_\perp \gamma_5 q \rightarrow q_+^+ \gamma_\perp \gamma_5 q_-$ is dependent
 $q_- \Rightarrow q_- [q_+, A_\perp]$ and actually involves gluons and QCD interactions

Many misguided interpretations of g_2 begin with this eqn.

- QCD EVOLUTION OF $g_2(x, Q^2)$

A complex story with a hopeful ending

General situation summarized diagrammatically



EXPERIMENTAL MEASUREMENT OF $g_2(x, Q_0^2)$ DOES NOT GIVE ENOUGH INFORMATION ($G(x_1, x_2, Q_0^2)$) TO EVOLVE.

TECHNICAL HISTORY

- AHMED & ROSS discovered $g_2(x, Q^2) \Leftrightarrow \bar{q} \gamma_\perp \gamma_5 D \dots D q$ and calculated anomalous dimensions $\{\gamma_n\}$ leading to straight forward evolution
- SHURAK & VANSHTEYN showed $g_2(x, Q^2)$ to be related by eqns of motion to tower of gluon-quark operators. And that (n-1) gluon-quark operators mix with g_2 under renormalization

- LIPATOV et al. } Calculated anomalous dimension
matrix for each moment n
RATCLIFF } $(\gamma_n)_{ij}$

MATRIX MIXING $\Rightarrow$ UNDERLYING TWO VARIABLE DISTRIBUTION
EVOLVING IN "STANDARD" FASHION

$$\frac{d}{d \ln Q^2} G(x_1, x_2, Q^2) = \frac{\alpha(Q^2)}{2\pi} \int_{x_1} dy_1 \int_{x_2} dy_2 P(x_1/y_1, x_2/y_2) G(y_1, y_2, Q^2)^*$$

* SCHEMATIC

- ALI, BRAUN, HILLER Showed that evolution simplifies in
 $N_c \rightarrow \infty$ limit.

$\bar{g}_2(x, Q^2)$ evolves in "standard" way with non-standard
anomalous dimension for $N_c \rightarrow \infty$

$$(\gamma_n^*)^{NS} = 2N_c \left(4(n+1) + \gamma_E - \frac{1}{4} + \frac{1}{2(n+1)} \right) + \mathcal{O}(1/N_c)$$

Although approximate it may be good enough to
transfer model predictions from model scales
($Q^2 = \mu_{\text{MODEL}}^2$) to experiment $Q^2 = 10 \text{ GeV}^2$.

END OF THEORETICAL INTERLUDE

III.4 BURKHARDT-COTTINGHAM SUM RULE

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda n) | PS \rangle = 2 \{ q_1(x) S \cdot n p^\mu + q_T(x) S_\perp^\mu + \dots \}$$

I • Contract with n_μ ($n \cdot p = 1$, $n^2 = 0$, $n \cdot S_\perp = 0$)

$$2 S \cdot n q_1(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{q}(0) \not{n} \gamma_5 q(\lambda n) | PS \rangle$$

• Integrate over x (restore Q^2 -label)

$$2 S \cdot n \int_{-1}^1 dx q_1(x, Q^2) = \langle PS | \bar{q}(0) \not{n} \gamma_5 q(0) | PS \rangle \Big|_{Q^2}$$

• In target rest frame choose $\vec{S} = \hat{e}_3$

$$\int_{-1}^1 dx q_1(x, Q^2) = \langle P \hat{e}_3 | \bar{q}(0) \gamma^3 \gamma_5 q(0) | P \hat{e}_3 \rangle \Big|_{Q^2}$$

II Let $S \cdot n = 0$, by choosing $\vec{S} = \hat{e}_1$

$$\int_{-1}^1 dx q_T(x, Q^2) = \langle P \hat{e}_1 | \bar{q}(0) \gamma^1 \gamma_5 q(0) | P \hat{e}_1 \rangle \Big|_{Q^2}$$

III Compare underlined equations $\Rightarrow \int_{-1}^1 dx q_T = \int_{-1}^1 dx q_1$

Or $\int_{-1}^1 dx q_2(x, Q^2) = 0$

APPARENTLY A CONSEQUENCE
OF ROTATION INVARIANCE

A POSSIBLE HITCH IN THE B.C. SUM RULE

$$\int_{-1}^1 dx g_2(x, Q^2) = 0 \quad \text{IS GOLD PLATED}$$

BUT $g_2(x, Q^2)$ IS A DISTRIBUTION IN x , i.e. A GENERALIZED FUNCTION & CAN HAVE $\delta(x)$

$$i \quad g_2(x, Q^2) = g_2^{\text{OBSERVABLE}}(x, Q^2) + C \delta(x) \quad ?$$

$$\text{WHENCE} \quad \int_0^1 dx g_2^{\text{OBSERVABLE}}(x, Q^2) = -\frac{1}{2} C$$

NOTE: SUCH A PATHOLOGY IS NOT SO ARBITRARY AS IT LOOKS

- IT IS AN EXAMPLE OF A TIME HONORED DISEASE KNOWN AS A "J=0 FIXED POLE WITH NON-POLYNOMIAL RESIDUE" FIRST STUDIED IN REGGE THEORY. EMERGES NATURALLY IN DERIVATION OF B.C. AS SUPERCONVERGENCE REL'N.
- TWO SIGNIFICANT INCIDENCES OF $\bar{t}=0$ F.P. RUINING SUM RULES ARE
 1. BURKHARDT COTTINGHAM
 2. DRELL-HEARN-GERASIMOV
 BOTH TESTIBLE IN NEAR FUTURE!

EVIDENCE ON B.C. SUM RULE

SEVERAL OTHER SIMILAR SUM RULES ARE
VIOLATED PERTURBATIVELY IN QCD

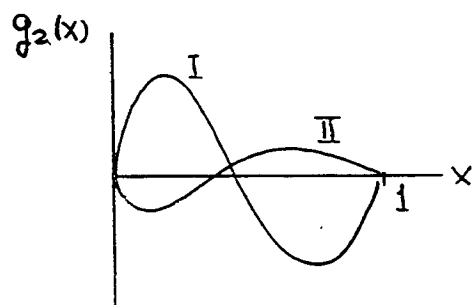
SEE

M. BURKARDT

NP B 373

FOR DISCUSSION

BURKHARDT COTTINGHAM DOES NOT APPEAR
TO BE VIOLATED IN PQCD AT 1-LOOP, BUT
NON-PERTURBATIVE EFFECTS $\neq$ HIGHER ORDERS
REMAIN UNSTUDIED



CASE I OR II ?

OR PERHAPS MORE NODES ?

LOOK AT MODELS

5 THE SHAPE OF $g_2(x, Q^2)$

AN EXPERIMENTAL AGENDA.

1. IS $\int_0^1 dx g_2(x, Q^2) = 0$?
2. IF YES, DOES $g_2(x, Q^2)$ HAVE JUST ONE (NON-TRIVIAL) ZERO ?

3. IF YES, WHAT IS THE SIGN OF

$$\int_0^1 dx x^2 g_2(x, Q^2) \equiv M_2[g_2]$$

$$M_2[g_2] < 0 \Rightarrow \begin{array}{c} | \\ \text{graph of } g_2(x, Q^2) \text{ starting positive, crossing zero, and ending negative} \\ x \end{array}$$

$$M_2[g_2] > 0 \Rightarrow \begin{array}{c} | \\ \text{graph of } g_2(x, Q^2) \text{ starting negative, crossing zero, and ending positive} \\ x \end{array}$$

4. CAN A SIGNAL OF TWIST-3 BE SEEN ?

re

$$\bar{g}_2(x, Q^2) = g_2(x, Q^2) - g_2^{WW}(x, Q^2) \neq 0 ?$$

RECALL

$$\Delta G(x, Q^2) = \frac{i}{\sqrt{2} p^+ x} \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle PS_z | \hat{F}^{+\alpha}(\lambda n) \mathcal{P}(\lambda n, 0) \tilde{F}_\alpha^+(0) | PS_z \rangle \Big|_{Q^2}$$

↑ NOTE $1/x$ RELATIVE TO Δq (ORIGINATES IN SPIN STRUCTURE OF $F^{\mu\nu}$)

$$\int_{-\infty}^{\infty} dx \Delta G(x, Q^2) \Rightarrow \int \frac{dx}{x} e^{i\lambda x} \quad \text{A LOCAL SUM RULE}$$

$$\int \frac{dx}{x} e^{i\lambda x} \Rightarrow \epsilon(\lambda) \quad \left(= \begin{matrix} +1, & \lambda > 0 \\ -1, & \lambda < 0 \end{matrix} \right)$$

$$\Delta G(Q^2) = \int_{-\infty}^{\infty} d\lambda \epsilon(\lambda) \langle PS_z | F^{+\alpha}(\lambda n) \mathcal{P}(\lambda n, 0) \tilde{F}_\alpha^+(0) | PS_z \rangle \Big|_{Q^2}$$

GAUGE INVARIANT BUT NOT LOCAL !

CHOOSE $A^+ = 0$ GAUGE (light-cone gauge, often used in parton model)

$$F^{+\alpha} = \partial^+ A^\alpha - \partial^\alpha A^+ + [A^+, A^\alpha] \Rightarrow \partial^+ A^\alpha \Rightarrow \frac{\partial}{\partial \lambda} A^\alpha$$

$$\mathcal{P}(\lambda n, 0) \Rightarrow 1$$

$$\Delta G(Q^2) = \int_{-\infty}^{\infty} d\lambda \epsilon(\lambda) \frac{\partial}{\partial \lambda} \langle \dots \rangle = 2 \langle \dots \rangle \Big|_{\lambda=0}$$

(Surface terms at $\lambda = \pm \infty$ can be dropped.)

$$\Delta G(Q^2) = \frac{1}{\sqrt{2} M} \langle PS_z | 2 \text{Tr} (A' F^{+2} - A^2 F^{+1}) | PS_z \rangle \Big|_{Q^2}$$

THE DILEMMA OF ORBITAL ANGULAR MOMENTUM

R.L. JAFFE

• $\Delta \Sigma \cong 0.2 - 0.3$

* IN SOME SENSE

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_Q + L_G$$

• AFTER CONSIDERABLE STRUGGLE WE KNOW

- WHAT ΔG IS
- HOW TO MEASURE IT

• $L_Q \neq L_G$ ARE POORLY UNDERSTOOD

- DEFINITION ?
- GAUGE INVARIANCE, UNIQUENESS ?
- DISTRIBUTION IN X_{Bj} ?
- MEASURABLE ?

ALL PROBLEMATIC, SUBJECT OF THIS TALK
HOPE: STIMULATE WORK.

COLLABORATORS, REFERENCES & OUTLINE

• THANKS TO SERGEI BASHINSKY
 XIANGDONG JI
 ANEESH MANOHAR

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9. OUTLINE

I. STANDARD PARTON DISTRIBUTIONS

II. MOMENTUM & ANGULAR MOMENTUM TENSOR
DENSITIES IN QCD

III. OPERATOR EXPRESSIONS FOR INTEGRATED
QUARK & GLUON CONTRIBUTIONS TO THE ANG. MOMENTUM

IV. LOCAL DISTRIBUTIONS IN X_{Bj}

V. THE PRESENT DILEMMA.

ESSENTIALLY ALL WORKERS AGREE THAT

$$\underline{\bar{q} \gamma_5 q}$$

IS QUARK SPIN OPERATOR. [AND IS MEASURABLE
IN POLARIZED D.I.S. — MODULO RENORMALIZATION
SCHEME CONTROVERSIES.]

- GLUON SPIN: APPEARS GAUGE DEPENDENT. HOWEVER
IT IS RELATED TO CORRECT OPERATOR DESCRIPTION
OF GLUON SPIN (SEE BELOW)
- QUARK & GLUON ORBITAL ANGULAR MOMENTUM
LOOK PHYSICAL, BUT GAUGE DEPENDENT
- NOTE: $M^{\mu\nu\lambda}$, IN TOTAL, IS GAUGE INVARIANT R/L/AM.
- "ANGULAR MOMENTUM SUM RULE" ?

IN SOME SENSE $M^{\mu\nu\lambda}$ IS A SUM OF 4 TERMS
HOW DOES IT IMPLY

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_Q + L_G \quad ?$$

GAUGE INVARIANT DECOMPOSITION OF $M^{\mu\nu\lambda}$

Ji

HEP-PH/9103249

REPLACE $\partial^\nu \rightarrow \partial^\nu - ig A^\nu$ IN QUARK
 ORBITAL TERM, USE EQ'S OF MOTION TO MASSAGE
 INTO TERM THAT SENDS $\partial^\nu \rightarrow \partial^\nu - ig A^\nu$ IN GLUON
 ORBITAL TERM:

$$M_{J1}^{\mu\nu\lambda} = \frac{i}{2} \bar{\Psi} \gamma^\mu (\underline{x^\lambda D^\nu} - \underline{x^\nu D^\lambda}) \Psi + \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\Psi} \gamma_\sigma \gamma_5 \Psi$$

$$- 2 \text{Tr} \{ F^{\mu\alpha} (\underline{x^\nu D^\lambda} - \underline{x^\lambda D^\nu}) A_\alpha \}$$

$$+ 2 \text{Tr} \{ F^{\mu\lambda} A^\nu - F^{\mu\nu} A^\lambda \} - \frac{1}{2} \text{Tr} F^2 (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu})$$

OBSERVATIONS (J1)

- Quark orbital term is now separately GI
- Gluon: only total spin + orbit is GI
- Quark orbital term is measurable (in principle) thru DVCS.

QUESTIONS

- What is physical significance of $\partial \rightarrow \partial - ig A$?
Relation to rotations etc?
- What is meaning of gluon spin that experimenters measure?
- What can be said about x_{Bj} distributions? 127

III. INTEGRATE PIECES OF THE ANGULAR MOMENTUM

Deep inelastic processes probe light-cone momentum distributions. The lowest moments of these may be local symmetry generators that appear in sum rules. Before studying light cone momentum distributions for spin and orbital angular momentum, we look here at:

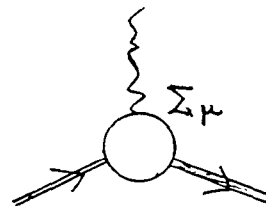
WHAT CAN BE DETERMINED @ EXPECTATION VALUES OF PIECES OF SYMMETRY GENERATORS?

• QUARK SPIN $\bar{\psi} \gamma_\mu \gamma_5 \psi \equiv \Sigma_\mu$

"FORM FACTOR" FOR Σ_μ : imagine an external probe that couples to Σ_μ

$$S_\mu(p, q, s) \equiv \int d^4x \langle p' s | \Sigma_\mu(x) | p s \rangle e^{iq \cdot x}$$

General covariant parameterization



$$-\langle p' s | \Sigma_\mu(0) | p s \rangle = S_\mu A(q^2) + q_\mu s \cdot q B(q^2) + \dots$$

$$S_\mu(p, q, s) = (2\pi)^4 \delta^4(p' + q - p) S_\mu A(q^2) + \dots$$

COMPARE $f_\mu(p, 0, s) = \int dx^0 \int d^3x \langle PS | \Sigma_\mu(x) | PS \rangle$

$$= (2\pi) \delta^4(0) \langle PS | \int d^3x \Sigma_\mu(x) | PS \rangle$$

USE $\langle P' | PS \rangle = (2\pi)^3 2E \delta^3(\vec{P} - \vec{P}')$ TO OBTAIN

$$A(0) \vec{S} = \frac{\langle PS | \int d^3x \vec{\Sigma}(x) | PS \rangle}{\langle PS | PS \rangle}$$

MEASURED BY

FORWARD SCATTERING

INTERPRET AS

THEORY

COMMENTS

- * OF COURSE NO CURRENT COUPLES DIRECTLY TO Σ_μ , BUT FLAVOR GENERALIZATIONS $\bar{q} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} q$ ARE MEASURED THIS WAY (IN β -DECAY)
- * WHY DID I BELABOR THIS OBVIOUS POINT? BECAUSE ITS NOT SO OBVIOUS FOR OPERATORS WITH EXPLICIT γ^μ .

* QUARK ORBITAL ANGULAR MOMENTUM

RLT $\neq$ AM

FIRST DO PEDAGOGICAL EXAMPLE OF MAGNETIC MOMENT.

7

MAGNETIC MOMENT

$$\vec{\mu}(x) = \frac{1}{2} \vec{x} \times \vec{j}(x)$$

MORE GENERALLY $\frac{1}{2} [x_\mu j_\beta(x) - x_\beta j_\mu(x)]$

$$F_{\beta\mu}(P,q,s) \equiv \frac{1}{2} \int d^4x e^{iqx} \langle P'S' | (x_\beta j_\mu(x) - x_\mu j_\beta(x)) | PS \rangle$$

General covariant parameterization $P^\alpha = \frac{1}{2}(P+P')^\alpha$

$$\langle P'S' | j_\mu(0) | PS \rangle = 2P_\mu F_1(q^2) + 2i\epsilon_{\mu\alpha\beta\gamma} P^\alpha q^\beta s^\gamma F_2(q^2)$$

Use $x_\beta = -i \partial / \partial q^\beta e^{iqx}$

$$\begin{aligned} F_{\beta\mu}(P,q,s) &= -i \frac{\partial}{\partial q^\beta} \left\{ (2\pi)^4 \delta^4(p'+q-p) \langle P'S' | j_\mu(0) | PS \rangle \right\} - (\mu \leftrightarrow \beta) \\ &= -i \frac{\partial}{\partial q^\beta} \left\{ (2\pi)^4 \delta^4(p'+q-p) [2P_\mu F_1 + 2i\epsilon_{\mu\alpha\beta\gamma} P^\alpha q^\beta s^\gamma F_2] \right\} \\ &\quad - (\mu \leftrightarrow \beta) \end{aligned}$$

TAKE $q^\mu \rightarrow 0$
AND COMPARE:

WAVE PACKET TERMS - NOT INTRINSIC
MAGNETIC MOMENT - DROP

$$\begin{aligned} F_{ij}(P,0,s) &= -i (2\pi)^4 \left\{ 2\partial_\beta \delta^4(0) P_\mu - 2\partial_\mu \delta^4(0) P_\beta \right\} \\ &\quad + 4 (2\pi)^4 \delta^4(0) \epsilon_{i\alpha j\gamma} P^\alpha s^\gamma F_2(0) \end{aligned}$$

IN THE END: $F_2(0) \vec{S} = \frac{\langle PS | \int d^3x \frac{1}{2} \vec{x} \times \vec{j}(x) | PS \rangle}{\langle PS | PS \rangle}$

MEASURED BY OFF-FORWARD
SCATTERING

INTERPRETABLE BY THEORY

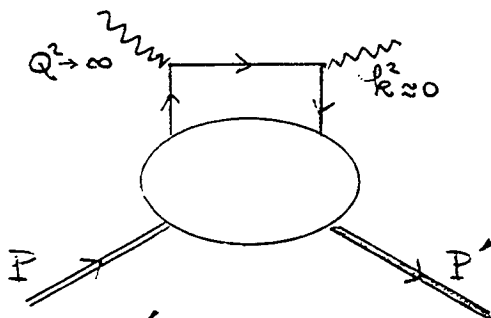
MORAL: WHEN x^λ APPEARS IN AN OPERATOR, IT IS
NECESSARY TO MEASURE OFF-FORWARD
MATRIX ELEMENTS

≠ Leave the usual world of "hard QCD"
HENCE DVCS.

J1'S PROPOSAL FOR MEASURING

$$x^\lambda T_Q^{\mu\nu} - x^\nu T_Q^{\mu\lambda} \Leftrightarrow \frac{1}{2} \Delta \Sigma + L_Q$$

$$x^\lambda T_G^{\mu\nu} - x^\nu T_G^{\mu\lambda} \Leftrightarrow \Delta G + L_G$$



$$P' - P = q$$

$$q^2 \equiv t \approx 0$$

USE PHOTON KINEMATICS TO
INTEGRATE SECOND MOMENT
OF $\bar{\psi} \gamma^\mu \psi$ ON LIGHT CONE

OFF FORWARD TO PICKUP MOMENT OF $T^{\mu\nu}$

TAKE STOCK

- $\Delta \Sigma$ Measure thru sum rules for polarized DIS
- L_Q Measure $\frac{1}{2} \Delta \Sigma + L_Q$ in DVCS where L_Q here is Ji's covariant version
- $L_G + \Delta G$ In principle measure this sum via DVCS.

PROBLEMS

- No place for ΔG . No local operator (GI)
- No physical interpretation for covariant L_Q vis a vis naive L_Q .
- No generalization of L_Q (or L_G) to $L_Q(x)$. Is a Bjorken- x distribution of orbital angular momentum meaningful?
- No general analysis of what quantities may (or may not) appear as parton distributions.
- No simple connection to partons in $A^+ = 0$ gauge (as exists for T^{uv} - see pg 10).

IV. LOCAL DISTRIBUTIONS IN X_{Bj}

THIS SECTION IS BASED ENTIRELY ON WORK

BY SERGEI BASHINSKY

SB & RLJ HEP-PH 9804397

- ANTICIPATE SIGNIFICANCE OF SPACE DIRECTION $x^3 \rightarrow x^+$, WHICH DEFINES PARTON MODEL
- DEFINE FIELDS CORRESPONDING TO NOTION OF QUARK OR GLUON PARTON w/ "MOMENTUM" $q^+ = xP^+$
- STUDY TRANSFORMATIONS OF THOSE FIELDS CORRESPONDING TO SPACE-TIME TRANSFORMS.
- PARTON DISTRIBUTIONS ARE NOETHER CHARGES CORRESP. TO THOSE TRANSFORMATIONS
- KEEP CLOSE CONTACT w/ $A^+ = 0$, BUT MAINTAIN GAUGE INV.
- ASSOCIATE PARTON DISTRIBUTIONS WITH FIELD TRANSFORMATIONS.
- DON'T WORRY @ EXPERIMENTAL MEASUREMENT TOO MUCH

RESULT

- PARTONIC DEFINITIONS OF ALL FOUR TERMS IN ANGULAR MOMENTUM S.R. AS FUNCTIONS OF X_{Bj}
- UNDERSTANDING OF ΔG AS LOCAL OPERATOR

1. WHAT IS A PARTON? (ILLUSTRATE FOR QUARK)

A QUANTUM WITH DEFINITE P^+ IN $A^+=0$ GAUGE

$$T_-^q(a^-) \psi(k^+, \underline{x}) = e^{-ik^+ a^-} \psi(k^+, \underline{x})$$

$\approx T_-^q(a^-)$ — TRANSLATION ALONG x^- BY a^-

$\approx \psi$ — QUARK FIELD IN L.C. GAUGE $\underline{x} = (x^+, x^1, x^2)$

SO QUARK PARTON IS DEFINED AS EIGENSTATE OF
TRANSLATION ALONG x^- IN $A^+=0$ GAUGE

SUPERPOSE PARTONS TO MAKE QUARK FIELD IN $A^+=0$ GAUGE

$$\psi(x) = \int \frac{dk^+}{2\pi} e^{-ik^+ x^-} \psi(k^+, \underline{x})$$

$$T_-^q(a^-) \psi(x) = \psi(x + a^-)$$

$\approx$ NAIVE TRANSLATION IN $A^+=0$ GAUGE

GENERALIZE TO ARBITRARY GAUGE

$$\psi(x) = e^{i\alpha(x)} \psi(x)$$

IN ARBITRARY GAUGE, PARTONS ARE DEFINED AS EIGENSTATES OF THE GAUGE COVARIANT TRANSLATION

$$T_-^g(a^-) \psi(x) = U(x, x+a^-) \psi(x+a^-)$$

$$U(x, x+a^-) = P \exp ig \int_{x+a^-}^x d\xi^- A^+(\xi) \quad \text{WILSON LINE}$$

GENERALIZE TO GLUONS

($D_\lambda(x)$ IS MATRIX IN FUNDAMENTAL)
 REP. $D_\lambda(x) \equiv \partial_\lambda - ig A_\lambda^a \lambda^a/2$

$$\star T_-^g(a^-) D_\lambda(x) = U(x, x+a^-) D_\lambda(x+a^-) U(x+a^-, x)$$

$$\equiv \text{INFINITESIMAL: } \delta_-^g A_\lambda = a^- F_{-\lambda} \quad (\delta_-^g A^+ = 0)$$

$$T_-^g(a^-) A_\lambda^\lambda(k^+, \underline{x}) = e^{-ik^+ a^-} A_\lambda^\lambda(k^+, \underline{x}) \quad \text{IN } A^+ = 0 \text{ GAUGE}$$

* ARBITRARY GAUGE

$$A_\lambda(x) = e^{i\alpha(x)} (A_\lambda(x) + \frac{i}{g} \partial_\lambda) e^{-i\alpha(x)}$$

PROPERTIES OF COVARIANT TRANSLATION

INDEPENDENT $[T_-^g(a^-), T_-^g(b^-)] = 0$

LEAVE GAUGE UNCHANGED $T_-^{g,g}(a^-): \alpha(x) \rightarrow \alpha(x)$

SEPARATE T_-^q AND T_-^g ARE NOT SYMMETRIES

BUT

$$T_-(a^-) = T_-^q(a^-) + T_-^g(a^-)$$

IS SYMMETRY OF

GAUGE INVARIANT LAGRANGIAN

WHOOOPS! RESIDUAL GAUGE SYMMETRY

Speaking of ψ and A^λ as physical was premature

If $\alpha = \alpha(\underline{x})$ only then

$$\psi(k^+, \underline{x}) \rightarrow e^{i\alpha(\underline{x})} \psi(k^+, \underline{x})$$

$$A_\lambda(k^+, \underline{x}) \rightarrow e^{i\alpha(\underline{x})} \left(A_\lambda(k^+, \underline{x}) + \frac{2\pi i \delta(k^+)}{g} \partial_\lambda \right) e^{-i\alpha(\underline{x})}$$

OBSERVABLES MUST BE INVARIANT UNDER RESIDUAL G.I.

BREAK OUT "RESIDUAL GAUGE FIELD"

$$A_\lambda(k^+, \underline{x}) \equiv 2\pi \delta(k^+) a_\lambda(\underline{x}) + G_\lambda(k^+, \underline{x})$$

$$\text{with } G_\lambda(k^+, \underline{x}) \Big|_{k^+=0} = 0 \quad \neq \quad G_\lambda \rightarrow e^{i\alpha(\underline{x})} G_\lambda e^{-i\alpha(\underline{x})}$$

$$a_\lambda \rightarrow e^{i\alpha(\underline{x})} \left(a_\lambda + \frac{i}{g} \partial_\lambda \right) e^{-i\alpha(\underline{x})}$$

2. WHAT ARE PARTON ATTRIBUTES?

NOETHER CHARGES OF TRANSFORMATIONS THAT COMMUTE WITH $T_1 \neq T_2$

- DEFINE A FIELD TRANSFORMATION T (NOT NECESSARILY A SYMMETRY OF QCD)
- ONLY ACCEPT $[T, T_1] = [T, T_2] = 0$
SO T IS DIAGONAL IN PARTON BASIS.
- THEN T CAN BE DEFINED INDEPENDENTLY IN EACH k^+ SUBSPACE. ASSOCIATED NOETHER CHARGE CARRIES k^+ LABEL.

IN $A^+ = 0$ GAUGE LET $\delta_{T_1}^{(q^+)}$ BE TRANSFORMATION ACTING ON $k^+ = q^+$ SUBSPACE:

$$\delta_{T_1}^{(q^+)} \psi(k^+) = i\epsilon T_1 \psi(q^+) \delta(q^+ - k^+)$$

$\times$ DEPENDENCE SUPPRESSED

SUPERPOSE k^+ SUBSPACES TO FIND FIELD TRANSFORMATION IN $A^+ = 0$ GAUGE

$$\delta_{T_1}^{(q^+)} \psi(x) = i\epsilon \int \frac{d\xi^-}{2\pi} e^{iq^+ \xi^-} T_1 \psi(x + \xi^-)$$

AND FOR GLUONS

$$\delta_{T^g}^{(q^+)} a_\beta = 0.$$

$$\delta_{T^g}^{(q^+)} A_\lambda(x) = i g \int \frac{d\xi^-}{2\pi} e^{i q^+ \xi^-} T_\lambda^{g\beta} (A_\beta(x+\xi^-) - A_\beta(x))$$

THESE TRANSFORMATIONS ARE GENERATED BY NOETHER CHARGES, WITH ASSOCIATED CURRENTS

$$J_{T^g}^\mu(x, x_{Bj}) = \frac{P^+}{2\pi} \int d\xi^- e^{i x_{Bj} P^+ \xi^-} \bar{\psi}(x) \gamma^\mu T^g \psi(x+\xi^-)$$

* WE HAVE SET $q^+ \equiv x_{Bj} P^+$

* $J_T^\mu = -i \frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi)} T \Phi$ A LA NOETHER

* J_T^+ IS GENERATOR OF T TRANSFORMATION BY COMMUTATION AT EQUAL x^+

SO $\int d^2 x_\perp dx^- J_T^+(x, x_{Bj})$ IS OBSERVABLE
CANONICALLY ASSOCIATED WITH T

PARTON DISTRIBUTION OF T

$$f_T(x_{Bj}) \equiv \frac{\langle P | Q_T^{(H)}(x_{Bj}) | P \rangle}{\langle P | P \rangle}$$

where $Q_T^{(H)} = \int dx^- d^2 x_\perp J_T^+(x, x_{Bj})$

TIME FOR EXAMPLES

IDEAL PARTON DISTRIBUTIONS

$$1 \quad T^q = 1 \quad \delta^{(q^+)} \psi(k^+) = i\epsilon \psi(q^+) \delta(q^+ - k^+)$$

$\Rightarrow$ NUMBER OPERATOR

$$f_1^q(x_{Bj}) \equiv q(x_{Bj}) = \frac{1}{\sqrt{2}} \int \frac{d\xi^-}{2\pi} e^{i x_{Bj} \xi^- P^+} \langle P | \psi_+^\dagger(0) \psi_+(\xi^-) | P \rangle$$

STANDARD UNPOLARIZED QUARK DISTRIBUTION

$$2. \quad T^q = \gamma^5 \quad \text{QUARK SPIN ROTATION}$$

$$f_{\gamma^5}(x_{Bj}) \equiv \Delta q(x_{Bj}) = \frac{1}{\sqrt{2}} \int \frac{d\xi^-}{2\pi} e^{i x_{Bj} \xi^- P^+} \otimes$$

STANDARD QUARK

$$\otimes \langle P | \psi_+^\dagger(0) \gamma^5 \psi_+(\xi^-) | P \rangle$$

HELICITY DISTRIBUTION

BOTH ARE WRITTEN IN $A^+ = 0$ GAUGE, BUT INTRODUCTION OF WILSON LINES PUTS THEM INTO ARBITRARY GAUGE.

$$3. \quad T^q = \delta_{\lambda}^{\beta} \quad \text{GLUON NUMBER OPERATOR}$$

$$f_{\Gamma=\delta_{\lambda}^{\beta}}(x_{Bj}) = \frac{i}{4\pi} \int d\xi^- e^{i x_{Bj} P^+ \xi^-} \langle P | F^{+\lambda}(0) A_{\lambda}(\xi^-) | P \rangle$$

WRITTEN IN AN (ARBITRARY) $A^+ = 0$ GAUGE.

NOTE: The reason it doesn't look GT is that it is written in a particular gauge.

RESTORE MANIFEST G.I.

- $e^{iX_{B_j} P^+ \xi^-} = \frac{1}{iX_{B_j} P^+} \frac{\partial}{\partial \xi^-} e^{iP^+ \xi^- X_{B_j}}$

- INTEGRATE BY PARTS

- USE $\partial_- A_\lambda = -F_\lambda^+$

$$G(X_{B_j}) = \frac{i}{4\pi X_{B_j} P^+} \int d\xi^- e^{iX_{B_j} P^+ \xi^-} \langle P | F^{+\lambda}(0) F_\lambda^+(\xi^-) | P \rangle$$

+ surface term that can be dropped

- Finally restore MANIFEST GI w/ Wilson line

$$G(X_{B_j}) = \frac{i}{4\pi P^+ X_{B_j}} \int d\xi^- e^{iX_{B_j} P^+ \xi^-} \langle P | F^{+\lambda}(0) U(0, \xi^-) F_\lambda^+(\xi^-) | P \rangle$$

3 APPLICATION TO ANGULAR MOMENTUM

22

ROTATIONS ABOUT $\hat{e}_3$ COMMUTE WITH T_-

$$\delta_J \psi = -\epsilon \left[-\frac{i}{2} \sigma^3 + (x^1 \partial_2 - x^2 \partial_1) \right] \psi$$

$$\delta_J G_\lambda = -\epsilon \left[-(\delta'_\lambda \delta_2^\beta - \delta_\lambda^2 \delta_1^\beta) + (x^1 \partial_2 - x^2 \partial_1) \right] G_\beta$$

CONSIDER EACH TRANSFORMATION INDEPENDENTLY

TERMS CONTAINING ∂_j ARE NOT INVARIANT UNDER RESIDUAL GAUGE GROUP. RESTORE GI TO EACH TERM (SUM IS G.I. REGARDLESS) BY

$$\partial_j \rightarrow \partial_j - i g A_j \equiv \mathcal{D}_j \quad (\text{RUSSIAN } \mathcal{D})$$

$$\Delta_\Sigma: \delta_\Sigma \psi = i \epsilon \sigma^{12} \psi \quad T_\Sigma = \gamma^0 \gamma^3 \gamma^5$$

$$L_Q: \delta_{L_Q} \psi = -\epsilon (x^1 \mathcal{D}_2 - x^2 \mathcal{D}_1) \psi \quad T_{L_Q} = (x^1 i \mathcal{D}_2 - x^2 i \mathcal{D}_1)$$

$$\Delta_G: \delta_{\Delta_G} G_\lambda = \epsilon (\delta'_\lambda \delta_2^\beta - \delta_\lambda^2 \delta_1^\beta) G_\beta \quad (T_{\Delta_G})_\lambda^\beta = -i \epsilon^{+-} \lambda^\beta$$

$$L_G: \delta_{L_G} G_\lambda = -\epsilon (x^1 \mathcal{D}_2 - x^2 \mathcal{D}_1) G_\lambda \quad (T_{L_G})_\lambda^\beta = (x^1 i \mathcal{D}_2 - x^2 i \mathcal{D}_1) \delta_\lambda^\beta$$

NOTE: $[T_+, T_-] = 0$ FOR ALL T 'S. HOWEVER, HAD WE

USED "NAIVE" COVARIANTIZATION $\partial_j \rightarrow \partial_j - i g A_j$ THEN

$[T_+, T_-] \neq 0$ AND PARTON INTERPRETATION WOULD BE RUINED. 141

① TEST ON GLUON SPIN

$$\Delta G(x_{Bj}) = \frac{1}{4\pi} \int d\xi^- e^{i x_{Bj} P^+ \xi^-} \langle P | F^{+\lambda}(0) \epsilon^+_{\lambda\beta} A_\beta(\xi^-) | P \rangle$$

- Restore manifest GI

$$\Delta G(x_{Bj}) = \frac{i}{4\pi x_{Bj} P^+} \int d\xi^- e^{i x_{Bj} P^+ \xi^-} \langle P | F^{+\lambda}(0) U(0, \xi^-) \tilde{F}_\lambda^+(\xi^-) | P \rangle$$

- This result agrees with standard parton model expression of Manohar (PL B255 (1991) 579) $\neq$ Balzaki, $\neq$ Braun (PL B 267 (1991) 405)

- Lowest moment, ΔG , is non-local in arbitrary gauge

$$\Delta G = \frac{1}{2P^+} \int d\xi^- \langle P | F^{+\lambda}(0) U(0, \xi^-) \tilde{F}_\lambda^+(\xi^-) | P \rangle \epsilon(\xi^-)$$

But in $A^+ = 0$ gauges

$$\Delta G = \frac{1}{2P^+} \langle P | 2(A^1 F^{+2} - A^2 F^{+1}) | P \rangle$$

- LOCAL
- IDENTICAL TO GENERATOR $\int d^3x M_{\Delta G}^{+12}$ FROM SYMMETRY ANALYSIS.

CONCLUSION ON GLUON SPIN

- $\exists$ A GI PARTON DISTRIBUTION IN x_{Bj} , $\Delta G(x)$
- THE LOWEST MOMENT OF ΔG MEASURES GLUON SPIN IN NUCLEON.
- OPERATOR FOR ΔG IS (IN GENERAL GAUGE) NON-LOCAL
- IN $A^+=0$ GAUGES IT IS LOCAL $\neq$ CORRESPONDS TO TRADITIONAL GLUON SPINGENERATOR IN $M^{\mu\nu\lambda}$
- SEE RLJ - PLB 365 (1996) 359 FOR EARLIER DEMONSTRATION FROM LESS FORMAL BEGINNINGS

② APPLY TO QUARK $\neq$ GLUON ORBITAL ANGULAR MOMENTUM.

$$L_Q(x_{Bj}) = \frac{1}{2\pi/2} \int d^2x_\perp \int d\xi^- e^{ix_{Bj}P^+\xi^-} \langle P | \int d^2x_\perp$$

$$\otimes \psi_+^\dagger(x_\perp) (x'^i \partial_2 - x'^2 \partial_1) U(x_\perp, x_\perp + \xi^-) \psi_+(x_\perp + \xi^-) |P\rangle$$

$$\otimes \mathcal{D}_i = \partial_i - ig A_i \quad (\text{IN } A^+=0 \text{ GAUGE})$$

$$A_i = \frac{\int dx^- A_{i-}(x, x^-)}{\int dx^-} \quad \text{SO } A_i = 0 \text{ IN ANY GAUGE}$$

WHERE $A_{i-} \rightarrow 0$ AS $x^+ \rightarrow \infty$

$$\text{IN SUCH WELL-BEHAVED GAUGES } \mathcal{D}_i \Rightarrow \partial_i$$

NOTE $[U(x_\perp, x_\perp + \xi^-), \partial_j^-] = 0$, SO THERE IS

NO AMBIGUITY IN PLACEMENT OF U .

CONTRAST FULL COVARIANT DERIVATIVE:

$$\psi_+^\dagger(0) : D_j \psi(\xi^-)$$

Actually involves 3 light-cone points

$$D_j^- = \partial_j^- - ig A_j^-(\eta^-),$$

$$\left. \begin{array}{l} x^- = 0 \\ x^- = \eta^- \\ x^- = \xi^- \end{array} \right\}$$

D_j HAS PARTON CONTENT, DOES NOT
COMMUTE W/ T_- . INTERPRETATION

AS QUARK ORBITAL ANGULAR MOMENTUM

IS LOST: D_j (AS PART OF $x^1 D_2 - x^2 D_1$)

GENERATES NO RECOGNIZABLE TRANSFORMATION
OF ψ .

LOWEST MOMENT

$$L_Q = \int dx L_Q(x) = \frac{1}{\sqrt{2} P^+ \int d^2 x_\perp} \langle P | \int d^2 x_\perp \psi_+^\dagger(x_\perp) (x^1 i \partial_2 - x^2 i \partial_1) \psi_+(x_\perp) | P \rangle$$

AGAIN CONNECTS TO PIECE OF M^{+12} IN $A^+ = 0$ GAUGE

= GLUON ORBITAL ANGULAR MOMENTUM.

(DIRECTLY IN COVARIANT FORM)

$$L_G(x_{Bj}) = \frac{1}{4\pi x_{Bj} P^+ \int d^2 x_\perp} \int d\xi^- e^{i x_{Bj} P^+ \xi^-} \langle P | \int d^2 x_\perp$$

$$\otimes F^{+\lambda}(x_\perp) (x^1 i \partial_2 - x^2 i \partial_1) U(x_\perp, x_\perp + \xi^-) F_\lambda^+(x_\perp + \xi^-) | P \rangle$$

LOWEST MOMENT

$$L_G = \frac{i}{2P^+ \int d^2 x_\perp} \langle P | \int d^2 x_\perp F^{+\lambda}(x_\perp) (x^1 i \partial_2 - x^2 i \partial_1) A_\lambda(x_\perp) | P \rangle$$

④ SUMMARY

- SIMPLE & ELEGANT WAY TO DEFINE A PARTON DISTRIBUTION FUNCTION WITH A FIELD TRANSFORMATION
- UNAMBIGUOUSLY DEFINES QUARK & GLUON SPIN & ORBITAL ANGULAR MOMENTUM IN GAUGE INVARIANT WAY THAT OBEYS SR. $\frac{1}{2} = \frac{1}{2} \Delta Z + \Delta G + L_Q + L_G$

- GIVE SAME RELATION AMONG * TENSOR DENSITY $M^{\mu\nu\lambda}$
* $A^+=0$ GAUGES & * PARTON DISTRIBUTIONS, AS WE FOUND AMONG * $T^{\mu\nu}$, * $A^+=0$ & * SPIN AVERAGE PARTON DISTRIBUTIONS IN CLASSICAL ENERGY-MOMENTUM ANALYSIS.

II. THE PRESENT DILEMMA

EASY TO STATE BUT HARD TO RESOLVE

REGARDING ORBITAL ANGULAR MOMENTUM

- Ⓐ • WHAT CAN BE INTERPRETED CANNOT BE MEASURED
- Ⓑ • WHAT CAN BE MEASURED CANNOT BE INTERPRETED

(YET)

DILEMMA BEST SET OUT BY HOODBHOT, J1 ≠ L1

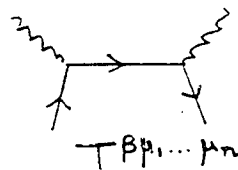
WHO STUDY HIGHER GENERALIZATIONS OF $X^{\lambda} T^{\mu\nu} - X^{\nu} T^{\mu\lambda}$

NAMELY:

$$M^{\alpha\beta\mu_1\ldots\mu_n} = X^{\alpha} T^{\beta\mu_1\ldots\mu_n} - X^{\beta} T^{\alpha\mu_1\ldots\mu_n}$$

$$\text{where } T^{\alpha\mu_1\ldots\mu_n} = S(\bar{\psi} \gamma^{\alpha} \vec{D}^{\mu_1} \vec{D}^{\mu_2} \ldots \vec{D}^{\mu_n} \psi)$$

$T^{\alpha\mu_1\ldots\mu_n}$ IS THE STANDARD LEADING TWIST OPERATOR THAT ARISES IN $J_{\lambda}(x) J_{\lambda}(0)$ AND CAN BE MEASURED OFF-FORWARD IN DVCS.



SO THE INVARIANTS ASSOCIATED WITH $M^{\alpha\beta\mu_1\ldots\mu_n}$ CAN BE MEASURED BUT THEY DON'T CORRESPOND TO ANGULAR MOMENTUM.

28.

$$\int d^3x M^{12++++\dots} = \underbrace{S^{++\dots}}_{\checkmark} + \underbrace{L_Q^{++\dots}}_{\checkmark} + \Delta L_Q^{++\dots} \quad ?$$

$$S^{++\dots} = \frac{2}{n+1} \int d^3\xi \bar{\psi} \gamma^+ \frac{\Sigma^3}{2} iD^+\dots iD^+ \psi$$

$$L_Q^{++\dots} = \frac{1}{n} \int d^3\xi \bar{\psi} \gamma^+ (x^1 iD^2 - x^2 iD^1) iD^+\dots iD^+ \psi \\ + \frac{1}{n} \int d^3\xi \bar{\psi} iD^+\dots iD^+ (x^1 iD^2 - x^2 iD^1) \psi$$

$$\Delta L_Q^{++\dots} = \frac{1}{n(n+1)} \int d^3\xi \left\{ \bar{\psi} \gamma^+ (x^1 \gamma^2 - x^2 \gamma^1) i g F^{\mu\nu} \gamma_\mu iD^+\dots iD^+ \psi \right. \\ \left. + \dots + \bar{\psi} iD^+ iD^+\dots iD^+ (x^1 \gamma^2 - x^2 \gamma^1) i g F^{\mu\nu} \gamma_\mu \psi \right\}$$

- NOTE
- $L_Q^{++\dots}$ IS NOT THE ORBITAL ANGULAR MOMENTUM
 - $\Delta L_Q^{++\dots}$ IS NOT INTERPRETABLE!

- SO — AT LEAST AT THIS TIME — THE MEASURABLE QUANTITY HAS NO DIRECT INTERPRETATION.

ON THE OTHER HAND

BASHINSKY'S METHOD YIELDS UNAMBIGUOUS
PARTON DISTRIBUTIONS ASSOCIATED WITH
ANY PARTONIC ($[T, T_{q,g}^-(\alpha)] = 0$) TRANSFORMATION
OF FIELDS.

CERTAINLY YIELDS DEFINITIONS OF QUARK $\neq$
GLUON DISTRIBUTIONS OF $L_Q \neq L_G$ IN X_{B_j}

IN MOST GAUGES THEY COINCIDE W/ THE
NAIVE EXPRESSIONS USED BY HAGLER $\neq$
SCHAFFER AND BY HARINDRANATH $\neq$ KUNDU

HOWEVER THESE DISTRIBUTIONS HAVE NOT (YET)
BEEN IDENTIFIED W/ EXPERIMENT.

FINALLY NOTE SOME EXPERIMENTS CAN BE DONE
ON COMPUTERS! WE HAVE GOOD (ie "IDEAL")
PARTON DISTRIBUTIONS FOR $\Delta\Sigma, \Delta G, L_Q \neq L_G$
SUCH THAT

$$\int dx_{B_j} \left\{ \frac{1}{2} \Delta\Sigma(\underline{x_{B_j}}) + \Delta G(\underline{x_{B_j}}) + L_Q(\underline{x_{B_j}}) + L_G(\underline{x_{B_j}}) \right\} = \frac{1}{2}$$

Quark and Gluon Structure of the Nucleon

Lectures Presented at the 9th RIKEN Winter School,

Shimoda, December 8-12, 1998

Klaus Rith

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Abstract

Recent results on unpolarised and polarised deep inelastic lepton nucleon scattering (DIS) and other hard QCD processes are presented. The first part of the lectures covers following topics:

- the definition of structure functions, early experimental determination of quark distributions;
- the flavour asymmetry of the light quark sea and its recent determination from the Drell-Yan process and from pion production in semi-inclusive DIS, both on hydrogen and deuterium targets;
- the Q^2 dependence of parton distributions and its description by perturbative QCD;
- recent results from HERA, especially the behaviour of the proton structure function F_2 at very low values of x and moderate Q^2 and at large x and very high Q^2 ;
- the determination of the gluon distribution from the Q^2 dependence of structure functions, from charm production and from direct photons at high transverse momenta in nucleon nucleon scattering.

The second part of the lectures summarizes the status of polarised DIS.

It covers:

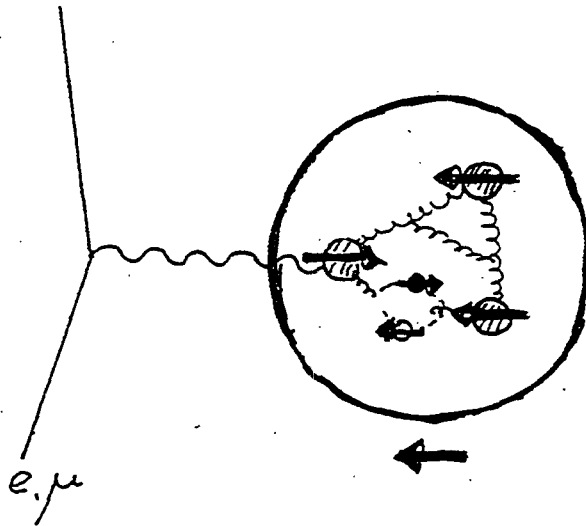
- the different experiments and their technological achievements;
- recent results on the asymmetry A_1 and the structure function g_1 for proton, deuteron and neutron;
- sum rules and their implications for the decomposition of the nucleon spin into its contributions from quarks, gluons and orbital angular momenta;
- determination of the individual polarised parton distributions from semi-inclusive DIS;
- measurements of the second structure function g_2 and the first attempts to derive the twist-3 matrix element d_2 .

The lectures close with an outlook on future measurements at HERMES, SLAC, COMPASS, RHIC and HERA with polarised protons.

Structure of the Nucleon

①

K. Rith



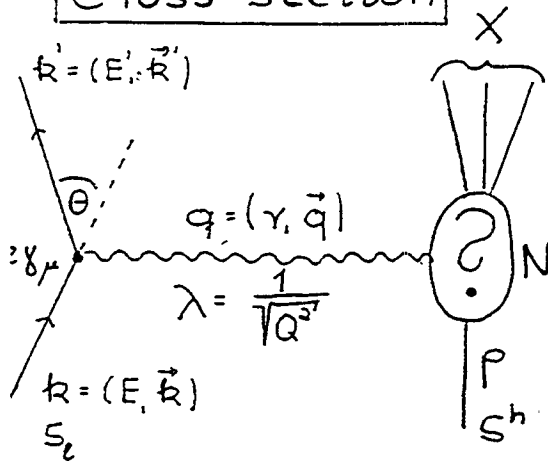
Selected results from
unpolarized and polarized
deep inelastic ℓ -N scattering

Outline

②

- ① Deep inelastic lepton nucleon scattering (DIS)
Structure functions
parton distributions
- ② DIS and perturbative QCD
scale breaking, $\alpha_s(Q^2)$
- ③ DIS at very high Q^2 — HERA
- ④ Polarised DIS
Spin of the nucleon — HERMES et

Cross section



③

$$q = (k - k')$$

$$Q^2 = -q^2$$

$$\stackrel{\text{LAB}}{=} 4EE' \sin^2 \frac{\theta}{2}$$

$$\gamma = \frac{p \cdot q}{M} \stackrel{\text{LAB}}{=} E - E'$$

$$X = \frac{Q^2}{2pQ} \stackrel{\text{LAB}}{=} \frac{Q^2}{2M\gamma}$$

$$M_{if}^2 = \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu}$$

unpolarized case

$$L_{\mu\nu} = \text{leptonic Tensor} = L_{\mu\nu}^{(S)} + L_{\mu\nu}^{(RS)}$$

$$= 2(k_\mu k'_\nu + k'_\mu k_\nu + (m_e^2 - g_{\mu\nu} k k')) - i \epsilon_{\mu\nu\alpha\beta} q^\alpha s^\beta$$

$$W^{\mu\nu} = \text{hadronic tensor} = W_{(S)}^{\mu\nu} + W_{(RS)}^{\mu\nu}$$

$$= \underline{W_1(\gamma, Q^2)} (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})$$

$$+ \underline{W_2(\gamma, Q^2)} \cdot \frac{1}{M^2} (p^\mu - \frac{p \cdot q}{q^2} q^\mu)(p^\nu - \frac{p \cdot q}{q^2} q^\nu)$$

$$+ \underline{G_1(\gamma, Q^2)} \cdot M \cdot i \epsilon^{\mu\nu\lambda\sigma} q_\lambda s_\sigma^h$$

$$+ \underline{G_2(\gamma, Q^2)} \frac{1}{M} \cdot i \epsilon^{\mu\nu\lambda\sigma} q_\lambda (p \cdot q \cdot s_\sigma^h - s^h \cdot q p_\sigma)$$

$s, s^h = \text{Polarisation vector of } e, N$

W_1, W_2, G_1, G_2 : Structure functions ④

$$M W_1(\gamma, Q^2) = F_1(x, Q^2) \quad \text{unpolarized}$$

$$\gamma W_2(\gamma, Q^2) = F_2(x, Q^2)$$

$$M^2 \gamma G_1(\gamma, Q^2) = g_1(x, Q^2) \quad \text{polarized} \rightarrow \text{later}$$

$$M \gamma^2 G_2(\gamma, Q^2) = g_2(x, Q^2)$$

$$\frac{d^2\sigma}{\Omega dE'} = \frac{4\pi^2 E'^2}{Q^4} \cos^2 \frac{\theta}{2} [W_2(\gamma, Q^2) + 2W_1(\gamma, Q^2) \tan^2 \frac{\theta}{2}]$$

Jacobian:

$$\frac{d^2\sigma}{\Omega dE'} = \frac{EE'}{\pi} \cdot \frac{x}{\gamma} \frac{d^2\sigma}{dQ^2 dx}$$

$$= \frac{EE'}{\pi} \frac{d^2\sigma}{dQ^2 d\gamma}$$

$$= \frac{1}{2M\gamma} \frac{E'}{\pi} \frac{d^2\sigma}{dx dy} \quad \text{with } \gamma = \frac{\gamma}{E}$$

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi^2 \pi^2}{Q^4} \left[(1-\gamma - \frac{Mxy}{2E}) \frac{F_2(x, Q^2)}{x} + \gamma^2 F_1(x, Q^2) \right]$$

Examples

6

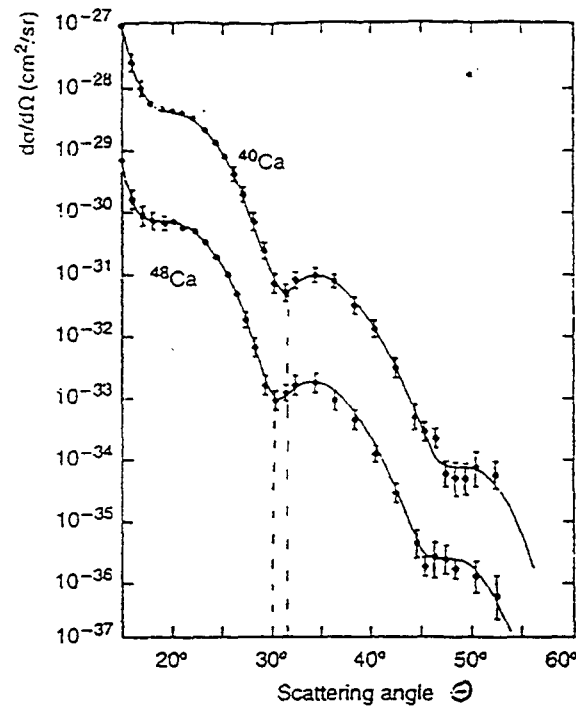
	$W_1(\gamma, Q^2)$	$W_2(\gamma, Q^2)$
elastic, nucleūs charge distr. $g(r)$ <u>Spin - 0</u>	0	$F_d^2(Q^2) \delta(\frac{Q^2}{2M_N} - \gamma)$
elastic, nucleon $g(r)$, spin - $\frac{1}{2}$	$\tau G_m^2(Q^2) \delta(\frac{Q^2}{2M} - \gamma)$ $\tau = \frac{Q^2}{4M^2}$	$\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} \delta(\frac{Q^2}{2M} - \gamma)$
elastic, muon pointlike, spin - $\frac{1}{2}$	$\frac{Q^2}{4M_\mu^2} \cdot 1 \cdot \delta(\frac{Q^2}{2M_\mu} - \gamma)$	$1 \delta(\frac{Q^2}{2M_\mu} - \gamma)$
elastic, quark pointlike, spin - $\frac{1}{2}$ Mass $M_q = x M$ charge $e_q e $	$e_q^2 \cdot \frac{Q^2}{4M^2 x^2} \delta(\frac{Q^2}{2Mx} - \gamma)$ $= e_q^2 \frac{1}{2M} \delta(\frac{Q^2}{2M\gamma} - x)$ $\Rightarrow 2MW_1 = \frac{1}{x} \gamma W_2 \quad \& \quad 2x F_1 = F_2$	$e_q^2 \delta(\frac{Q^2}{2Mx} - \gamma)$ $= e_q^2 \frac{x}{\gamma} \delta(\frac{Q^2}{2M\gamma} - x)$

$$F_d(q^2) = \text{Formfactor} = \int_0^\infty e^{i\vec{q}\vec{r}} f(\vec{r}) d^3r \quad \left| \begin{array}{l} g(\vec{r}) = Z \cdot e \cdot f \\ \end{array} \right.$$

$$= 1 - \frac{1}{6} q^2 \langle r^2 \rangle + \dots$$

$$\left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0} = -\frac{1}{6} \langle r^2 \rangle$$

Elastic e-Nucleus scattering



$$q(1st\ min) \cdot R \approx 4.5$$

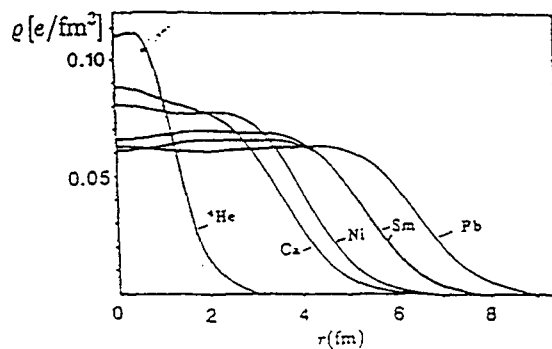
$$R_{^{48}\text{Ca}} > R_{^{40}\text{Ca}}$$

$$q \approx 2E \sin \frac{\Theta}{2}$$

$$S(r) = \frac{S(0)}{1 + e^{(r-c)/a}}$$

$$c = 1.07 \text{ fm } A^{1/3}$$

$$a = 0.54 \text{ fm}$$



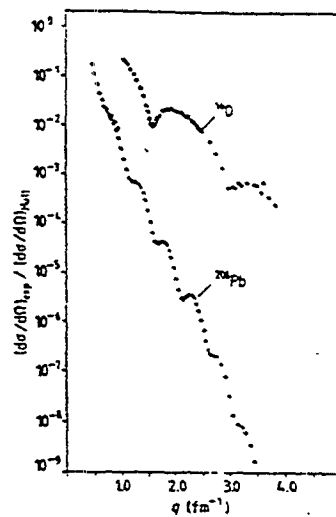
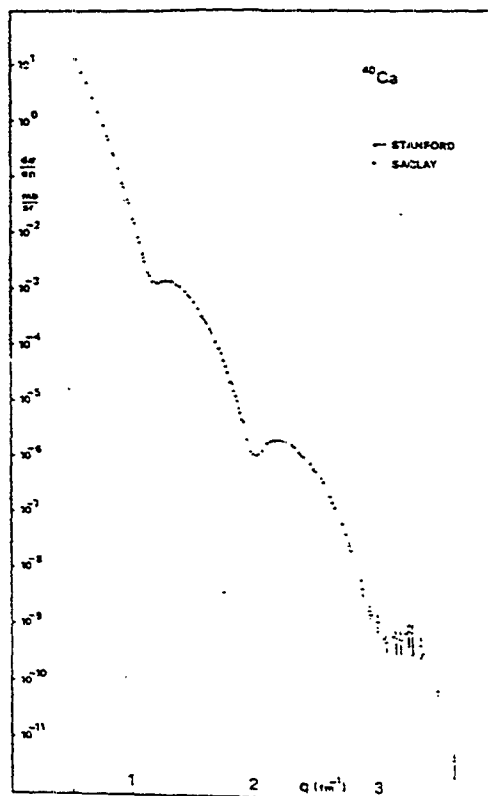
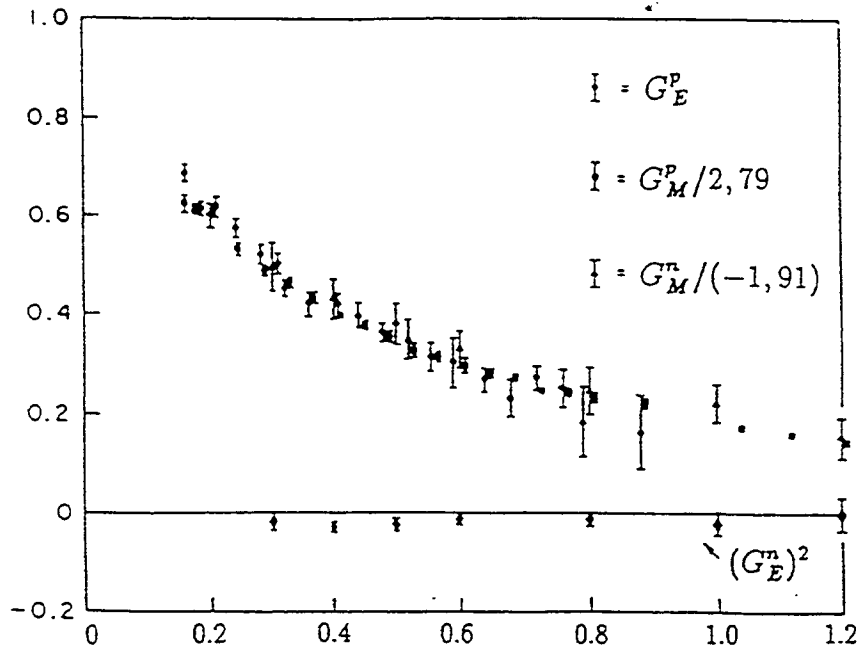


Fig. 6.5. Elastic scattering cross section of electrons from ^{40}Ca from experiments performed at Stanford and Saclay, France. [Courtesy I. Sick, *Phys. Lett.* 88B, 245 (1979).]



Elastic e-Nucleon scattering

⑧



$$G_E^p(Q^2) \approx \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2} \quad (\text{Dipole})$$

$$g(\tau) = g(0) e^{-a\tau} \quad a = 4.27 \text{ fm}^{-1}$$

$$\sqrt{\langle \tau_p^2 \rangle} = 0.862 \text{ fm}$$

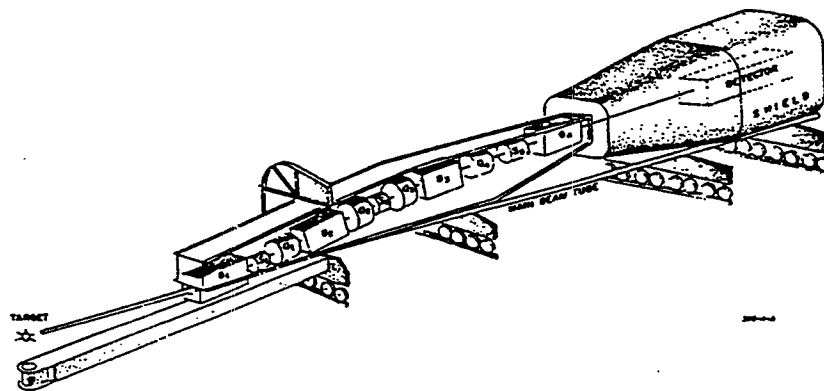
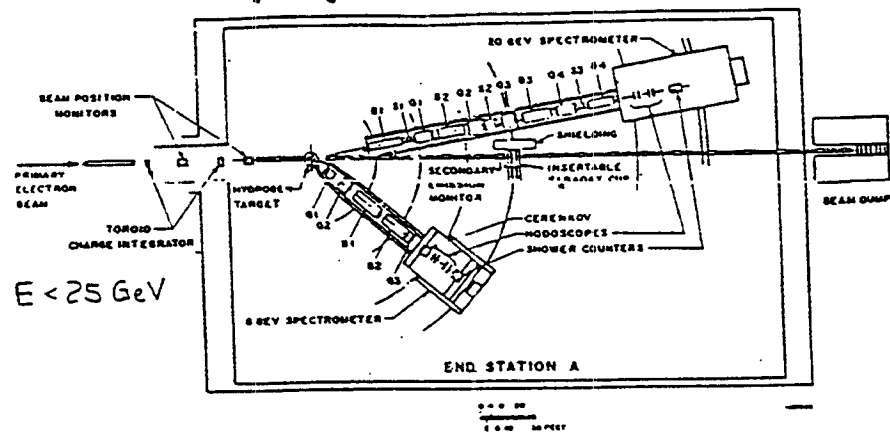


Fig. 5 SLAC 20 GeV spectrometer.

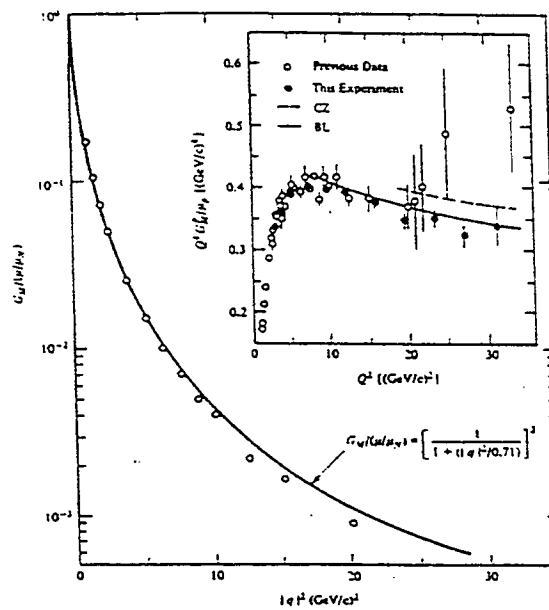
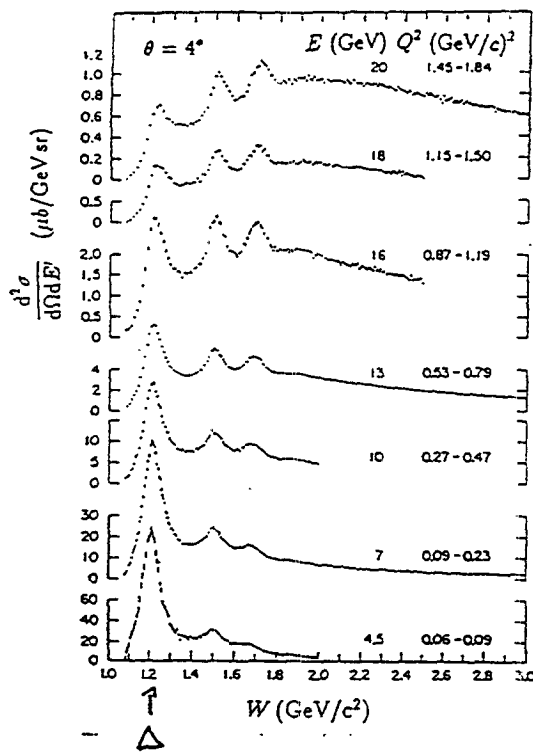


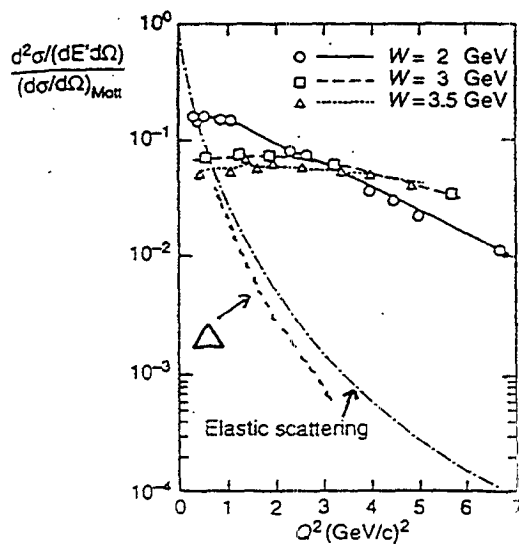
Fig. 6.15. Values of the proton magnetic form factor G_M , normalized by division with the proton magnetic moment, plotted against the squared momentum transfer q^2 . An empirical dipole fit to the data is shown as a solid line. The inset shows $Q^2 G_M/\mu_p$ as a function of Q , where $Q^2 = -q^2$, together with some theoretical curves. [From R. Arnold et al., *Phys. Rev. Lett.* 57, 174 (1986).]

Inelastic e-Nucleon scattering ⑪

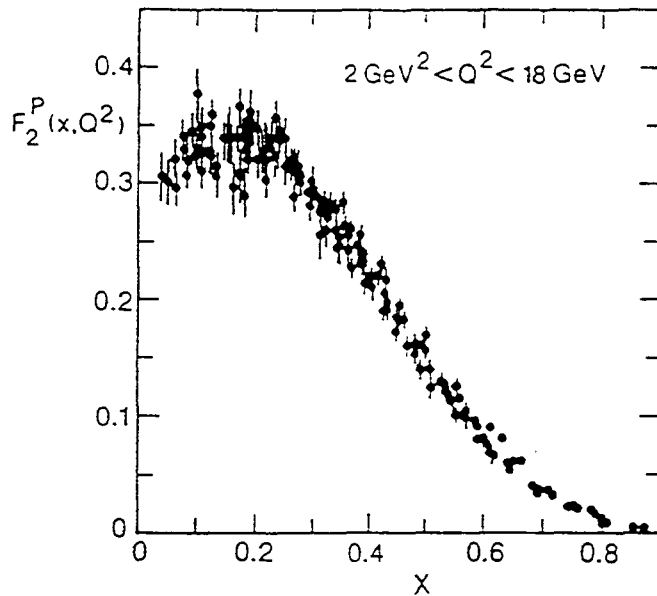


$$W^2 = (p+q)^2$$

$$= M^2 + 2MY - Q^2$$



Deep inelastic e-Nucleon scattering (12)

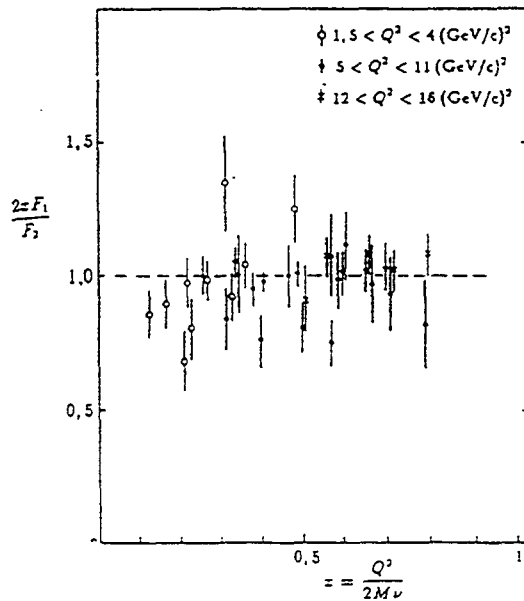


$$F_2(x, Q^2) \Big|_{x=\text{const}} \neq f(Q^2)$$

$\times$ scaling

→ nucleon has pointlike sub-structure

(Partons, Quarks)



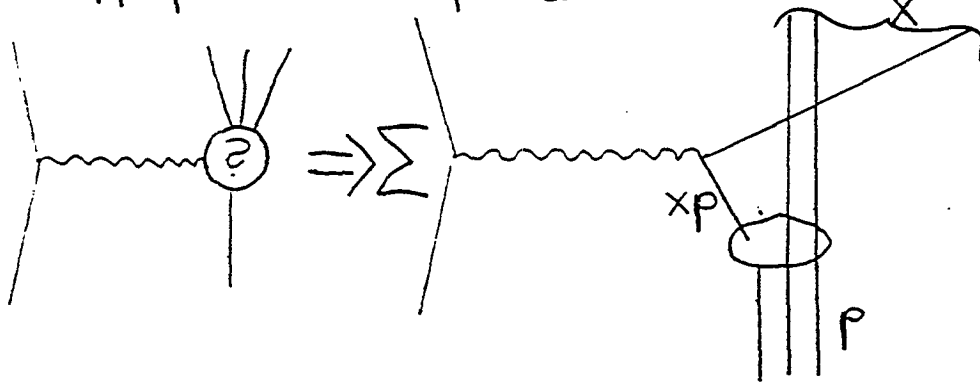
$$2xF_1(x) \approx F_2(x)$$

→ spin- $\frac{1}{2}$

Deep inelastic e-N-scattering

②

$\hat{=}$ incoherent sum of elastic scattering
off pointlike spin- $\frac{1}{2}$ constituents



Light cone :

$$X = \frac{p_0^q + p_z^q}{p_0^N + p_z^N} = \frac{\sqrt{m_q^2 + p_z^q^2 + p_t^q^2} + p_z^q}{\sqrt{M^2 + p_z^N^2} + p_z^N}$$

$$\xrightarrow{p_z \rightarrow 0} \frac{m_q}{M} \quad (\text{nucleon at rest})$$

$$\xrightarrow{p_z \rightarrow \infty} \frac{p_z^q}{p_z^N} \quad (\text{infinite momentum fr.})$$

$q_f(x) dx$ = probability to find quark
with flavor f in $[x, x+dx]$

$q_f(x)$ = quark distribution
= quark number density

$$F_2(x) = \sum_f e_f^2 \times [q_f(x) + \bar{q}_f(x)]$$

Proton: $|p\rangle = |u_v u_v d_v\rangle + \text{sea}$

$$\frac{1}{x} F_2^p(x) = \frac{4}{9} (u(x) + \bar{u}(x)) + \frac{1}{9} (d + \bar{d}) + \frac{1}{9} (s + \bar{s}) + \frac{4}{9} (c + \bar{c})$$

$$u_v(x) = u(x) - \bar{u}(x) \quad u_s(x) = \bar{u}(x)$$

$$\int_0^1 (u - \bar{u}) dx = \int_0^1 u_v dx = 2$$

$$\int_0^1 (d - \bar{d}) dx = \int_0^1 d_v dx = 1$$

$$\int_0^1 (s_s - \bar{s}) dx = \int_0^1 (d_s - \bar{d}) dx = \int_0^1 (c_s - \bar{c}) dx = 0$$

Neutron $|n\rangle = |d_v d_v u_v\rangle + \text{sea}$

isospin symmetry: $u^p = d^n = u$

$$d^p = u^n = d$$

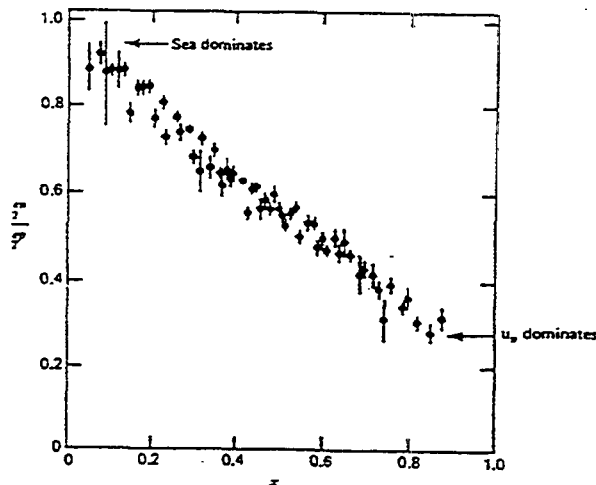
$$\frac{1}{x} F_2^n(x) = \frac{1}{9} (u + \bar{u}) + \frac{4}{9} (d + \bar{d}) + \frac{1}{9} (s + \bar{s}) + \frac{4}{9} (c + \bar{c})$$

$$\boxed{F_2^n(x)/F_2^p(x)}$$

(15)

assumption: $\bar{u}=\bar{d}=\bar{s}=\bar{c}=S$

$$\Rightarrow \frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_v + u_v + 20S}{4u_v + d_v + 20S}$$

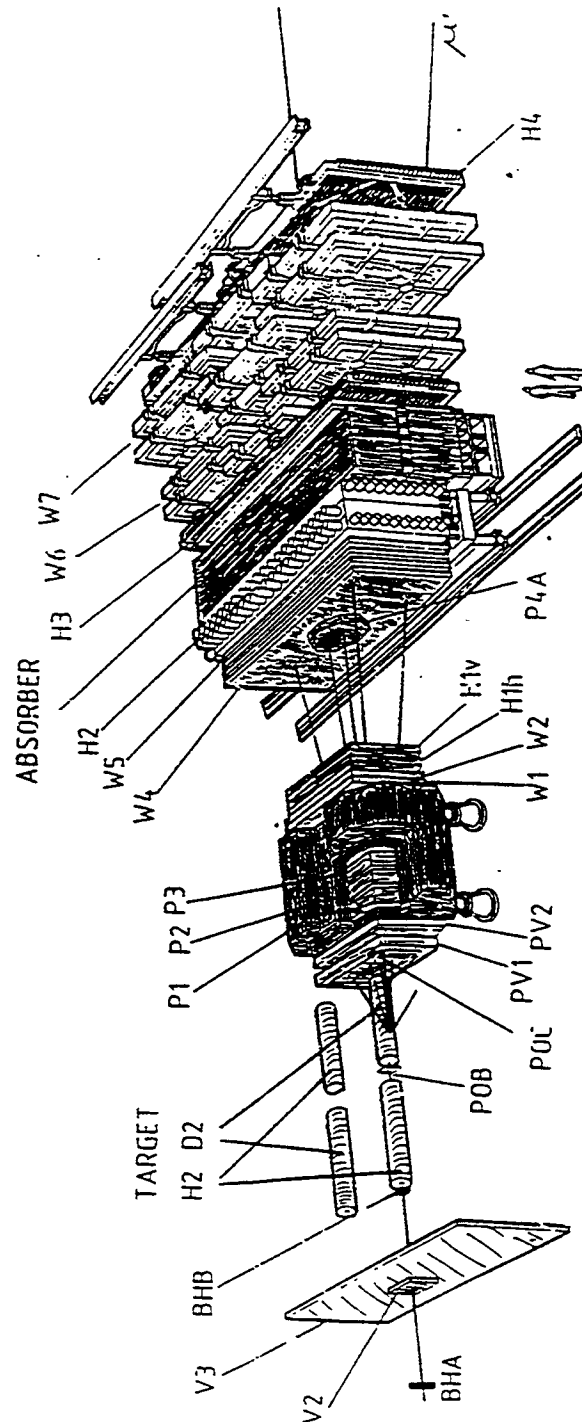


	F_2^n/F_2^p
$S=0, u_v=2d_v$	$\frac{2}{3}$
$S=0, u_v=0$	$\frac{1}{4}$
$S=0, d_v=0$	$\frac{1}{4}$
$u_v, d_v \ll S$	1

At large x : quark which defines
quantum numbers dominates
i.e. u in p , d in n

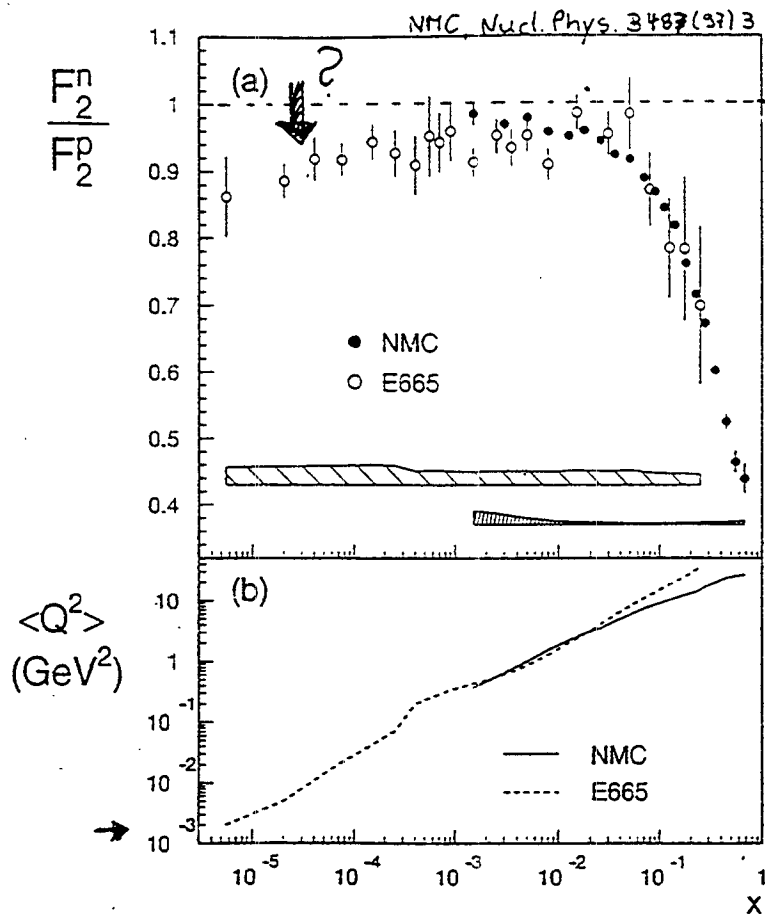
At small x : seaquarks dominate

NMC spectrometer



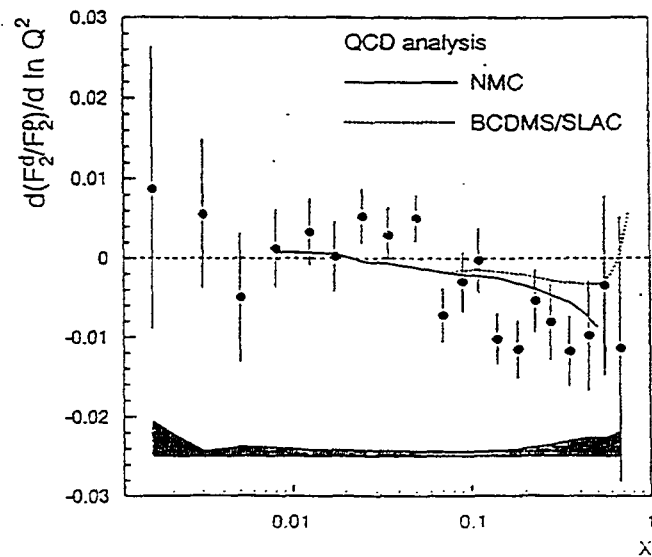
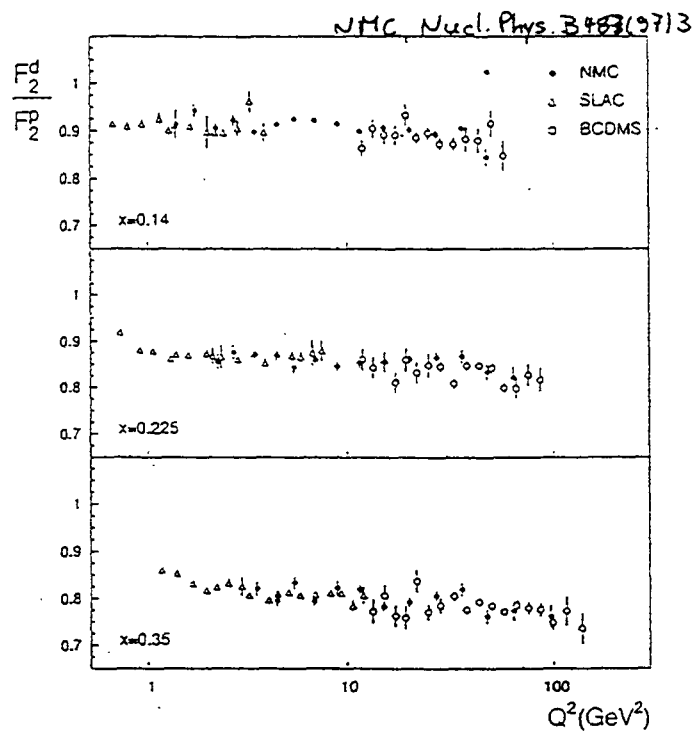
replaced drift chambers by proportional chambers
 added many wire chambers
 optimised for ratios

(16)



'Shadowing' in the deuteron?

