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Nonlinear Modeling of Macroscopic Plasma Dynamics

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1. Public Abstract

The project entitled, "Nonlinear Modeling of Macroscopic Plasma Dynamics," applied numerical computation to solve mathematical models of plasma behavior in laboratory systems. Plasma is the state of matter with sufficiently large internal energy density such that electrons dissociate from atoms to form gas of electrically charged particles. It occurs naturally in stars, planetary magnetospheres, and interstellar and intergalactic media. It is also used for industrial purposes and in lighting. Fusion energy science seeks to create and sustain plasma conditions where atomic nuclei fuse to create energy that can be captured for power production. Many laboratory experiments for this effort use magnetic field to contain plasma and to thermally insulate it from its surroundings. Possible plasma dynamics in magnetized systems include collective behavior over spatial scales that are comparable to the configuration size, and these dynamics often include evolution of the magnetic field that is used for confinement. Solving the mathematical models that describe these dynamics in specific experiments often requires computer simulations, as done in this project. This project analyzed plasma magnetic relaxation dynamics in the laboratory configurations known as the reversed-field pinch (RFP), the spheromak, and the spherical torus (ST). It also developed capabilities for modeling disruptive plasma dynamics in the tokamak, which has achieved the highest performance for magnetized plasma and is the design of the large international ITER experiment (https://www.iter.org) that is under construction and is expected to be the first to achieve self-heating conditions.

Dynamic evolution to states of low energy, the aforementioned magnetic relaxation, is known to be important for generating the RFP and spheromak configurations. Nonlinear computational modeling with the Non-Ideal Magnetohydrodynamics with Rotation, Open Discussion code (NIMROD, https://nimrodteam.org) was applied to understand specific aspects of magnetic relaxation in RFPs, spheromaks, and STs. The RFP study considered how the nonlinear transfer of power among different fluctuation harmonics is altered due to the application of pulses of electric field, as done experimentally in the Madison Symmetric Torus RFP device at the University of Wiscosin-Madison. It also investigated the separation of electron and ion dynamics, known as two-fluid effects, including effects on the evolution of the plasma flow profile. The spheromak study considered the influence of these two-fluid effects in conditions relevant to the Sustained Spheromak Experiment (SSPX), formerly at Lawrence Livermore National Laboratory. Our study found that two-fluid effects enhance stability with respect to interchange in SSPX-relevant conditions, but having a complete model with two-fluid effects in separate electron and ion temperature equations is important. The study for STs focused on plasma startup in the Pegasus Toroidal Experiment at UW-Madison, which uses DC voltage applied through a small plasma gun. The simulations show how the resulting helical current stream relaxes through repetitive merging, forming current rings that accumulate into a tokamak-like configuration. Analytical theory was also developed to help explain behavior that was observed in the RFP and spheromak computations.

Although the tokamak configuration has outperformed other magnetic confinement configurations, it has a tendency to undergo discharge-terminating dynamics that release energy onto plasma-facing surfaces and exert electromechanical forces on magnetic coils and structures. Our efforts have improved NIMROD's capabilities to model tokamak disruption by incorporating a resistive wall model that can be applied to axisymmetric, as well as asymmetric, distortions of the magnetic field configuration. Computations addressed whether mathematical conditions applied to plasma flow at the wall influence disruptive dynamics.

2. Project Summary

2.0. Introduction

The primary objective of the Nonlinear Modeling of Macroscopic Plasma Dynamics project was to improve understanding of magnetic-topology-changing behavior in magnetic confinement experiments. The behavior of interest included current-driven magnetic relaxation in reversed-field pinches (RFPs), spheromaks, and non-inductive startup of spherical tokamaks (STs). Dynamics that occur during disruptions in tokamaks are phenomenologically similar, and the project also developed simulation capability for modeling tokamak disruptions. Most of the research effort involved use and development of the NIMROD simulation code [1, 2]. The funding provided by U.S. Department of Energy grant DE-FG02-06ER54850 supported graduate research assistant appointments for James Reynolds, Jacob King, John O'Bryan, Eric Howell, Joshua Sauppe, Zz Riford, and Kyle Bunkers at the University of Wisconsin-Madison. It also provided summer and partial academic-year support for the principal investigator Prof. Carl Sovinec.

The remainder of this summary is organized as follows: Section 2.1 describes accomplishments on modeling pulsed parallel current drive (PPCD) in RFPs. Section 2.2 describes analysis of two-fluid effects during magnetic relaxation in RFPs, and Sect. 2.3 considers instabilities during the post-formation phase of spheromaks. Section 2.4 presents our results on non-inductive startup in an ST, and Sect. 2.5 describes developments for modeling tokamak disruption. Section 2.6 is a list of references, apart from those that are direct products of this effort; they are listed separately in Section 3.

2.1. Simulations of Pulsed Parallel Current Drive

When RFPs are powered by conventional loop-voltage current drive, the electrical current in the plasma is large enough to concentrate toroidal magnetic flux toward the center of the cross section. The resulting magnetic shear impedes current drive in the edge region, and current-density-gradient-driven magnetic tearing modes tend to be unstable. Their nonlinear evolution spreads the parallel current density but also leads to chaotic magnetic field topology over much of the volume. A major effort to improve RFP performance focused on current profile control, and experiments performed on the Madison Symmetric Torus (MST) proved that pulses of electric field (the PPCD technique) could transiently alter the current profile and reduce the magnetic fluctuation level [3]. Optimizing this control scheme led to a factor of five increase in energy confinement time [4].

