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Gas-Induced Motion of an Object in a Liquid-Filled Housing during Vibration: I. Analysis
(II. Experiments is the next talk)

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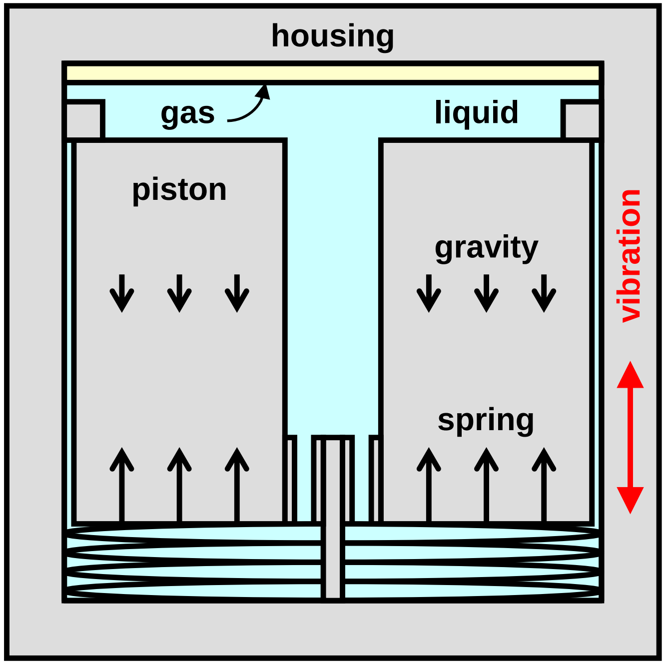
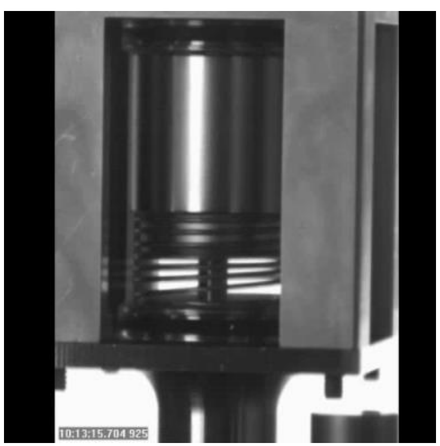
Strange Vibration-Induced Dynamics



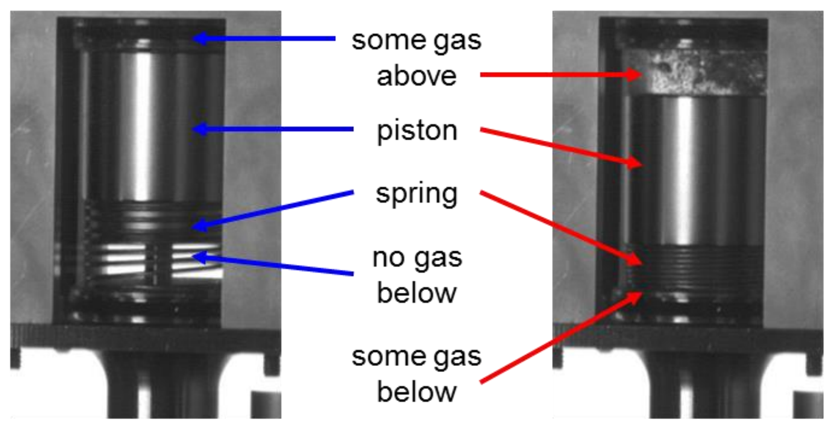


Spring-mass-dashpot system

- Piston moves vertically in housing
- Spring supports it against gravity
- Viscous liquid provides damping
- Small amount of gas is present

Housing is vibrated vertically

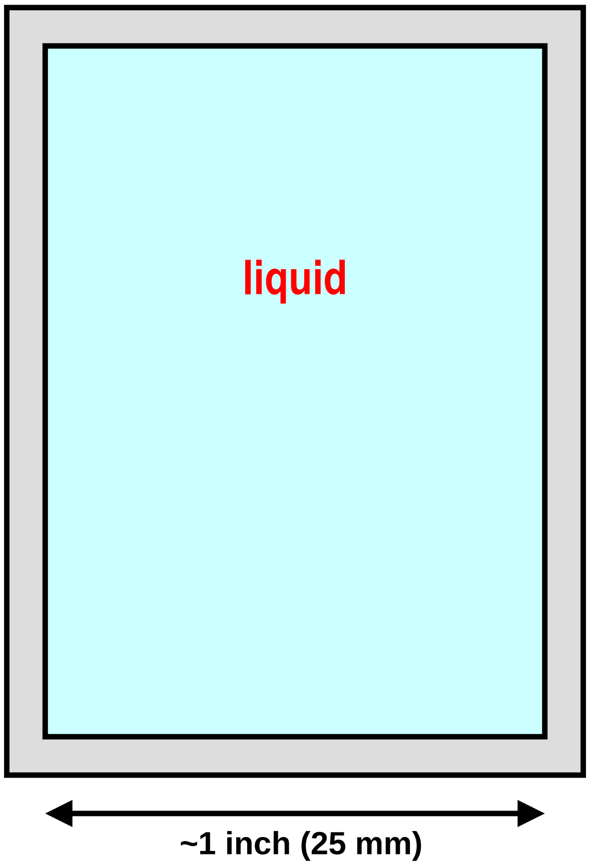
- Gas moves down below piston
- Piston moves down against spring



Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Fill a Housing with a Liquid