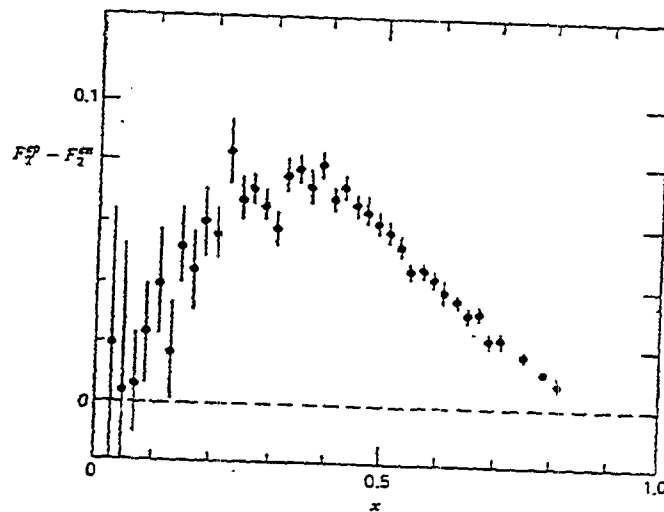
$$\text{But } \lambda = \frac{\hbar c}{\sqrt{Q^2}} \approx 6 \text{ fm} \gg R_d$$



$$\begin{aligned} \frac{1}{x} (F_2^p(x) - F_2^n(x)) &= \frac{1}{3} (u_v + u_s + \bar{u}) - \frac{1}{3} (d_v + d_s + \bar{d}) \\ &= \frac{1}{3} (u_v - d_v) + \frac{2}{3} (\bar{u} - \bar{d}) \end{aligned}$$

Gottfried Sum Rule:

$$S_G = \int_0^1 \frac{1}{x} (F_2^p(x) - F_2^n(x)) dx = \frac{1}{3} \quad \text{if } \bar{u} \equiv \bar{d}$$



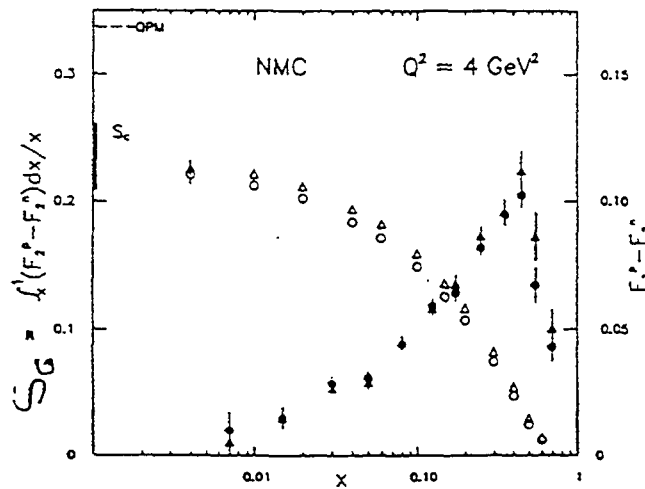
$$S_G = 0.28 \pm 2 \quad (\text{extrapolation } x \rightarrow 0 \hat{C})$$

~~Quarks~~ Quarks carry fractional charges

$$e_u = \frac{2}{3}, \quad e_d = -\frac{1}{3}$$

$$\begin{aligned}
 S_G &= \int_0^1 \frac{1}{x} (F_2'(x) - F_2''(x)) dx \\
 &= \frac{1}{3} \int_0^1 (u_v(x) - d_v(x)) dx + \frac{2}{3} \int_0^1 (\bar{u}(x) - \bar{d}(x)) dx \\
 &= \frac{1}{3} \text{ if } \bar{u} \equiv \bar{d}
 \end{aligned}$$

NMC, P.R. 50 (1994), R1



$$S_G(0.004 - 0.8) = 0.221 \pm 0.008(\text{stat}) \pm 0.019(\text{sys})$$

$$S_G(Q^2 = 4 \text{ GeV}^2) = 0.235 \pm 0.026$$



$$\int_0^1 dx (\bar{u}(x) - \bar{d}(x)) = -0.147 \pm 0.039$$



$$* u_v > d_v$$

$$* \bar{d} > \bar{u}$$

Why $\bar{u} \neq \bar{d}$?

21

Feynman + Field (1977)

$$x \cdot \bar{u} = 0.17 (1-x)^{10}$$

$$x = 0.17 (1-x)^7 \Rightarrow \int \frac{dx}{x} (\bar{F}_2^p - \bar{F}_2^n) = 0.2$$

Reason: Pauli blocking

$$(uud)(u\bar{u}) < (uud)(d\bar{d})$$

equivalent to:

$$\left. \begin{array}{l} 2 \text{ Flavors} \\ 3 \text{ colors} \end{array} \right\} 6 \text{ p}$$

$$\Rightarrow 4 \text{ more } u\bar{u}$$

$$5 \text{ more } d\bar{d}$$

$$\Rightarrow \frac{\bar{d}}{\bar{u}} \approx 5/4$$

$$p \leftrightarrow p + \pi^0$$

$$\pi^0 = (u\bar{u} + d\bar{d})$$

$$p \leftrightarrow n + \pi^+$$

$$\pi^+ = (u\bar{d})$$

$$p \leftrightarrow \Delta^{++} + \pi^-$$

$$\pi^- = (\bar{u}d) \text{ suppressed}$$

➡ More $\bar{d}$ than $\bar{u}$ in proton

$$\text{correspondingly } n \leftrightarrow p + \pi^-$$

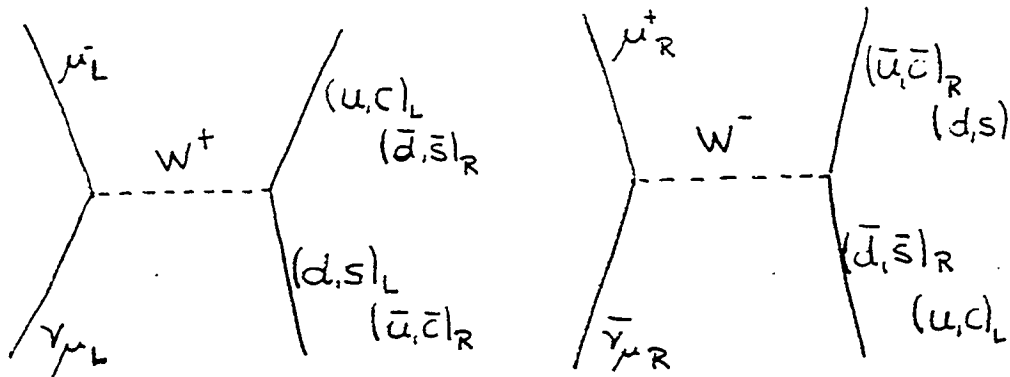
$$n \leftrightarrow \Delta^- + \pi^+$$

➡ Sea not only perturbative

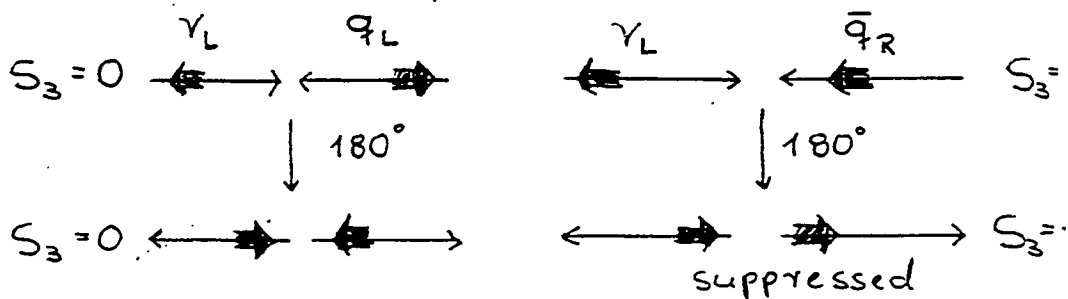
Antiquark distributions

$\gamma, \bar{\gamma}$ -N scattering, charged currents

			q_f		q_f
$\begin{pmatrix} \gamma_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \gamma_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \gamma_\tau \\ \tau^- \end{pmatrix}_L$	0 -1	$\begin{pmatrix} \bar{\gamma}_e \\ e^+ \end{pmatrix}_R$	0 +1
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\frac{2}{3}$ $-\frac{1}{3}$	$\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_R$	$-\frac{2}{3}$ $\frac{1}{3}$



Angular momentum conservation:



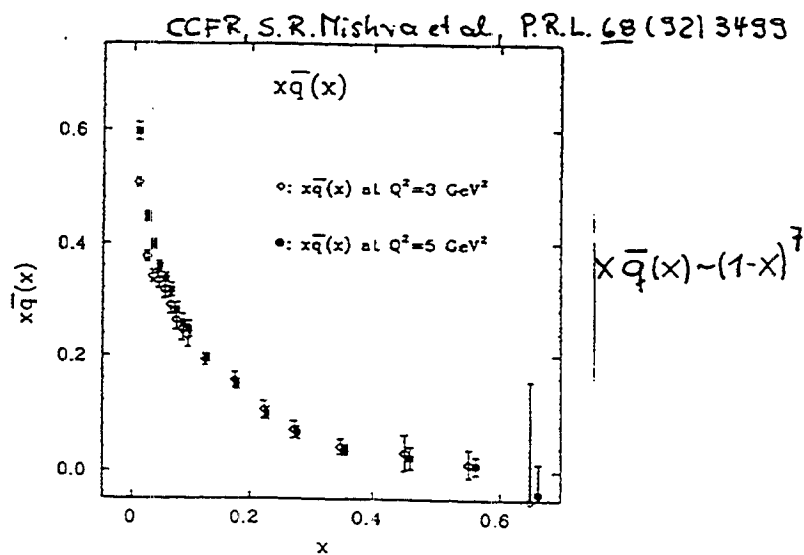
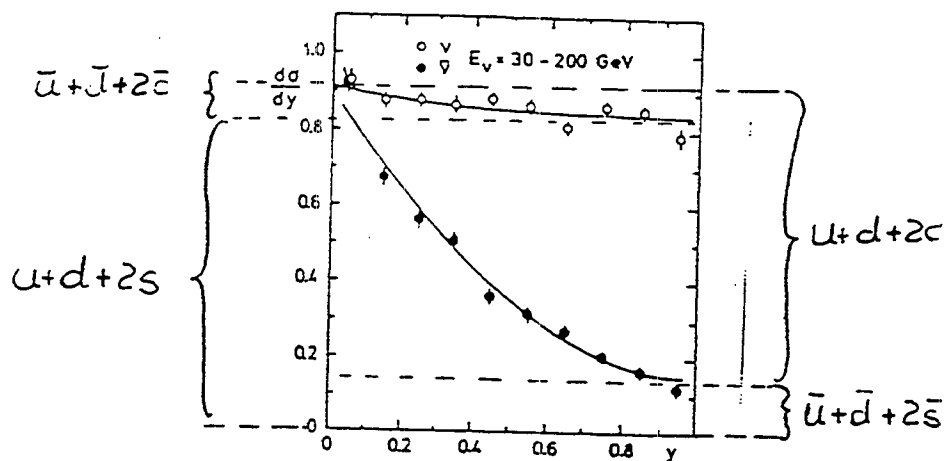
$$\Rightarrow \sigma_{\gamma_L \bar{q}_R, \bar{\gamma}_R q_L} \sim (1-y)^2 \quad y = \frac{E_\nu - E_\mu}{E_\nu}$$

Isoscalar target (Fe, C, ...)

$$P = \frac{M_W^2}{M_W^2 + Q^2} \quad (23)$$

$$\frac{d^2\sigma^{\bar{\nu}}}{dx dy} = \frac{G_F^2 \cdot P^2}{\pi} \cdot \underbrace{ME_Y}_{S_q} 2x [(u+d+2s) + (1-y)^2(\bar{u}+\bar{d}+2\bar{c})]$$

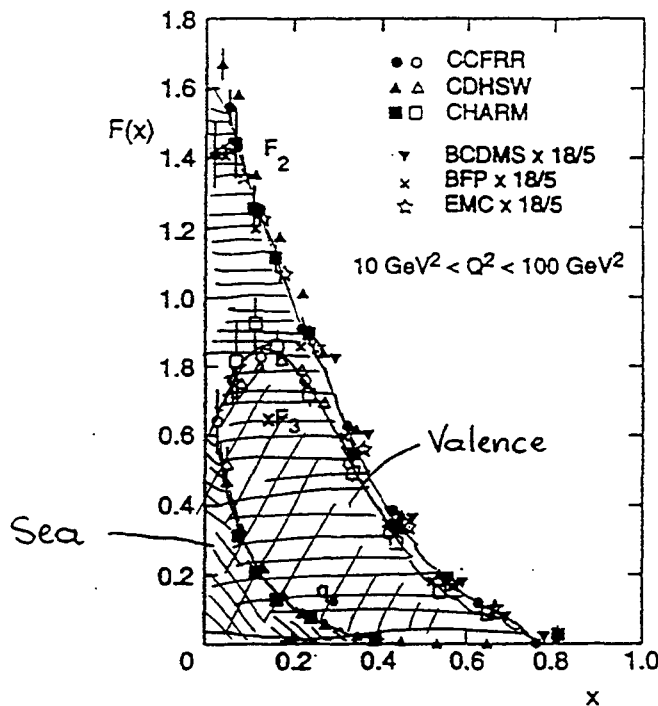
$$\frac{d^2\sigma^{\bar{\nu}}}{dx dy} = \dots \dots \dots [(\bar{u}+\bar{d}+2\bar{s}) + (1-y)^2(u+d+2c)]$$



$$F_2^{\gamma N} = x [u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x)], N = \frac{1}{2}(p, \bar{p})$$

$$F_2^{\mu N} = \frac{5}{18} x [u + \bar{u} + d + \bar{d} + s + \bar{s} + c + \bar{c}] + \frac{1}{6} [c + \bar{c} - s - \bar{s}]$$

$$\approx \frac{5}{18} F_2^{\gamma N} \quad (9)$$



Expectation:

$$\int_0^1 F_2^{\gamma N}(x) dx = \sum_f \int_0^1 x [q_f(x) + \bar{q}_f(x)] dx = 1$$

Experiment:

$$\dots \approx 0.48$$

→ Quarks carry only 48% of nucleon's momentum → Rest: Gluons

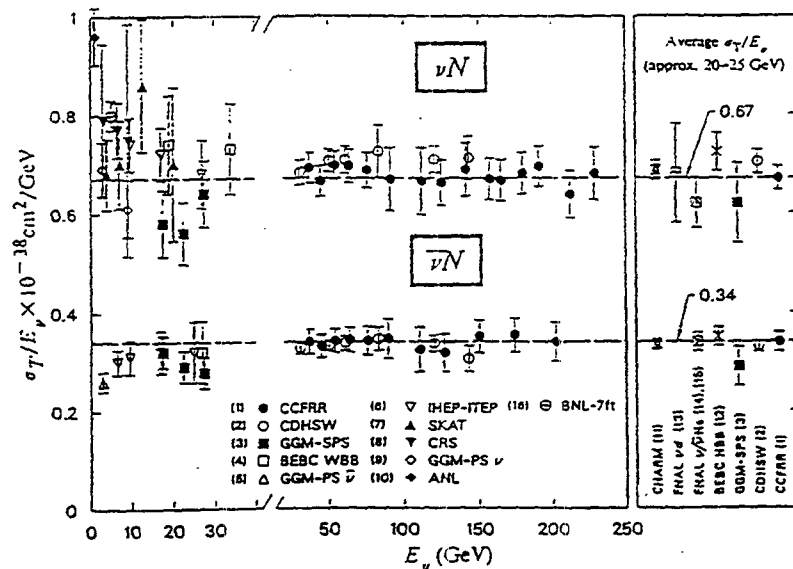
Total $\nu, \bar{\nu}$ -N cross sections

(25)

$$V = \int_0^1 (u_v(x) + d_v(x)) x dx, \quad \int_0^1 (1-y)^2 dy = \frac{1}{3}$$

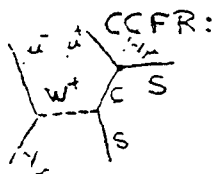
$$Q_s = \bar{Q} = \int_0^1 \sum_f x \bar{q}_f(x) dx$$

$$G = \int_0^1 x g(x) dx, \quad \int_0^1 F_2^{\nu N}(x) dx = V + Q_s + \bar{Q} = 0.48$$



$$\frac{\sigma_T(\bar{\nu})}{\sigma_T(\nu)} = \frac{\bar{Q} + \frac{1}{3}(V + Q_s)}{(V + Q_s) + \frac{1}{3}\bar{Q}} = \frac{1 + 4\bar{Q}/V}{3 + 4\bar{Q}/V} = \frac{0.34}{0.67}$$

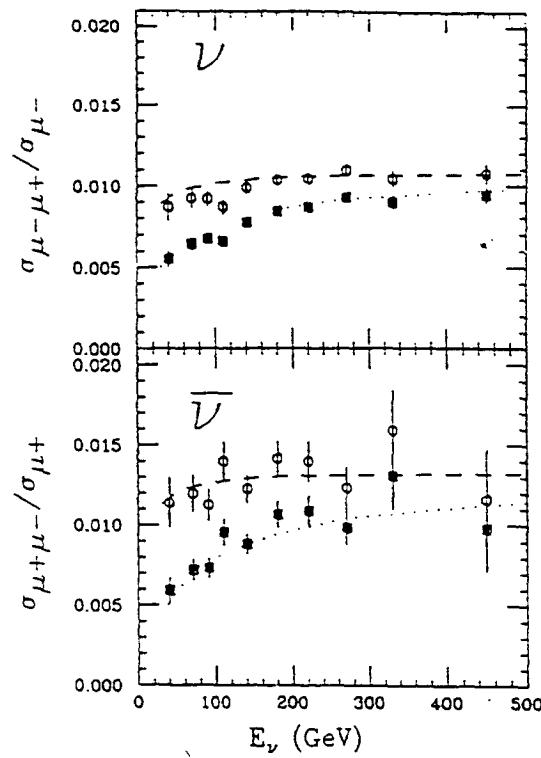
$$\rightarrow \boxed{V = 0.31; S = Q_s + \bar{Q} = 0.17; G = 0.52}$$



$$\frac{2 \int x \bar{s}(x) dx}{\int x (\bar{u}(x) + \bar{d}(x)) dx} = 0.477 \begin{matrix} +0.051 + 0.017 \\ -0.050 - 0.036 \end{matrix}$$

↓

0.477 ± 0.036

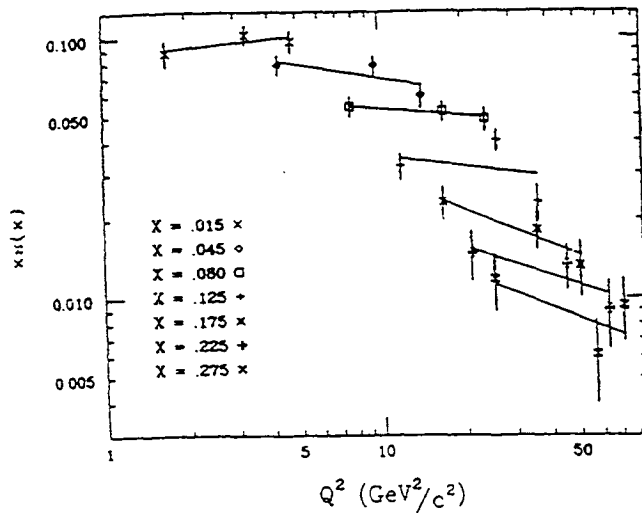


CCFR, S. R. Reberowicz et al.
P.R.L. 70 (1993) 134

(26)

■ expt. Daten
○ Daten nach 'slow
Rescaling'

'Slow Rescaling': $\frac{d^2\sigma}{dx dy}(\gamma N \rightarrow \mu^+ \mu^- X) = \dots \left[1 - \frac{m_c^2}{2ME_v y}\right] D(z)$



$$m_c = 1.31 \pm 0.236$$

$$B_c = 0.105 \pm 0.01$$

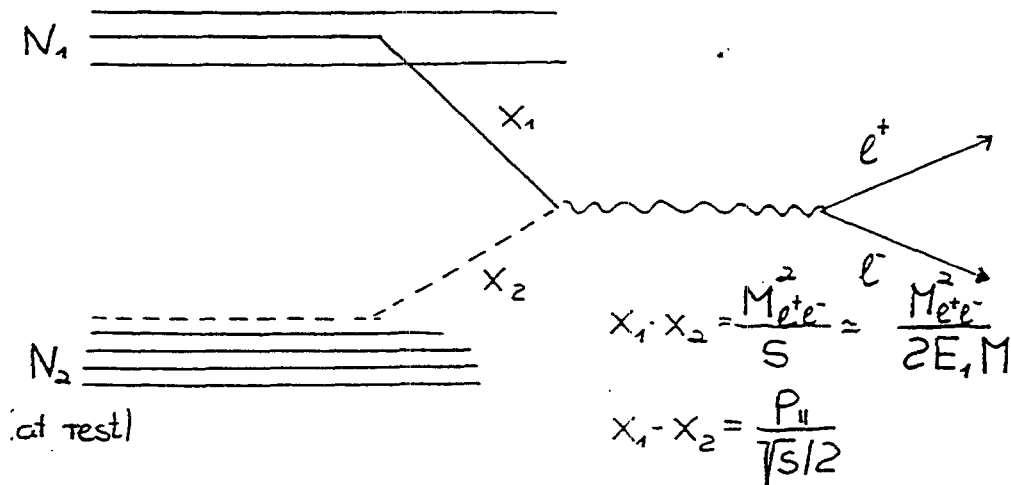
$$f = x \left(1 + \frac{m_c^2}{Q^2}\right)$$

$$x \bar{S}(x) \sim (1-x)^3$$

$$\partial Q^2 = 22.2 \text{ GeV}$$

The Drell-Yan process

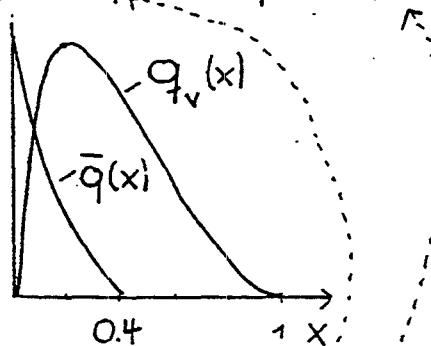
(27)



$$\frac{d^2\sigma}{dx_1 dx_2} = \sigma_0 \times \sum_f e_f^2 [\bar{q}_f(x_1) q_f(x_2) + q_f(x_1) \bar{q}_f(x_2)]$$

$$\sigma_0 = \frac{4\pi\alpha^2}{3S} \cdot \frac{1}{x_1 x_2} \cdot \frac{3}{9}$$

$S_{q\bar{q}}$ colour

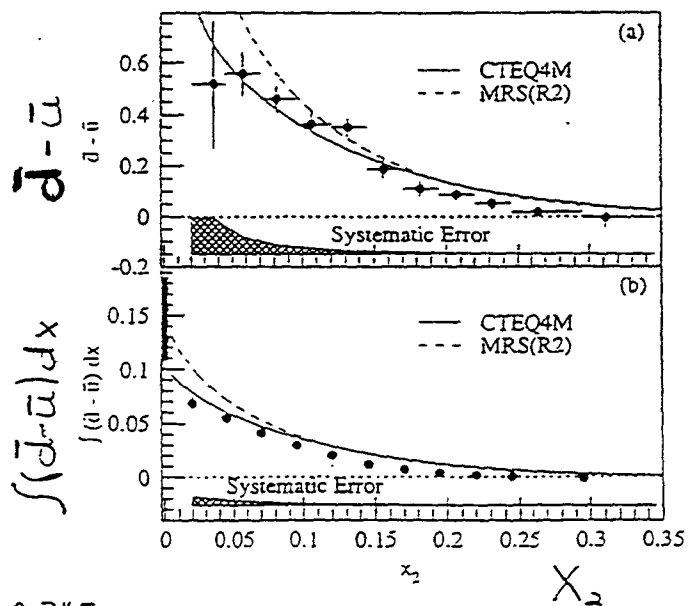
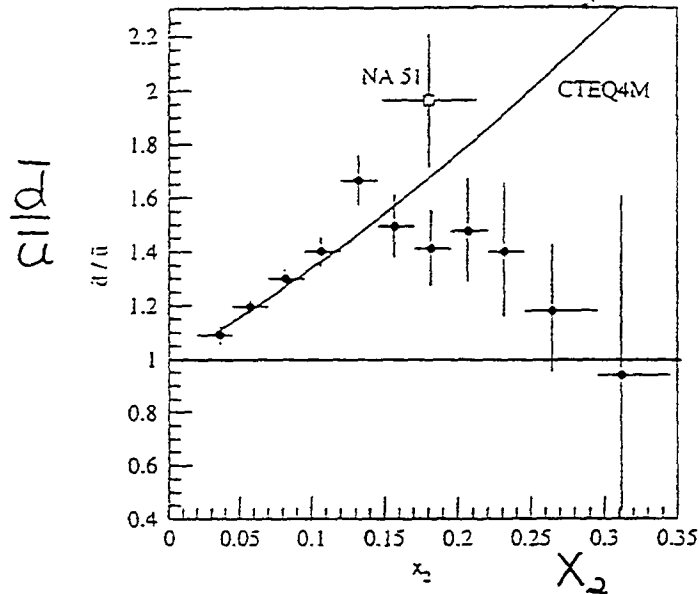


Select $x_1 > 0.4 \rightarrow \bar{q}(x_1) \approx 0$

$\rightarrow$ Measure $\bar{q}(x_2)$

p, n target $\rightarrow \bar{d}(x)/\bar{u}(x)$

$$\left. \frac{\sigma_{pd}}{2\sigma_{pp}} \right|_{x_1 \gg x_2} \approx \frac{1}{2} \frac{\left(1 + \frac{1}{4} \frac{d_1}{u_1}\right)}{\left(1 + \frac{1}{4} \frac{d_1}{u_1} \cdot \frac{\bar{d}_2}{\bar{u}_2}\right)} \cdot \left(1 + \frac{\bar{d}_2}{\bar{u}_2}\right) \quad (28)$$



0.345

$$\int_{0.020}^{0.345} (\bar{d} - \bar{u}) dx = 0.068 \pm 0.007(\text{stat}) \pm 0.008(\text{sys})$$

(II) Deep inelastic lepton-nucleon scattering and perturbative QCD

- Q.P.M: Scaling for $Q^2, \gamma \rightarrow \infty$

$$F_2(x, Q^2) \text{ for fixed } x = \frac{Q^2}{2M\nu}$$

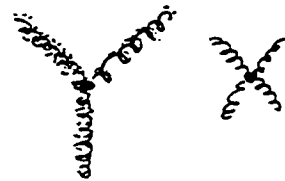
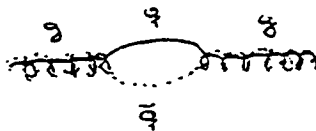
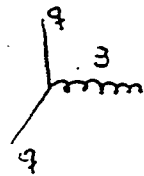
Data: Scale breaking

- Quarks carry only ~50% of nucleons momentum

$$\sum_f \int_0^1 x (q(x) + \bar{q}(x)) dx \approx 0.5$$



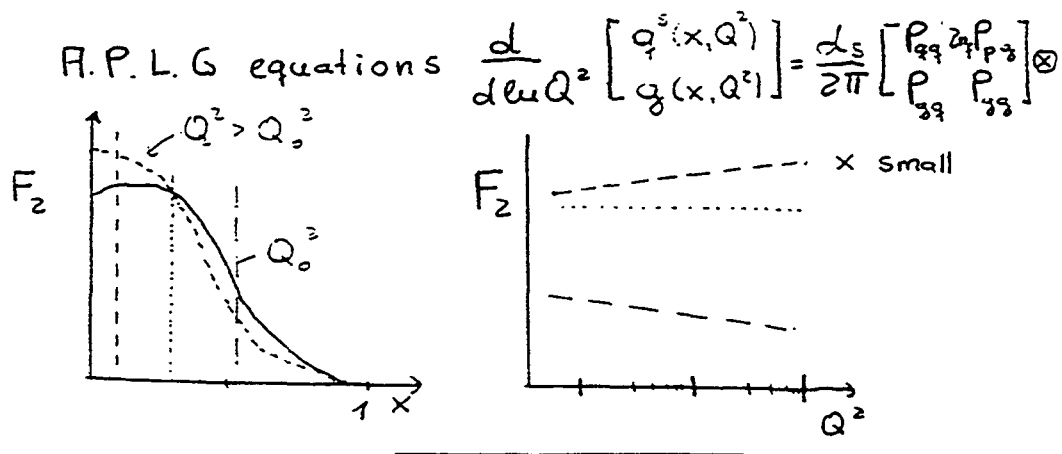
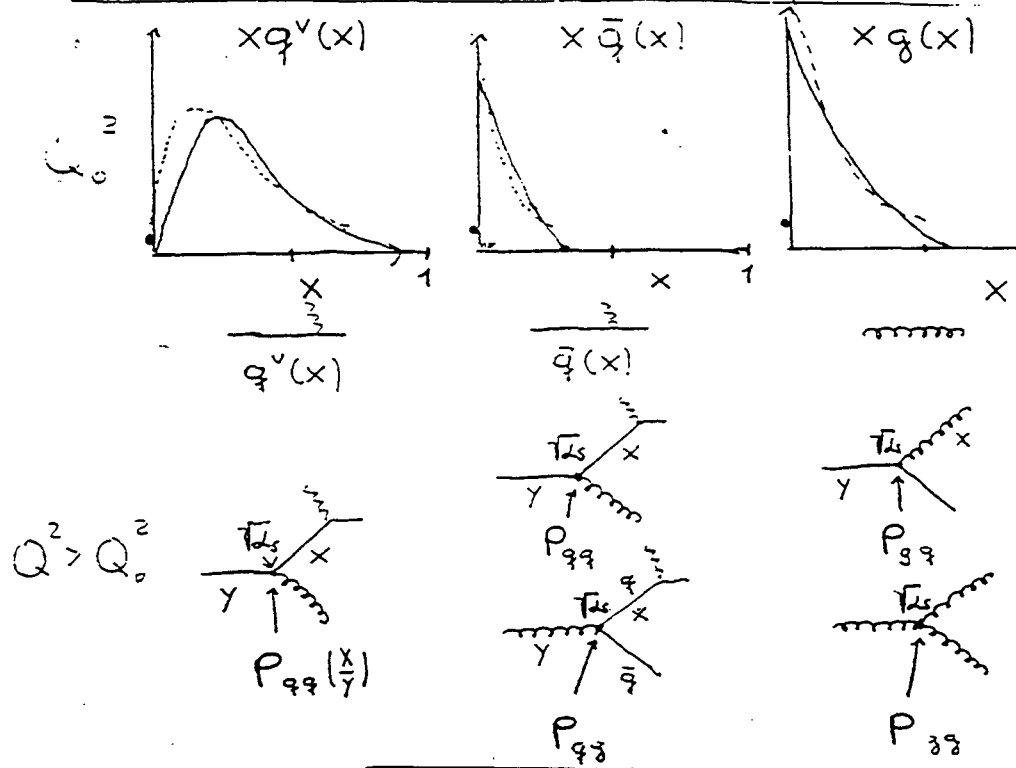
strong interaction



→ asymptotic freedom:

$$\alpha_s(Q^2)^{l.o.} = \frac{12\pi}{(33-2n_f) \ln(Q^2/\Lambda^2)} \xrightarrow{Q^2 \rightarrow \infty} 0 \text{ for } n_f =$$

QCD evolution of parton-distr

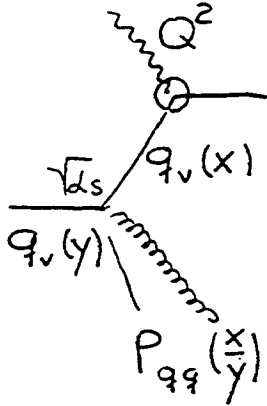


From pattern of scale breaking $\rightarrow \underline{x_s, g(x)}$

DGLAP equations

(31)

a) Non-Singlet (Valence like: $F_2^p - F_2^n, xF_3$)



$$\frac{d q_v(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q_v(y, Q^2) P_{qg}\left(\frac{x}{y}\right)$$

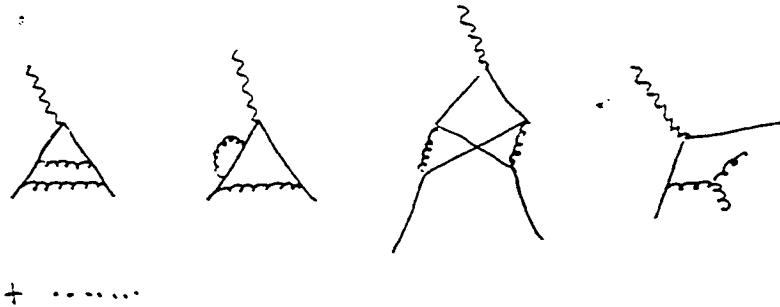
$$= \frac{\alpha_s}{2\pi} P_{qg} \otimes q_v$$

b) Singlet ($F_2^{\mu, \gamma-N}$)

$$\frac{d}{d \ln Q^2} \begin{bmatrix} q^s(x, Q^2) \\ g(x, Q^2) \end{bmatrix} = \frac{\alpha_s}{2\pi} \begin{bmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{bmatrix} \otimes \begin{bmatrix} q^s \\ g \end{bmatrix}$$

Higher order corrections

(32)



■ Expressions for 'Splitting functions' $P_{ij}(\frac{x}{y})$ are more complicated

$$\alpha_s^{n.l.o.} = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left(1 - \frac{\beta_1}{\beta_0} \frac{\ln \ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} \right)$$

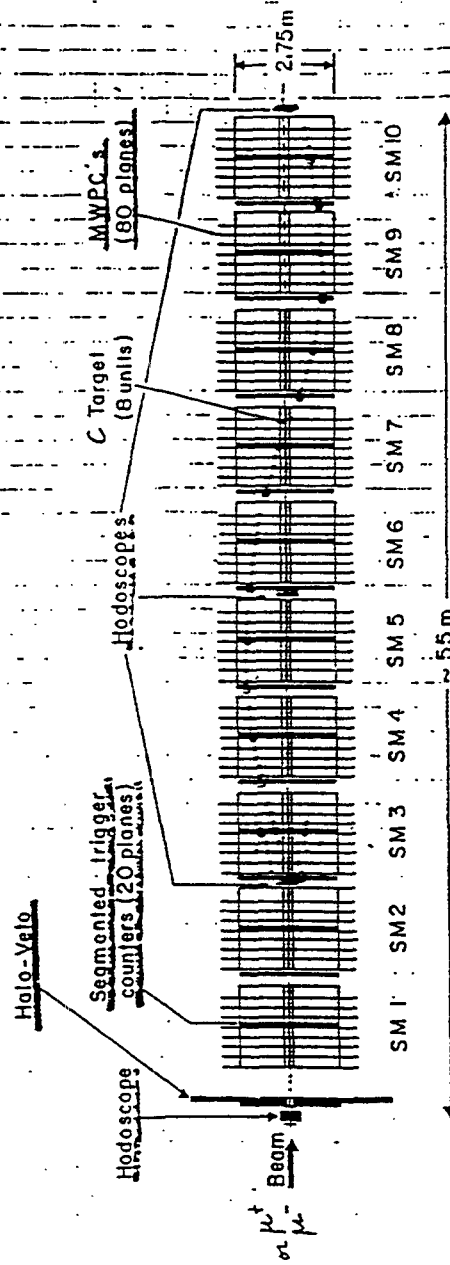
$$\beta_0 = 11 - \frac{2}{3} n_f \quad ; \quad \beta_1 = 102 - \frac{38}{3} n_f$$

$$\hookrightarrow \alpha_s^{n.l.o.} \approx \alpha_s^{l.o.} (1 - 0.2) \quad \text{at } Q^2 = 100 \text{ GeV}^2$$

■ Λ depends on renormalisation-scheme

usually: $\Lambda_{\overline{MS}}$

($\hookrightarrow$ Roberts, Ch.5)

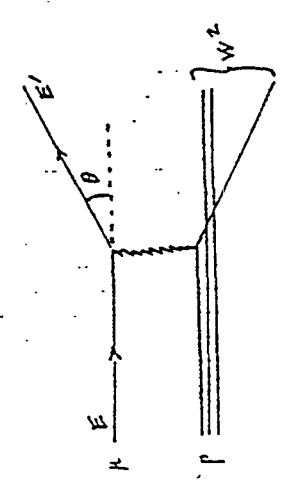


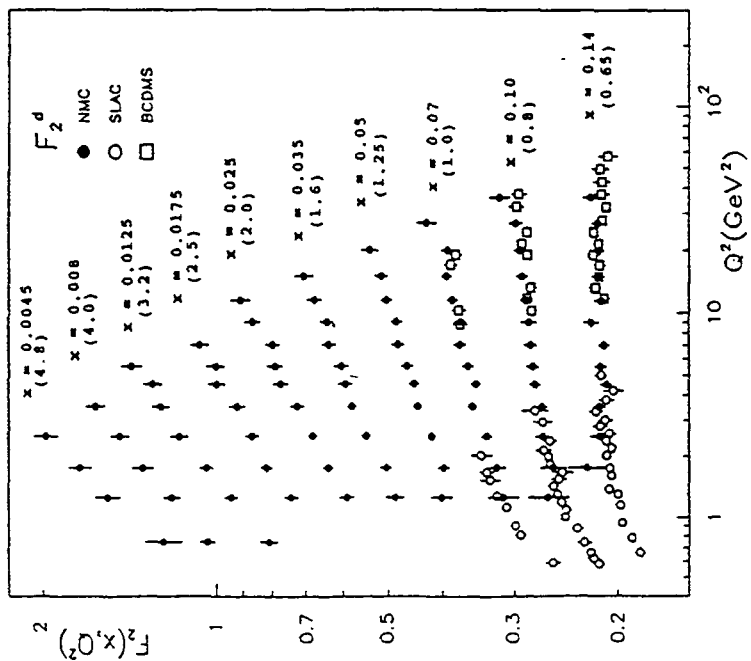
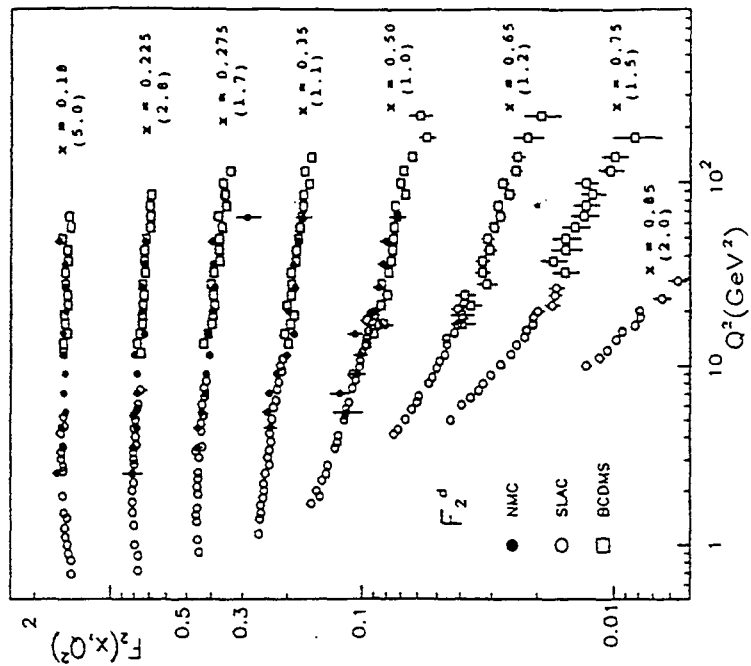
$$Q^2 = 2EE'(\sin\theta)$$

$$y = \frac{E-E'}{E}$$

$$x = \frac{Q^2}{2E'}$$

BCDMS





NLO perturbative QCD fits to NMC

(35)

Fitted are:

• xq^{NS}, xq^d, xq at $Q^2 = 7 \text{ GeV}^2$

$$xq^{NS} \equiv x \{ (u+\bar{u}) - (d+\bar{d}) \}$$

$$xq^d \equiv xq^{SI} - \frac{3}{5} x\tilde{q}^{NS}$$

$$xq^{SI} \equiv x \{ u+\bar{u} + d+\bar{d} + s+\bar{s} + c+\bar{c} \}$$

$$x\tilde{q}^{NS} \equiv x \{ (s+\bar{s}) - (c+\bar{c}) \} \quad (\text{From KIRS-3p})$$

$$\text{QPM: } F_2^p = \frac{5}{18} xq^d + \frac{3}{18} xq^{NS}$$

$$F_2^d = \frac{5}{18} xq^d$$

$$F_2^p - F_2^n = \frac{1}{3} xq^{NS}$$

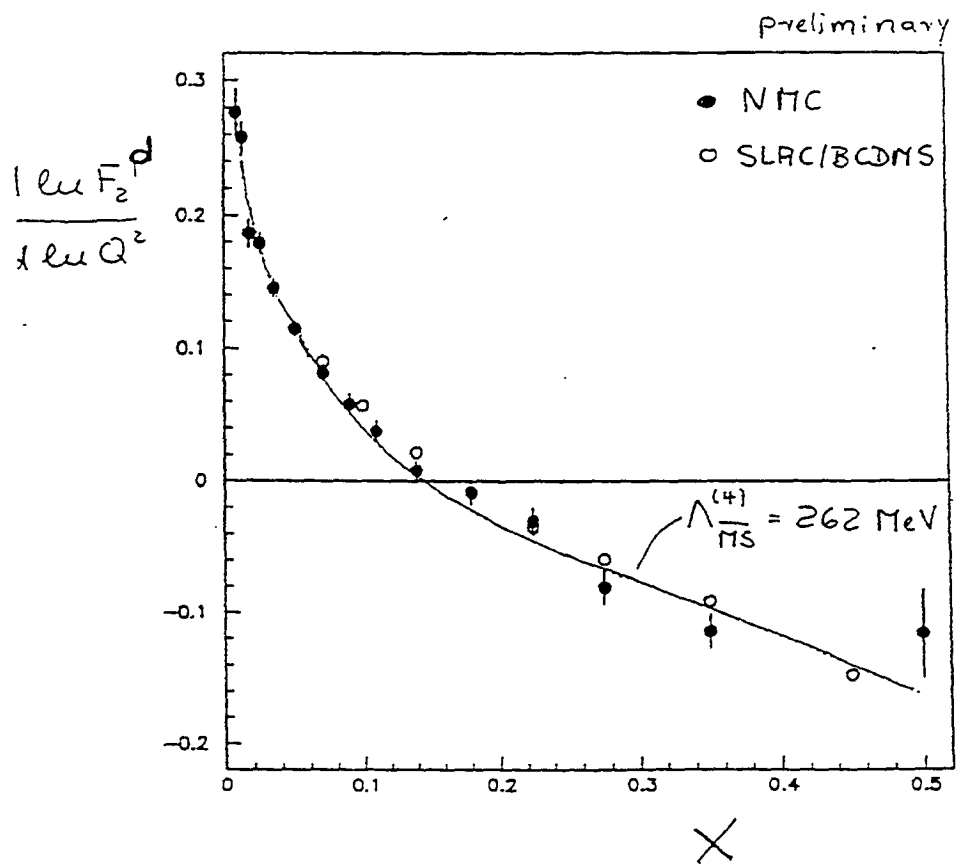
- NLO Q^2 evolution program (Vivchaux, Ouraou, Botje)

- Fix $\Lambda_{\overline{MS}}^{(4)} = 263 \pm 42 \text{ MeV}$

- $Q_{\min}^2 = 1 \text{ GeV}^2$

- Impose momentum S.R. $\int_0^1 dx (xq^{SI} + xq) = 1$

— QCD fit to NMC data
(Λ fixed)

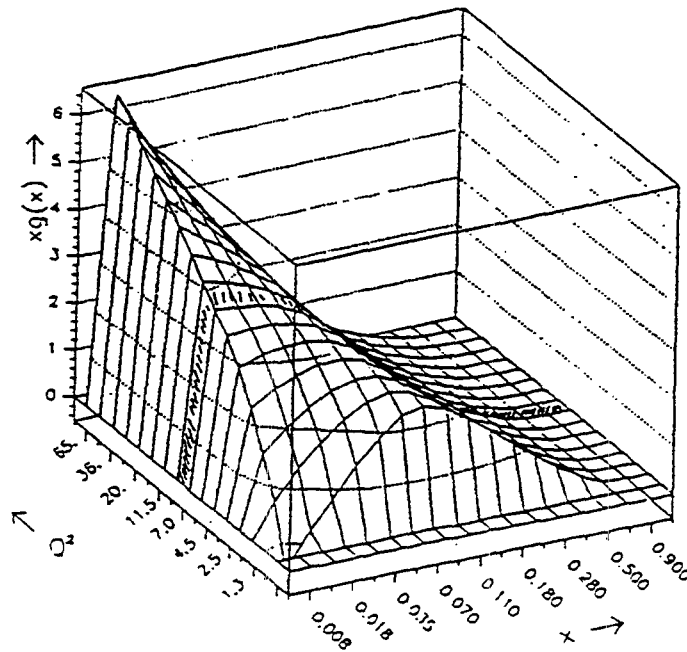
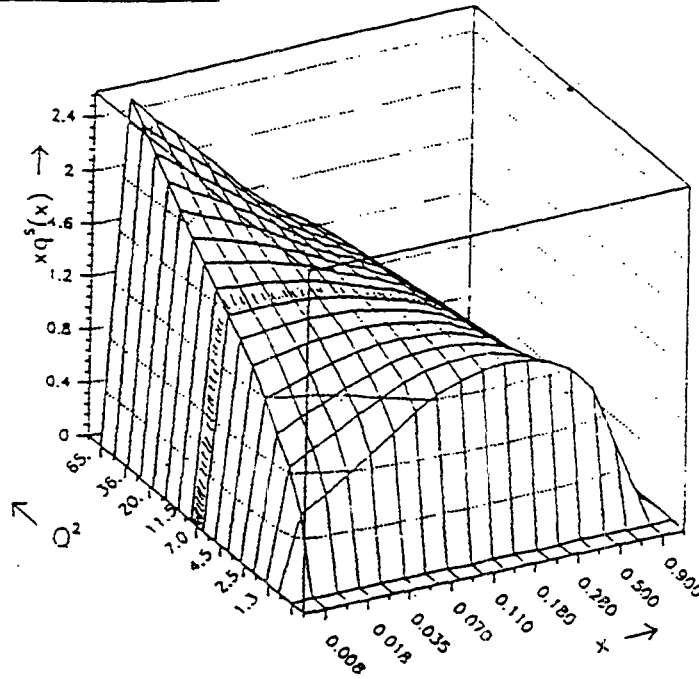


x and Q^2 evolution of parton distributions

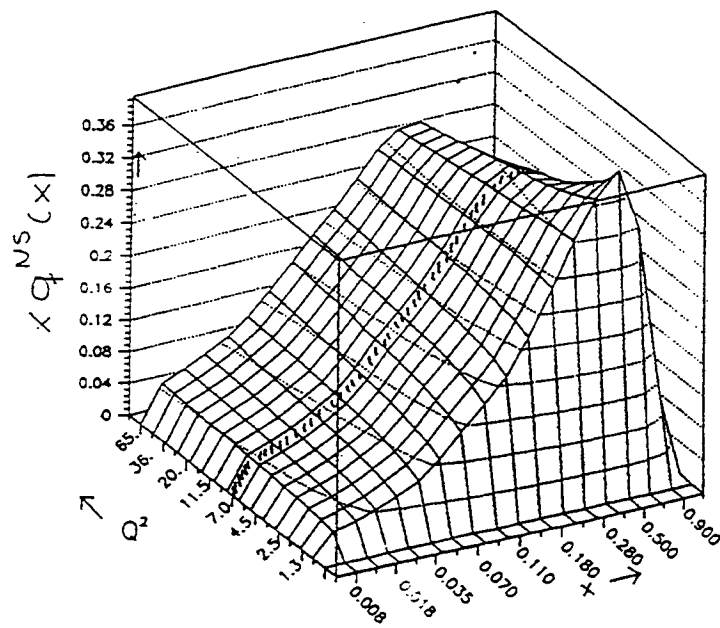
A.P. evolution equations

(37)

Fit at $Q^2 = 7 \text{ GeV}^2$



38



$$\Lambda_{\overline{MS}} : \mathcal{L}_S(Q^2)$$

(39)

Quantity	Experiment	$\Lambda_{\overline{MS}}^{(4)}$ (MeV)	$\mathcal{L}_S$ ($Q^2 = M_Z^2$)
$\times F_3^{\gamma N}$	CCFR	$179 \pm 36 \pm 41$	
$\times F_3^{\nu N} ; F_2^{\nu N}(x > 0.25)$	CCFR	$210 \pm 28 \pm 41$	0.111 ± 0.002 $\downarrow$ ± 0.004 $0.113 \pm \dots$
$F_2^{e\mu N}$	SLAC-BCDMS	$263 \pm 42 \pm 55$	0.113 ± 0.003 ± 0.004
$F_2^{\mu N}$	EMC	$211 \pm 80 \pm 80$	
$F_2^{\mu N}$	NMC		$Q^2 = 7.8 \text{ GeV}^2$ 0.264 ± 0.018 $\pm 0.070 \pm 0.013$
SF	world average	$234 \pm 26 \pm 50$	$Q^2 = M_Z^2$ 0.1175 $\pm 0.002 \pm 0.004$
e^+e^-	world average		0.121 ± 0.006

Rule of the thumb : $\hbar c \approx 200 \text{ MeV fm}$

$$R_N \approx 0.86 \text{ fm}$$

$$\Rightarrow \Lambda \approx \frac{\hbar c}{R_N}$$

(40)

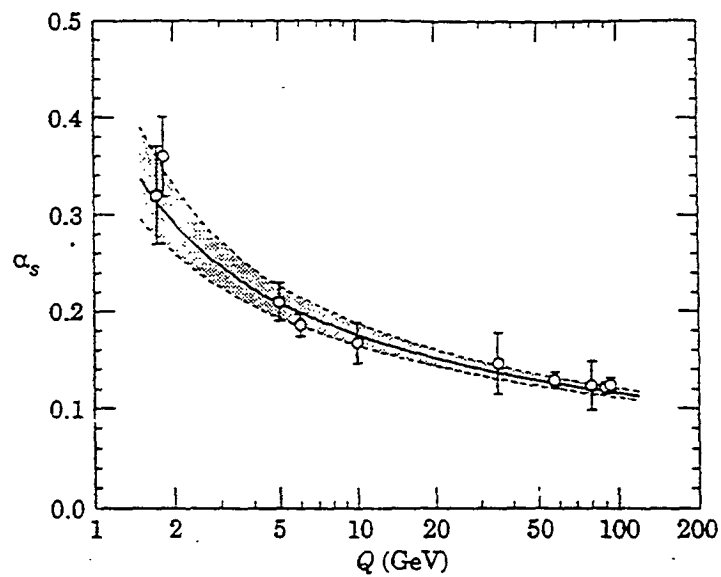


Figure 25.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ .

Rule of the thumb:

$$\alpha_s(Q=5 \text{ GeV}) \approx \frac{1}{5}$$

$$\alpha_s(Q=30 \text{ GeV}) \approx \frac{1}{9}$$

$$\boxed{R(x, Q^2)}$$

(4)

$$\boxed{\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{xQ^4} \cdot F_2(x, Q^2) \left\{ 1 - \gamma \frac{Q^2}{4E^2} + \frac{\gamma^2 + Q^2/E^2}{2(1+R(x, Q^2))} \right\}}$$

$$R(x, Q^2) = \frac{F_L(x, Q^2)}{2 \times F_1(x, Q^2)}$$

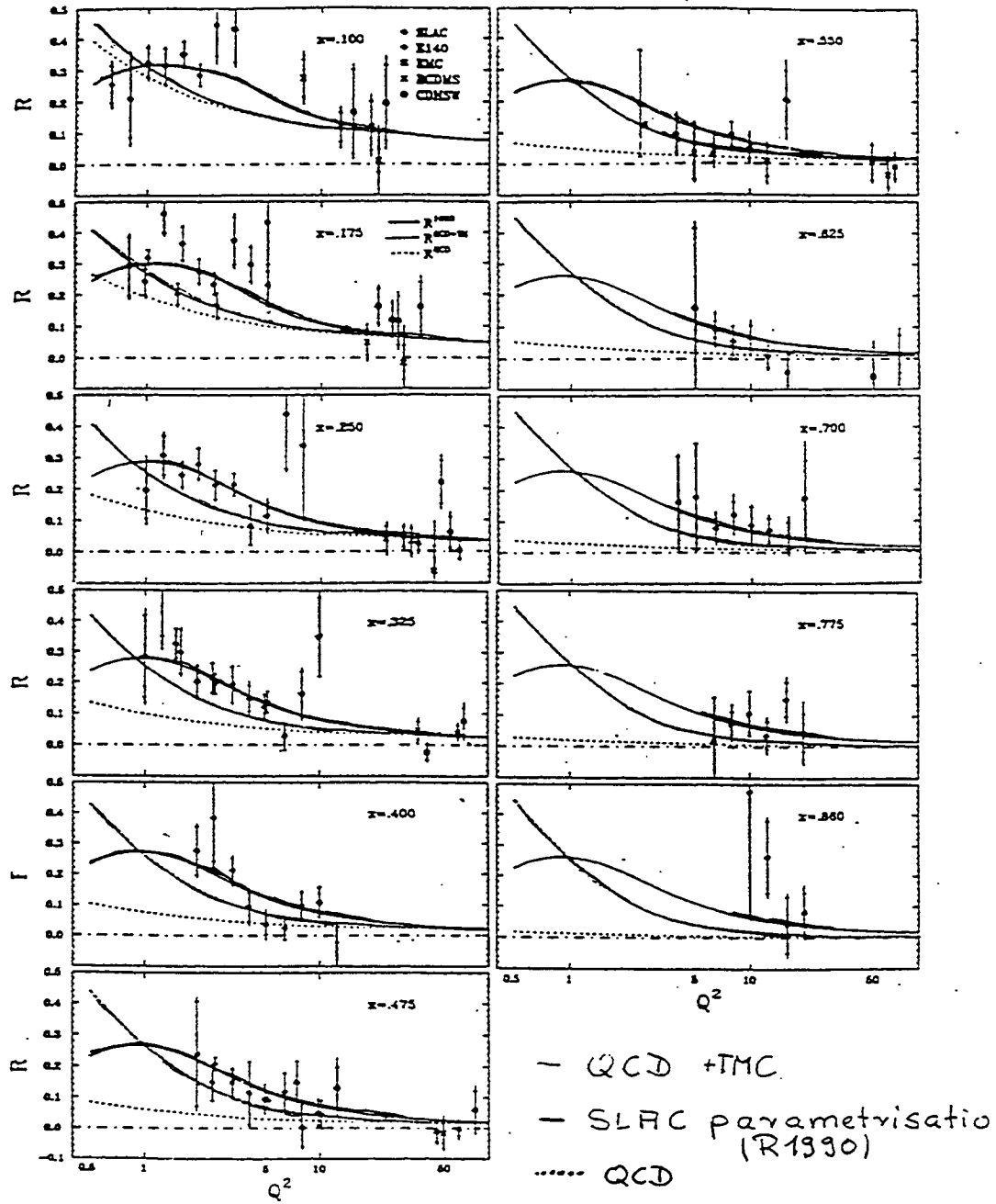
$$\boxed{F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left[\frac{8}{3} F_2(y, Q^2) + \frac{40}{9} \left(1 - \frac{x}{y}\right) y G(y, Q^2) \right]}$$

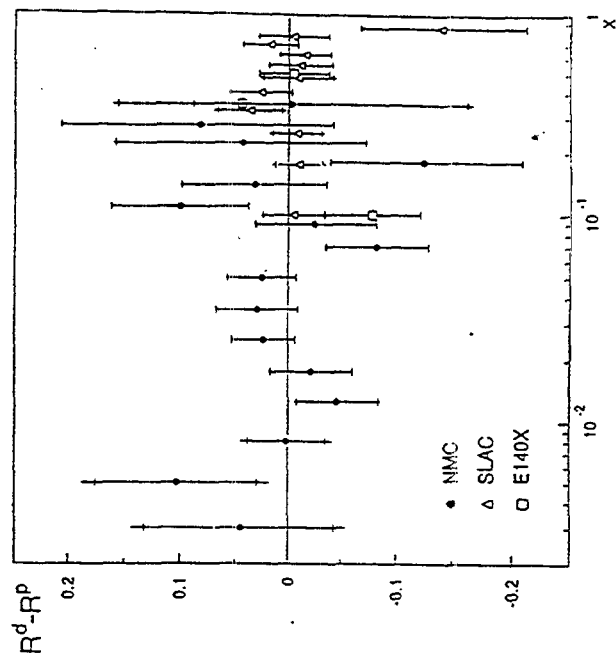
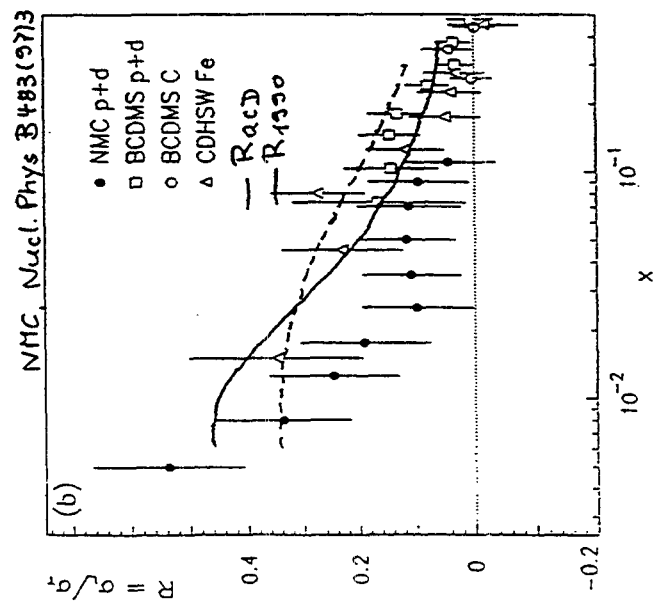
- Knowledge of $R(x, Q^2)$ needed for determination of F_2 from $\frac{d^2\sigma}{dQ^2 dx}$
Large contribution to syst. error of F_2
- dito : polarised s.f. $g_1(x, Q^2) \sim \frac{F_1 \cdot F_2}{2x(1+R)}$
- In principle : tool for det. of $g(x, Q^2)$
- In reality : data 'compatible' with QCI

➤ Substantial improvement !
urgently needed

(Several beam energies, systematics!)

W. Whitlow et al, P.L 2508 (1990) 193. 42



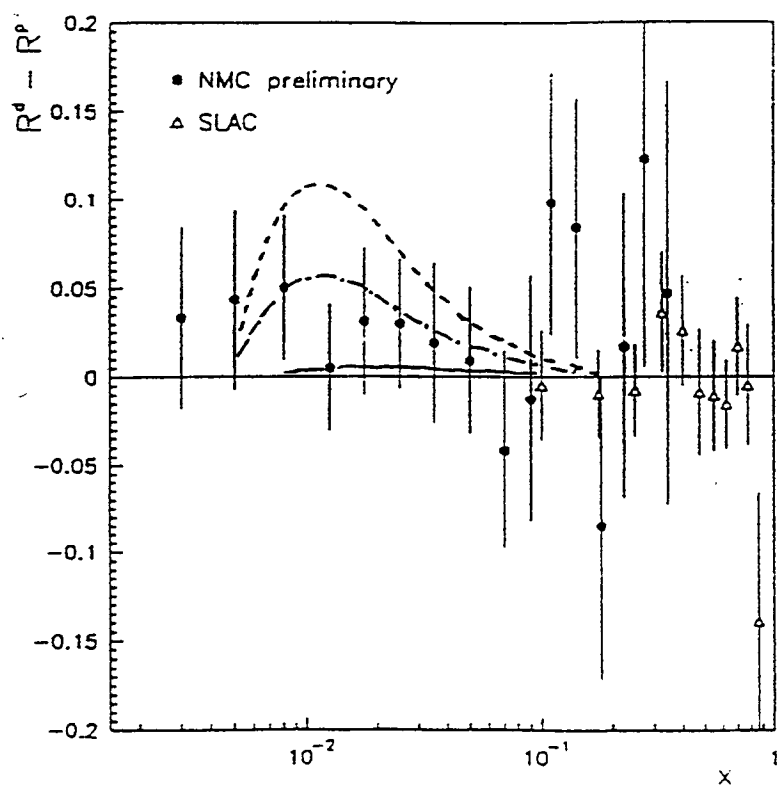


(43)

$$F_L(x, Q^2) = \frac{\alpha_s}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left\{ \frac{8}{3} F_2(y, Q^2) + \frac{40}{9} (1 - \frac{x}{y}) y G(y, Q^2) \right\} \quad (5)$$

$\downarrow$
 different for p, n, d?

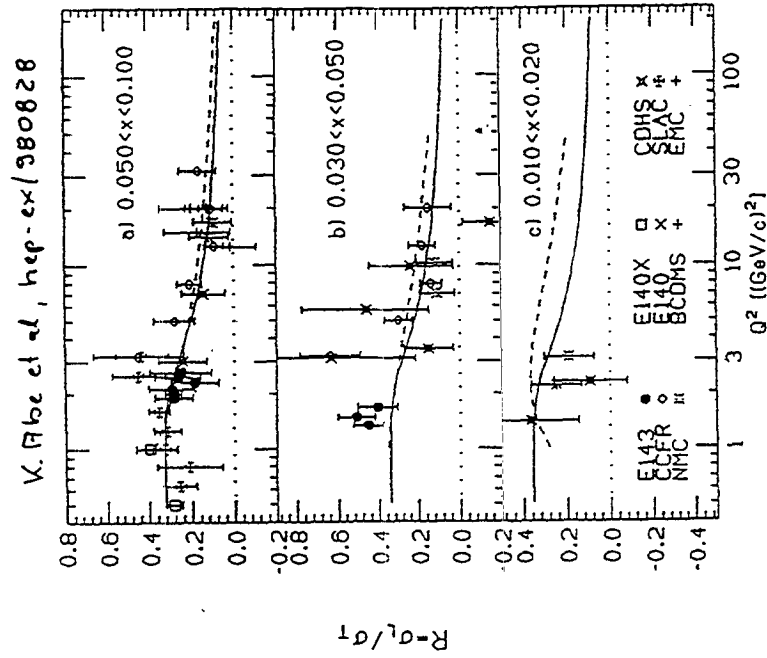
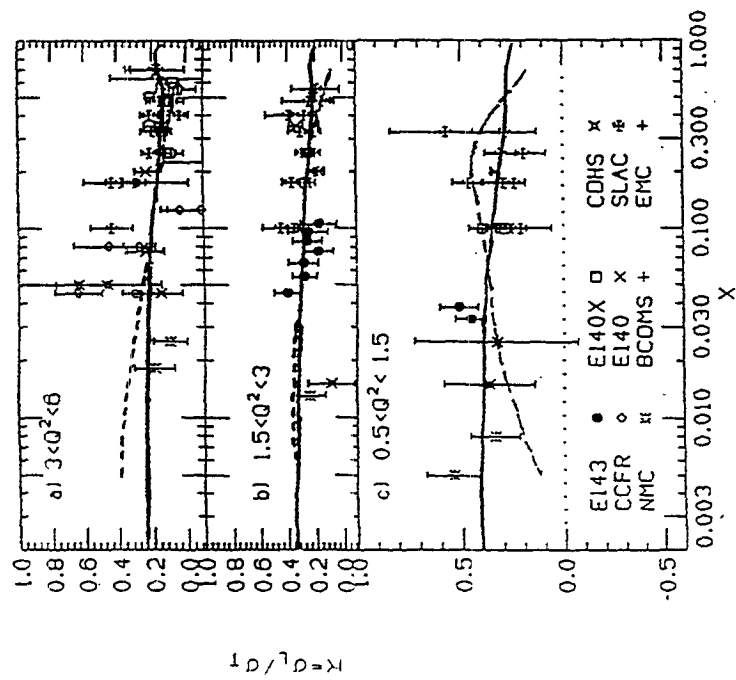
$$\frac{G}{F}^d = \frac{F_2^d}{F_2^p} \quad \text{if} \quad R^d \equiv R^p$$



New data from SLAC-E143 for $x < 0.1$

----- NNLO pQCD

— R1998



46

③ DIS at very high Q^2 - HERA

④

Very high Q^2, W^2 only possible with colliding beams

Fixed target:

$\longrightarrow \otimes$
 $k = (E_e, \vec{k}) \quad p = (M, 0)$
 $s = (k+p)^2 = 2E_e M$
 FNAL-E665 $E_e \sim 5506$
 $\sqrt{s} \approx 32 \text{ GeV}$
 $Q^2 \lesssim 500 \text{ GeV}^2$

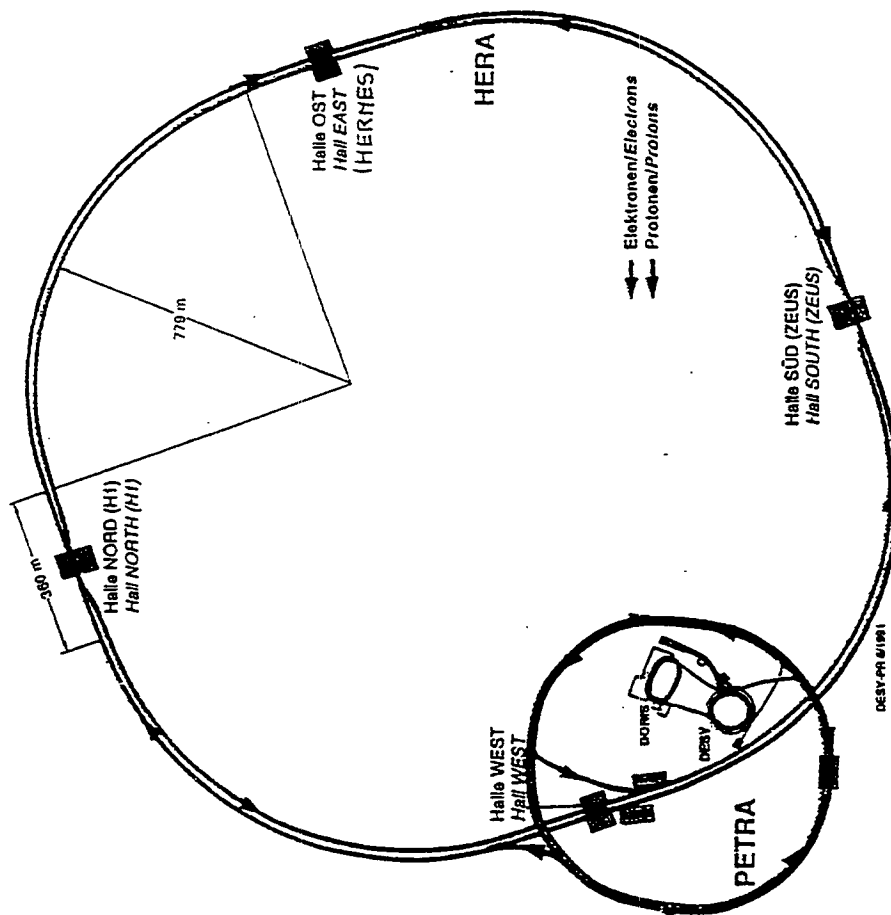
collider:

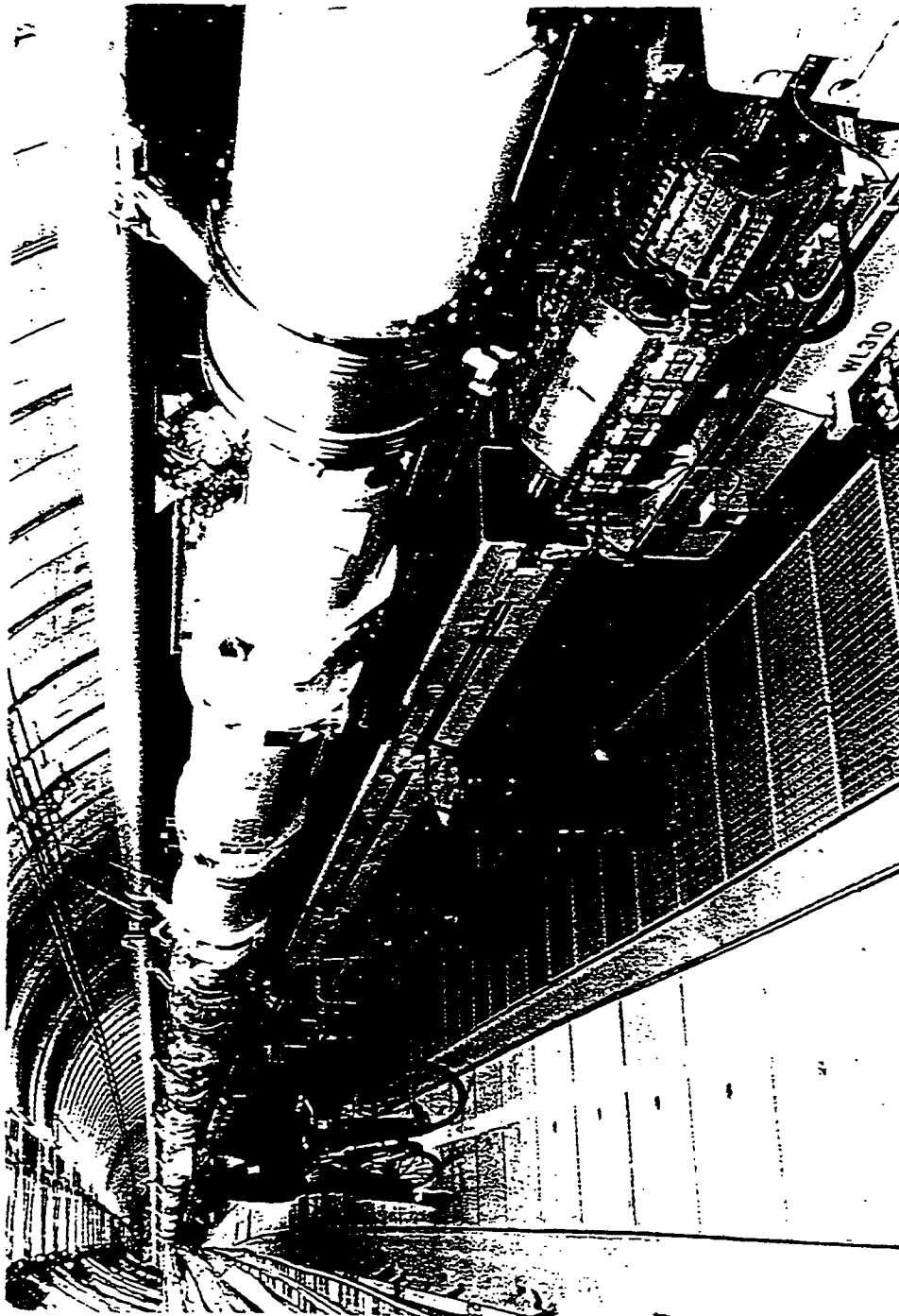
$\longrightarrow \longleftarrow$
 $k = (E_e, \vec{k}) \quad p = (E_p, \vec{p})$
 $s = (k+p)^2 = 2E_e E_p$

HERA 1994-1997:

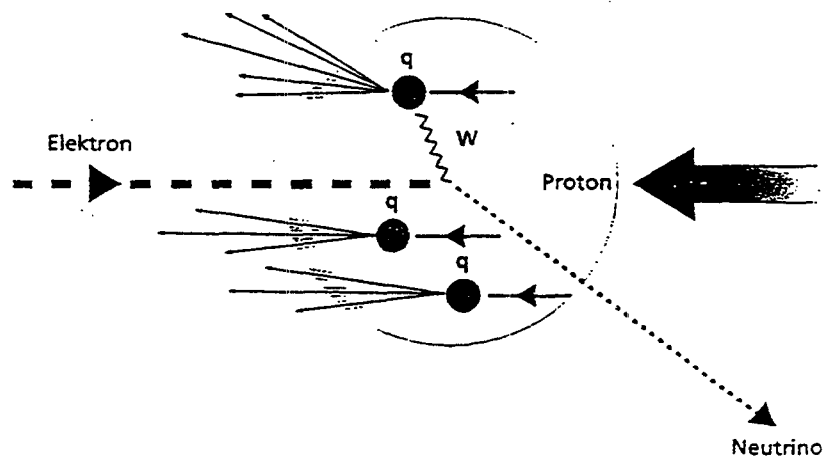
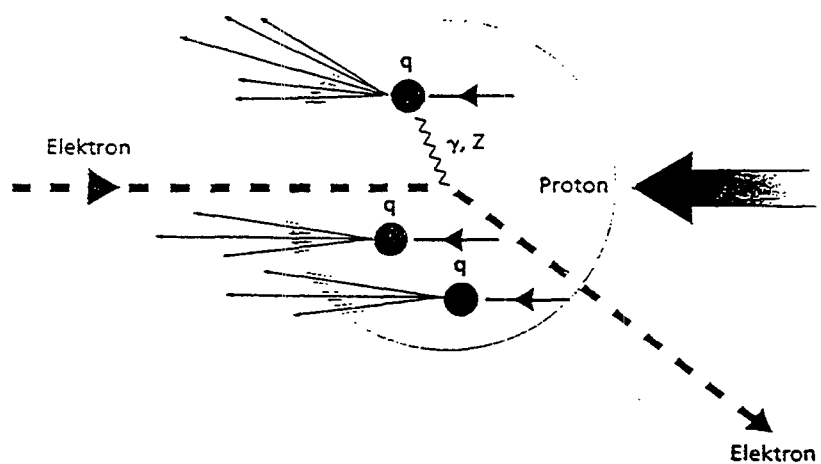
$E_e = 27.5 \text{ GeV}; E_p = 820 \text{ GeV} \rightarrow \sqrt{s} = 300 \text{ GeV}$

$Q^2 = s \cdot x \cdot y$
 $\approx 30\,000 \text{ GeV}^2$
 $x \approx 10^{-5}$





DESY GRAF 94-003



Study $\left. \begin{array}{l} \text{electromagnetic } (\gamma) \\ \text{strong } (g) \\ \text{weak } (W^\pm, Z^0) \end{array} \right\} \text{interactions}$

ZEUS (HERA) (8)

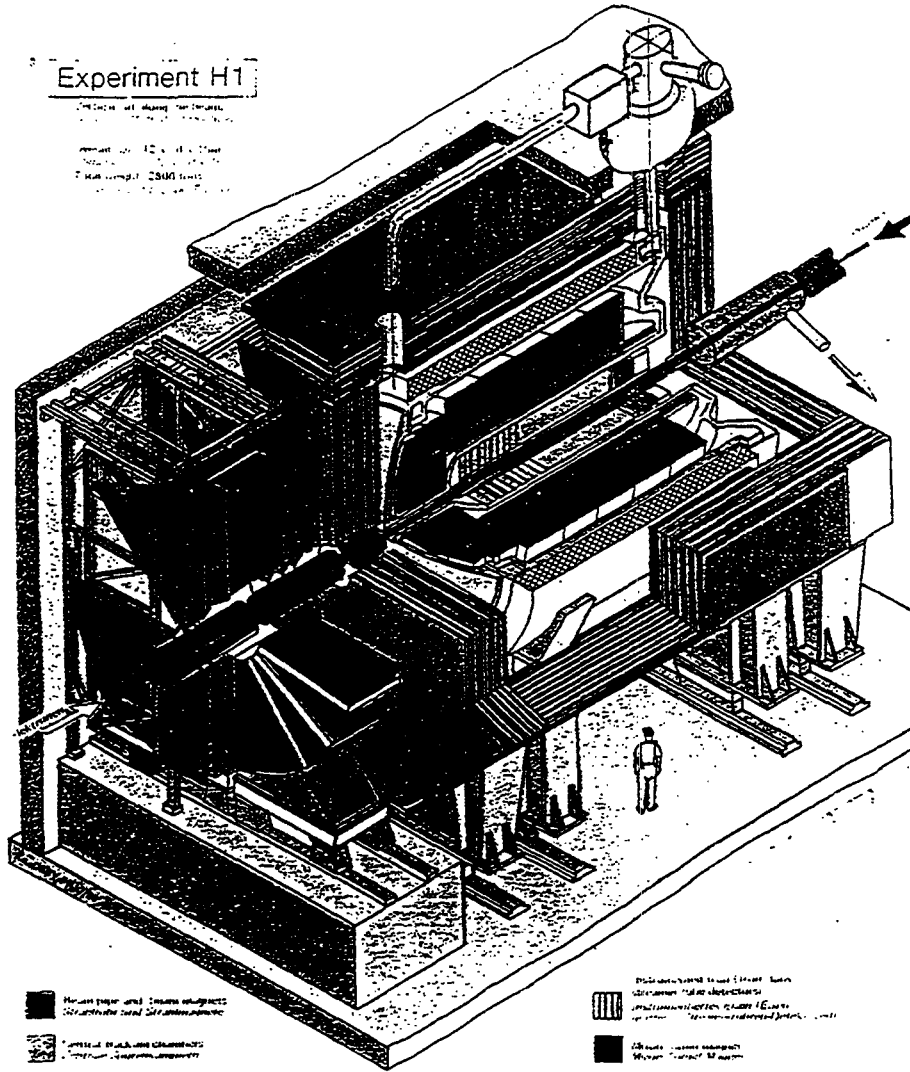
Zeus (Hera) (8) (9)

(9)

Experiment H1

Production of strong materials

Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons



Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons

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 Weight: 100 kg
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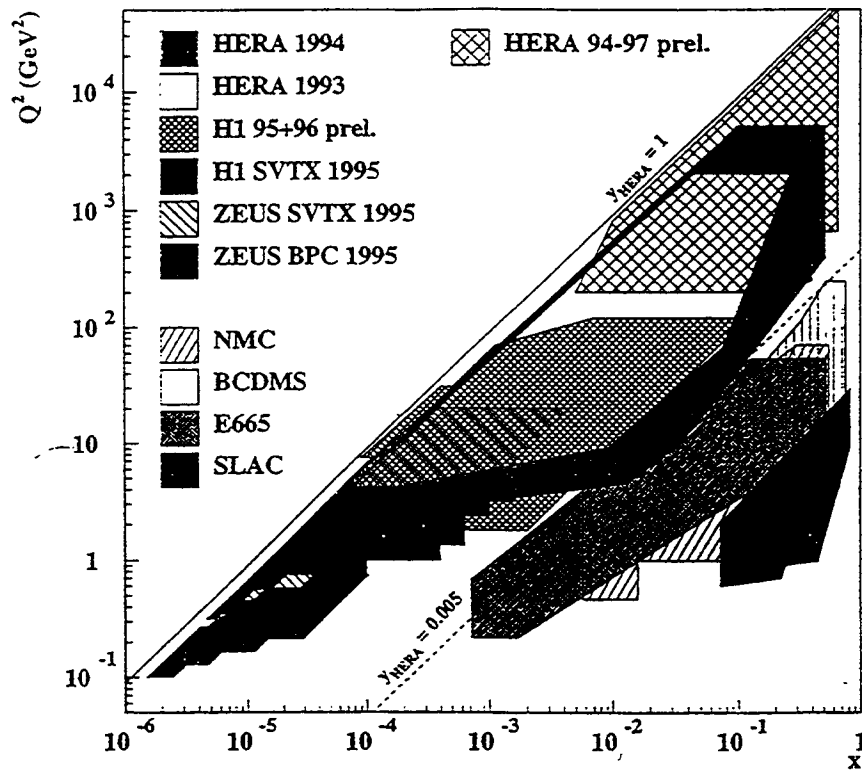
Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons

Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons

Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons

Kinematic reach

(53)

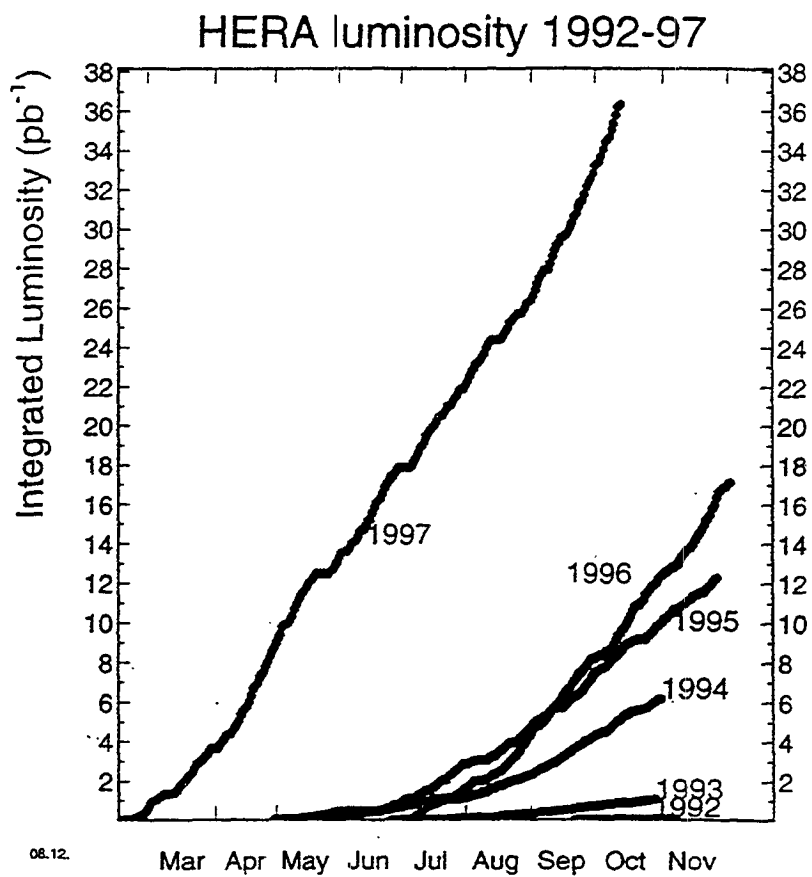


Several upgrades

$$10^{-6} < x < 0.7$$

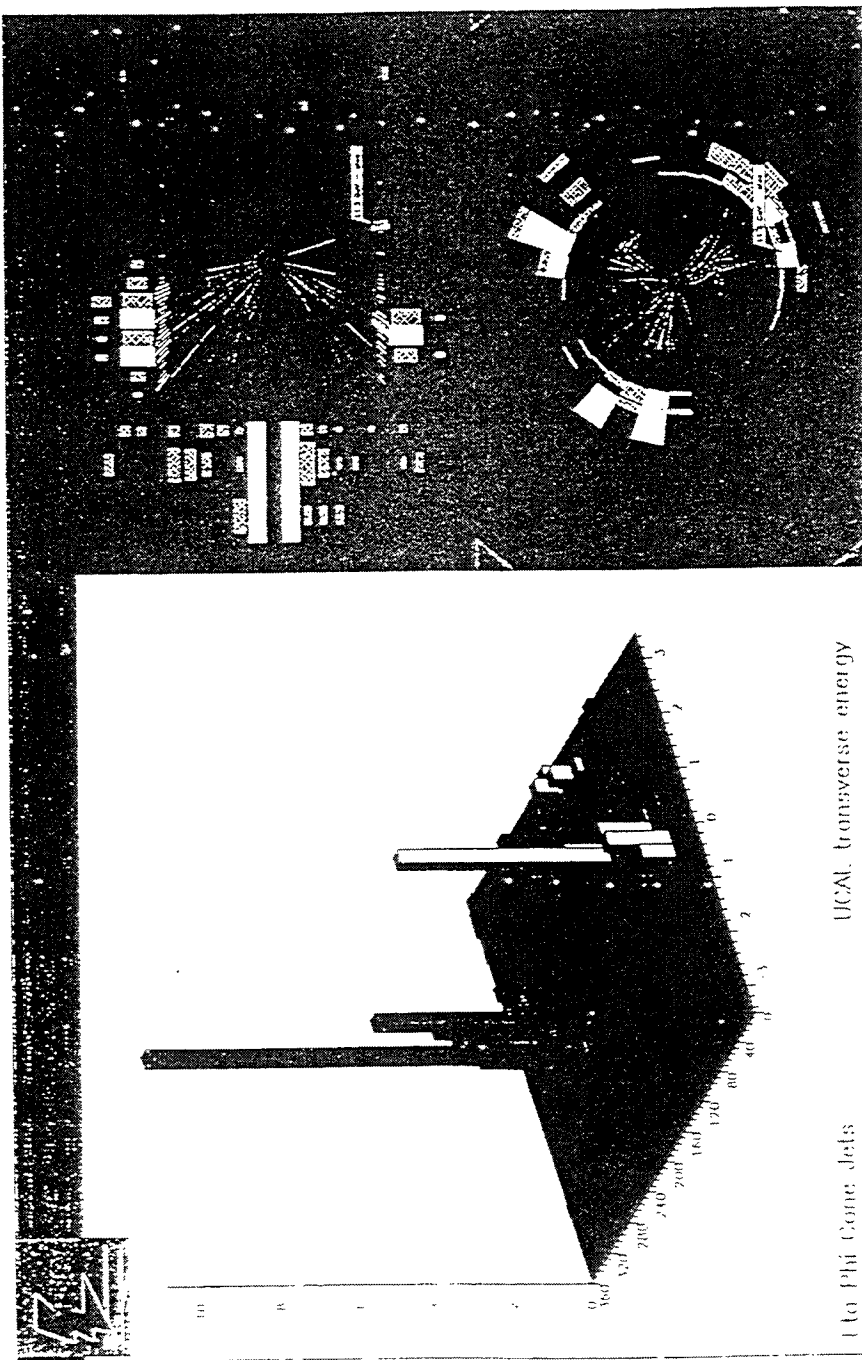
$$10^{-1} \text{ GeV}^2 \lesssim Q^2 \lesssim 30\,000 \text{ GeV}^2$$

54



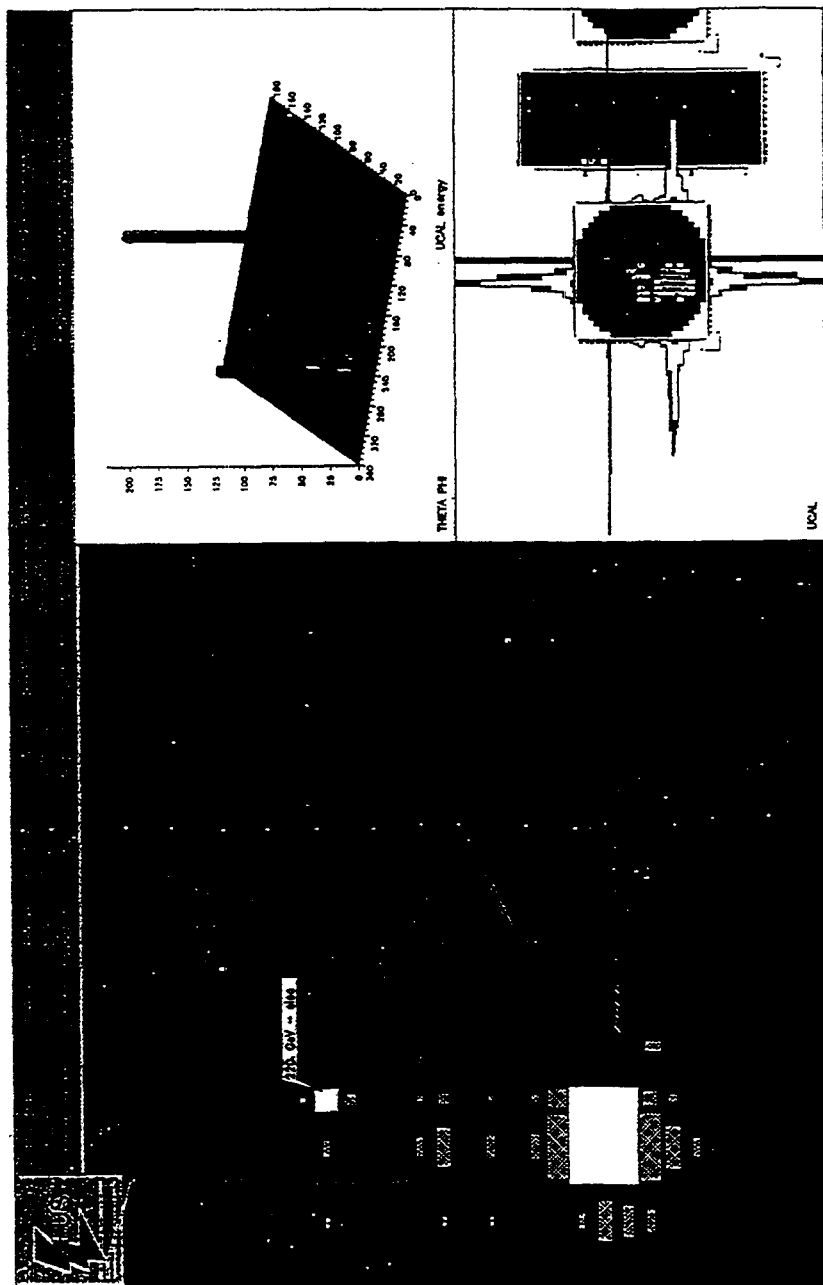
08.12.

