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FROM SANDIA NATIONAL LABS'

ROBIN
BLUME-KOHOUT

CCR
CENTER FOR COMPUTING RESEARCH

FANTASTIC ERRORS AND WHERE TO FIND THEM

U.S. DEPARTMENT OF ENERGY

Sandia National Laboratories

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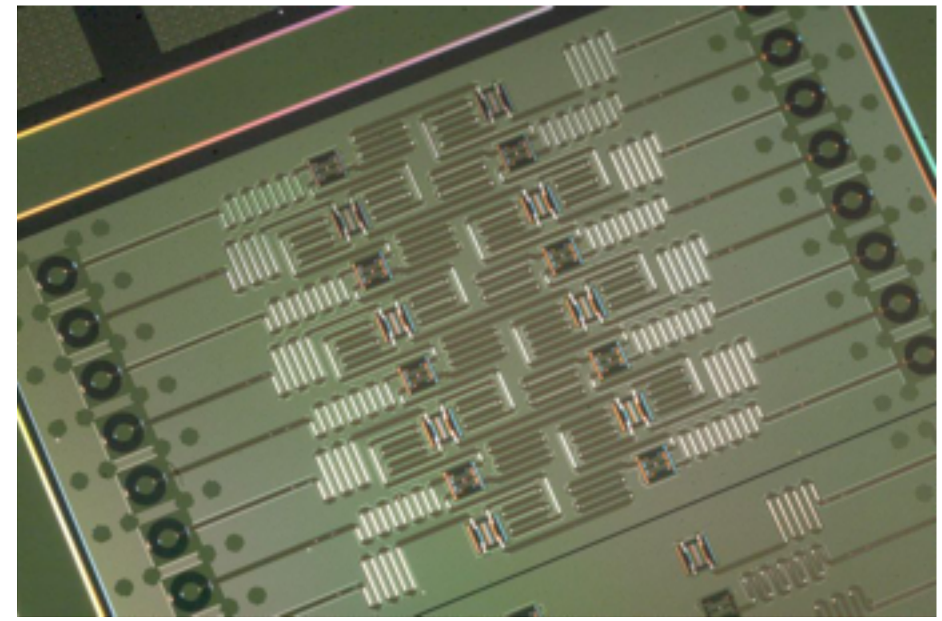
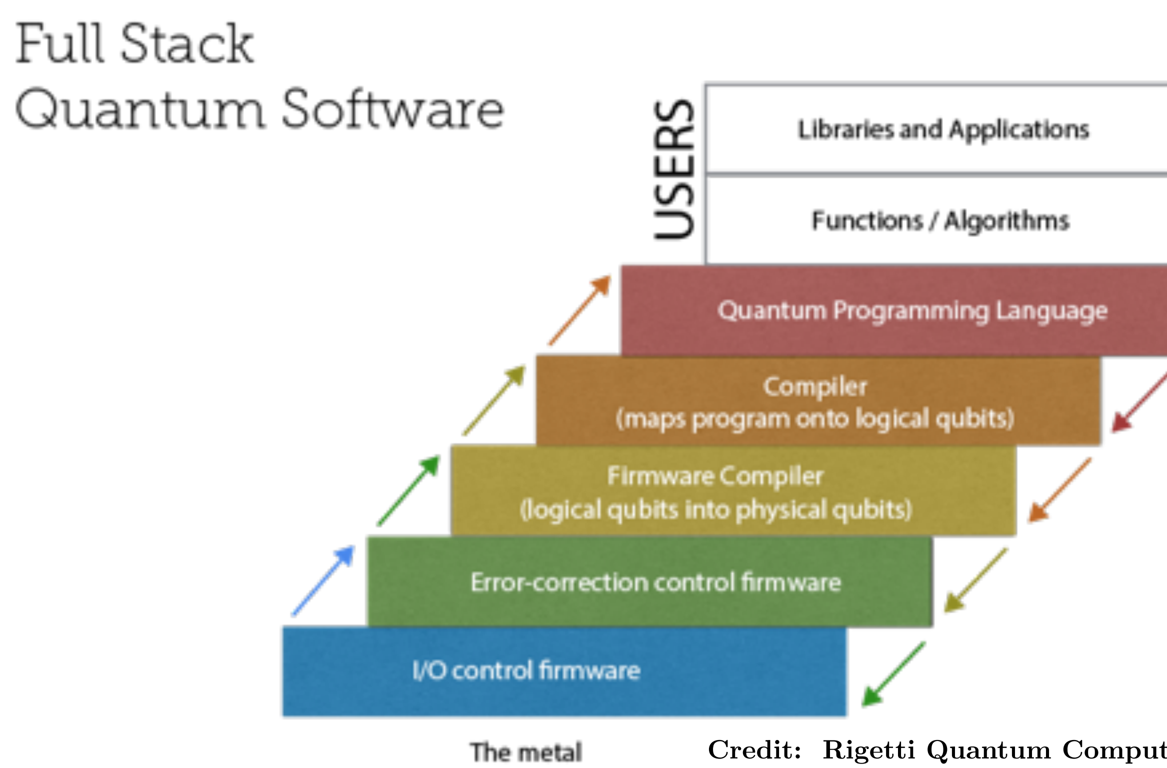
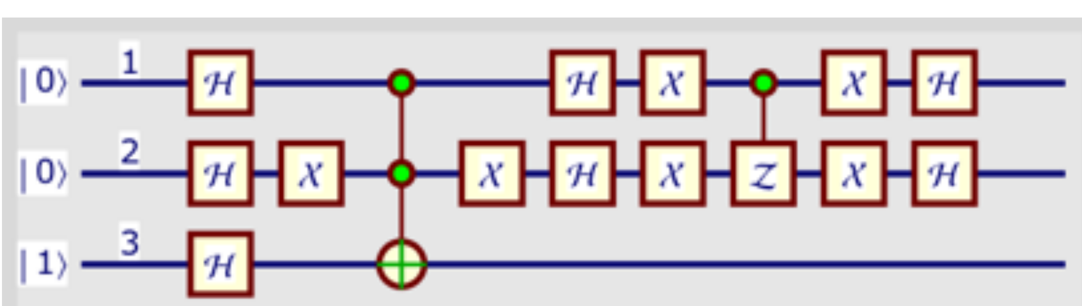
SOME CONTEXT