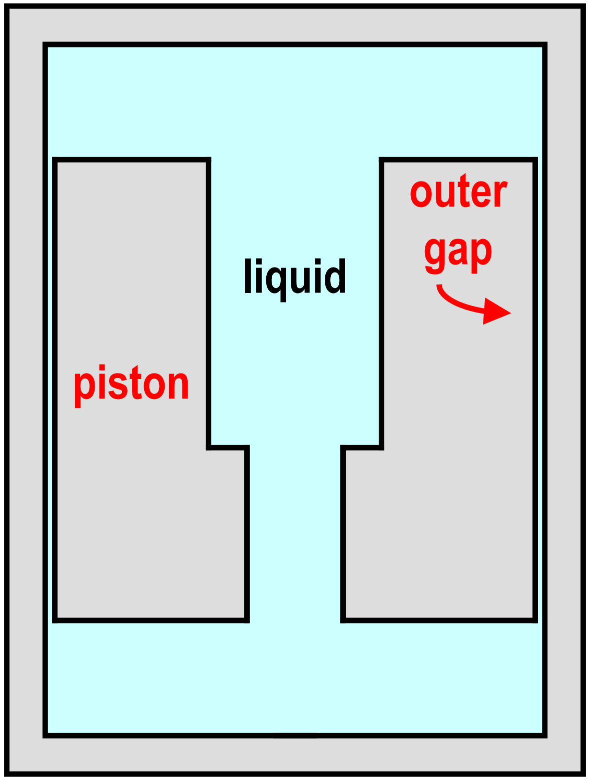
Make a cylindrical housing

- Stainless steel, completely rigid
- ID ~1 inch (25 mm)
- Height ~2 inch (50 mm)

Fill it with incompressible liquid

- Typically silicone oil (20-cSt PDMS)
- Density ~ water density
- Viscosity ~20x water viscosity

Put a Piston Inside the Housing



Piston is basically cylindrical

- Stainless steel, ~8x liquid density
- Real piston complex but same mass

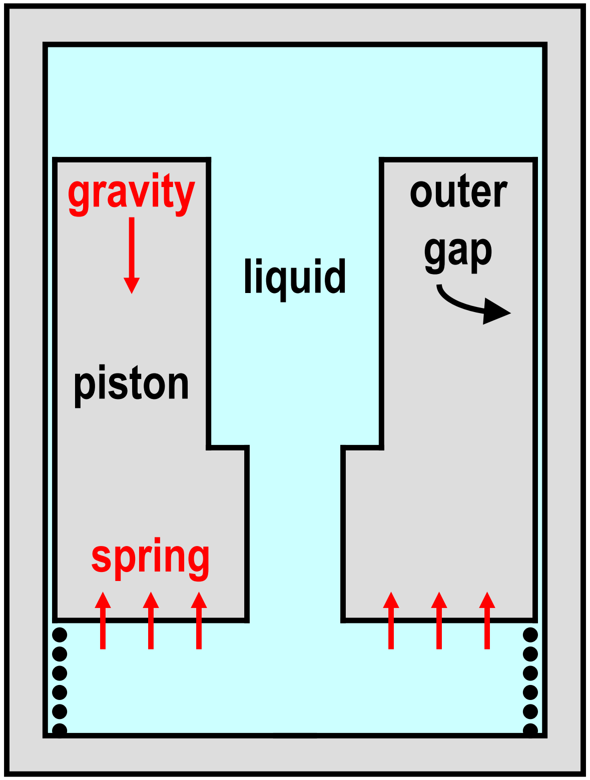
Piston and housing define outer gap

- Outer gap is ~0.001x piston diameter
  - Typically ~0.001 inch (0.025 mm)
- Piston can move only vertically