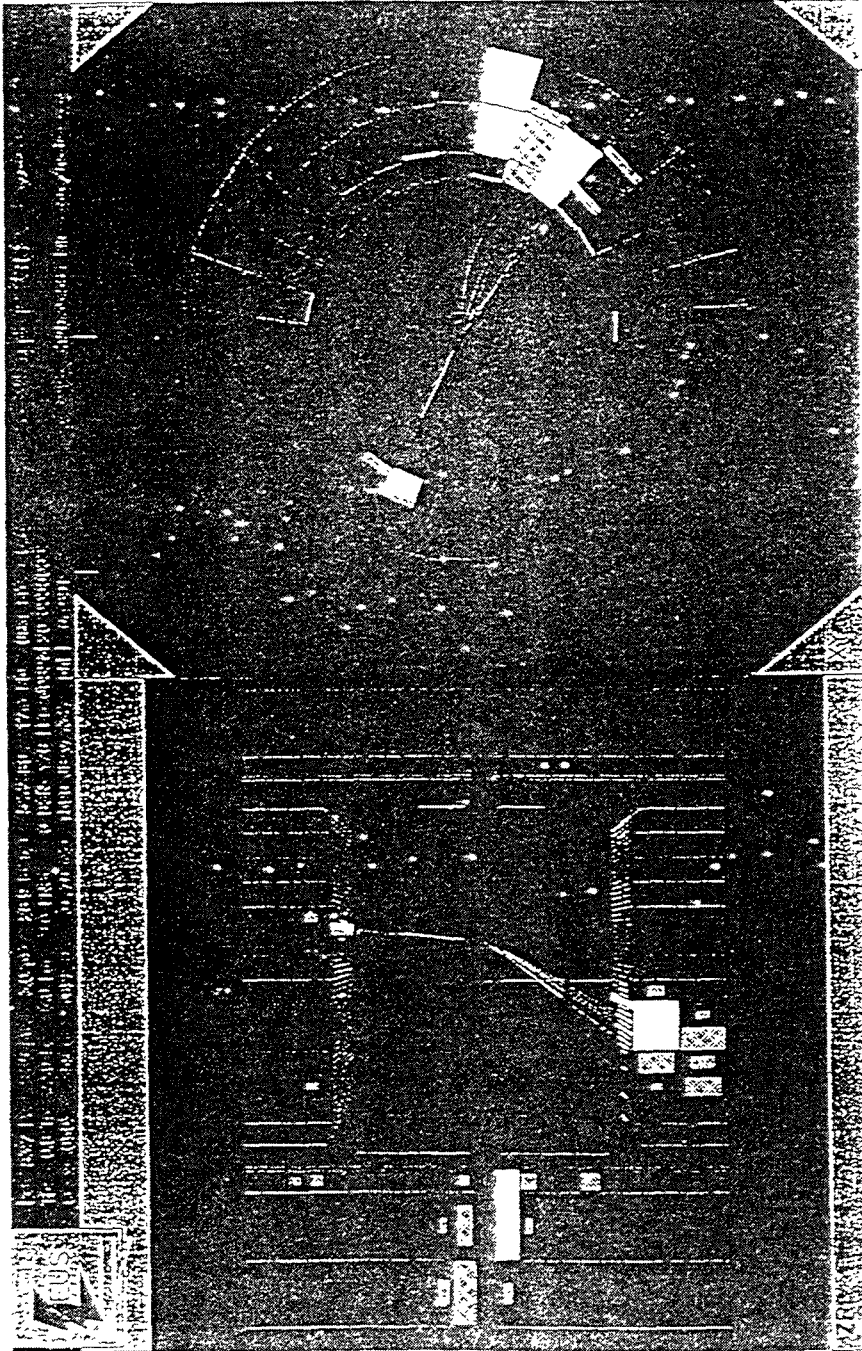
γ jet event



(b)

$$\psi^2 = 22,860 \text{ GeV}^2$$





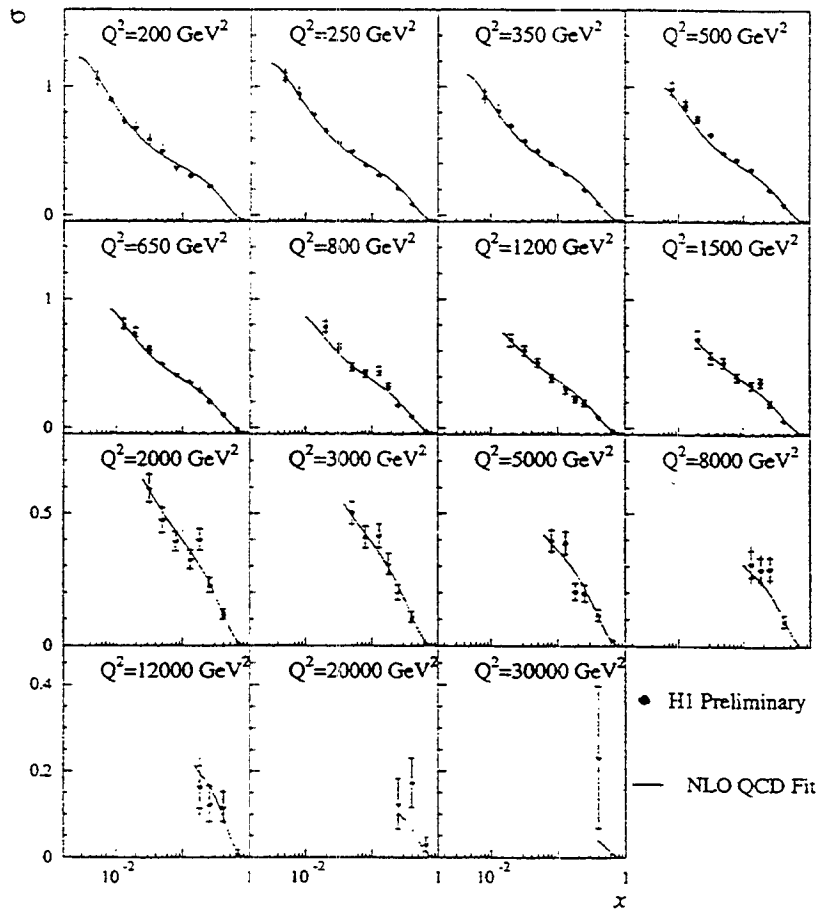
H1 data, 1994-1997, neutral current

Reduced cross section

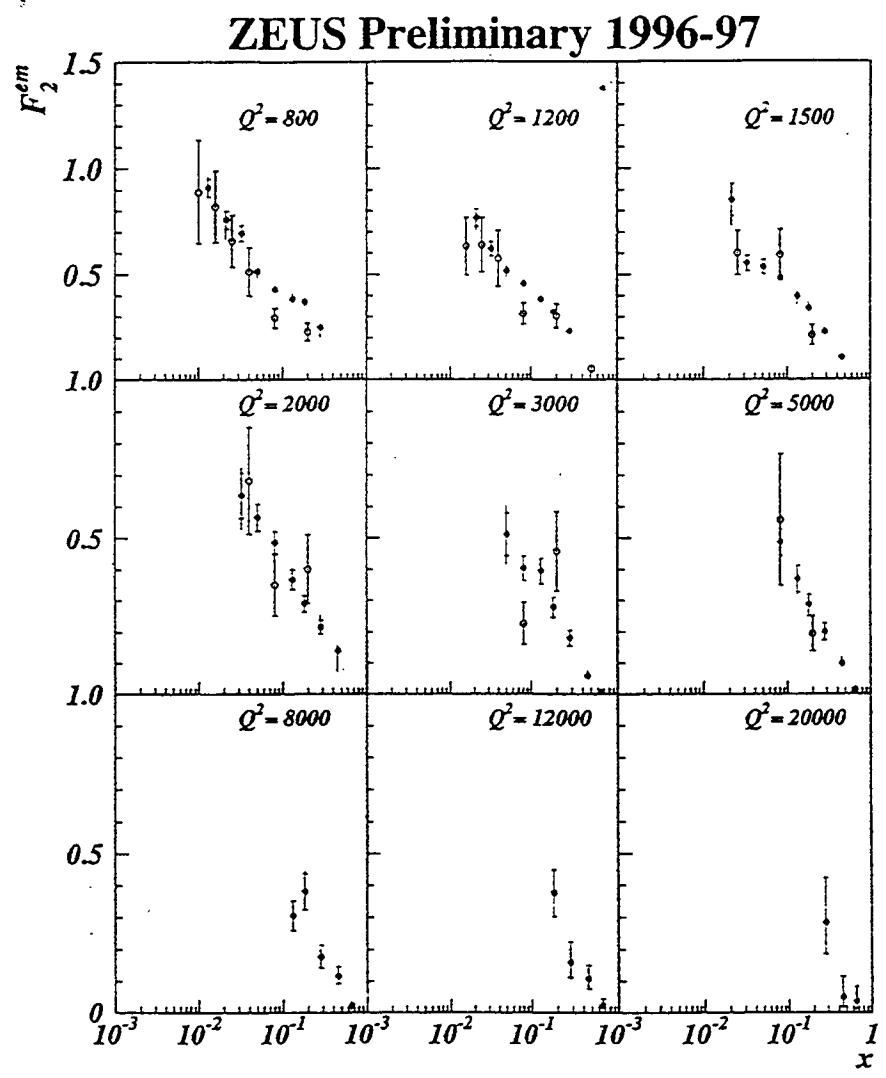
(58)

$$\sigma := \frac{xQ^4}{2\pi\alpha^2 Y_+} \frac{d^2\sigma^{NC}}{dx dQ^2} = F_2 - \frac{y^2}{Y_+} F_L - \frac{Y_-}{Y_+} x F_3$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

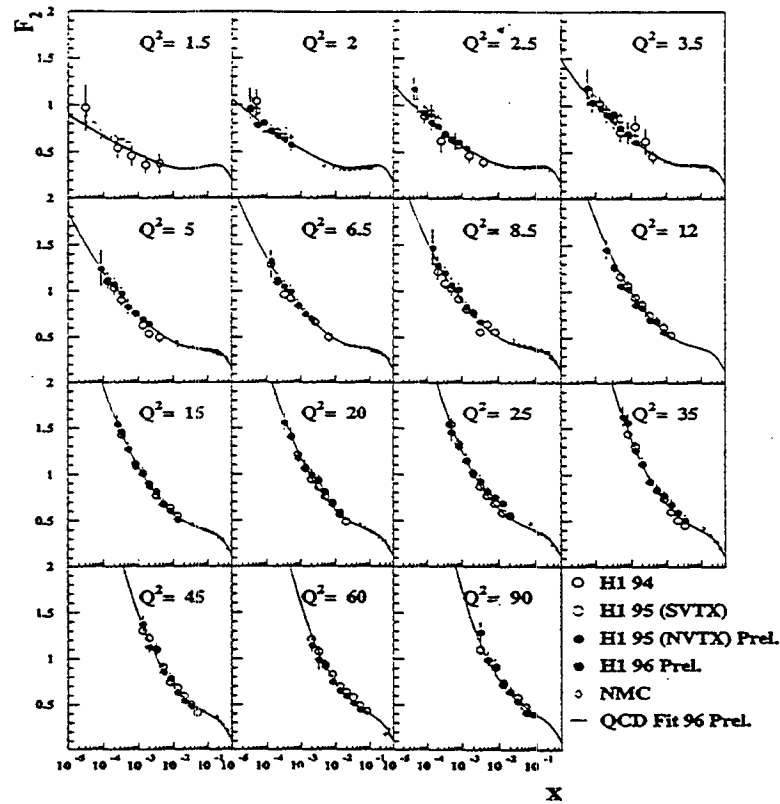


Steep rise at low x



F_2 at 'low Q^2 '

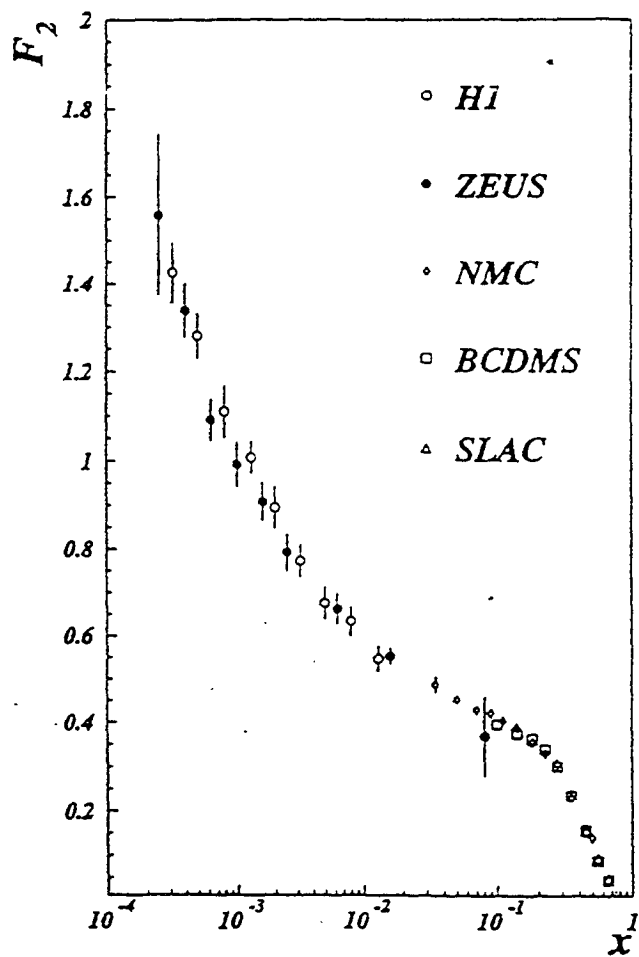
(60)



Steep rise of F_2 at low x

➡ strong increase of parton densities in the proton at low x

61

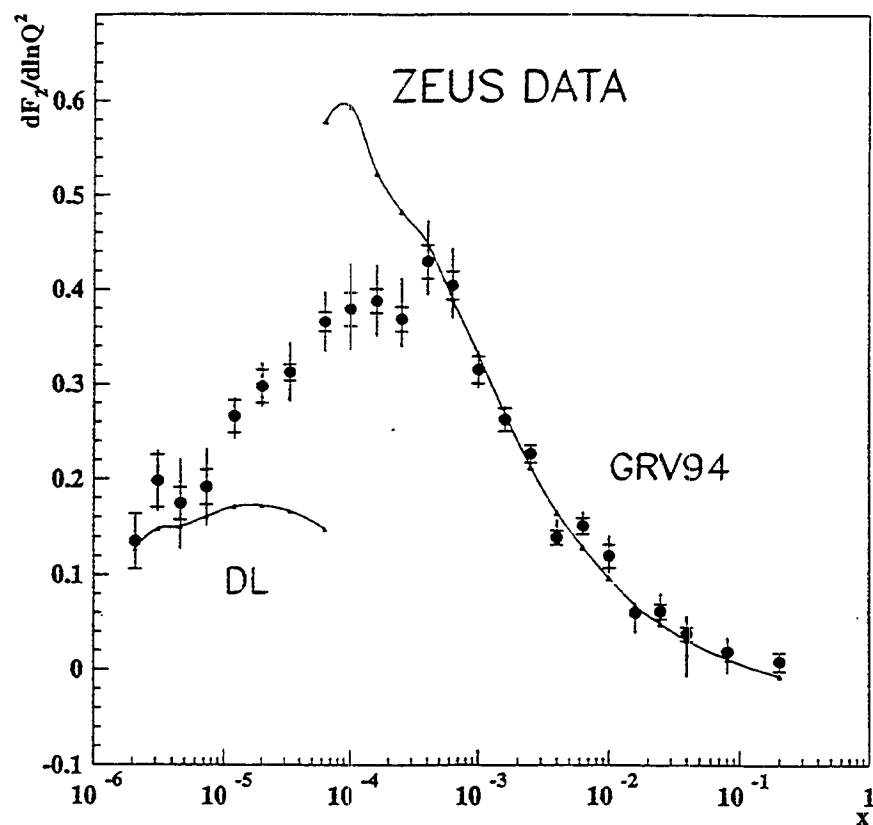


low x : $F_2(x) = \frac{5}{18} x \cdot q(x) \leadsto q(x=2.5 \cdot 10^{-3}) = 2300$

$\leadsto$ rather crowded!

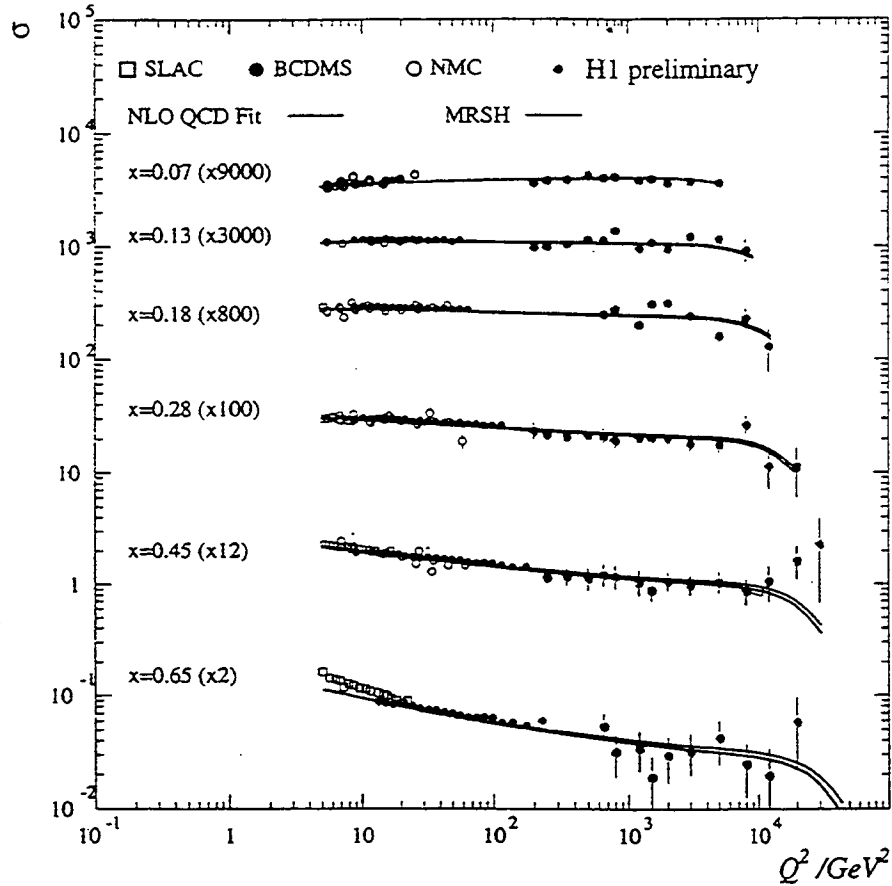
ZEUS 1995 Preliminary

$\langle Q^2 \rangle =$
 0.13 0.18 0.20 0.23 0.29 0.37 0.52 1.10 2.45 3.85 5.24 8.76 10.63 13.52 14.57 19.26 26.15 24.58 56.27 19.37 45.01 167.61 156.59 388.03 GeV²



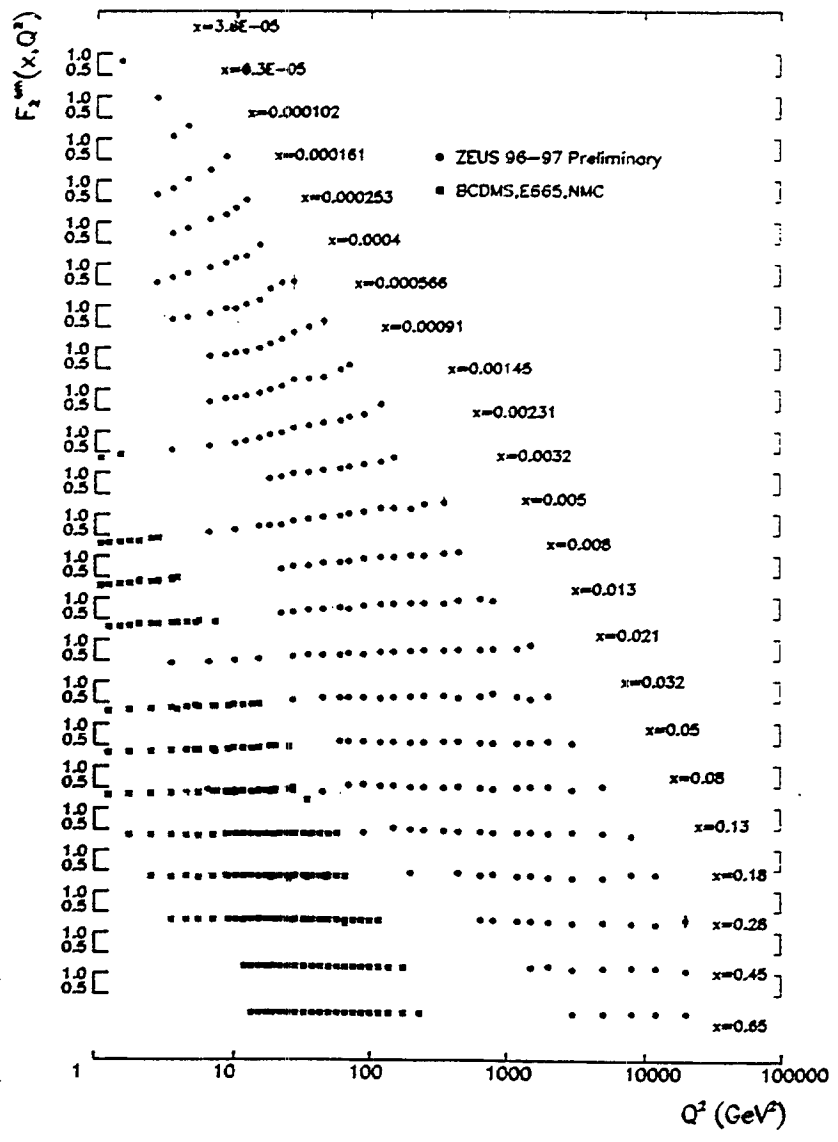
Reduced Cross-section at High x

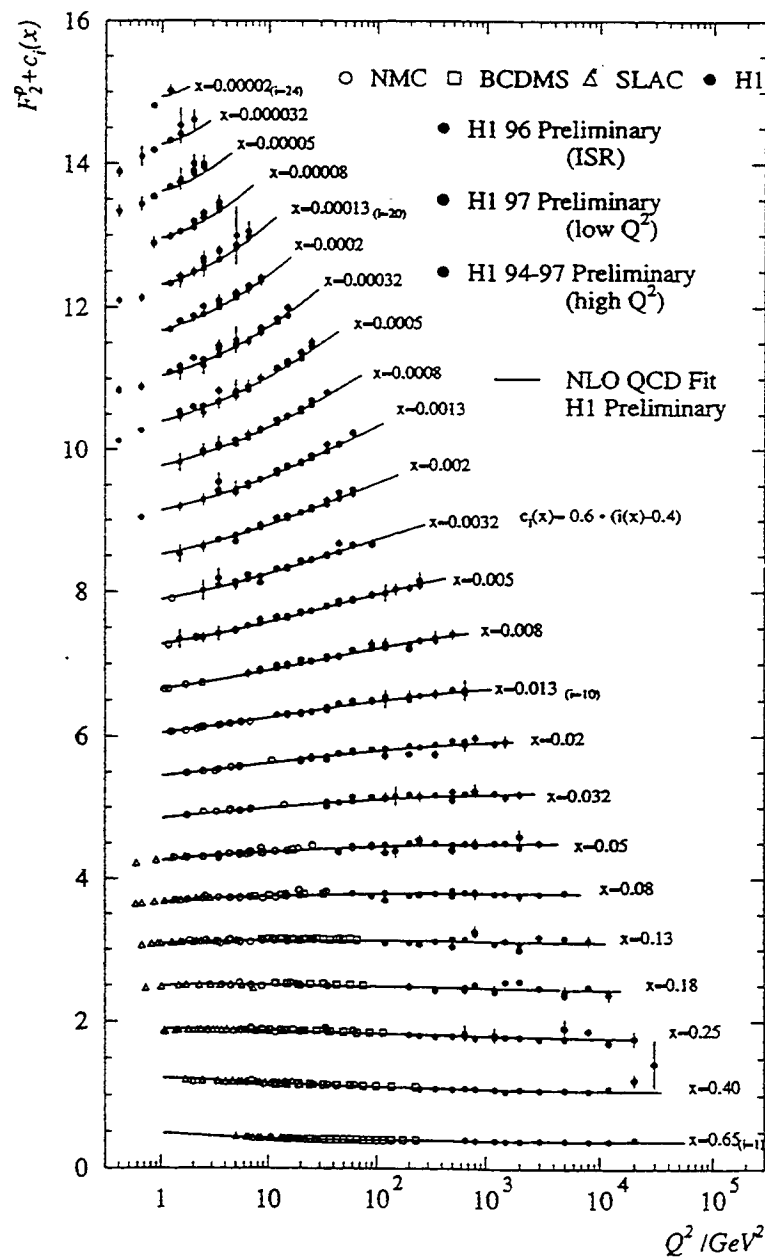
63



- Difference visible in the QCD fit when the high Q^2 data is or is not included.
- High Q^2 HERA data now also have an influence at high x .

67





Gluondistribution $g(x)$

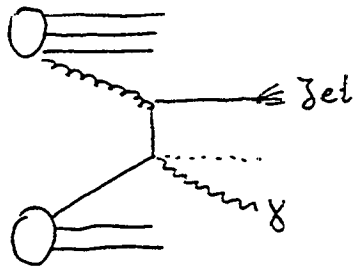
(69)

1) From scalebreaking

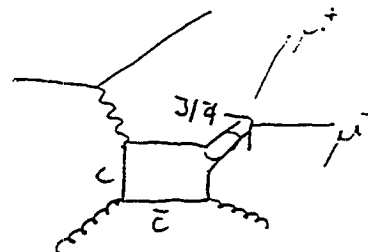
→ coupled Altarelli-Parisi equations

(Problem $g(x) \leftrightarrow \Lambda$ correlated)

2) Direct photons at high p_t
(QCD - Compton)



3) J/ψ - Production
Photon - gluon - Fusion

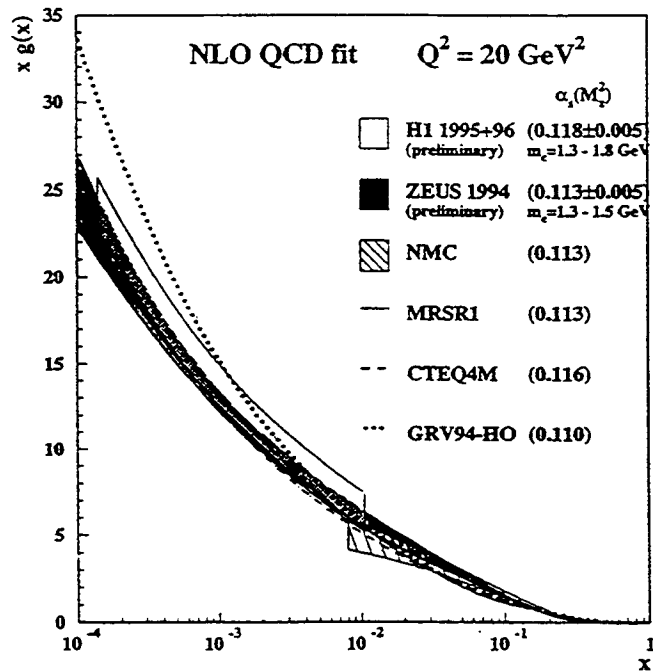


$$4) (F_L(x, Q^2) \leftrightarrow R(x, Q^2) = \frac{\sigma_L}{\sigma_t})$$

$g(x)$ from scaling violations

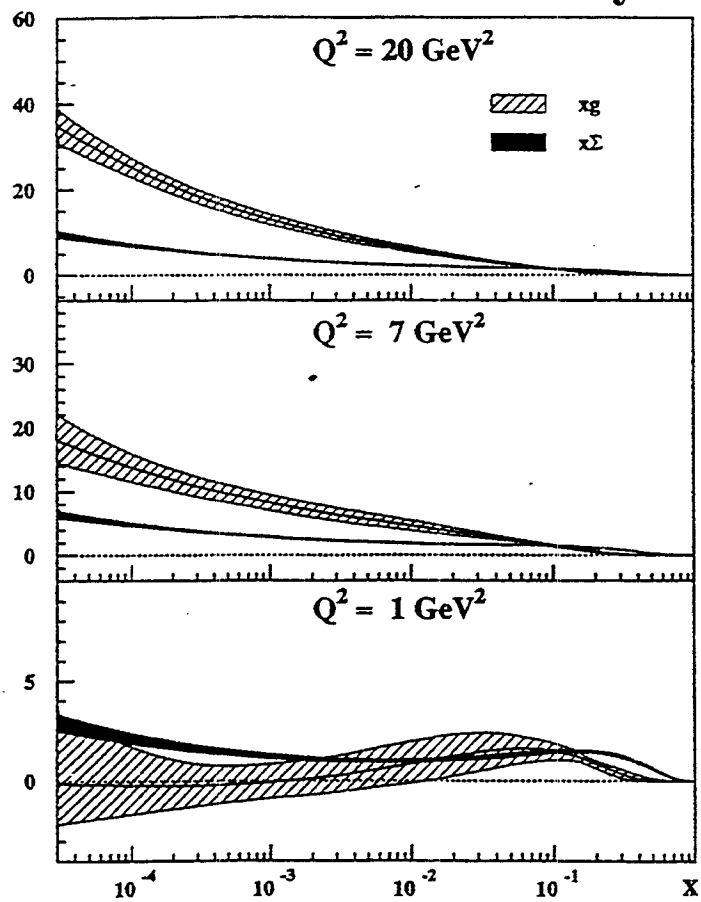
(70)

$$\frac{dF_2}{d\ln Q^2} = \frac{\alpha_2(Q^2)}{2\pi} \left[\int_x^1 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qq}\left(\frac{x}{z}\right) F_2(x, Q^2) \right. \\ \left. + \int_x^1 2 \sum_q e_q^2 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qg}\left(\frac{x}{z}\right) g(x, Q^2) \right]$$



71

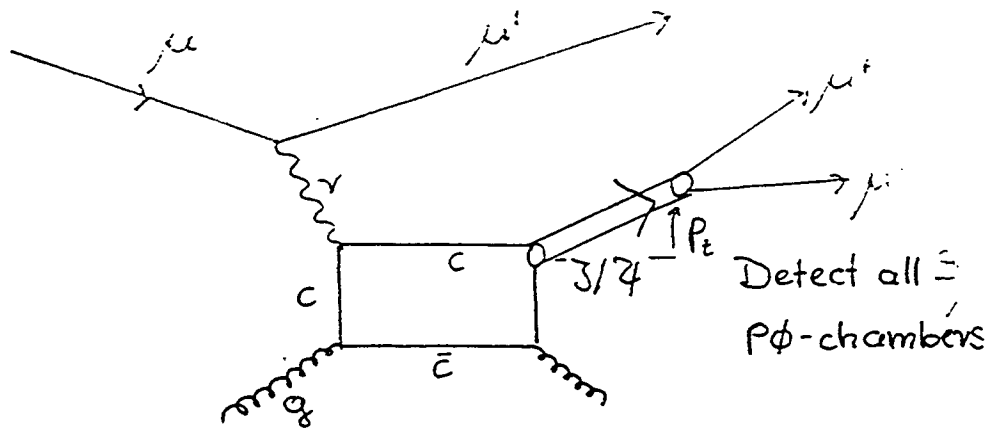
ZEUS 1995 Preliminary



(72)

J/ψ production from H.D
S n, C

Photon - gluon - fusion model



$$\sigma_{\text{inelastic}}^{J/\psi} \sim q(x)$$

$$\text{inelastic: } z = \frac{E^{J/\psi}}{\gamma} < 0.9$$

$$P_t^2 \lesssim 0.2 \text{ GeV}^2$$

$$X = \frac{S_X \cdot \alpha_s}{S_X \cdot N} = \frac{1}{W^2} \left[\frac{M_{J/\psi}^2}{z} + \frac{P_t^2}{z(1-z)} \right]$$

$$\stackrel{z \rightarrow 1}{=} \frac{M_{J/\psi}^2}{W^2}$$

$$E_{\mu} = 280 \text{ GeV}$$

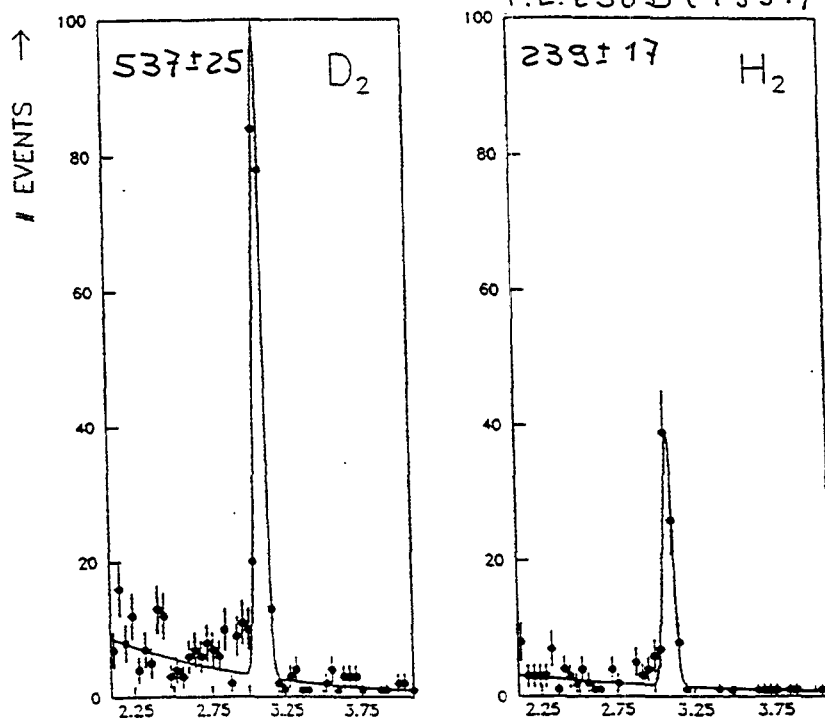
(73)

$$I = (3.05 \pm 0.06) \cdot 10^{12} \mu \text{ } \curvearrowright \mathcal{L} = 125 \text{ pb}^{-1}$$

$$\frac{\sigma^{J/\psi}(D)}{\sigma^{J/\psi}(H)} = 0.96 \pm 0.08$$

Thesis: M. de Jong
C. Mariotti

NM/CERN-PPE/90-178
P.L. 258 B (1991) 493.



$$m_{J/\psi} = 3.095 \pm 0.002 \mu^+ \mu^- \text{ Mass (GeV}/c^2) \rightarrow$$

$$\text{FIG. 2 } m_{J/\psi} = 3.093 \pm 0.003$$

Inelastic J/ψ -Production (78)

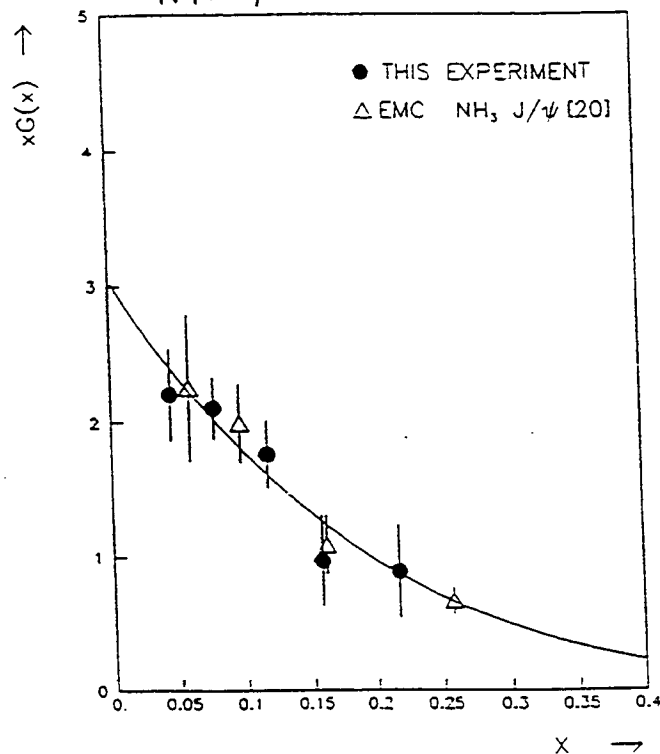
$$z = \frac{E_{J/\psi}}{\gamma} \leq 0.9; \quad p_t^2 > 0.1 \text{ GeV}^2$$

(279 ± 25 events)

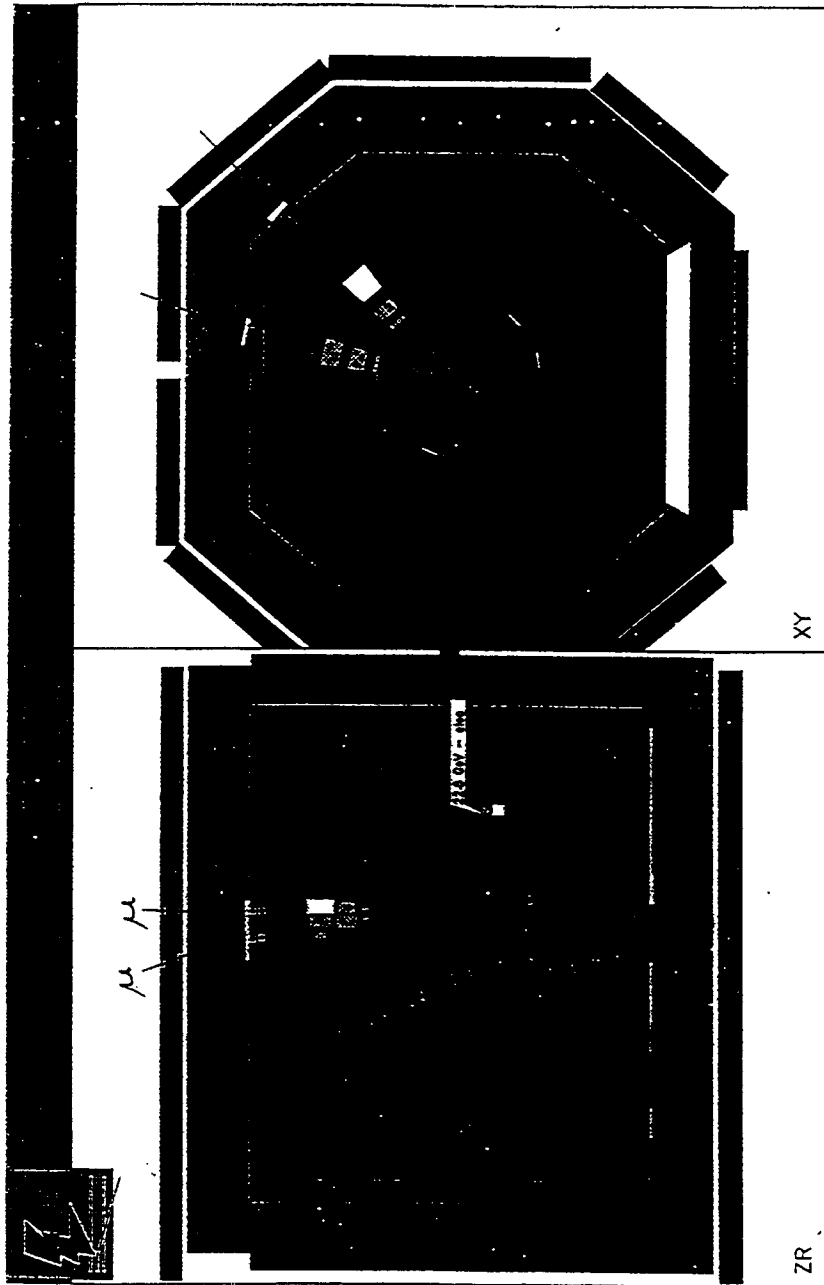
$$\rightarrow xG(x) \sim \frac{\eta+1}{2} (1-x)^\eta; \quad \eta = 5.1 \pm 0.9$$

P.L. B258 (1991) 493.

NMC; CERN-PPE/90-178



3/4 → $\mu^+ \mu^-$

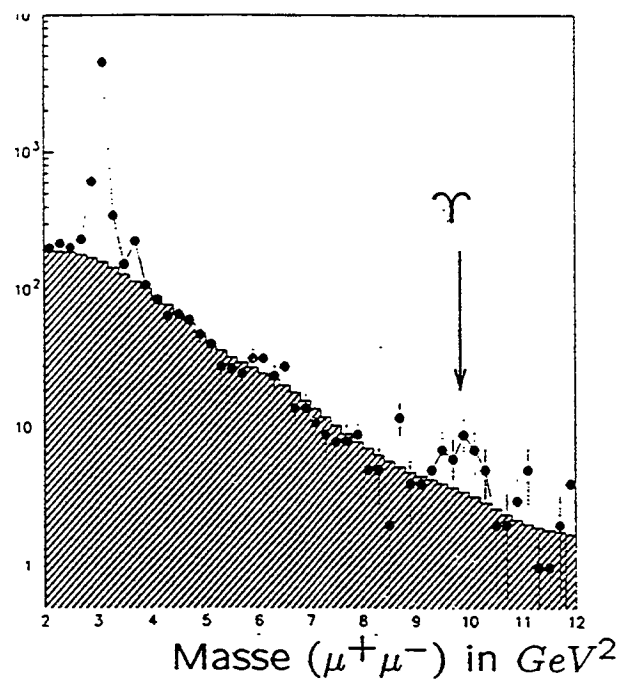


75

J/ψ and Υ production at HERA

80

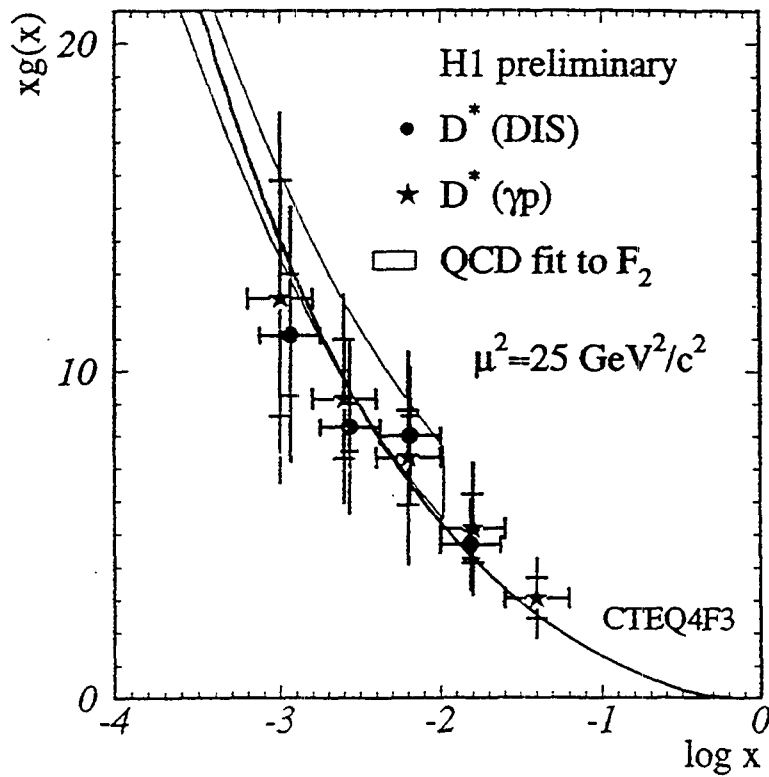
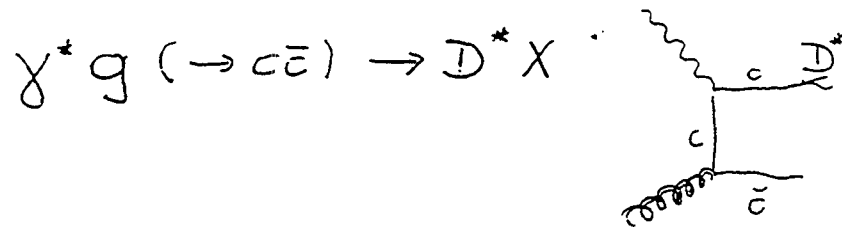
ZEUS 95-97 Preliminary



Υ cross section larger than expected

$g(x)$ from open charm

(81)



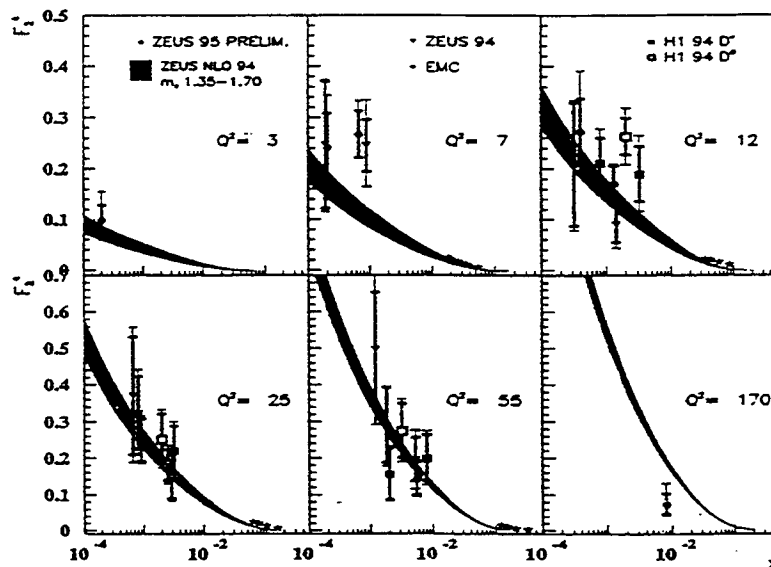
F_2 - charm

(82)

NLO QCD: $g(x, Q^2), P_{g \rightarrow c\bar{c}} \rightarrow F_2^{c\bar{c}}$

Use D^* events

(Transition probabilities $c \rightarrow D^*$, $\bar{c} \rightarrow \bar{D}^*$
and $D^*, \bar{D}^*$ decays known from LEP)



Measurement of F_2^{charm} consistent
with NLO prediction

At low x contribution to F_2 20-25%

Higher Twist Effects

(83)

$$F_2(x, Q^2) = \begin{cases} F_2(x, Q^2)^{LT} \left(1 + \frac{C(x)}{Q^2}\right) \\ F_2(x, Q^2)^{LT} + \frac{C'(x)}{Q^2} + \dots \end{cases}$$

- Measure of quark-quark-correlations ~~quark-quark-correlations~~
(Less clean than quark-gluon-corr.
in $g_2(x, Q^2)$ - Twist-3)

- Effects largest at high x

- H.T. - effects different for bound and free quarks?

Nuclear binding : Correlations

between quarks of different nucleons?



Fits: $C^p(x) \approx C^d(x)$

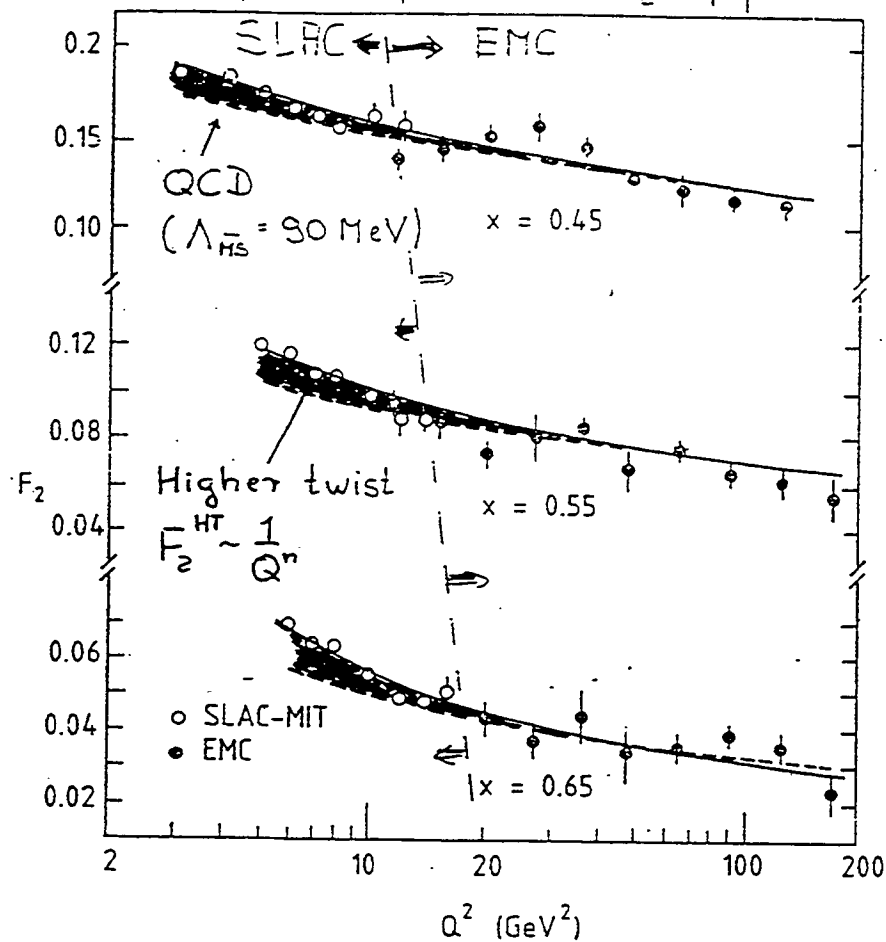
$$\frac{F_2^n}{F_2^p}(Q^2) \rightarrow C^n(x) > C^p(x) \quad \begin{array}{l} \text{small effect} \\ \text{Similar to EMC effect} \end{array}$$

→ Need high lumi

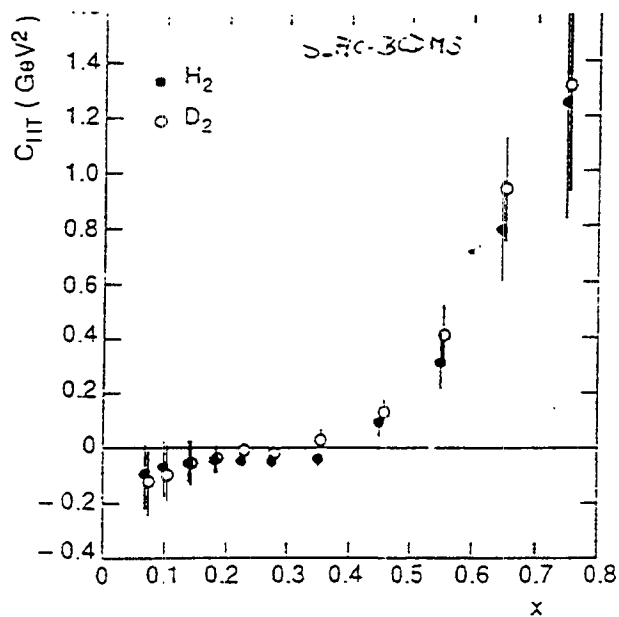
$$100 < \Lambda_{\overline{MS}} < 300 \text{ MeV}$$

(87)

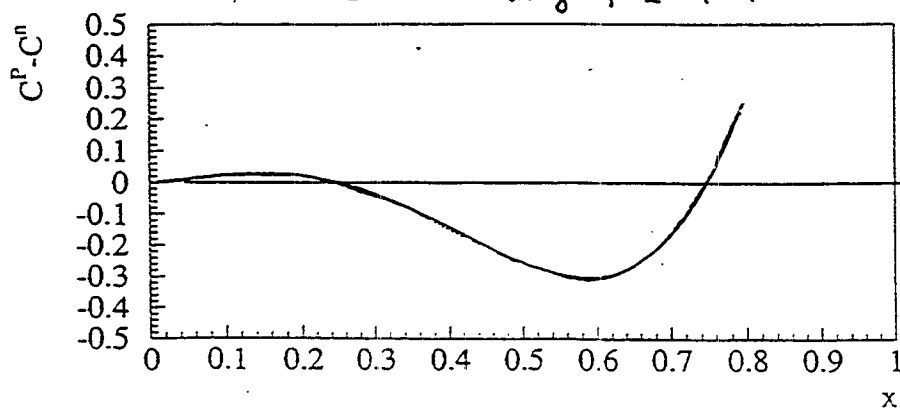
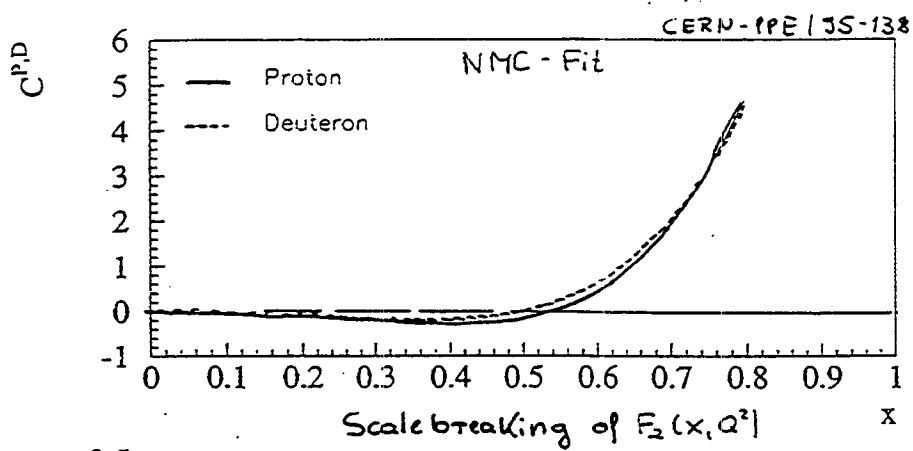
Structure function F_2 of proton.



Have to increase Q^2 range
to test QCD!
One measurement!

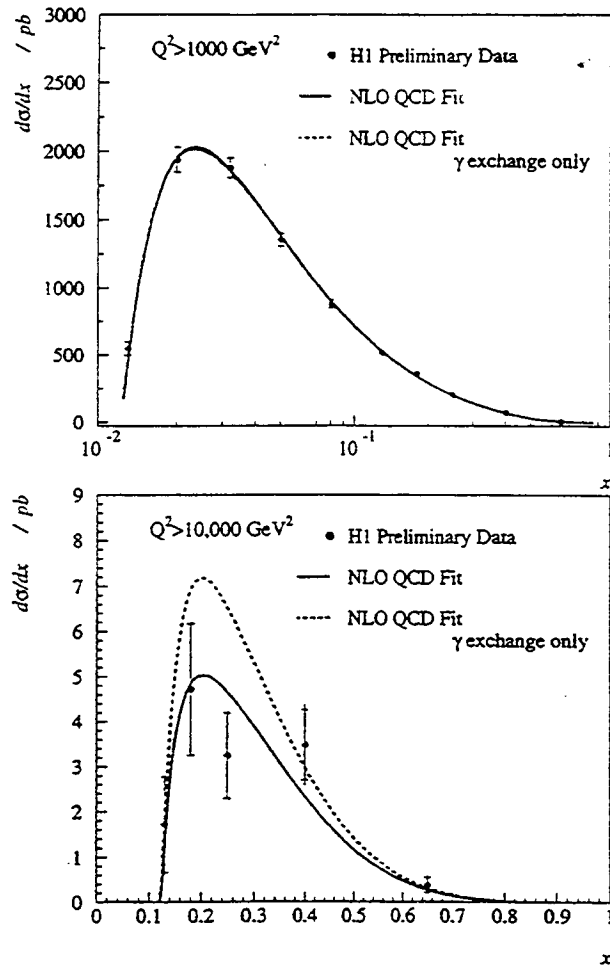


98

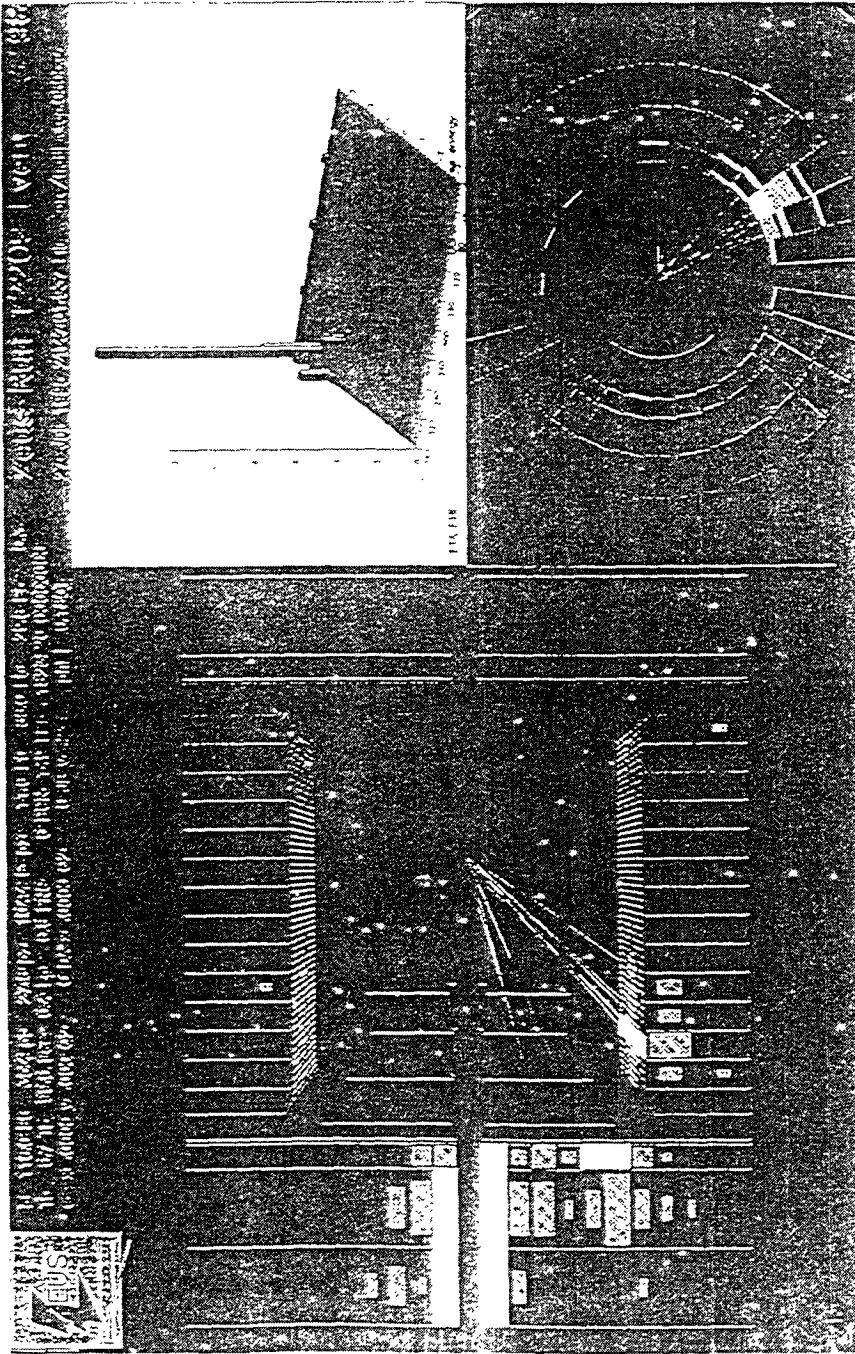


$d\sigma^{NC}/dx$ für $Q^2 > 1000$ und $> 10000 \text{ GeV}^2$

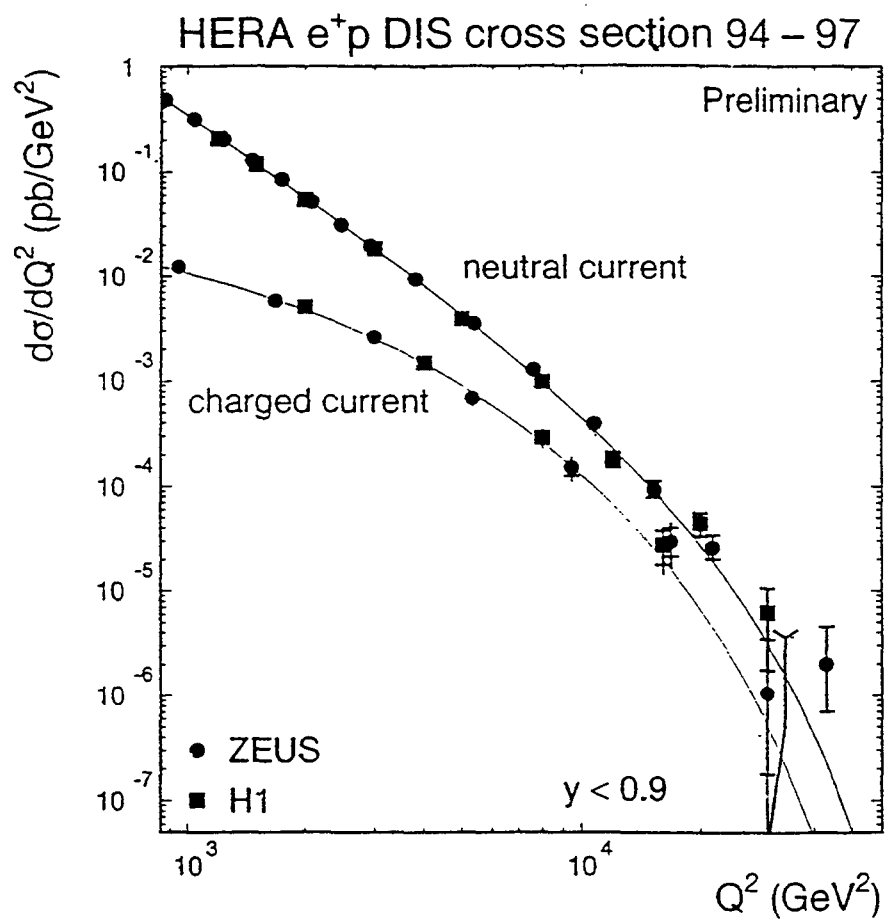
(89)



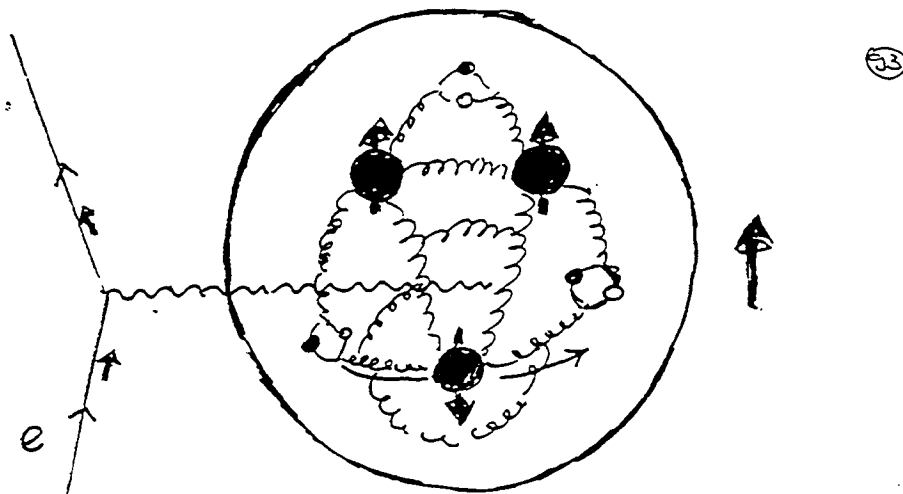
- Gute Beschreibung der Daten durch das Standardmodell mit destruktiver γ - Z^0 -Interferenz für e^+p .