Credit: The Economist

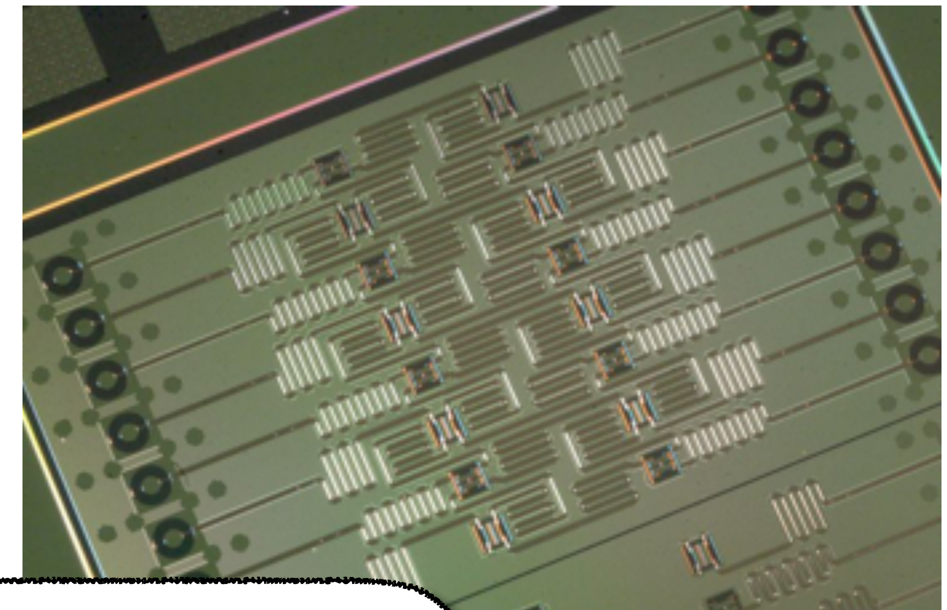

Credit: IBM





Credit: Rigetti Quantum Computing

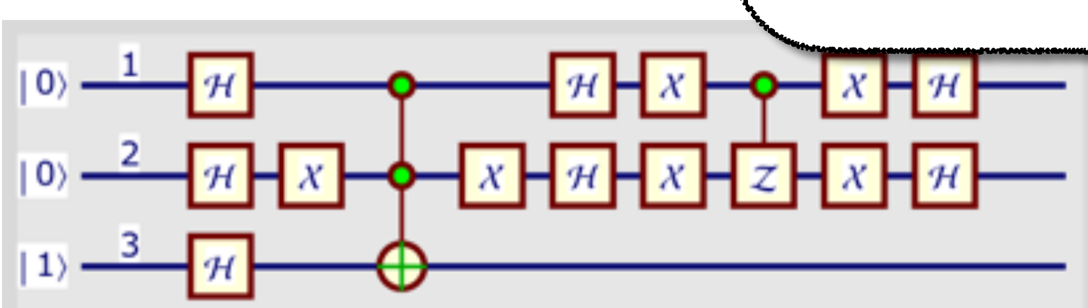
SOME CONTEXT

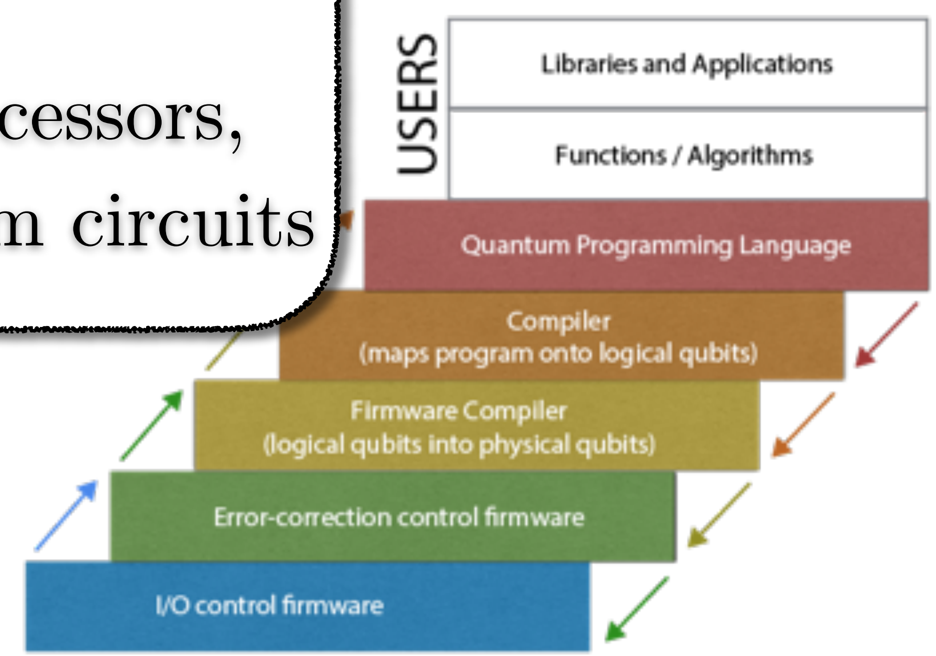


Credit: IBM

Our world today:

Real quantum processors, running real quantum circuits





The metal

Credit: Rigetti Quantum Computing

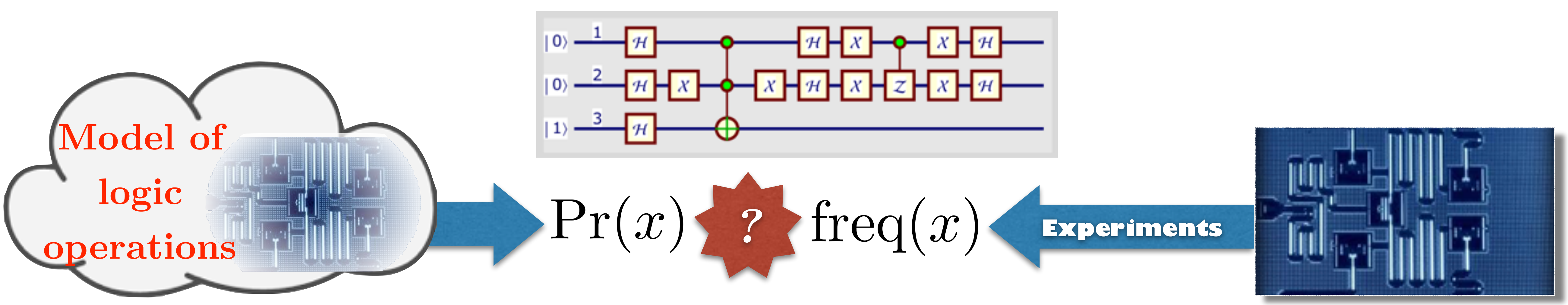
WHAT THIS TALK IS ABOUT

- When we run quantum circuits on *real* quantum processors, we don't get always get the right (expected) outcome.
- **This may be the least surprising thing ever.**
- But the *reasons why* we get wrong outcomes are surprising.
- This is about our ongoing quest to <u>completely</u> model and understand (phenomenologically) *errors* in qubit processors.
- **Error**: Any process that changes circuit outcome probabilities.
- **Goal**: To be able to predict (perfectly) the outcome probabilities of *any* circuit, based on learning from only *a few* experiments.

ERROR MODELS

- **Error**: Any process that changes circuit outcome probabilities.
- **Goal**: To be able to predict (perfectly) the outcome probabilities of *any* circuit, based on learning from only *a few* experiments.

- We want to enable *modeling and simulation* of arbitrary circuits on small (≤17 qubit) quantum processors. You ought to be able to *predict* outcome probabilities, then *verify* by running those circuits.

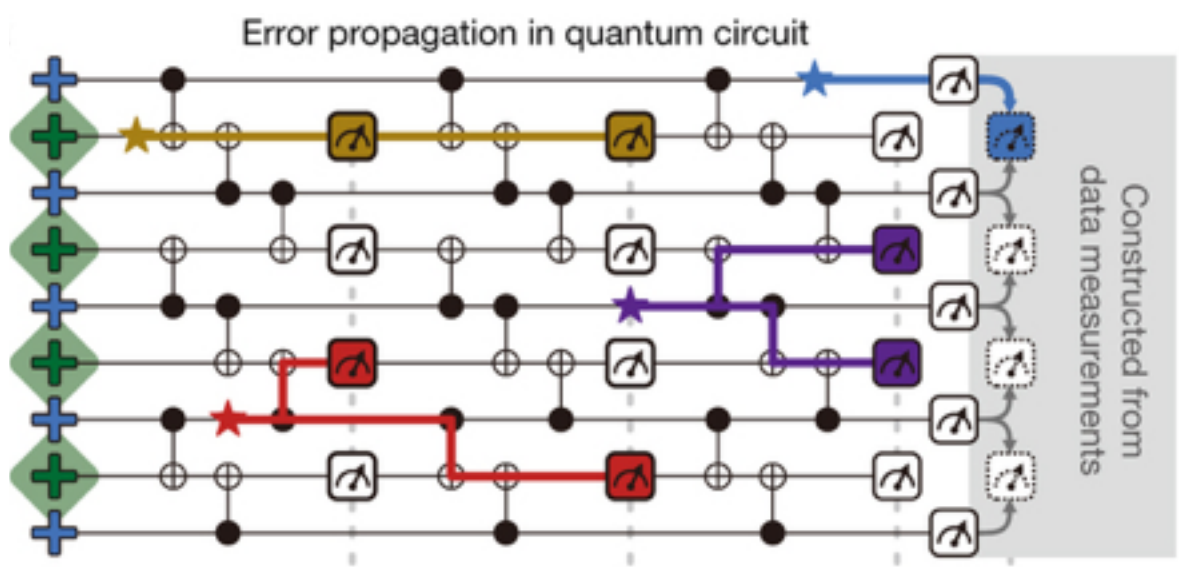


TECHNICAL OUTLINE

- **The “standard” models of quantum errors**
  - The QEC Theorist’s Model (“Pauli errors”)
  - The Control Theorist’s Model (“Hamiltonian noise”)
  - The Quantum Fundamentalist’s Model (“CPTP maps”)
- **How we test and validate error models**
- **Fantastic Errors (“Exotics”)**
  - Leakage
  - Crosstalk
  - Intermediate measurements as processes
  - Drift $\Leftrightarrow$ non-Markovianity
  - Context dependence
- **Where (and how) to find (and characterize) them**

ERROR MODELING IN QEC

- QEC theory typically concerned with *lots* of qubits. Simple models — easy to calculate with — at a premium.
- Most common: *Pauli stochastic errors.*
  - ☛ At each circuit location (gate), X or Y or Z happen with small probabilities p_x, p_y, p_z. Often $p_x{=}p_z$ and $p_y{=}p_z$ or 0.
  - ☛ Very easy to simulate, calculate, and think about effects of errors.