Piston has hole along axis

- Hole diameter varies with position

Support the Piston with a Spring



Piston wants to sink to bottom

- Gravity pulls down on everything
- Buoyancy is much less than weight

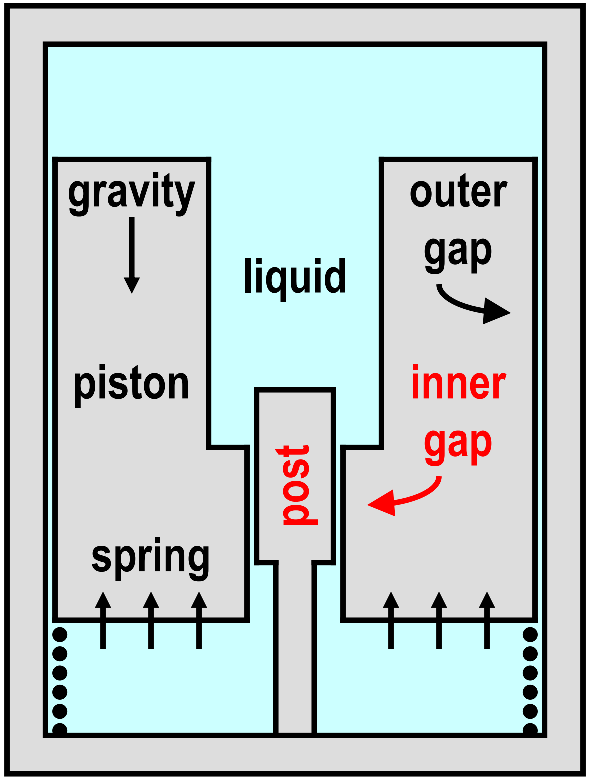
Support it from below with a spring

- Helical coil of very narrow wire
- Diagram shows slice through coils

Here, piston freely suspended in liquid

- Reality: piston pressed against stop
- Ignore preload and stop for now

Add a Post to Specify the Damping



Post is fixed firmly to housing

- Post diameter varies with position

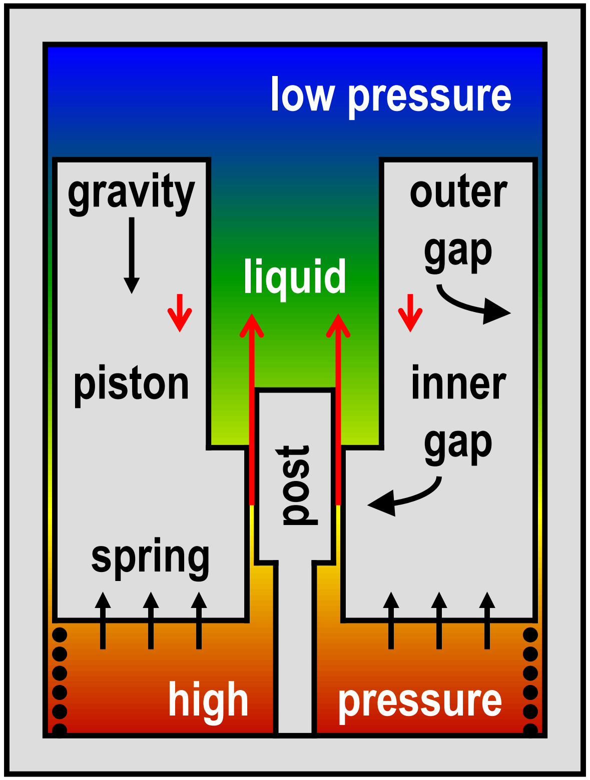
Piston and post define inner gap

- Inner gap ~4x as wide as outer gap
  - Typically ~0.004 inch (0.1 mm)
- Flow resistance: inner ~0.01x outer

Damping depends on piston position

- Damping proportional to gap length
- Gap shortens as piston moves down

Try to Move the Piston



Piston and liquid motions are coupled

- Suppose piston moves down
- Liquid flows up through inner gap

Resistance to piston motion is large

- Inner gap has small cross section
- Liquid velocity in gap is very large
- Opposing pressure drop is large

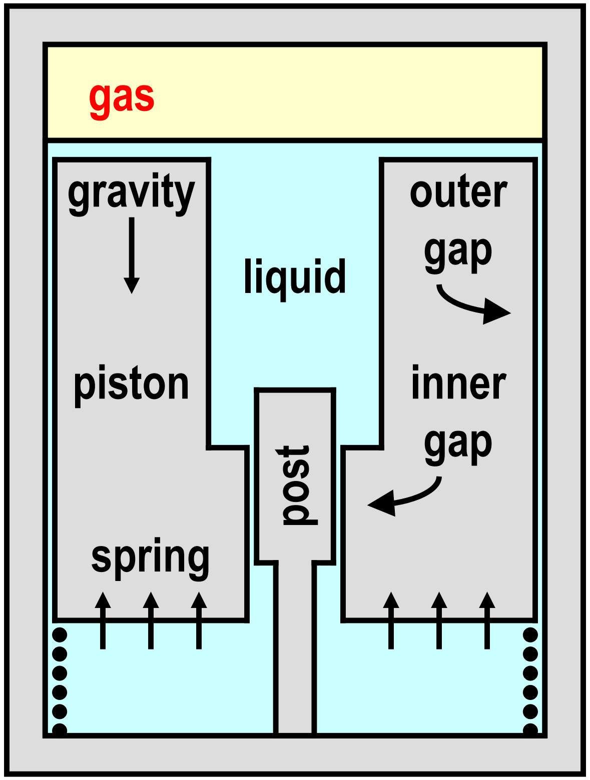
Liquid-filled system is overdamped

- Acting as intended: a dashpot

Piston-spring “resonance” irrelevant

$$\omega = \sqrt{\frac{K_{\text{spring}}}{M_{\text{piston}}}}$$ is highly overdamped

Now Add Some Gas



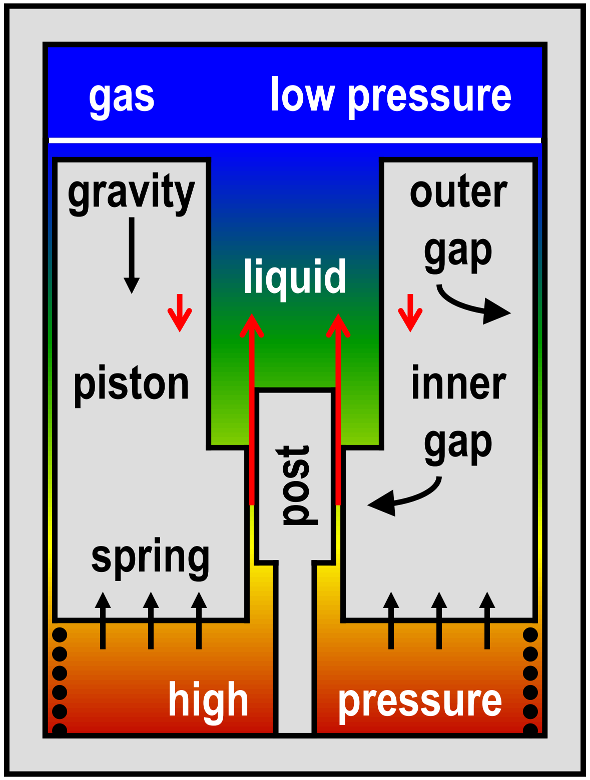
Air, nitrogen, argon; oil vapor minimal
- Gas is filtered, humidity is controlled

Gas prevents housing from bursting
- Liquid has large thermal expansion
- Solids have small thermal expansion

Gas is generally at top of system
- Buoyancy and minor agitation
- Dissolves and diffuses in liquid

Some gas might be under piston
- Recesses on piston bottom surface
- For now, suppose no gas underneath