The PPCD technique was motivated by linear stability considerations, but nonlinear effects play a large role in RFP dynamics. We hypothesized that nonlinear numerical simulations would provide a reasonably comprehensive cause-and-effect description of what happens during the transients. Former graduate student Dr. James Reynolds used NIMROD to solve the nonlinear equations of resistive magnetohydrodynamics (MHD), simplified by not evolving plasma pressure, with boundary conditions that model the pulse of surface electric field. He found that the initial increase in symmetric edge current density had a nearly instantaneous effect on the core-resonant tearing mode fluctuations [JP1[a]]. He used linear analysis to understand how the linear stability of these and other modes change during the pulse. He also computed the power that is transferred nonlinearly among different groups of fluctuations. The largest impact on the

[a]Citations that start with "JP" and "CWP" refer to Journal Publications and Conference and Workshop Presentations, respectively, that are listed as products of the project in Section 3.

edge fluctuations was shown to be the reduction in nonlinear power transferred to them from the core-resonant modes, and the edge modes decay after a slight delay while this coupling power changes. Reducing the loop voltage drive during PPCD was found to improve performance in the experiment, and the simulations showed that the benefit comes from reducing the destabilizing core current density that otherwise tends to grow during the transient. More details on the results of this study can be found in Ref. [JP1].

A proposed aspect of this study that was not completed is to model plasma pressure in the PPCD simulations. The NIMROD computations had difficulty with numerical interchange prior to numerical developments that came later [5]. This is not considered a major shortcoming of the study, because the RFP fluctuations being considered are dominantly driven by gradients of current density and not by gradients of plasma pressure.

2.2. Two-fluid Effects on RFP Relaxation

Single-fluid modeling (MHD) describes many characteristics of magnetic relaxation in RFPs, including the sustainment of magnetic-field reversal, the tearing-mode spectrum, and nonlinear coupling between core- and edge-resonant modes. However, effects that are outside the scope of the MHD model, had been measured in MST. Fast laser polarimetry had been used to measure current density and magnetic field fluctuations in the core of MST [6]. Their correlation during relaxation events amounts to a Hall dynamo effect, $\left\langle \tilde{\mathbf{J}} \times \tilde{\mathbf{B}} \right\rangle_{\parallel} / ne$, where averaging is over the poloidal and toroidal directions, and parallel is with respect to $\langle \mathbf{B} \rangle$. The Hall dynamo effect can influence magnetic relaxation and is distinct from the MHD dynamo effect, $-\left\langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \right\rangle_{\parallel}$, in that Hall dynamo requires significant decoupling of electron dynamics from ion dynamics. Because the Hall dynamo effect implies fluctuation-induced force density, it also influences flow profiles. Measurements of magnetic fluctuations using probes in the edge of MST found Hall dynamo effect near the reversal surface [7]. That experimental effort also used Rutherford scattering in the core and Mach probes in the edge to measure plasma flow fluctuations. Their correlation was used to infer Reynolds stress, hence another force density that acts on the average flow profile.

Theoretically, linear and quasilinear analysis of two-fluid tearing without drift effects had been developed and applied to conditions of MST [8,9]. For tearing conditions with large guide field and not near the threshold for ideal instability, i.e. at "small Δ'", electron dynamics separate from ion dynamics and lead to increased tearing growth rates if the reconnection layer is smaller than the "sound gyro-radius," $\rho_s = c_s / \Omega_i$ [8], where c_s is the ion-acoustic speed and Ω_i is the ion gyro-frequency. The sound gyro-radius is proportional to $\sqrt{\beta_e} d_i$, where β_e is the ratio of electron thermal pressure to magnetic-field pressure, and d_i is the ion skin depth. The quasilinear analysis shows that two-fluid tearing modes at finite amplitude induce Hall dynamo effect very near the tearing-mode resonance, and they induce MHD dynamo effect outside the Hall layer [9].

A self-consistent nonlinear analysis of two-fluid effects on RFP relaxation had not been undertaken prior to our work. Our hypothesis was that nonlinear modeling would also show that the Hall dynamo contributes to relaxation and that it drives flow profile relaxation. We applied the extended-MHD model, which is described in Ref. [2] and includes the Hall $\mathbf{J} \times \mathbf{B} / ne$ and electron pressure-gradient $-\nabla p_e / ne$ terms in its Ohm's law and the magnetized limit of Braginskii ion gyroviscous stress [10] in the momentum-density relation. Equilibria with

uniform plasma pressure were considered in order to model the core region of RFPs and to avoid additional numerical challenges. Former graduate student Dr. Jacob King[b] first verified NIMROD linear computations with the results of Ref. [8]. When adding gyroviscous stress to model warm ions, however, he found an intermediate-ρ_s regime where the growth rates are smaller than the small-ρ_s, resistive-MHD limit. His analysis identified this effect as drift-tearing behavior resulting from average ∇B and poloidal magnetic curvature [JP2], which are significant in RFP profiles but not in tokamak profiles. When he applied this model to nonlinear conditions for single-mode evolution, he found that the Hall dynamo effect is active during the growth of the magnetic island. It remains active at saturation when the warm ion effects are included, because they preclude alignment of **J** and **B** as all forces over the island come into balance, whereas cold-ion two-fluid modeling reverts to the MHD result as **J** and **B** become aligned [JP2]. King also considered nonlinear multi-helicity conditions, and his simulations found significant Hall dynamo effect and its influence on flow-profile evolution [JP3].