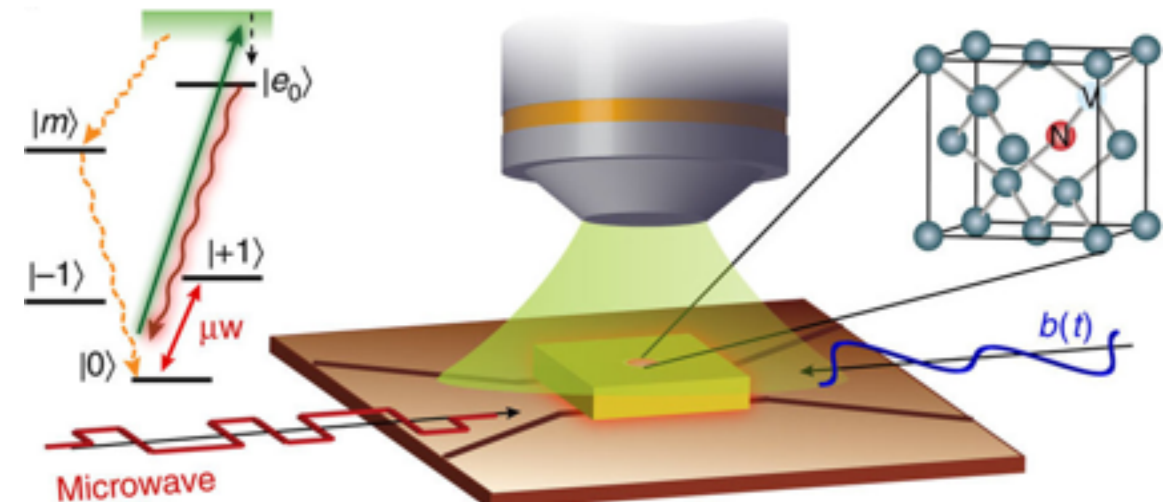


Credit: Martinis group

- Where is the dividing line between questions that this model represents well, and those where it fails. **We don't know**.
  - This model *probably* captures high-level truths about QEC.
  - This model is *never* accurate for individual circuits today.

THE HAMILTONIAN MODEL

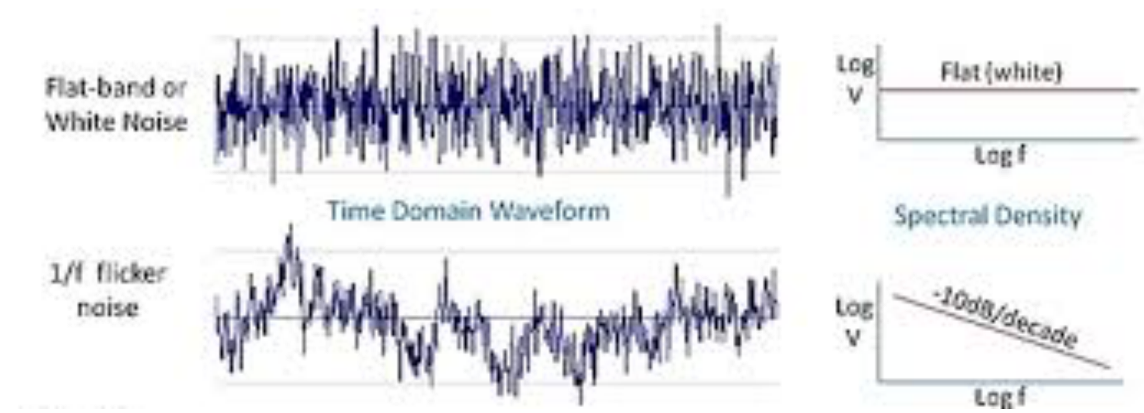
- Control theorists specify *parameters of Hamiltonians.*
  ==> all quantum evolution is
  described by Hamiltonians.



Credit: Nat. Comms. *5*, 3141 (2014)

- So *noise* (causes errors) means the wrong Hamiltonian.
  - Implicitly/explicitly, expand around desired Hamiltonian
  ==> so $\boldsymbol{H}$=0 corresponds to no noise, no errors.

- In this paradigm, the "stochastic" aspect of noise comes from $\boldsymbol{H}$ *varying* (fluctuating) over time. Properties of noise are described by *correlation function* of Hamiltonian parameters at different times, and its *power spectrum.*



- $\langle \boldsymbol{H} \rangle \neq 0$ corresponds to *systematic* (coherent) errors.

THE QUANTUM FUNDAMENTALIST'S MODEL (PROCESS MATRICES)

- The "gold standard" of error models: describe each gate by a d^2 x d^2 *process matrix* (CPTP map) $\boldsymbol{G}$: $\rho_{\text{init}} \rightarrow \rho_{\text{final}}$.

- Process matrices have a reputation for *universality* (thanks to Stinespring's theorem) — everything that can happen!
  - ☛ This is true in a sense — includes coherent errors, Pauli stochastic errors, amplitude damping (T1) decay, and various other things.
  - ☛ But... "*There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy.*" —***Shakey Bill***

$$G_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\rho = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

- The process matrix model of errors actually relies on a lot of assumptions — e.g. Markovianity — that aren't true in real qubits.

A TAXONOMY OF SMALL MARKOVIAN ERRORS

- The point of this talk is jump off from "standard" errors that are described by process matrices, and tackle "exotic" errors.

- First, though, I want to devote a little attention to the *Markovian* errors that <u>are</u> described by process matrices...

