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Gas-Induced Rectified Motion of a Solid Object in a Liquid-Filled Housing during Vibration: Analysis and Experiments

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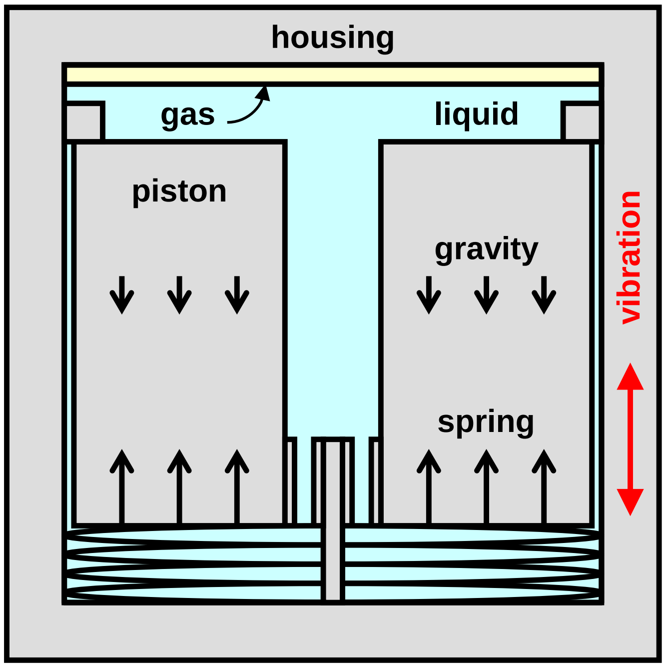
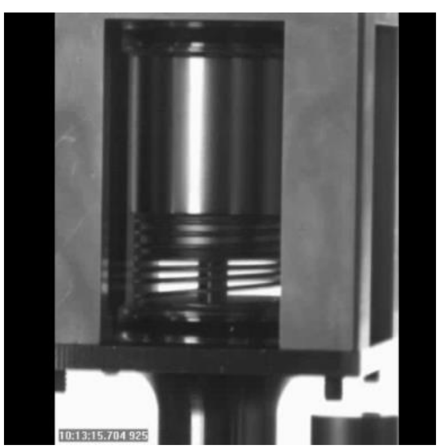
Strange Vibration-Induced Dynamics



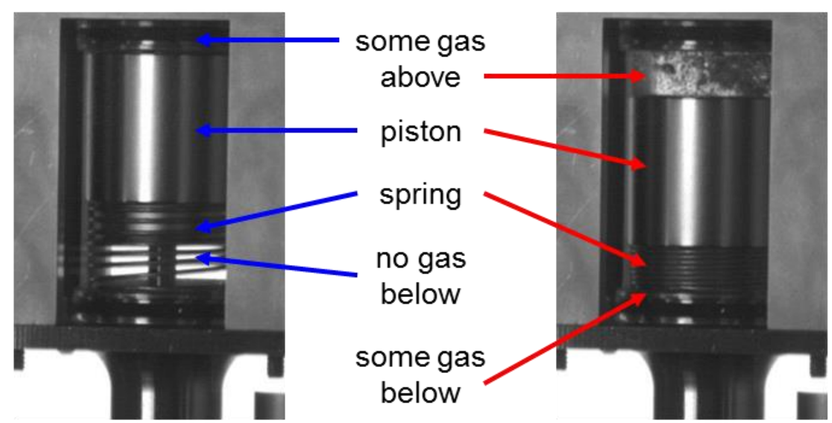


Simple spring-mass-dashpot system

- Piston moves vertically in housing
- Spring supports it against gravity
- Viscous liquid provides damping
- Small amount of gas is present

Housing is vibrated vertically

- Gas moves down below piston
- Piston moves down against spring



Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Vibration Makes Gas Move Down