Initial comparisons of the multi-helicity results with MST observations appeared to be consistent in terms of the magnitude of the Hall dynamo effect and changes in parallel flow velocity. However, the fact that the current density runs anti-parallel to the magnetic field in the experiments for Refs. [6-7] was not appreciated at the time, and the simulations effectively predicted changes in core flow in the opposite direction to what is observed in MST. Former graduate student Dr. Joshua Sauppe[c] numerically investigated the coupling between magnetic relaxation and flow-profile relaxation further. Analytical two-fluid relaxation theory made predictions of flow-profile flattening [11], so Sauppe checked how well the simulations respected approximate constraints of the model. He found that global magnetic and hybrid helicities are well conserved during relaxation events in both MHD and two-fluid simulations [JP6]. The three components of the hybrid helicity are the magnetic, cross, and kinetic helicities, and their respective scalings are ε^0, ε^1, and ε^2, where $\varepsilon = d_i V_0 / a V_A$, V_0/V_A is the plasma background flow speed normalized by the Alfven speed, and a is a characteristic scale. The parameter ε is much less than unity in most magnetic confinement experiments. Because the magnetic energy density is much larger than the kinetic energy density, relaxation of the free energy in the magnetic field completely dominates that of kinetic energy, so the theoretical considerations that imply flow relaxation are not realized. Sauppe carried out his multi-helicity RFP simulations over multiple relaxation events. He found that after the first somewhat artificial event of a two-fluid simulation, the Hall dynamo effect can act opposite to an enhanced MHD dynamo effect [JP8]. These events produce the change in core flow-profile with the orientation that is observed in MST. They are also consistent with other findings in Ref. [7].

Many of the objectives of this study were accomplished, but investigating the effects from radially decreasing ρ_s-profiles, which is realistic with radially decreasing plasma pressure, was not accomplished. Simulations of these conditions produced short-wavelength instabilities that impeded progress. These instabilities were studied in greater detail in the context of spheromak stability, which is discussed in the following section.

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[b]Part of King's work was supported by the Center for Magnetohydrodynamics Modeling through U.S. Dept. of Energy grant DE-FC02-08ER54975.
[c]Part of Sauppe's work was supported by the Center for Magnetic Self-Organization, an N.S.F. Frontiers of Physics center, grant PHY-0821899.

2.3. Post-formation Spheromak Stability

Prior to the start of this project, resistive-MHD modeling with NIMROD had been applied to understand formation and sustainment via magnetic relaxation in spheromaks that are driven by DC helicity injection [12, 13]. NIMROD modeling was also applied to specific conditions of the SSPX experiment to investigate synergies among the macroscopic modes, the waveform of the current-drive system, magnetic topology evolution, and energy confinement [14, 15]. While the effort was successful in explaining significant aspects of the experiment, modeling of later optimized discharges tended to predict confinement-degrading modes more frequently than what was observed in the laboratory [16]. We hypothesized that this resulted from the lack of potentially stabilizing drift influences, and during this project, we applied NIMROD's two-fluid modeling to examine these effects. To focus on the controlled decay phase, former graduate student Dr. Eric Howell attempted to use equilibria that were fitted to experimental discharges. However, interpolation was not sufficiently accurate for linear computations, so Howell developed the NIMEQ code to solve the Grad-Shafranov (GS) force-balance equation with NIMROD's spectral-element representation [CWP5, JP5]. In addition to the basis functions, NIMEQ was unique in also being able to represent simply connected, i.e. topologically spherical, domains. It has since become a staple of pre-processing for NIMROD studies in a wide variety of configurations.

Spheromaks have magnetic safety factor (q) less than unity across their entire profile, like RFPs, so there are no regions of good curvature. Expectations from the fundamental two-fluid theory of Roberts and Taylor [17] were that two-fluid effects would stabilize interchange, which Howell sought to confirm with his study. He computed the linear stability of spheromak-relevant profiles in the SSPX geometry and in idealized cylindrical and slab geometries [for example CWP12, CWP18, CWP35], comparing results from resistive-MHD with two-fluid models. A model with the resistive-MHD Ohm's law and ion gyroviscosity did find reduced growth rates for short wavelength modes, but none of the two-fluid computations found stabilization for the longer wavelength modes that affect confinement in the MHD simulations. Over this time, limitations on the Roberts and Taylor analysis were investigated, and that study found stabilizing mechanism to be profile-dependent [18].

Howell also investigated a cylindrical configuration, where two-fluid effects had been considered, previously [19]. He was able to reproduce Fig. 10 of Ref. [19] over a large extent of the Hall parameter (Λ), but he encountered a second instability at the largest Λ−values [20], where complete stabilization had been expected from the previous results. Howell then considered SSPX-relevant profiles in cylindrical computations, comparing different possible models of 1) single-temperature, resistive-MHD Ohm's law with gyroviscous stress, 2) single-temperature, two-fluid Ohm's law with gyroviscous stress, and 3) a complete two-fluid, two-temperature model including diamagnetic heat flow. The first model reduces growth rates, relative to resistive MHD, both below and above the threshold for ideal interchange instability, but gyroviscous stress does not fully stabilize either the ideal mode or the resistive mode. The single-temperature two-fluid model has a weak stabilizing effect for small values of Λ, but a second instability appears at moderate Λ−values. The two-temperature model is the most complete and shows significant stabilization of interchange in the experimentally relevant ranges of Λ and interchange parameters [20, JP9]. This model also exhibits a second instability, but it occurs at significantly larger Λ-values for the experimentally relevant interchange conditions. Thus, we infer that plasma drift contributed to SSPX's stability and that a relatively complete two-fluid model is needed to reproduce the effect.