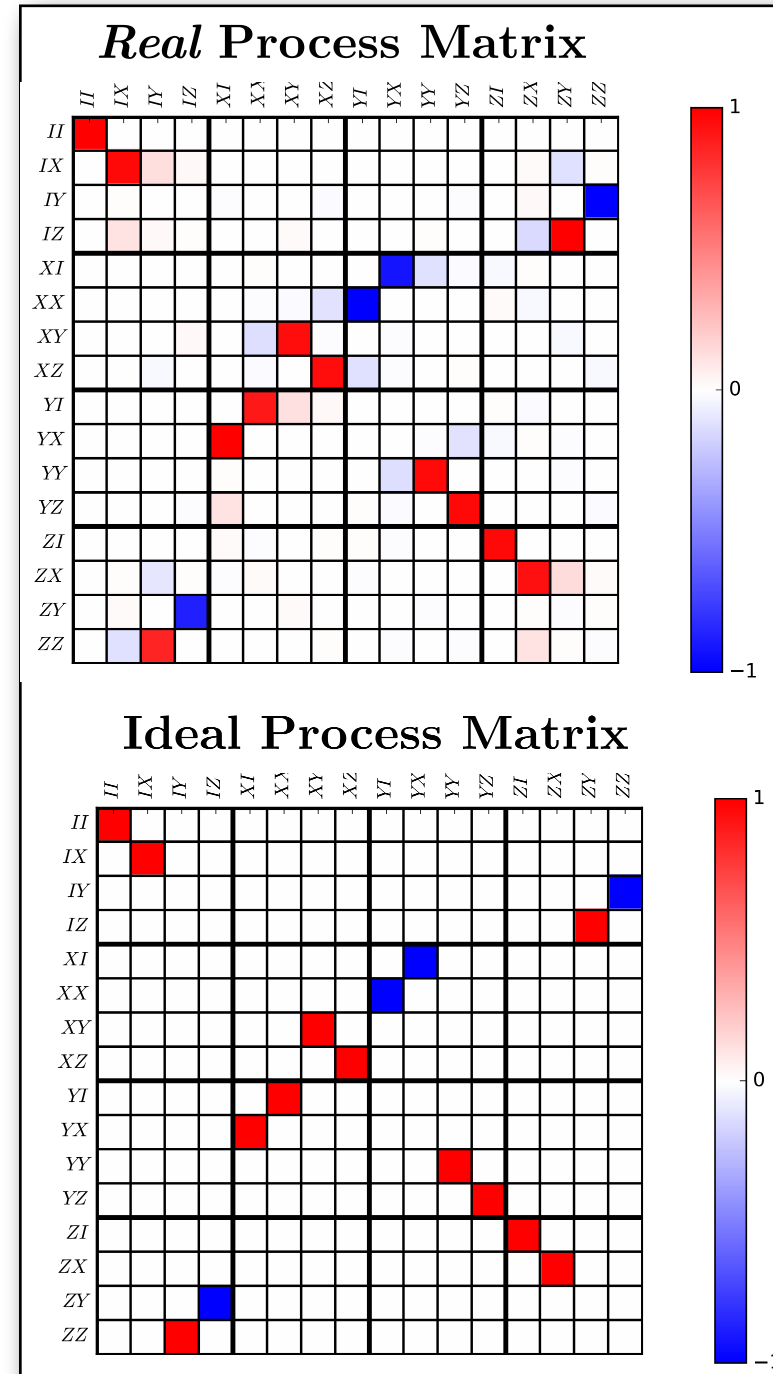
  ...because although in *principle* we understand process matrices completely, in *practice* it turns out that we don't.

- (Why do I know this? Because when we present folks with 2-qubit process matrices from GST, the invariable question is "So, what do all those numbers actually *mean*?"

A TAXONOMY
OF SMALL MARKOVIAN ERRORS

- Suppose I show you a process matrix describing how a real, noisy gate acts on 1 or 2 qubits. How do you read it?

Gate	Superoperator (Pauli basis)
Gi	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 4\times10^{-5} & 0.9936 & -0.0023 & -0.0014 \\ 0.0003 & 0.0023 & 0.9948 & -0.0006 \\ -0.0002 & 0.0011 & 0.0017 & 1 \end{pmatrix}$
Gx	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -0.0001 & 1.0003 & 0.0052 & -0.0023 \\ 5\times10^{-5} & 0.0029 & -0.005 & 0.9967 \\ -5\times10^{-5} & -0.0001 & -0.9967 & -0.0091 \end{pmatrix}$
Gy	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 5\times10^{-5} & -0.0001 & 0.0058 & -0.9949 \\ 3\times10^{-5} & 0.0034 & 1.0004 & -0.0007 \\ 5\times10^{-5} & 0.9949 & -0.0017 & -0.003 \end{pmatrix}$