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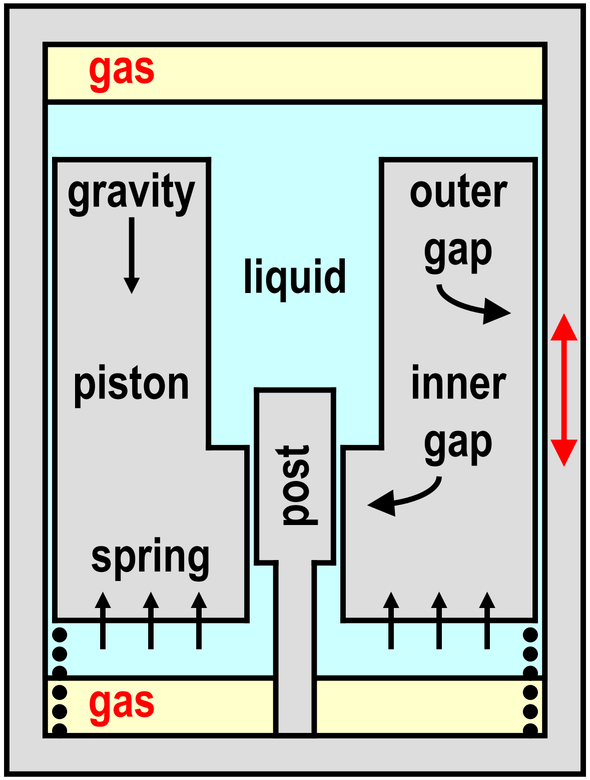
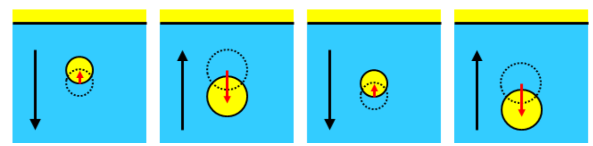


Some gas moves down below piston!

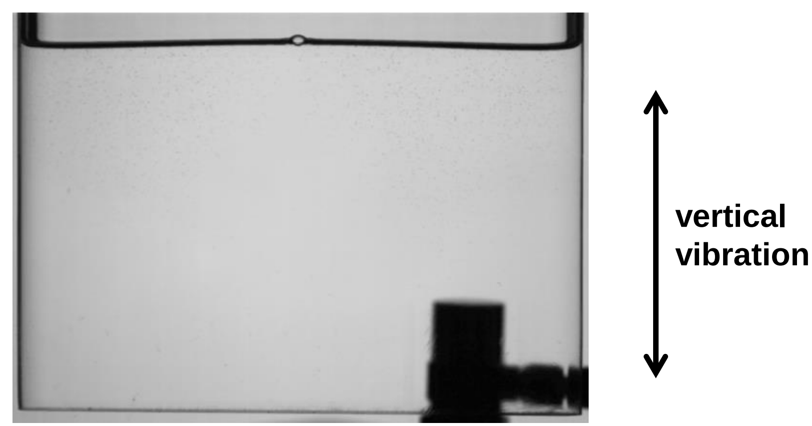
- **Bjerknes forces push bubbles down**
- **Create & stabilize a lower gas region**