Howell examined the gravitational interchange (*g*-) mode in slab geometry to better understand the origins of the second instability. He was able to extend the analytical work of Ref. [18] to investigate a wide range of conditions in normalized parameter space. Howell's analytical work shows that the second instability at large-Λ results from the interaction of an ion drift wave with the stabilized *g*-mode [20, JP9]. Its stability criterion depends on the ratio of density and temperature length scales. This criterion is similar to the ion temperature gradient (ITG) criterion, but the second mode here is distinct from the ITG mode.

The original objectives for our spheromak study included nonlinear two-fluid simulations of driven and decaying conditions in SSPX. Nonlinear computations were attempted at different times in the project, but they never proved to be reliable. The detailed linear analysis led to a much better appreciation of properties of various simplified models, and the unexpected second instability likely caused at least part of the difficulty with the nonlinear simulations.

2.4. Non-inductive Startup in Spherical Tokamaks

A major thrust of the research on the Pegasus Toroidal Experiment is the use of DC-injected current for creating plasma in the ST configuration, which has limited space for a conventional central solenoid. DC current injection through toroidally symmetric electrodes had been investigated in HIT, HIT-II [21], NSTX [22], and other ST experiments, but the Pegasus approach is unique in drawing current from spatially localized plasma guns [23, 24]. Former undergraduate student Dr. Thomas Bird and former graduate student Dr. John O'Bryan[d] applied NIMROD simulations to investigate the detailed physics of the relaxation. They used a localized current source to induce current along helical streams. O'Bryan found that it was necessary to distinguish the electrically conducting streams by modeling heating and heat transport with variable magnetization in the thermal conductivity tensor, so that heat is not confined in spaces between the current streams. His simulation results showed repeated merger of the helical current streams, via a process that is related to the island coalescence instability [25]. O'Bryan found that each merger releases a ring of current, whose magnetic flux builds over multiple events to form a tokamak-like configuration [JP4]. The prediction of this unique phenomenology was later supported by laboratory probe measurements [26] and by visible-light imaging. O'Bryan also found that removing the current drive allows the configuration to become more symmetric and to form nested toroidal flux surfaces.

2.5. Development and Results on Tokamak Disruption

Many forms of macroscopic dynamics arise during disruptive transients in tokamaks, but ones that involve vertical displacement pose the greatest risk for thermal and electromechanical damage in large experiments. Thus, we focused on modeling vertical displacement. First, graduate student Kyle Bunkers verified [CWP34] that large aspect-ratio axisymmetric NIMROD computations reproduce the decay-index stability criterion of Mukhovatov and Shafranov [27]. We then added a new resistive-wall capability that uses a numerical computation of the magnetic-field response outside the wall. An implicit implementation of the thin-wall equation couples the external solution with the magnetic advance in the plasma region [CWP33], avoiding the conditional stability constraint of an explicit implementation. The implementation was verified on linear computations of a basic cylindrical resistive wall mode (RWM) stability problem, as shown in Fig. 1. In addition to the resistive-wall modeling, we added free-boundary

[d]Part of O'Bryan's work was supported by this grant. Most of the effort was supported by the Plasma Science and Innovation Center through U. S. DOE grant DE-FC02-05ER54813.

capability to the NIMEQ GS solver. Free-boundary computations self-consistently solve the equilibrium and poloidal flux distribution from the plasma current density distribution and from current in external coils. A unique feature of the implementation is that it simultaneously determines coefficients of expansions for the poloidal flux function and for J_ϕ/R as separate field. The J_ϕ/R field has a linear integral relation to the surface flux [CWP45], and solving the two fields simultaneously avoids the extra iterative step that is used in other GS solvers.

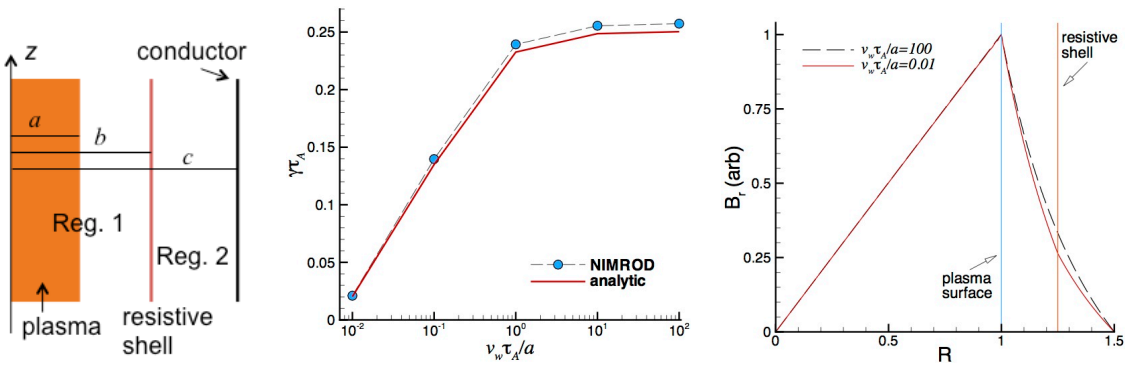

Figure 1. RWM configuration (left), verification of numerically computed growth rates (center) and computed eigenfunctions (right) for large aspect-ratio resistive wall modes in a cylinder. The configuration has $R/a = 10$, $c/b = 1.25$, $S = 10^6$, Pm=1, and uniform J_z in the plasma region. Note that $v_w \equiv \eta_w/\mu_0\delta x$. [From CWP33]