Try Again to Move the Piston



Piston and liquid motions still coupled

- Suppose piston moves down
- Liquid still must flow up through gap

Gas volume cannot change, no effect

- Liquid and solids are incompressible

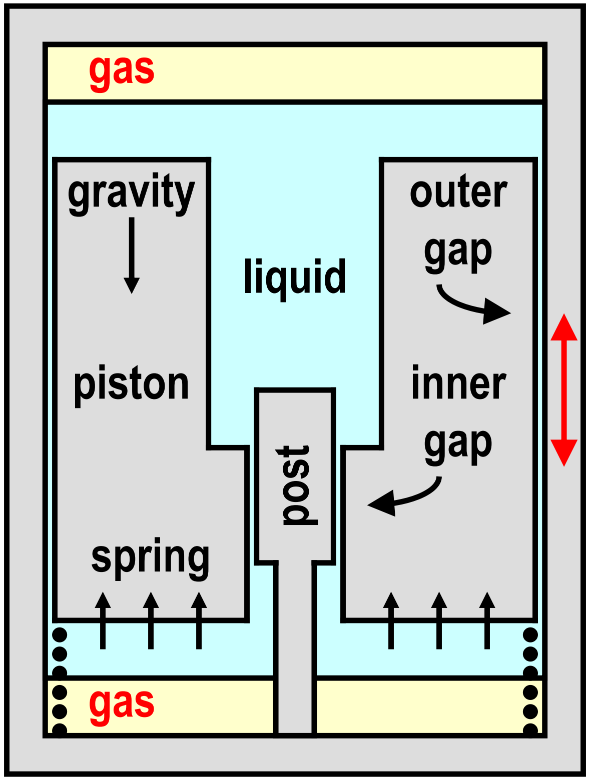
Resistance to piston motion still large

- Inner gap has small cross section
- Liquid velocity in gap still very large
- Opposing pressure drop still large

Gas-at-top system still overdamped

- Acting as intended: still a dashpot

Now Vibrate System Vertically

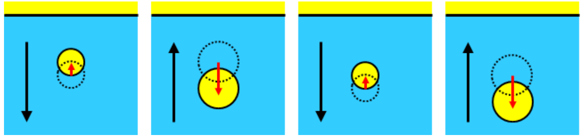


Some gas moves down below piston!

- **Bjerknes forces push bubbles down**
- **Create & stabilize a lower gas region**

Two gas regions: upper and lower

- **Both are quasi-stable (stationary)**



L. A. Romero et al., *Phys. Fluids*, 053301 (2014).

T. J. O'Hern et al., *Phys. Fluids*, 091108 (2012).

Vibration Makes Piston Move Down

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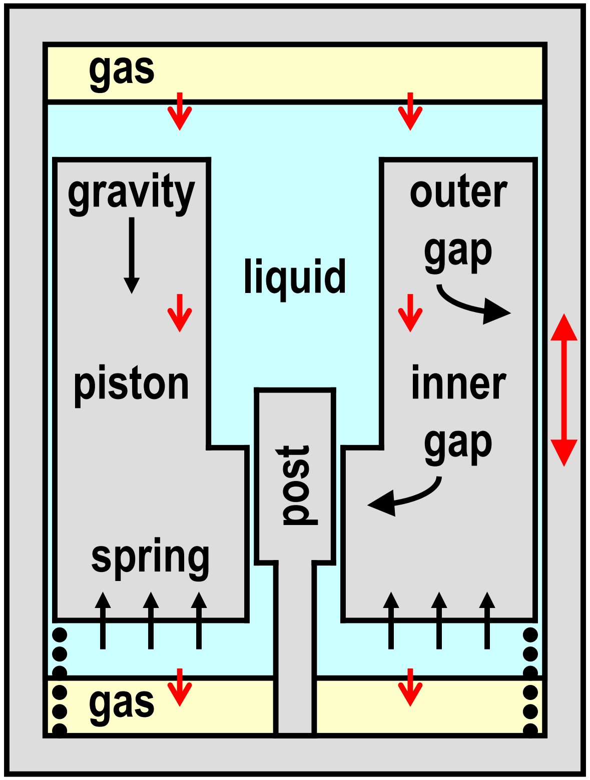