Two gas regions: upper and lower

- **Both are quasi-stable (stationary)**

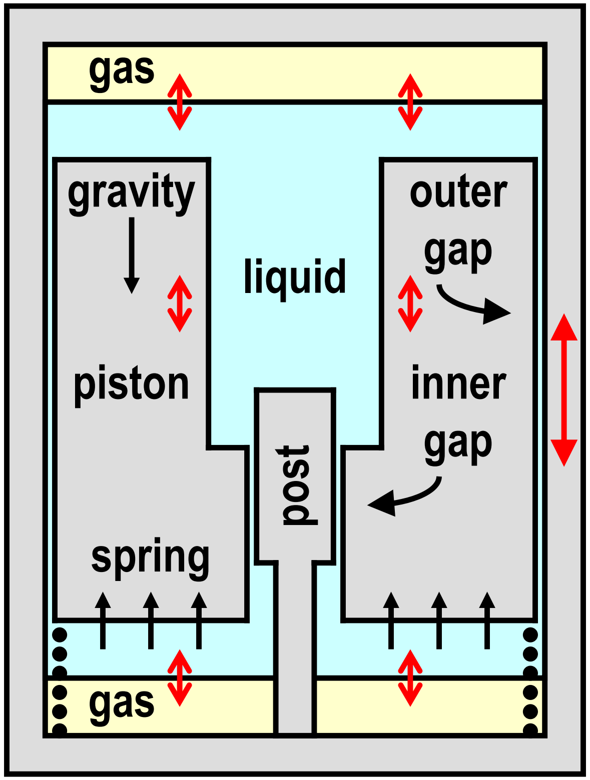


L. A. Romero et al., *Phys. Fluids*, 053301 (2014).



T. J. O’Hern et al., *Phys. Fluids*, 091108 (2012).

Vibration Makes Piston Move Down



L. A. Romero et al., *ASME J. Fluids Eng.*, 061302 (2016).

Gas regions form pneumatic spring

- One expands, the other compresses
- Stiffness is ~100x helical spring

Enables new mode with low damping

- Piston and interfaces move together
- No liquid is forced through inner gap

Low damping gives strong resonance

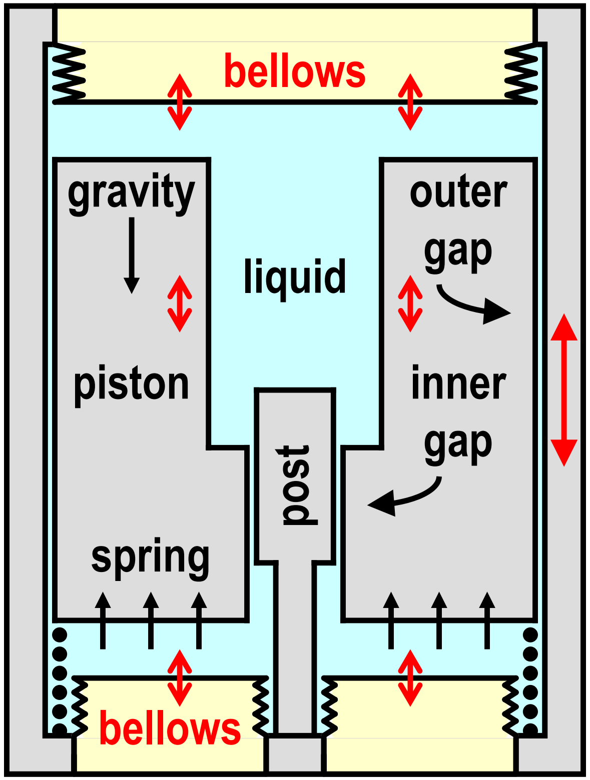
- Piston + liquid mass and gas spring

Gap nonlinearity produces net force

- Damping depends on piston position
- Piston moves down to shorten gap

$\omega_{\text{res}} = \sqrt{\dfrac{K_{\text{gas}}}{M_{\text{total}}}}$ has very low damping

Better System for Analysis



Gas regions are hard to analyze

- Upper/lower split of gas is not known
- Motion is transient and complicated

So replace gas regions with bellows

- Compressibility is well characterized
- Choose to be similar to gas regions

Well suited for theory & simulation

- Liquid: incompressible Navier-Stokes equations with moving boundaries
- Solids: Newton's 2nd Law ("F = ma")

Analysis of Rectified Piston Motion

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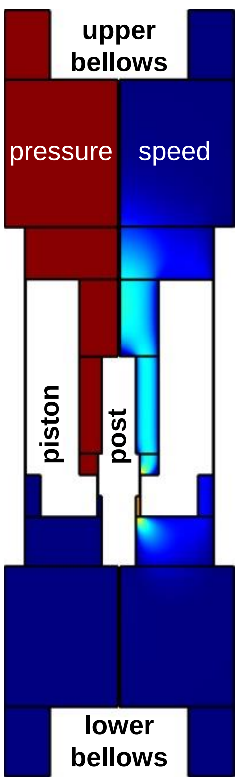
Theory gives 2-DOF nonlinear damped harmonic oscillator

- **Quasi-steady Stokes & Newton's 2nd Law: PDEs → ODEs**
- **Liquid added mass & damping depend on piston position**

Piston motion agrees with Navier-Stokes ALE simulation



Navier-Stokes Eqns.
Newton's 2nd Law

$$\lambda = \frac{A_B}{A_P + (A_G/2)}$$

$$F_{\text{rect}} = -\frac{dB_{12}}{dz_1}\langle z_1 \dot{z}_2 \rangle$$

$$U_{\text{rect}} = \frac{F_{\text{rect}}}{B_{11}} \text{ (drift)}$$

liquid added masses — liquid damping coefficients — gravity-buoyancy

$$\left(\tilde{\mathbf{M}} + \mathbf{M}\right)\ddot{\mathbf{Z}} + \left(\tilde{\mathbf{B}} + \mathbf{B}\right)\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{F}$$

object masses — object damping coefficients — object spring constants

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_i = 0, \quad \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_i = 0$$

$$\mathbf{u}_i = \begin{cases} U\hat{\mathbf{e}}_z \text{ on } S_i \\ 0 \text{ on other walls} \end{cases}$$