94



④ Polarised DIS ⑤2
Spin of the nucleon



$$\boxed{\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_z^q + \Delta G + L_z^g}$$

↑
Quarks

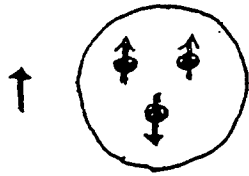
↑
Gluons
↑
orbital angular momentum

$$\begin{aligned} \Delta \Sigma &= \Delta u_v + \Delta d_v + 2(\Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s}) \\ &= 1 \text{ QPM} \\ &\approx 0.6 \text{ rel. QPM} \end{aligned}$$

$$\left. \begin{aligned} \Delta \Sigma &\approx 0.2 ? \\ \Delta S &\approx -0.1 ? \end{aligned} \right\} \text{EMC/SLAC: Spin-'Puzzle'}$$

Quark-Modell (1961) - SU(6)

(34)



$$p = |uud\rangle$$

$$e_u = \frac{2}{3}|e|$$

$$n = |ddu\rangle$$

$$e_d = -\frac{1}{3}|e|$$

$$|p^\uparrow\rangle = \frac{1}{\sqrt{18}} \left\{ 2|\overset{\uparrow}{u}\overset{\uparrow}{u}\overset{\downarrow}{d}\rangle - |\overset{\uparrow}{u}\overset{\downarrow}{u}\overset{\uparrow}{d}\rangle - |\overset{\downarrow}{u}\overset{\uparrow}{u}\overset{\uparrow}{d}\rangle + \dots \right\}$$

$$\begin{aligned} \rightarrow u^{\uparrow\uparrow} &= u^+ = \frac{5}{3} ; u^{\uparrow\downarrow} = \bar{u} = \frac{1}{3} ; \Delta u = u^+ - \bar{u} = \frac{4}{3} \\ d^{\uparrow\uparrow} &= d^+ = \frac{1}{3} ; d^{\uparrow\downarrow} = \bar{d} = \frac{2}{3} ; \Delta d = d^+ - \bar{d} = -\frac{1}{3} \end{aligned}$$

$$\langle \mu^p \rangle = \frac{1}{3} \{ 4 \langle \mu^u \rangle - \langle \mu^d \rangle \}$$

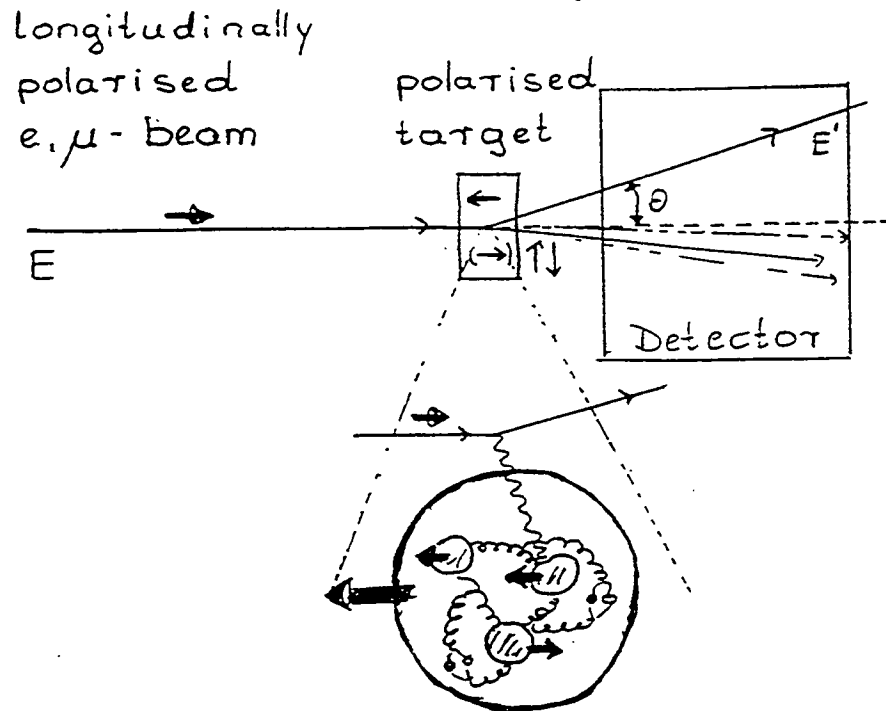
$$\langle \mu^n \rangle = \frac{1}{3} \{ 4 \langle \mu^d \rangle - \langle \mu^u \rangle \}$$

$$m_u = m_d = m_q$$

$$\rightarrow \frac{\langle \mu^n \rangle}{\langle \mu^p \rangle} = -\frac{2}{3} \approx -\frac{1.91}{2.73} ; m_q \approx \frac{M_p}{2.73}$$

100L: Polarised deep inelastic
lepton-nucleon scattering

35



Measure probability for quark-spin
and nucleon-spin $\uparrow\uparrow$ or $\downarrow\uparrow$



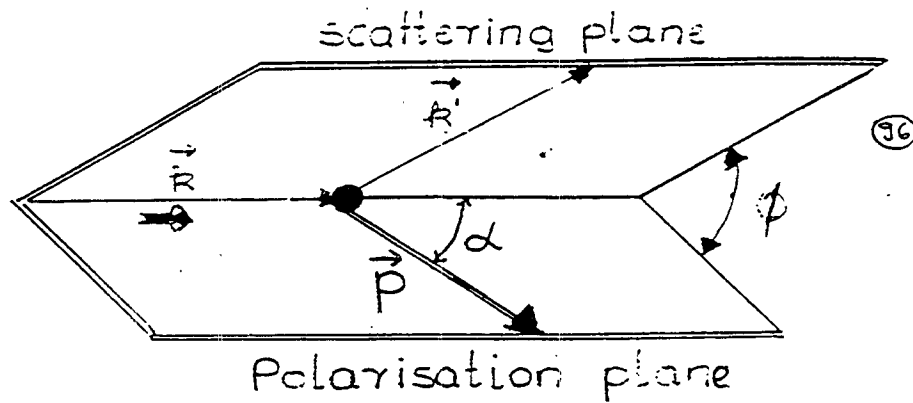
Spin-Structure Function $\boxed{g_1^{p,n}(x, Q^2)}$

In addition:

$g_2^{p,n}(x, Q^2)$, b_1^d , Δ^d , h_1 ,

ΔV , ΔS , ΔG , L_2

Polarised lepton nucleon scattering



$$\left. \frac{d^3 \sigma(\alpha)}{dx dy d\phi} \right|_{(e)} = \frac{d^3 \bar{\sigma}}{dx dy d\phi} \begin{pmatrix} + \\ - \end{pmatrix} \frac{d^3 \sigma^*(\alpha)}{dx dy d\phi}$$

$\uparrow$ $\uparrow$
 $\sim f(E_1, E_2)$ $\sim f(g_1, g_2)$

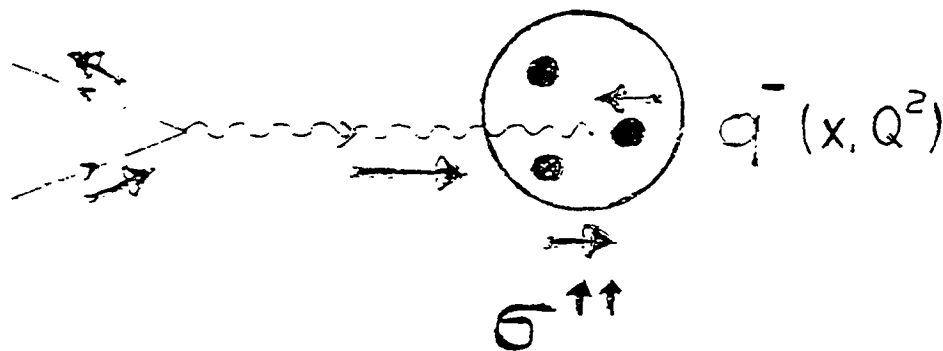
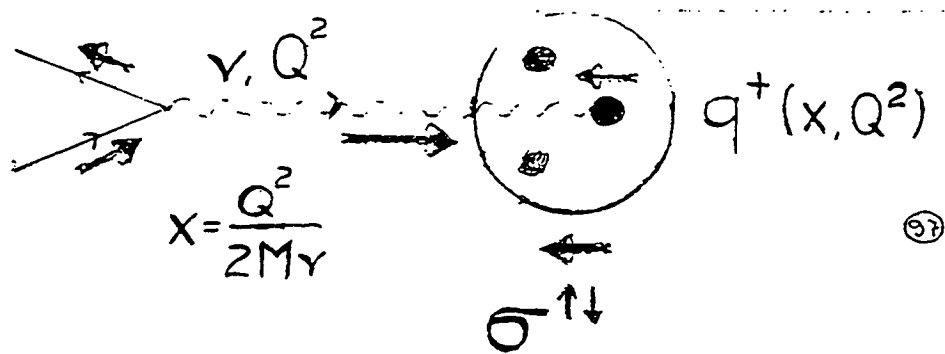
$$\frac{d^3 \sigma^*(\alpha)}{dx dy d\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \cos \alpha \cdot [a \cdot g_1(x) + b g_2(x)] - \cos \phi \sin \alpha \cdot c \left[\frac{Y}{2} g_1(x) + g_2(x) \right] \right\}$$

$$a \gg b \quad \left(a = 1 - \frac{Y}{2} - \frac{Y^2}{4}(\kappa - 1); b = -\frac{Y}{2}(\kappa - 1), c = \sqrt{(\kappa - 1)\alpha} \right)$$

$\alpha = 0^\circ$: measure dominantly $g_1(x)$

$\alpha = 90^\circ$: " both $g_1(x), g_2(x)$

$$\text{Asymmetries: } A = \frac{\sigma(\alpha + \pi) - \sigma(\alpha)}{\sigma(\alpha + \pi) + \sigma(\alpha)}$$



$$\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow} \sim \boxed{g_1(x) = \frac{1}{2} \sum_f e_f^2 \{q_f^+(x, Q^2) - q_f^-(x, Q^2)\}}$$

$$\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow} \sim F_1(x) = \dots$$

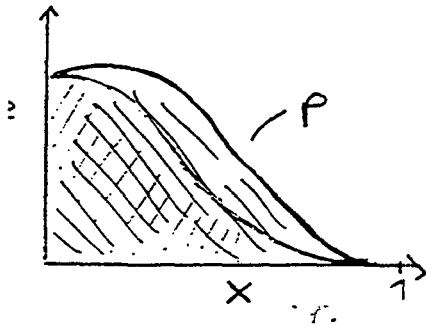
$$\frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} \sim \boxed{R_1(x) \approx \frac{g_1(x)}{F_1(x)}} \quad \begin{aligned} (R_{11} &= D(R_1 + \eta f) \\ (2 \times F_1 &= F_2 / (1+i) \end{aligned}$$

$$\Delta q_f = \int_0^1 \{q_f^+(x) - q_f^-(x)\} dx$$

$$\Delta q_f \cdot S_\mu = \langle P, S | \bar{q}_f \gamma_\mu \gamma_5 q_f | P, S \rangle \frac{1}{2M}$$

Integrals

a) un-polarised

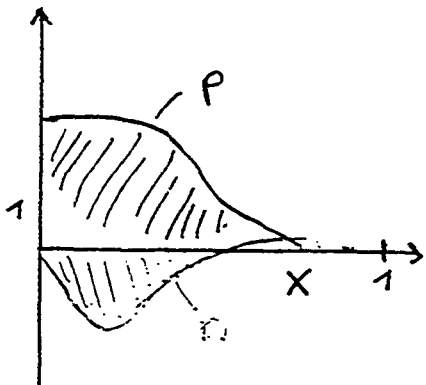


$$F_2(x) = \sum_f e_f^2 x (q_f(x) + \bar{q}_f(x))$$

$$\Gamma_2 = \int_0^1 F_2(x) dx \quad (95)$$

- $\Gamma_2^P + \Gamma_2^N \sim$ quark-momenta
only ~ 50% of N-momentum
- $\Gamma_1^P - \Gamma_1^N \approx \frac{1}{3}$ (≈ 0.24)
 $\sim$ quark charges: $\bar{u} \neq \bar{d}$

b) polarised



$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x)$$

$$I_1 = \int_0^1 g_1(x) dx$$

- $I_1^P + I_1^N \sim$ quark-spins
 $\sim$ quarks contribute little to N-spin
- $I_1^P - I_1^N$ (Bj-SR)
 $\sim$ axial charge; g_a

But: Only global informations

Detailed test of QCD from

$$F_2(x, Q^2)$$

(last ~25 years)

Sum Rules:

① Bjorken (66) Fundamental! ☹

$$\int_0^1 dx (g_1^p(x) - g_1^n(x)) = \frac{1}{6} \left| \frac{g_R}{g_V} \right|_{np} * \text{QCD Korr.}$$

$$= 0.191 \pm 0.003$$

g_R, g_V : weak coupl. const. | from Gamow-Teller β -decay

② Ellis-Jaffe - SU(3)

$$\int_0^1 dx g_1^{p,(n)}(x) = \frac{1}{12} \left| \frac{g_R}{g_V} \right|_{np} \left\{ \begin{matrix} + \\ (-) \end{matrix} 1 + \frac{5}{3} \cdot \frac{3F-D}{F+D} \right\} * \text{QCD Korr.}$$

$\downarrow \Delta S=0, F/D = 0.579$
 $\begin{matrix} p \\ n \end{matrix} \begin{matrix} = 0.175 \pm 0.007 \\ (-) = -0.016 \pm \dots \end{matrix}$

F, D : SU(3) coupling const.; baryon decays:

np : $F+D$; Λp : $F+\frac{1}{3}D$; $\Sigma \Lambda$: $F-\frac{1}{3}D$

Sum Rules

$$\Gamma_1^{p,n}(Q_0^2) \equiv \int_0^1 g_1(x, Q_0^2) dx$$

(100)

(via QCD fit

- Expt:
- Evolve data from $\bar{Q}_m^2(x) \rightarrow Q_0^2$
 - Extrapolations: $\int_0^{x_{\min}}$, $\int_{x_{\max}}^1$
- $\uparrow$ big? $\uparrow$ small

Theorie:

$\alpha_s(Q^2)$ - Corrections:

(Larin, Phys. Lett. B334 (1994)1

(Kataev, Starshenko, Mod. Phys. Lett

A10 (1995)

Known to $O(\alpha_s^3)$, estimates $O(\alpha_s^4)$

$$\Gamma_1^{p,n}(Q^2) = \frac{1}{12} \left(\left(\frac{1}{2} a_3 + \frac{1}{3} a_8 \right) C^{NS}(Q^2) + \frac{1}{9} a_0 C^S(Q^2) \right)$$

$$a_3 = g_R/g_V = F+D = (\Delta u - \Delta d) \quad n\text{-decay}$$

$$a_8 = (3F-D) = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon-}$$

$$a_0 = \Delta \Sigma = (\Delta u + \Delta d + \Delta s) \quad \text{decay}$$

$$g_R/g_V = 1.2573 \pm 0.0028$$

$$3F-D = 0.579 \pm 0.025$$

$\alpha_s(Q^2)$ CORRECTIONS

LAREN Phys. Lett. B334(1994)192-198

(101)

→ Bjorken Sum Rule

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \frac{g_a}{g_v} \left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 \right]$$

→ Ellis-Jaffe Sum Rules

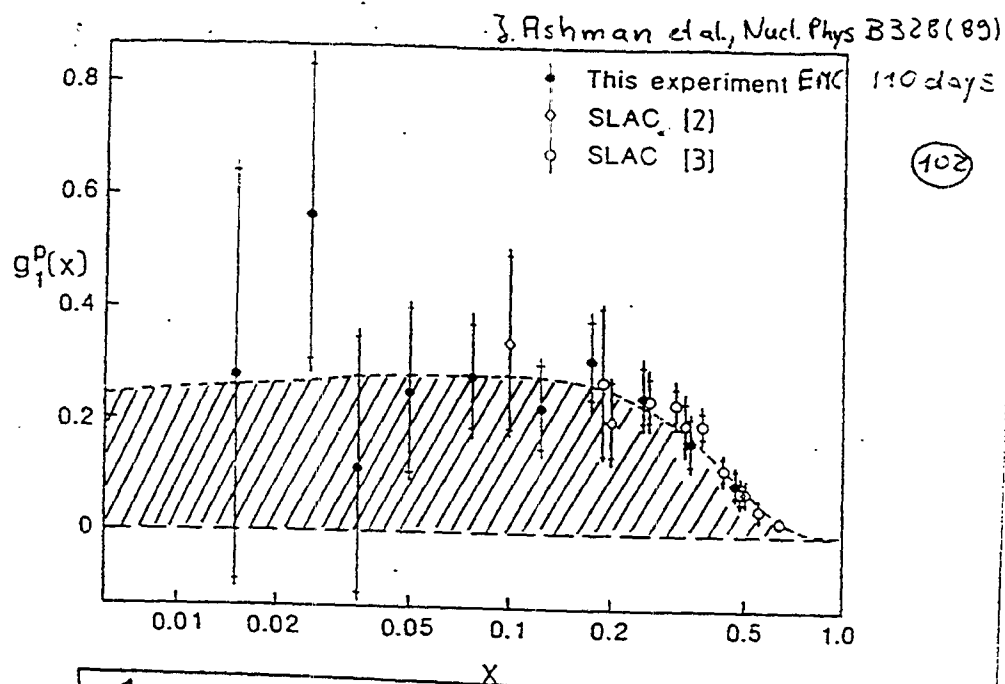
$$\begin{aligned} \Gamma_1^{p(n)} &= \Gamma_1^{NS} + \Gamma_1^S \\ &= \frac{1}{12} \left(+(-)a_3 + \frac{a_8}{\sqrt{3}} \right) \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s}{\pi} \right)^3 \right] \\ &\quad + \frac{1}{9} \left[1 - \frac{\alpha_s}{\pi} - 0.55 \left(\frac{\alpha_s}{\pi} \right)^2 \right] \Delta\Sigma \end{aligned}$$

$$a_3 = \frac{g_a}{g_v} = F + D = (\Delta u - \Delta d) \quad \text{neutron decay}$$

$$a_8 = \frac{3F - D}{\sqrt{3}} = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon decay}$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

+ higher twist corrections



(102)

$$\int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015$$

$$\delta A_{\chi^2} = \frac{1}{\sqrt{N_H + N_H}} \left(f_N \cdot P^B \cdot P^T \cdot D \right)^{-1}$$

0.15 0.8 0.8 0.2-0.7

$$f(\text{NH}_3) = \frac{3}{17} ;$$

$$f(\text{C}_4\text{H}_9\text{OH}) = \frac{10}{74}$$

But:

$$f(\text{H}) = f(\text{D}) = 1 !$$

$$\delta g_1 \sim \delta A_{\chi^2} F_1 \sim \delta A_{\chi^2} \cdot F_2 / x !$$

Consequences from $\Gamma_1' = 0.126 \pm 0.018$

(LO - QCD)

(103)

$$\Gamma_{1 \text{ exp}}^P = \Gamma_{1 \text{ EJ}}^P + \frac{1}{3} C_2 \Delta S$$

$$0.126 = 0.175 + \frac{1}{3} \cdot 0.997 \Delta S$$

QPM

$$\Rightarrow \boxed{\Delta S = -0.147}$$

0

$$1) \Delta u - \Delta d = 1.2572 \pm \dots$$

$$2) \Delta u + \Delta d - 2\Delta S = 0.5796 \pm \dots$$

$$3) \Delta S = -0.147 \pm$$

$$\leadsto \boxed{\Delta u = 0.771 \pm \dots}$$

$\frac{4}{3}$

$$\boxed{\Delta d = -0.486 \pm \dots}$$

$-\frac{2}{3}$

$$\blacktriangleright \boxed{\Delta \Sigma = \Delta u + \Delta d + \Delta S = 0.138 \pm 0.170} \quad ? \quad 1$$

'Spin puzzle'

■ Very difficult experiments, tiny asymmetry
tremendous effort (104)

■ Phantastic progress in very different
new technologies:

● Polarized (strained GaAs) e^- sources with
 $p_e \sim 85\%$ (E-143, E-154)

● Long. beam polarization (Sokolov-Ternov)
in HERA with $p_e \nearrow 70\%$

● Cryogenic polarized targets ($\text{NH}_3^\uparrow, \text{C}_4\text{H}_9\text{OH}^\uparrow, \text{h}$
(SMC, E143, TJNAL, ELSA, ...) highly developed

● Polarized ^3He gas targets $\boxed{+ \text{Li}^\uparrow \text{D}^\uparrow}$
▲ Spin exchange with optically pumped $\text{Rb}^\uparrow$
@ 10 atm (E142, E-154) $\uparrow$

high power laser diode

▲ metastability exchange (HERMES)

● Internal storage cell target works perfectly
(HERMES)

● Atomic Beam Source for $\text{H}^\uparrow, \text{D}^\uparrow$ h.d. (HERMES)

● Laser Driven Source for $\text{H}^\uparrow, \text{D}^\uparrow$ coming

● Polarimetry

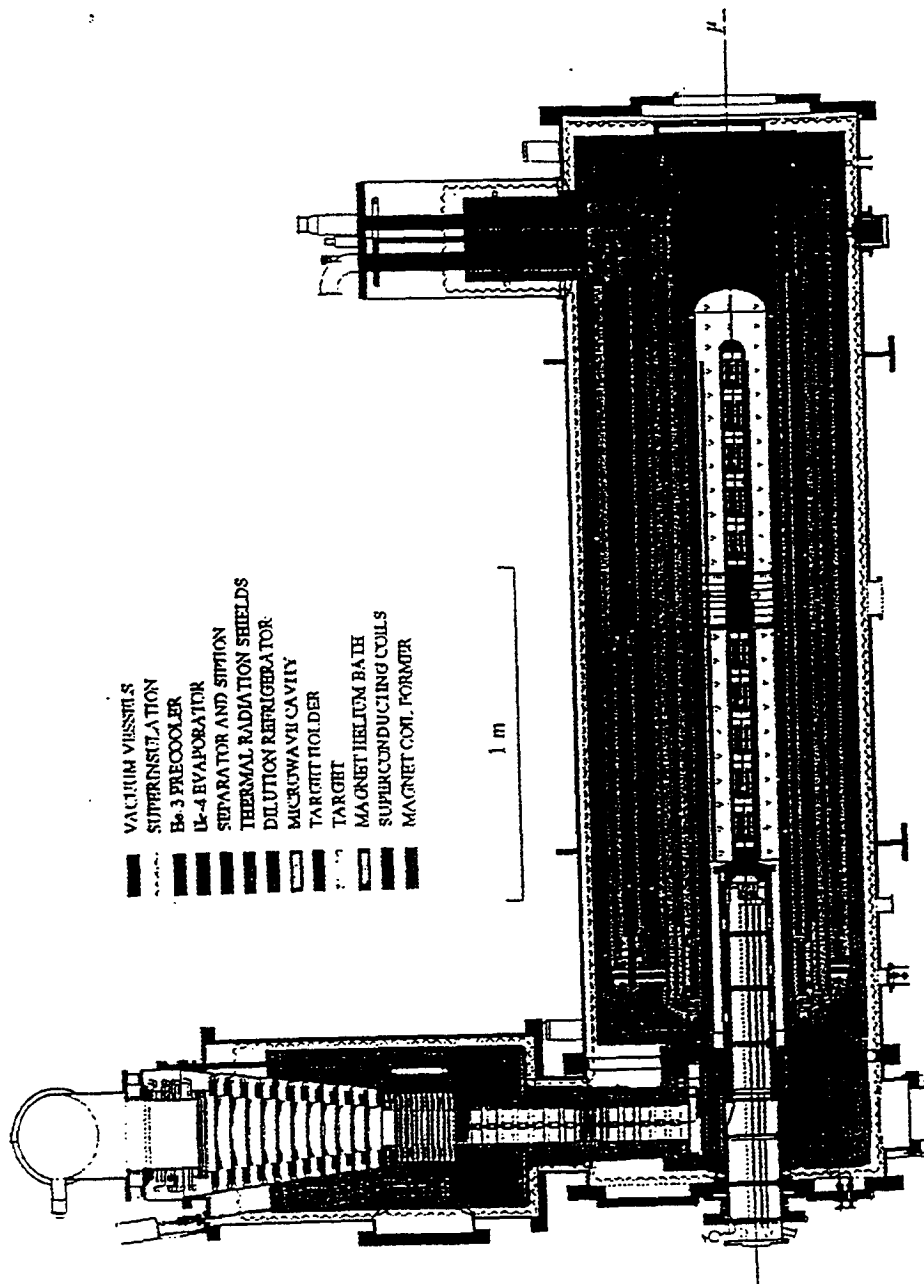
●
●
●
●

Spin experiments		P_T	f
SMC/CERN	$C_4 H_3^{\uparrow} OH^{\uparrow}$	~ 0.85	$\sim 1/3$
	$C_4 D_3^{\uparrow} OD^{\uparrow}$	< 0.5	$\sim 1/3$
	$NH_3^{\uparrow}$	0.89	$\sim 1/3$
π^- decay in flight	$\mu^{\uparrow}$ $P_B \sim 0.8$ $\bar{I} \sim \frac{1}{2} \mu A$ $E = 190 \text{ GeV}$		(10)

E-142 / E-154 / SLAC		$^3He^{\uparrow} (10 \text{ atm})$ (optically pumped Rb)	$\sim .33$ $\sim 1/3$ $\sim .38$ $\sim 1/3$
Ga-As strained "	$e^{\uparrow}$ $P_B \sim 0.4, \bar{I} \sim 10 \mu A$ $P_B \sim 0.8, \bar{I} \sim 100 nA$ $E = 26/48 \text{ GeV}$		

E-143 / E-155 / SLAC		$^{15}NH_3$ $^{15}ND_3$ LiD	< 0.8 $\sim 1/3$ < 0.3 $\sim 1/3$ ~ 0.5 $\sim 1/3$
strained Ga-As	$e^{\uparrow}$ $P_B \sim 0.8, \bar{I} \sim 80 nA$ $E = 29/48 \text{ GeV}$		

HERMES / DESY		$H^{\uparrow}, D^{\uparrow}$ $^3He^{\uparrow}$ (metastability exch.)	~ 0.9 1 ~ 0.5 $1/3$
HERA (Sokolov-Ternov)	$e^{\uparrow}$ $P_B = 0.5-0.7$ $\bar{I} \sim 20 \text{ mA}$ $E = 28 \text{ GeV}$		



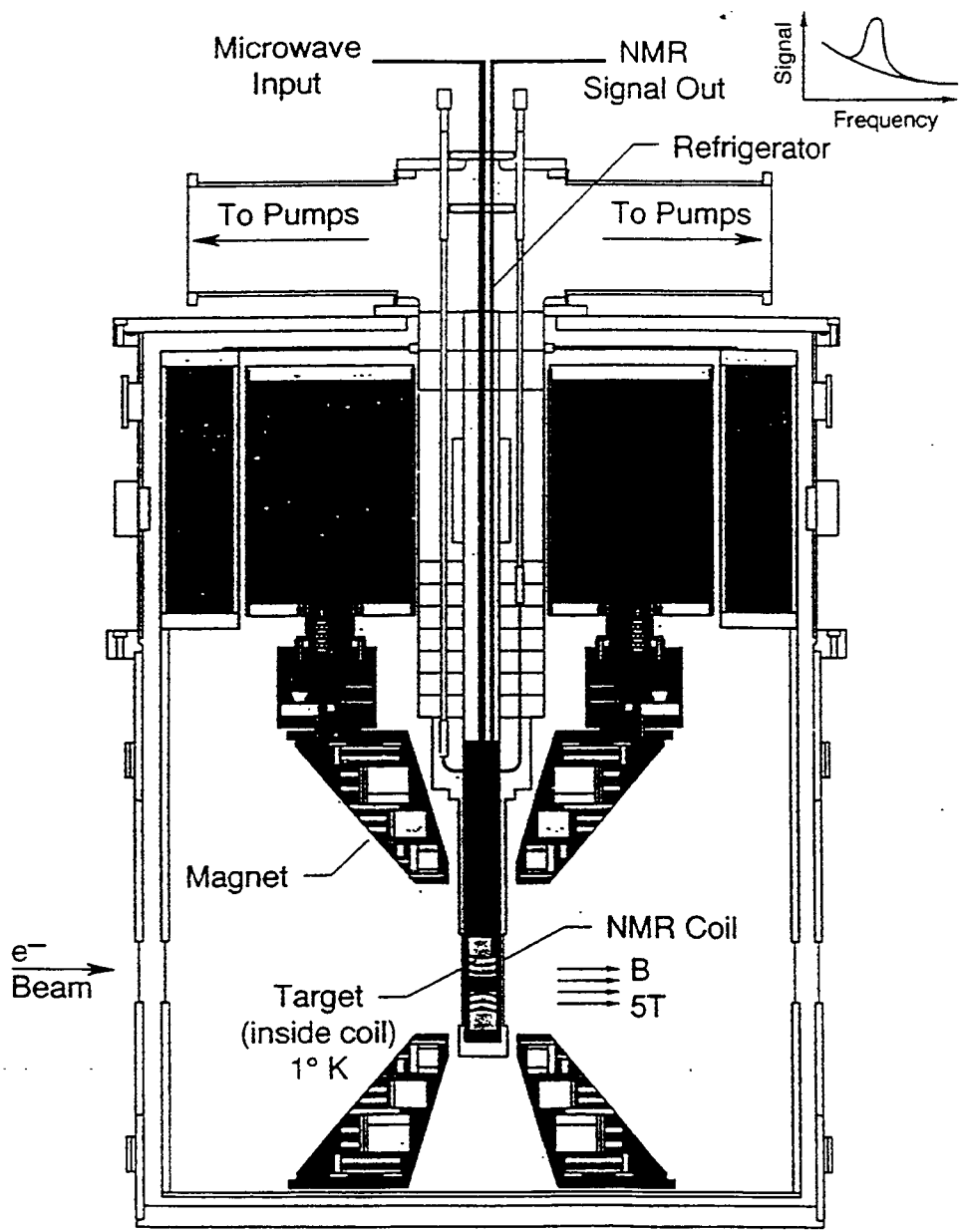
- VACUUM VESSELS
- SUPERINSULATION
- He-3 PRECOOLER
- U-4 EVAPORATOR
- SEPARATOR AND SIFTON
- THERMAL RADIATION SHIELDS
- DILUTION REFRIGERATOR
- MICROWAVE CAVITY
- TARGET HOLDER
- TARGET
- MAGNET HELIUM BATH
- SUPERCONDUCTING COILS
- MAGNET COIL FORMER

1 m

102

File 1004
Not listed
Classification

(107)

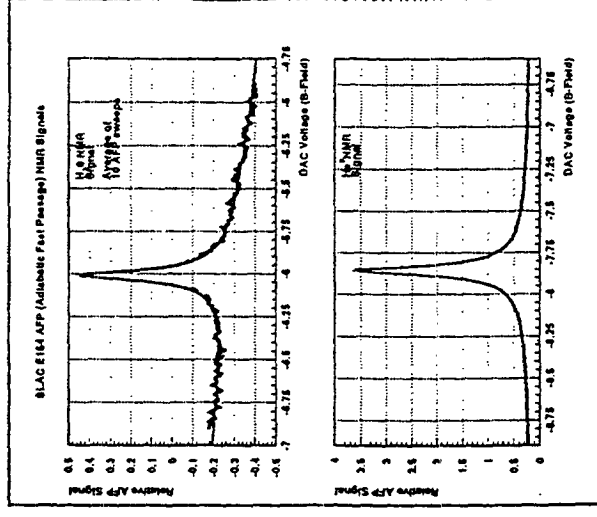
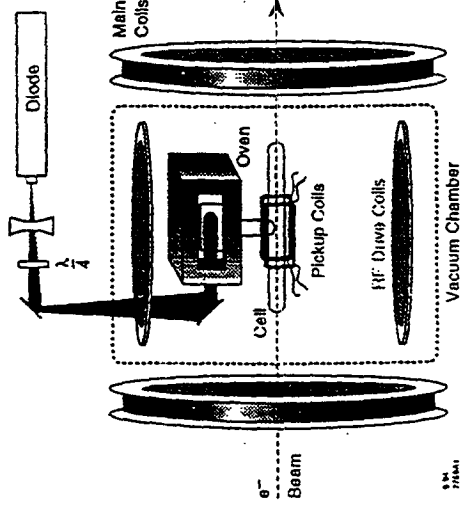


4-94

7656A1

E154 ³He Target Assembly & NMR

- 4 Ti:Sapphire Laser + 3 Laser Diode Arrays,
- 30 cm long target vessel, 50 μm windows,
- ~10 Amagat ³He density.

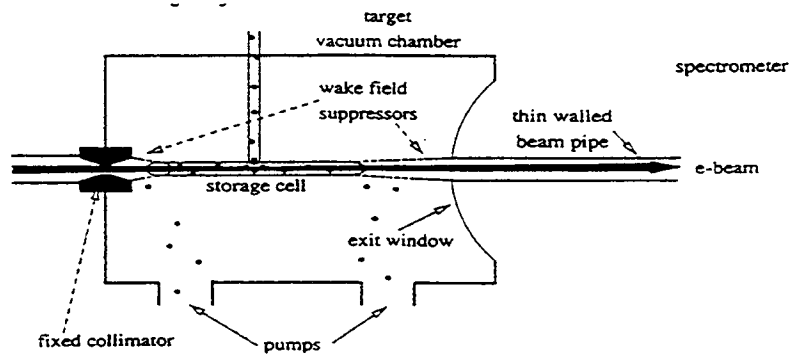


NIM A366, 148-162 (1995)

The HERMES target



- Internal storage cell:



- undiluted target atoms: $\vec{H}$, $\vec{D}$, $\vec{{}^3\text{He}}$



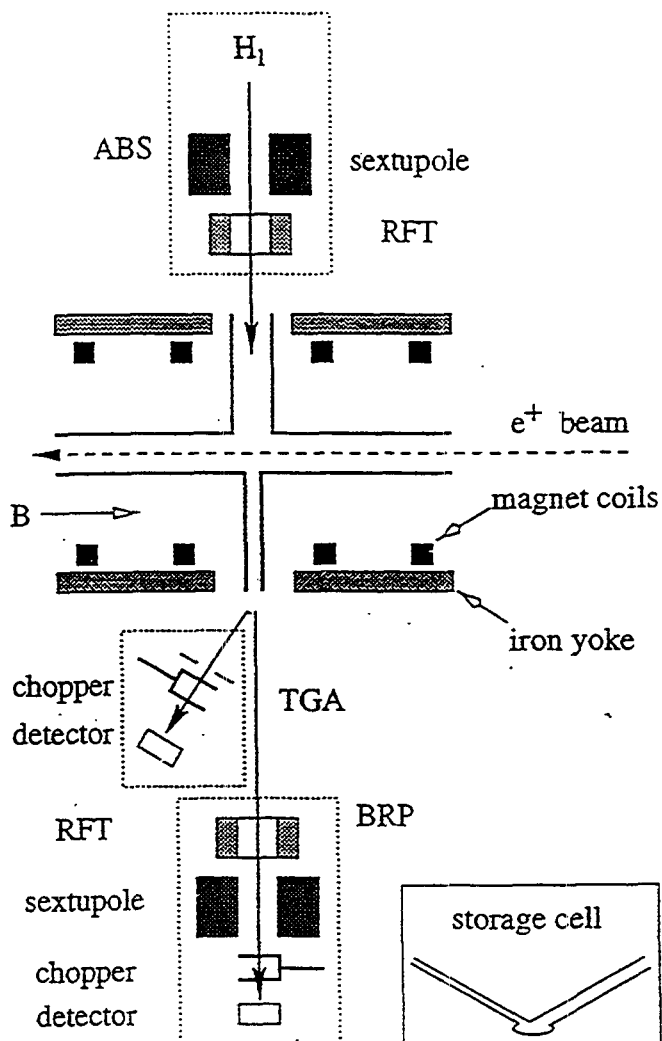
- Gas targets:

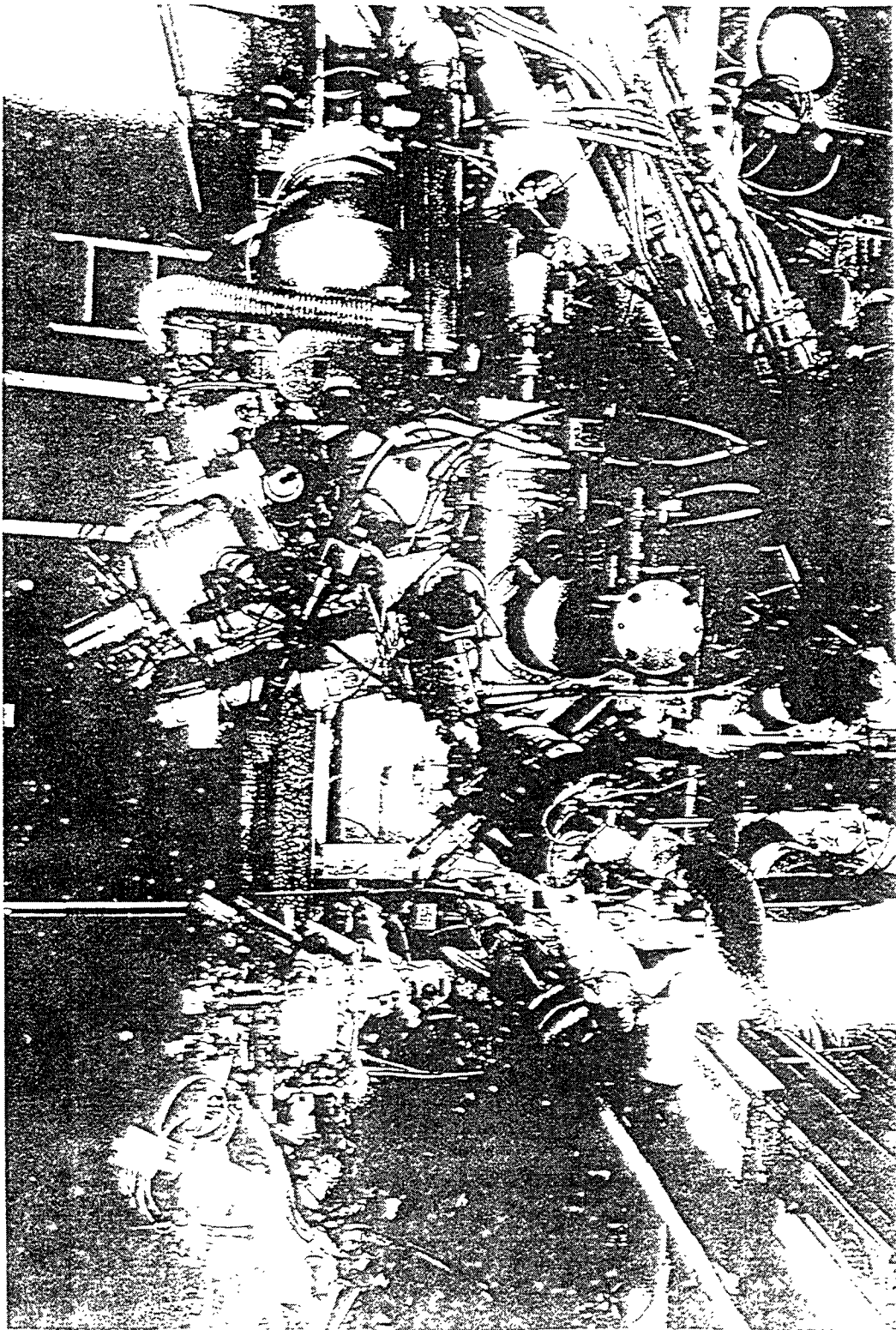
- Laser driven source: $\vec{{}^3\text{He}}$
 $\rho = 1 \cdot 10^{15} \text{ N/cm}^2$ $\Delta t_{flip} \sim 10 \text{ min}$ $\overline{P_T} = 46\%$

%

- Atomic beam source: $\vec{H}$
 $\rho = 7 \cdot 10^{13} \text{ N/cm}^2$ $\Delta t_{flip} \sim 1 \text{ min}$ $\overline{P_T} = 92\%$

- Unpolarized gasses: ${}^1\text{H}$, ${}^2\text{D}$, ${}^3\text{He}$ and ${}^{14}\text{N}$.

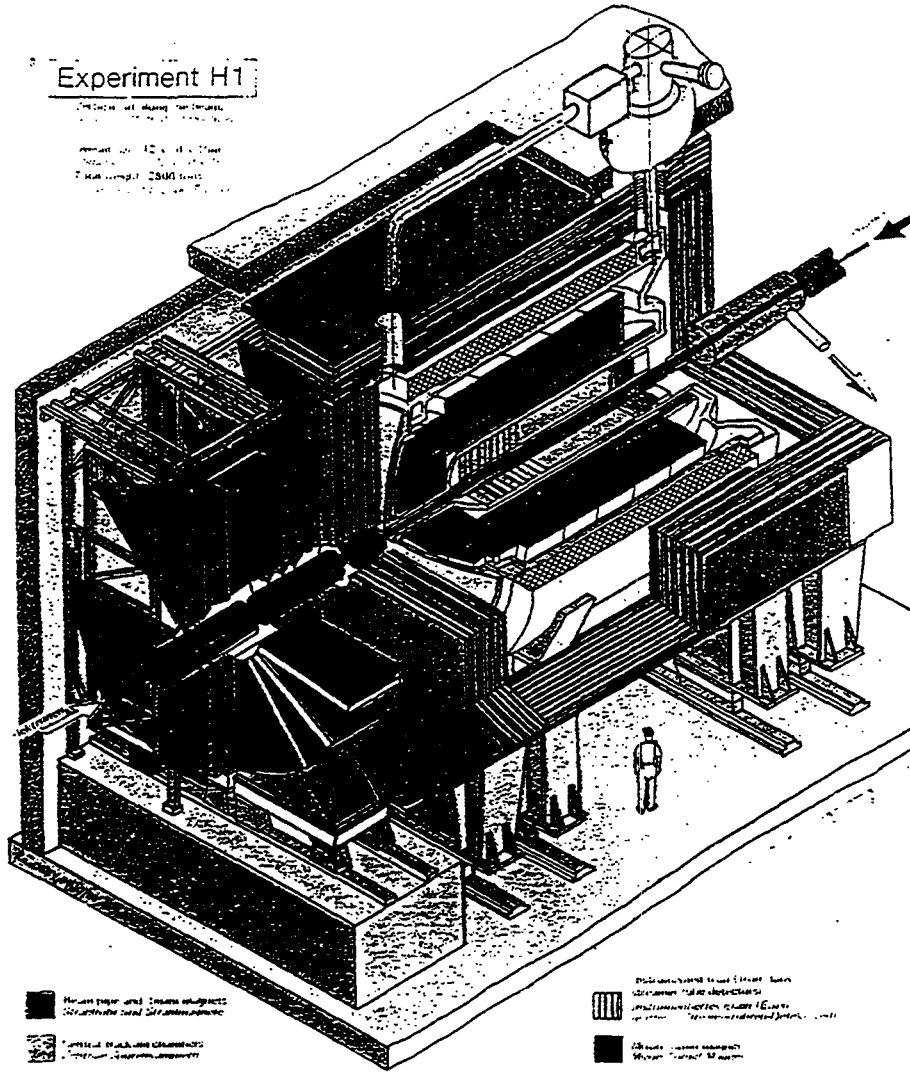




Experiment H1

Production of strong materials

Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons



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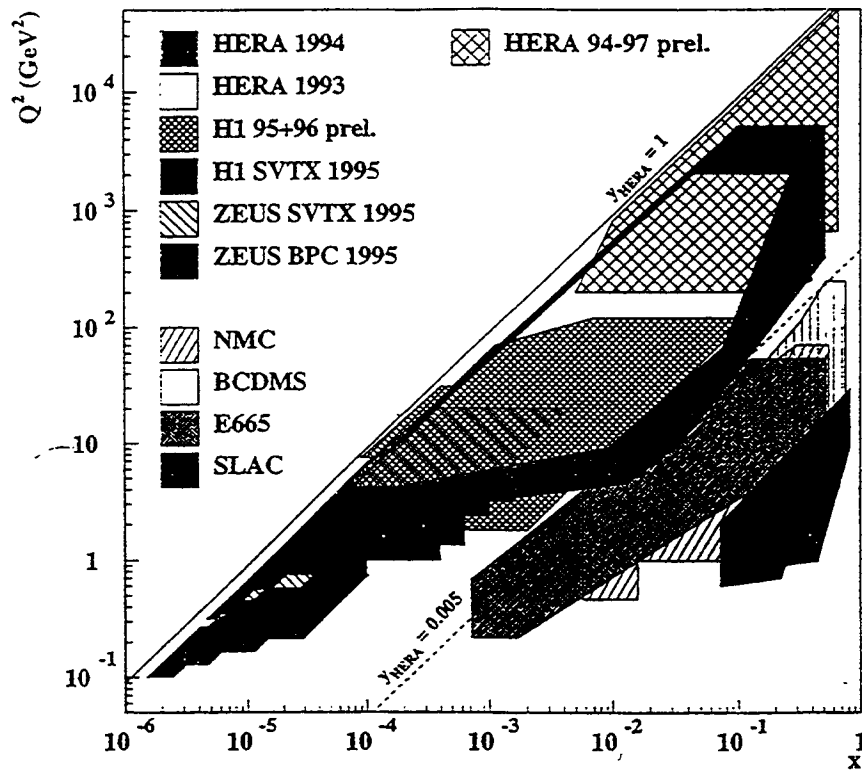
Material: 12 x 12 x 12
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 Weight: 100 kg
 Tensile strength: 2800 tons

Material: 12 x 12 x 12
 Weight: 100 kg
 Tensile strength: 2800 tons

Kinematic reach

(53)

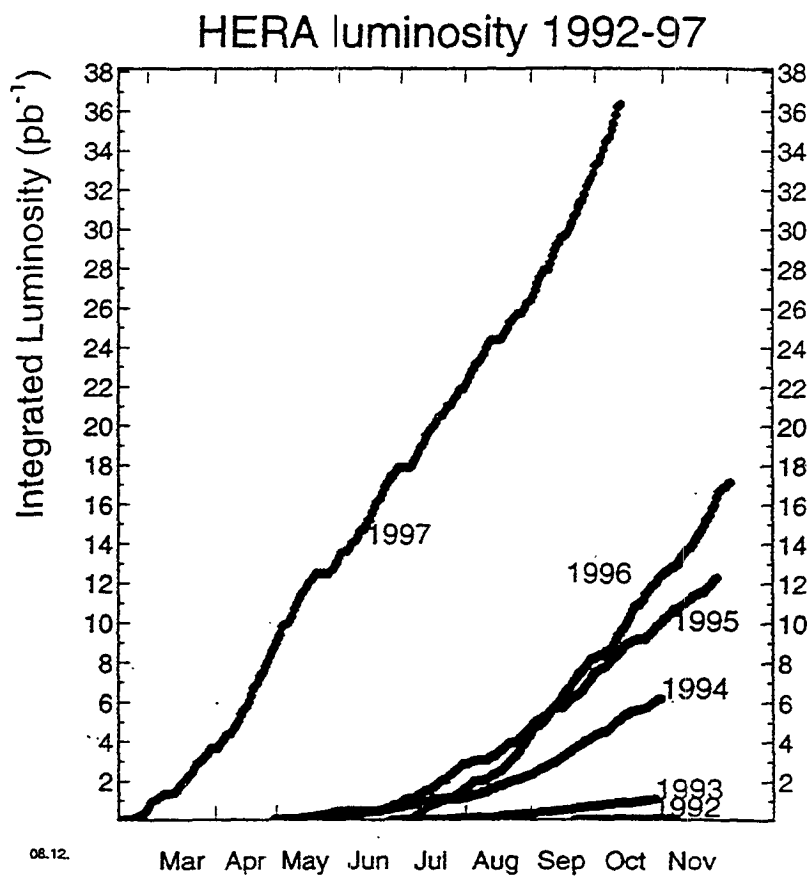


Several upgrades

$$10^{-6} < x < 0.7$$

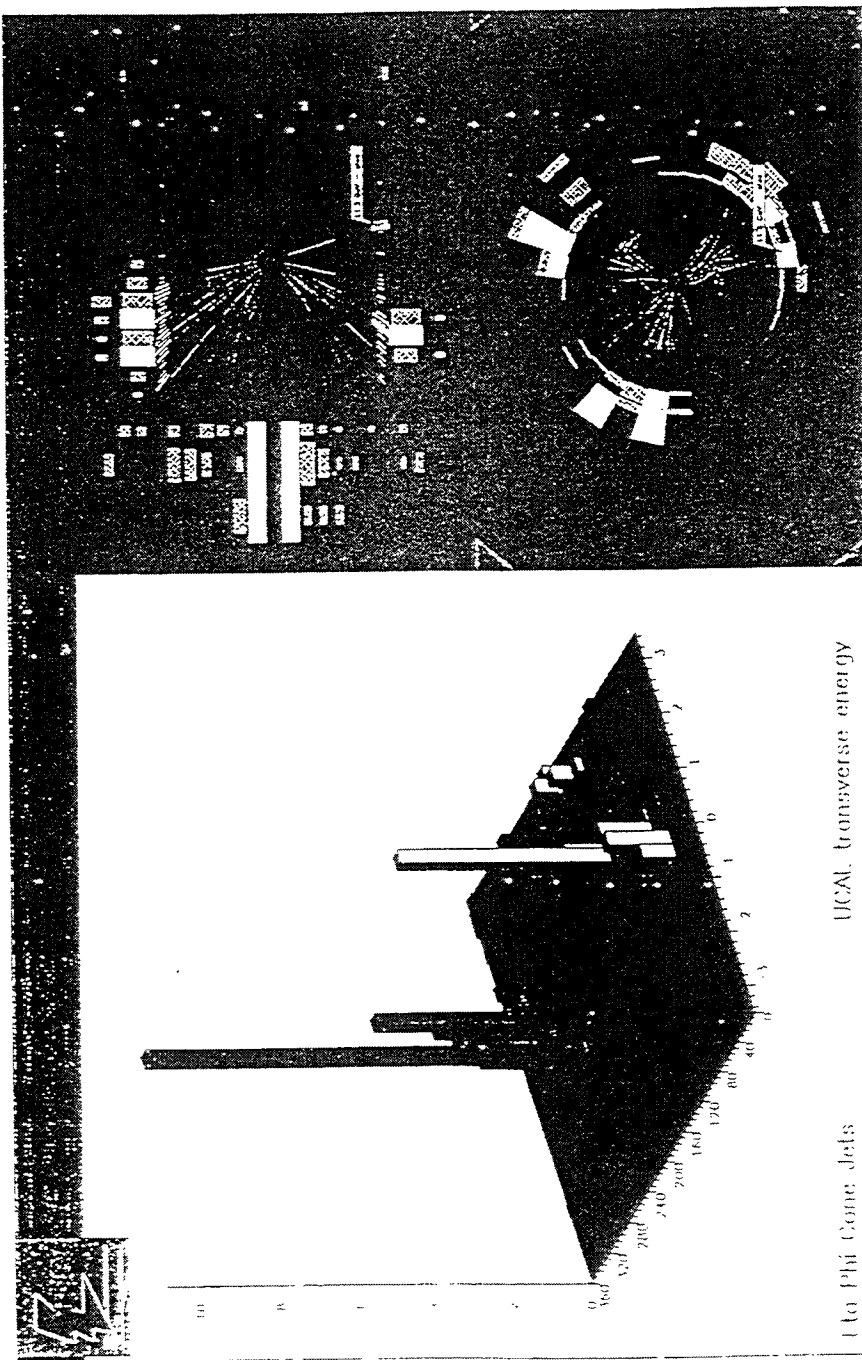
$$10^{-1} \text{ GeV}^2 \lesssim Q^2 \lesssim 30\,000 \text{ GeV}^2$$

54

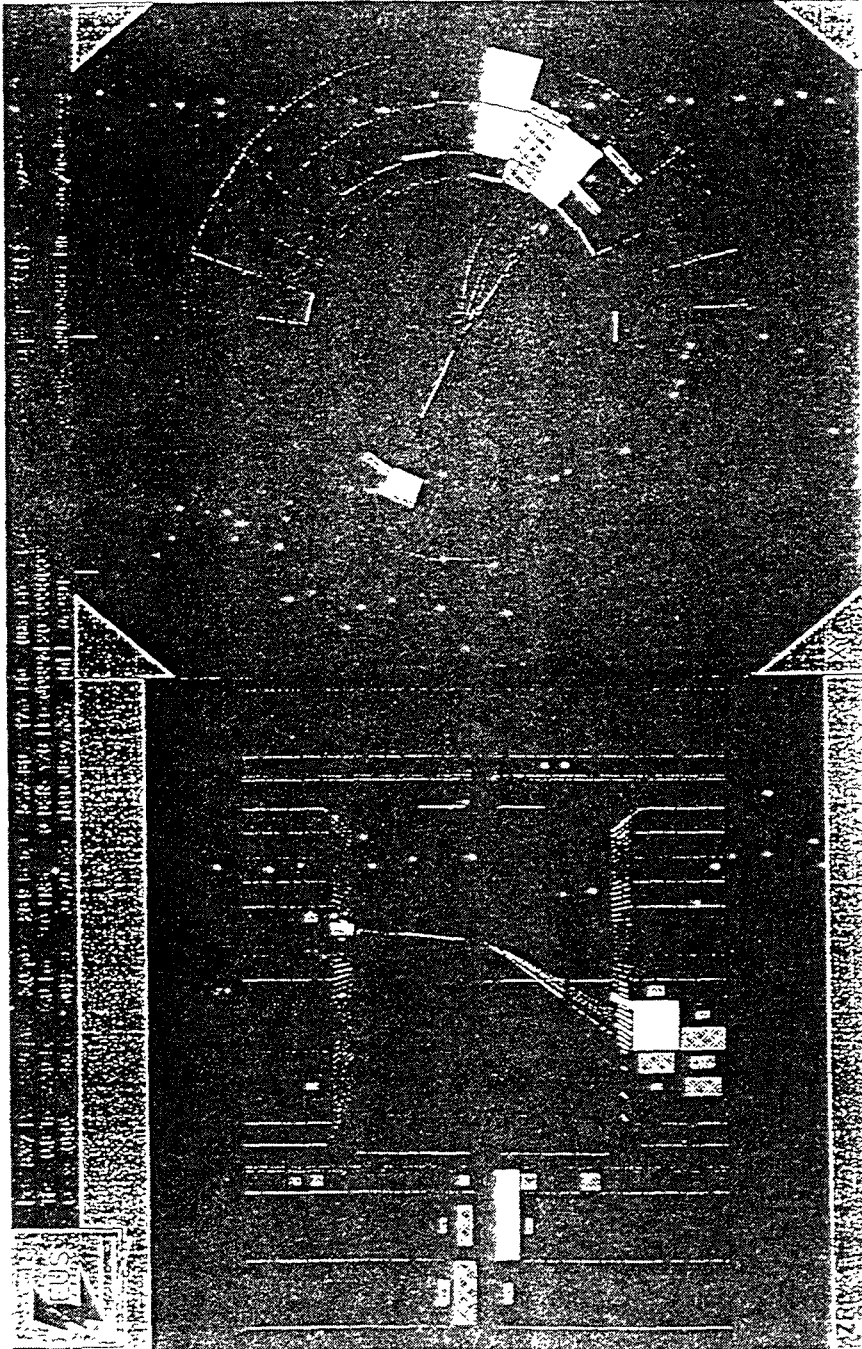


08.12.

γ jet event



(b)



(3)

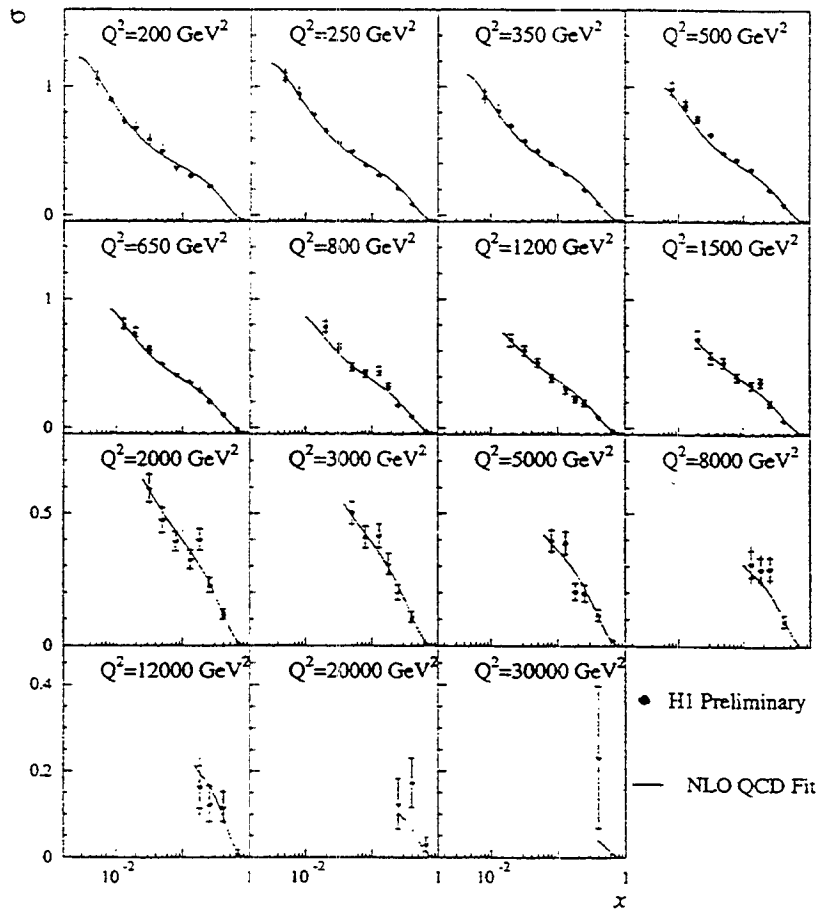
H1 data, 1994-1997, neutral current

Reduced cross section

(58)

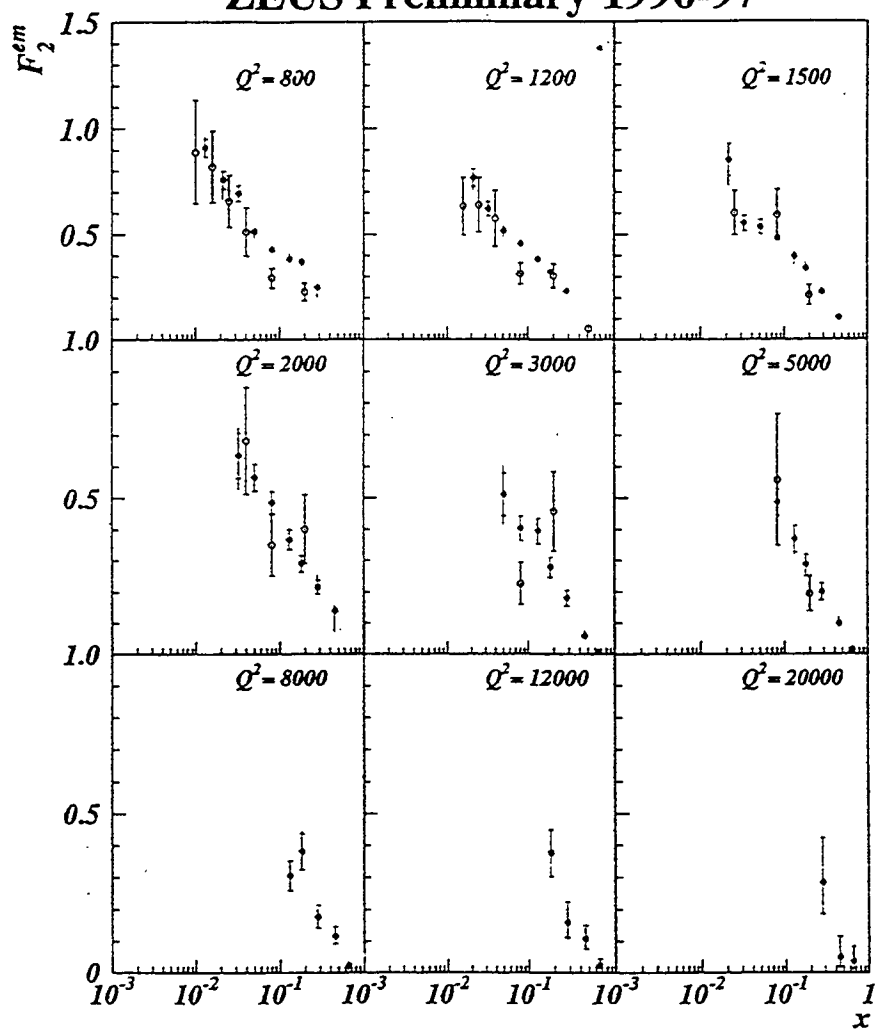
$$\sigma := \frac{xQ^4}{2\pi\alpha^2 Y_+} \frac{d^2\sigma^{NC}}{dx dQ^2} = F_2 - \frac{y^2}{Y_+} F_L - \frac{Y_-}{Y_+} x F_3$$

$$Y_{\pm} = 1 \pm (1-y)^2$$



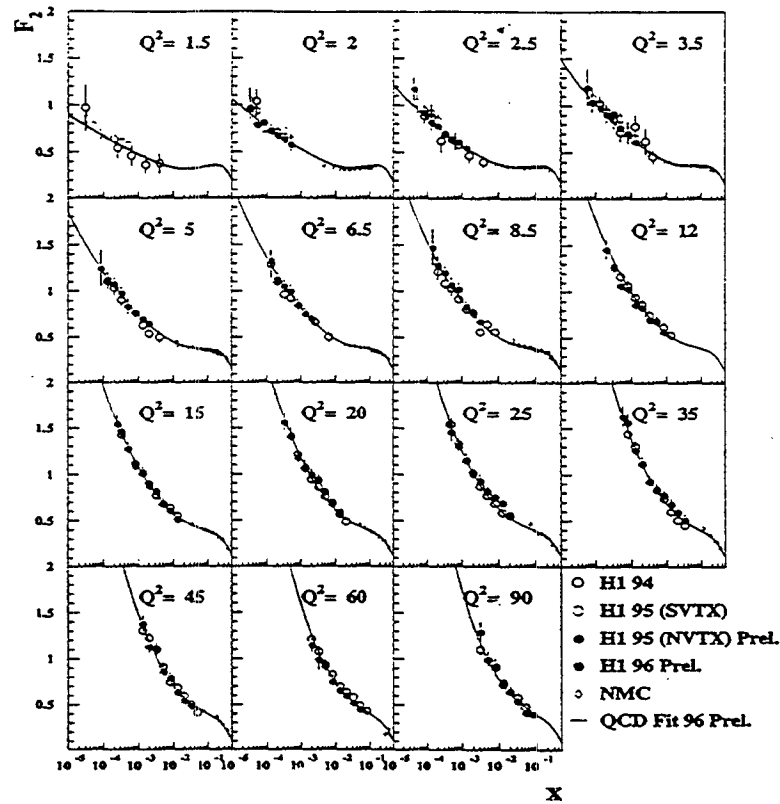
Steep rise at low x

ZEUS Preliminary 1996-97



F_2 at 'low Q^2 '

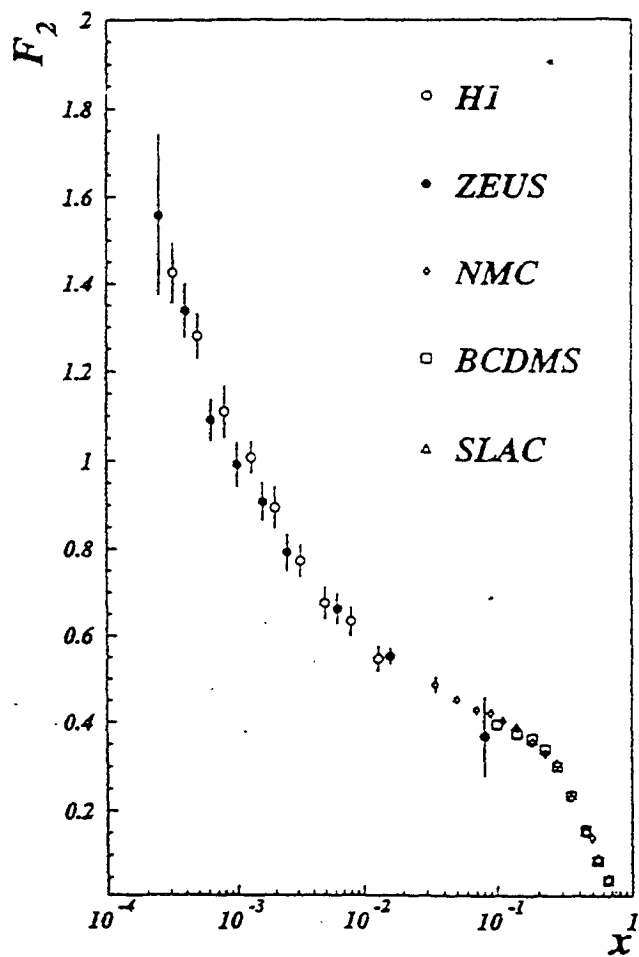
(60)



Steep rise of F_2 at low x

➡ strong increase of parton densities in the proton at low x

61

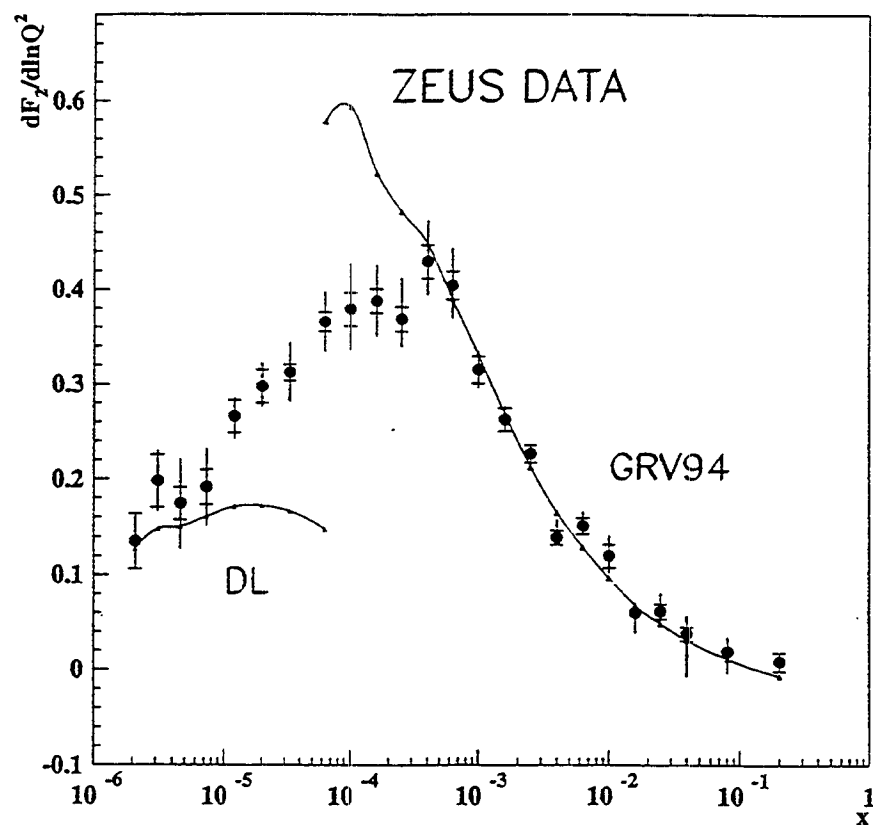


low x : $F_2(x) = \frac{5}{18} x \cdot q(x) \leadsto q(x=2.5 \cdot 10^{-3}) = 2300$

$\leadsto$ rather crowded!

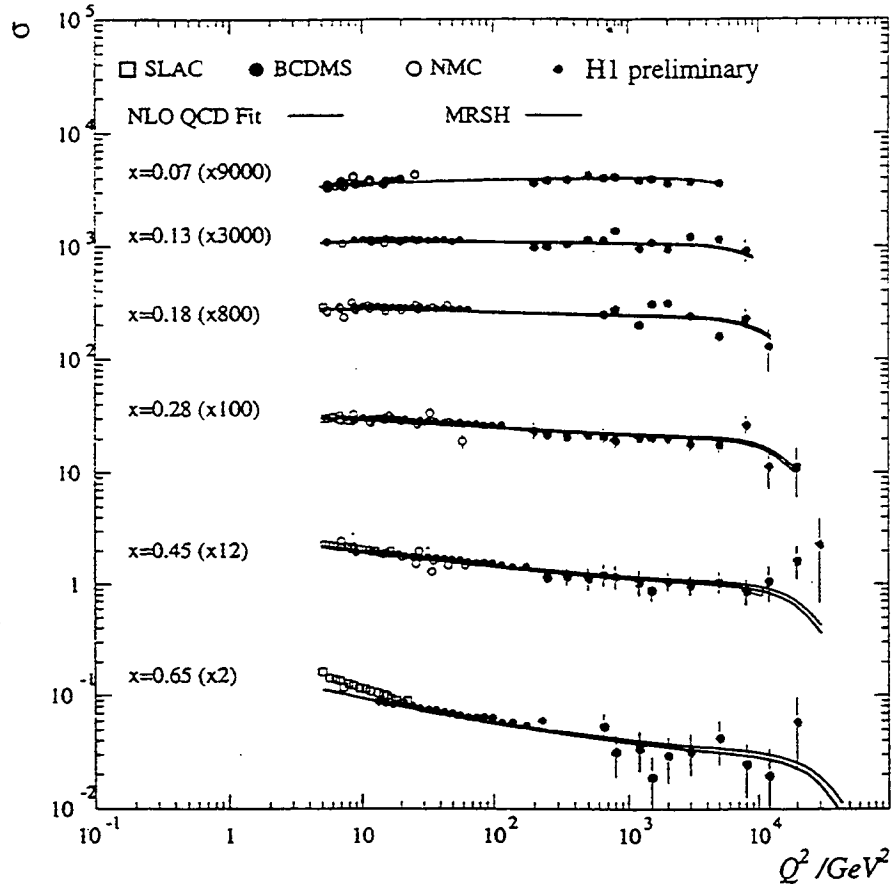
ZEUS 1995 Preliminary

$\langle Q^2 \rangle =$
 0.13 0.18 0.20 0.23 0.29 0.37 0.52 1.10 2.45 3.85 5.24 8.76 10.63 13.52 14.57 19.26 26.15 24.58 56.27 19.37 45.01 167.61 156.59 388.03 GeV²



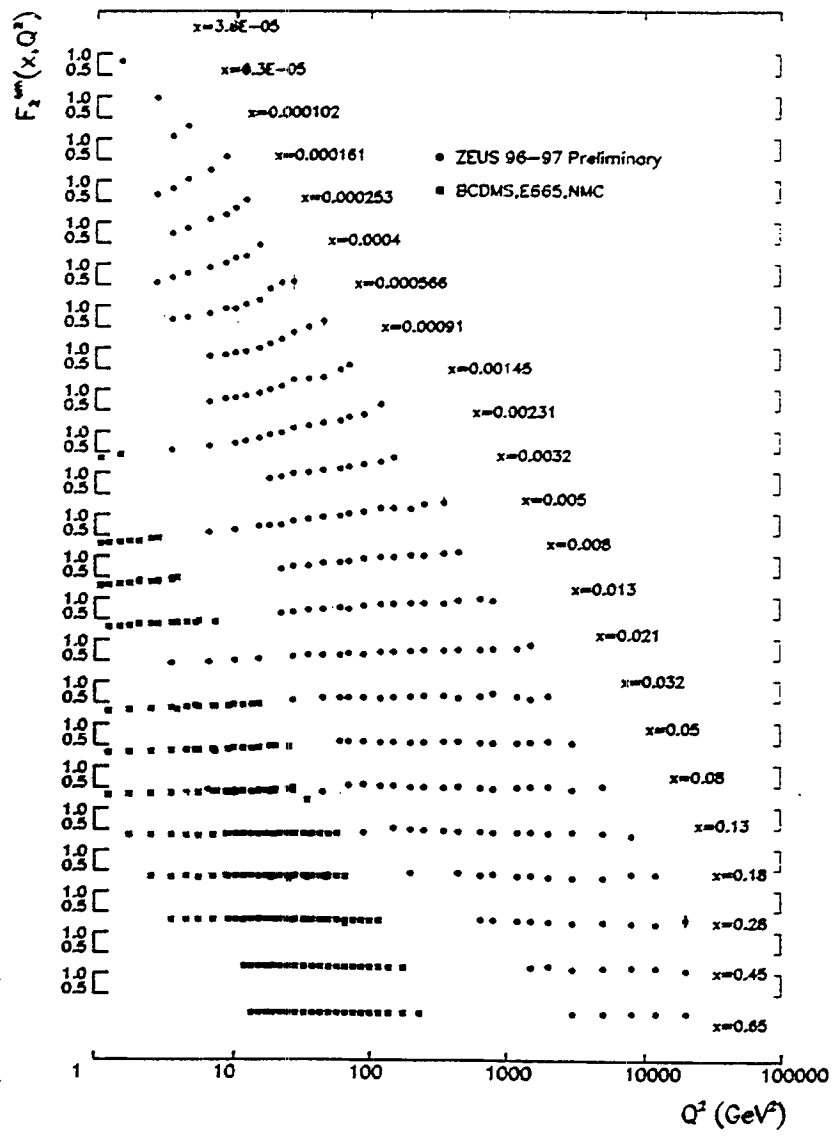
Reduced Cross-section at High x

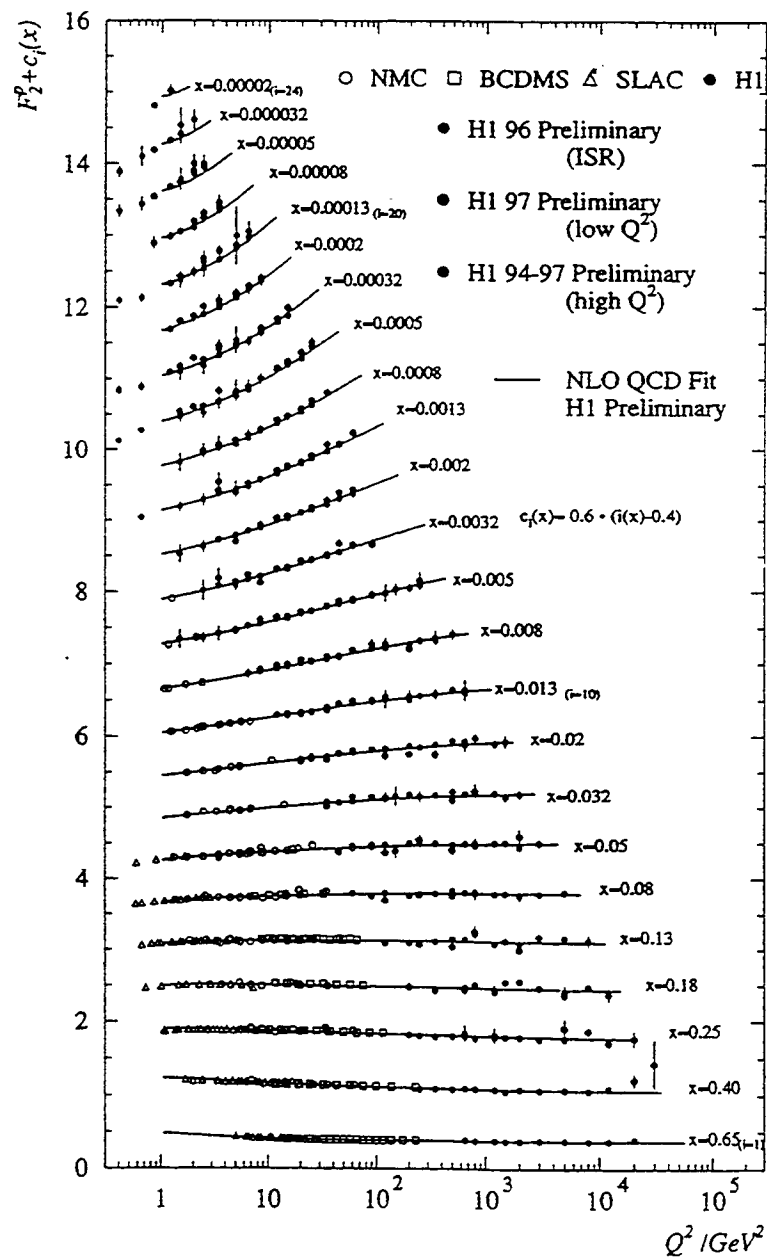
63



- Difference visible in the QCD fit when the high Q^2 data is or is not included.
- High Q^2 HERA data now also have an influence at high x .

67





Gluondistribution $g(x)$

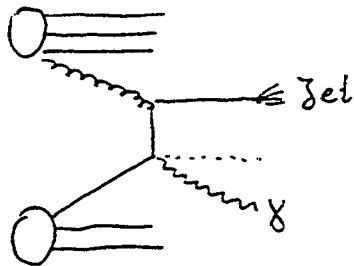
(69)

1) From scalebreaking

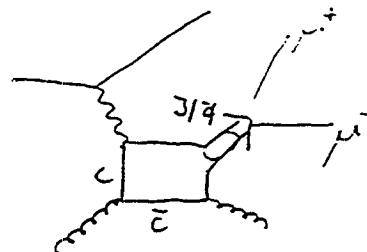
→ coupled Altarelli-Parisi equations

(Problem $g(x) \leftrightarrow \Lambda$ correlated)

2) Direct photons at high p_t
(QCD - Compton)



3) J/ψ - Production
Photon - gluon - Fusion

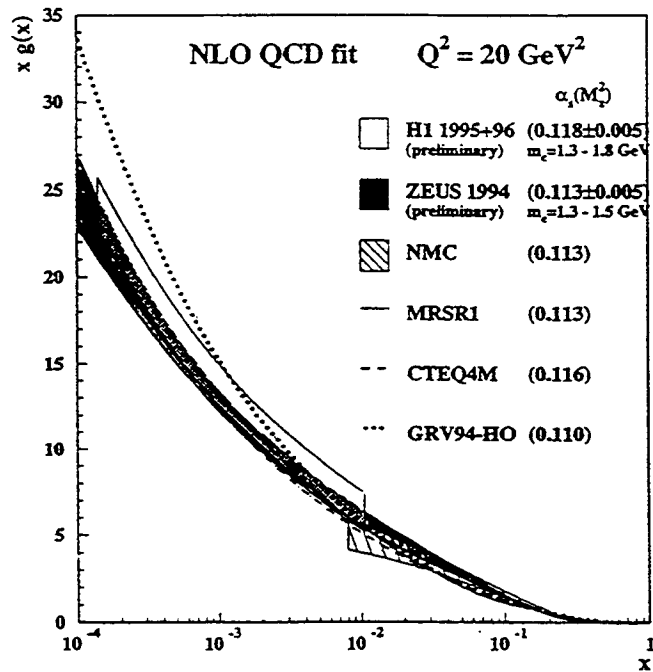


$$4) (F_L(x, Q^2) \leftrightarrow R(x, Q^2) = \frac{\sigma_L}{\sigma_t})$$

$g(x)$ from scaling violations

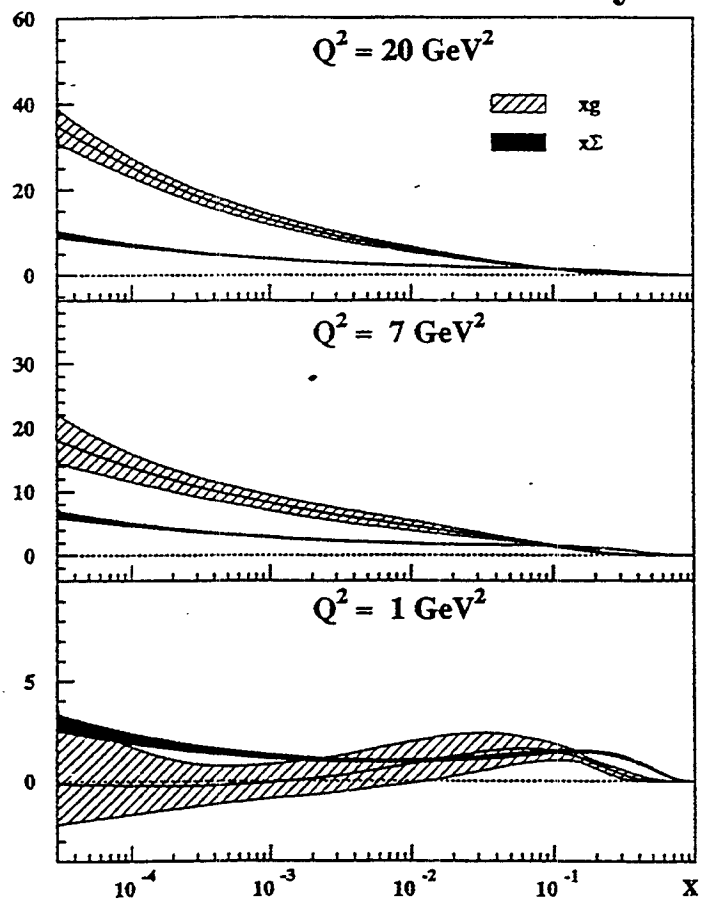
(70)

$$\frac{dF_2}{d\ln Q^2} = \frac{\alpha_2(Q^2)}{2\pi} \left[\int_x^1 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qq}\left(\frac{x}{z}\right) F_2(x, Q^2) \right. \\ \left. + \int_x^1 2 \sum_q e_q^2 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qg}\left(\frac{x}{z}\right) g(x, Q^2) \right]$$



71

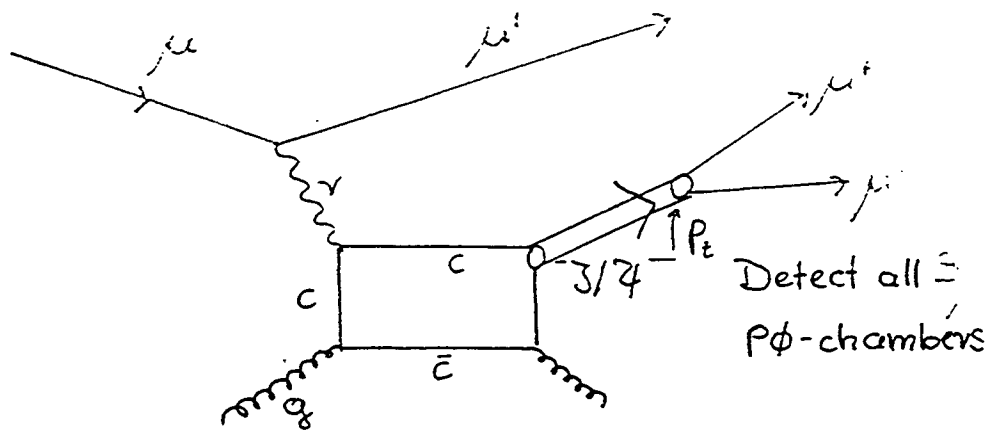
ZEUS 1995 Preliminary



(72)

J/ψ production from H.D
S n, C

Photon - gluon - fusion model



$$\sigma_{\text{inelastic}}^{J/\psi} \sim g(x)$$

$$\text{inelastic: } z = \frac{E^{J/\psi}}{\gamma} < 0.9$$

$$P_t^2 \lesssim 0.2 \text{ GeV}^2$$

$$X = \frac{S_X \cdot \alpha_s}{S_X \cdot N} = \frac{1}{W^2} \left[\frac{M_{J/\psi}^2}{z} + \frac{P_t^2}{z(1-z)} \right]$$

$$\stackrel{z \rightarrow 1}{\approx} \frac{M_{J/\psi}^2}{W^2}$$

$$E_\mu = 280 \text{ GeV}$$

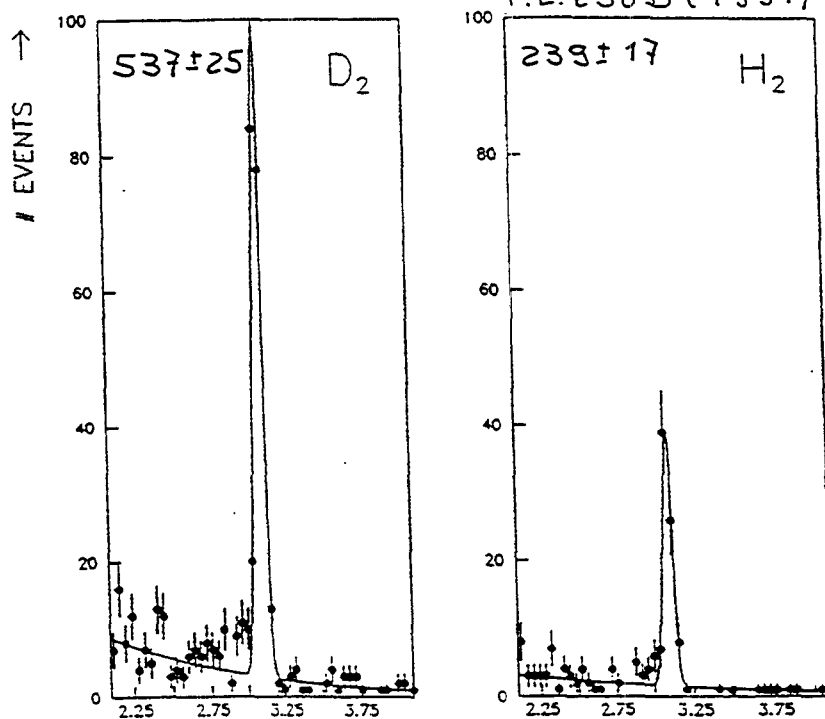
(73)

$$I = (3.05 \pm 0.06) \cdot 10^{12} \mu \text{ } \curvearrowright \mathcal{L} = 125 \text{ pb}^{-1}$$

$$\frac{\sigma^{J/\psi}(D)}{\sigma^{J/\psi}(H)} = 0.96 \pm 0.08$$

Thesis: M. de Jong
C. Mariotti

NM/CERN-PPE/90-178
P.L. 258 B (1991) 493.



$$m_{J/\psi} = 3.095 \pm 0.002 \mu^+ \mu^- \text{ Mass (GeV/c}^2\text{)} \rightarrow$$

$$\text{FIG. 2 } m_{J/\psi} = 3.093 \pm 0.003$$

Inelastic J/ψ -Production (78)

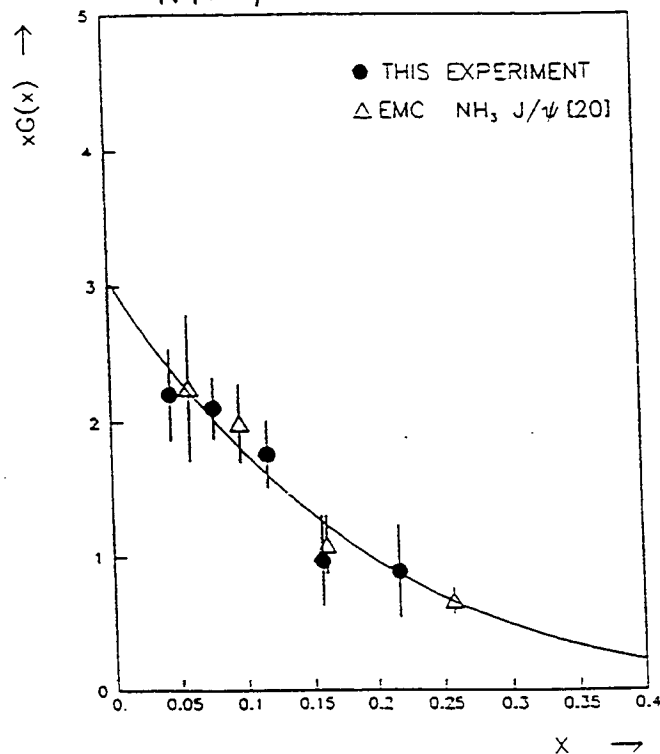
$$z = \frac{E_{J/\psi}}{\gamma} \leq 0.9; \quad p_t^2 > 0.1 \text{ GeV}^2$$

(279 ± 25 events)

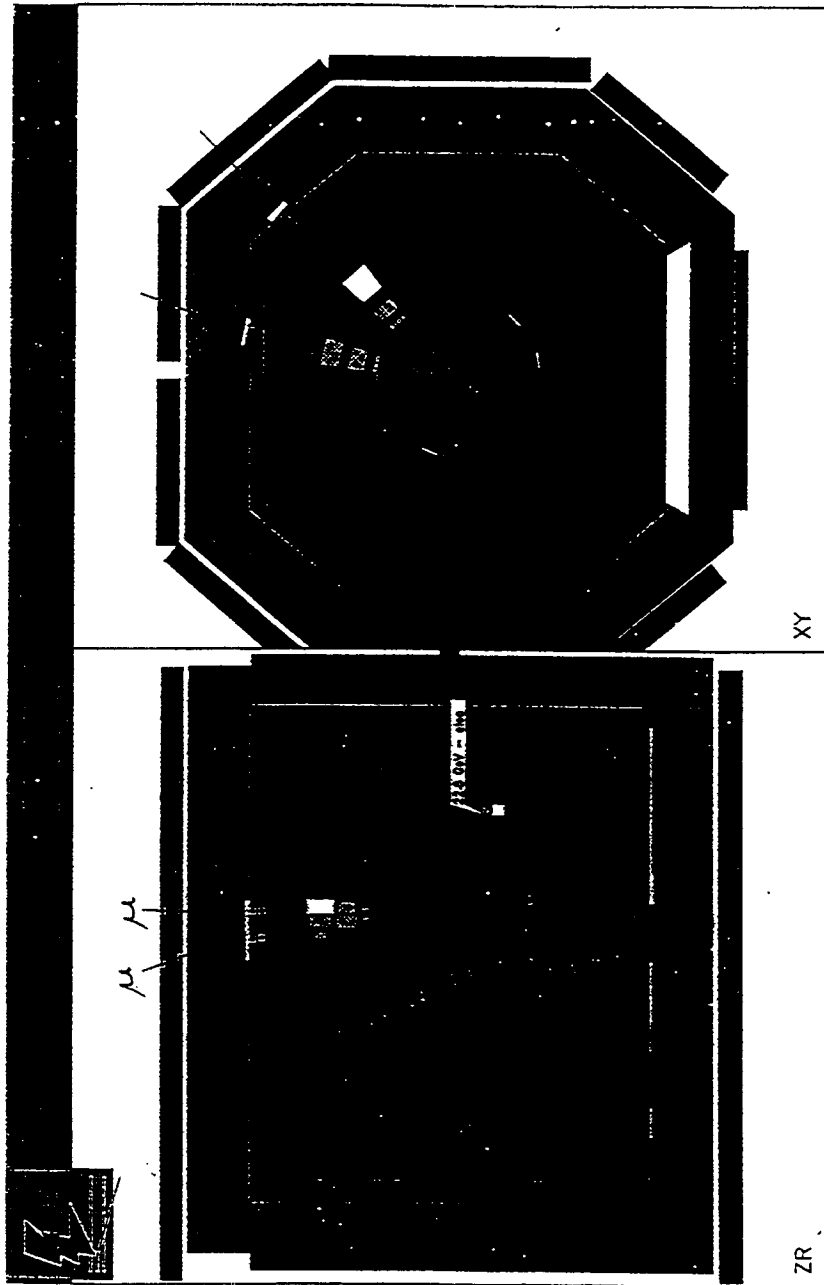
$$\rightarrow xG(x) \sim \frac{\eta+1}{2} (1-x)^\eta; \quad \eta = 5.1 \pm 0.9$$

P.L. B258 (1991) 493.