Gas regions form pneumatic spring

- One expands, the other compresses
- Stiffness is ~100x helical spring

Enables new mode with low damping

- Piston and interfaces move together
- No liquid is forced through inner gap

Low damping gives strong resonance

- Piston + liquid mass and gas spring

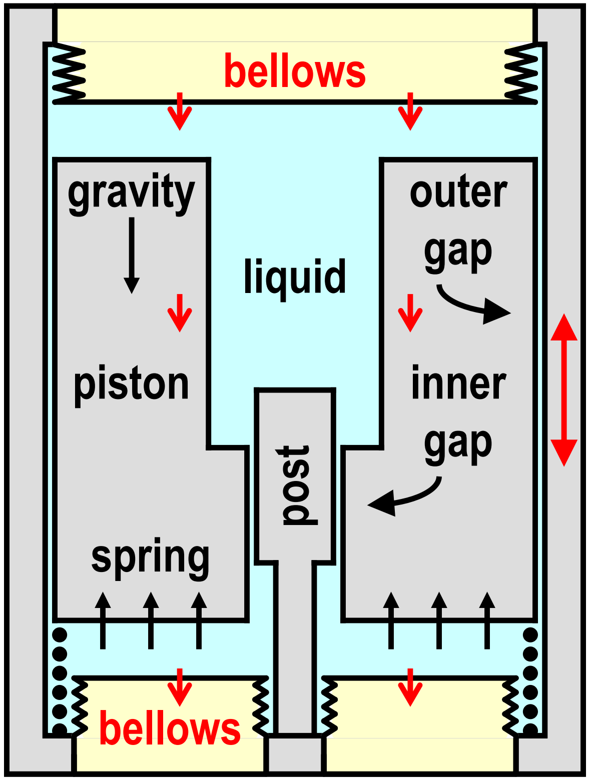
Gap nonlinearity produces net force

- Damping depends on piston position
- Piston moves down to shorten gap

$\omega_{res} = \sqrt{\frac{K_{gas}}{M_{total}}}$ has very low damping

L. A. Romero et al., *ASME J. Fluids Eng.*, 061302 (2016).

Better System for Analysis



Gas regions are hard to analyze

- Upper/lower split of gas is not known
- Motion is transient and complicated

So replace gas regions with bellows

- Compressibility is well characterized
- Choose to be similar to gas regions

Well suited for theory & simulation

- Liquid: incompressible Navier-Stokes equations with moving boundaries
- Solids: Newton's 2nd Law ("F = ma")

Analysis of Rectified Piston Motion

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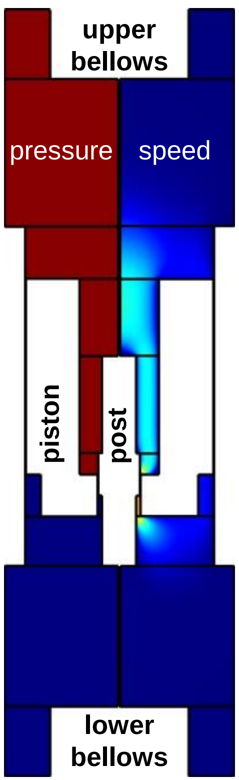
Theory gives 2-DOF nonlinear damped harmonic oscillator

- **Quasi-steady Stokes & Newton's 2nd Law: PDEs → ODEs**
- **Liquid added mass & damping depend on piston position**

Piston motion agrees with Navier-Stokes ALE simulation



Navier-Stokes Eqns.
Newton's 2nd Law

$$\lambda = \frac{A_B}{A_P + (A_G/2)}$$

$$F_{\mathrm{rect}} = -\frac{dB_{12}}{dz_1}\langle z_1 \dot{z}_2 \rangle$$

$$U_{\mathrm{rect}} = \frac{F_{\mathrm{rect}}}{B_{11}} \text{ (drift)}$$

liquid added masses; liquid damping coefficients; gravity-buoyancy

$$\left(\tilde{\mathbf{M}} + \mathbf{M}\right)\ddot{\mathbf{Z}} + \left(\tilde{\mathbf{B}} + \mathbf{B}\right)\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{F}$$

object masses; object damping coefficients; object spring constants

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_i = 0, \quad \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_i = 0$$

$$\mathbf{u}_i = \begin{cases} U\hat{\mathbf{e}}_z \text{ on } S_i \\ 0 \text{ on other walls} \end{cases}$$

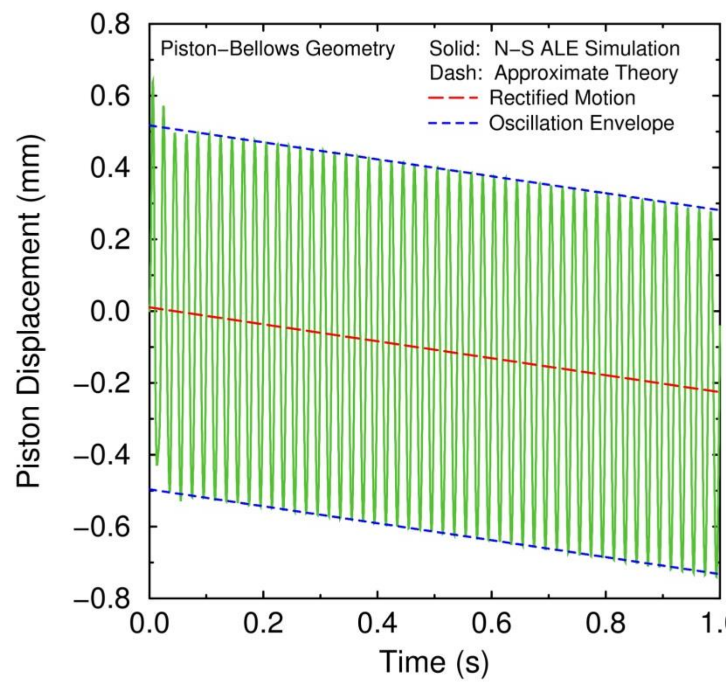
$$\mathbf{S}_i = \frac{1}{2}\left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}}^T\right)$$

$$\mathbf{Z} = \begin{pmatrix} Z_P \\ Z_B \end{pmatrix} \qquad \mathbf{F} = -\begin{pmatrix} M_{PG} \\ M_{BG} \end{pmatrix} g_1 \sin \omega t$$

$$\mathbf{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \qquad B_{ij} = \frac{2\mu}{U^2}\int_V \mathbf{S}_i : \mathbf{S}_j \, dV$$