A significant outcome of our early results on modeling vertical displacement events (VDEs) is that the evolution is largely independent of boundary conditions on plasma flow velocity and particle density [CWP39]. This contradicts previous claims [28, 29], which can be understood by the fact that flows in NIMROD's resistive-MHD modeling are not strictly tied to the magnetic flux, as they are in ideal-MHD models. In resistive-MHD modeling, the component of flow velocity that is parallel to open magnetic field lines is the primary contributor to mass outflow. Speed reaches sonic levels, as expected from sheath theory and from analogy with fluid dynamics [30]. We have found that simulation results are sensitive to boundary conditions on temperature, however, because it affects the electrical resistivity in the open-field halo region. This prompted us to adapt boundary relations that model conditions at the magnetic pre-sheath entrance [31] for our VDE computations [CWP49]. Bunkers is implementing this set of boundary relations in NIMROD.

Three-dimensional computations are being applied to investigate the physics of asymmetric VDEs (AVDEs). A computation that starts with weak edge instabilities and has substantial separation of Alfvénic, resistive wall, and plasma resistive diffusion times, $\tau_\eta \sim 1000\tau_{\text{wall}} \sim 10^6\tau_A$, is shown in Fig. 2. This computation represents a forced VDE, because the initial state removes the current in the upper divertor coil, which excites eddy currents on the exterior of the resistive wall. The simulated tokamak displaces vertically as the eddy currents decay over a resistive-wall time. Our AVDE simulations reproduce important characteristics of tokamak disruption, including a thermal quench that is fast relative to the current quench, the positive bump in plasma current during the thermal quench, and horizontal forcing on the resistive wall.

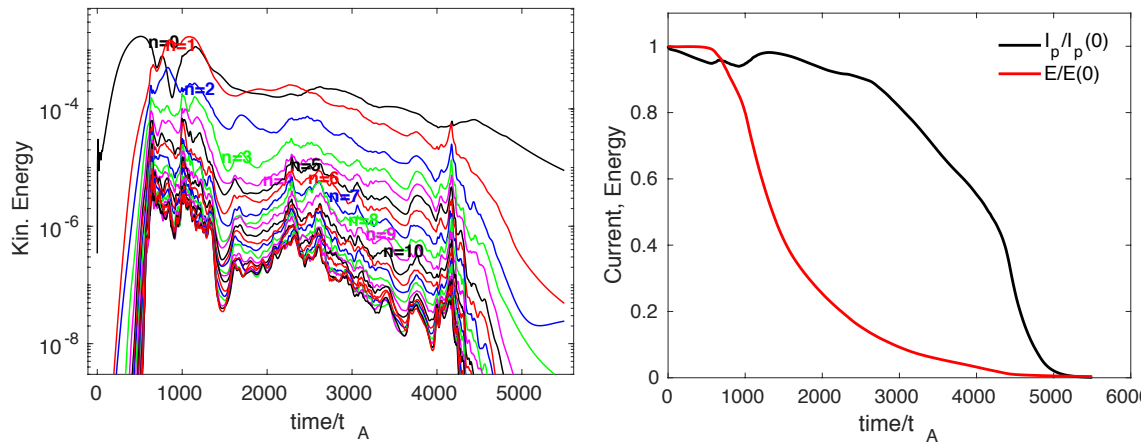

Figure 2. Results from a NIMROD AVDE computation of kinetic energy fluctuations (left) and plasma current and internal energy (right) [CWP51].

Our modeling effort on tokamak disruptions has achieved many of the milestones for the topic that were proposed for the second two funding cycles of this grant. They are the resistive-wall implementation, pre-processing for experimentally fitted equilibria, verification of 2D VDE results, completing nonlinear 3D simulations of AVDEs, and implementing force diagnostics. We have not made direct comparisons with specific experimental discharges. Our attempts uncovered a computational difficulty in representing reentrant corners (ones with angles larger than 180°). Having this capability is important for modeling experiments, and two current development efforts seek to address the issue. One is the implementation of the vector finite elements that have been used in computational electromagnetics, and the other is the implementation of singular basis functions to augment NIMROD's continuous polynomial basis functions.

2.7. References

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3. Products

The work supported by this grant appears in the following peer-reviewed journal articles and conference/workshop presentations.

Journal Publications

1. J. M. Reynolds, C. R. Sovinec, and S. C. Prager, “Nonlinear Magnetohydrodynamics of Pulsed Parallel Current Drive in Reversed-field Pinches,” Physics of Plasmas **15**, 062512 (2008). DOI 10.1063/1.2937770
2. J. R. King, C. R. Sovinec, and V. V. Mirnov, “First-Order Finite-Larmor-Radius Effects on Magnetic Tearing in Pinch Configurations,” Physics of Plasmas **18**, 042303 (2011). DOI 10.1063/1.3571599
3. J. R. King, C. R. Sovinec, and V. V. Mirnov, “First-Order Finite-Larmor-Radius Modeling of Tearing and Relaxation in a Plasma Pinch,” Physics of Plasmas **19**, 055905 (2012). DOI 10.1063/1.3695346
4. J. B. O’Bryan, C. R. Sovinec, and T. M Bird, “Simulation of Current-Filament Dynamics and Relaxation in the Pegasus Spherical Tokamak,” Physics of Plasmas **19**, 080701 (2012). DOI 10.1063/1.4746089
5. E. C. Howell and C. R. Sovinec, "Solving the Grad-Shafranov equation with spectral elements," Computer Physics Communications **185**, 1415 (2014). DOI 10.1016/j.cpc.2014.02.008