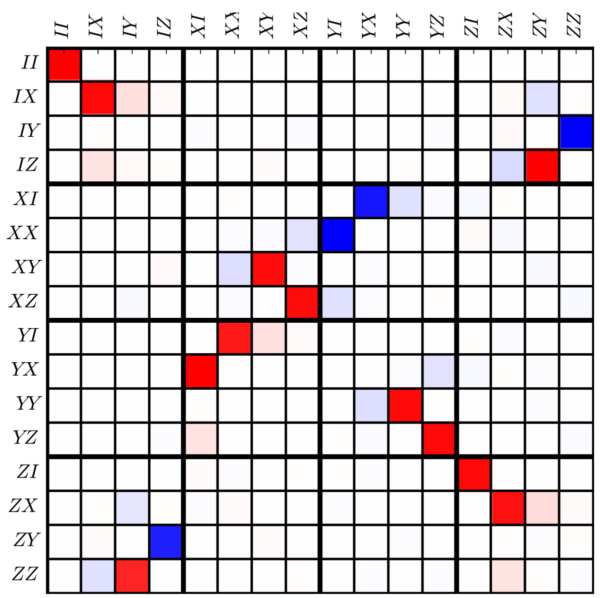


- A good start: ***Kueng et al*, PRL 117, 170502 (2016)**

A TAXONOMY OF SMALL MARKOVIAN ERRORS

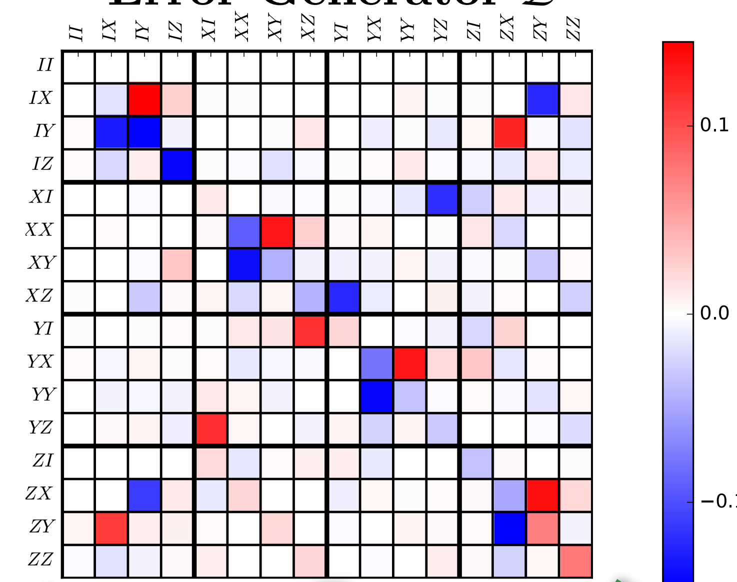
1. Map *processes* to *error generators*. They form a *semi-algebra*.
2. Split the generator space into *meaningful subspaces*.

Process Matrix G



$$\mathcal{L} \equiv \log\left(G_0^{-1} G\right)$$

Error Generator $\mathcal{L}$



Q1 \ Q2	I	X	Y	Z
I	-0	0	0	0
X	-0	-0	-0	-0
Y	0	-0	0	0
Z	-0	0	0	0

Decay Projection (Γ)

Q1 \ Q2	I	X	Y	Z
I	0	.02	.07	-.39
X	.04	-.06	-0	0
Y	-.07	.03	-.01	-.02
Z	-.02	.01	.04	.33

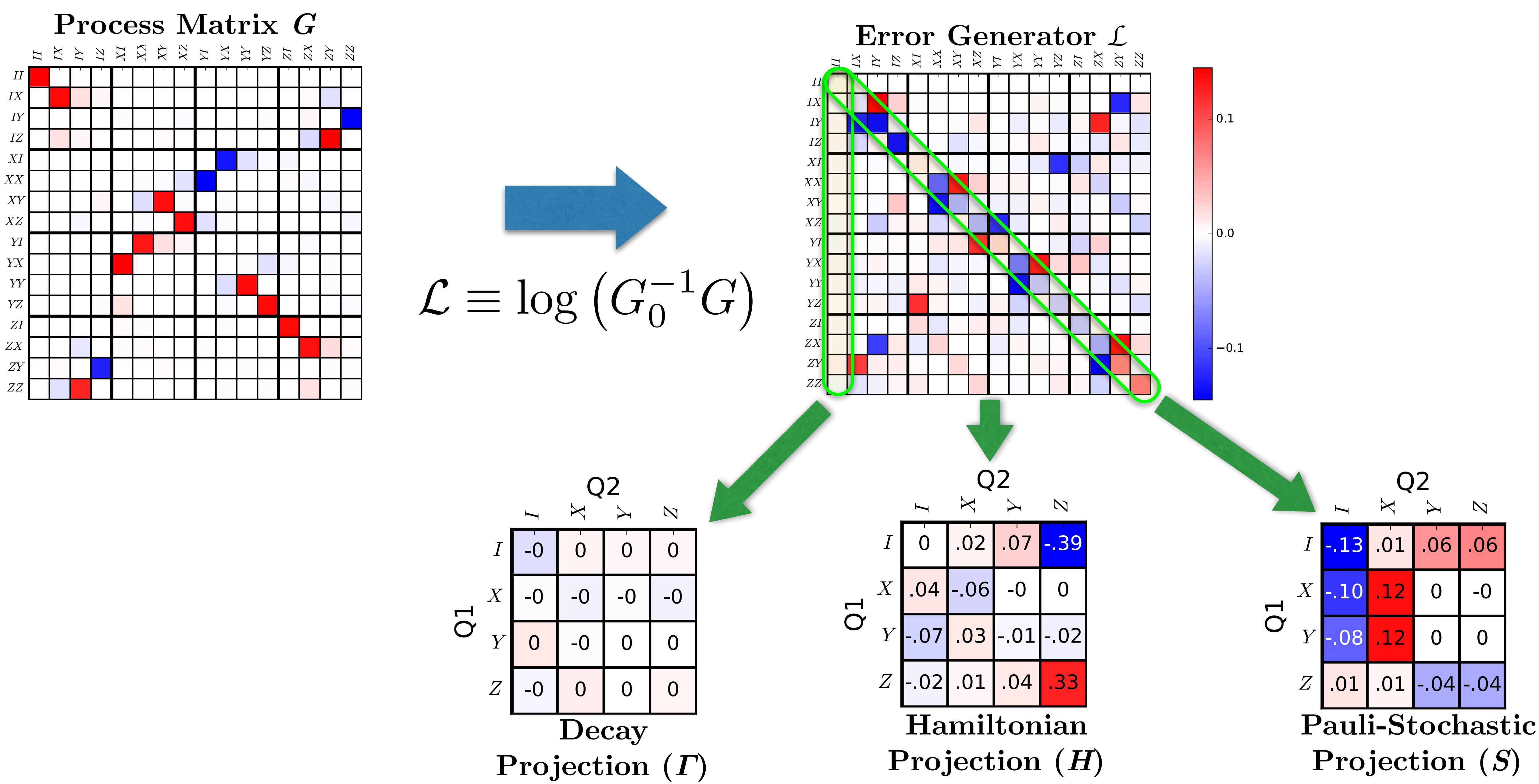
Hamiltonian Projection (H)