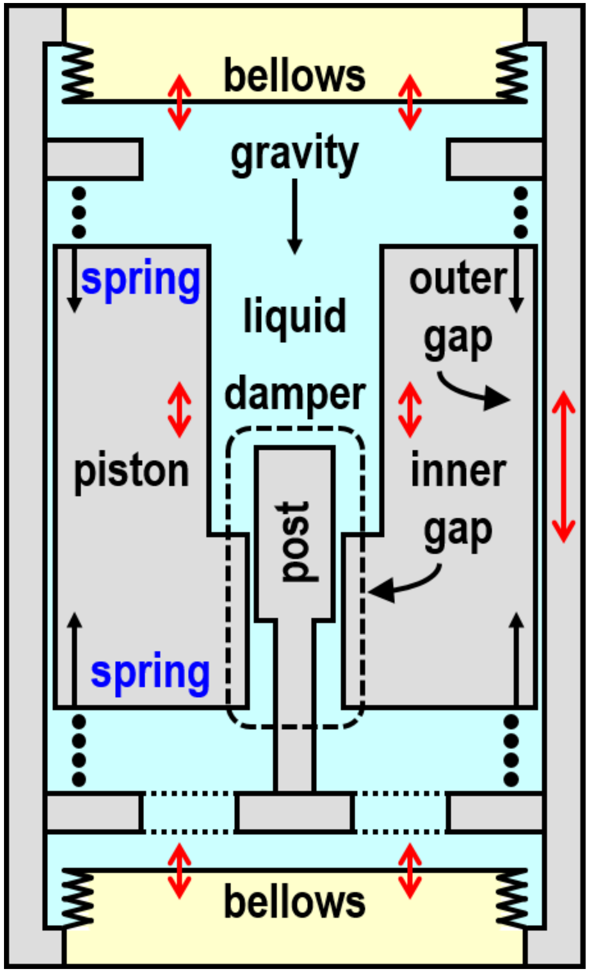
$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \qquad m_{ij} = \frac{\rho}{U^2}\int_V \mathbf{u}_i \cdot \mathbf{u}_j \, dV$$

$$\tilde{\mathbf{K}} = \begin{pmatrix} K_P & 0 \\ 0 & K_B \end{pmatrix} \qquad \tilde{\mathbf{B}} = \begin{pmatrix} B_P & 0 \\ 0 & B_B \end{pmatrix} \qquad \tilde{\mathbf{M}} = \begin{pmatrix} M_P & 0 \\ 0 & M_B \end{pmatrix}$$



Theory and Simulation

Better System for Comparison



Interaction with stop is complicated

- Flat surfaces with liquid in between
- Squeeze-film damping from liquid
- Asperities control solid-solid contact

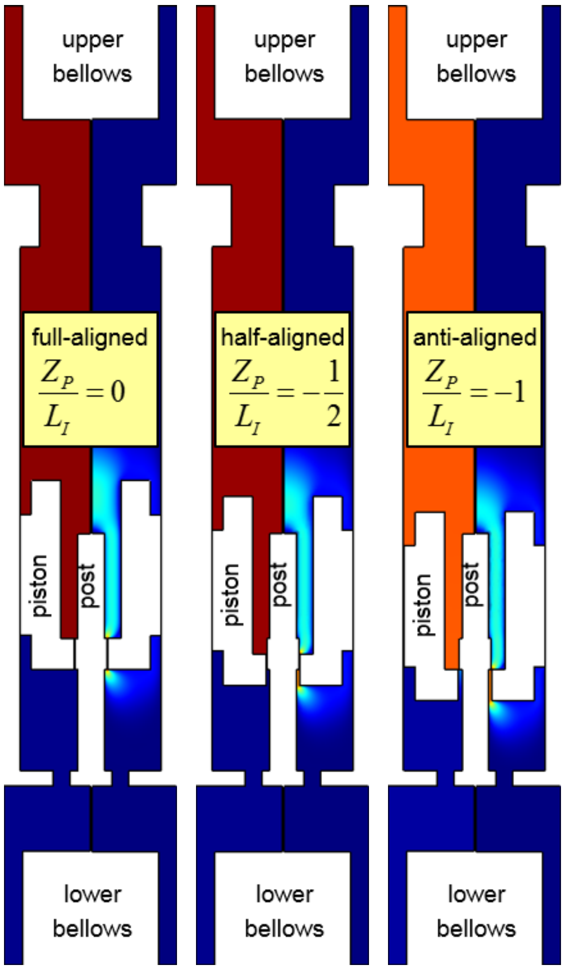
So replace stop with a second spring

- Two-spring suspension holds piston
- Spring force is well characterized

Well suited for analysis & experiment

- Focus on rectification nonlinearity
- Study stop interaction subsequently

Two-Spring System



maximum damping | **zero net force** | **minimal damping**

Piston positions of significance

- **Full-aligned: maximum damping**
- **Half-aligned: zero net force**
- **Anti-aligned: minimal damping**

Quasi-steady equilibrium analysis

Navier-Stokes and Newton's 2nd Law

$$\rho \frac{D\mathbf{u}}{Dt} = -\frac{\partial p}{\partial \mathbf{x}} + \mu \nabla^2 \mathbf{u} + \rho(\mathbf{g} - \mathbf{a}), \quad \nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} = \mathbf{u}_{\text{wall}};$$

$$\tilde{\mathbf{M}}\ddot{\mathbf{Z}} = -\tilde{\mathbf{B}}\dot{\mathbf{Z}} - \tilde{\mathbf{K}}\mathbf{Z} + \tilde{\mathbf{M}}(\mathbf{g} - \mathbf{a}) + \mathbf{F}_{\text{liquid}}, \quad \mathbf{u}_{\text{wall}} = \mathbf{u}_{\text{wall}}\left[\mathbf{Z}, \dot{\mathbf{Z}}\right]$$

Full ODE (quasi-steady Stokes)

$$\left(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}]\right)\ddot{\mathbf{Z}} + \left(\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}]\right)\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{F}, \quad \mathbf{F} = \mathbf{F}_0 \sin[\omega t]$$