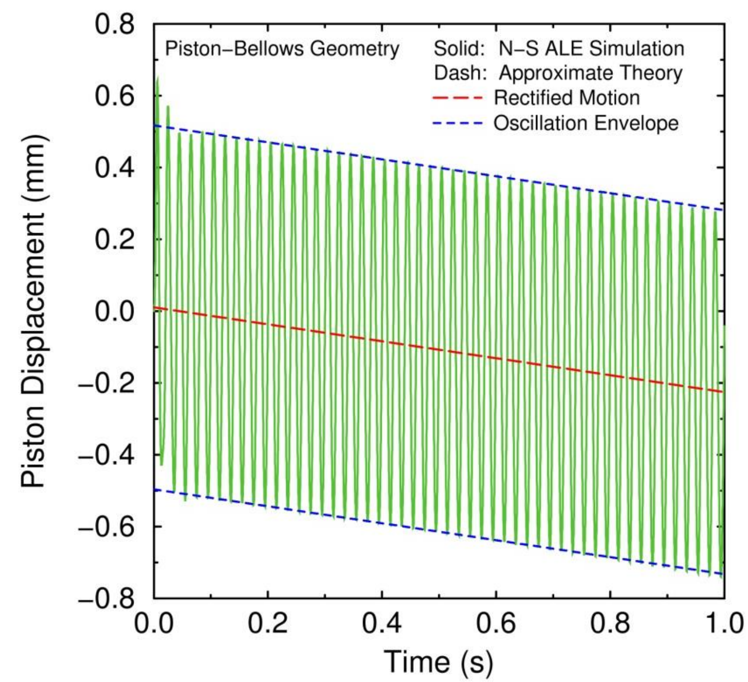
$$\mathbf{S}_i = \frac{1}{2}\left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}}^T\right)$$

$$\mathbf{Z} = \begin{pmatrix} Z_P \\ Z_B \end{pmatrix} \qquad \mathbf{F} = -\begin{pmatrix} M_{PG} \\ M_{BG} \end{pmatrix} g_1 \sin \omega t$$

$$\mathbf{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \qquad B_{ij} = \frac{2\mu}{U^2}\int_V \mathbf{S}_i : \mathbf{S}_j \, dV$$

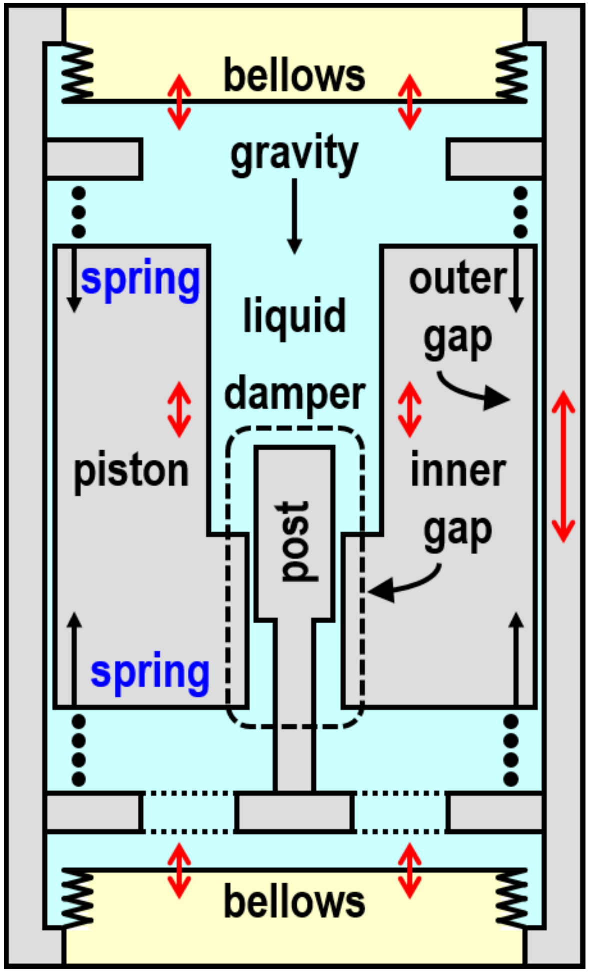
$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \qquad m_{ij} = \frac{\rho}{U^2}\int_V \mathbf{u}_i \cdot \mathbf{u}_j \, dV$$

$$\tilde{\mathbf{K}} = \begin{pmatrix} K_P & 0 \\ 0 & K_B \end{pmatrix} \qquad \tilde{\mathbf{B}} = \begin{pmatrix} B_P & 0 \\ 0 & B_B \end{pmatrix} \qquad \tilde{\mathbf{M}} = \begin{pmatrix} M_P & 0 \\ 0 & M_B \end{pmatrix}$$



Theory and Simulation

Better System for Comparison



Interaction with stop is complicated

- Flat surfaces with liquid in between
- Squeeze-film damping from liquid
- Asperities control solid-solid contact