Q1 \ Q2	I	X	Y	Z
I	-.13	.01	.06	.06
X	-.10	.12	0	-0
Y	-.08	.12	0	0
Z	.01	.01	-.04	-.04

Pauli-Stochastic Projection (S)

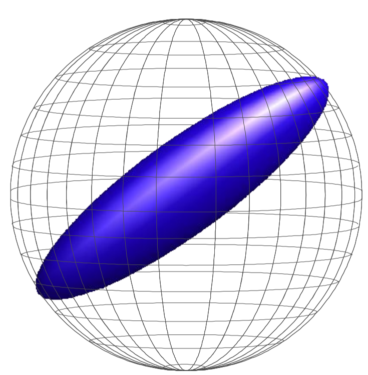
A TAXONOMY OF SMALL MARKOVIAN ERRORS

1. Map *processes* to *error generators*. They form a *semi-algebra*.
2. Split the generator space into *meaningful subspaces*.

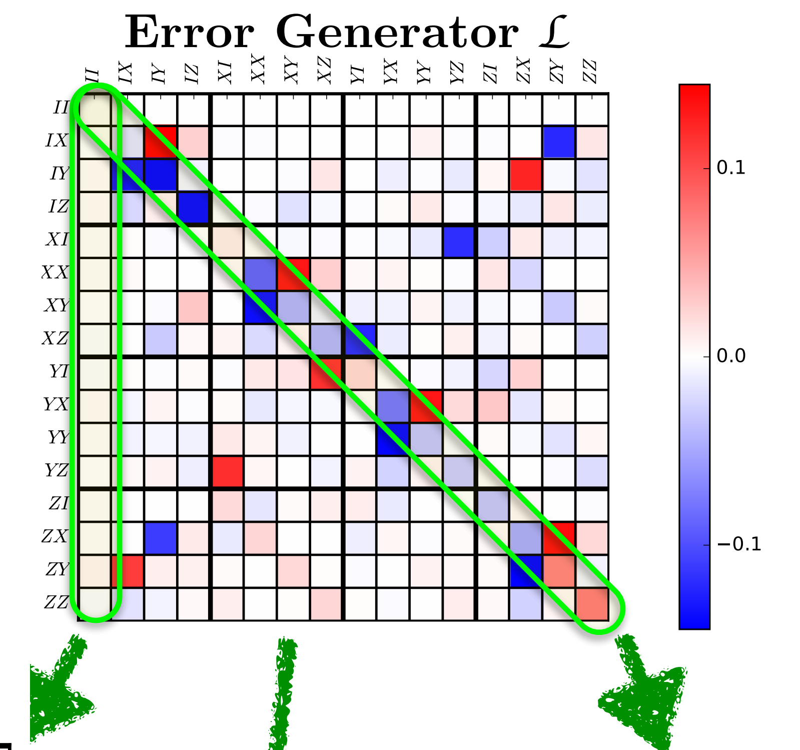


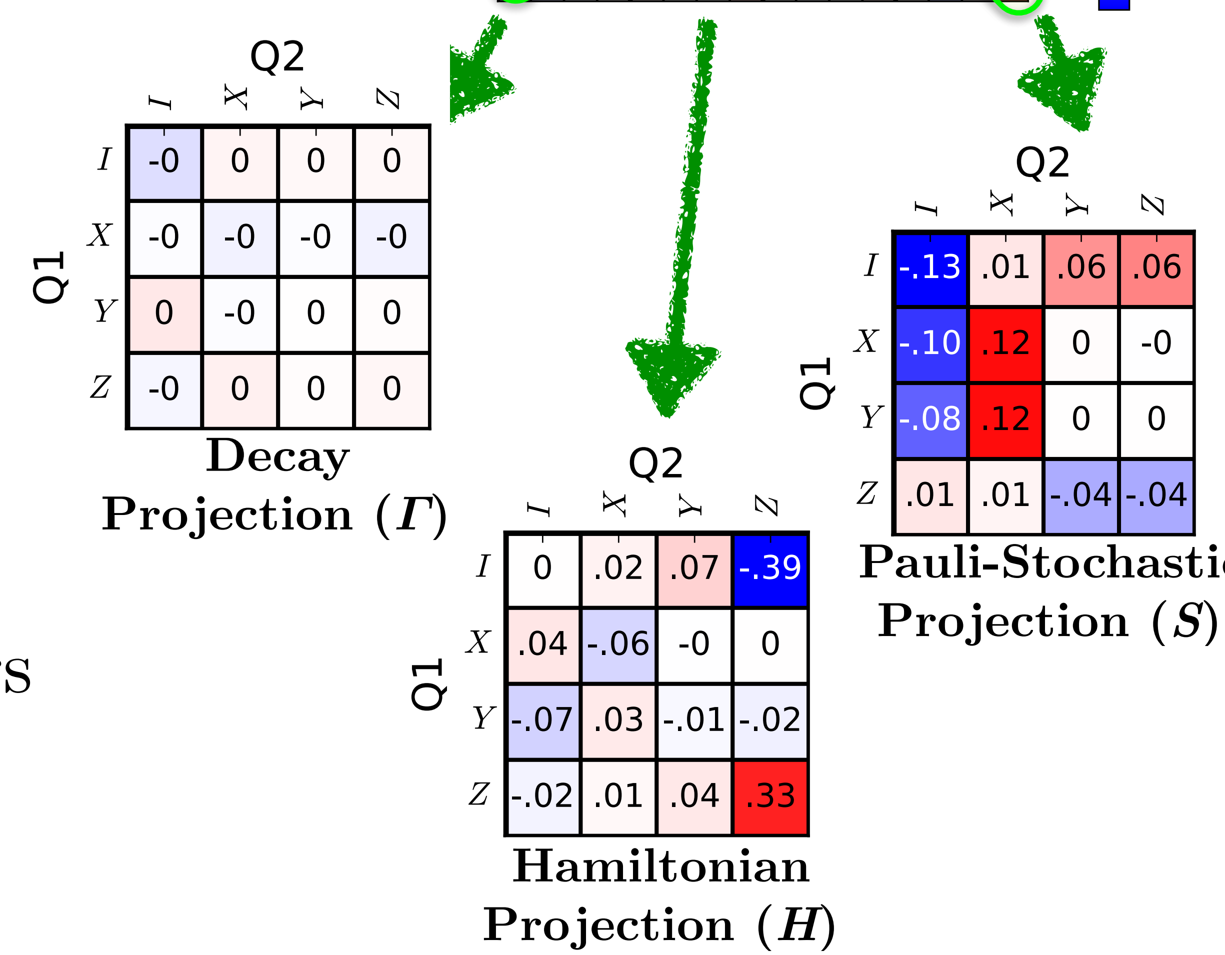
A TAXONOMY OF SMALL MARKOVIAN ERRORS

- 240 parameters => 3x16=48 parameters. So what's left (in those 192 params)?