NMC; CERN-PPE/90-178



3/4 → $\mu^+ \mu^-$

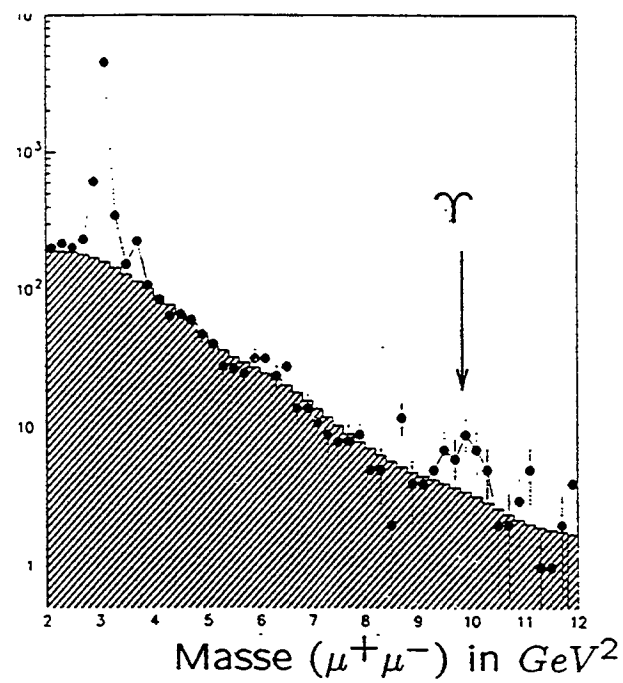


70

J/ψ and Υ production at HERA

80

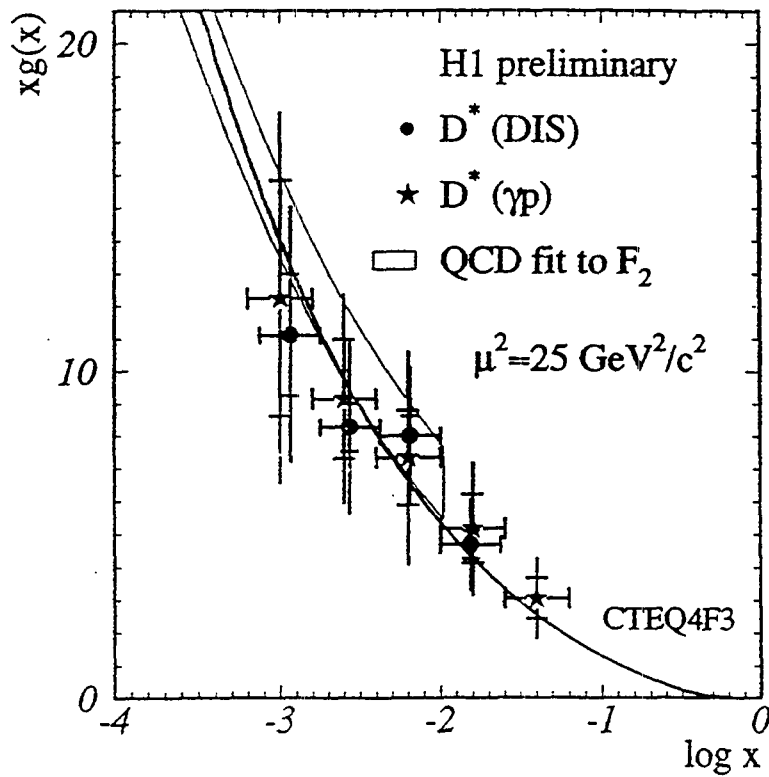
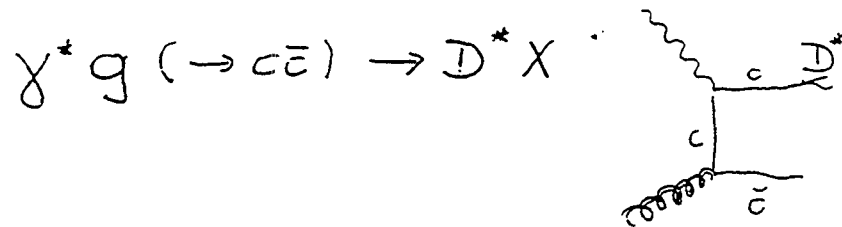
ZEUS 95-97 Preliminary



Υ cross section larger than expected

$g(x)$ from open charm

(81)



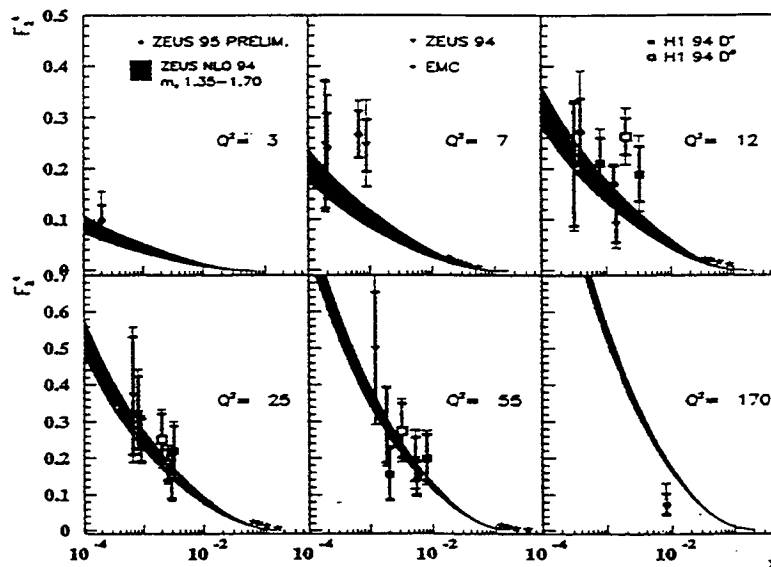
F_2 - charm

(82)

NLO QCD: $g(x, Q^2), P_{g \rightarrow c\bar{c}} \rightarrow F_2^{c\bar{c}}$

Use D^* events

(Transition probabilities $c \rightarrow D^*$, $\bar{c} \rightarrow \bar{D}^*$
and $D^*, \bar{D}^*$ decays known from LEP)



Measurement of F_2^{charm} consistent
with NLO prediction

At low x contribution to F_2 20-25%

Higher Twist Effects

(83)

$$F_2(x, Q^2) = \begin{cases} F_2(x, Q^2)^{LT} \left(1 + \frac{C(x)}{Q^2}\right) \\ F_2(x, Q^2)^{LT} + \frac{C'(x)}{Q^2} + \dots \end{cases}$$

- Measure of quark-quark-correlations ~~quark-quark-correlations~~
(Less clean than quark-gluon-corr.
in $g_2(x, Q^2)$ - Twist-3)

- Effects largest at high x

- H.T. - effects different for bound and free quarks?

Nuclear binding : Correlations

between quarks of different nucleons?



Fits: $C^p(x) \approx C^d(x)$

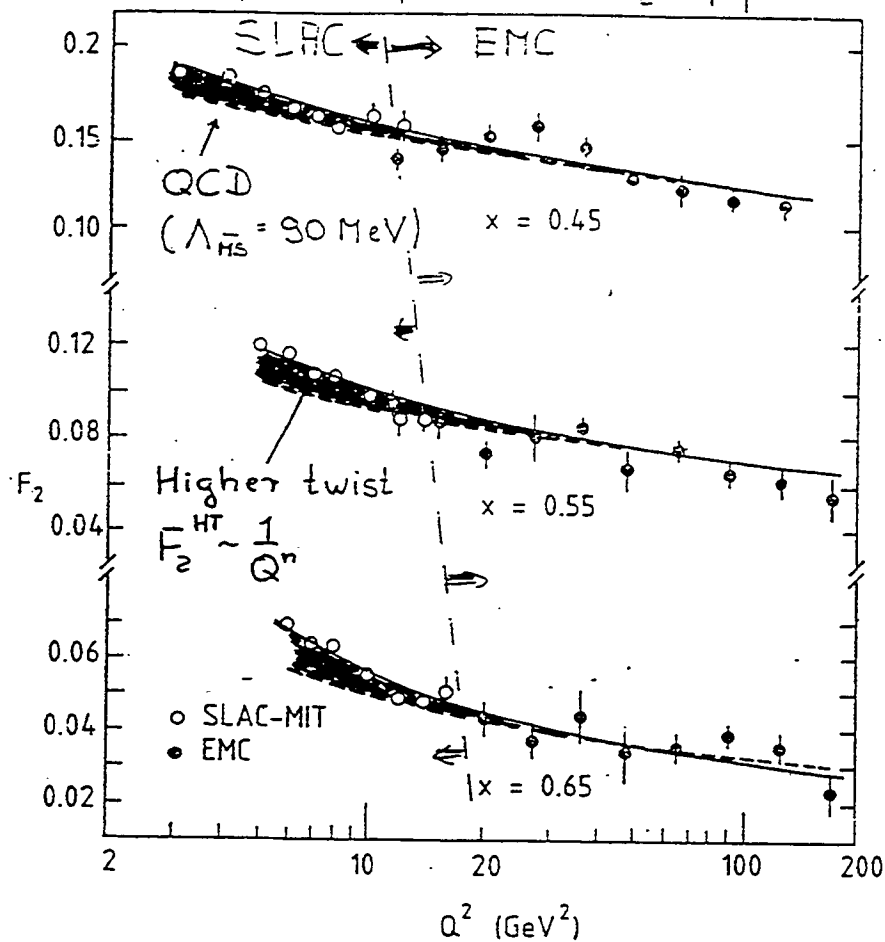
$$\frac{F_2^n}{F_2^p}(Q^2) \rightarrow C^n(x) > C^p(x) \quad \begin{array}{l} \text{small effect} \\ \text{Similar to EMC effect} \end{array}$$

→ Need high lumi

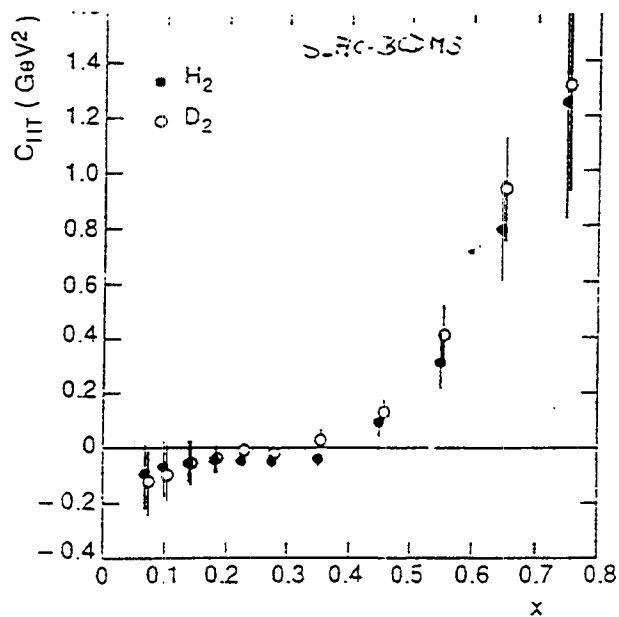
$$100 < \Lambda_{\overline{MS}} < 300 \text{ MeV}$$

(87)

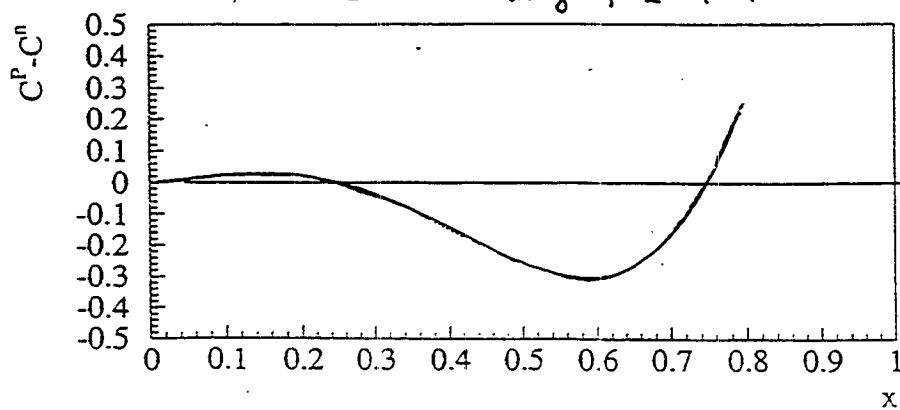
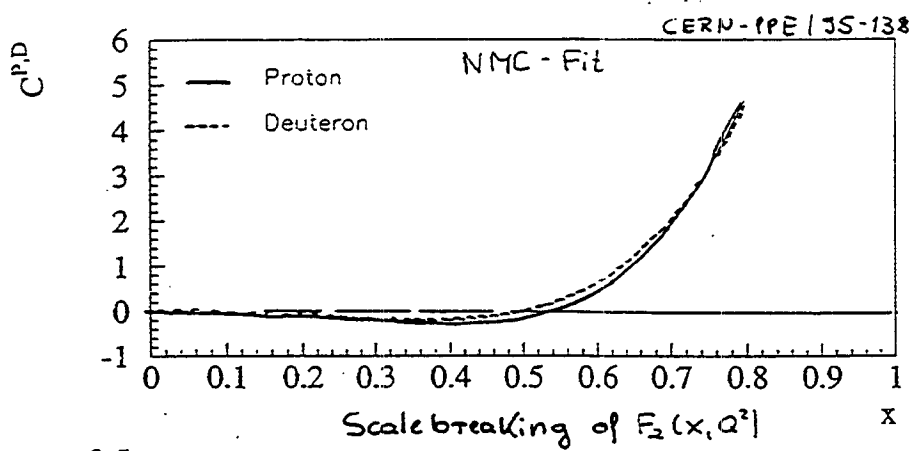
Structure function F_2 of proton.



Have to increase Q^2 range
to test QCD!
One measurement!

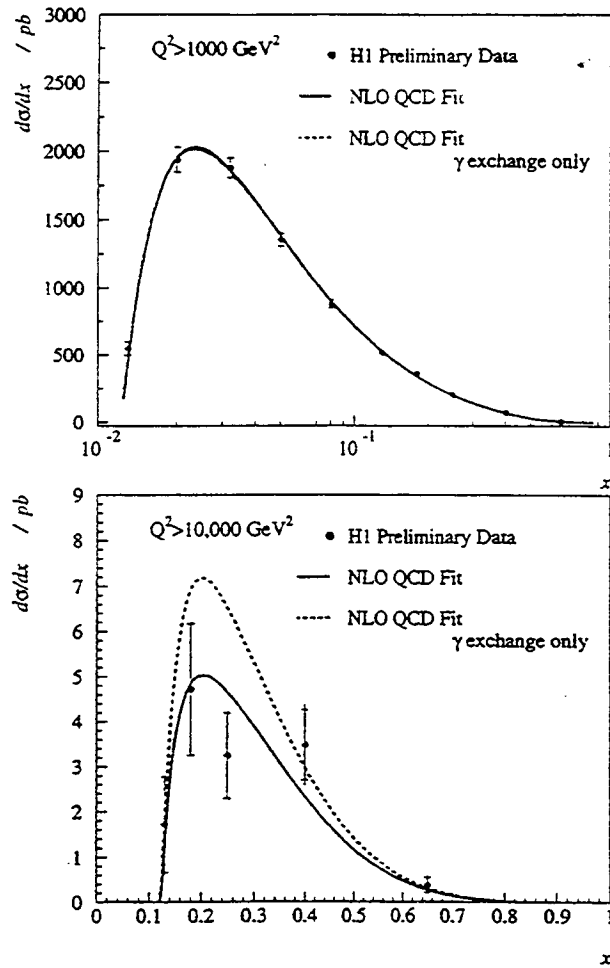


98

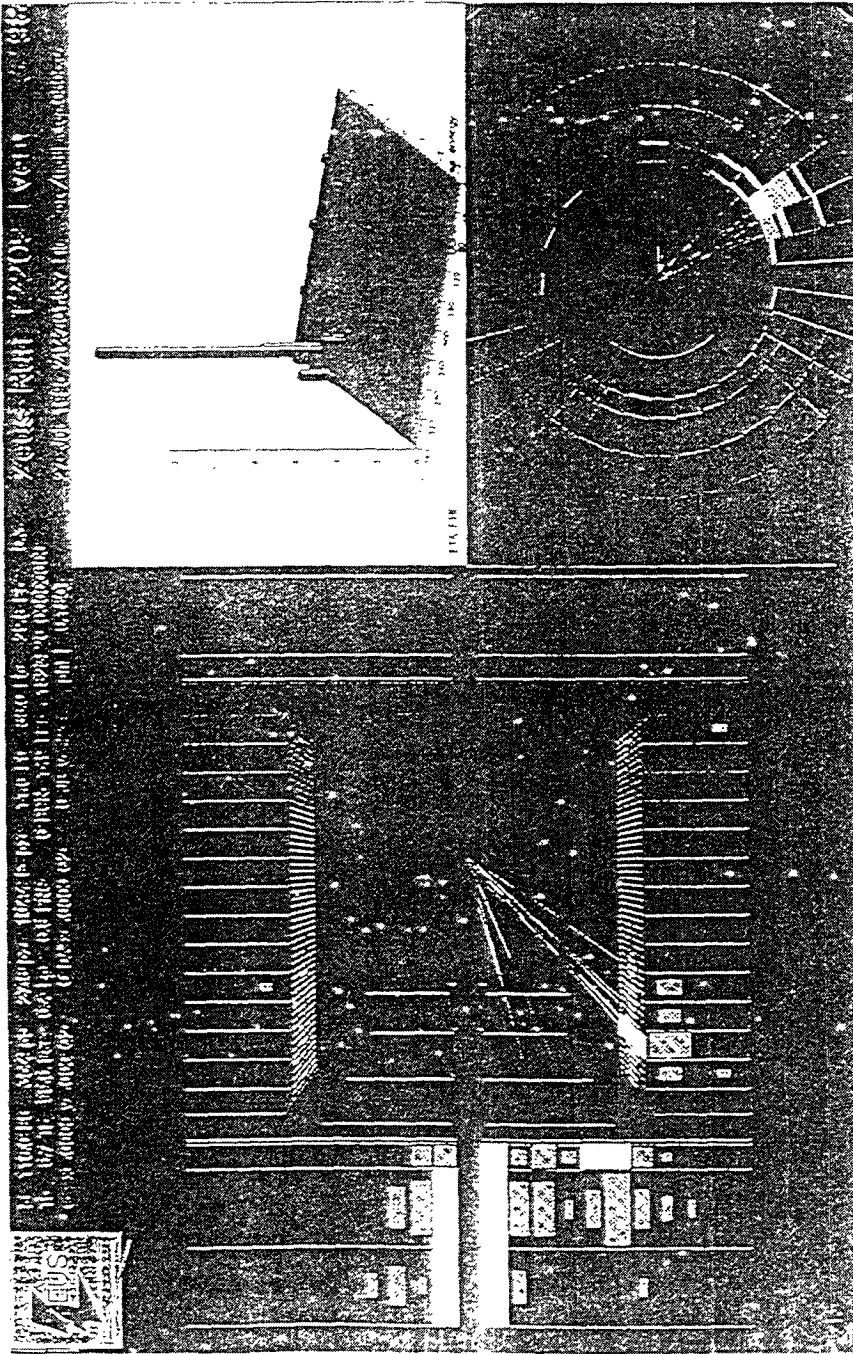


$d\sigma^{NC}/dx$ für $Q^2 > 1000$ und $> 10000 \text{ GeV}^2$

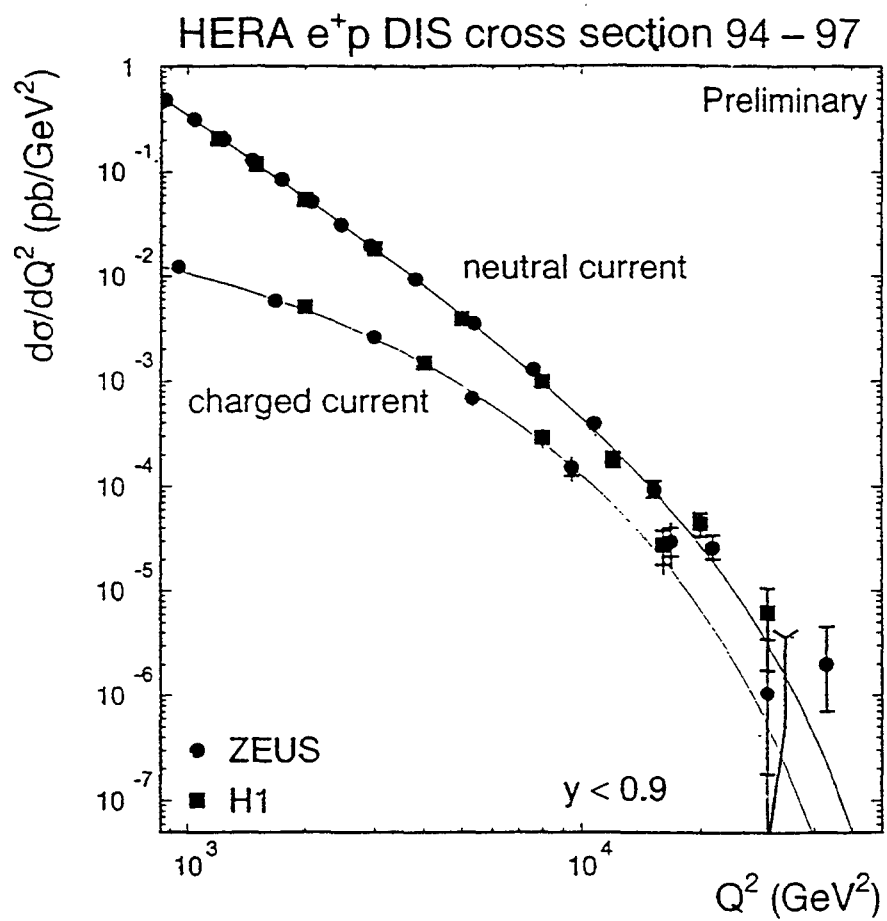
(89)



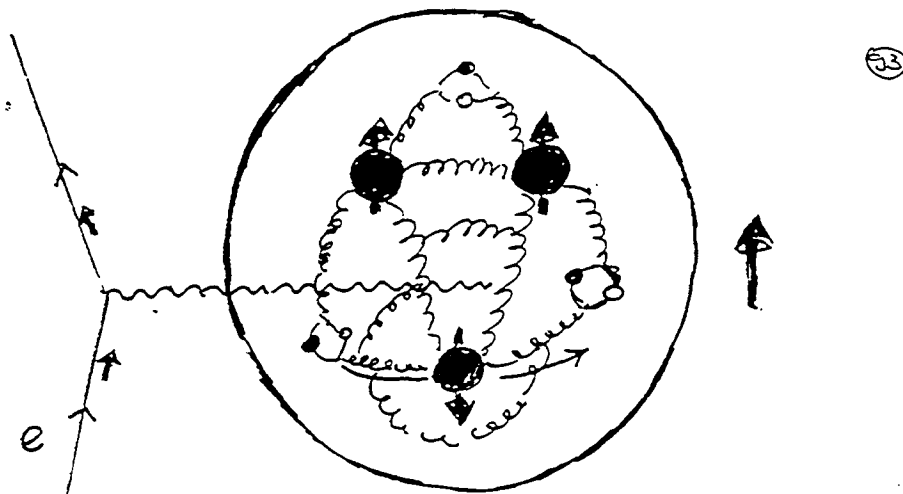
- Gute Beschreibung der Daten durch das Standardmodell mit destruktiver γ - Z^0 -Interferenz für e^+p .



94



④ Polarised DIS ⑤2
Spin of the nucleon



$$\boxed{\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_z^q + \Delta G + L_z^g}$$

↑
Quarks

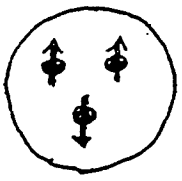
↑
Gluons
↑
orbital angular momentum

$$\begin{aligned} \Delta \Sigma &= \Delta u_v + \Delta d_v + 2(\Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s}) \\ &= 1 \text{ QPM} \\ &\approx 0.6 \text{ rel. QPM} \end{aligned}$$

$$\left. \begin{aligned} \Delta \Sigma &\approx 0.2 ? \\ \Delta S &\approx -0.1 ? \end{aligned} \right\} \text{EMC/SLAC: Spin-'Puzzle'}$$

Quark-Modell (1961) - SU(6)

(34)

↑  $p = |uud\rangle$ $e_u = \frac{2}{3}|e|$
 $n = |ddu\rangle$ $e_d = -\frac{1}{3}|e|$

$$|p^\uparrow\rangle = \frac{1}{\sqrt{18}} \left\{ 2|\overset{\uparrow}{u}\overset{\uparrow}{u}\overset{\downarrow}{d}\rangle - |\overset{\uparrow}{u}\overset{\downarrow}{u}\overset{\uparrow}{d}\rangle - |\overset{\downarrow}{u}\overset{\uparrow}{u}\overset{\uparrow}{d}\rangle + \dots \right\}$$

→ $u^{\uparrow\uparrow} = u^+ = \frac{5}{3}$; $u^{\uparrow\downarrow} = u^- = \frac{1}{3}$; $\Delta u = u^+ - u^- = \frac{4}{3}$
 $d^{\uparrow\uparrow} = d^+ = \frac{1}{3}$; $d^{\uparrow\downarrow} = d^- = \frac{2}{3}$; $\Delta d = d^+ - d^- = -\frac{1}{3}$

$$\langle \mu^p \rangle = \frac{1}{3} \{ 4 \langle \mu^u \rangle - \langle \mu^d \rangle \}$$

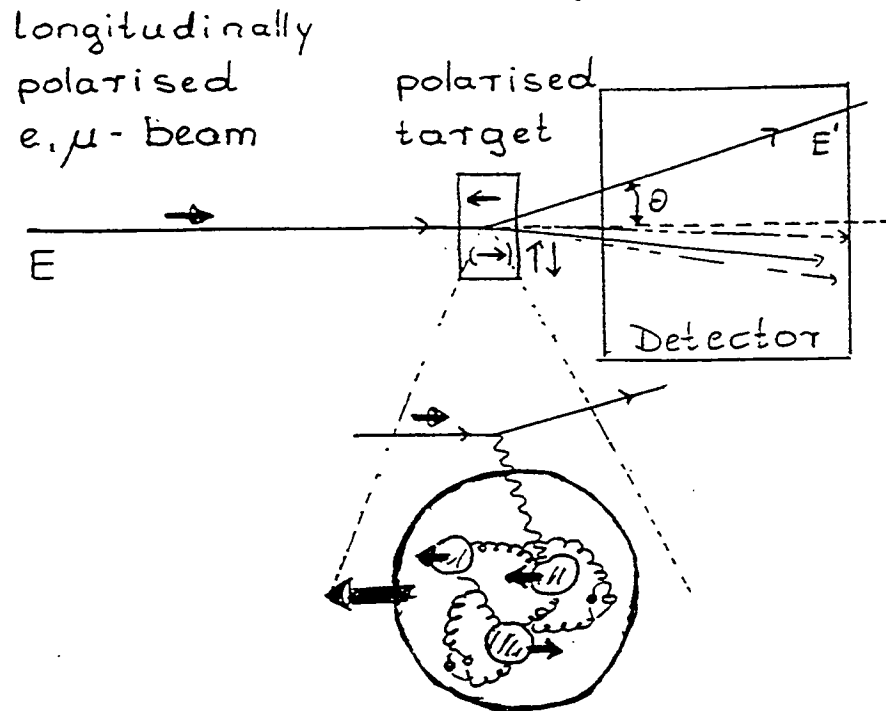
$$\langle \mu^n \rangle = \frac{1}{3} \{ 4 \langle \mu^d \rangle - \langle \mu^u \rangle \}$$

$$m_u = m_d = m_q$$

→ $\frac{\langle \mu^n \rangle}{\langle \mu^p \rangle} = -\frac{2}{3} \approx -\frac{1.91}{2.73} ; m_q \approx \frac{M_p}{2.73}$

100L: Polarised deep inelastic
lepton-nucleon scattering

35



Measure probability for quark-spin
and nucleon-spin $\uparrow\uparrow$ or $\downarrow\uparrow$



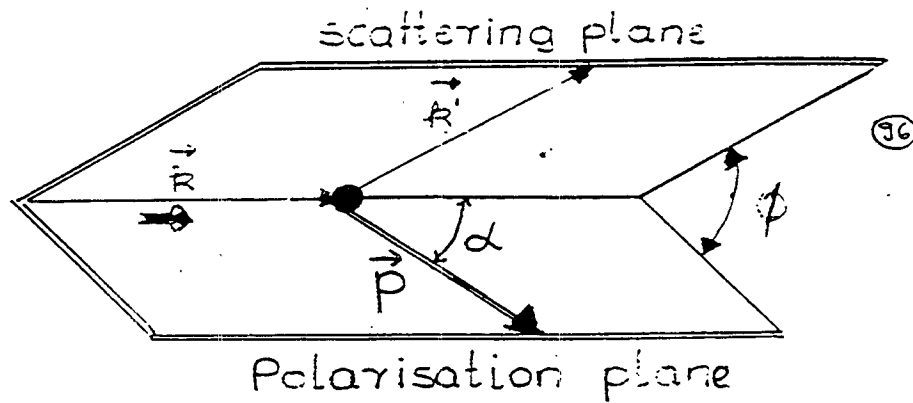
Spin-Structure Function $\boxed{g_1^{p,n}(x, Q^2)}$

In addition:

$$g_2^{p,n}(x, Q^2), b_1^d, \Delta^d, h_1,$$

$$\Delta V, \Delta S, \Delta G, L_2$$

Polarised lepton nucleon scattering



$$\left. \frac{d^3 \sigma(\alpha)}{dx dy d\phi} \right|_{(e)} = \frac{d^3 \bar{\sigma}}{dx dy d\phi} \begin{pmatrix} + \\ - \end{pmatrix} \frac{d^3 \sigma^*(\alpha)}{dx dy d\phi}$$

$\uparrow$ $\uparrow$
 $\sim f(E_1, E_2)$ $\sim f(g_1, g_2)$

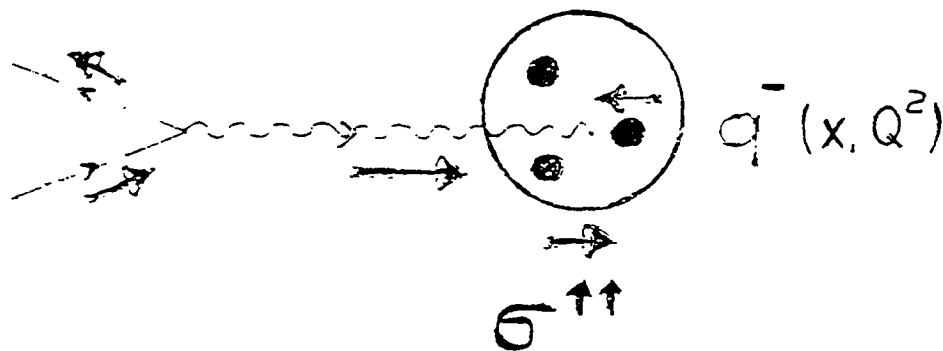
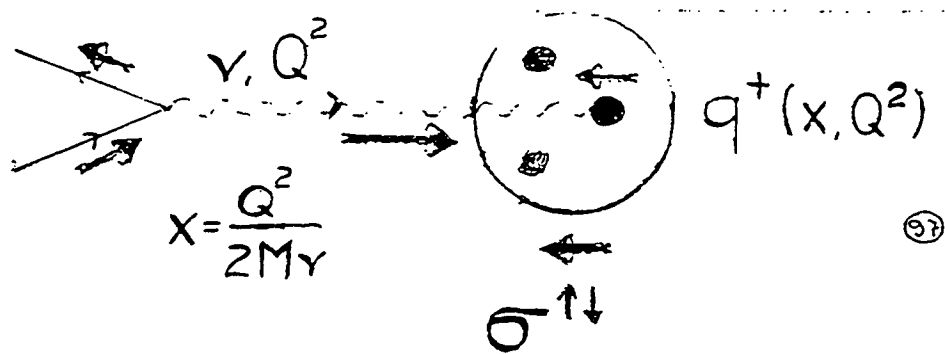
$$\frac{d^3 \sigma^*(\alpha)}{dx dy d\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \cos \alpha \cdot [a \cdot g_1(x) + b g_2(x)] - \cos \phi \sin \alpha \cdot c \left[\frac{Y}{2} g_1(x) + g_2(x) \right] \right\}$$

$$a \gg b \quad \left(a = 1 - \frac{Y}{2} - \frac{Y^2}{4}(\kappa - 1); b = -\frac{Y}{2}(\kappa - 1), c = \sqrt{(\kappa - 1)\alpha} \right)$$

$\alpha = 0^\circ$: measure dominantly $g_1(x)$

$\alpha = 90^\circ$: " both $g_1(x), g_2(x)$

$$\text{Asymmetries: } A = \frac{\sigma(\alpha + \pi) - \sigma(\alpha)}{\sigma(\alpha + \pi) + \sigma(\alpha)}$$



$$\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow} \sim \boxed{g_1(x) = \frac{1}{2} \sum_f e_f^2 \{q_f^+(x, Q^2) - q_f^-(x, Q^2)\}}$$

$$\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow} \sim F_1(x) = \dots$$

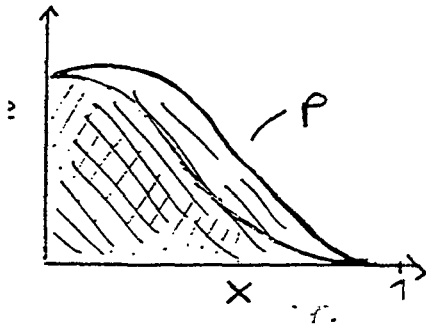
$$\frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} \sim \boxed{R_1(x) \approx \frac{g_1(x)}{F_1(x)}} \quad \begin{aligned} (R_{11} &= D(R_1 + \eta f) \\ (2 \times F_1 &= F_2 / (1+i) \end{aligned}$$

$$\Delta q_f = \int_0^1 \{q_f^+(x) - q_f^-(x)\} dx$$

$$\Delta q_f \cdot S_\mu = \langle P, S | \bar{q}_f \gamma_\mu \gamma_5 q_f | P, S \rangle \frac{1}{2M}$$

Integrals

a) un-polarised

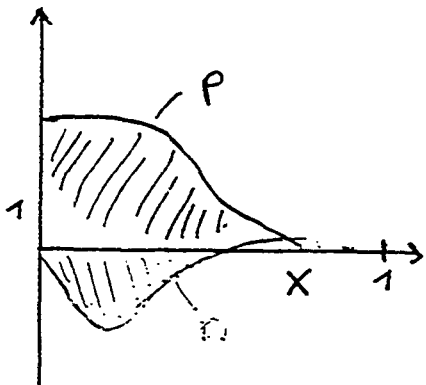


$$F_2(x) = \sum_f e_f^2 x (q_f(x) + \bar{q}_f(x))$$

$$\Gamma_2 = \int_0^1 F_2(x) dx \quad (95)$$

- $\Gamma_2^P + \Gamma_2^N \sim$ quark-momenta
only ~ 50% of N-momentum
- $\Gamma_1^P - \Gamma_1^N \approx \frac{1}{3}$ (≈ 0.24)
 $\sim$ quark charges: $\bar{u} \neq \bar{d}$

b) polarised



$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x)$$

$$I_1 = \int_0^1 g_1(x) dx$$

- $I_1^P + I_1^N \sim$ quark-spins
 $\sim$ quarks contribute little to N-spin
- $I_1^P - I_1^N$ (Bj-SR)
 $\sim$ axial charge; g_a

But: Only global informations

Detailed test of QCD from

$$F_2(x, Q^2)$$

(last ~25 years)

Sum Rules:

① Bjorken (66) Fundamental! ☹

$$\int_0^1 dx (g_1^p(x) - g_1^n(x)) = \frac{1}{6} \left| \frac{g_R}{g_V} \right|_{np} * \text{QCD Korr.}$$

$$= 0.191 \pm 0.003$$

g_R, g_V : weak coupl. const. | from Gamow-Teller β -decay

② Ellis-Jaffe - SU(3)

$$\int_0^1 dx g_1^{p,(n)}(x) = \frac{1}{12} \left| \frac{g_R}{g_V} \right|_{np} \left\{ \begin{matrix} + \\ (-) \end{matrix} 1 + \frac{5}{3} \cdot \frac{3F-D}{F+D} \right\} * \text{QCD Korr.}$$

$\downarrow \Delta S=0, F/D = 0.579$
 $\begin{matrix} p \\ n \end{matrix} \begin{matrix} = 0.175 \pm 0.007 \\ (-) = -0.016 \pm \dots \end{matrix}$

F, D : SU(3) coupling const.; baryon decays:

np : $F+D$; Λp : $F+\frac{1}{3}D$; $\Sigma \Lambda$: $F-\frac{1}{3}D$

Sum Rules

$$\Gamma_1^{p,n}(Q_0^2) \equiv \int_0^1 g_1(x, Q_0^2) dx$$

(100)

(via QCD fit

- Expt:
- Evolve data from $\bar{Q}_m^2(x) \rightarrow Q_0^2$
 - Extrapolations: $\int_0^{x_{\min}}$, $\int_{x_{\max}}^1$
- $\uparrow$ big? $\uparrow$ small

Theorie:

$\alpha_s(Q^2)$ - Corrections:

(Larin, Phys. Lett. B334 (1994)1

(Kataev, Starshenko, Mod. Phys. Lett

A10 (1995)

Known to $O(\alpha_s^3)$, estimates $O(\alpha_s^4)$

$$\Gamma_1^{p,n}(Q^2) = \frac{1}{12} \left(\left(\frac{1}{2} a_3 + \frac{1}{3} a_8 \right) C^{NS}(Q^2) + \frac{1}{9} a_0 C^S(Q^2) \right)$$

$$a_3 = g_R/g_V = F+D = (\Delta u - \Delta d) \quad n\text{-decay}$$

$$a_8 = (3F-D) = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon-decay}$$

$$a_0 = \Delta \Sigma = (\Delta u + \Delta d + \Delta s)$$

$$g_R/g_V = 1.2573 \pm 0.0028$$

$$3F-D = 0.579 \pm 0.025$$

$\alpha_s(Q^2)$ CORRECTIONS

LAREN Phys. Lett. B334(1994)192-198

(101)

→ Bjorken Sum Rule

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \frac{g_a}{g_v} \left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 \right]$$

→ Ellis-Jaffe Sum Rules

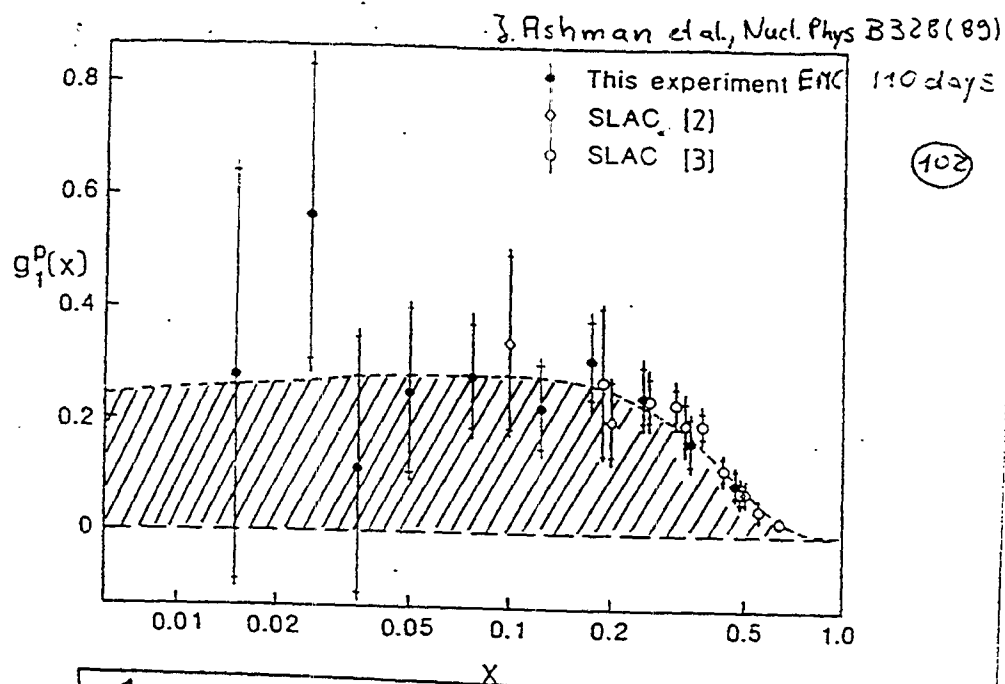
$$\begin{aligned} \Gamma_1^{p(n)} &= \Gamma_1^{NS} + \Gamma_1^S \\ &= \frac{1}{12} \left(+(-)a_3 + \frac{a_8}{\sqrt{3}} \right) \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s}{\pi} \right)^3 \right] \\ &\quad + \frac{1}{9} \left[1 - \frac{\alpha_s}{\pi} - 0.55 \left(\frac{\alpha_s}{\pi} \right)^2 \right] \Delta\Sigma \end{aligned}$$

$$a_3 = \frac{g_a}{g_v} = F + D = (\Delta u - \Delta d) \quad \text{neutron decay}$$

$$a_8 = \frac{3F + D}{\sqrt{3}} = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon decay}$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

+ higher twist corrections



(102)

$$\int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015$$

$$\delta A_{\chi^2} = \frac{1}{\sqrt{N_H + N_H}} \left(f_N \cdot P^B \cdot P^T \cdot D \right)^{-1}$$

0.15 0.8 0.8 0.2-0.7

$$f(\text{NH}_3) = \frac{3}{17} ;$$

$$f(\text{C}_4\text{H}_9\text{OH}) = \frac{10}{74}$$

But:

$$f(\text{H}) = f(\text{D}) = 1 !$$

$$\delta g_1 \sim \delta A_{\chi^2} F_1 \sim \delta A_{\chi^2} \cdot F_2 / x !$$

Consequences from $\Gamma_1' = 0.126 \pm 0.018$

(LO - QCD)

(103)

$$\Gamma_{1 \text{ exp}}^P = \Gamma_{1 \text{ EJ}}^P + \frac{1}{3} C_2 \Delta S$$

$$0.126 = 0.175 + \frac{1}{3} \cdot 0.997 \Delta S$$

QPM

$$\Rightarrow \boxed{\Delta S = -0.147}$$

0

$$1) \Delta u - \Delta d = 1.2572 \pm \dots$$

$$2) \Delta u + \Delta d - 2\Delta S = 0.5796 \pm \dots$$

$$3) \Delta S = -0.147 \pm$$

$$\leadsto \boxed{\Delta u = 0.771 \pm \dots}$$

$\frac{4}{3}$

$$\boxed{\Delta d = -0.486 \pm \dots}$$

$-\frac{2}{3}$

$$\blacktriangleright \boxed{\Delta \Sigma = \Delta u + \Delta d + \Delta S = 0.138 \pm 0.170} \quad ? \quad 1$$

'Spin puzzle'

■ Very difficult experiments, tiny asymmetries
tremendous effort (104)

■ Fantastic progress in very different
new technologies:

● Polarized (strained GaAs) e^- sources with
 $p_e \sim 85\%$ (E-143, E-154)

● Long. beam polarization (Sokolov-Ternov)
in HERA with $p_e \rightarrow 70\%$

● Cryogenic polarized targets ($\text{NH}_3^\uparrow, \text{C}_4\text{H}_9\text{OH}^\uparrow, \text{h}$
(SMC, E143, TJNAL, ELSA, ...) highly developed

● Polarized ^3He gas targets $\boxed{+ \text{Li}^\uparrow \text{D}^\uparrow}$
▲ Spin exchange with optically pumped $\text{Rb}^\uparrow$
@ 10 atm (E142, E-154) $\uparrow$

high power laser diode

▲ metastability exchange (HERMES)

● Internal storage cell target works perfectly
(HERMES)

● Atomic Beam Source for $\text{H}^\uparrow, \text{D}^\uparrow$ h.d. (HERMES)

● Laser Driven Source for $\text{H}^\uparrow, \text{D}^\uparrow$ coming

● Polarimetry

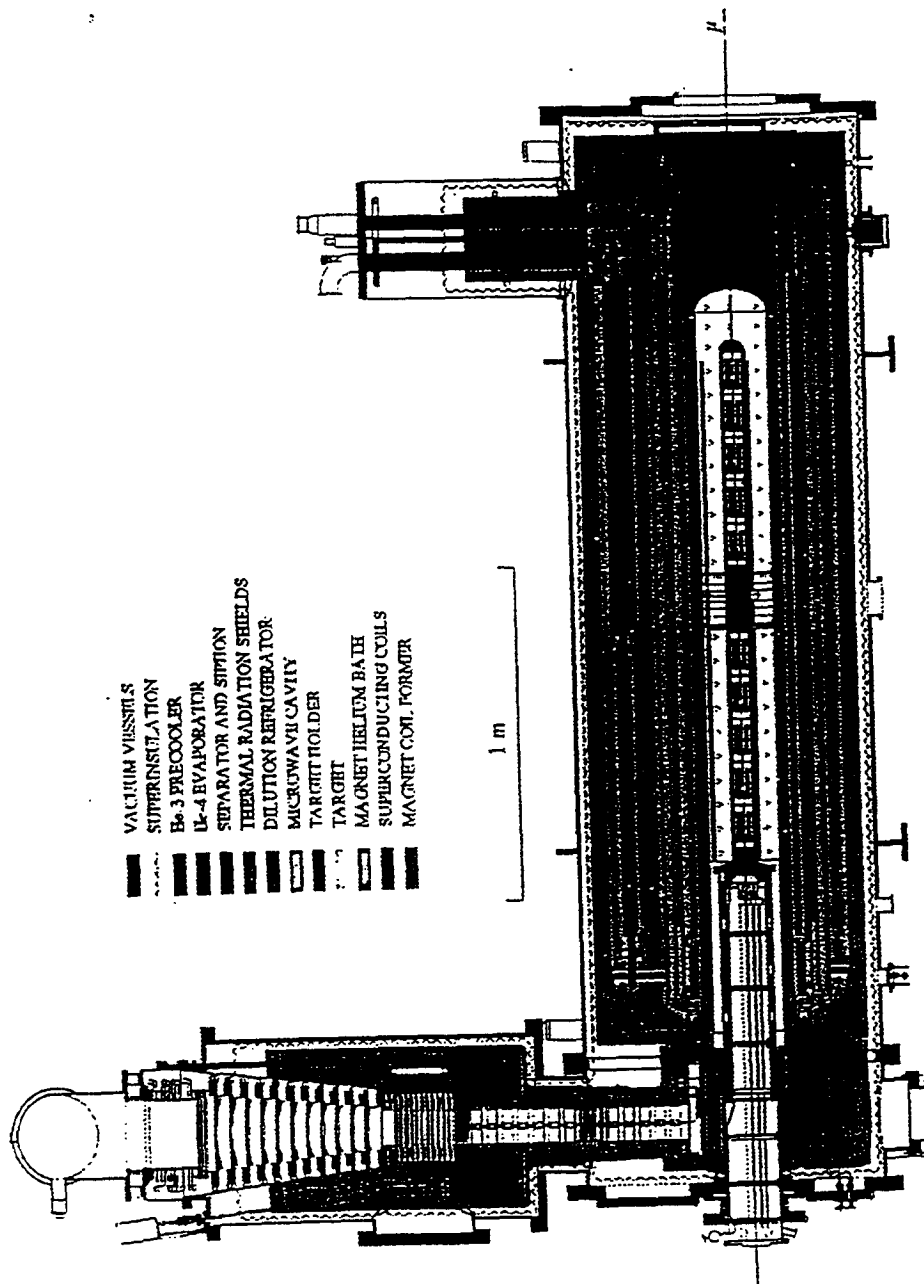
●
●
●
●

Spin experiments		P_T	f
SMC/CERN	$C_4 H_3^{\uparrow} OH^{\uparrow}$	~ 0.85	$\sim 1/3$
	$C_4 D_3^{\uparrow} OD^{\uparrow}$	< 0.5	$\sim 1/3$
	$NH_3^{\uparrow}$	0.89	$\sim 1/3$
π -decay in flight	$\mu^{\uparrow} \xrightarrow{P_B \sim 0.8} \left[\leftarrow \right] \left[\rightarrow \right]$ $\bar{I} \sim \frac{1}{2} \mu A$ $E = 190 \text{ GeV}$	$T \sim 4 \cdot 10^{25} \text{ cm}^{-2}$	(10%)

E-142 / E-154 / SLAC		$^3He^{\uparrow} (10 \text{ atm})$ (optically pumped Rb)	$\sim .33$ $\sim 1/3$ $\sim .38$ $\sim 1/3$
Ga-As strained "	$e^{\uparrow} \xrightarrow{P_B \sim 0.4, \bar{I} \sim 10 \mu A} \left[\leftarrow \right] \left[\rightarrow \right]$ $P_B \sim 0.8, \bar{I} \sim 100 nA$ $E = 26/48 \text{ GeV}$	$T \sim 3 \cdot 10^{22} \text{ cm}^{-2}$	

E-143 / E-155 / SLAC		$^{15}NH_3$ $^{15}ND_3$ LiD	< 0.8 $\sim 1/3$ < 0.3 $\sim 1/3$ ~ 0.5 $\sim 1/3$
Strained Ga-As	$e^{\uparrow} \xrightarrow{P_B \sim 0.8, \bar{I} \sim 80 nA} \left[\leftarrow \right]$ $E = 29/48 \text{ GeV}$	$T \sim 10^{24} \text{ cm}^{-2}$	

HERMES / DESY		$H^{\uparrow}, D^{\uparrow}$ $^3He^{\uparrow}$ (metastability exch.)	~ 0.9 1 ~ 0.5 $1/3$
HERA (Sokolov-Ternov)	$e^{\uparrow} \xrightarrow{P_B = 0.5-0.7} \left[\leftarrow \right] \left[\rightarrow \right]$ $\bar{I} \sim 20 \text{ mA}$ $E = 28 \text{ GeV}$	$T = (1 \rightarrow 10) \cdot 10^{14} \text{ cm}^{-2}$	



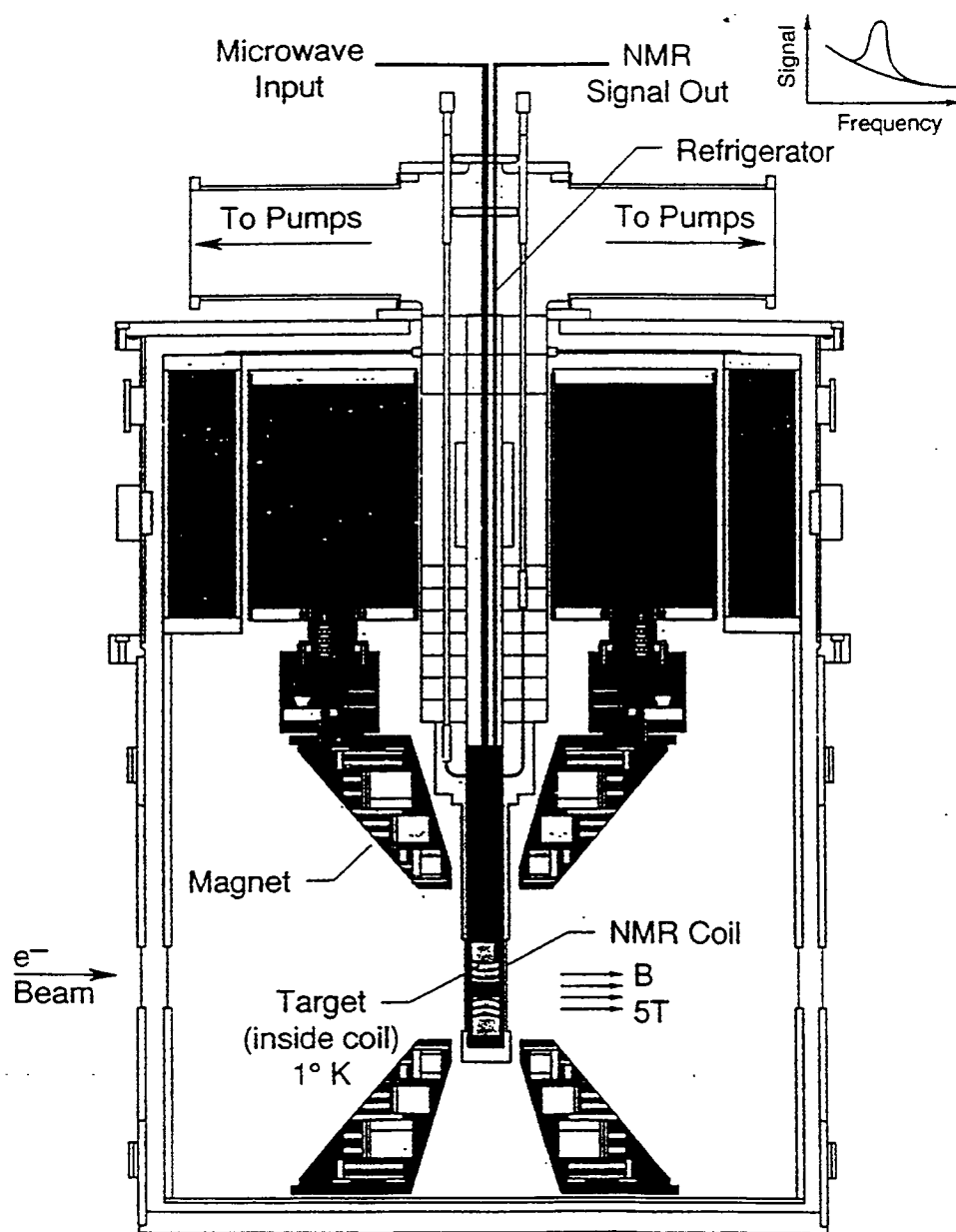
- VACUUM VESSELS
- SUPERINSULATION
- He-3 PRECOOLER
- U-4 EVAPORATOR
- SEPARATOR AND SHIM
- THERMAL RADIATION SHIELDS
- DILUTION REFRIGERATOR
- MICROWAVE CAVITY
- TARGET HOLDER
- TARGET
- MAGNET HELIUM BATH
- SUPERCONDUCTING COILS
- MAGNET COIL FORMER

1 m

102

File 1004
Not listed
Classified

(107)

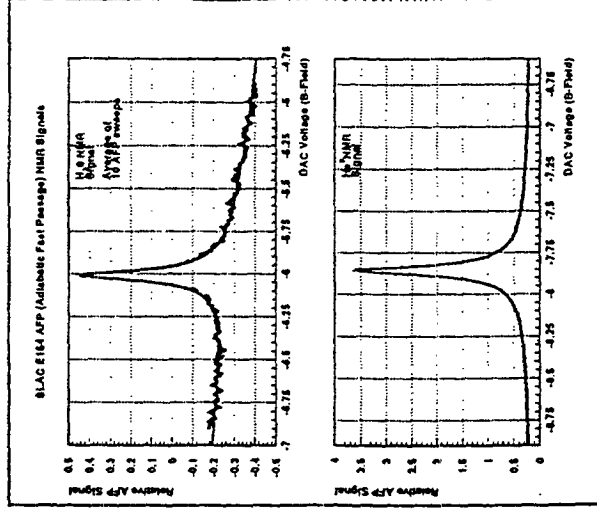
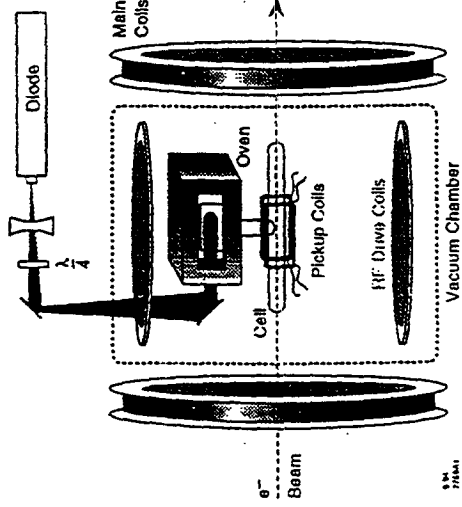


4-94

7656A1

E154 ^3He Target Assembly & NMR

- 4 Ti:Sapphire Laser + 3 Laser Diode Arrays,
- 30 cm long target vessel, 50 μm windows,
- ~10 Amagat ^3He density.

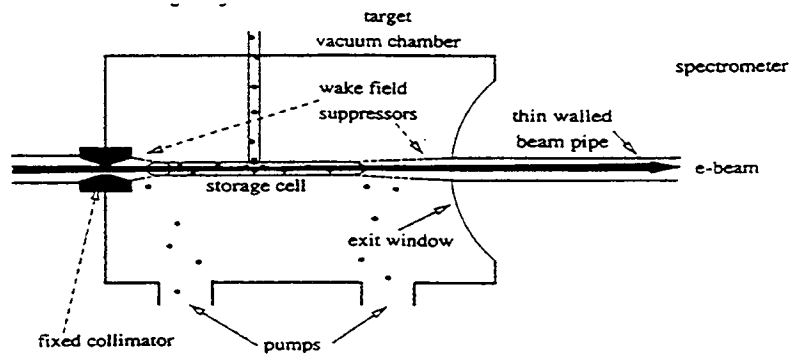


NIM A366, 148-162 (1995)

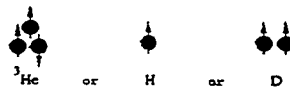
The HERMES target



- Internal storage cell:



- undiluted target atoms: $\vec{H}$, $\vec{D}$, $\vec{{}^3\text{He}}$



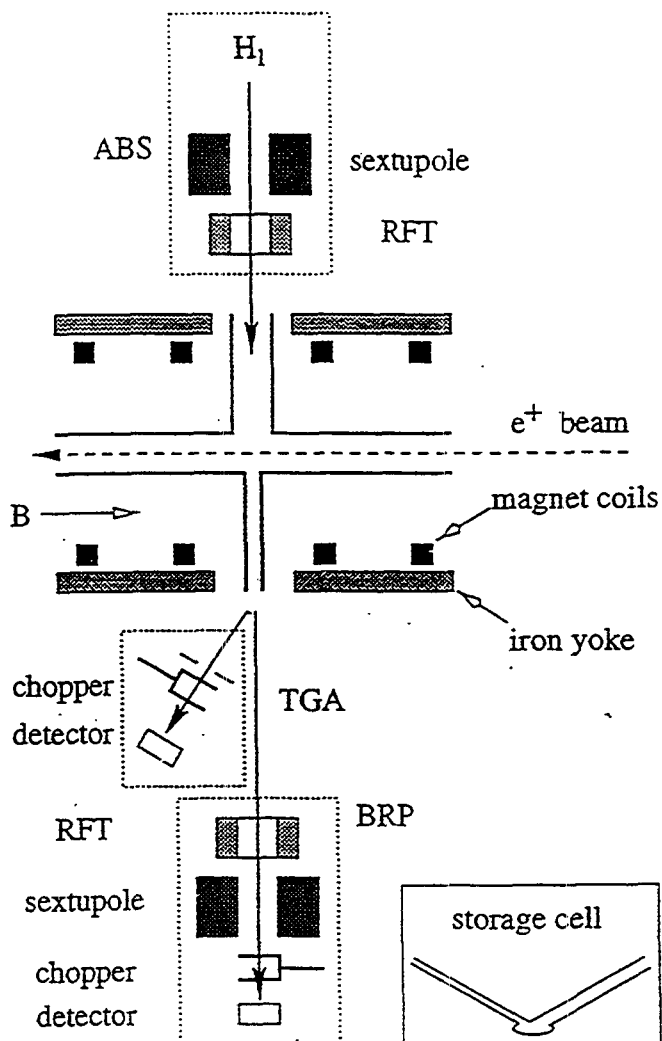
- Gas targets:

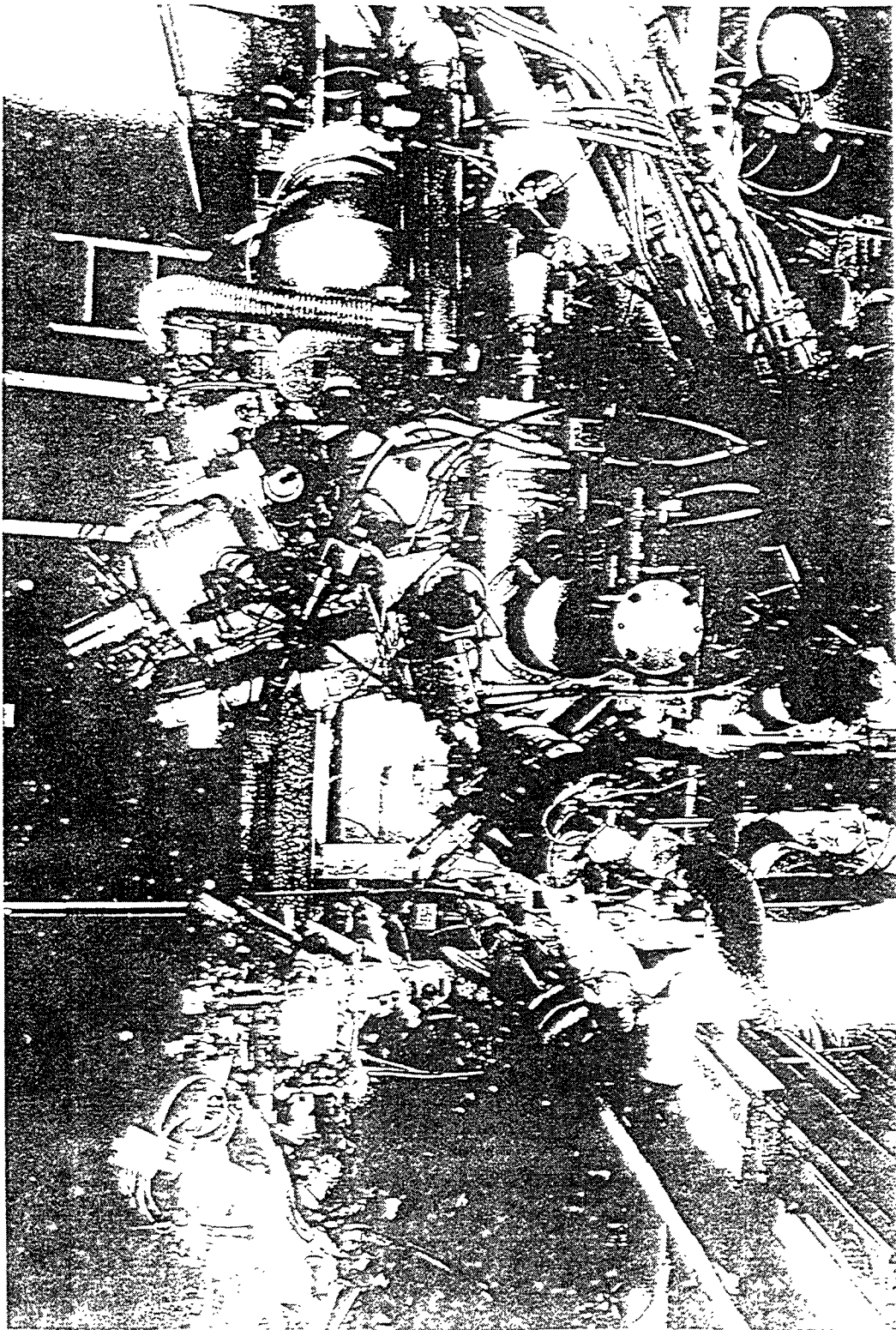
- Laser driven source: $\vec{{}^3\text{He}}$
 $\rho = 1 \cdot 10^{15} \text{ N/cm}^2$ $\Delta t_{flip} \sim 10 \text{ min}$ $\overline{P_T} = 46\%$

%

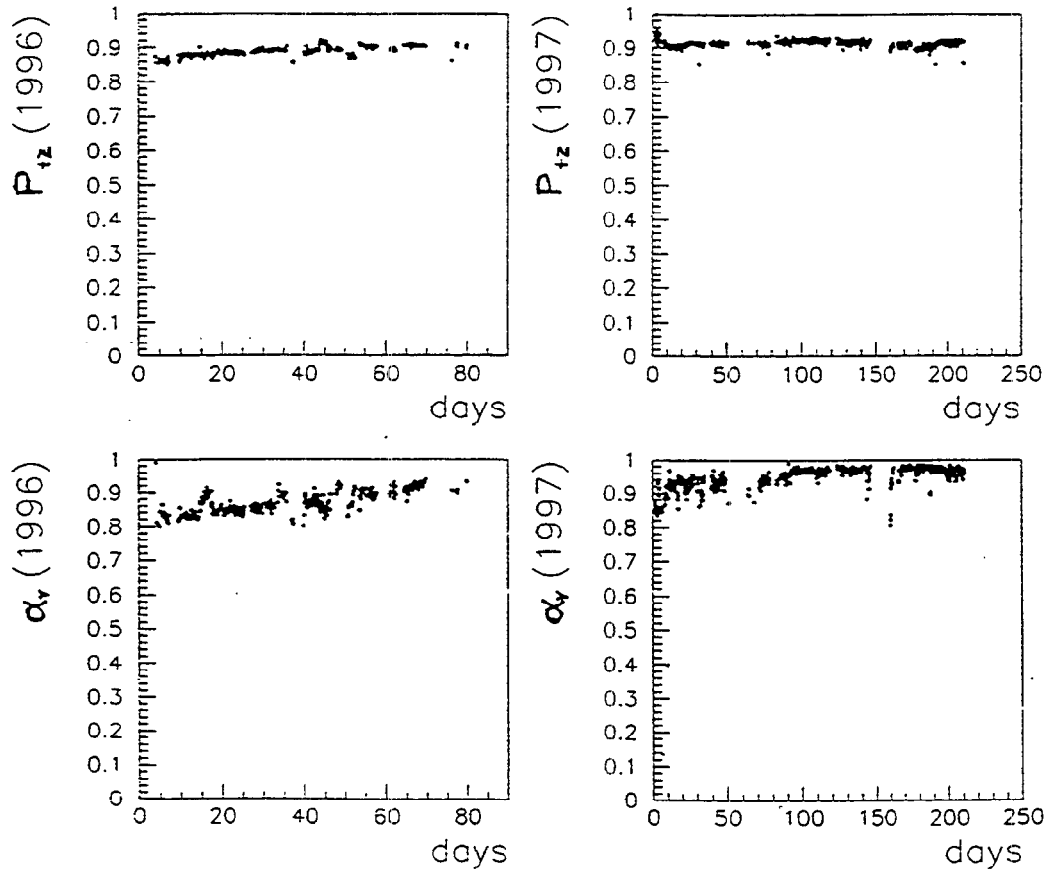
- Atomic beam source: $\vec{H}$
 $\rho = 7 \cdot 10^{13} \text{ N/cm}^2$ $\Delta t_{flip} \sim 1 \text{ min}$ $\overline{P_T} = 92\%$

- Unpolarized gasses: ${}^1\text{H}$, ${}^2\text{D}$, ${}^3\text{He}$ and ${}^{14}\text{N}$.