Oscillation + drift model

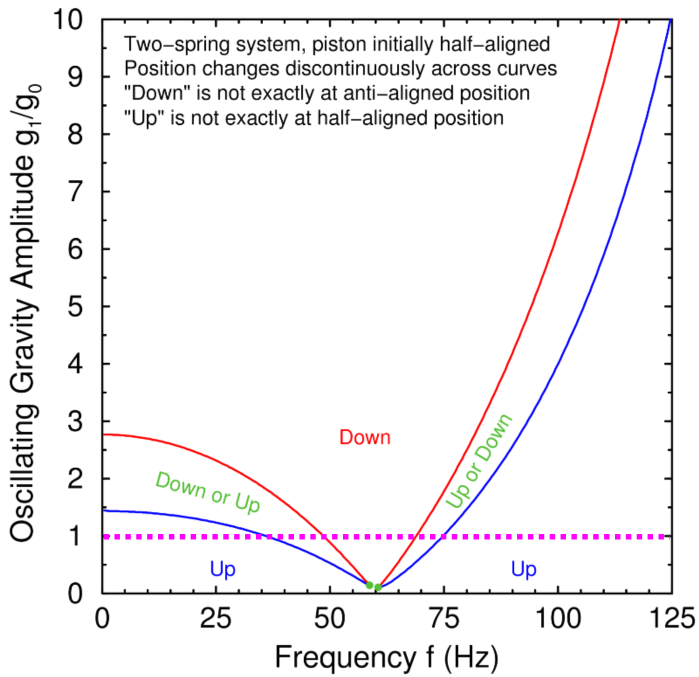
$$\left(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}]\right)\ddot{\mathbf{Z}}_{\text{oscil}} + \left(\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}]\right)\dot{\mathbf{Z}}_{\text{oscil}} + \tilde{\mathbf{K}}\mathbf{Z}_{\text{oscil}} = \mathbf{F}_{\text{oscil}}$$

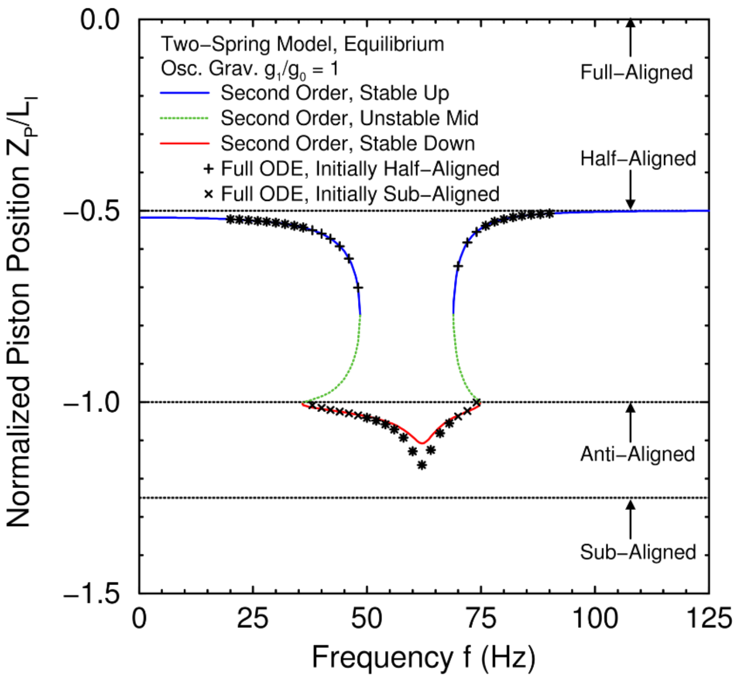
$$\left(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}]\right)\ddot{\mathbf{Z}}_{\text{drift}} + \left(\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}]\right)\dot{\mathbf{Z}}_{\text{drift}} + \boxed{\tilde{\mathbf{K}}\mathbf{Z}_{\text{drift}} = \mathbf{F}_{\text{drift}}}$$

$$\mathbf{F}_{\text{oscil}} = \mathbf{F}_0 \sin[\omega t], \quad \mathbf{F}_{\text{drift}} = -\left\langle \mathbf{Z}_{\text{oscil}} \frac{\partial \mathbf{B}}{\partial \mathbf{Z}} \dot{\mathbf{Z}}_{\text{oscil}} \right\rangle$$

Quasi-steady equilibrium

Multiple Equilibrium Piston Positions





Equilibrium piston position versus amplitude & frequency

- Two stable states: up, down (unstable state between: mid)
- Up & down regions separated by multi-state regions