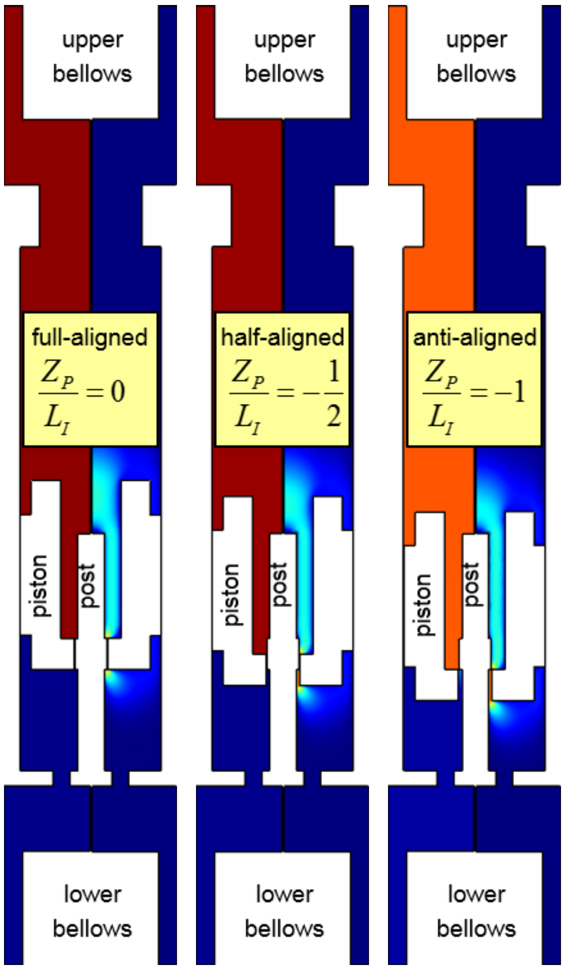
So replace stop with a second spring

- Two-spring suspension holds piston
- Spring force is well characterized

Well suited for analysis & experiment

- Focus on rectification nonlinearity
- Study stop interaction subsequently

Two-Spring System



maximum damping | zero net force | minimal damping

Piston positions of significance

- **Full-aligned: maximum damping**
- **Half-aligned: zero net force**
- **Anti-aligned: minimal damping**

Quasi-steady equilibrium analysis

Navier-Stokes and Newton's 2nd Law

$$\rho \frac{D\mathbf{u}}{Dt} = -\frac{\partial p}{\partial \mathbf{x}} + \mu \nabla^2 \mathbf{u} + \rho(\mathbf{g} - \mathbf{a}), \quad \nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} = \mathbf{u}_{\text{wall}};$$

$$\tilde{\mathbf{M}}\ddot{\mathbf{Z}} = -\tilde{\mathbf{B}}\dot{\mathbf{Z}} - \tilde{\mathbf{K}}\mathbf{Z} + \tilde{\mathbf{M}}(\mathbf{g} - \mathbf{a}) + \mathbf{F}_{\text{liquid}}, \quad \mathbf{u}_{\text{wall}} = \mathbf{u}_{\text{wall}}[\mathbf{Z}, \dot{\mathbf{Z}}]$$

⇩

Full ODE (quasi-steady Stokes)

$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}])\ddot{\mathbf{Z}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}])\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{F}, \quad \mathbf{F} = \mathbf{F}_0 \sin[\omega t]$$

⇩

Oscillation + drift model

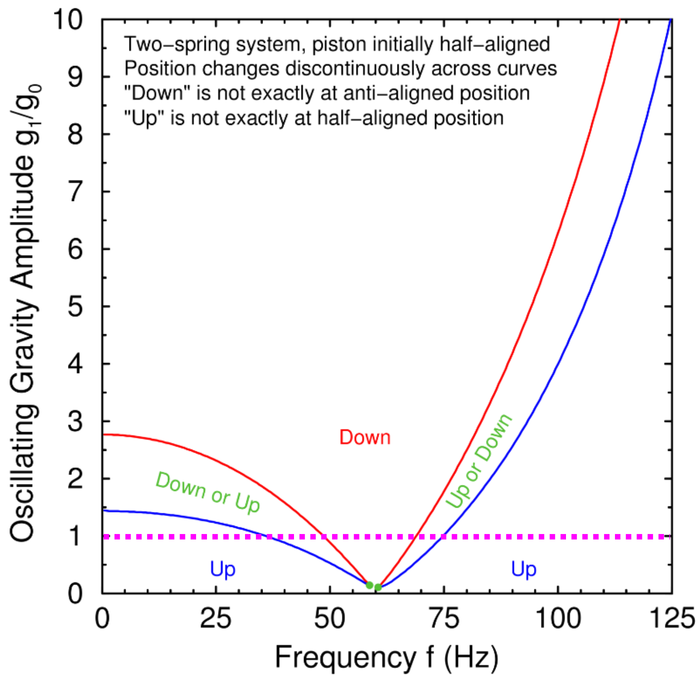
$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}])\ddot{\mathbf{Z}}_{\text{oscil}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}])\dot{\mathbf{Z}}_{\text{oscil}} + \tilde{\mathbf{K}}\mathbf{Z}_{\text{oscil}} = \mathbf{F}_{\text{oscil}}$$