6. J. P. Sauppe and C. R. Sovinec, "Two-fluid and finite Larmor radius effects on helicity evolution in a plasma pinch," Physics of Plasmas **23**, 032303 (2016). DOI 10.1063/1.4942761
7. V. V. Mirnov, J. P. Sauppe, C. C. Hegna, and C. R. Sovinec, "Analytical and numerical treatment of resistive drift instability in a plasma slab," Plasma Physics Reports **42**, 440 (2016). DOI 10.1134/S1063780X16050123
8. J. P. Sauppe and C. R. Sovinec, "Extended MHD modeling of tearing-driven magnetic reconnection," Physics of Plasmas **24**, 056107 (2017). DOI 10.1063/1.4977540
9. E. C. Howell and C. R. Sovinec, "Parameter-space survey of linear g-mode and interchange in extended magnetohydrodynamics," Physics of Plasmas **24**, 102117 (2017). DOI 10.1063/1.4993440

Conference and Workshop Presentations

1. J. Reynolds and C. Sovinec, "Numerical Studies of Tearing Mode Reduction and Suppression of the MHD Dynamo During Pulsed Poloidal Current Drive in the Reversed Field Pinch," 48th Annual Meeting of the APS Division of Plasmas Physics, presentation CP1.00031, Philadelphia, Pennsylvania, Oct. 30-Nov. 3 (2006).
2. J. R. King, C. R. Sovinec, and V. V. Mirnov, "Numerical Studies of Linear Two Fluid Tearing Modes in Slab and Cylindrical Geometries," 49th Annual Meeting of the APS Division of Plasmas Physics, presentation GP8.00143, Orlando, Florida, Oct. 12-16 (2007).
3. J. M. Reynolds, C. R. Sovinec, and S. C. Prager, "Numerical Simulation of Pulsed Parallel Current Drive in RFPs," 49th Annual Meeting of the APS Division of Plasmas Physics, presentation JP8.00140, Orlando, Florida, Oct. 12-16 (2007).
4. J. R. King, C. R. Sovinec, and V. V. Mirnov, "Numerical Studies of Two Fluid Tearing Modes in Slab and Cylindrical Geometries," 2008 International Sherwood Fusion Theory Conference, contributed presentation, Boulder, Colorado, March 31-April 2 (2008).
5. E. C. Howell and C. R. Sovinec, "NIMEQ: MHD Equilibrium Solver for NIMROD," 50th Annual Meeting of the APS Division of Plasma Physics, presentation BP6.00041, Dallas, Texas, Nov. 17-21 (2008).
6. T. M. Bird, C. R. Sovinec, D. J. Battaglia, and J. B. O'Bryan, "Numerical Simulation of MHD Relaxation during Non-Inductive Startup of Spherical Tokamaks," 50th Annual Meeting of the APS Division of Plasma Physics, presentation JP6.00012, Dallas, Texas, Nov. 17-21 (2008).
7. J. R. King, C. R. Sovinec, and V. V. Mirnov, "Numerical Studies of Nonlinear Two Fluid Tearing Modes in Cylindrical RFPs," 50th Annual Meeting of the APS Division of Plasma Physics, presentation NP6.00071, Dallas, Texas, Nov. 17-21 (2008).
8. J. B. O'Bryan, C. R. Sovinec, D. J. Battaglia, and T. M Bird, "Computational study of a non-ohmic flux compression startup method for spherical tokamaks," 50th Annual Meeting of the APS Division of Plasma Physics, presentation NP6.00139, Dallas, Texas, Nov. 17-21 (2008).