Position is multi-valued versus frequency at fixed amplitude

- Quasi-steady equilibrium agrees well with full ODE

More Amplitudes and Bellows

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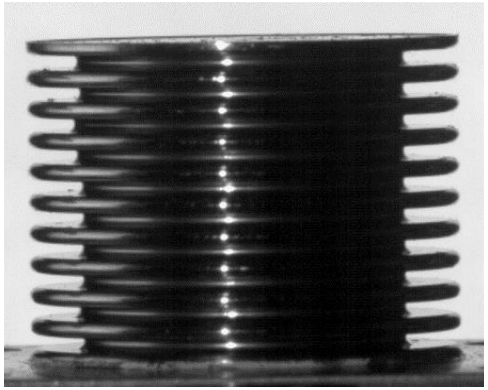
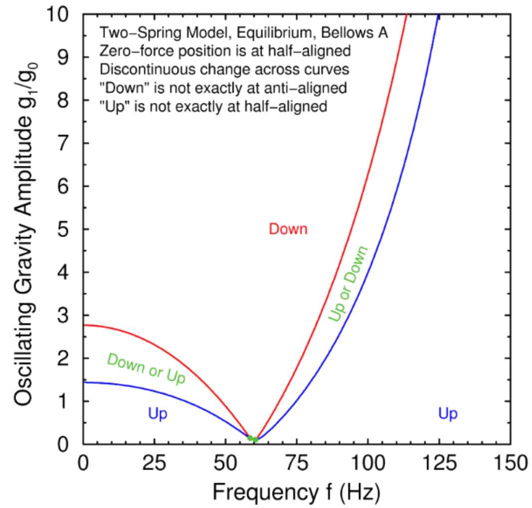
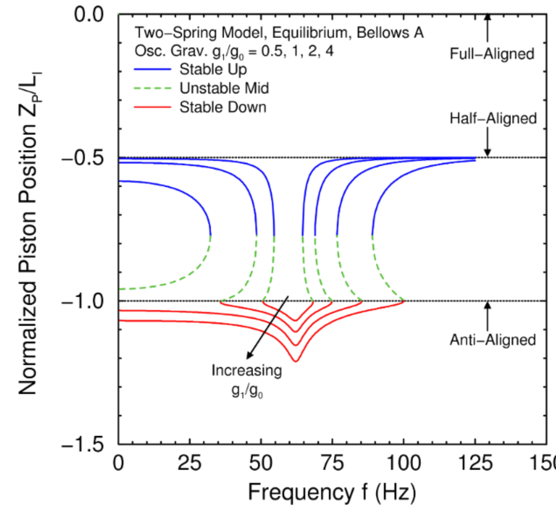
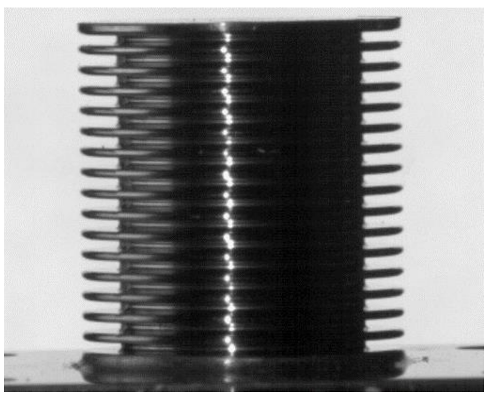


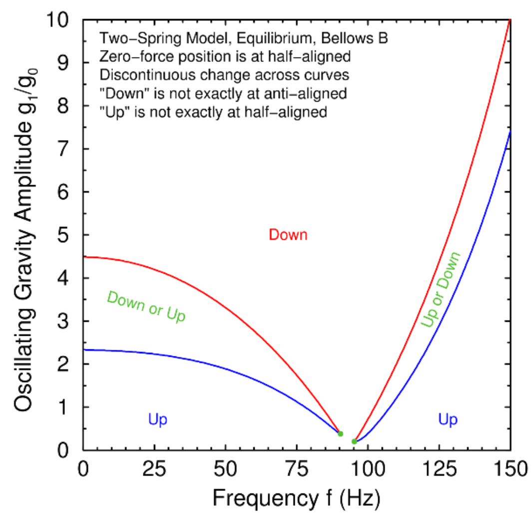
Servometer FC-16 Bellows "A"
(like a bigger gas bubble)

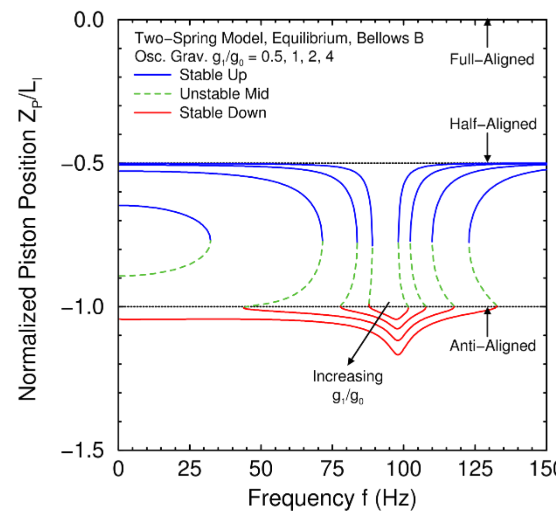






Servometer FC-13 Bellows "B"
(like a smaller gas bubble)





Apply model to determine regime maps of stable states and piston position versus frequency for fixed amplitude

Summary and Future Work

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gas above + vibration = gas below

⇩

gas above + gas below = gas spring

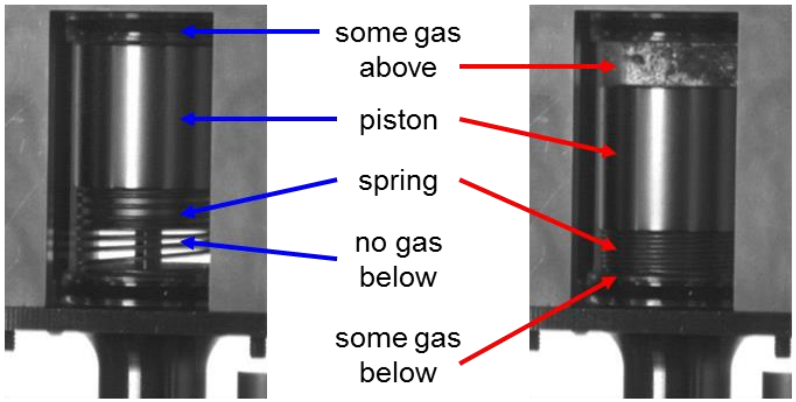
⇩

gas spring + total mass = resonance

⇩

resonance + nonlinearity = net motion

⇨



Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Cause of vibration-induced piston motion determined

- **Clear physical picture of route to net motion (rectification)**
- **Good agreement between theory & simulation (bellows)**

Much work remains to obtain a complete understanding

- **Investigate effects of friction & contact forces (the stop)**
- **Study how gas divides between upper and lower regions**

Next talk will compare results from theory and experiment