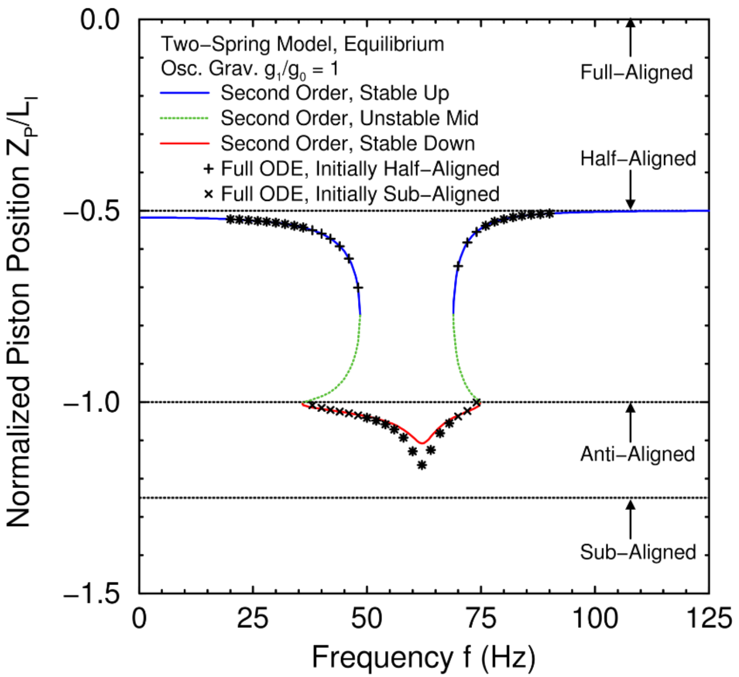
$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}])\ddot{\mathbf{Z}}_{\text{drift}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}])\dot{\mathbf{Z}}_{\text{drift}} + \boxed{\tilde{\mathbf{K}}\mathbf{Z}_{\text{drift}} = \mathbf{F}_{\text{drift}}}$$

$$\mathbf{F}_{\text{oscil}} = \mathbf{F}_0 \sin[\omega t], \quad \mathbf{F}_{\text{drift}} = -\left\langle \mathbf{Z}_{\text{oscil}} \frac{\partial \mathbf{B}}{\partial \mathbf{Z}} \dot{\mathbf{Z}}_{\text{oscil}} \right\rangle$$

↑ Quasi-steady equilibrium

Multiple Equilibrium Piston Positions





Equilibrium piston position versus amplitude & frequency

- **Two stable states: up, down (unstable state between: mid)**
- **Up & down regions separated by multi-state regions**