9. J. R. King, C. R. Sovinec, and V. V. Mirnov, "Numerical Studies of Two-Fluid Tearing and Dynamo in a Cylindrical Pinch," 2009 International Sherwood Fusion Theory Conference, contributed presentation, Denver, Colorado, May 3-5 (2009).
10. J. B. O'Bryan, C. R. Sovinec, D. J. Battaglia, and T. M. Bird, "Numerical Simulation of Non-inductive Startup and Flux Compression in the Pegasus Toroidal Experiment," 51st Annual Meeting of the APS Division of Plasma Physics, presentation GP8.00050, Atlanta, Georgia, Nov. 2-6 (2009).
11. J. King, C. Sovinec, and V. Mirnov, "Numerical Studies of Two-Fluid Tearing and Dynamo in a Cylindrical Pinch," 51st Annual Meeting of the APS Division of Plasma Physics, presentation TP8.00071, Atlanta, Georgia, Nov. 2-6 (2009).
12. E. C. Howell and C. R. Sovinec, "Extended MHD Simulations of Interchange Modes in Spheromak Equilibria," 51st Annual Meeting of the APS Division of Plasma Physics, presentation UP8.00034, Atlanta, Georgia, Nov. 2-6 (2009).
13. T. M. Bird, C. R. Sovinec, and J. B. O'Bryan, "Simulations of Helical Current- Channel Relaxation," 2010 International Sherwood Fusion Theory Conference, contributed presentation, Seattle, Washington, April 19-21 (2010).
14. E. C. Howell and C. R. Sovinec, "Two-Fluid Numerical Studies of Interchange Modes in Spheromak Equilibria," 2010 International Sherwood Fusion Theory Conference, contributed presentation, Seattle, Washington, April 19-21 (2010).
15. J. King, C. Sovinec, and V. Mirnov, "Numerical Studies of Two-Fluid Tearing and Dynamo in a Cylindrical Pinch," 2010 International Sherwood Fusion Theory Conference, contributed presentation, Seattle, Washington, April 19-21 (2010).
16. J. B. O'Bryan, C. R. Sovinec, and T. M Bird, "Numerical Simulations of Non-inductive Startup and Flux Compression in the Pegasus Toroidal Experiment," 2010 International Sherwood Fusion Theory Conference, contributed presentation, Seattle, Washington, April 19-21 (2010).
17. J. B. O'Bryan, C. R. Sovinec, and T. M. Bird, "Numerical Simulation of Non-inductive Startup in the Pegasus Toroidal Experiment," 52nd Annual Meeting of the APS Division of Plasma Physics, presentation CP9.00072, Chicago, Illinois, Nov. 8-12 (2010).
18. E. C. Howell and C. R. Sovinec, "Calculation of two-fluid resonant modes in spheromaks," 52nd Annual Meeting of the APS Division of Plasma Physics, presentation GP9.00129, Chicago, Illinois, Nov. 8-12 (2010).
19. J. R. King, C. R. Sovinec, and V. V. Mirnov, "Studies of the Two-Fluid RFP Dynamo and Mode Coupling," 52nd Annual Meeting of the APS Division of Plasma Physics, presentation PP9.00070, Chicago, Illinois, Nov. 8-12 (2010).
20. J. R. King, C. R. Sovinec, and V. V. Mirnov, "First-order finite-Larmor-radius effects on magnetic tearing in pinch configurations," 2011 International Sherwood Fusion Theory Conference, contributed presentation, Austin, Texas, May 2-4 (2011).
21. J. P. Sauppe and C. R. Sovinec, "Convergence Properties of Spectral Elements on Thermal Transport in Chaotic Magnetic Fields," 2011 International Sherwood Fusion Theory Conference, contributed presentation, Austin, Texas, May 2-4 (2011).

22. E. C. Howell and C. R. Sovinec, "Extended MHD Simulations of Spheromaks," 53rd Annual Meeting of the APS Division of Plasma Physics, presentation CP9.00119, Salt Lake City, Utah, Nov. 14-18 (2011).
23. J. R. King, C. R. Sovinec, and V. V. Mirnov, "First-Order FLR Effects on Magnetic Tearing and Relaxation in Pinch Configurations," 53rd Annual Meeting of the APS Division of Plasma Physics, invited oral presentation JI2.00004 Salt Lake City, Utah, Nov. 14-18 (2011).
24. C. R. Sovinec, J. P. Sauppe, and J. R. King, "Flow Profile Relaxation in Two-Fluid RFP Modeling," 2012 International Sherwood Fusion Theory Conference, contributed presentation, Atlanta, Georgia, April 1-3 (2012).
25. C. R. Sovinec, J. R. King, J. P. Sauppe, V. V. Mirnov, J. S. Sarff, and W. X. Ding, "Coupling of Current and Flow Relaxation in Reversed-Field Pinches Due to Two-Fluid Effects," 2012 IAEA Fusion Energy Conference, selected poster presentation, San Diego, California, Oct. 8-12 (2012).
26. J. Sauppe and C. Sovinec, "Two-Fluid Modeling of Current and Flow Relaxation in the Reversed-Field Pinch," 54th Annual Meeting of the APS Division of Plasma Physics, presentation JP8.00147, Providence, Rhode Island, Oct. 29-Nov. 2 (2012).
27. E. C. Howell and C. R. Sovinec, "Extended MHD Simulations of Spheromaks," 54th Annual Meeting of the APS Division of Plasma Physics, presentation PP8.00063, Providence, Rhode Island, Oct. 29-Nov. 2 (2012).
28. E. C. Howell and C. R. Sovinec, "Two-Fluid Simulations of CHI Spheromak Formation," 2013 International Sherwood Fusion Theory Conference, contributed presentation, Santa Fe, New Mexico, April 15-17 (2013).
29. E. C. Howell and C. R. Sovinec, "Extended MHD Stabiliy Calculations of Spheromak Equilibria," 55th Annual Meeting of the APS Division of Plasma Physics, presentation GP8.00069, Denver, Colorado, Nov. 11-15 (2013).
30. J. Sauppe, C. Sovinec, C. Hegna, and V. Mirnov, "Helicity Conservation and Two-Fluid Relaxation Modeling for Reversed-Field Pinches," 55th Annual Meeting of the APS Division of Plasma Physics, presentation JP8.00123, Denver, Colorado, Nov. 11-15 (2013).
31. E. C. Howell and C. R. Sovinec, "Nonlinear Simulations of Interchange Modes in Spheromaks," 2014 International Sherwood Fusion Theory Conference, contributed presentation, San Diego, California, March 24-26 (2014).
32. J. P. Sauppe, C. R. Sovinec, C. C. Hegna, and V. V. Mirnov, "Helicity Conservation and Two-Fluid Relaxation Modeling for Reversed-Field Pinches," 2014 International Sherwood Fusion Theory Conference, contributed presentation, San Diego, California, March 24-26 (2014).
33. C. R. Sovinec, K. J. Bunkers, and A. L. Becerra, "Development and Verification Tests for Vertical Displacement Studies with NIMROD," 2014 International Sherwood Fusion Theory Conference, contributed presentation, San Diego, California, March 24-26 (2014).
34. K. Bunkers and C. R. Sovinec, "Initial Computations of Vertical Displacement Events with NIMROD," 56th Annual Meeting of the APS Division of Plasma Physics, presntation BP8.00016, New Orleans, Louisiana, Oct. 27-31 (2014).