HERMES Hydrogen target performance, 1996-97

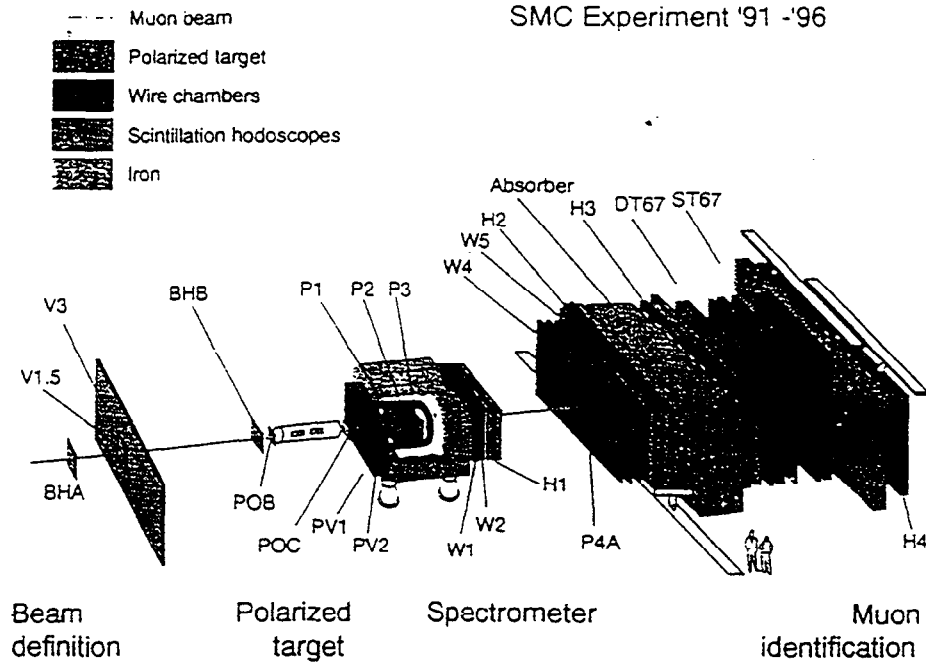


$$\alpha = \text{degree of dissociation}$$

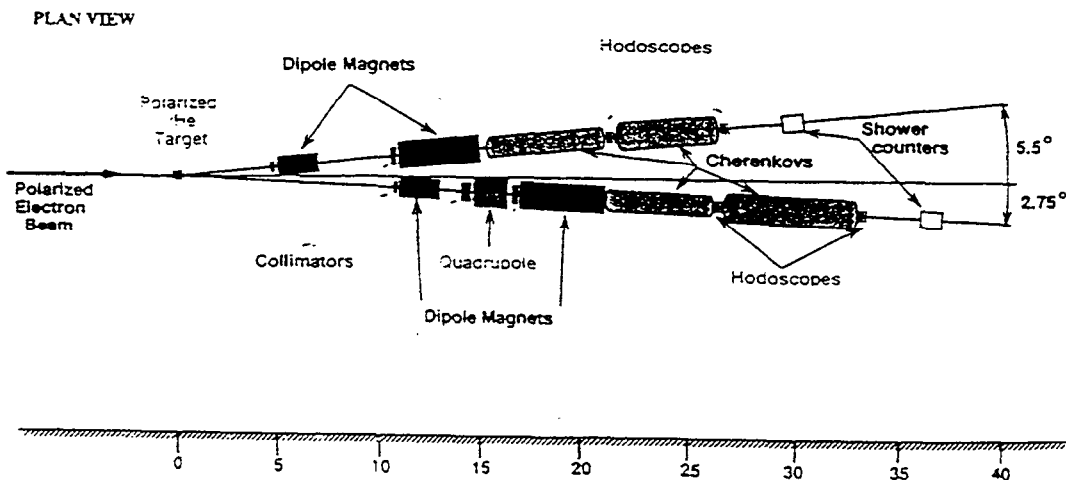
$$= \frac{N_H}{N_H + 2N_{H_2}}$$

113

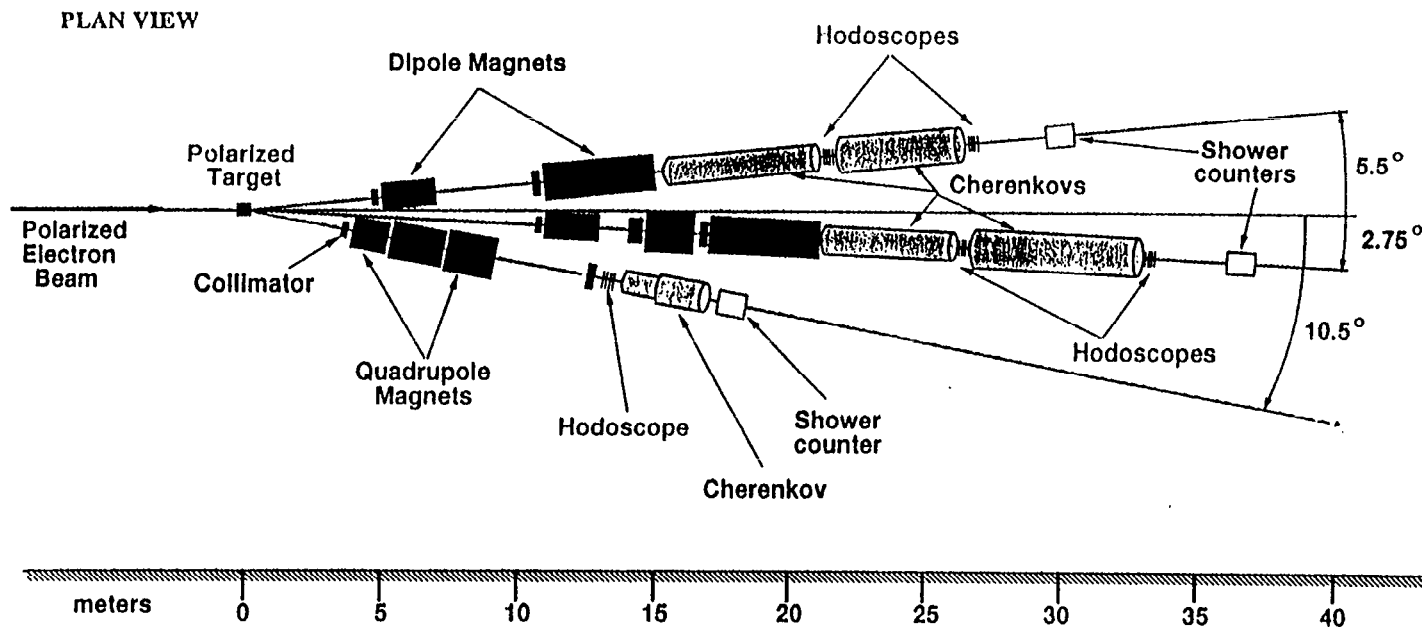
SMC Experiment '91 -'96



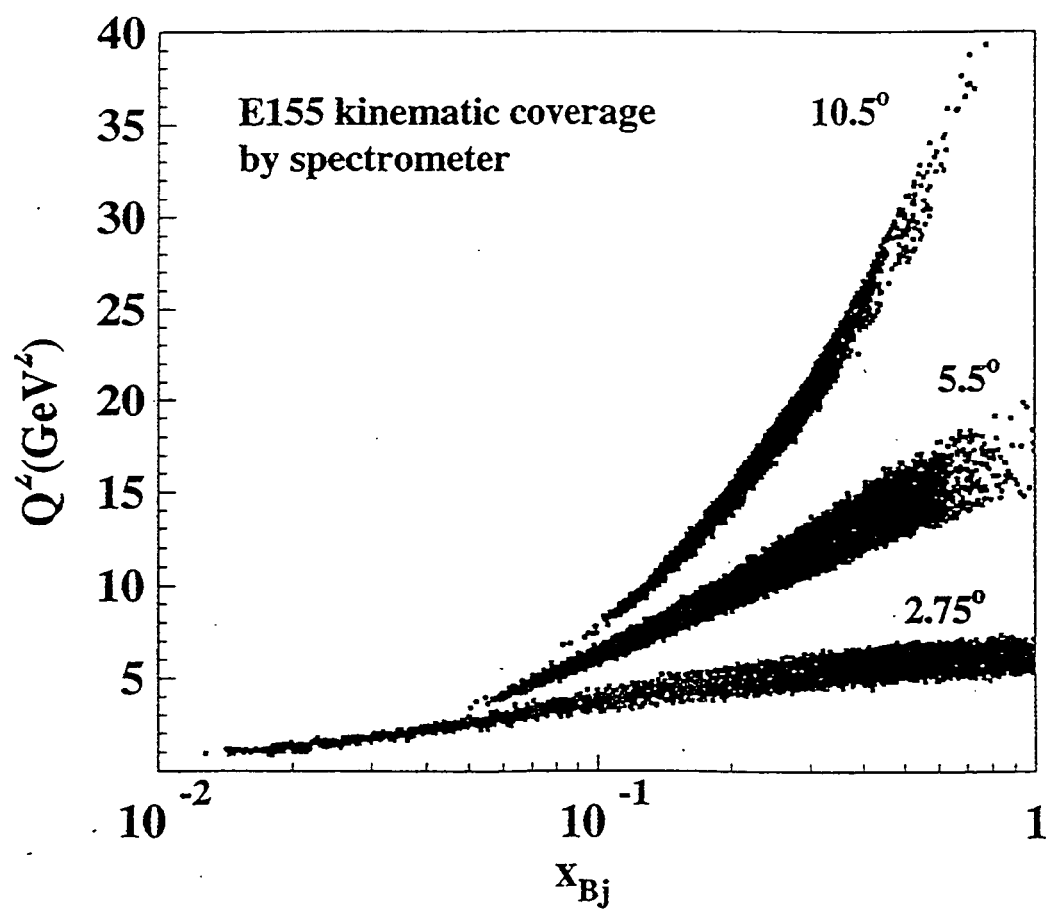
E154 Setup

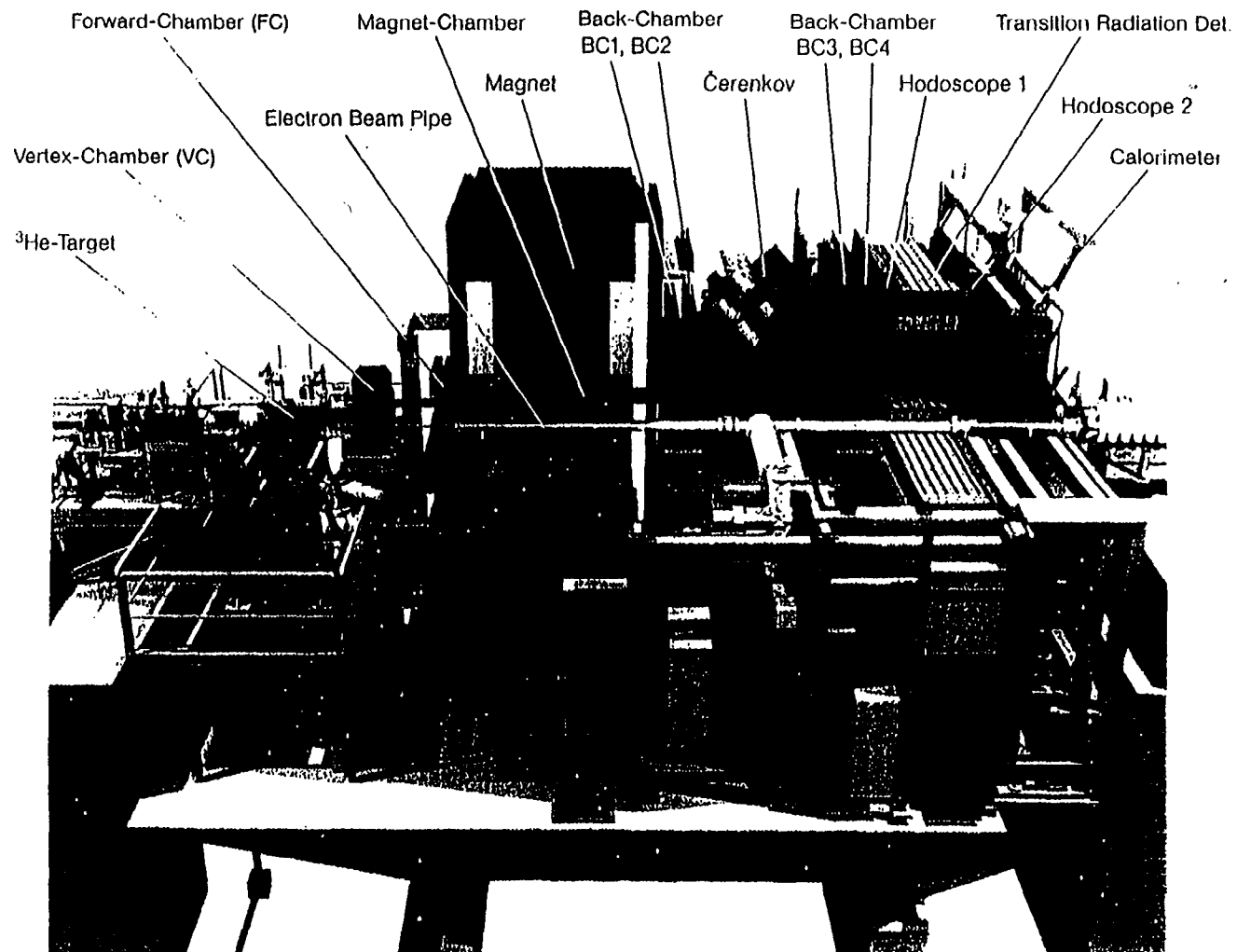


E155 Spectrometers



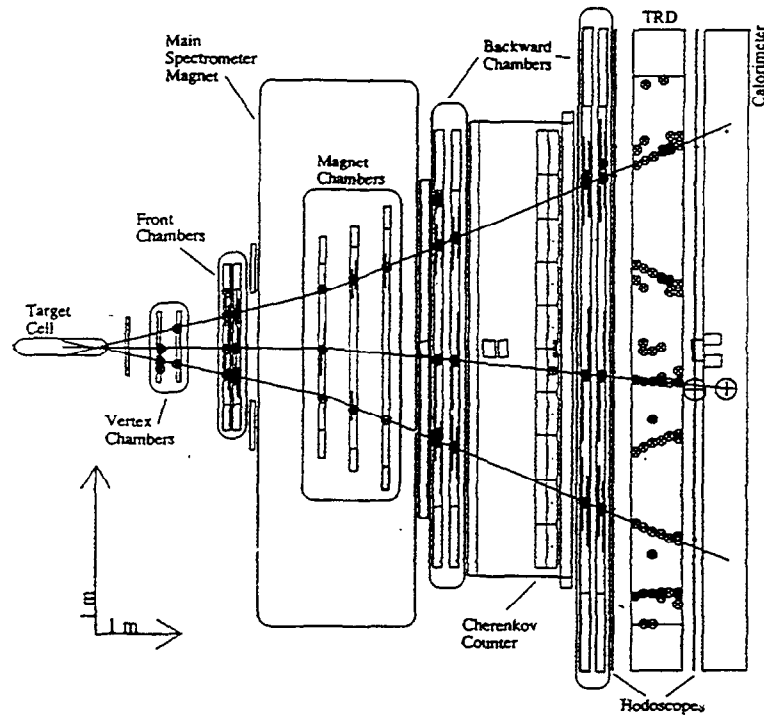
11S



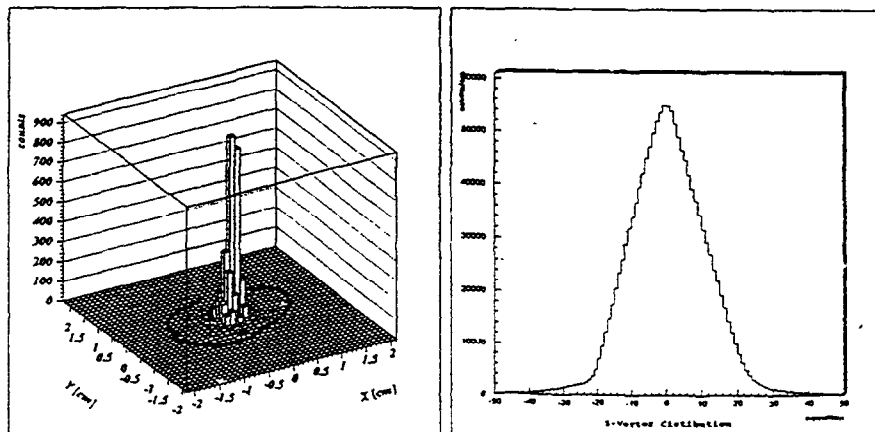


416

Event Reconstruction



transverse and longitudinal vertex distribution:

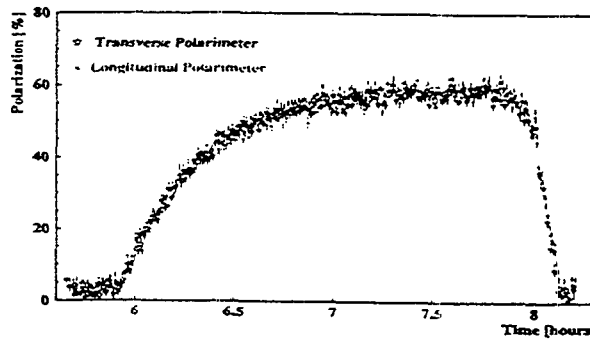


no background from wall scattering!

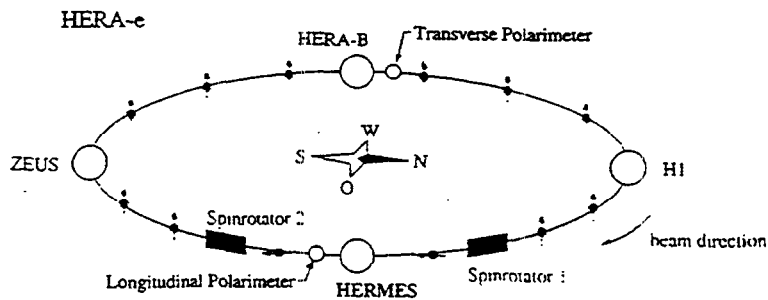
Beam polarisation and polarimetry

Transverse polarisation: through emission of synchrotron radiation in the curved sections ('Sokolov-Ternov effect'):

$$P(t) = P_{max} \cdot (1 - e^{-t/\tau})$$



Longitudinal polarisation: achieved by a 90° spin rotation in the region of the HERMES experiment:



annual mean: $\overline{P_B} = .55$

$$\sigma_{P_B} = 5.4\% ('95)$$

$$\sigma_{P_B} = 4.1\% ('96, '97)$$

SMC

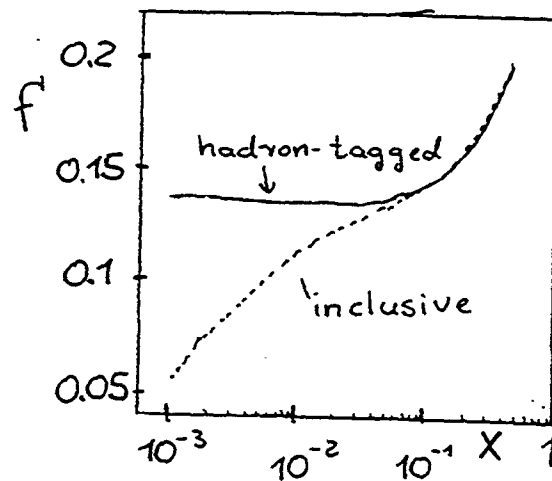
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- Final data sets - CERN-EP/98-85
- Improved beam polarisation

$$\rightarrow R_{x, \text{new}} = 0.965 R_{\text{old}}$$

- Hadron tagging

- $\rightarrow$ eliminate elastic contribution to DIS rate
- $\rightarrow$ reduced radiative corrections
- $\rightarrow$ smaller dilution factor at low x
- $\rightarrow$ extend low x , low Q^2 range

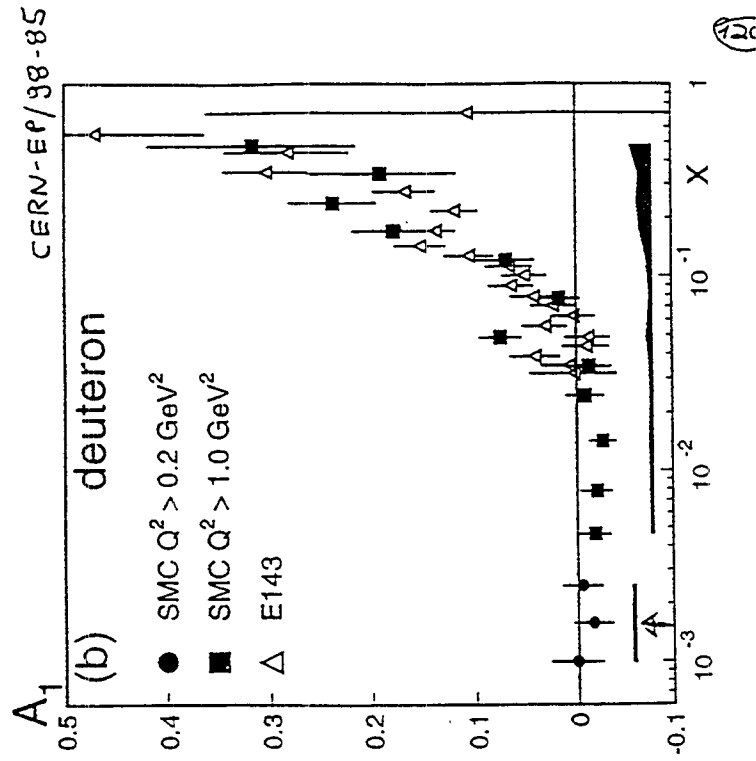
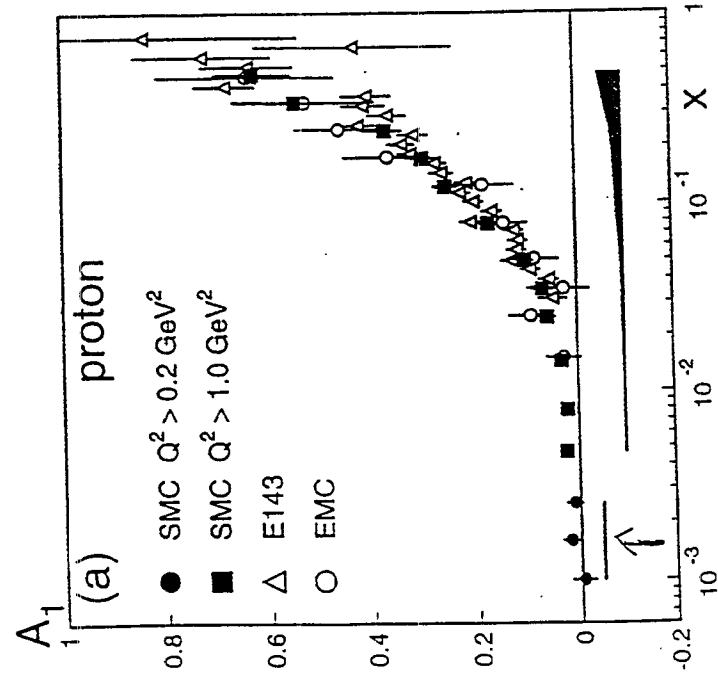


- Hadrons: flavor decomposition $\Delta\zeta$
- QCD analysis CERN-EP/98-86

SMC - updated, final results

* Normalisation change (p^3): -3,5%

* low x , only events with hadrons $\rightarrow$ reduced rad. corr.,
 $\rightarrow$ smaller dilution



SLAC - E155

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- High statistics (200 M events)
inclusive results from
 $\vec{N}\vec{H}_3$, $\vec{Li}\vec{D}$ targets

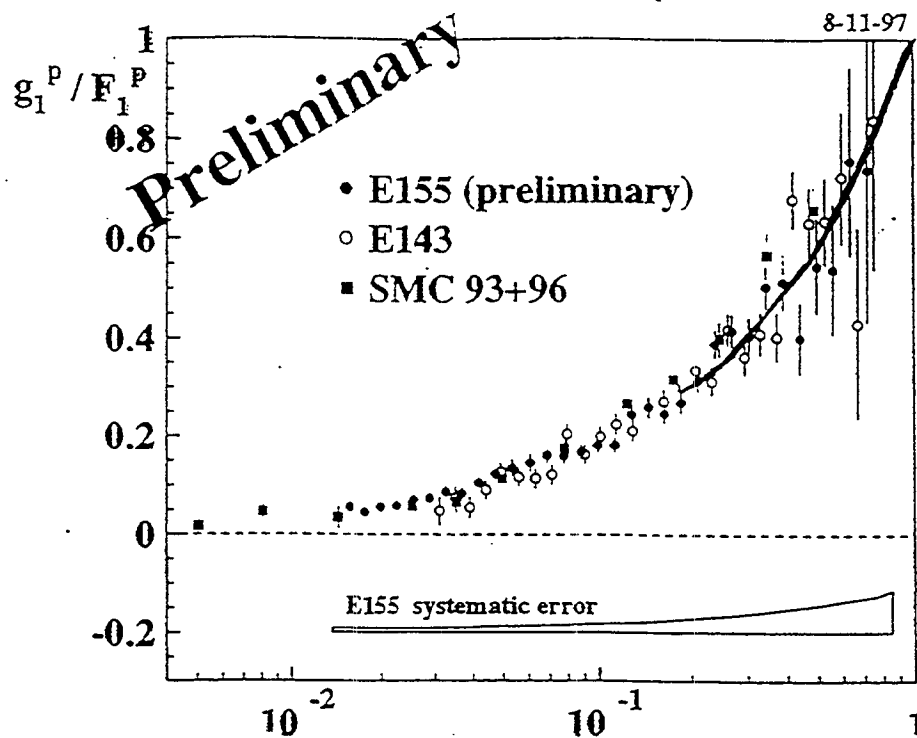
$$\Rightarrow F_{1,2}^{p,D}(x), g_{1,2}^{p,D,n}(x)$$

- 3 spectrometers ($2.75^\circ, 5.5^\circ, 10.5^\circ$)



$$f = 0.5$$

$$P^T \approx 0.2 - 0.25$$



- * Very impressive statistical accuracy
- * Also results for g_1^d / F_1^d from LiD target

HERMES

- F_1^p, g_1^p from '97 $\vec{H}$ data
- Hadron asymmetries from
 '95 $^3\vec{He}$ data
 '96 $\vec{H}$ "

$$\Rightarrow \Delta u_v(x), \Delta d_v(x), \Delta \bar{q}(x)$$

'97 proton results

$$p^B = 0.55 \pm 0.02$$

$$Q^2 > 0.8 \text{ GeV}^2$$

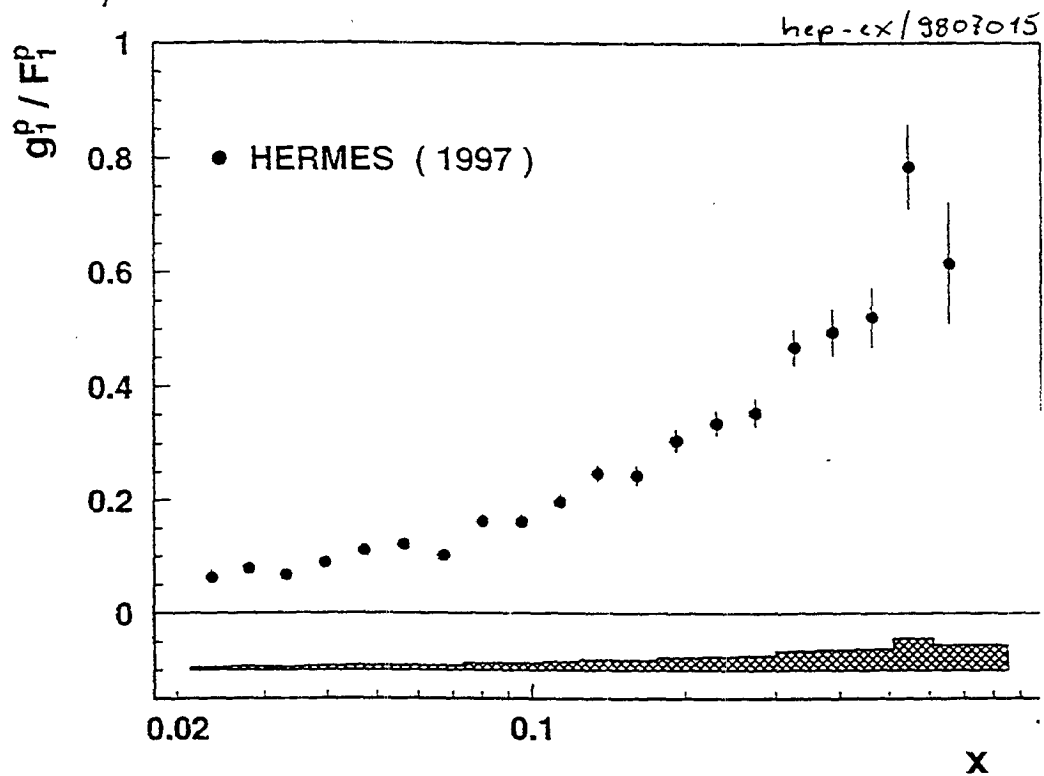
$$p^T = 0.88 \pm 0.04$$

$$W^2 > 3.24 \text{ GeV}^2$$

$$\Rightarrow 1.73 \times 10^6 \text{ events}$$

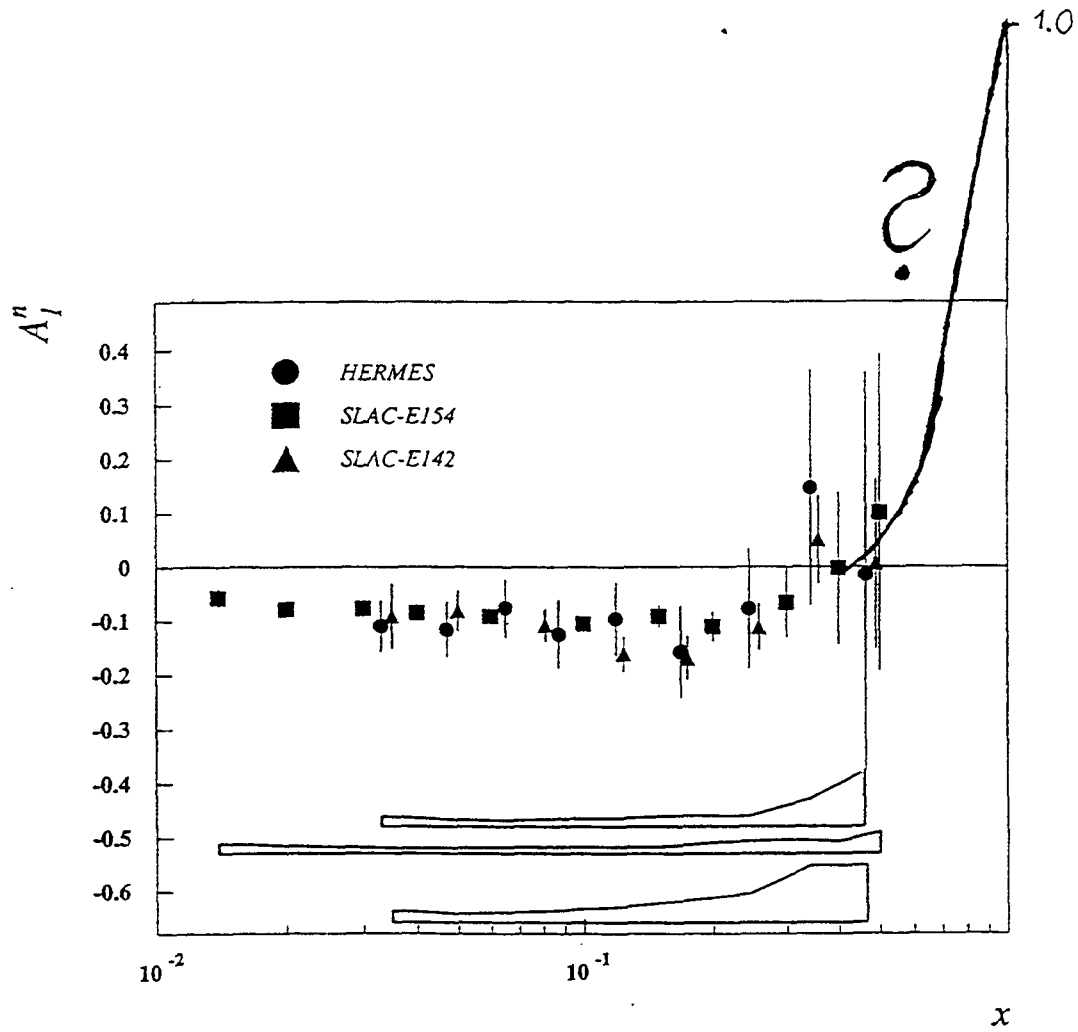
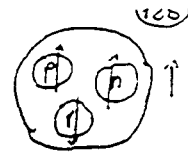
$$0.04 \text{ rad} < \Theta < 0.22 \text{ rad} \quad ({}^{36}\text{Fe} : -0.3 \times 10^6 \text{ events})$$

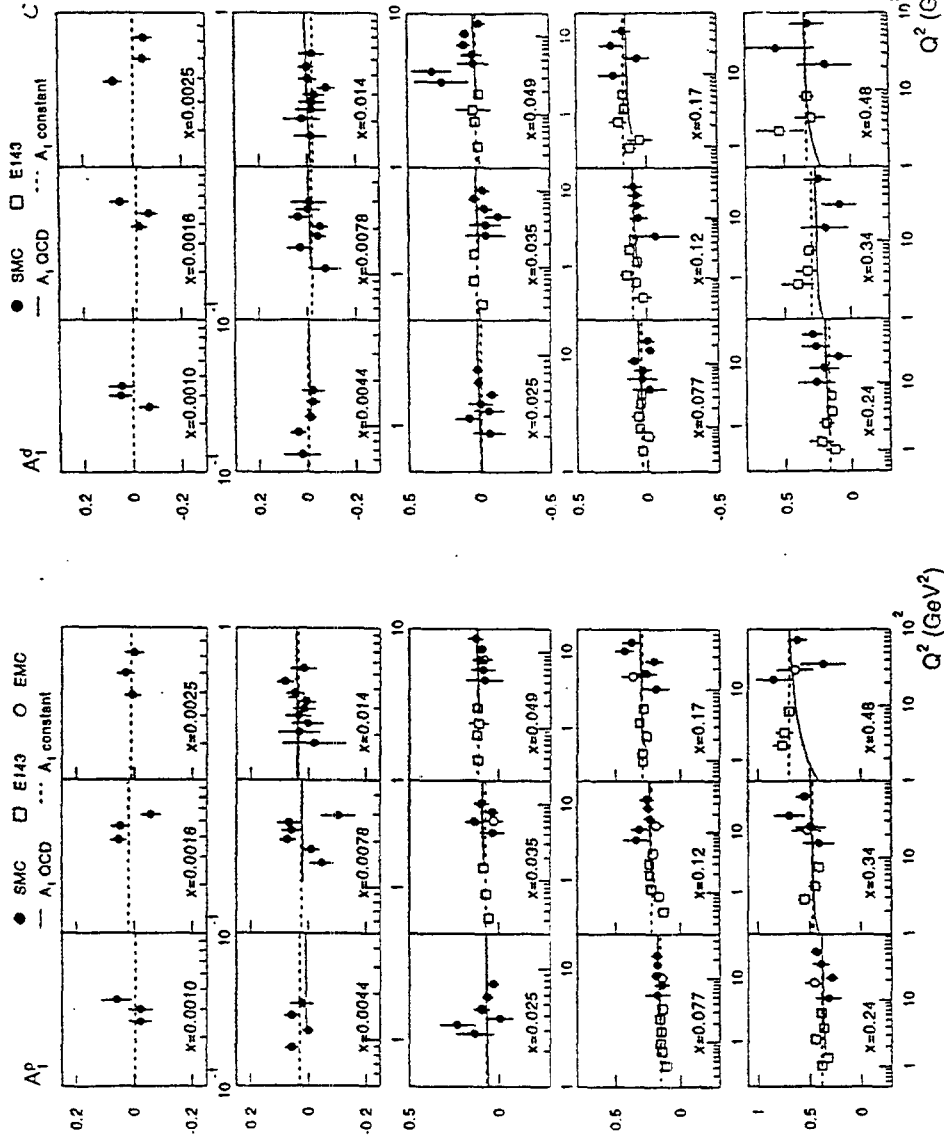
$$0.1 \leq y \leq 0.8$$



$$\frac{g_1}{F_1} = \frac{1}{(1+y^2)} \left(\frac{A_{||}}{D} - (\gamma - \gamma) A_2 \right)$$

neutron results from ${}^3\vec{\text{He}}$





$$F(x, Q^2) \rightarrow g_1(x, Q^2)$$

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$$\bullet \quad F(x, Q^2) = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D(F_1 + \gamma F_2)$$

↑
small correction

$$\bullet \quad g_1(x, Q^2) \approx F_1(x, Q^2) \cdot \frac{F_2(x, Q^2)}{2 \times (1 + R(x, Q^2))}$$

■ $F_2(x, Q^2)$: NMC - parametrisation

■ $R(x, Q^2)$: SLAC - parametrisation: R1:
(Recently new one from E143 for 0.03
R 1998, hep-ex/9808028)

▲ Q^2_{measured} different for each x-bin

Integrals: require data at fixed Q_0^2

* Assumption: $F_1(x, Q^2) \neq f(Q^2)$ QCD: small
 Q^2 -depend.
data: ok within e

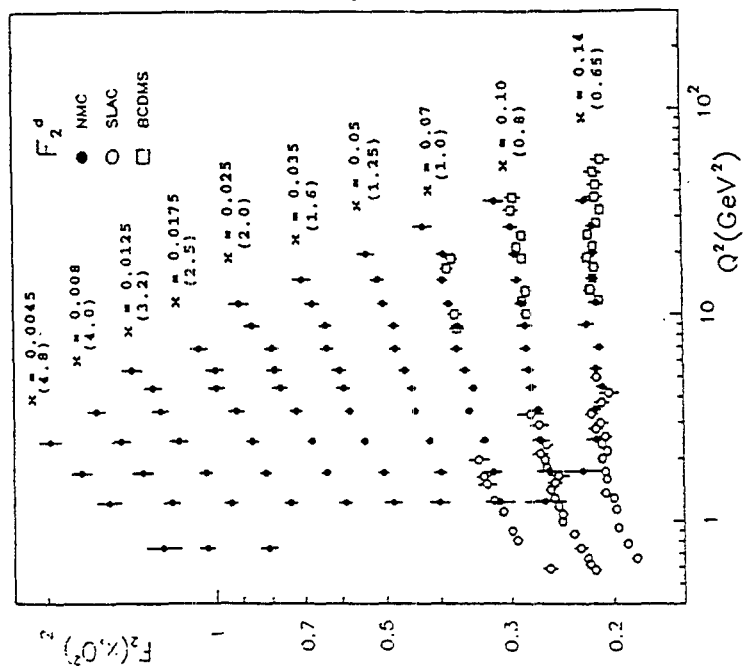
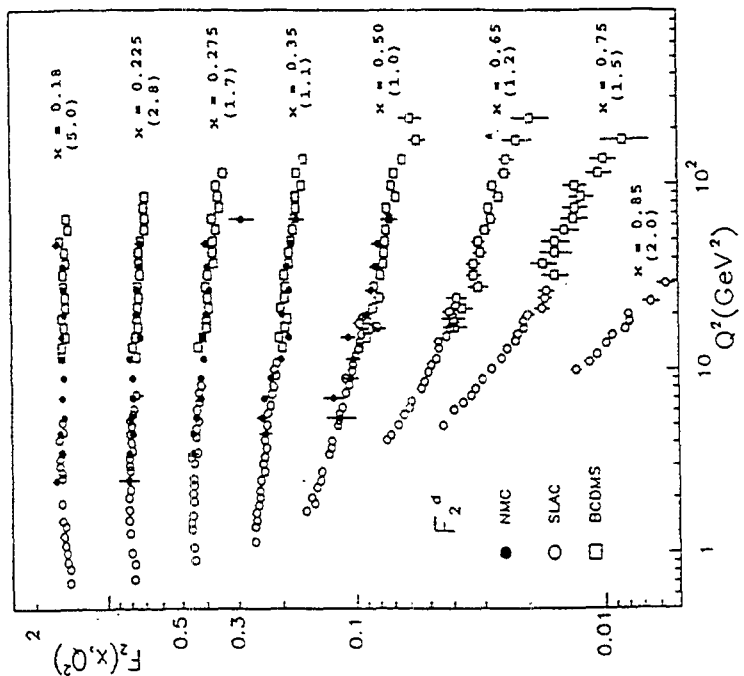
↪ Extrapolation via

Q^2 dependence of $F_2(x, Q^2), R(x, Q^2)$

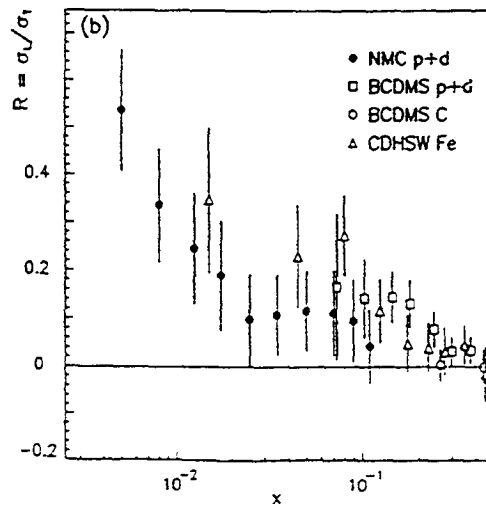
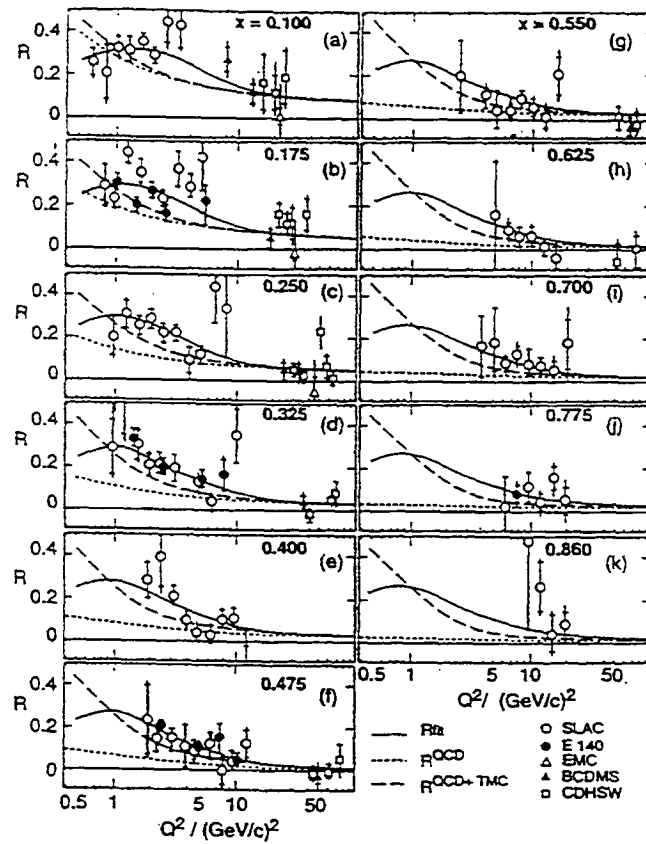
or

* Use results of NLO QCD analysis

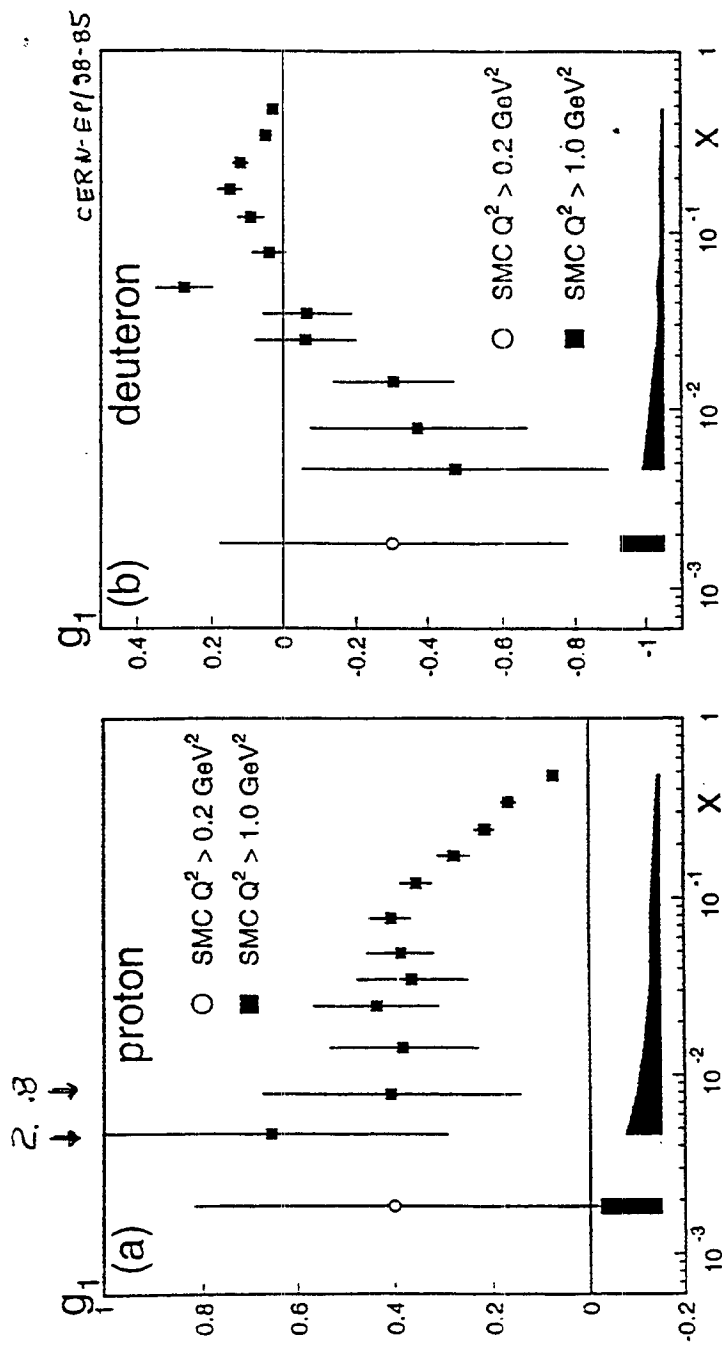
(However: many assumptions, esp. $\Delta g(x, Q^2)$)



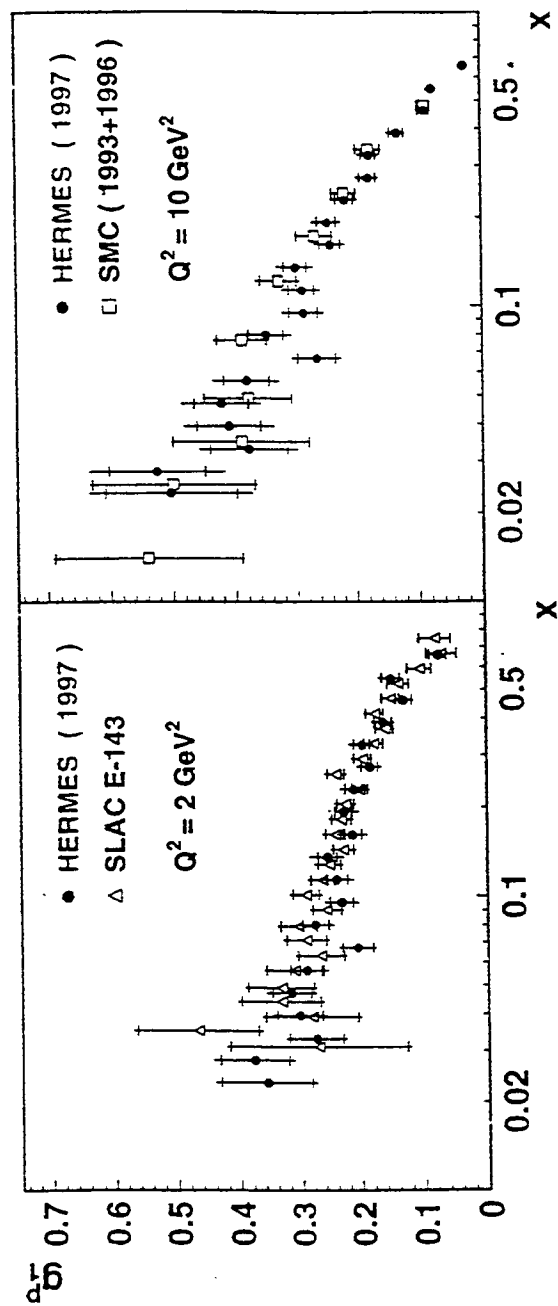
129



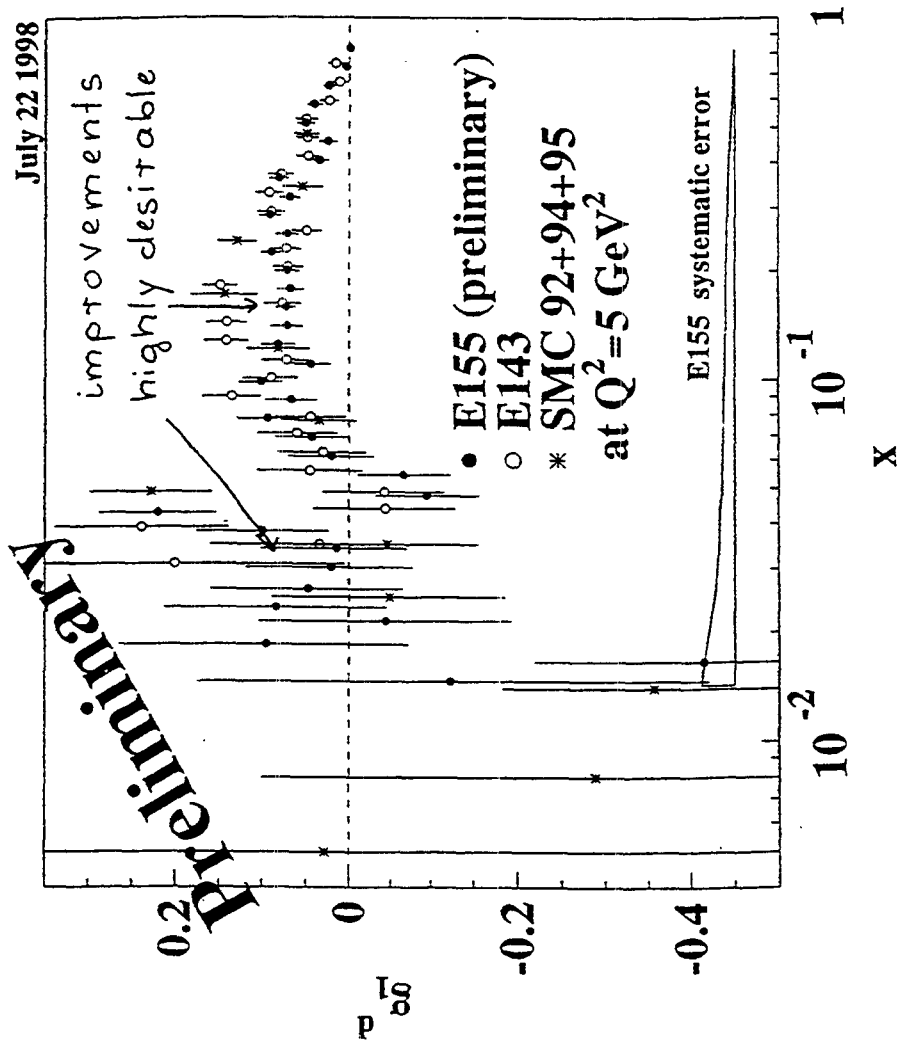
SMC - final results



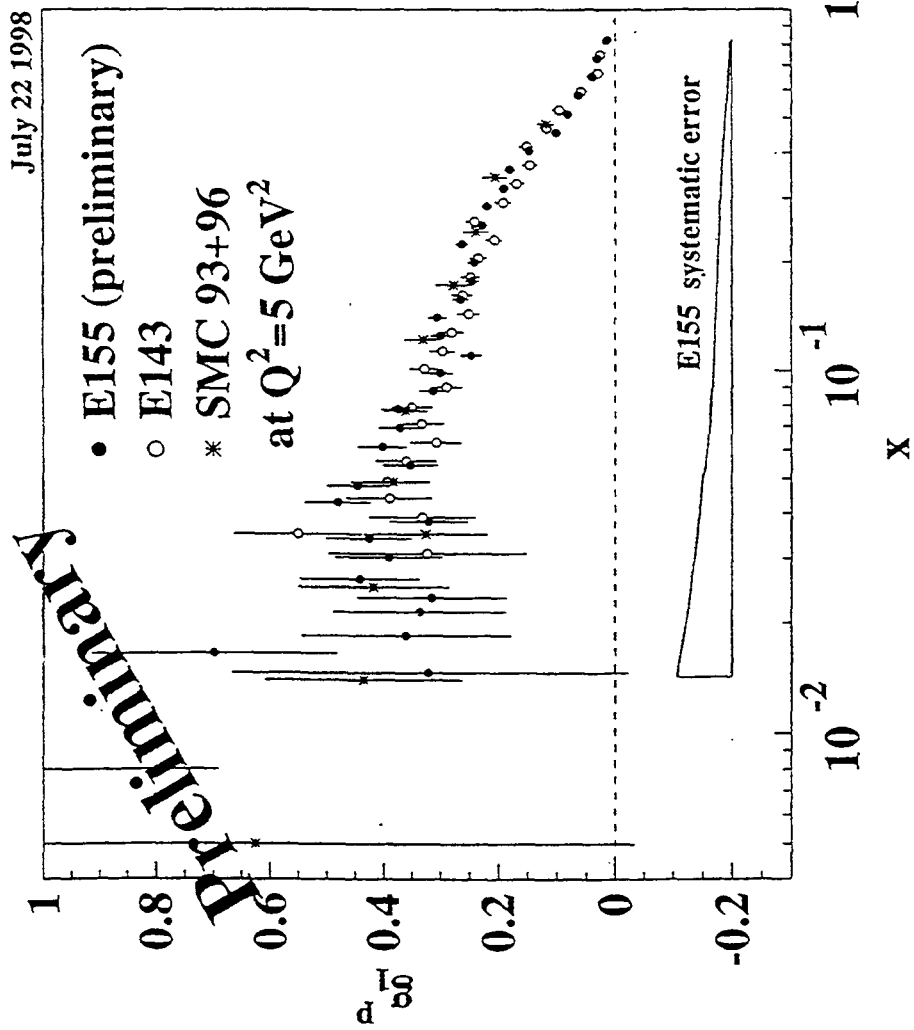
230



$\vec{\text{LiD}}$ - target; nuclear effects understood?

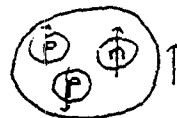


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^3He results



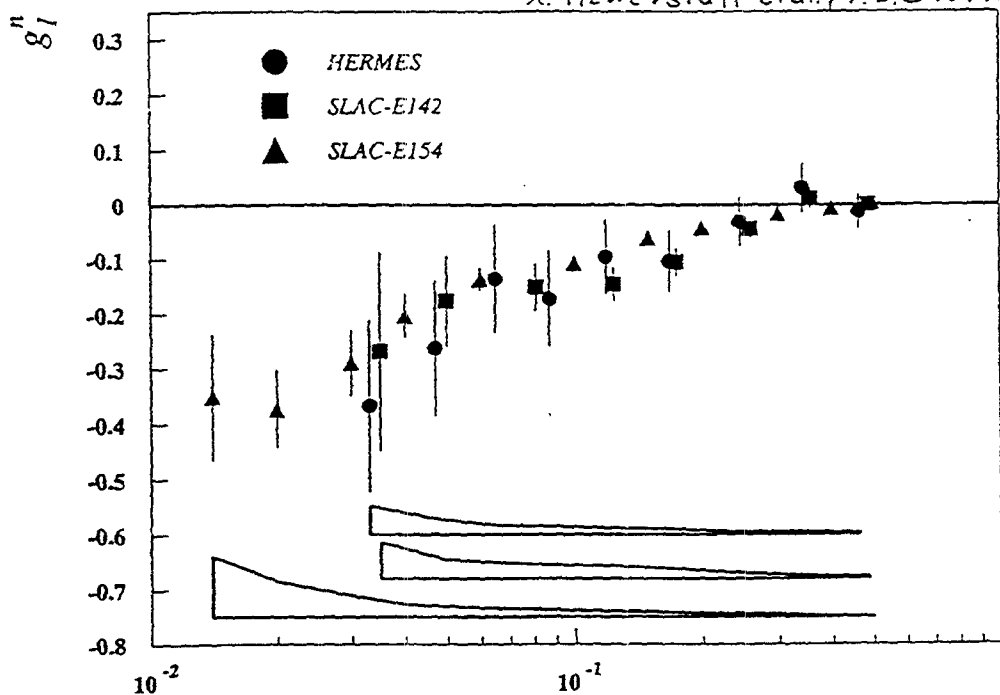
(134)

HERMES: $W^2 > 4 (\text{GeV}/c^2)^2$

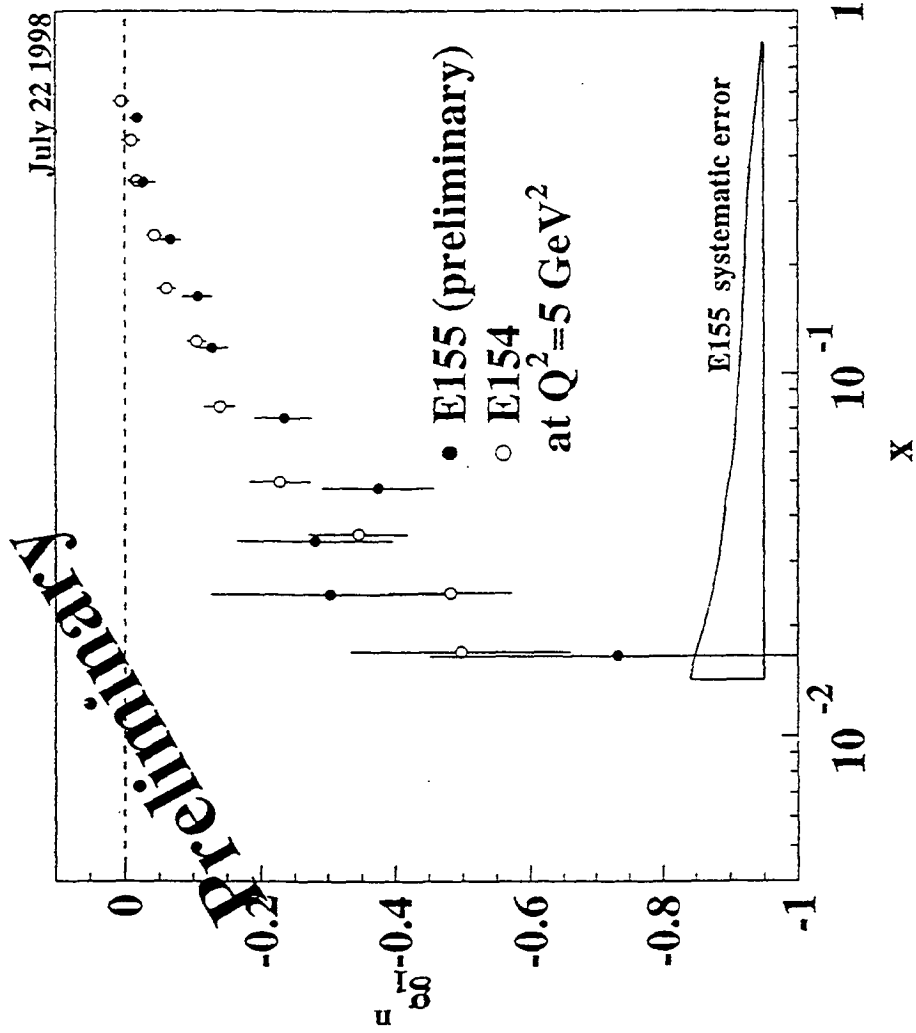
$Q^2 > 1 (\text{GeV}/c)^2$ 2.7×10^6 events

$y < 0.85$

X. Ruckstuhl et al., P.L.B404(87)38

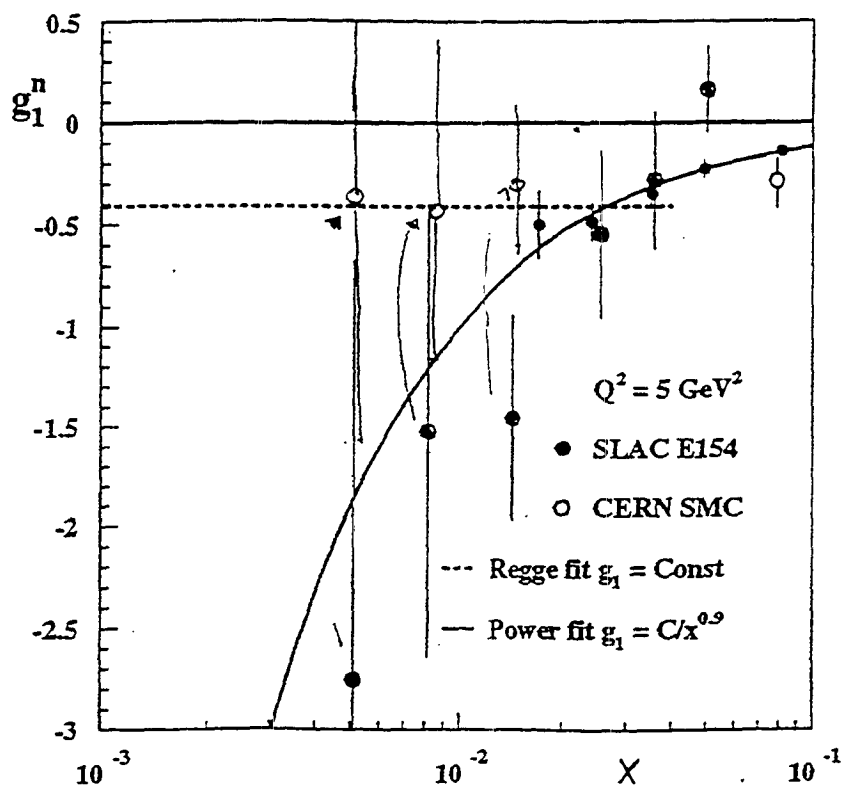


x



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$g_1^n(x)$ strongly decreasing
for $x \rightarrow 0$?

low x extrapolation?

New SMC results much flatter!!

The 'small x problem':

(137)

SLAC - E154 result:

- measured region:

$$\int_{0.014}^{0.7} g_1^n(x) dx = 0.036 \pm 0.004 \pm 0.005$$

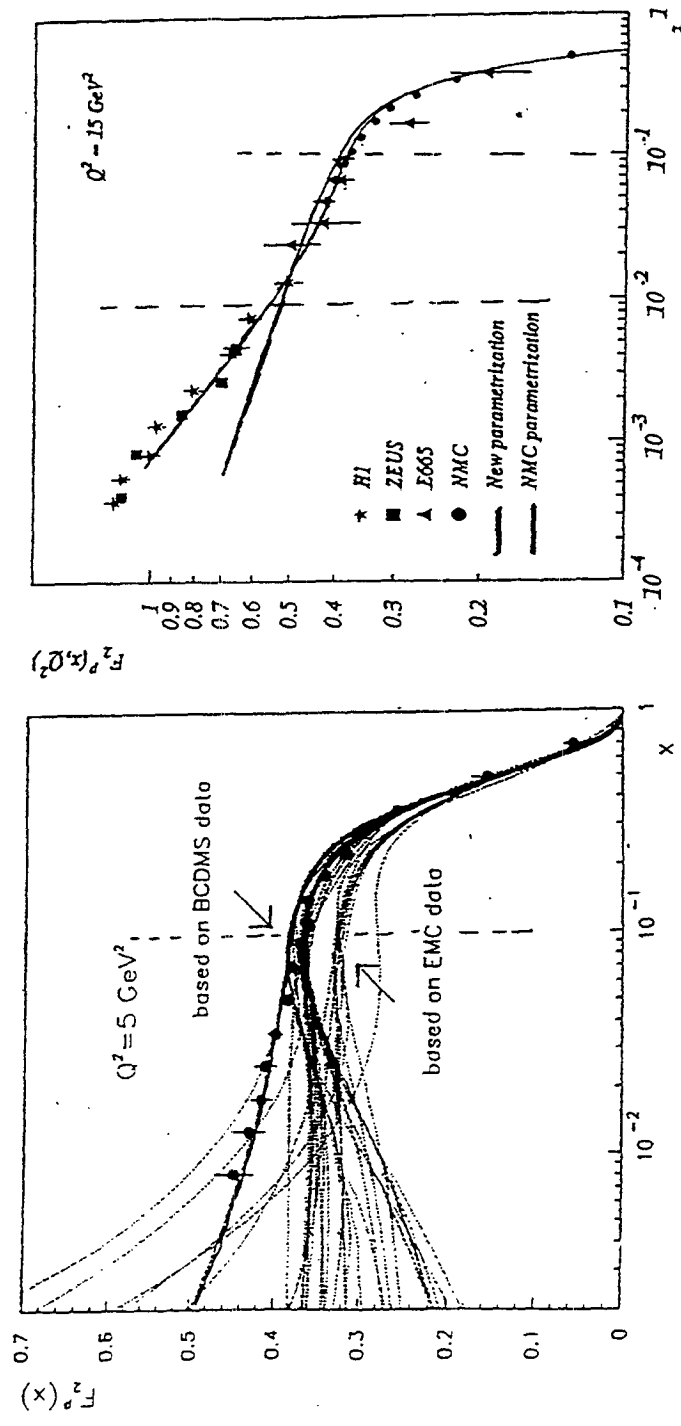
- power law fit: $g_1^n(x) = -0.02 x^{-0.8}$
 $\int_{0.014}^0 g_1^n(x) dx = -0.041$ ⚡

↪ 100% error from low x extrapolation?

But: Maybe 'problem' is over-emphasized
by logarithmic presentation of data



- Need more precise data @ very low x
 - HERA ($\vec{e} \rightarrow e \vec{p}$) (unfort. D very small)
 - RHIC
 - TeV-Linear-Collider



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Results for integrals $\Gamma_1 \equiv I_1 = \int_0^1 dx g_1(x)$

Exp.	Target	Q^2 GeV ²	I_1^{meas}	I_1^{high}	I_1^{low}	I_1	I_1^{EJ}
SMC	p('93)	10	0.130 (0.013)(0.009)	0.0015 (0.0007)	0.0042 (0.0016)	0.136 (0.013)(0.010)	0.170 (0.005)
SMC	p('93+'96)	10	0.145 (0.006)(0.009)	0.0015 (0.0007)	0.0029 (0.0007)	0.149 (0.006)(0.010)	0.170 (0.005)
SMC	d	10	0.0407 (0.0059)(0.0046)	0.0006 (0.0009)	0.0000 (0.0009)	0.0414 (0.0059)(0.0048)	0.071 (0.003)
	n('93p+d)					-0.046 (0.013)(0.010)	-0.017 (0.005)
E143	p	3	0.120 (0.004)(0.008)	0.001 (0.001)	0.006 (0.006)	0.127 (0.004)(0.010)	0.160 (0.006)
	d	3	0.040 (0.003)(0.004)	0.000 (0.001)	0.001 (0.001)	0.042 (0.003)(0.004)	0.069 (0.004)
	n	3				-0.037 (0.008)(0.011)	-0.011 (0.005)
E142	n(³ He)	3	-0.028 (0.006)(0.006)	0.003 (0.003)	-0.0053 (0.0053)	-0.031 (0.006)(0.009)	-0.011 (0.005)
E154	n(³ He)	5	-0.036 (0.004)(0.005)			-0.041 (0.004)(0.006)	
HERMES	n(³ He)	2.5	-0.036 (0.013)(0.005)	0.002 (0.002)	-0.005 (0.005)	-0.041 (0.013)(0.007)	-0.011 (0.005)

Table 4. Integrals over the polarised structure function g_1 from recent experiments.

Results for integrals @ $Q^2 = 5 \text{ GeV}^2$

World data (without HERMES p $\bar{p}$
E155 d)

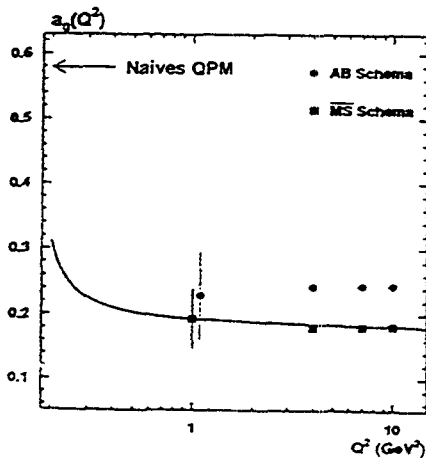
(Recent SMC analysis)

Γ_1	
proton	$0.121 \pm 0.003 \pm 0.005 \pm 0.026$
neutron	$-0.068 \pm 0.007 \pm 0.005 \pm 0.023$
deuteron	$0.021 \pm 0.004 \pm 0.003 \pm 0.023$
	(stat.) (syst.) (theor.) Extrapol.

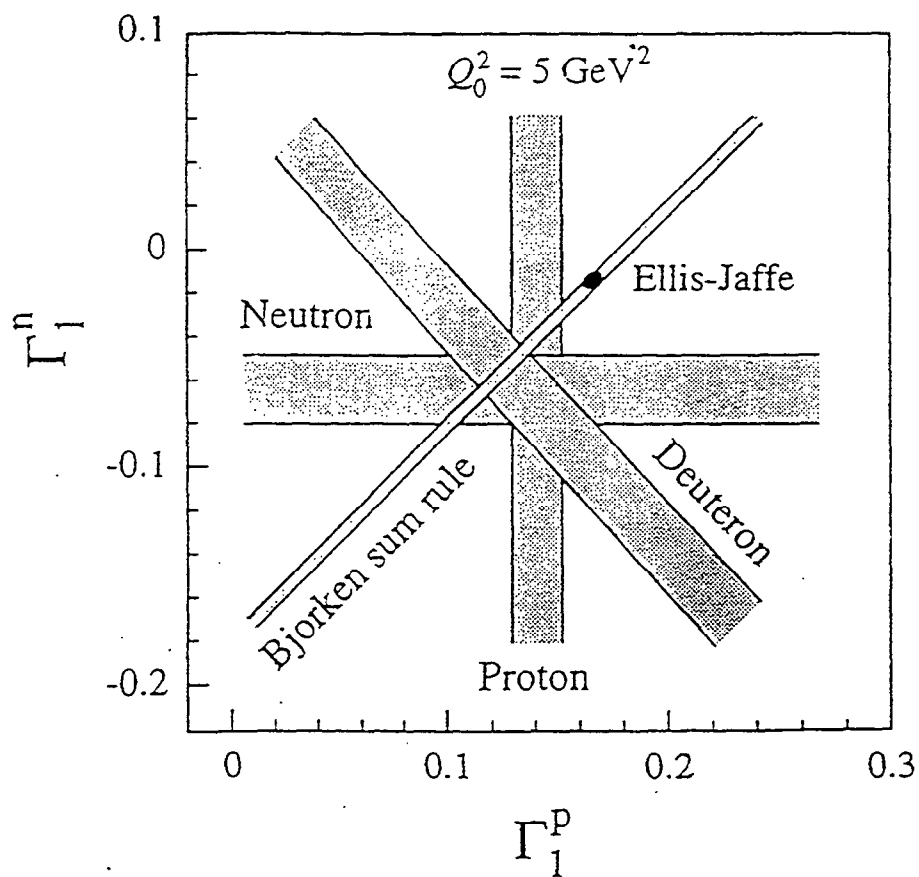
The singlet axial matrix element $a_0(Q^2)$

$$\overline{\text{MS}}\text{-scheme} : a_0(Q^2) = \Delta \Sigma(Q^2) = 0.19 \pm 0.05 \pm$$

$$\text{FB-scheme} : a_0(Q^2) = \Delta \Sigma(Q^2) - n_f \frac{\Delta \gamma(Q^2)}{2\pi} \Delta g$$

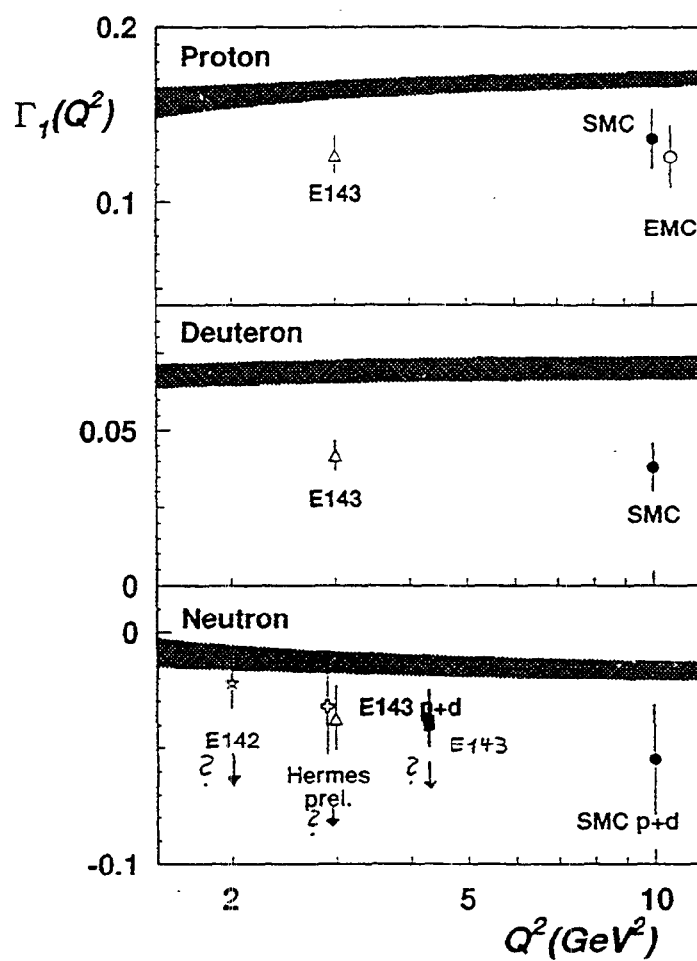


$$0.38 \pm 0.03 = 0.24 \pm 0.07 \pm$$



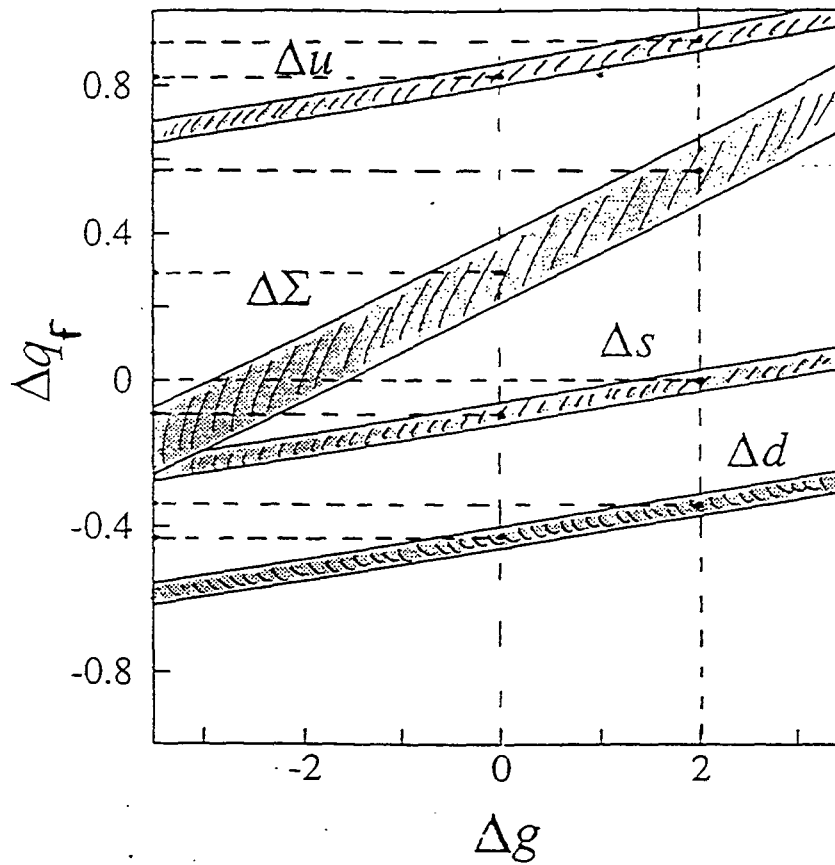
- Bjorken Sum Rule satisfied
- Ellis-Jaffe S.R. violated by 2-3 σ

Ellis-Jaffe Sum Rule



- Ellis-Jaffe Sum Rule is violated
- Deuteron data most significant
 4.6σ (E-143, 3 GeV^2) and 3.5σ (SMC, 10 GeV^2)

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anomaly : $\rightarrow$ gluon contribution

Adler-Bardeen - fact scheme:

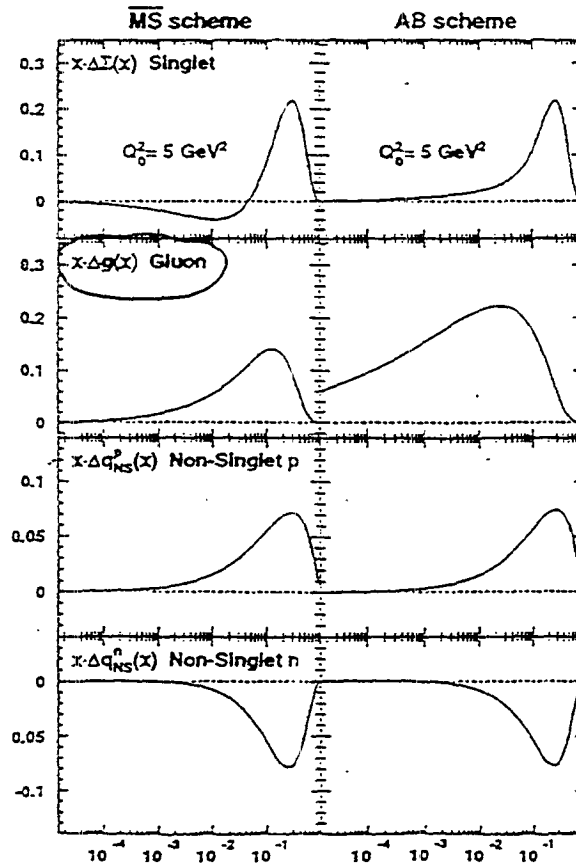
$$\begin{aligned} a_o(Q^2) &= \Delta\Sigma - n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2) \\ a_{o,f}(Q^2) &= \Delta q_f - \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2) \end{aligned}$$

QCD Fits

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Results are very scheme dependent

Comparison $\overline{MS}$ and Adler-Bardeen sch.



$$AB: \Delta g = \int_0^1 \Delta g(x) dx = 0.99^{+1.17}_{-0.31} + 0.42^{+1.4}_{-0.4} \quad \text{(stat.) (syst.) (theo.)}$$

$$\overline{MS}: \Delta g \approx 0.3 \pm \dots$$

→ Gluon polarisation badly known

$\Delta\Sigma \approx 0.2-0.4$ Spin Puzzle ⁽¹⁴⁾
confirmed by recent inclusive data!

What is the origin of the nucleon Spin?

Many models and theor. explanations

↪ Need additional experimental info

$\Delta u_v(x) ?$
 $\Delta \bar{u}(x) ?$
 $\Delta d_v(x) ?$
 $\Delta \bar{d}(x) ?$

$\pi^+, \pi^-, \pi^+\pi^-$

$\Delta S(x) ?$

$K^-(\bar{u}s), \Lambda(uds), \bar{\Lambda}$

$\Delta g(x, Q^2) ?$

$\gamma^*g \rightarrow c\bar{c} X \rightarrow J/\psi, D, \bar{D}$

$L_2^g(Q^2) ?$

ϕ distr. of hadrons, $p_T^{\perp} \rightarrow K\pi$
 $\gamma^*P \rightarrow \gamma P, DVCS$

↪ Semi-inclusive data.

Domain of **HERMES** (good PID)

(COMPASS → 2001)

Orbital angular momenta L_z^q, L_z^g ⁽¹⁴⁷⁾

(Ji et al., P.R.L. 76 (196) 740, Ratcliff, ...)

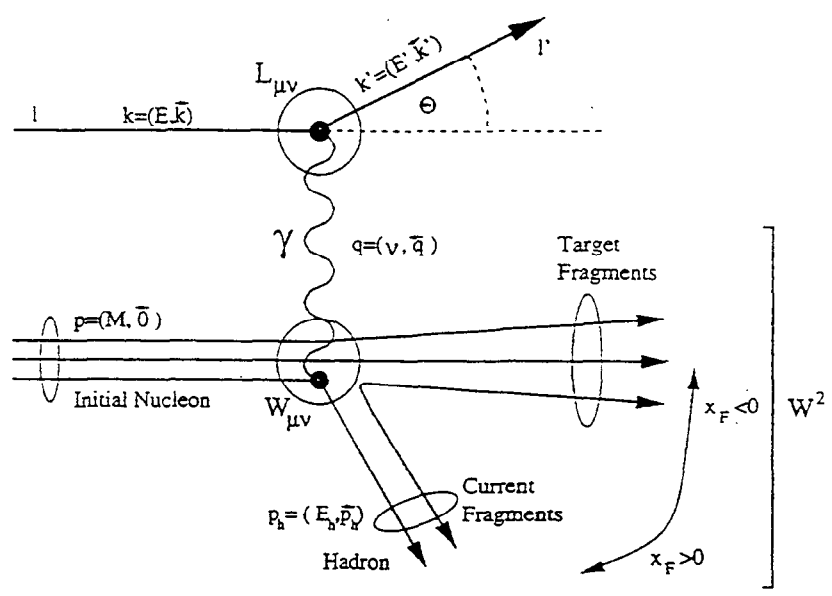
$$Q^2 \rightarrow \infty \quad \Delta \Sigma + 2 L_z^q = \frac{3 n_f}{16 + 3 n_f} = 2 \gamma^q = 0.43$$

$$2 \Delta g + 2 L_z^g = \frac{16}{16 + 3 n_f} = 2 \gamma^g = 0.57$$

$$\Rightarrow L_z^q (Q^2 = 3 \cdot 10^6 \text{ eV}^2) \approx 0.075$$

Experimentally very difficult
to attack (DVC deeply virtual Compton scatt.²),

Semi-Inclusive DIS



$$\frac{1}{\sigma_{tot}} \frac{d\sigma^h(x, Q^2, z)}{dz} = \frac{\sum_q e_q^2 q(x, Q^2) D_q^h(z)}{\sum_q e_q^2 q(x, Q^2)}$$

Four-momentum Transfer	Q^2	
Bjorken's scaling variable	$x = Q^2/2M\nu$	incl.
Parton density function	$q_f(x, Q^2)$	
Energy fraction of hadron	$z \stackrel{lab}{=} E_h/\nu$	semi-
Fragmentation function	$D_q^h(z)$	incl.

Light Quark Sea



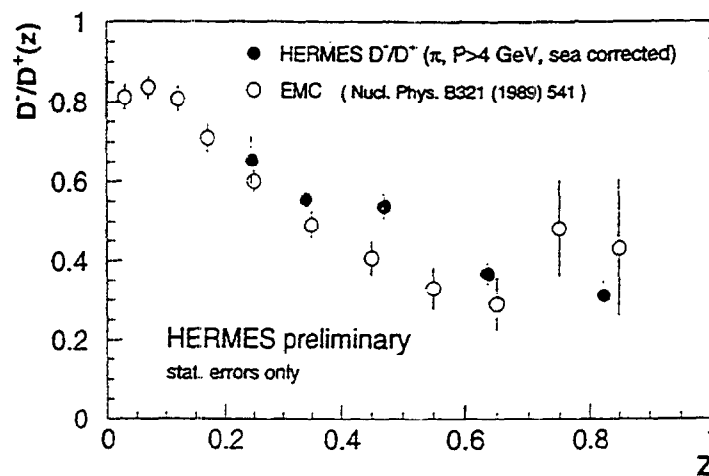
- Proton - neutron pion multiplicity difference:

$$r(x, z) = \frac{N_p^{\pi^-} - N_n^{\pi^-}}{N_p^{\pi^+} - N_n^{\pi^+}}$$

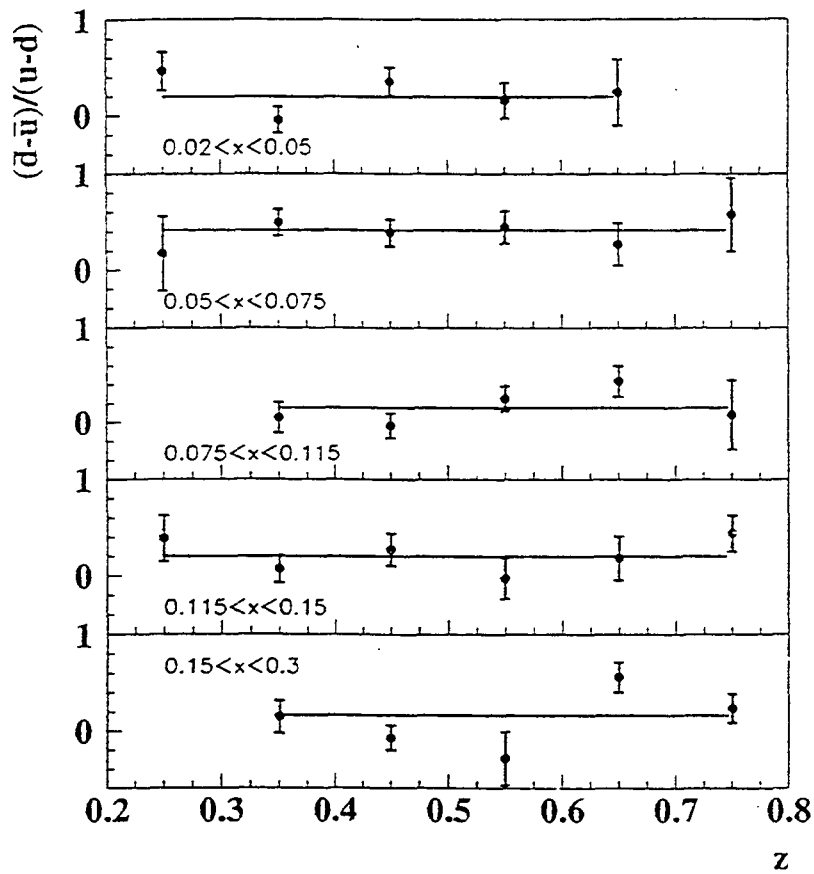
- Extract flavour asymmetry of the light sea:

$$\frac{\bar{d}(x) - \bar{u}(x)}{u(x) - d(x)} = \frac{(\frac{1+D^-/D^+}{1-D^-/D^+})[1 - r(x, z)] - [1 + r(x, z)]}{(\frac{1+D^-/D^+}{1-D^-/D^+})[1 - r(x, z)] + [1 + r(x, z)]}$$

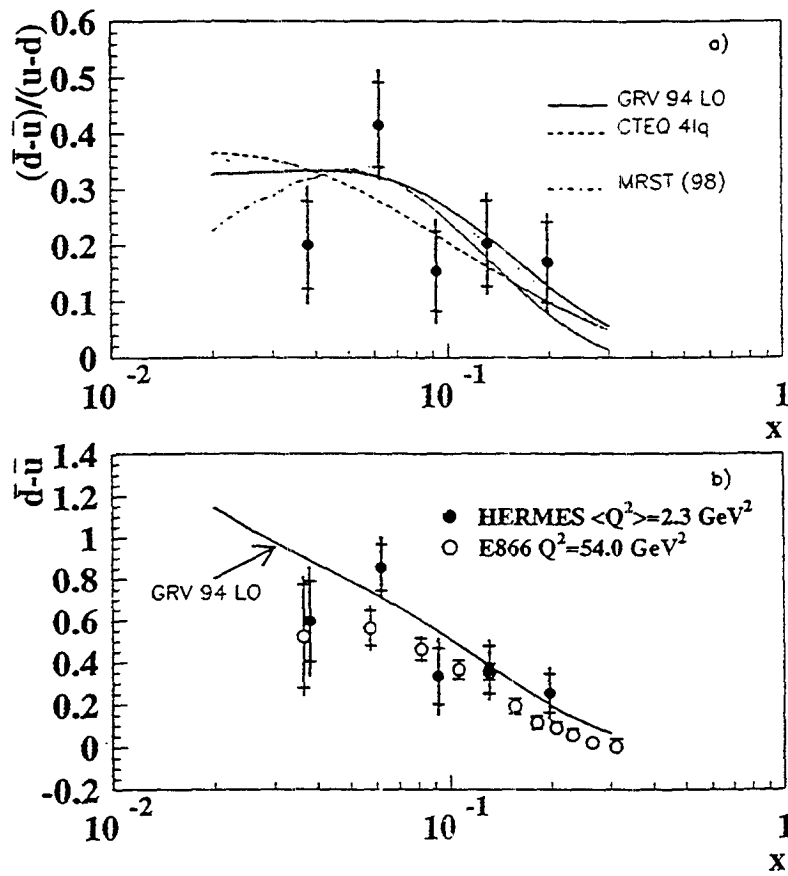
- Ratio of fragmentation functions D^-/D^+ :



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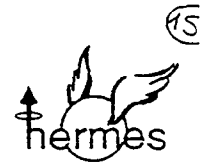


✦ Assumption of factorisation o.k.