Position is multi-valued versus frequency at fixed amplitude

- **Quasi-steady equilibrium agrees well with full ODE**

Model and Experiment Agree

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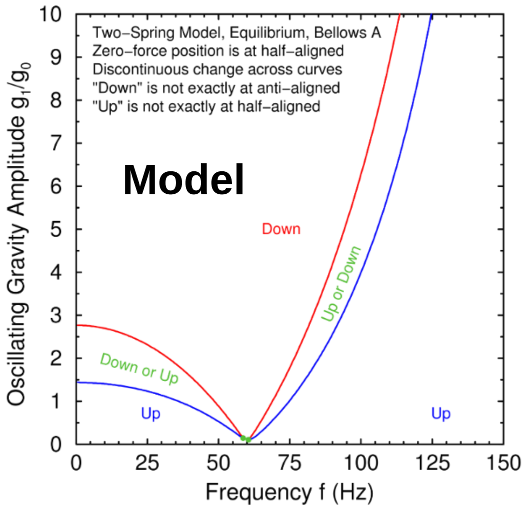
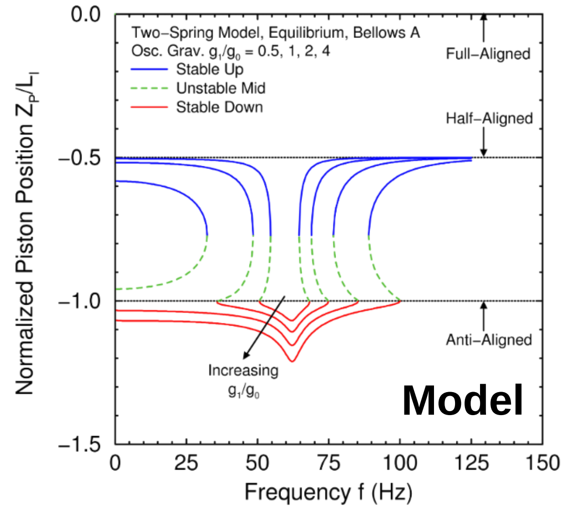
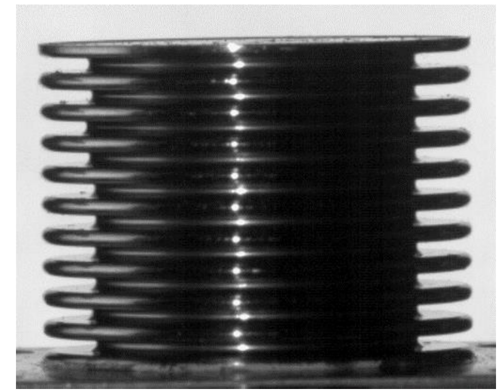
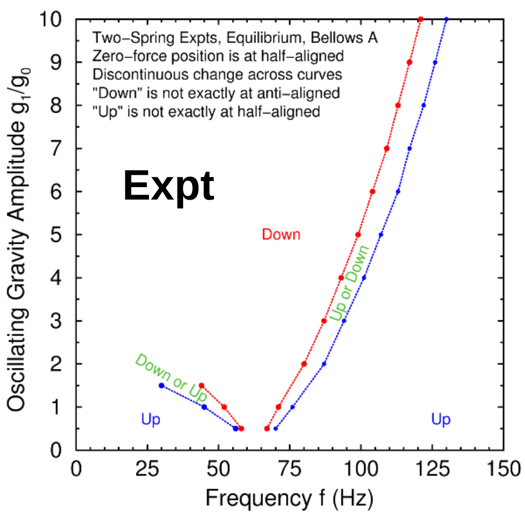
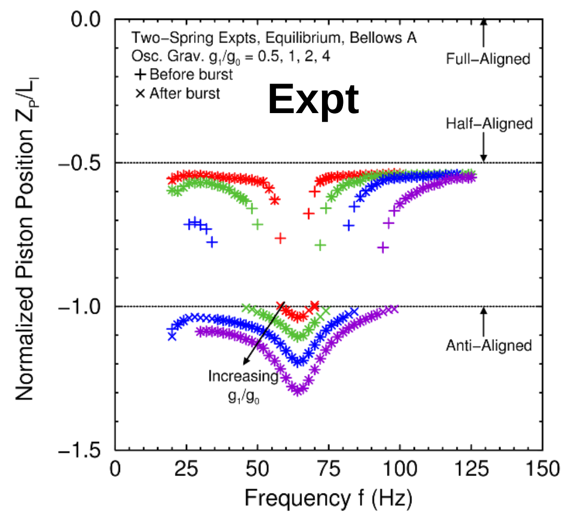






Servometer FC-16 Bellows "A"
(like a bigger gas bubble)