35. E. C. Howell and C. R. Sovinec, "Study of extended MHD effects on interchange modes in spheromak equilibria," 56th Annual Meeting of the APS Division of Plasma Physics, presentation BP8.00025, New Orleans, Louisiana, Oct. 27-31 (2014).
36. J. Sauppe, C. Sovinec, J. Sarff, and J. Triana, "NIMROD Extended MHD Simulations of Reversed-Field Pinch Relaxation Dynamics," 56th Annual Meeting of the APS Division of Plasma Physics, presentation PP8.00035, New Orleans, Louisiana, Oct. 27-31 (2014).
37. E. C. Howell and C. R. Sovinec, "Extended MHD Analysis of the Gravitational Interchange," 2015 International Sherwood Fusion Theory Conference, contributed presentation, New York, New York, March 16-18 (2015).
38. J. P. Sauppe, C. R. Sovinec, C. C. Hegna, and V. V. Mirnov, "Accounting of Magnetic, Cross, and Kinetic Helicities in Nonlinear Two-Fluid Relaxation Simulations," 2015 International Sherwood Fusion Theory Conference, contributed presentation, New York, New York, March 16-18 (2015).
39. C. R. Sovinec and K. J. Bunkers, "Axisymmetric Vertical-Displacement Event Modeling with NIMROD," 2015 International Sherwood Fusion Theory Conference, contributed presentation, New York, New York, March 16-18 (2015).
40. C. R. Sovinec, J. P. Sauppe, and J. R. King, "Numerical Two-Fluid Studies of Magnetic-Field and Flow Relaxation in RFPs," invited oral presentation, Midwest Magnetic Fields Workshop, Madison, Wisconsin, May 21-22 (2015).
41. C. R. Sovinec and K. J. Bunkers, "Initial Simulations of Hot Vertical Displacement Events with NIMROD," IEA Workshop: Theory and Simulation of Disruptions, invited oral presentation, Princeton Plasma Physics Laboratory, Princeton, New Jersey, July 13-15 (2015).
42. J. Sauppe and C. Sovinec, "Analysis of Helicities and Hall and MHD Dynamo Effects in Two-Fluid Reversed-Field Pinch Simulations," 57th Annual Meeting of the APS Division of Plasma Physics, presentation CP12.00031, Savannah, Georgia, Nov. 16-20 (2015).
43. K. J. Bunkers and C. R. Sovinec, "Numerical Simulations of Hot Vertical Displacement Events," 57th Annual Meeting of the APS Division of Plasma Physics, presentation NP12.00014, Savannah, Georgia, Nov. 16-20 (2015).
44. K. J. Bunkers and C. R. Sovinec, "Simulations of Nonlinear External Kinks for Disruption Studies with NIMROD," 2016 International Sherwood Fusion Theory Conference, contributed presentation, Madison, Wisconsin April 4-6 (2016).
45. C. R. Sovinec, J. R. King, and E. C. Howell, "Computational Enhancements of the NIMEQ Grad-Shafranov Solver," 2016 International Sherwood Fusion Theory Conference, contributed presentation, Madison, Wisconsin April 4-6 (2016).
46. C. Sovinec, A. Almagri, M. Beidler, K. Bunkers, B. Chapman, B. Cornille, A. DuBois, J. Goetz, K. McCollam, and S. Munaretto, "Overview of disruption-related research at UW-Madison," IEA Workshop: Theory and Simulation of Disruptions, invited oral presentation, Princeton Plasma Physics Laboratory, Princeton, New Jersey, July 20-22 (2016).

47. K. J. Bunkers and C. R. Sovinec, "Nonlinear External Kink Computing with NIMROD," 58th Annual Meeting of the APS Division of Plasma Physics, presentation GP10.00077, San Jose, California, Oct. 31-Nov. 4, (2016).

48. J. P. Sauppe, C. R. Sovinec, J. S. Sarff, J. C. Triana, J. R. King, K. J. McCollam, and V. V. Mirnov, "Extended MHD Modeling of Tearing-Driven Magnetic Relaxation," 58th Annual Meeting of the APS Division of Plasma Physics, invited oral presentation YI3.00003, San Jose, California, Oct. 31-Nov. 4, (2016).

49. K. J. Bunkers and C. R. Sovinec, "Model Development for VDE Computations in NIMROD," 59th Annual Meeting of the APS Division of Plasma Physics, presentation CP11.00104, Milwaukee, Wisconsin, October 23-27 (2017).

50. C. R. Sovinec and K. J. Bunkers, "Computations of Vertical Displacement Events with Toroidal Asymmetry," 59th Annual Meeting of the APS Division of Plasma Physics, presentation CP11.00105, Milwaukee, Wisconsin, October 23-27 (2017).

51. C. R. Sovinec, "Tokamak disruption simulation: progress toward comprehensive modeling," 2018 International Sherwood Fusion Theory Conference, invited plenary review presentation, Auburn, AL, April 25, 2018.