$\Rightarrow u_v(x) > d_v(x)$
 $d\bar{d}\text{-sea} > u\bar{u}\text{-sea}$

Particle Asymmetries



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- Photon-Nucleon Asymmetry:

$$\begin{aligned}
 A_1^h(x, z) &= \frac{\sigma_h^+ - \sigma_h^-}{\sigma_h^+ + \sigma_h^-} \\
 &= \frac{\sum_q e_q^2 (q^+(x) - q^-(x)) D_q^h(z)}{\sum_{q'} e_{q'}^2 (q'^+(x) + q'^-(x)) D_{q'}^h(z)}
 \end{aligned}$$

- Measured Asymmetry:

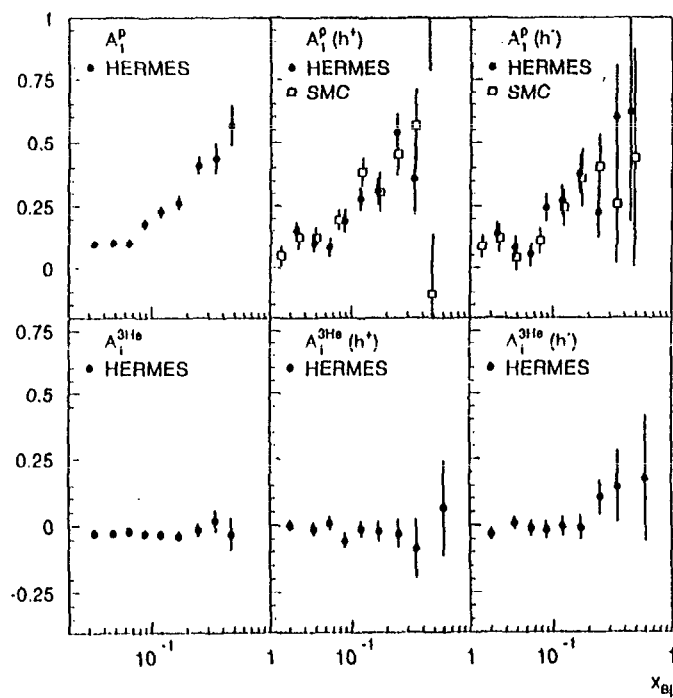
$$\begin{aligned}
 A_{||}^h(x, z) &= \frac{\sigma_h^{\vec{\epsilon}} - \sigma_h^{\vec{\Xi}}}{\sigma_h^{\vec{\epsilon}} + \sigma_h^{\vec{\Xi}}} \\
 &= A_1^h(x, z) \cdot D,
 \end{aligned}$$

with photon depolarization factor

$$D(x, Q^2) = \frac{y(2-y)}{y^2 + 2(1-y)(1+R(x, Q^2))}.$$

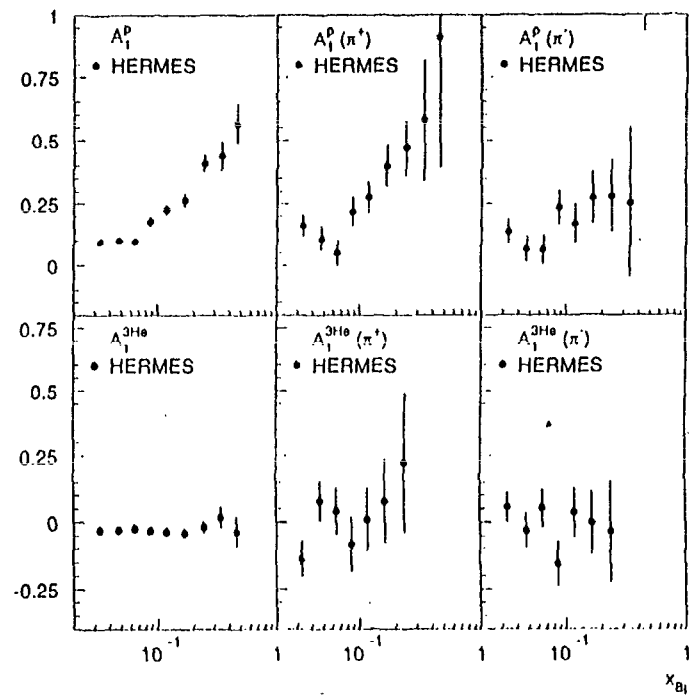
- Here we assumed the naive QPM.
- Also possible in improved QPM, factors of $(1 + R(x, Q^2))$ and $(1 + Q^2/\nu^2)$ have to be added.

Hadron Asymmetries



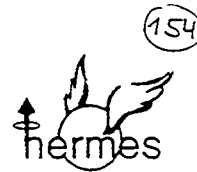
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Pion Asymmetries



Marc Andre Funk
Bryan Tipton

Purities



- Rewrite Photon-Nucleon Asymmetry:

$$\begin{aligned} A_1^h(x, z) &= \sum_q \frac{e_q^2 q(x) D_q^h(z)}{\sum_{q'} e_{q'}^2 q'(x) D_{q'}^h(z)} \cdot \frac{\Delta q(x)}{q(x)} \\ &= \sum_q P_q^h(x, z) \cdot \frac{\Delta q(x)}{q(x)} \end{aligned}$$

- The hadron quark-purity $P_q^h(x, z)$ is the probability that a quark q was struck in an event $l + N \longrightarrow l' + h + X$.
- Purities are spin-independent (unpolarized) quantities!
- Define

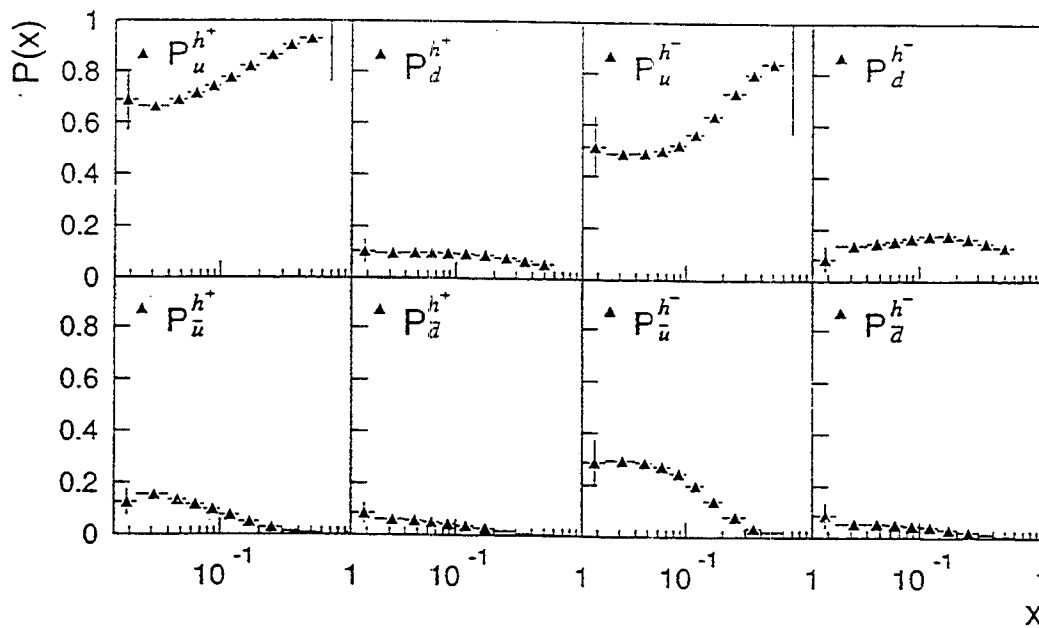
$$\vec{A} = \begin{pmatrix} A^{h1}(x) \\ \dots \\ A^{hm}(x) \end{pmatrix}, \vec{Q} = \begin{pmatrix} \Delta q_1(x)/q_1(x) \\ \dots \\ \Delta q_n(x)/q_n(x) \end{pmatrix}, \mathcal{P} = [P_q^h(x)].$$

To measure quark polarizations invert:

$$\vec{A} = \mathcal{P} \vec{Q}$$

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Purities for proton target



dominated by $u(x), \bar{u}(x)$
 z dependence very small

Polarised quark distributions ('95+'96)

(15)

$$Q^2 > 1 \text{ GeV}^2$$

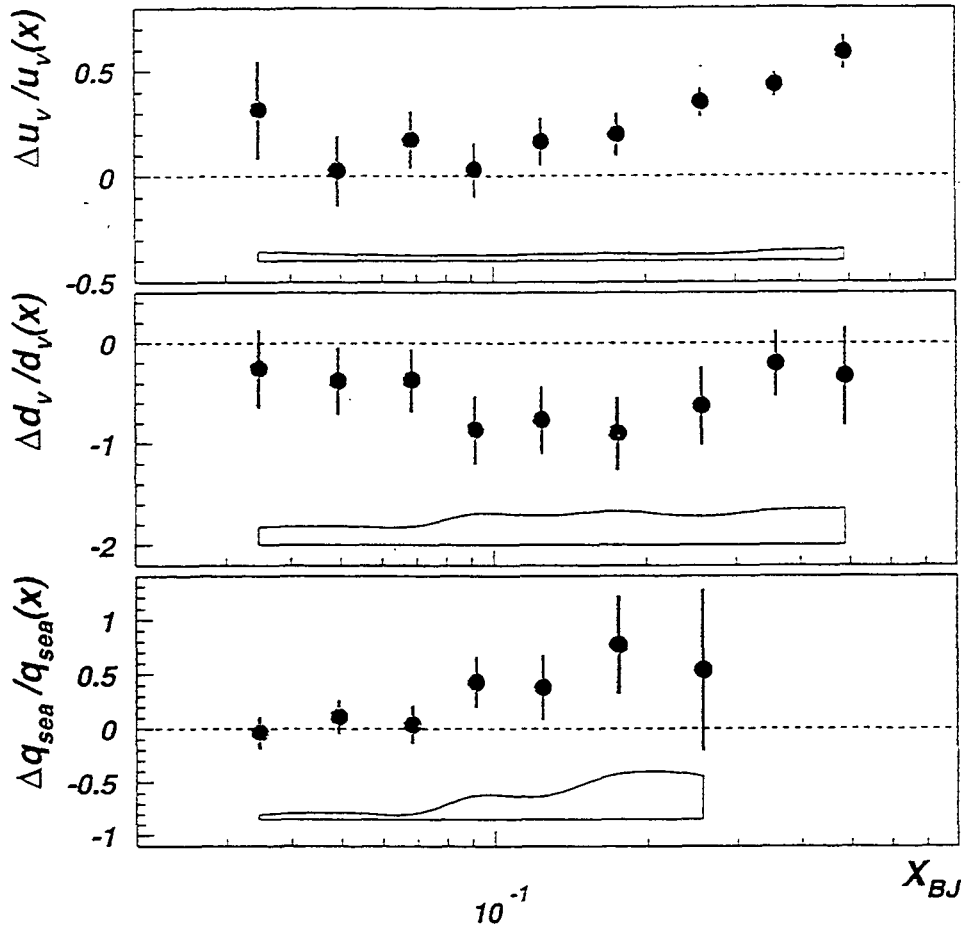
$$W^2 > 4 \text{ GeV}^2$$

$$0.023 < x < 0.6$$

$$95': z > 0.1$$

$$96': z > 0.2, x_F > 0.1$$

HERMES preliminary ($\Delta u_{\text{sea}}/u_{\text{sea}} = \Delta d_{\text{sea}}/d_{\text{sea}} = \Delta s/s$)



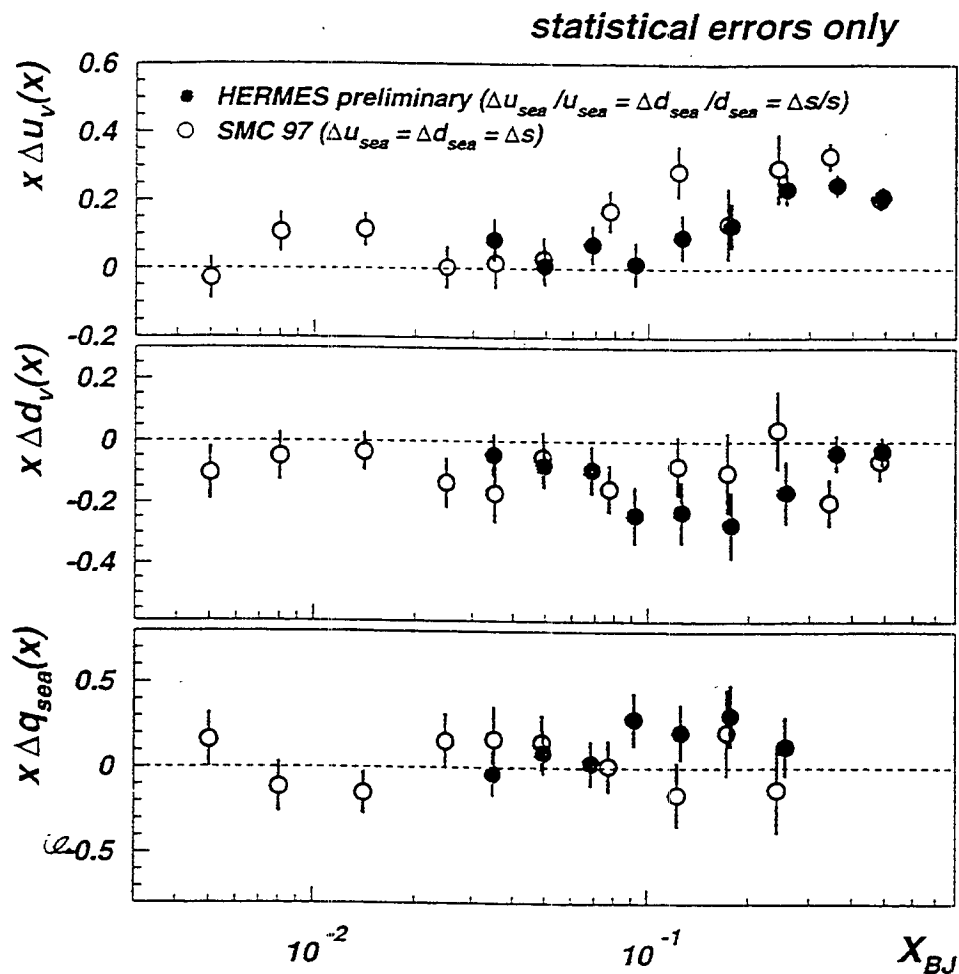
Similar results for

* assumption $\Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s}$

* different choice of $D_n^h(z)$

(157)

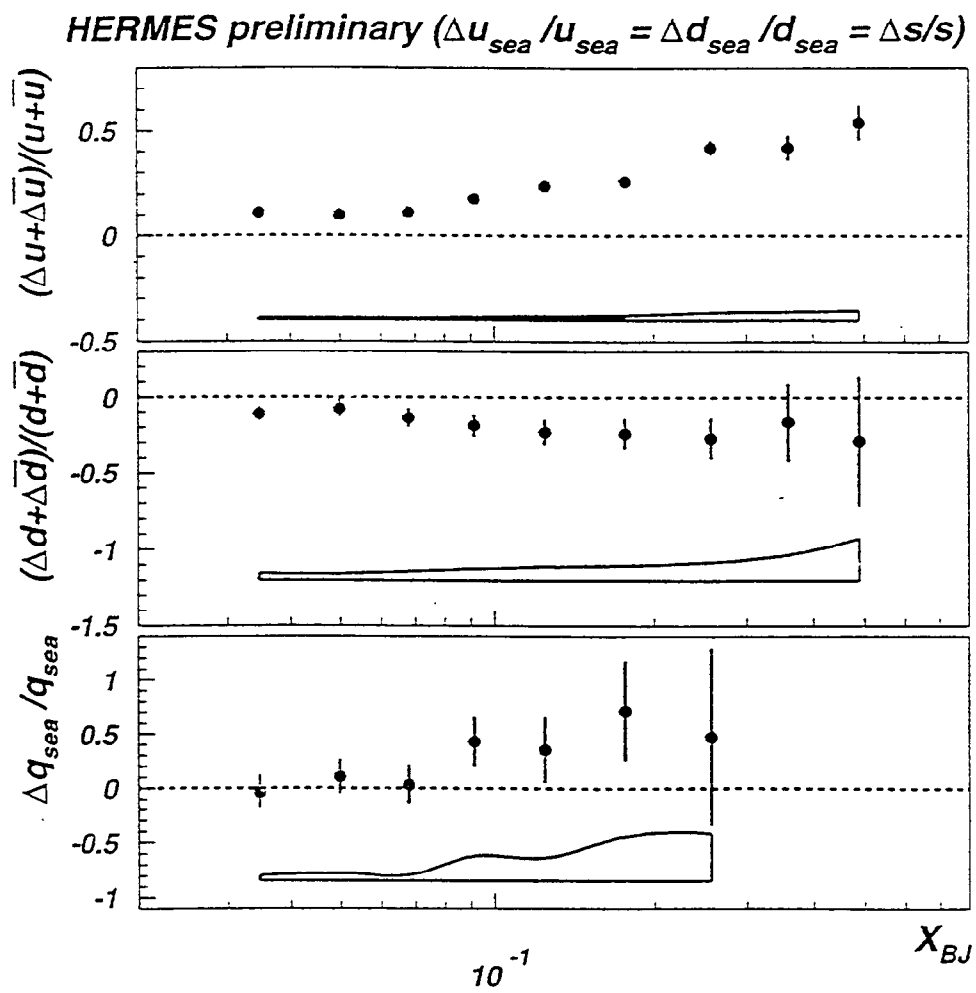
Errors dominated by correlations!



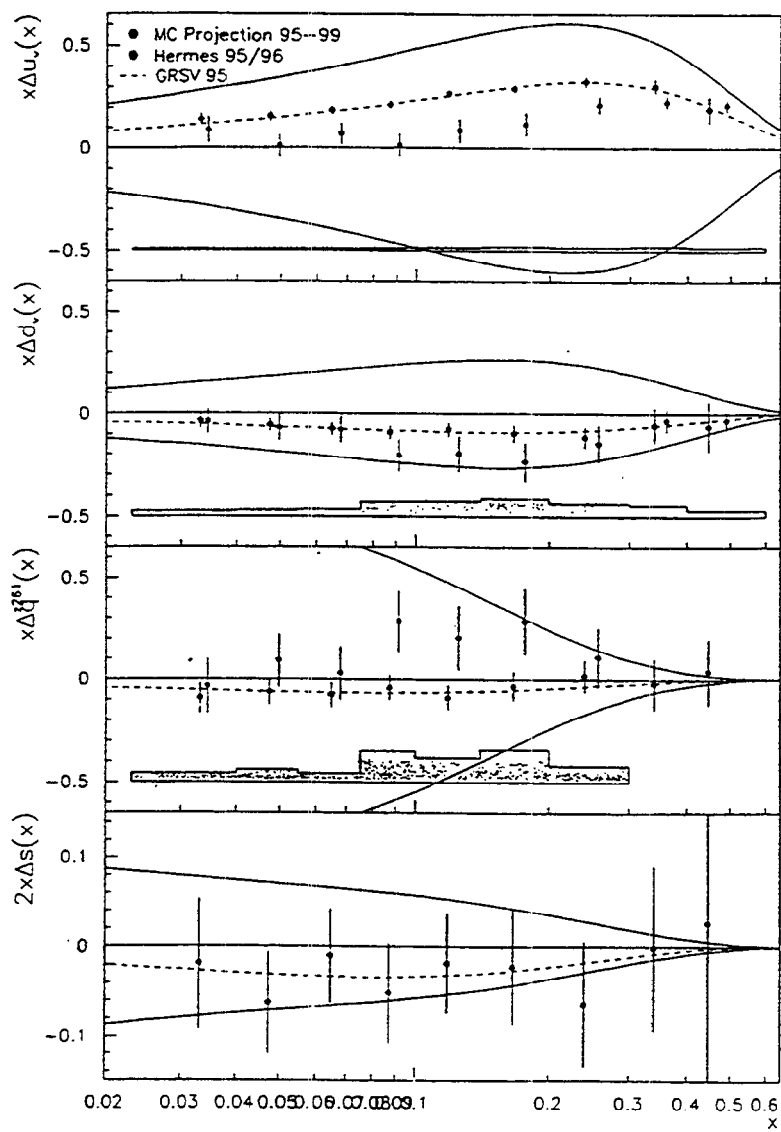
- * Reasonable agreement with SMC
- * '97 data : 3-times higher statistics for p
- * Need high quality deuteron data → '98, '99

H. Reh

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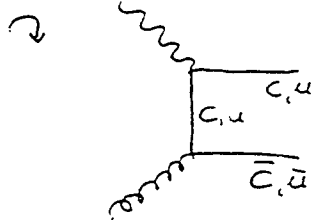
159



$$\boxed{\Delta g}$$

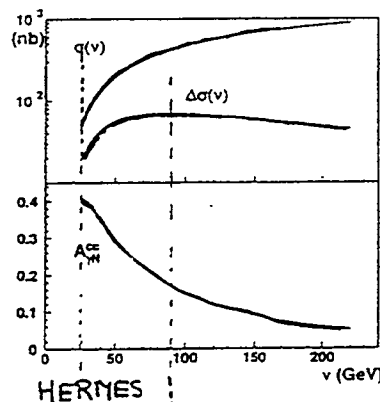
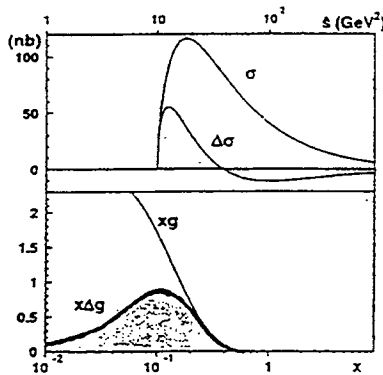
(160)

- $\Delta g(x, Q^2)$ rather weakly constrained by present data
- Study processes where $\Delta g(x)$ occurs directly already in L.O. $\gamma^*(\gamma) \Delta g \rightarrow c\bar{c}$



* open charm production
 $D \rightarrow K\pi$, $D \rightarrow K\mu\nu$

* high p_t pions, kaons



HERMES

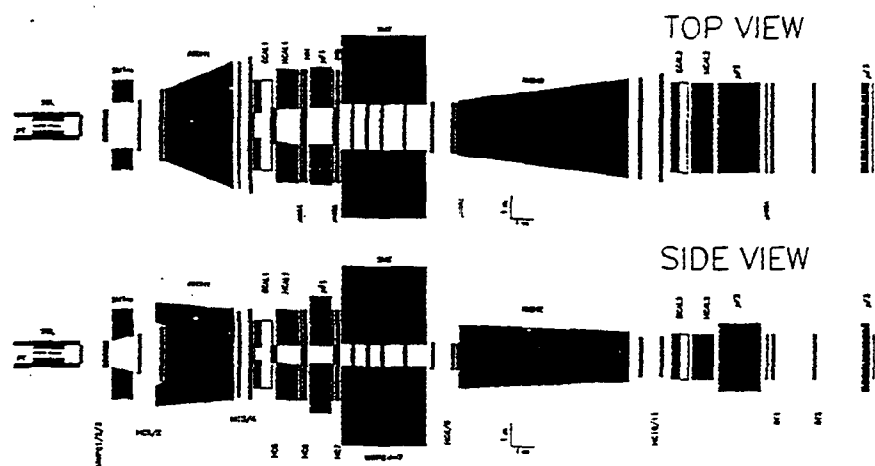
COMPASS

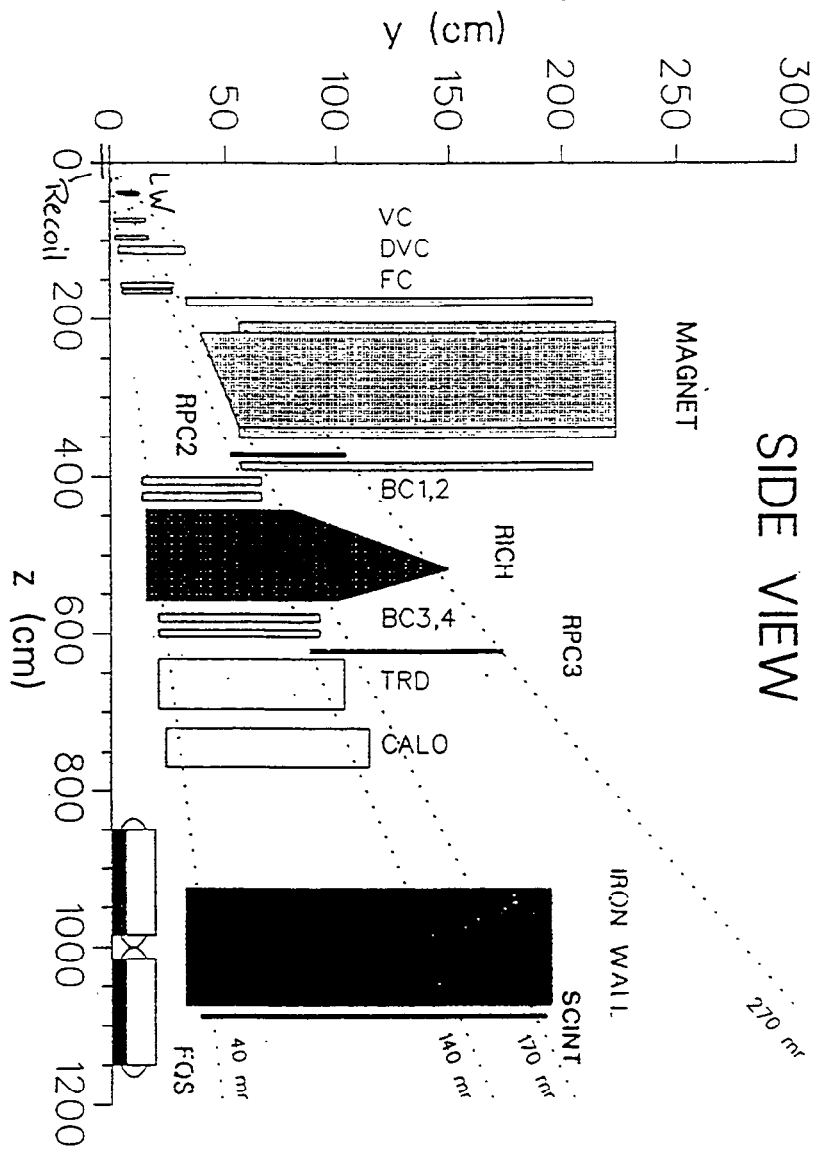
- $\vec{p}\vec{p} \rightarrow \gamma + \text{jet}$; $\vec{p}\vec{p} \rightarrow \chi_{2(355)} + X$ RHIC
 - $\rightarrow \gamma + X$
 - $\rightarrow \text{jet} + X$
 - $\rightarrow \text{jet} + \text{jet}$
- HERA $\vec{p}\vec{p}$ fixed

The COMPASS Spectrometer

- Spectrometer Components

Tracking detectors, Rich Detectors,
electromagnetic and hadronic Calorimeters,
Magnets, Muon filters (iron)





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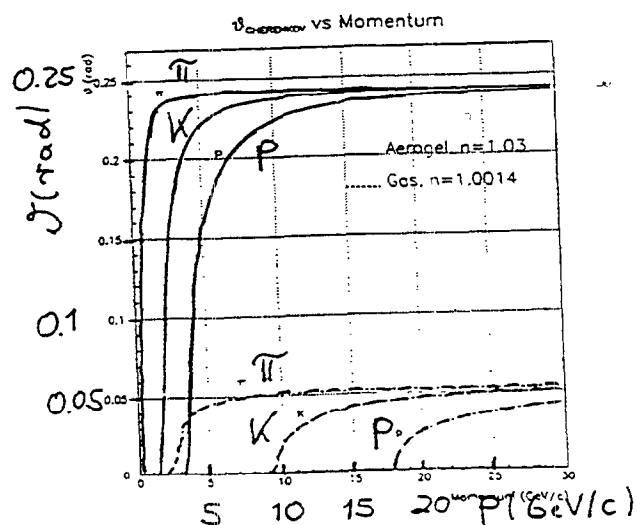
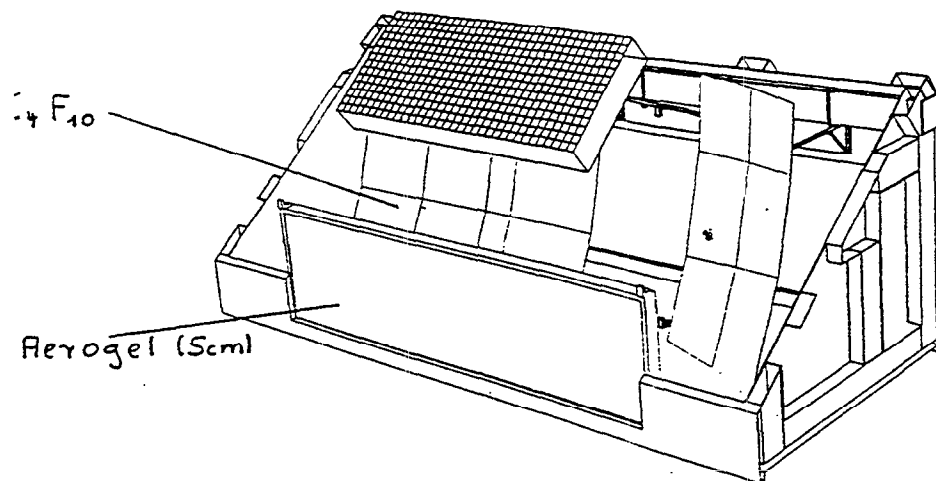
Upgrade I: Dual radiator RICH

(162)

► Clear silica aerogel ($n=1.03$; $\gamma_{\pi}=4.2$)

► C_4F_{10} ($n=1.0005$; $\gamma_{\pi}=32$)

	Aerogel	Gas
Threshold (π, K, p) GeV/c	0.6, 2.1, 3.9	4.4, 15.8, 30
Separation ($\pi/K; K/p$) GeV/c	1-9; 2-15.5	4-23.5, 10-25



COURIER

VOLUME 38 NUMBER 8 NOVEMBER 1998

HERA experiments get RICH

GRAN SASSO

The Italian laboratory's MACRO
neutrino detector is about to be
upgraded for neutrino oscillations

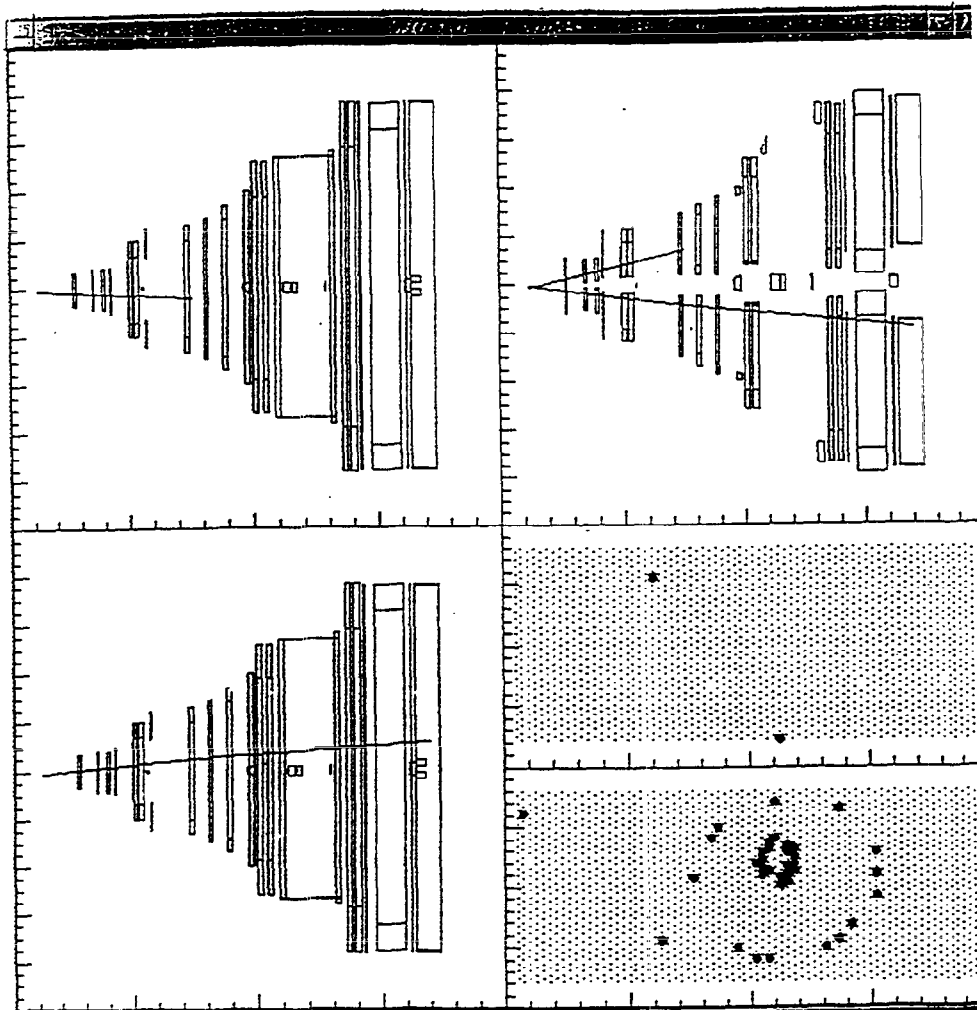
HIGH-ENERGY CULTURE

Why is it so difficult to convey the
excitement of new developments in
particle physics to the general public?

PARTICLES FOR EXPORT

In the quest for new particles, some
particle beams have already been
sent 730 kilometers a day

764



$$g_2(x)$$

Wandzura, Wilczek

(166)

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dz}{z} g_1(z) + \tilde{g}_2(x)$$

Quark-Gluon-Correlation
(Twist-3 Operator)

Long. pol. beam

Long. + transverse pol. target

Interesting Sum Rules:

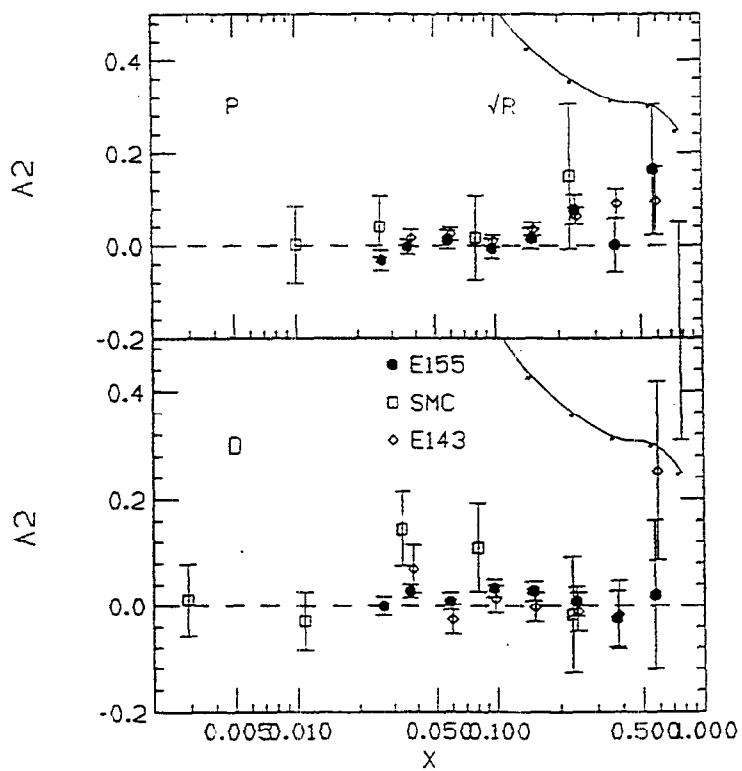
$$\Delta \int_0^1 g_2(x) dx = 0 \quad (\text{Burkhard-Cottingham})$$

$$\Delta \int_0^1 x^n g_2(x) dx = \frac{1}{4} \frac{n}{n+1} (-a_n + d_n) \quad n=2,4,\dots$$

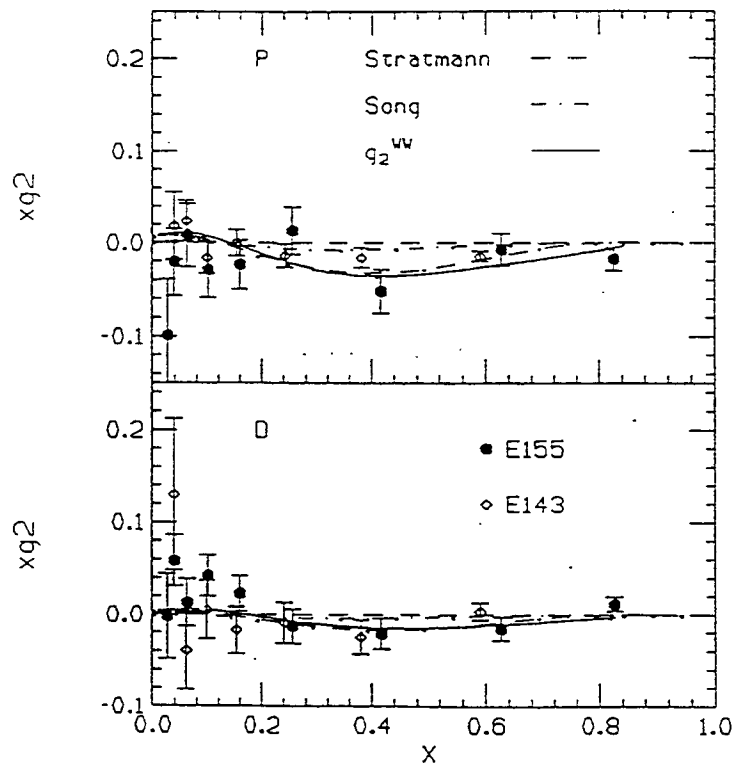
$$\Delta \int_0^1 x^n g_1(x) dx = \frac{a_n}{4} \quad \begin{array}{l} \text{Twist-2} \\ \text{Twist-3} \end{array} \quad \begin{array}{l} n=0,2,4,\dots \end{array}$$

Precise measurement of $g_2(x)$ very helpful in determining the nucleon wave function

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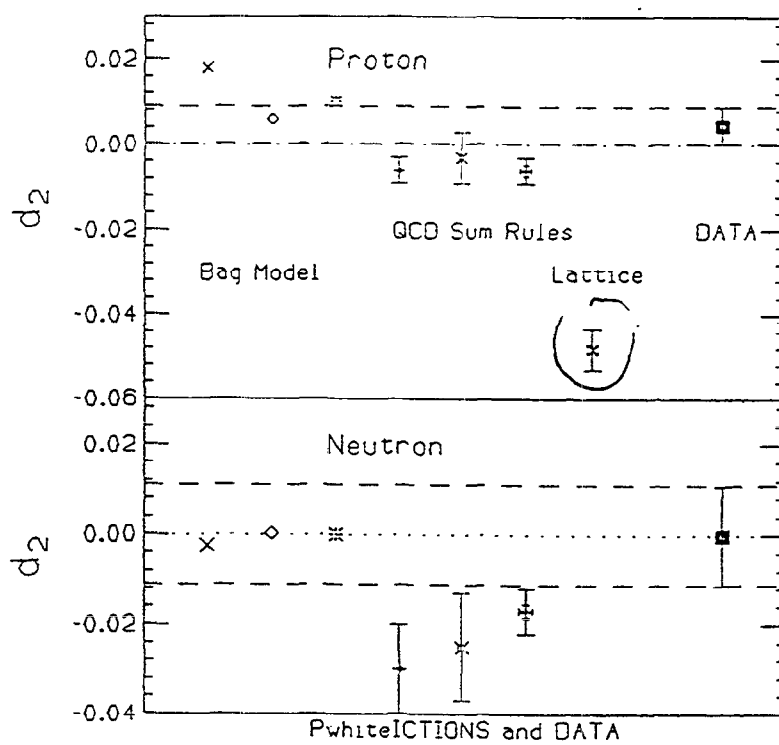
P_2 very small!



$g_2(x)$ small

data compatible with twist-2 (WW)

Twist-3 very small



Perspectives

HERMES

(170)

1998/99 : $\vec{D}$ (+ RICH, ω -wall, FQS, Λ -wheel)

■ g_1^d, g_1^n , Bjorken S.R

■ $\Delta u_v, \Delta d_v, \Delta \bar{u}, \Delta \bar{s}$

■ Δg open charm $D \rightarrow K\pi, K\mu\nu$ $\langle x_g \rangle \sim 0.3$
high p_t π $\langle x_g \rangle \sim 0.15$

$$\sigma\left(\frac{\Delta g}{g}\right) \approx 0.45 \sim 0.2$$

■ Λ - polarisation (l-transfer)

■ nuclear (unpol.) targets ($D_2, {}^3\text{He}, N_2, Kr$)

* hadronisation in nuclei

* g, ϕ - production (ang. distr, nucl. transp.)

2000 $\rightarrow$ 200?

■ H (+ RICH....)

■ transverse target pol.

$g_2(x), b_1^d(x), \Delta^d(x), h_1(x) \dots$

SLAC - E15X

1999: transverse target pol.: g_2, d_2

COMPASS / CERN

2000/1 →

(77)

- 2-stage spectrometer (ext. of SMC)
hadron-ID with 2 RICH,
high luminosity, $E_\mu = 100-200$ GeV
 $LiD^\uparrow$, $NH_3^\uparrow$ - targets

- Main aim: ΔQ_g

open charm ($0.07 < x_g < 0.4$) $\delta\left(\frac{\Delta Q_g}{Q_g}(0.14)\right) = 0.1$

high- p_t $K, H^\pm$ ($0.04 < x_g < 0.2$) $\delta\left(\frac{\Delta Q_g}{Q_g}(0.1)\right) = 0.05$

- $g_1, h_1, \dots$

RHIC 2000 →

- $\vec{p} \vec{p}$, $\sqrt{s} = 50-500$ GeV; PHENIX, STAR

- ΔQ_g $\vec{p} \vec{p} \rightarrow \gamma + \text{jet} \not\rightarrow q + q \rightarrow \gamma + q$

200 GeV : ($0.07 < x_g < 0.2$) $\delta\left(\frac{\Delta Q_g}{Q_g}\right) = 0.05 \rightarrow 0.4$

500 GeV ($0.03 < x_g < 0.1$) " $= 0.05 \rightarrow 0.2$

- $\frac{\Delta u/u}{\Delta d/d}$ $F_L^{W^\pm}$ $x > 0.05$

HERA - $\vec{p}$ 200? $\rightarrow$

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$\vec{e} \rightarrow e \vec{p}$ $\sqrt{s} = 300 \text{ GeV}$
 27.5 GeV 820 GeV

■ $g_1(x) \times \sim 5 \cdot 10^{-5}$

(but: depd. factor D large $\rightarrow$ errors)

■ Δg * NLO-fits
 * 2-jet events $\times \sim 5 \cdot 10^{-4}$
 *

Conclusions

(17)

■ Many new and interesting results:

■ Still plenty of open questions

* $\Delta G(x)$

* $L_2^{q,g}$

* $\Delta q_f(x)$

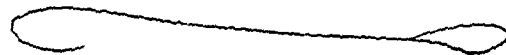
* low- x extrapolation

* $g_2(x) \rightarrow d_2$

* Q^2 -dependence of
polarised quark distributio

*
:
:

■ Plenty of exciting results to be
expected in the future



RHIC Spin Program

N.HAYASHI
RIKEN

The RHIC spin program is a new type of the experiments to study the spin-dependent structure function of the nucleon and probe possible new phenomena.

The RHIC (Relativistic Heavy Ion Collider) is now under constructing at BNL will be the first polarized proton-proton collider. Previously it was not possible to keep the polarization inside the synchrotron, but special sets of magnets, Siberian snake, make this type of experiment possible. The polarized proton acceleration study at AGS, which will be used as an injector, has made a big progress and now it has more than 0.50 polarization at over 20 GeV/c. The proton beam polarimetry is also important and many activities are going on.

The quark spin contributions to the nucleon spin have been studied extensively in polarized deep inelastic scattering (pDIS) experiments. However, it was found that the quark contribution is very small and the gluon spin contribution has a very weak constraint. The gluon contribution can be studied with prompt photon which mostly comes from quark-gluon compton reaction at RHIC energy. Fine granular PHENIX EM Calorimeter is suitable to suppress a large background and large acceptance of STAR EMCAL allows to detect aside jet together with photon. Moreover, pDIS inclusive reaction alone is not possible to separate individual quark or anti-quark flavor contribution. The flavor decomposition can be done with W -boson production at RHIC and high transverse momentum lepton is a W decay signature.

In addition to these important subjects, this program can measure new structure function, the transversity distribution, through the transversely polarized proton collision. Studying parity violation may give us a hint for a beyond standard model physics. The experiments contain those programs will start taking data in the year 2000.

Introduction

- Polarized Nucleon Structure Function

- Proton Spin Crisis: (SLAC, 1980s)

$$\Delta\Sigma \sim 0, \Delta s \sim -0.2$$

- More pol-DIS experiments — Update on $\Delta\Sigma$:

(SMC, SLAC, HERMES 1992~)

$$\Delta\Sigma \simeq 0.3$$

- We know

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

- Rest of the nucleon spin in ΔG ??

QCD NLO analysis $\Rightarrow \Delta G \sim 0 \sim 2 \pm 2$

* Current knowledge on ΔG is not satisfactory

- Some information about flavor decomposition

$(\Delta u, \Delta d, \Delta s)$. But always $\Delta\bar{u}(x) = \Delta\bar{d}(x)$?

C.Bourrely et al., hep-ph/9803229

Unpolarized case $\Rightarrow \bar{u}(x) \neq \bar{d}(x)$ (NMC, NA71, E802)

- Polarized RHIC (PHENIX, STAR) will measure

$$\Delta G(x), \Delta\bar{q}(x)...$$

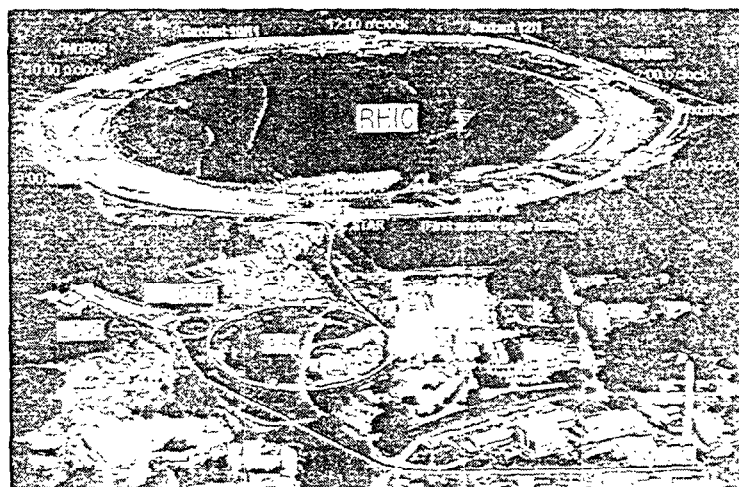
- Transversity Distribution

Relativistic Heavy Ion Collider (RHIC)

as The First Polarized Proton-Proton Collider

RIKEN-BNL Collaboration

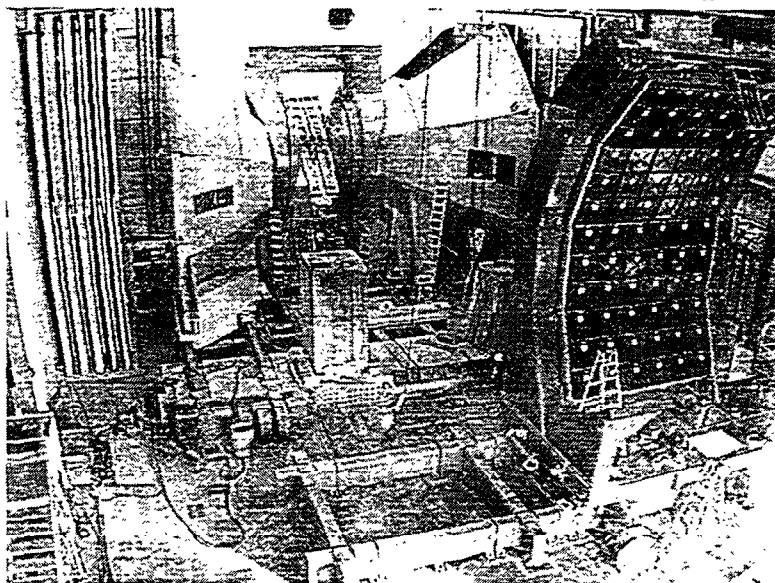
Since 1995



- $\sqrt{s} = 50 \sim 500 \text{ GeV}$
- Beam polarization: $P_B \simeq 0.7$
- Luminosity:
 - $\mathcal{L} = 8 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ at $\sqrt{s} = 200 \text{ GeV}$
 - $\mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ at $\sqrt{s} = 500 \text{ GeV}$
- Integrated Luminosity:
 - 10 weeks, 70% efficiency with full luminosity
 - $\int \mathcal{L} dt = 320 \text{ pb}^{-1}$ at $\sqrt{s} = 200 \text{ GeV}$
 - $\int \mathcal{L} dt = 800 \text{ pb}^{-1}$ at $\sqrt{s} = 500 \text{ GeV}$

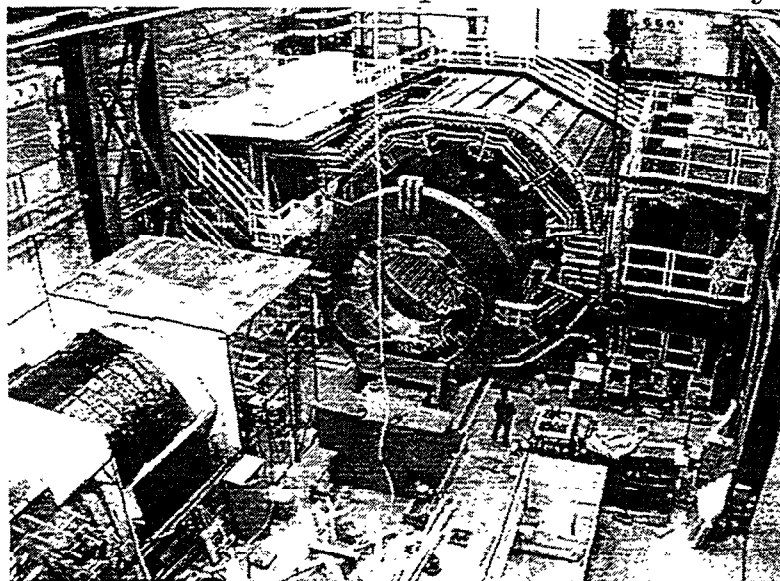
Todays Status (Nov '98)

Current PHENIX Experimental Hall picture



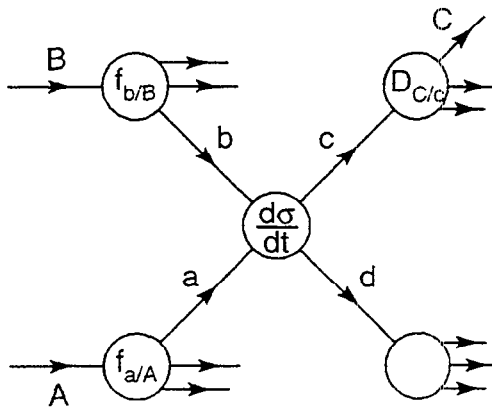
<http://www.chic.knl.gov/phenix/FWW/figures/mfh/todays-mfh.jpg>

STAR construction picture of the day



Hadron-Hadron Reaction

$$A + B \rightarrow C + X$$



- Parton distributions with x_a, x_b :
 $f_{a/A}(x_a), f_{b/B}(x_b)$
- Parton reaction: $\frac{d\hat{\sigma}_{cd}^{ab}}{dt}$
- Fragmentation:
 $D_{C/c}(z)$

$$E \frac{d^3\sigma}{dp^3} \sim \sum_{abcd} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes \frac{d\hat{\sigma}_{cd}^{ab}}{dt} \otimes D_{C/c}(z)$$

Asymmetry for $A + B \rightarrow C + X$

$$A = E \frac{d^3\Delta\sigma}{dp^3} / E \frac{d^3\sigma}{dp^3} \simeq \frac{\Delta f_{a/A}(x_a)}{f_{a/A}(x_a)} \otimes \frac{\Delta f_{b/B}(x_b)}{f_{b/B}(x_b)} \otimes \hat{a}_{LL}(ab \rightarrow cd)$$

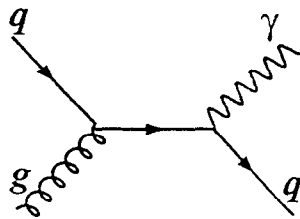
- A: Experimentally measured asymmetry
- Polarized parton distributions with x_a, x_b :
 - $\Delta f_{a/A}(x_a), \Delta f_{b/B}(x_b)$
- Parton level asymmetry: $\hat{a}_{LL}(ab \rightarrow cd)$
 - calculated in pQCD
- Fragmentation:
 - Not relevant for γ, W
 - Cancel for asymmetry in principle

Prompt Photon

$$\vec{p}\vec{p} \rightarrow \gamma + X$$

- Two processes to produce γ

(a)



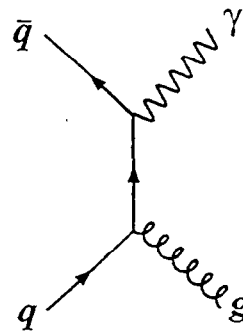
gluon Compton

$$g + q \rightarrow \gamma + q$$

80 ~ 90 %

dominant

(b)

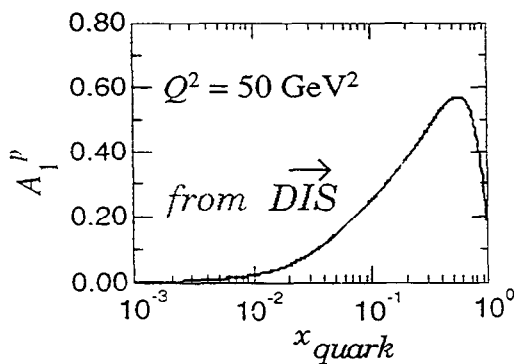


annihilation

$$q + \bar{q} \rightarrow \gamma + g$$

10 ~ 20 %

$$A_{LL}(p_T) = \frac{\Delta G(x)}{G(x)} \frac{g_1(x)}{F_1(x)} \hat{a}_{LL}(g + q \rightarrow \gamma + q)$$

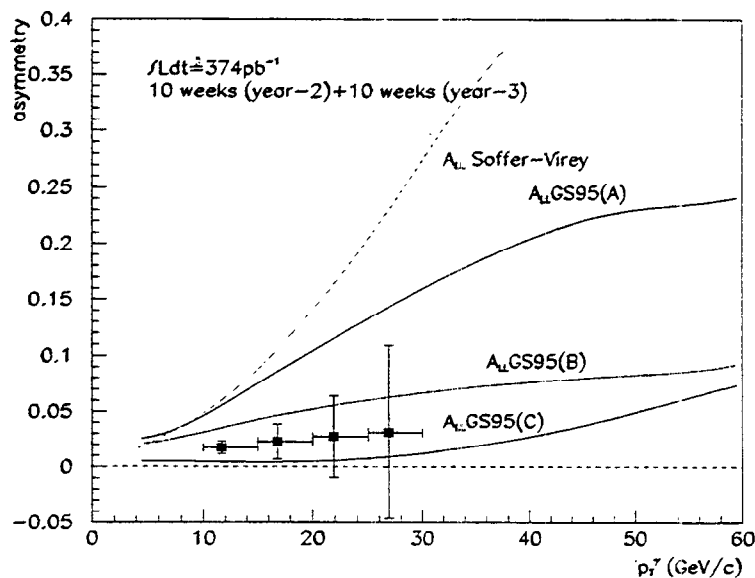


$$A_1 \simeq \frac{g_1(x)}{F_1(x)} \simeq \frac{\sum_i e_i^2 \Delta q_i(x)}{\sum_i e_i^2 q_i(x)}$$

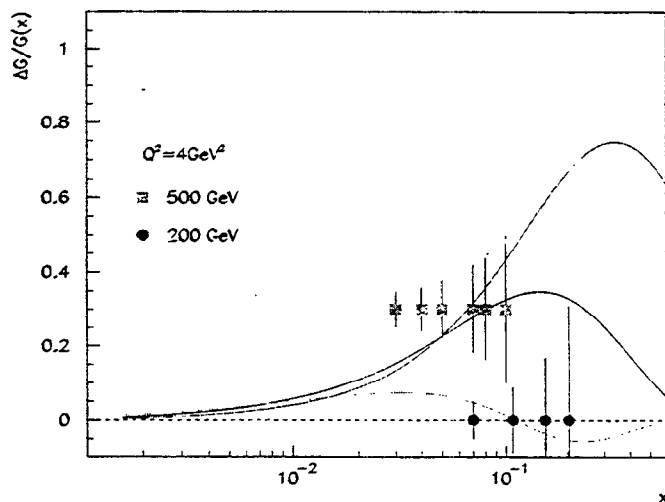
from pDIS experiment

- Theoretically rather clear process

Prompt Photon Asymmetry and Sensitivity



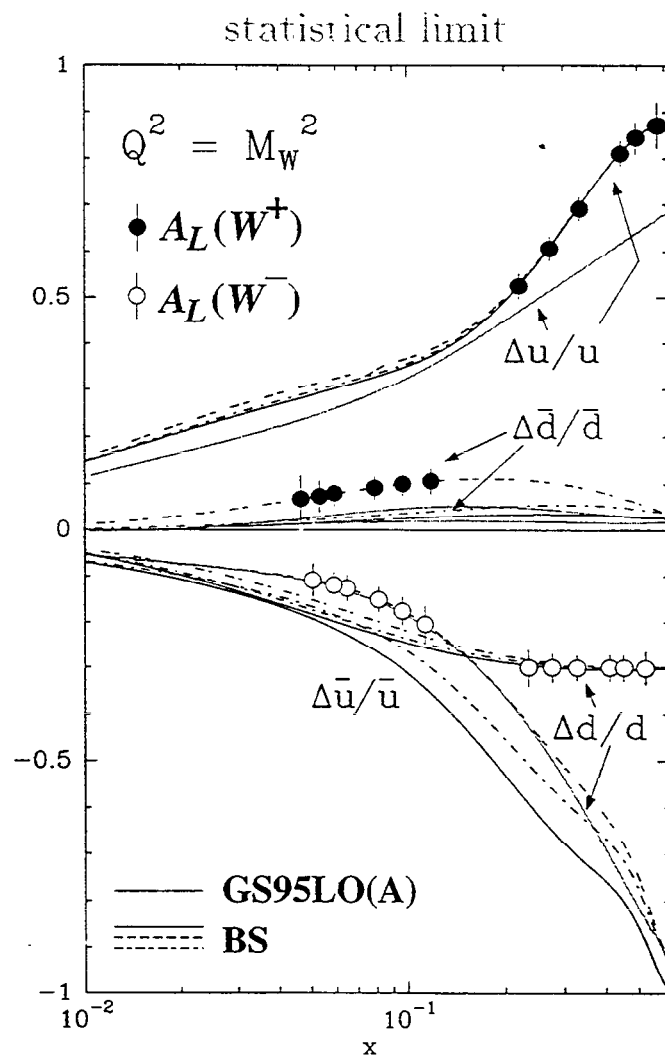
- GS95(A): $\Delta G = 1.71$
- GS95(B): $\Delta G = 1.63$
- GS95(C): $\Delta G = 1.02$



Phys. Rev. Lett. 67, 1253 (1991)

- 5-30% at $\sqrt{s} = 200$ GeV
- 5-20% at $\sqrt{s} = 500$ GeV

Sensitivity



courtesy of Jacques Soffer & Claude Bourrely

- Anti-quark polarization measured with A_L^W
- More systematic studies are underway
 - background from π/K decays
 - background from Z^0 decays

Summary

- Significant Progress on Polarized proton acceleration study
- Many Polarimetry works going on
- Detectors installation progress
- Spin Structure Function Studies
 - $\Delta G(x)$ measurements:
 - * prompt γ^* 5-30% at $\sqrt{s} = 200$ GeV,
5-20% at $\sqrt{s} = 500$ GeV (PHENIX)
 - * $\gamma + jet$, di-jets (STAR)
 - * π^0 inclusive, open heavy quark, quarkonium production
 - $\Delta \bar{q}(x)$ measurements:
 - * 2% error through A_L^W
- further systematic studies are underway ...
- New measurements
 - Transversity
 - Single spin asymmetry A_N
 - Search for new parity violated interaction

RHIC/Spin Plan

- RHIC turn on (Engineering Run), January to July 1999
- Siberian Snakes installation (1999 summer)
- RHIC Physics Run, October 1999!
 - Polarized proton commissioning
 - $1 \sim 10\%$ luminosity (?)
- Spin rotators installation (2000 summer)
- First polarized proton-proton physics run (2000-01)
 - Heavy Ion pp comparison run
- Full capabilities: $\sqrt{s} = 500$ GeV (2002-?)

Twist-3 Effects in Hard Processes

Yuji Koike

Department of Physics, Niigata University, Ikarashi, Niigata 950-21, Japan

Abstract

The quark and gluon distribution functions in the nucleon can be classified by the “twist”. Twist-2 distributions have a simple parton model interpretation and contribute to the cross sections with no power suppression with respect to the the hard momentum. On the other hand, higher twist distributions represent complicated quark-gluon correlation and appear in the cross section as a power suppressed contribution. In general, it is difficult to measure the higher twist contributions in hard processes, since they are hidden behind the leading twist-2 contributions. However, when the leading twist-2 contribution is absent due to a kinematics, one can measure some twist-3 contributions. As an example for this type of interesting “twist-3 processes”, We discuss (1) Longitudinal-transverse spin-asymmetry in the nucleon-nucleon polarized Drell-Yan process, and (2) Semi-inclusive production of the polarized spin-1/2 baryon in the deep-inelastic scatterings.

In the first part of this talk, we present a first estimate for the longitudinal-transverse spin asymmetry (A_{LT}) in the nucleon-nucleon polarized Drell-Yan process at RHIC and HERA- $\bar{N}$ energies in comparison with A_{LL} and A_{TT} . A_{LT} receives contribution from g_1 , the transversity distribution h_1 , and the twist-3 distributions g_T and h_L . For the twist-3 contribution we use the bag model prediction evolved to a high energy scale by the large- N_c evolution equation. We found that A_{LT} (normalized by the asymmetry in the parton level) is much smaller than the corresponding A_{TT} . Twist-3 contribution given by the bag model also turned out to be negligible.

In the second part of this talk, we analyze the semi-inclusive deep-inelastic scattering from the nucleon detecting a polarized spin-1/2 baryon in the final state. It turns out that with the unpolarized electron beam and the transversely (longitudinally) polarized nucleon target, detection of the longitudinally (transversely) polarized spin-1/2 baryon gives rise to a leading twist-3 contribution to the spin-asymmetries.

* Twist Expansion for Inclusive Cross Sections

$$\sigma(Q^2) \sim \sigma_0(\ln Q^2) + \frac{\Lambda_{QCD}}{Q} \sigma_1(\ln Q^2) + \frac{\Lambda_{QCD}^2}{Q^2} \sigma_2(\ln Q^2) + \dots$$

• Twist-2

• "Parton model"

• Incoherent scatt.
from each parton

• Twist-3

• Quark-Gluon correlation

• Peculiar to spin-dependant
processes

Twist-4

* Interesting "Twist-3" processes

$$\sigma(Q^2) \sim \cancel{\sigma_0(\ln Q^2)} + \frac{\Lambda_{QCD}}{Q} \sigma_1(\ln Q^2) + \dots$$

Kinematic
suppression

New test of p-QCD
beyond twist-2

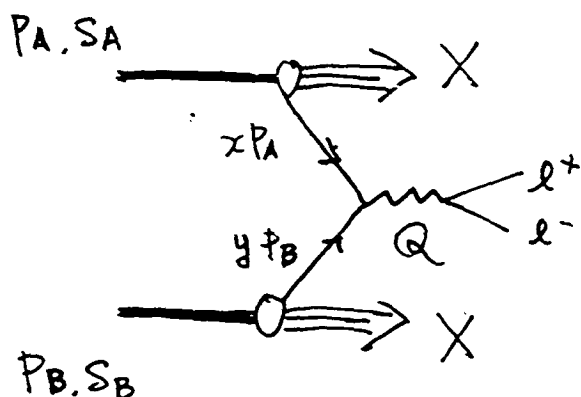
$$\cdot \vec{e} + p \uparrow \rightarrow e' + X \quad : \quad g_T(x, Q^2)$$

$$\cdot \vec{\gamma} + p \uparrow \rightarrow e^+ e^- + X \quad : \quad h_L(x, Q^2), g_T(x, Q^2)$$

• semi-inclusive DIS

P.L. B430 (98) 195

★ Polarized N-N Drell-Yan



$$S = (P_A + P_B)^2 \quad (= (\text{CM energy})^2)$$

$$Q^2 = (xP_A + yP_B)^2 \cong xyS \quad (M_N^2 \ll S)$$

• Double spin asymmetry

$$A_{S_A S_B} = \frac{\sigma(S_A S_B) - \sigma(S_A, -S_B)}{\sigma(S_A S_B) + \sigma(S_A, -S_B)}$$

$S_A, S_B = +, -$: Longitudinal pol.

$\uparrow \downarrow$: Transverse pol.

$$A_{LL} = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)} = \frac{\sum_a e_a^2 g_1^a(x, Q^2) g_1^{\bar{a}}(y, Q^2)}{\sum_a e_a^2 f_1^a(x, Q^2) f_1^{\bar{a}}(y, Q^2)}$$

$$A_{TT} = \frac{\sigma(\uparrow\uparrow) - \sigma(\uparrow\downarrow)}{\sigma(\uparrow\uparrow) + \sigma(\uparrow\downarrow)} = \frac{\sin^2\theta \cos 2\phi}{1 + \cos^2\theta} \frac{\sum_a e_a^2 h_1^a(x, Q^2) h_1^{\bar{a}}(y, Q^2)}{\sum_a e_a^2 f_1^a(x, Q^2) f_1^{\bar{a}}(y, Q^2)}$$

$\equiv a_{LL}$

$$A_{LT} = \frac{\sigma(+\uparrow) - \sigma(+\downarrow)}{\sigma(+\uparrow) + \sigma(+\downarrow)}$$

$$= \frac{2 \sin 2\theta \cos \phi}{1 + \cos^2\theta} \frac{M}{Q} \frac{\sum_a e_a^2 [g_1^a(x, Q^2) y g_T^{\bar{a}}(y, Q^2) + x h_L^a(x, Q^2) h_1^{\bar{a}}(y, Q^2)]}{\sum_a e_a^2 f_1^a(x, Q^2) f_1^{\bar{a}}(y, Q^2)}$$

$\equiv a_{LT}$

Quark distribution functions (Jaffe-Ji, '92)

twist \ spin	Average	Longitudinal	Transverse
twist-2	$f_1(x, \mu^2)$	$g_1(x, \mu^2)$	$h_1(x, \mu^2)$
twist-3	$e(x, \mu^2)$	$h_L(x, \mu^2)$	$g_T(x, \mu^2)$

Distribution functions in QCD

= Light-cone correlation functions in a Nucleon

- Assume $\vec{p} \parallel \hat{e}_3$
- Put $\phi(z) = \psi_+(z)$: Light-cone "good" component

$$\left(\begin{array}{l} z^+ = z_\perp = 0 \\ z^2 = 0 \end{array} \right)$$
 Dynamically independent variables.

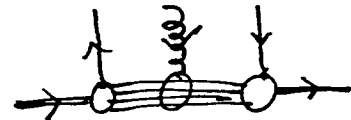
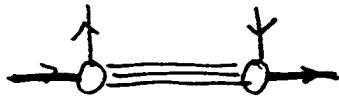
$$\bar{z}^\pm \equiv \frac{1}{\sqrt{2}}(z^0 \pm z^3) \quad (\psi_\pm \equiv p_\pm \psi, p_\pm \equiv \frac{1}{2} \gamma^\mp \gamma^\pm)$$

★ Twist-2

$$g_1(x, \mu^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^-} \langle PS_{11} | \bar{\phi}(0) \gamma^+ \gamma_5 \phi(z) |_{\mu^2} | PS_{11} \rangle$$

$\downarrow p \exp \left\{ i g \int_z^0 d\bar{z}_\mu A^\mu(\bar{z}) \right\}$

$$h_1(x, \mu^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^-} \langle PS_{11} | \bar{\phi}(0) \gamma^\perp \gamma^+ \gamma_5 \phi(z) |_{\mu^2} | PS_{11} \rangle$$



★ Twist-3

$$g_T(x) = \underbrace{\int_x^1 \frac{g_1(y)}{y} dy}_{\text{twist-2}} + \underbrace{\tilde{g}_T(x)}_{\text{purely twist-3 part}} \equiv g_T^{ww}(x) + \tilde{g}_T(x)$$

$$h_L(x) = 2x \underbrace{\int_x^1 \frac{h_1(y)}{y^2} dy}_{\text{Wandzura-Wilczek part}} + \underbrace{\tilde{h}_L(x)}_{\text{purely twist-3 part}} \equiv h_L^{ww}(x) + \tilde{h}_L(x)$$

Wandzura-Wilczek part purely twist-3 part

$$\tilde{h}_L(x, \mu^2) = \frac{iP^+}{M} \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{-2ixP^-} \int_0^1 u du \int_{-u}^u t dt (z^-)^2$$

$$\times \langle PS_{11} | \bar{\phi}(uz) \gamma_5 \gamma^\perp \underline{g_G^{+1}}(tz) \phi(-uz) |_{\mu^2} | PS_{11} \rangle$$

quark-gluon correlation

(similarly for $\tilde{g}_T(x, \mu^2)$)

* Estimate ALT , using Bag model and available parametrization for distribution functions

- $\left. \begin{array}{l} f_1 \\ g_1 \end{array} \right\}$ - GRSV standard distribution
 (Glück et al, Phys. Rev. D53, 4775 (96))
 - GS(A) distribution
 (Gehrmann-Stirling, Phys. Rev. D53, 6100 (96))

h_1 : Assume at Low Q_0^2

$$h_1(x, Q_0^2) = g_1(x, Q_0^2)$$

GRSV @ $Q_0^2 = 0.23 \text{ GeV}^2$

GS @ $Q_0^2 = 1 \text{ GeV}^2$

- This is suggested by a low energy effective model for the nucleon

↓ LO Evolution equation

$h_1(x, Q^2)$ @ High Q^2

Twist-3 distributions

$$g_T(x, Q^2) = g_T^{tw}(x, Q^2) + \tilde{g}_T(x, Q^2)$$

$$h_L(x, Q^2) = h_L^{tw}(x, Q^2) + \tilde{h}_L(x, Q^2)$$

↑ ↑
Evolution, separately

* $\tilde{g}_T(x, Q^2), \tilde{h}_L(x, Q^2)$

Low scale ← MIT bag model

bag scale $\mu_{bag}^2 = 0.081 \text{ GeV}^2$

$\mu_{bag}^2 = 0.25 \text{ GeV}^2$

Large- N_c evolution

(Ali et al, P.L. B266, 117 (91))

(Balitsky et al. P.R.L. 77, 3078 (96))

• As simple as twist-2 evolution

• Accurate to $O(1/N_c^2) \sim 10\%$

High scale

(Kanazawa-Koike, P.L. B403, 357 (97))

NB. Ignore mixing with gluon distribution
for $\tilde{g}_T(x, \mu^2)$

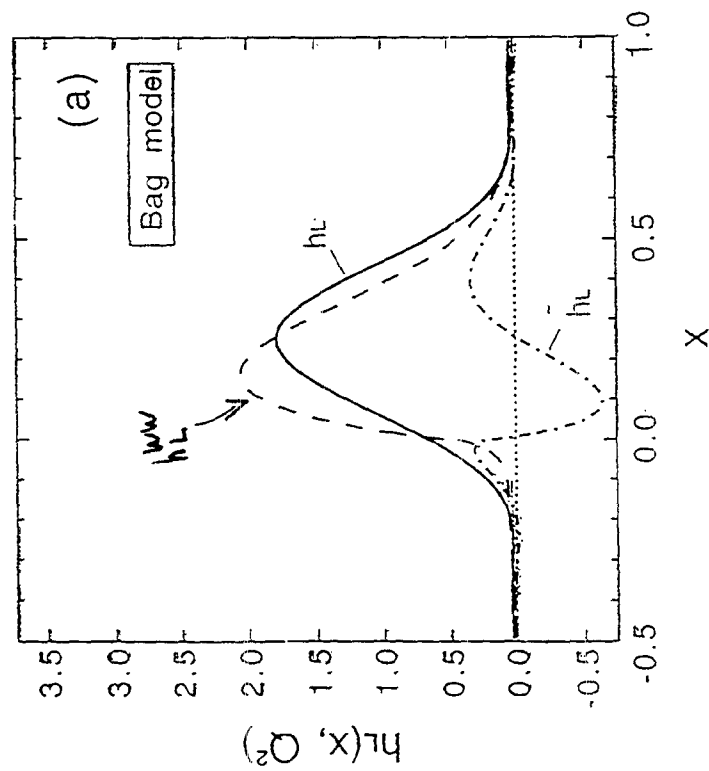


Fig.2(a)

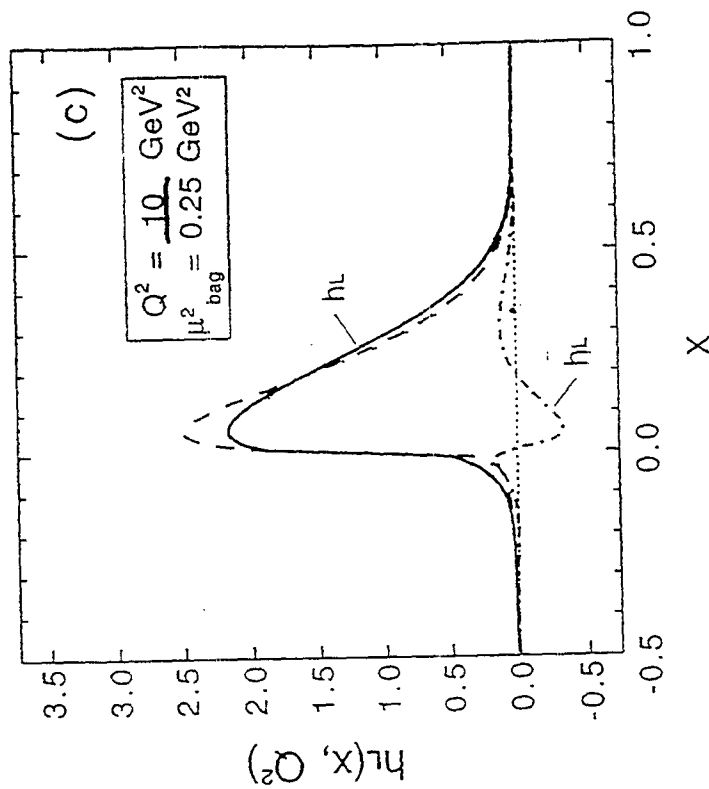


Fig.2(c)

Results for the asymmetries

◦ $-A_{LT}/a_{LT}$ in comparison with $-A_{LL}$, $-A_{TT}/a_{TT}$

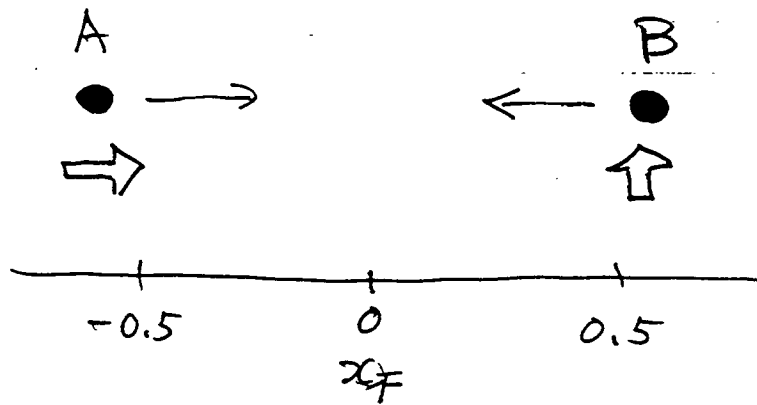
◦ Kinematic variables

S , Q^2 and $x_F = x - y$

$$Q^2 = xyS \Rightarrow x = \frac{1}{2} \left[x_F + \sqrt{x_F^2 + \frac{4Q^2}{S}} \right]$$

$$y = \frac{1}{2} \left[-x_F + \sqrt{x_F^2 + \frac{4Q^2}{S}} \right]$$

◦ CM frame



$$\sqrt{S} = 50, 200 \text{ GeV}$$

(cf. RHIC, $50 < \sqrt{S} < 500 \text{ GeV}$
HERA- $\vec{N}$, $\sqrt{S} = 40 \text{ GeV}$)

GRSV + Bag

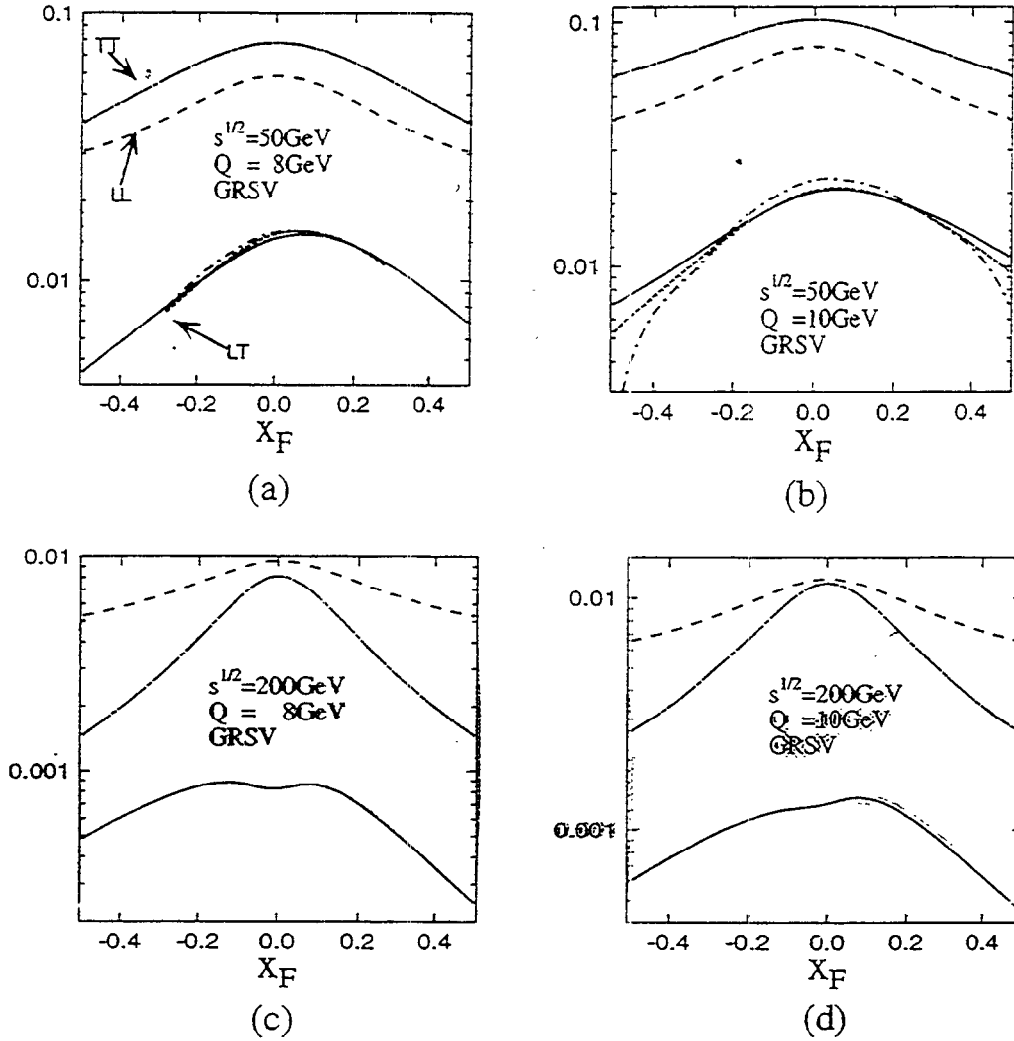


Figure 2: Double spin asymmetries, $\tilde{A}_{LL}$, $\tilde{A}_{TT}$, $\tilde{A}_{LT}$, for the polarized Drell-Yan using the GRSV parton distribution and the bag model at $Q = 8, 10 \text{ GeV}$ and $\sqrt{s} = 50, 200 \text{ GeV}$. The solid line denotes $\tilde{A}_{LT}$ with only the Wandzura-Wilczek contributions in g_T and h_L . The short dash-dot line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.25 \text{ GeV}^2$, and the dotted line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.081 \text{ GeV}^2$. The long dashed line corresponds to $\tilde{A}_{LL}$, and the long dash-dot line corresponds to $\tilde{A}_{TT}$.