Piston position

- **Equilibrium**
- **Bellows "A"**

Stable states

- **Regime maps**
- **Fixed-amplitude frequency slices**

Model and Experiment Agree

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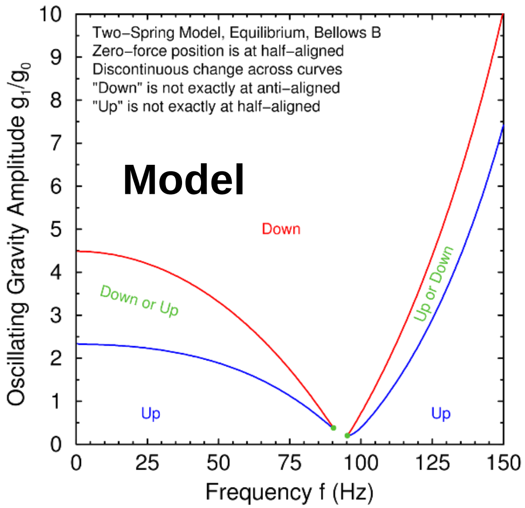
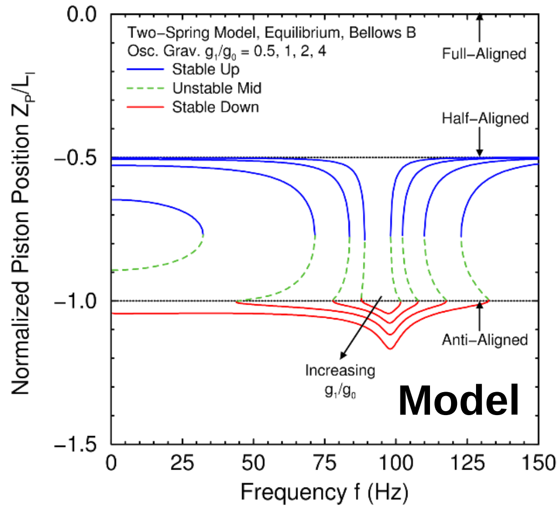
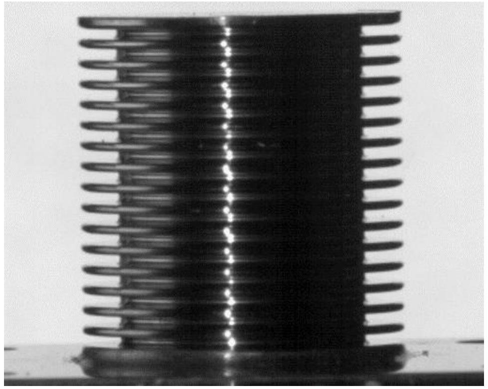
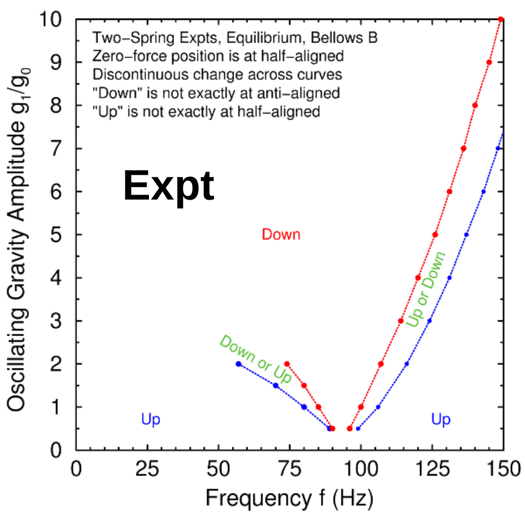
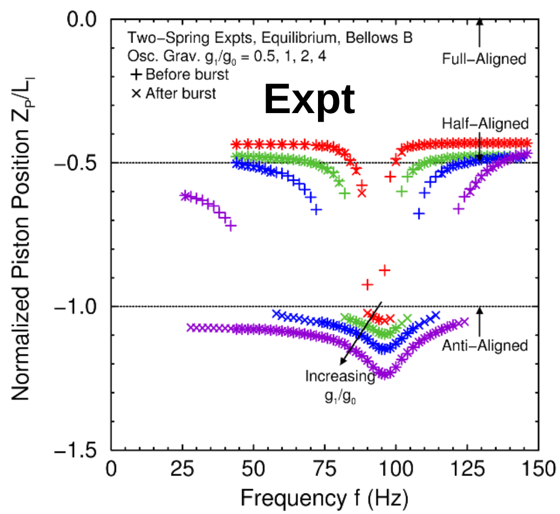






Servometer FC-13 Bellows "B"
(like a smaller gas bubble)





Piston position

- **Equilibrium**
- **Bellows "B"**

Stable states

- **Regime maps**
- **Fixed-amplitude frequency slices**

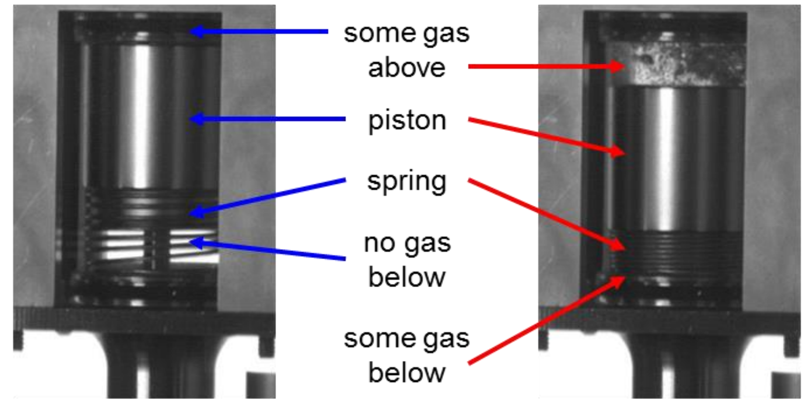
Summary and Future Work

gas above + vibration = gas below

gas above + gas below = gas spring

gas spring + total mass = resonance

resonance + nonlinearity = net motion



Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Cause of vibration-induced piston motion determined

- **Clear physical picture of route to net motion (rectification)**
- **Good agreement between theory & experiment (bellows)**

Much work remains to obtain a complete understanding

- **Investigate effects of friction & contact forces (the stop)**
- **Study how gas divides between upper and lower regions**