Summary

- Estimate of $-A_{LT}/a_{LT}$, $-A_{LL}$, $-A_{TT}/a_{TT}$ using GRSV & GS parton distributions and Bag model
 - Asymmetries $\uparrow$ as $Q^2/s \uparrow$
 - $|A_{LT}/a_{LT}| < |A_{LL}|, |A_{TT}/a_{TT}|$
 - Bag model input for the purely twist-3 parts $\tilde{h}_L(x, Q^2)$, $\tilde{g}_T(x, Q^2)$
 $\rightarrow$ small
 Because of strong Q^2 -dependence of those pieces. (evolution)
- $\Rightarrow$ WAIT FOR EXP. AT RHIC & HERA- $\vec{N}$

Polarized proton-deuteron Drell-Yan processes

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ABSTRACT

In order to investigate spin structure of spin-1 hadrons, we study general formalism of proton-deuteron Drell-Yan processes [1]. Because of the spin-1 nature of the deuteron, there are new structure functions which cannot be measured in the proton-proton reactions. Imposing Hermiticity, parity conservation, and time-reversal invariance, we find that 108 structure functions exist in the Drell-Yan processes. However, the number reduces to 22 after integrating the cross section over the virtual-photon transverse momentum $\vec{Q}_T$ or after taking the limit $Q_T \rightarrow 0$. There are 11 new structure functions in addition to the 11 ones in the proton-proton Drell-Yan processes. The additional structure functions are associated with the tensor structure of the deuteron, and they could be measured by quadrupole spin asymmetries. We show a number of spin asymmetries for extracting the polarized structure functions.

Next, we analyze the polarized proton-deuteron Drell-Yan processes in a naive parton model [2]. Quark and antiquark correlation functions are expressed in terms of possible combinations of Lorentz vectors and pseudovectors with the constraints of Hermiticity, parity conservation, and time-reversal invariance. Then, we find tensor polarized distributions for the deuteron. There are only four finite structure functions and the others vanish after integrating the cross section over the virtual-photon transverse momentum $\vec{Q}_T$ or after taking the limit $Q_T \rightarrow 0$. One of the finite structure functions is related to the tensor structure function b_1 , and it does not exist in the proton-proton reactions. The vanishing structure functions should be associated with higher-twist physics. The tensor distributions, particularly the tensor-polarized antiquark distributions, can be measured by the quadrupole polarization measurements. The proton-deuteron reactions may be realized in the RHIC-Spin project and other future ones.

References

- [1] S. Hino and S. Kumano, preprint SAGA-HE-136-98 (Phys. Rev. D in press).
- [2] S. Hino and S. Kumano, preprint SAGA-HE-139-98.

Research purpose

description of proton-deuteron Drell-Yan

- Spin-1/2 hadrons (proton)

Proton spin structure:

well investigated theoretically and experimentally

- Spin-1 hadrons
 - tensor spin structure as a new ingredient
 - ed scattering: some theoretical studies
 - pd Drell-Yan: no paper
 - investigate polarized pd Drell-Yan process

Purposes

- 1) test our knowledge of spin physics
by another observable
- 2) understand tensor structure in a parton model
- 3) establish theoretical formalism
for the next-generation RHIC-Spin project

$$\int dx b_1^D(x) = \frac{5}{6} \left[\Gamma_{0,0} - \frac{1}{2}(\Gamma_{1,1} + \Gamma_{-1,-1}) \right] + \frac{1}{9} (\delta Q + \delta \bar{Q})_{\text{sea}}$$

macroscopically

$$\Gamma_{0,0} = \lim_{t \rightarrow 0} \left[F_c(t) - \frac{t}{3 M^2} F_Q(t) \right] \quad \begin{array}{c} t \rightarrow 0 \\ \updownarrow \\ \text{---} \end{array}$$

$$\Gamma_{+1,+1} = \Gamma_{-1,-1} = \lim_{t \rightarrow 0} \left[F_c(t) + \frac{t}{6 M^2} F_Q(t) \right]$$

F.E.Close & SK,
Phys. Rev. D42 (1990)

$$\int dx b_1^D(x) = \lim_{t \rightarrow 0} -\frac{5}{12} \frac{t}{M^2} F_Q(t) + \frac{1}{9} (\delta Q + \delta \bar{Q})_{\text{sea}}$$

$$\Rightarrow \lim_{t \rightarrow 0} -\frac{5}{12} \frac{t}{M^2} F_Q(t)$$

Gottfried sum rule

$$\int \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} \int dx [u_v - d_v] + \frac{2}{3} \int dx [\bar{u} - \bar{d}]$$

Experimental possibilities

HERA, ELFE, RCNP ?

$$\begin{aligned} \frac{d\sigma}{d^4Q d\Omega_0} &= \frac{1}{4\pi} \sum_{L,M} f(L) D_{M0}^L(\phi_0, \theta, 0) \sum_{\lambda, \lambda'} \sum_X \delta^4(P_A + P_B - Q - P_X) \\ &\quad \times \langle 1\lambda : LM | 1\lambda' \rangle \sum_{\mu, \mu'} \sum_{\nu, \nu'} \rho(AB)_{\mu\nu: \mu'\nu'} F_{\lambda'\mu'\nu'}^\dagger F_{\lambda\mu\nu} \\ F_{\lambda\mu\nu} &= \sqrt{\frac{\pi \alpha^2}{3 s \vec{k}^2}} \langle X | \vec{\epsilon}^*(\lambda) \cdot \vec{J}(0) | A(\mu) B(\nu) \rangle \end{aligned}$$

Spin density matrix:

$$\rho(AB)_{\mu\nu: \mu'\nu'} = \rho(A)_{\mu\mu'} \rho(B)_{\nu\nu'}$$

$$\rho = \frac{1}{2S+1} \sum_{K,N} \langle \tau_{KN}(S) \rangle \tau_{KN}^\dagger(S)$$

$$\tau_{KN}(S) = \sqrt{2S+1} \sum_{\mu, \mu'} (-1)^{S-\mu} \langle S\mu' : S-\mu | KN \rangle | S\mu' \rangle \langle S\mu |$$

$$\begin{aligned} \rho_{\mu\mu'} &= \langle S\mu | \rho | S\mu' \rangle \\ &= \sum_{K,N} \frac{\sqrt{2K+1}}{2S+1} \langle S\mu' : KN | S\mu \rangle \langle \tau_{KN}^\dagger(S) \rangle \end{aligned}$$

$$\langle \tau_{00}^\dagger(S) \rangle = 1, \quad \langle \tau_{1N}^\dagger(1/2) \rangle = |\vec{S}_A| D_{N0}^1(\alpha_A - \Phi, \beta_A, 0)$$

$$\langle \tau_{1N}^\dagger(1) \rangle = \sqrt{\frac{3}{2}} |\vec{S}_B| D_{N0}^1(\alpha_B + \Phi, \beta_B, 0), \quad \langle \tau_{2N}^\dagger(1) \rangle = \sqrt{2} \vec{S}_B^2 D_{N0}^2(\alpha_B + \Phi, \beta_B, 0)$$

$$\begin{aligned} \frac{d\sigma}{d^4Q d\Omega_0} &= \frac{1}{4\pi} \sum_{L=0,2} f(L) \sum_M D_{M0}^L(\phi_0, \theta, 0) \left[R_{L00}^{M00} + \sqrt{3} |\vec{S}_A| \sum_{N_1} R_{L10}^{MN_10} D_{N_10}^1(\alpha_A - \Phi, \beta_A, 0) \right. \\ &\quad + \frac{3}{\sqrt{2}} |\vec{S}_B| \sum_{N_2} R_{L01}^{M0N_2} D_{N_20}^1(\alpha_B + \Phi, \beta_B, 0) + \sqrt{10} \vec{S}_B^2 \sum_{N_2} R_{L02}^{M0N_2} D_{N_20}^2(\alpha_B + \Phi, \beta_B, 0) \\ &\quad + \frac{3\sqrt{3}}{\sqrt{2}} |\vec{S}_A| |\vec{S}_B| \sum_{N_1, N_2} R_{L11}^{MN_1N_2} D_{N_10}^1(\alpha_A - \Phi, \beta_A, 0) D_{N_20}^1(\alpha_B + \Phi, \beta_B, 0) \\ &\quad \left. + \sqrt{30} |\vec{S}_A| \vec{S}_B^2 \sum_{N_1, N_2} R_{L12}^{MN_1N_2} D_{N_10}^1(\alpha_A - \Phi, \beta_A, 0) D_{N_20}^2(\alpha_B + \Phi, \beta_B, 0) \right] \quad 336 \end{aligned}$$

Structure functions

$$R_{L K_1 K_2}^{M N_1 N_2} \equiv \frac{1}{6} \sum_{\lambda, \mu, \nu} \sum_X \delta^4(P_A + P_B - Q - P_X) \langle 1\lambda : LM | 1\lambda' \rangle$$

$$\times F_{\lambda'\mu'\nu'}^\dagger F_{\lambda\mu\nu} \langle \frac{1}{2}\mu' : K_1 N_1 | \frac{1}{2}\mu \rangle \langle 1\nu' : K_2 N_2 | 1\nu \rangle$$

$$R_{L K_1 K_2}^{M N_1 N_2} = (-1)^{M+N_1+N_2} \left(R_{L K_1 K_2}^{-M -N_1 -N_2} \right)^* \quad (\text{Hermiticity})$$

$$R_{L K_1 K_2}^{M N_1 N_2} = (-1)^{L+M+K_1+K_2+N_1+N_2} R_{L K_1 K_2}^{-M -N_1 -N_2} \quad (\text{Parity, Time-reversal})$$

$$R_{L K_1 K_2}^{M N_1 N_2} = \begin{cases} \text{real} & L + K_1 + K_2 = \text{even} \\ \text{imaginary} & L + K_1 + K_2 = \text{odd} \\ 0 & L + K_1 + K_2 = \text{odd and } M + N_1 + N_2 = 0 \end{cases}$$

Imposing these constraints, we obtain **108** independent structure functions.

Integration over Φ (azimuthal angle of $\vec{Q}_T$) : **22**

$$2 \int d\Phi \frac{d\sigma}{d^4Q d\Omega} = f(0) \left[R_{000}^{000} + \frac{3\sqrt{3}}{\sqrt{2}} |\bar{S}_A| |\bar{S}_B| \left\{ \cos\beta_A \cos\beta_B R_{011}^{000} + \sin\beta_A \sin\beta_B \cos(\alpha_A + \alpha_B) R_{011}^{011} \right\} \right.$$

$$\left. + \frac{\sqrt{5}}{\sqrt{2}} \bar{S}_B^2 (3 \cos^2 \beta_B - 1) R_{002}^{000} - 3\sqrt{10} |\bar{S}_A| \bar{S}_B^2 \sin\beta_A \sin\beta_B \cos\beta_B \sin(\alpha_A + \alpha_B) i R_{012}^{011} \right]$$

$$+ f(2) (3 \cos^2 \theta - 1) \left[\frac{1}{2} R_{200}^{000} + \frac{3\sqrt{3}}{2\sqrt{2}} |\bar{S}_A| |\bar{S}_B| \left\{ \cos\beta_A \cos\beta_B R_{211}^{000} + \sin\beta_A \sin\beta_B \cos(\alpha_A + \alpha_B) R_{211}^{011} \right\} \right.$$

$$\left. + \frac{\sqrt{5}}{2\sqrt{2}} \bar{S}_B^2 (3 \cos^2 \beta_B - 1) R_{202}^{000} - \frac{3\sqrt{5}}{\sqrt{2}} |\bar{S}_A| \bar{S}_B^2 \sin\beta_A \sin\beta_B \cos\beta_B \sin(\alpha_A + \alpha_B) i R_{212}^{011} \right]$$

$$+ f(2) \sin\theta \cos\theta \left[\sin(\phi - \alpha_A) \left\{ 3 |\bar{S}_A| \sin\beta_A i R_{210}^{1-10} + \frac{3\sqrt{5}}{\sqrt{2}} |\bar{S}_A| \bar{S}_B^2 \sin\beta_A (3 \cos^2 \beta_A - 1) i R_{212}^{1-10} \right\} \right.$$

$$\left. - \sin(\phi + \alpha_B) \left\{ \frac{3\sqrt{3}}{\sqrt{2}} |\bar{S}_B| \sin\beta_B i R_{201}^{101} + 3\sqrt{30} |\bar{S}_A| \bar{S}_B^2 \cos\beta_A \sin\beta_B \cos\beta_B i R_{212}^{101} \right\} \right.$$

$$\left. - \sin(\phi + \alpha_A + 2\alpha_B) \frac{3\sqrt{15}}{2} |\bar{S}_A| \bar{S}_B^2 \sin\beta_A \sin^2 \beta_B i R_{212}^{112} \right.$$

$$\left. - \cos(\phi - \alpha_A) \frac{9}{\sqrt{2}} |\bar{S}_A| |\bar{S}_B| \sin\beta_A \cos\beta_B R_{211}^{1-10} \right.$$

$$\left. + \cos(\phi + \alpha_B) \left\{ 3\sqrt{10} \bar{S}_B^2 \sin\beta_B \cos\beta_B R_{202}^{101} + \frac{9}{\sqrt{2}} |\bar{S}_A| |\bar{S}_B| \cos\beta_A \sin\beta_B R_{211}^{101} \right\} \right]$$

$$+ f(2) \sin^2 \theta \left[\cos(2\phi + 2\alpha_B) \frac{3\sqrt{5}}{2\sqrt{2}} \bar{S}_B^2 \sin^2 \beta_B R_{202}^{202} - \cos(2\phi - \alpha_A + \alpha_B) \frac{9}{4} |\bar{S}_A| |\bar{S}_B| \sin\beta_A \sin\beta_B R_{211}^{2-11} \right.$$

$$\left. + \sin(2\phi - \alpha_A + \alpha_B) \frac{3\sqrt{15}}{2} |\bar{S}_A| \bar{S}_B^2 \sin\beta_A \sin\beta_B \cos\beta_B i R_{212}^{2-11} \right.$$

$$\left. - \sin(2\phi + 2\alpha_B) \frac{3\sqrt{15}}{2\sqrt{2}} |\bar{S}_A| \bar{S}_B^2 \cos\beta_A \sin^2 \beta_B i R_{212}^{202} \right] \quad 337$$

Hadron tensor and polarization states of proton and deuteron

$$W^{\mu\nu} \equiv \int \frac{d^4\xi}{(2\pi)^4} e^{iQ\cdot\xi} \langle P_A S_A P_B S_B | J^\mu(0) J^\nu(\xi) | P_A S_A P_B S_B \rangle$$

- $X^\mu = P_A^\mu Q^2 Z \cdot P_B - P_B^\mu Q^2 Z \cdot P_A + Q^\mu (Q \cdot P_B Z \cdot P_A - Q \cdot P_A Z \cdot P_B)$
- $Y^\mu = \epsilon^{\mu\alpha\beta\gamma} P_{A\alpha} P_{B\beta} Q_\gamma$
- $Z^\mu = P_A^\mu Q \cdot P_B - P_B^\mu Q \cdot P_A$

- $S_A^\mu = \lambda_A P_A^\mu / M_A + S_{AT}^\mu - \delta_-^\mu (\lambda_A M_A / P_A^+)$
- $S_B^\mu = \lambda_B P_B^\mu / M_B + S_{BT}^\mu - \delta_+^\mu (\lambda_B M_B / P_B^-)$
- $S_T^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) S_\nu$
- $T^\mu = \epsilon^{\mu\alpha\beta\gamma} S_\alpha Z_\beta Q_\gamma$

- $[W^{\nu\mu}(Q; P_A S_A; P_B S_B)]^* = W^{\mu\nu}(Q; P_A S_A; P_B S_B)$ (Hermiticity)
- $W_{\mu\nu}(\bar{Q}; \bar{P}_A - \bar{S}_A; \bar{P}_B - \bar{S}_B) = W^{\mu\nu}(Q; P_A S_A; P_B S_B)$ (Parity)
- $W_{\mu\nu}(\bar{Q}; \bar{P}_A \bar{S}_A; \bar{P}_B \bar{S}_B) = [W^{\mu\nu}(Q; P_A S_A; P_B S_B)]^*$ (Time reversal)

$$\bar{P}^\mu \equiv (P^0, -\vec{P})$$

$$\begin{aligned} (W^{\mu\nu})_{Q_T=0} = & -g^{\mu\nu} A - \frac{Z^\mu Z^\nu}{Z^2} B' + Z^{\{\mu} T_A^{\nu\}} C + Z^{\{\mu} T_B^{\nu\}} D \\ & + Z^{\{\mu} S_{AT}^{\nu\}} E + Z^{\{\mu} S_{BT}^{\nu\}} F - S_{BT}^\mu S_{BT}^\nu G' - S_{AT}^{\{\mu} S_{BT}^{\nu\}} H' \\ & + T_A^{\{\mu} S_{BT}^{\nu\}} I' + S_{BT}^{\{\mu} T_B^{\nu\}} J + Q^\mu Q^\nu K + Q^{\{\mu} Z^{\nu\}} L \\ & + Q^{\{\mu} S_{AT}^{\nu\}} M + Q^{\{\mu} S_{BT}^{\nu\}} N + Q^{\{\mu} T_A^{\nu\}} O + Q^{\{\mu} T_B^{\nu\}} P \end{aligned}$$

where $Q^{\{\mu} Z^{\nu\}} \equiv Q^\mu Z^\nu + Q^\nu Z^\mu$

$$\begin{aligned}
A_1 &= W_{0,0}, & A_2 &= V_{0,0}^{LL}, & A_3 &= V_{0,0}^{TT}, & A_4 &= V_{0,0}^{UQ_0}, & A_5 &= V_{0,0}^{TQ_1}, \\
B_1 &= W_{2,0}, & B_2 &= V_{2,0}^{LL}, & B_3 &= V_{2,0}^{TT}, & B_4 &= V_{2,0}^{UQ_0}, & B_5 &= V_{2,0}^{TQ_1}, \\
C_1 &= U_{2,1}^{TU}, & C_2 &= U_{2,1}^{TQ_0}, & D_1 &= U_{2,1}^{UT}, & D_2 &= U_{2,1}^{LQ_1}, & D_3 &= U_{2,1}^{TQ_2}, \\
E_1 &= U_{2,1}^{TL}, & F_1 &= U_{2,1}^{LT}, & F_2 &= U_{2,1}^{UQ_1}, & & & \\
H_1 &= U_{2,2}^{TT}, & G_1 &= U_{2,2}^{UQ_2}, & I_1 &= U_{2,2}^{TQ_1}, & J_1 &= U_{2,2}^{LQ_2}.
\end{aligned}$$

$$Q_0 \quad \text{for the term } 3 \cos^2 \beta_B - 1 \sim Y_{20}$$

$$Q_1 \quad \text{for} \quad \sin(2\beta_B) \sim Y_{21}$$

$$Q_2 \quad \text{for} \quad \sin^2(\beta_B) \sim Y_{22}$$

$$\begin{aligned}
W^{\mu\nu} = & - \left[g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right] \left\{ W_{0,0} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B V_{0,0}^{LL} - S_{AT} \cdot S_{BT} V_{0,0}^{TT} \right. \\
& \left. - \left(\frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) V_{0,0}^{UQ_0} + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_{BT} V_{0,0}^{TQ_1} \right\} \\
& - \left[\frac{Z^\mu Z^\nu}{Z^2} - \frac{1}{3} \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) \right] \left\{ W_{2,0} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B V_{2,0}^{LL} - S_{AT} \cdot S_{BT} V_{2,0}^{TT} \right. \\
& \left. - \left(\frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) V_{2,0}^{UQ_0} + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_{BT} V_{2,0}^{TQ_1} \right\} \\
& - Z^{\{\mu} T_A^{\nu\}} \frac{1}{\sqrt{Q^2} Z^2} \left\{ U_{2,1}^{TU} - \left(\frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) U_{2,1}^{TQ_0} \right\} \\
& - Z^{\{\mu} T_B^{\nu\}} \frac{1}{\sqrt{Q^2} Z^2} \left\{ U_{2,1}^{UT} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B U_{2,1}^{LQ_1} + S_{AT} \cdot S_{BT} U_{2,1}^{TQ_2} \right\} \\
& + Z^{\{\mu} S_{AT}^{\nu\}} \frac{\sqrt{Q^2} M_B}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,1}^{TL} \\
& + Z^{\{\mu} S_{BT}^{\nu\}} \left\{ - \frac{\sqrt{Q^2} M_A}{Z^2 Q \cdot P_A} Z \cdot S_A U_{2,1}^{LT} + \frac{\sqrt{Q^2} M_B}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,1}^{UQ_1} - \frac{1}{\sqrt{Q^2} Z^2} T_A \cdot S_{BT} U_{2,1}^{TQ_2} \right\} \\
& - \left[2 S_{BT}^\mu S_{BT}^\nu - S_{BT}^2 \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{UQ_2} \\
& - \left[S_{AT}^{\{\mu} S_{BT}^{\nu\}} - S_{AT} \cdot S_{BT} \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{TT} \\
& - \left[T_A^{\{\mu} S_{BT}^{\nu\}} - T_A \cdot S_{BT} \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,2}^{TQ_1} \\
& + S_{BT}^{\{\mu} T_B^{\nu\}} \frac{M_A}{Z^2 Q \cdot P_A} Z \cdot S_A U_{2,2}^{LQ_2}
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma}{d^4Q d\Omega} &= \frac{\alpha^2}{2 s Q^2} \left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2} \right) W_{ij} \\
&= \frac{\alpha^2}{2 s Q^2} \left\{ 2 \left[W_{0,0} + \frac{1}{4} \lambda_A \lambda_B V_{0,0}^{LL} + |\vec{S}_{AT}| |\vec{S}_{BT}| \cos(\phi_A - \phi_B) V_{0,0}^{TT} \right. \right. \\
&\quad \left. \left. + \frac{2}{3} \left(2 |\vec{S}_{BT}|^2 - \lambda_B^2 \right) V_{0,0}^{UQ_0} + |\vec{S}_{AT}| \lambda_B |\vec{S}_{BT}| \sin(\phi_A - \phi_B) V_{0,0}^{TQ_1} \right] \right. \\
&\quad \left. + \left(\frac{1}{3} - \cos^2 \theta \right) \left[W_{2,0} + \frac{1}{4} \lambda_A \lambda_B V_{2,0}^{LL} + |\vec{S}_{AT}| |\vec{S}_{BT}| \cos(\phi_A - \phi_B) V_{2,0}^{TT} \right. \right. \\
&\quad \left. \left. + \frac{2}{3} \left(2 |\vec{S}_{BT}|^2 - \lambda_B^2 \right) V_{2,0}^{UQ_0} + |\vec{S}_{AT}| \lambda_B |\vec{S}_{BT}| \sin(\phi_A - \phi_B) V_{2,0}^{TQ_1} \right] \right. \\
&\quad \left. + 2 \sin \theta \cos \theta \left[\sin(\phi - \phi_A) |\vec{S}_{AT}| \left(U_{2,1}^{TU} + \frac{2}{3} \left(2 |\vec{S}_{BT}|^2 - \lambda_B^2 \right) U_{2,1}^{TQ_0} \right) \right. \right. \\
&\quad \left. \left. + \sin(\phi - \phi_B) |\vec{S}_{BT}| \left(U_{2,1}^{UT} + \frac{1}{4} \lambda_A \lambda_B U_{2,1}^{LQ_1} \right) \right. \right. \\
&\quad \left. \left. + \sin(\phi + \phi_A - 2\phi_B) |\vec{S}_{AT}| |\vec{S}_{BT}|^2 U_{2,1}^{TQ_2} \right. \right. \\
&\quad \left. \left. + \cos(\phi - \phi_A) |\vec{S}_{AT}| \lambda_B U_{2,1}^{TL} \right. \right. \\
&\quad \left. \left. + \cos(\phi - \phi_B) |\vec{S}_{BT}| \left(\lambda_A U_{2,1}^{LT} + \lambda_B U_{2,1}^{UQ_1} \right) \right] \right. \\
&\quad \left. + \sin^2 \theta \left[\cos(2\phi - 2\phi_B) |\vec{S}_{BT}|^2 U_{2,2}^{UQ_2} \right. \right. \\
&\quad \left. \left. + \cos(2\phi - \phi_A - \phi_B) |\vec{S}_{AT}| |\vec{S}_{BT}| U_{2,2}^{TT} \right. \right. \\
&\quad \left. \left. + \sin(2\phi - \phi_A - \phi_B) |\vec{S}_{AT}| \lambda_B |\vec{S}_{BT}| U_{2,2}^{TQ_1} \right. \right. \\
&\quad \left. \left. + \sin(2\phi - 2\phi_B) \lambda_A |\vec{S}_{BT}|^2 U_{2,2}^{LQ_2} \right] \right\}
\end{aligned}$$

Possible spin asymmetries

pp Drell-Yan: $\langle \sigma \rangle$, A_{LL} , A_{TT} , A_{LT} , A_T

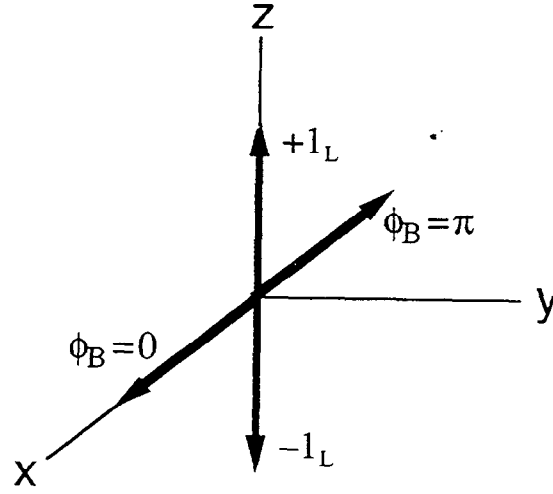


pd Drell-Yan:

$$\begin{array}{ccccccccc} \langle \sigma \rangle, & A_{LL}, & A_{TT}, & A_{LT}, & A_{TL}, & & & & \\ A_{UT}, & A_{TU}, & A_{UQ_0}, & A_{TQ_0}, & A_{UQ_1}, & & & & \\ A_{LQ_1}, & A_{TQ_1}, & A_{UQ_2}, & A_{LQ_2}, & A_{TQ_2} & & & & \end{array}$$

The quadrupole spin asymmetries are new ones in spin-1 hadron reactions.

Quadrupole Q_0 spin asymmetries



$$A_{UQ_0} = \frac{1}{2 \langle \sigma \rangle} \left[\sigma(\bullet, 0_L) - \frac{\sigma(\bullet, +1_L) + \sigma(\bullet, -1_L)}{2} \right]$$

$$= \frac{2 V_{0,0}^{UQ_0} + (\frac{1}{3} - \cos^2 \theta) V_{2,0}^{UQ_0}}{2 W_{0,0} + (\frac{1}{3} - \cos^2 \theta) W_{2,0}}$$

$$A_{TQ_0} = \frac{1}{2 \langle \sigma \rangle} \left[\{ \sigma(\phi_A = 0, 0_L) - \sigma(\phi_A = \pi, 0_L) \} / 2 \right.$$

$$\quad - \{ \sigma(\phi_A = 0, +1_L) + \sigma(\phi_A = 0, -1_L) \\$$

$$\quad \left. - \sigma(\phi_A = \pi, +1_L) - \sigma(\phi_A = \pi, -1_L) \} / 4 \right]$$

$$= \frac{2 \sin \theta \cos \theta \sin \phi U_{2,1}^{TQ_0}}{2 W_{0,0} + (\frac{1}{3} - \cos^2 \theta) W_{2,0}}$$

Spin asymmetries in the parton model $\left(\int d\vec{Q}_T \text{ case} \right)$

$$f_1 \rightarrow q_a, \quad g_{1L} \rightarrow \Delta q_a, \quad h_1 \rightarrow \Delta_T q_a, \quad b_1 \rightarrow \delta q_a$$

Unpolarized cross section

$$\begin{aligned} \left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle &= \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \bar{W}_T \\ &= \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)] \end{aligned}$$

Spin asymmetries

$$\begin{aligned} A_{LL} &= -\frac{\bar{V}_T^{LL}}{4\bar{W}_T} = \frac{\sum_a e_a^2 [\Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{(T)T} = A_{T(T)} = A_{TT}^{\parallel} &= \frac{\sin^2 \theta \cos 2\phi}{1 + \cos^2 \theta} \frac{\bar{U}_{2,2}^{TT}}{\bar{W}_T} \\ &= \frac{\sin^2 \theta \cos(2\phi)}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 [\Delta_T q_a(x_A) \Delta_T \bar{q}_a(x_B) + \Delta_T \bar{q}_a(x_A) \Delta_T q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{TT}^{\perp} &= \tan(2\phi) A_{TT}^{\parallel} \\ A_{UQ_0} &= \frac{\bar{V}_T^{UQ_0}}{\bar{W}_T} = \frac{\sum_a e_a^2 [q_a(x_A) \delta \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} = A_{LQ_1} \\ &= A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0 \end{aligned}$$

Advantage of the hadron reaction ($\delta \bar{q}$ measurements)

$$A_{UQ_0}(\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)} \quad 343$$

Spin asymmetries in the parton model $\left(\int d\vec{Q}_T \text{ case} \right)$

$$f_1 \rightarrow q_a, \quad g_{1L} \rightarrow \Delta q_a, \quad h_1 \rightarrow \Delta_T q_a, \quad b_1 \rightarrow \delta q_a$$

Unpolarized cross section

$$\begin{aligned} \left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle &= \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \overline{W}_T \\ &= \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)] \end{aligned}$$

Spin asymmetries

$$\begin{aligned} A_{LL} &= -\frac{\overline{V}_T^{LL}}{4\overline{W}_T} = \frac{\sum_a e_a^2 [\Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{(T)T} = A_{T(T)} = A_{TT}^{\parallel} &= \frac{\sin^2 \theta \cos 2\phi \overline{U}_{2,2}^{TT}}{1 + \cos^2 \theta \overline{W}_T} \\ &= \frac{\sin^2 \theta \cos(2\phi)}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 [\Delta_T q_a(x_A) \Delta_T \bar{q}_a(x_B) + \Delta_T \bar{q}_a(x_A) \Delta_T q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{TT}^{\perp} &= \tan(2\phi) A_{TT}^{\parallel} \\ A_{UQ_0} &= \frac{\overline{V}_T^{UQ_0}}{\overline{W}_T} = \frac{\sum_a e_a^2 [q_a(x_A) \delta \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]} \\ A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} = A_{LQ_1} \\ &= A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0 \end{aligned}$$

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Hard Exclusive Processes and Vector Meson Wave Functions in QCD

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The light-cone wave functions (distribution amplitudes) of vector mesons describe a “long distance part” for hard exclusive processes involving vector meson in the final state[1], such as exclusive semileptonic and radiative B decays, hard electroproduction of vector mesons, etc. In particular, the higher twist distribution amplitudes are relevant for understanding preasymptotic corrections to the hard exclusive amplitudes.

We discuss a systematic study of light-cone distribution amplitudes of vector mesons in QCD[2]. We give operator definitions of quark-antiquark distribution amplitudes up to twist-4 and quark-antiquark-gluon distribution amplitudes of twist-3, based on matrix elements of nonlocal light-cone operators between the vacuum and the vector meson state. A detailed operator expansion analysis is performed for the twist-3 distribution amplitudes, and the constraints among them from the QCD equations of motion are derived and solved. The solutions give explicit relations between the quark-antiquark and the quark-antiquark-gluon distribution amplitudes.

Based on (approximate) conformal invariance in massless QCD, we introduce conformal partial wave expansion as a powerful framework for systematic treatment of these solutions. A complete set of twist-3 distribution amplitudes is constructed, which satisfies all (exact) equations of motion and constraints from conformal expansion. The renormalization scale dependence of the distribution amplitudes is also worked out in the leading logarithmic approximation, utilizing conformal symmetry.

Taking into account a few low order terms in the conformal expansion, we obtain a consistent set of distribution amplitudes which involve a minimum number of independent nonperturbative parameters as expansion coefficients. These nonperturbative parameters are calculated from QCD sum rules. We also take into account the SU(3) flavor violation effects induced by quark masses, and construct models for the distribution amplitudes of ρ , ω , K^* , and ϕ mesons up to twist-3, which satisfy all QCD constraints.

References

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V.L. Chernyak and A.R. Zhitnitsky, Phys. Rept. **112** (1984) 173.
- [2] P. Ball, V.M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. **B529** (1998) 323.

hard processes in QCD

light-cone nonlocal operators

$$\bar{\Psi}(z) \Gamma \lambda^a \Psi(-z)$$

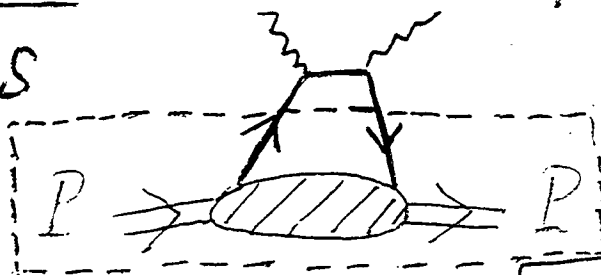
$$z^2=0; \quad z^\mu = (0, z^+, \mathbf{z}_\perp)$$

λ^a : flavor matrix

Inclusive

$$\Gamma = \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu}, 1, \gamma_5$$

DIS



structure fn.

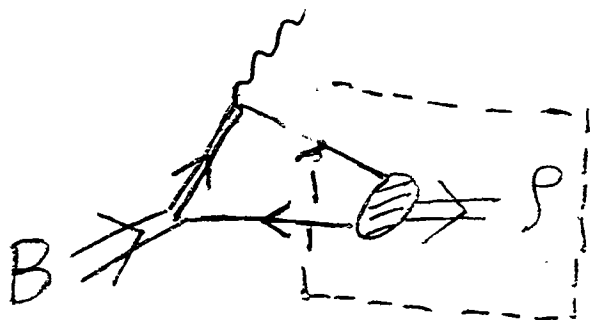
$$\langle P | \bar{\Psi}(z) \Gamma \lambda^a \Psi(-z) | P \rangle$$

Exclusive

heavy meson (semileptonic radiative) decay

$$B \rightarrow f e \bar{\nu}$$

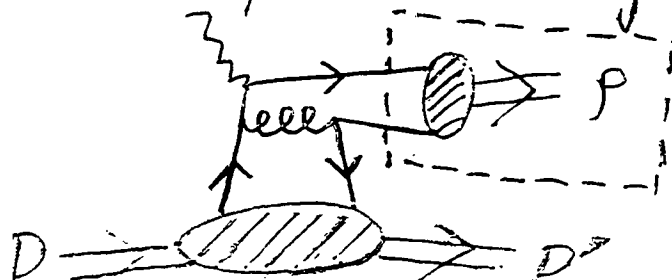
$$B \rightarrow f + \gamma$$



LC distribution amplitudes

$$\langle 0 | \bar{u}(z) \Gamma' d(-z) | f^- \rangle$$

electroproduction of vector meson



higher twist distribution amplitudes

- preasymptotic corrections $1/Q^2$
- new information on QCD and hadron structure

$$\langle PS | \bar{\Psi}(z) \Gamma \Psi(-z) | PS \rangle$$

Jaffe-Ji ('92)

wist	spin ave.	$S_{ }$	$S_{\perp}$	LC-proj.
$2 \ 0(1)$	f_1	g_1	$\underline{h_1}$	++
$3 \ 0(1/2)$	$\underline{e}$	$\underline{h_4}$	g_T	+-
$4 \ 0(1/2^2)$	f_4	g_3	$\underline{h_3}$	--

$$\langle 0 | \bar{u}(z) \Gamma d(-z) | P(P, E) \rangle$$

Ball, Braun, Koike & Tanaka ('98)

wist	$E_{ }$	$E_{\perp}$	LC-proj.
$2 \ 0(1)$	$\phi_{ }$	$\underline{\phi_{\perp}}$	++
$3 \ 0(1/2)$	$\underline{h_{ }^{(L)}}, \underline{h_{ }^{(S)}}$	$g_{\perp}^{(v)}, g_{\perp}^{(a)}$	+-
$4 \ 0(1/2^2)$	g_3	$\underline{h_3}$	--

$$|\psi\rangle = \psi + \psi$$

Classification of chiral-odd DA's

$$P^\mu = p^\mu + \frac{1}{2} m_f^2 \frac{z^\mu}{(p \cdot z)}; \quad p^\mu = (p^+, 0, \mathbf{0}_\perp), \quad z^\mu = (0, z^-, \mathbf{0}_\perp)$$

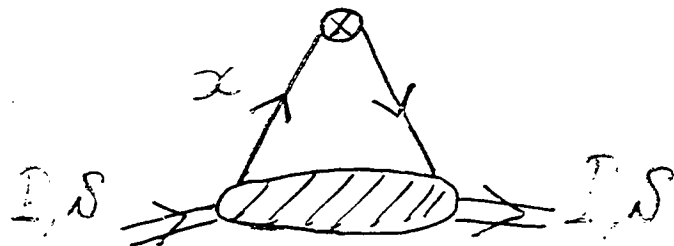
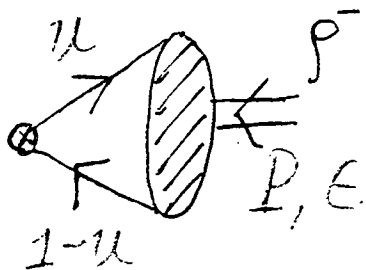
$$\epsilon^\mu = \frac{(\epsilon \cdot z)}{(p \cdot z)} p^\mu + \frac{(\epsilon \cdot p)}{(p \cdot z)} z^\mu + \epsilon_\perp^\mu; \quad \epsilon^\mu \epsilon_\mu = -1$$

$$\langle 0 | \bar{u}(z) \sigma_{\mu\nu} d(-z) | f^-(p, \epsilon) \rangle \quad \begin{aligned} \bar{z} &= u - (1-u) \\ &= 2u - 1 \end{aligned}$$

$$\begin{aligned} &= i f_f^T \left[(\epsilon_\perp^\nu p_\nu - \epsilon_\perp^\mu p_\mu) \int_0^1 du e^{i\bar{z} p \cdot z} \phi_\perp(u) \right. \\ &\quad + (p_\mu z_\nu - p_\nu z_\mu) \frac{(\epsilon \cdot z)}{(p \cdot z)^2} m_f^2 \int_0^1 du e^{i\bar{z} p \cdot z} h_{11}(u) \\ &\quad \left. + \frac{1}{2} (\epsilon_\perp^\nu z_\nu - \epsilon_\perp^\mu z_\mu) \frac{1}{(p \cdot z)} m_f^2 \int_0^1 du e^{i\bar{z} p \cdot z} h_3(u) \right] \end{aligned}$$

$$\langle PS | \bar{\psi}(z) \sigma_{\mu\nu} i \gamma_5 \psi(-z) | PS \rangle$$

$$\begin{aligned} &= 2 \left[(S_{1\mu} p_\nu - S_{1\nu} p_\mu) \int_{-1}^1 dx e^{2ix p \cdot z} h_1(x) \right. \\ &\quad + (p_\mu z_\nu - p_\nu z_\mu) \frac{(\epsilon \cdot z)}{(p \cdot z)^2} M^2 \int_{-1}^1 dx e^{2ix p \cdot z} h_L(x) \\ &\quad \left. + (S_{1\mu} z_\nu - S_{1\nu} z_\mu) \frac{1}{(p \cdot z)} M^2 \int_{-1}^1 dx e^{2ix p \cdot z} h_3(x) \right] \end{aligned}$$



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$$\langle 0 | \bar{u}(0) \sigma_{\mu\nu} d(0) | f^-(p, \epsilon) \rangle = i f_f^T (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)$$

$$\langle 0 | \bar{u}(z) d(-z) | P^-(P, \epsilon) \rangle$$

$$= -i \left(f_p^T - f_p \frac{m_u + m_d}{m_p} \right) (\epsilon \cdot z) m_p^2 \int_0^1 du e^{i \frac{z}{2} p \cdot z} h_{//}^{(s)}(u)$$

$$\langle PS | \bar{\psi}(z) \psi(-z) | PS \rangle$$

$$= 2M \int_{-1}^1 dx e^{i x p \cdot z} e(x)$$

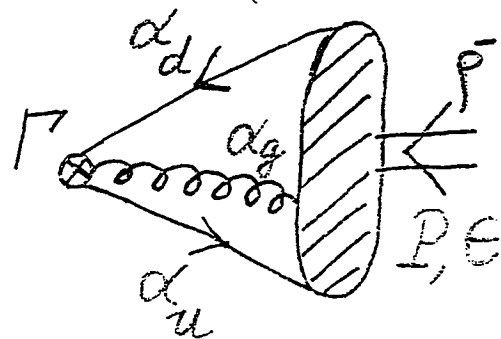
$$\langle 0 | \bar{u}(0) \gamma_\mu d(0) | P^-(P, \epsilon) \rangle = f_p m_p \epsilon_\mu$$

$$\int_0^1 du h_{//}^{(s)}(u) = 1$$

- particle DA's

$$\langle \bar{u}(z) \Gamma g G_{\mu\nu}(vz) d(-z) | P^-(P, \epsilon) \rangle$$

$$= \int_0^1 \underline{\Delta \alpha} e^{-i P \cdot z (\alpha_d - \alpha_u + v \alpha_g)} \Phi(\alpha_d, \alpha_u, \alpha_g; \mu^2) d\alpha_d d\alpha_u d\alpha_g \delta(1 - \alpha_d - \alpha_u - \alpha_g)$$



<u>w.</u>	3 O(1/2)
<u>//</u>	<u>T(α_d, α_u, α_g; μ²)</u>

$$\Gamma = \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}$$

V A T

Constraints on twist-3 DA's

Using QCD-EOM $(i\not{D}-m_u)u=0$ etc.

$$\begin{aligned} & \frac{\partial}{\partial x_\mu} [\bar{u}(x) \sigma_{\mu\nu} x_\nu d(-x)] && h_{\parallel}^{(\pm)}, \phi_{\perp} \\ &= i \int_{-1}^1 dv v \bar{u}(x) \sigma_{\alpha\beta} \boxed{g G_{\mu\beta}(vx)} x_\alpha x_\mu d(-x) && T \\ & - i x_\beta \partial_\beta [\bar{u}(x) d(-x)] && h_{\parallel}^{(S)} \\ & - (m_u - m_d) \bar{u}(x) \gamma_\mu x_\mu d(-x) && \phi_{\parallel} \end{aligned}$$

$$\begin{aligned} & \bar{u}(x) d(-x) - \bar{u}(0) d(0) && h_{\parallel}^{(S)} \\ &= \int_0^1 dt \int_{-t}^t dv \bar{u}(tx) \sigma_{\alpha\beta} \boxed{g G_{\mu\beta}(vx)} x_\alpha x_\mu d(-tx) && T \\ & + i \int_0^1 dt \partial_\alpha [\bar{u}(tx) \sigma_{\alpha\beta} x_\beta d(-tx)] && h_{\parallel}^{(\pm)} \\ & + i \int_0^1 dt (m_u + m_d) \bar{u}(tx) \gamma_\mu x_\mu d(-tx) && \phi_{\parallel} \end{aligned}$$

Solution via the conformal expansion ($m_u = m_d = 0$)

$$\rho(t, s) h_{||}(u) = \boxed{\rho(t, s)_{WW} h_{||}(u)} + \boxed{\rho(t, s)_g h_{||}(u)}$$

$$\begin{aligned} & T(\alpha_d, \alpha_u, 1 - \alpha_d - \alpha_u) \\ &= 360 \alpha_d \alpha_u (1 - \alpha_d - \alpha_u)^2 \sum_{k, l=0}^{\infty} \omega_{k, l} J_{kl}(\alpha_d, \alpha_u) \end{aligned}$$

$$\Phi_{\perp}(u) = 6u(1-u) \sum_{n=0}^{\infty} a_n C_n^{3/2}(2u-1)$$

$$\begin{aligned} \mathcal{L}_n &\sim \langle 0 | \Omega_{\mathcal{L}_n}^c(0) | \mathcal{P}(P, \epsilon_{\perp}) \rangle, \quad \omega_{kl} \sim \langle 0 | \mathcal{H}_{kl}^c(0) | \mathcal{P}(P, \epsilon_{||}) \rangle \\ \mathcal{L}_n^c(0) &= \bar{u}(0) \sigma_{\perp \nu} z_{\nu} C_n^{3/2}(z \cdot \vec{D}) d(0) \quad j = n+2 \\ \mathcal{H}_{kl}^c(0) &= \bar{u}(0) (-i \vec{D} \cdot \vec{z})^l \sigma_{\mu \beta} g G_{\nu \beta}^{(0)} z_{\mu} z_{\nu} (z \cdot i \vec{D})^k d(0) \\ &\quad + (\text{total derivatives}) \quad j = k+l+7/2 \end{aligned}$$

$$\begin{aligned} \rho(t) h_{||}(u) &= \sum_{n=0}^{\infty} (H_n - H_{n-1}) C_n^{1/2}(2u-1) \\ \rho(s) h_{||}(u) &= 4u(1-u) \sum_{n=0}^{\infty} \frac{H_n - H_{n+1}}{(n+1)(n+2)} C_n^{3/2}(2u-1) \end{aligned}$$

$$H_n = \underbrace{H_n^{WW}}_{\propto a_n \text{ or } a_{n-1}} + {}^{351} \underbrace{H_n^g}_{\propto \mathcal{L}(1) \dots | b+0 = n-9/2} \quad j = n+3/2$$

Models for $\Psi(\omega)$, K^* , ϕ DA's

truncation of conformal expansion: $j \leq 1/2$

$$P_{\perp}(u) = 6u(1-u) \sum_{n=0}^{\infty} a_n C_n^{3/2}(2u-1)$$

$$\Gamma(\alpha_d, \alpha_u, 1-\alpha_d-\alpha_u) = 360 \alpha_d \alpha_u (1-\alpha_d-\alpha_u)^2 \sum_{k,l=0}^{\infty} \omega_{kl} J_{kl}^{(\alpha_d, \alpha_u)} \quad k+l \leq 1$$

$$p_{\perp}^{(t)}(u) = \sum_{n=0}^{\infty} (H_n - H_{n-1}) C_n^{1/2}(2u-1) \quad + h_{\perp}^{(t)m}$$

$$p_{\perp}^{(s)}(u) = 4u(1-u) \sum_{n=0}^{\infty} \frac{H_n - H_{n+1}}{(n+1)(n+2)} C_n^{3/2}(2u-1) \quad + h_{\perp}^{(s)m}$$

$\propto m_q$

nonperturbative matrix elements

QCD-SR

$$\lambda_n \sim \langle 0 | \bar{u}(0) \sigma_{1\nu} z_{\nu} C_n^{3/2}(z \cdot \vec{D}) d(0) | P(P, \epsilon_{\perp}) \rangle \quad n=0,1,2$$

$$\nu_{10} \sim \langle 0 | \bar{u}(0) \sigma_{\mu\beta} z_{\mu} z_{\nu} [g G_{\nu\beta}^{(0)}(z \cdot i \vec{D}) - (z \cdot i \vec{D}) g G_{\nu\beta}^{(0)}] d(0) | P(P, \epsilon_{\perp}) \rangle$$

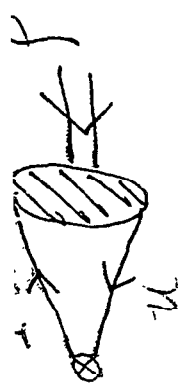
chiral-even

$$\begin{aligned} \phi_{\perp} &\longleftrightarrow \phi_{\perp} \\ g_{\perp}^{(v)}, g_{\perp}^{(a)} &\longleftrightarrow h_{\perp}^{(t)}, h_{\perp}^{(s)} \end{aligned}$$

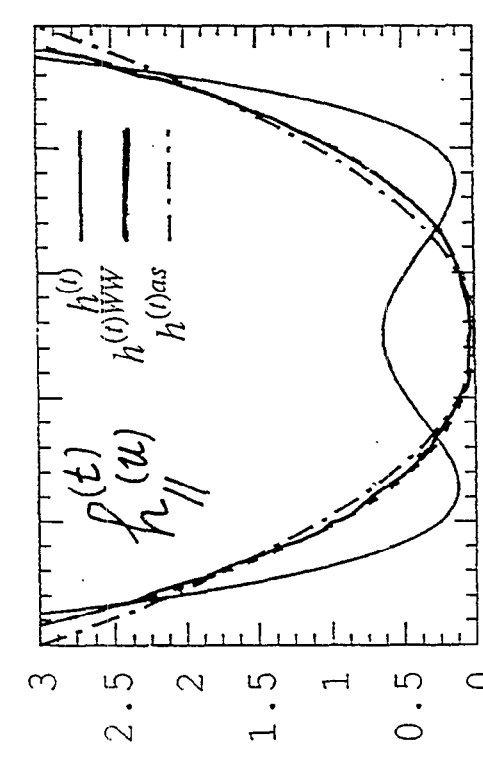
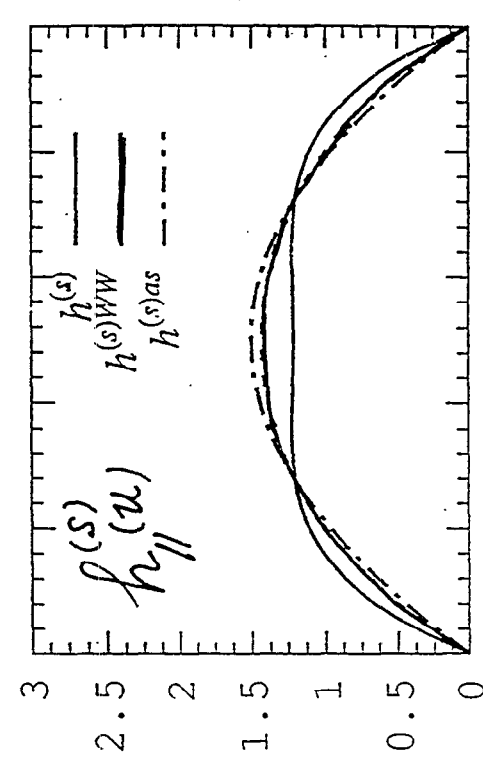
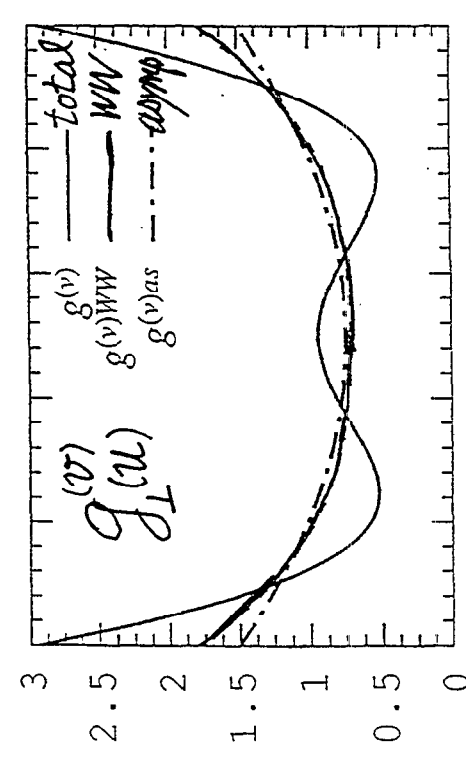
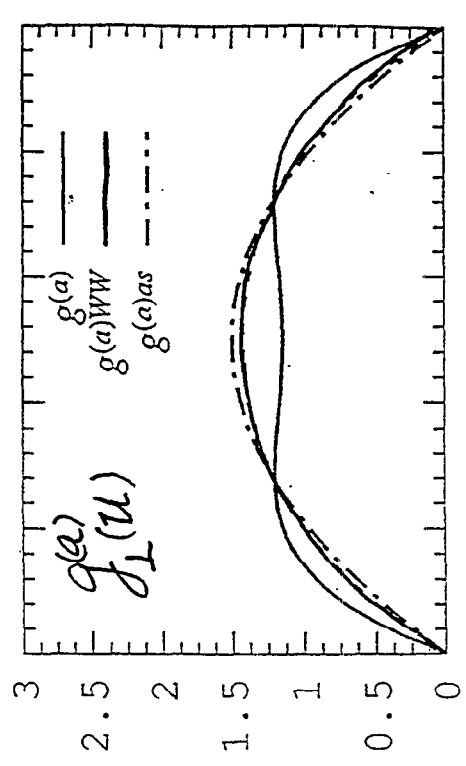
352

Renormalization:
Keike, Nishiyama, Tanaka

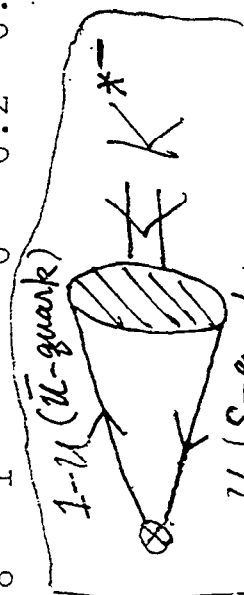
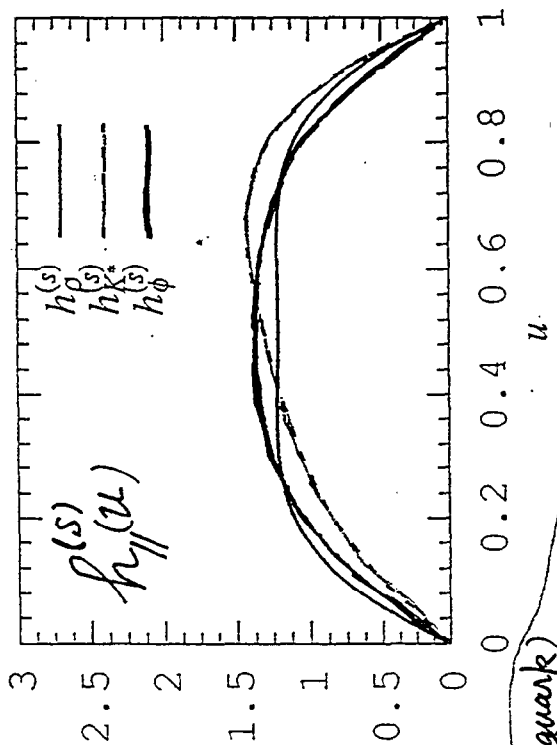
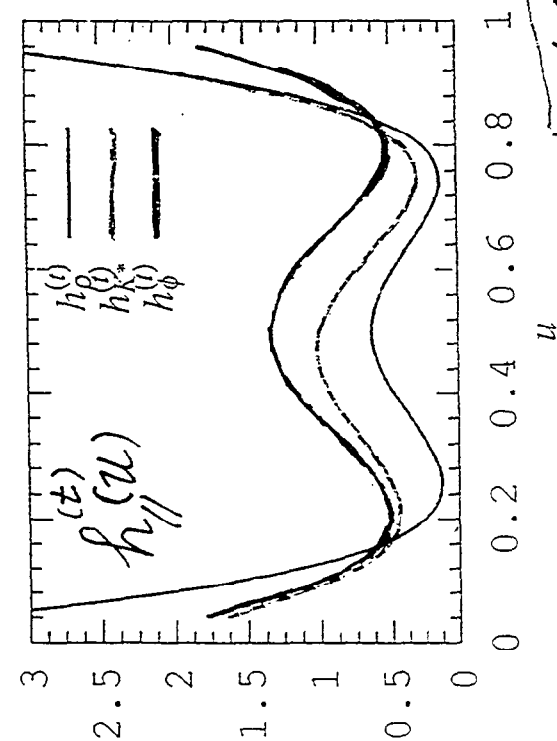
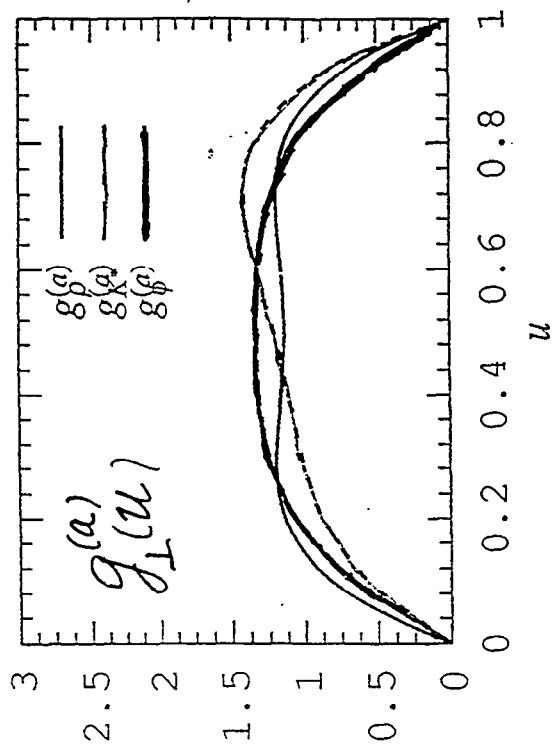
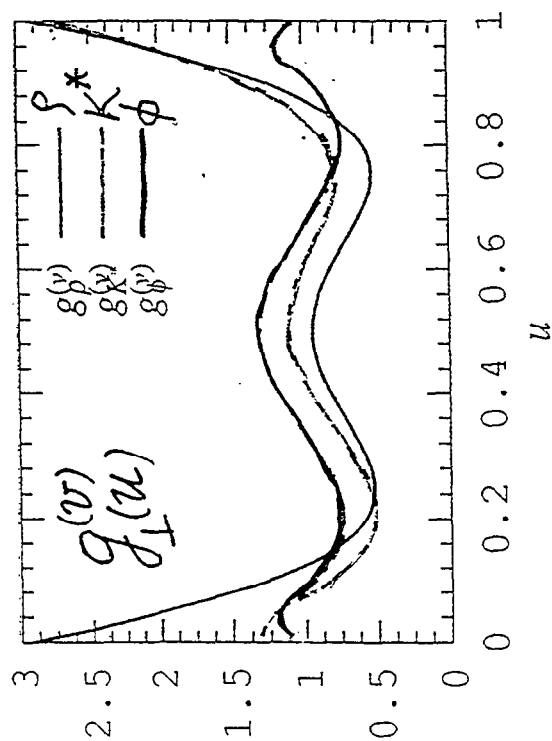
$$\mu = 1 \text{ GeV}$$



P muon DA's
(twist-3)



K^0, ϕ meson DA's (Twist-3)



$\mu = 1 \text{ GeV}$

Summary

Exclusive processes

{ heavy meson decay
electroproduction

Light-cone DA's of vector mesons ρ, ω, K^*, ϕ

complete classification of 2-particle DA's

3-particle DA's

chiral-odd & chiral-even

Systematic analysis of twist-3 DA's

QCD-EOM & conformal partial wave expansion

$$\left. \begin{array}{l} h_{\parallel}^{(t,s)} \\ g_{\perp}(v,a) \end{array} \right\} = (WW) + (3\text{-part.}) + O(m_q)$$


Resolution order by order in conformal spin j

j : good quantum number up to $O(\alpha^2)$

Renormalization based on conformal operator basis

{ all anomalous dimensions are determined!
3-particle DA's: complicated mixing of many $\bar{u}Gd$ ops.

$N_c \rightarrow \infty$ or $n \rightarrow \infty$: no complicated mixing!

$$\gamma_n \sim \ln n \quad (n \rightarrow \infty)$$

Explicit models of DA's for ρ, ω, K^*, ϕ

{ truncation of conformal expansion
estimates of nonperturbative matrix elements by QCD SR
SU(3) flavor violation effects due to quark masses; asymmetry
• broadening by 3-particle contributions

Powerful framework for hard exclusive processes:

All info. predictable based on QCD

Model building consistent with all QCD constraints

nonperturbative estimates of matrix elements

Problems for J.Kodaira's Lectures

1. Understand the following facts:

- A spin-1/2 fermion field ψ is decomposed into the two components due to the eigenvalues of γ_5 ($= \pm 1$, called "chirality") as $\psi_R \equiv (1 + \gamma_5)\psi/2$ and $\psi_L \equiv (1 - \gamma_5)\psi/2$. Show that $\bar{\psi}\Gamma\psi = \bar{\psi}_L\Gamma\psi_L + \bar{\psi}_R\Gamma\psi_R$ for $\Gamma = \gamma_\mu, \gamma_\mu\gamma_5$ and $\bar{\psi}\Gamma\psi = \bar{\psi}_R\Gamma\psi_L + \bar{\psi}_L\Gamma\psi_R$ for $\Gamma = 1, \gamma_5, \sigma_{\mu\nu}$. From this we understand that the standard model lagrangian preserves chirality of the quarks except quark mass effect.
- For massless fermions, chirality is identical to helicity. (For anti-fermions, they are opposite.)
- In the high-energy e^+e^- collision, the positive-helicity (right-handed) electron can annihilate only in the collision with the negative helicity (left-handed) positron, and accordingly the spin of the virtual photon is transverse.

2. Consider the nucleon structure function in the deep-inelastic scattering

$$W_{\mu\nu}(q, P, S) = \frac{1}{4\pi} \int d\xi e^{iq\xi} \langle PS | [J_\mu^\dagger(\xi), J_\nu(0)] | PS \rangle. \quad (1)$$

- Show $W_{\mu\nu}(q, P, S) = W_{\nu\mu}^*(q, P, S)$. (Hint: Take complex conjugate of the above equation.)
- Show that the parity invariance requires $W_{\mu\nu}(q, P, S) = W^{\mu\nu}(\bar{q}, \bar{P}, -\bar{S})$ where $\bar{P}$ denotes $\bar{P} = (P^0, -\vec{P})$ for $P = (P^0, \vec{P})$ etc.
- Show that the time reversal invariance requires $W_{\mu\nu}(q, P, S) = W^{\nu\mu}(\bar{q}, \bar{P}, \bar{S})$.
- For the unpolarized case (i.e. $|PS\rangle = |P\rangle$), the most general decomposition of $W_{\mu\nu}$ can be written as

$$\begin{aligned} W_{\mu\nu}(x, Q^2) = & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1 + \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) W_2 \\ & + \frac{i}{2M^2} \epsilon_{\mu\nu\lambda\sigma} P^\lambda q^\sigma W_3 + \frac{1}{M^2} q_\mu q_\nu W_4 + \frac{1}{2M^2} (P_\mu q_\nu + P_\nu q_\mu) W_5 \\ & - \frac{i}{2M^2} (P_\mu q_\nu - P_\nu q_\mu) W_6. \end{aligned} \quad (2)$$

Show that (i) W_{1-6} are real, (ii) Parity invariance kills W_3 , (iii) T-invariance kills W_6 , (iv) The current conservation $q^\mu W_{\mu\nu} = 0$ kills W_{4-6} .

3. Define the scaling functions as $F_1(x, Q^2) = W_1(x, Q^2)$ and $F_2(x, Q^2) = \nu W_2(x, Q^2)/M^2$ ($\nu = P \cdot q$, $x = Q^2/2\nu$). Prove that the Callan-Gross relation, $F_2(x) = 2xF_1(x)$, is a direct consequence of the fact that the parton is a spin-1/2 particle.
4. Solve the renormalization group (RG) equation for the coupling constant

$$\mu \frac{d}{d\mu} g(\mu) = \beta(g) = -\frac{g^3}{16\pi^2} b_0 - \frac{g^5}{(16\pi^2)^2} b_1 - \dots, \quad (3)$$

to obtain

$$\frac{g^2(Q^2)}{16\pi^2} = \frac{1}{b_0 \ln(Q^2/\Lambda^2)} \left[1 - \frac{b_1 \ln \ln(Q^2/\Lambda^2)}{b_0^2 \ln(Q^2/\Lambda^2)} + \dots \right], \quad (4)$$

where $b_0 = 11 - \frac{2}{3}N_f$ and $b_1 = 102 - \frac{38}{3}N_f$. The RG equation for an operator $O_n(\mu)$ is given as

$$\mu \frac{d}{d\mu} O_n(\mu) + \gamma_n(g(\mu)) O_n(\mu) = 0. \quad (5)$$

Solve this equation when the anomalous dimension $\gamma_n(g)$ is given as

$$\gamma_n(g) = \frac{g^2}{16\pi^2} \gamma_n^{(0)} + \frac{g^4}{(16\pi^2)^2} \gamma_n^{(1)} + \dots \quad (6)$$

Note that in the $\overline{\text{MS}}$ scheme $O_n(\mu)$ depends on μ only through $g(\mu)$. The answer is

$$O_n(Q^2) = \left(\frac{g^2(Q^2)}{g^2(\mu^2)} \right)^{\gamma_n^{(0)}/2b_0} \left[1 + \frac{g^2(Q^2) - g^2(\mu^2)}{16\pi^2} \frac{b_1}{b_0} \left(\frac{\gamma_n^{(1)}}{2b_1} - \frac{\gamma_n^{(0)}}{2b_0} \right) \right] O_n(\mu^2). \quad (7)$$

(Suggestion: Try to solve the leading order result first by putting $b_1 = \gamma_n^{(1)} = 0$.)

5. The parton distribution functions are defined as

$$\Phi(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \Gamma \psi(\lambda n) | PS \rangle. \quad (8)$$

Answer to the following questions.

- (a) The anti-quark distribution $\bar{\Phi}(x)$ is defined by the replacement $\psi \rightarrow C \bar{\psi}^T$, where $C = i\gamma^2 \gamma^0$ is a charge conjugation matrix. Show that $\bar{\Phi}(-x) = -\bar{\Phi}(x)$ for $\Gamma = \gamma_\mu, \sigma_{\mu\nu}$ and $\bar{\Phi}(-x) = \bar{\Phi}(x)$ for $\Gamma = \gamma_\mu \gamma_5, 1, i\gamma_5$. (Hint: Use the following relations; $C = -C^\dagger = -C^T$, $CC^\dagger = 1$, $C\gamma_\mu C^{-1} = -\gamma_\mu^T$, $C\gamma_5 C^{-1} = \gamma_5^T$, $C\sigma_{\mu\nu} C^{-1} = -\sigma_{\mu\nu}^T$, $C\gamma_\mu \gamma_5 C^{-1} = (\gamma_\mu \gamma_5)^T$.)

- (b) (Similar to the problem 2. So you may skip.) Show that the time-reversal invariance prohibits the presence of the following $e_L(x)$, $h(x)$ and $f_T(x)$ although they are allowed by parity:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) i\gamma_5 \psi(\lambda n) | PS \rangle = M(S \cdot n) e_L(x), \quad (9)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) i\gamma_5 \sigma_{\mu\nu} \psi(\lambda n) | PS \rangle = \dots + M \epsilon_{\mu\nu\lambda\sigma} p^\lambda n^\sigma h(x) + \dots, \quad (10)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_\mu \psi(\lambda n) | PS \rangle = \dots + \epsilon_{\mu\nu\lambda\sigma} p^\nu n^\lambda S_\perp^\sigma f_T(x) + \dots \quad (11)$$

The complete set of the quark distribution functions defined from (8) which respect time-reversal and parity invariance is listed in Jaffe & Ji, Nucl. Phys. B375 ('92) 527.

6. (Not easy) Understand the origin of the Gauge-link operators in the quark distribution functions. (Answer: As an example, take the deep inelastic scattering. Consider the diagrams in which the usual quark-hadron diagram has additional gluon lines connecting short-distance and long-distance parts. Decompose the gluon fields A^μ as $A^\mu = \omega^{\mu\lambda} A_\lambda + p^\mu (n \cdot A)$ where $\omega^{\mu\lambda} = g^{\mu\lambda} - p^\mu n^\lambda$. Then the $p^\mu (n \cdot A)$ piece ("scalar polarized gluon") contributes to the process in the same order with respect to $1/Q$ as the simplest hadron diagram. Using the technique of Ward identity, one can show that this scalar polarized gluon contributions can be recast into the form of the gauge-link operator. This gauge-link operator is reduced to unit operator in $n \cdot A = 0$ -gauge (light-cone gauge).

Problem set for R. L. Jaffe's lectures (except for the last part)

[1] Spin, orbital, and total angular momenta in the Dirac theory.

Prove that the orbital and spin angular momenta are not separately conserved in the Dirac theory although the total angular momentum is conserved:

$$[\vec{L}, H] \neq 0, \quad [\vec{S}, H] \neq 0, \quad [\vec{J}, H] = 0,$$

where

$$H = \vec{\alpha} \cdot \vec{p} + \beta m, \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

[2] Energy-momentum and angular-momentum tensors in scalar field theory.

(2.1) Consider the translation $x \rightarrow x + a$ in the Lagrangian of scalar field theory:

$$\mathcal{L}(x + a) = \mathcal{L}[\phi(x + a), \partial_\mu \phi(x + a)]$$

Then, the variations of ϕ and $\partial_\mu \phi$ are given by

$$\begin{aligned} \delta\phi &= \delta a^\nu(x) \partial_\nu \phi(x), \\ \delta\partial_\mu \phi(x) &= \delta a^\nu \partial_\nu \partial_\mu \phi(x) + \partial_\mu [\delta a^\nu(x)] \partial_\nu \phi(x), \end{aligned}$$

for an infinitesimal x -dependent transformation. Suppose that the variation of the action is invariant under the transformation. Prove the following conservation law for the energy-momentum tensor:

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

(2.2) Next, suppose that the action is invariant under the infinitesimal Lorentz transformation $\delta x^\mu = \delta\omega^{\mu\nu} x_\nu$, where $\delta\omega^{\mu\nu}$ is the antisymmetric ($\delta\omega^{\nu\mu} = -\delta\omega^{\mu\nu}$). Show the following conservation law for the angular-momentum tensor:

$$\begin{aligned} \partial_\mu M^{\mu\nu\lambda} &= 0, \quad M^{\mu\nu\lambda} = x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}, \\ &= \partial^\mu \phi (x^\nu \partial^\lambda - x^\lambda \partial^\nu) \phi - (g^{\mu\lambda} x^\nu - g^{\mu\nu} x^\lambda) \mathcal{L}, \end{aligned}$$

where the Lagrangian is given by $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi / 2 - m^2 \phi^2 / 2 - V(\phi)$.

[3] Gauge invariance of the angular momentum tensor in Abelian gauge theory.

(3.1) For the electromagnetic field with

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu,$$

show that the angular-momentum tensor is given by

$$\begin{aligned} M^{\mu\nu\lambda} = & -F^{\mu\rho}(x^\nu \partial^\lambda - x^\lambda \partial^\nu) A_\rho && \text{orbital} \\ & + F^{\mu\lambda} A^\nu + F^{\nu\mu} A^\lambda && \text{spin} \\ & - \frac{1}{4} F^2 (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}), \end{aligned}$$

where $F^2 = F_{\mu\nu} F^{\mu\nu}$.

(3.2) Then, prove that the orbital and spin terms are not separately gauge invariant although the whole angular momentum tensor is invariant under the local gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$.

[4] Longitudinal and transverse structure functions.

From the antisymmetric part of the hadron tensor

$$W_{\mu\nu}^A = i \varepsilon_{\mu\nu\lambda\sigma} \frac{1}{\nu} q^\lambda S^\sigma g_1 + i \varepsilon_{\mu\nu\lambda\sigma} \frac{1}{\nu^2} q^\lambda (\nu S^\sigma - q \cdot S P^\sigma) g_2,$$

show

$$W_{\mu\nu}^A = i \varepsilon_{\mu\nu\lambda\sigma} (n \cdot S) n^\lambda p^\sigma g_1 + i \varepsilon_{\mu\nu\lambda\sigma} n^\lambda S_\perp^\sigma (g_1 + g_2),$$

so that g_1 and $g_T = g_1 + g_2$ are interpreted as the longitudinal and transverse structure functions, respectively.

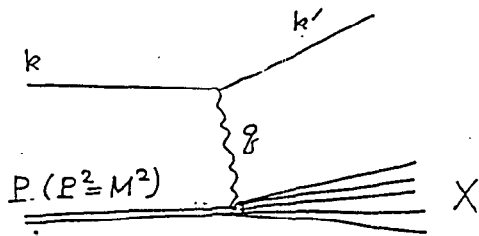
[5] “Good” and “bad” components.

From the Dirac equation $(i\partial - gA)\psi = m\psi$. show that $i\partial\psi_-/\partial x^-$ is expressed by $A_\perp$ and ψ_+ :

$$i \frac{\partial}{\partial x^-} \psi_- = \frac{1}{\sqrt{2}} [(i\partial_{\perp j} - g A_{\perp j}) \gamma_\perp^j + m] \gamma^0 \psi_+,$$

so that ψ_- is not independent of $A_\perp$ and ψ_+ .

[1] Kinematics of Deep Inelastic Scattering.



- (1) Write the expressions in the laboratory system for the frequently used Lorentz invariants:

$$v = \frac{q \cdot P}{M},$$

$$Q^2 = -q^2,$$

$$y = \frac{q \cdot P}{k \cdot P}.$$

The lepton 4-momenta in the lab. system are given by

$$k^\mu = (E, \vec{k}) \quad , \quad k'^\mu = (E', \vec{k}'), \quad \theta : \vec{k} \wedge \vec{k}',$$

and the lepton mass is assumed to be negligible.

- (2) Express the invariant mass squared of the final state hadrons $W^2 = (P + q)^2$ by the invariants given in (1) and show that the Bjorken variable, $x = Q^2/2P \cdot q$ satisfies $0 \leq x \leq 1$. Also, confirm that $0 \leq y \leq 1$.

- (3) Show $S = (k + P)^2 = Q^2/xy + M^2$.

- [2] Use the previous results to answer the following.

- (1) Consider a fixed target deep inelastic scattering experiment (e, e') on proton at $E = 30$ GeV. Obtain the minimum accessible x for $Q^2 \geq 1 (\text{GeV}/c)^2$. Assume for simplicity that the proton mass is approximately 1 GeV. What is the electron scattering angle θ at $y = 0.9$?
- (2) Repeat the same calculation for $E = 200$ GeV.
- (3) In the case of a HERA type experiment ($E_p = 820$ GeV, $E_e = 30$ GeV), obtain the total energy in the center-of-mass system. Also obtain $x_{\min}$ for $Q^2 \geq 1 (\text{GeV}/c)^2$.

- [3] Calculate the lepton tensor, $L^{\mu\nu}$, defined by

$$L^{\mu\nu} = \frac{1}{2} \sum_{s, s'} \langle k, s | j^\mu(0) | k', s' \rangle \langle k', s' | j^\nu(0) | k, s \rangle,$$

$$= \frac{1}{2} \sum_{s, s'} \bar{u}(k, s) \gamma^\mu u(k', s') \bar{u}(k', s') \gamma^\nu u(k, s).$$

The 4-vector, S^μ , is given by

$$\bar{u}(k, s) \gamma^\mu \gamma_5 u(k, s) = 2S^\mu,$$

and the lepton mass is again assumed to be negligible.

- [4] An experiment is performed at the luminosity $L = 10^{32} (\text{cm}^{-2} \text{s}^{-1})$ for 100 days. Obtain the integrated luminosity $\int L dt$ (pb^{-1}). Obtain the expected number of events for the reaction with the cross section $\sigma = 1 (\text{pb})$. What is the statistical error in this case?

The RIKEN Winter School

Structure of Hadrons - Introduction to QCD Hard Processes -

Organizers: N. Saito, RIKEN
T.-A. Shibata, Tokyo Inst. Tech./RIKEN
K. Yazaki, Univ. Tokyo/RIKEN

Lecturers: R. Jaffe, MIT
J. Kodaira, Hiroshima Univ.
K. Rith, Erlangen-Nuernberg Univ.

Tutors: N. Hayashi, RIKEN
Y. Koike, Niigata Univ.
S. Kumano, Saga Univ.
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Program of the School

	Dec. 9	Dec. 10	Dec. 11	Dec. 12
9:15- 9:30	Opening			
9:30-10:30	Kodaira	Rith	Jaffe	Jaffe
10:45-11:45	Kodaira	Rith	Jaffee	Rith
		lunch		
13:00-14:00	Jaffe	Kodaira	Koike	Tanaka
14:15-15:15	Rith	Kodaira	Hayashi	Kumano
		coffee		
15:45-17:45	*****	tutorial session	*****	
	(Koike Tanaka)	(Hayashi Tanaka)	(Kumano Tanaka)	
	dinner	dinner	banquet	
20:00-22:00	** evening session **			