- *Note: often, the <u>magnitude</u> of what's left is negligible — whatever it is, doesn't happen!*
- About 105 of them describe *non-Pauli stochastic errors.*



- We're still figuring out what errors the other 87 params represent.





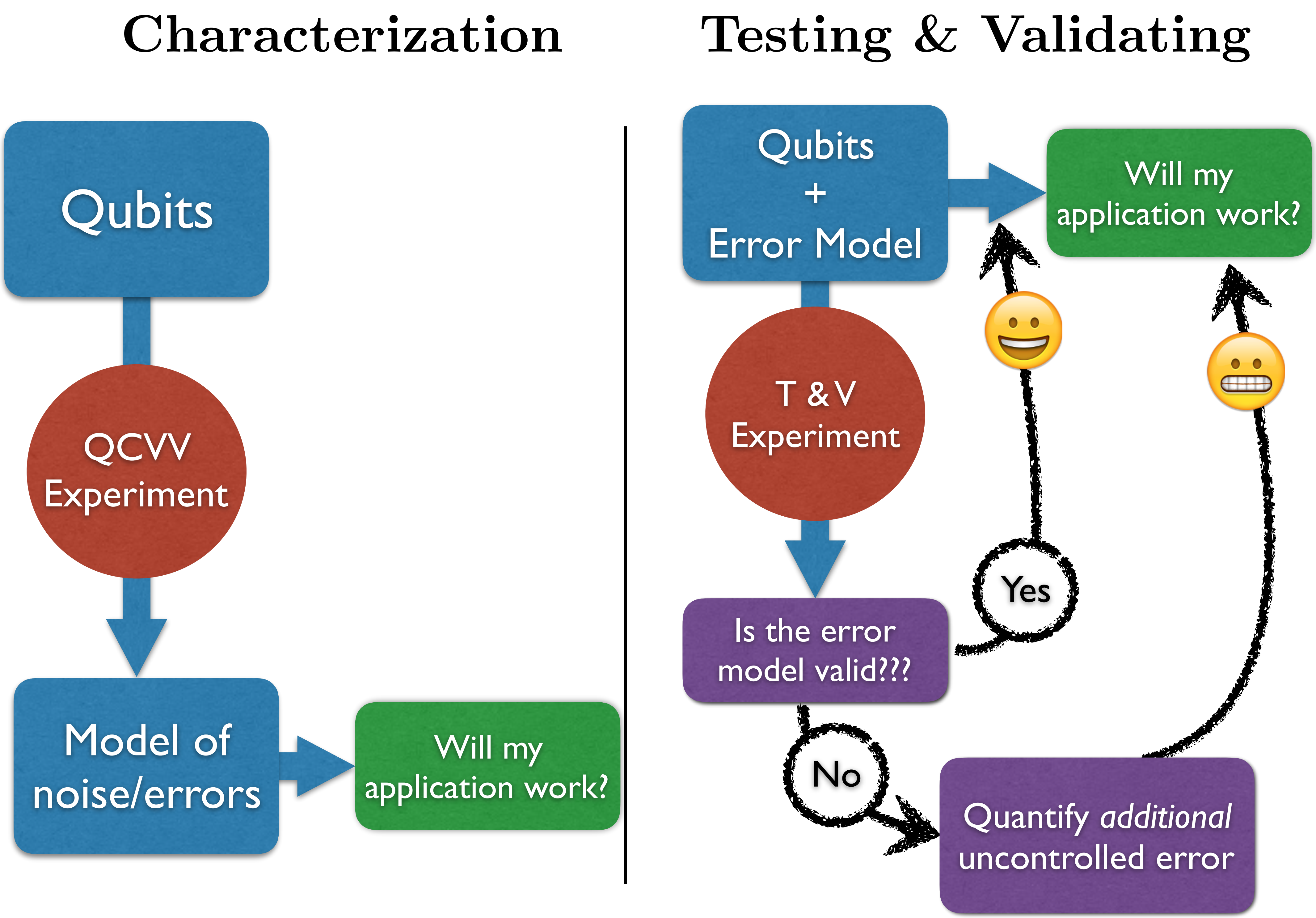
TESTING & VALIDATING ERROR MODELS

- Processors should come with an error model — part of the "spec"!
  ==> Then users can use that model to predict performance:

- There are lots of valid ways to *build* an error model
  - RB, GST, other QCVV techniques
  - theory, etc.

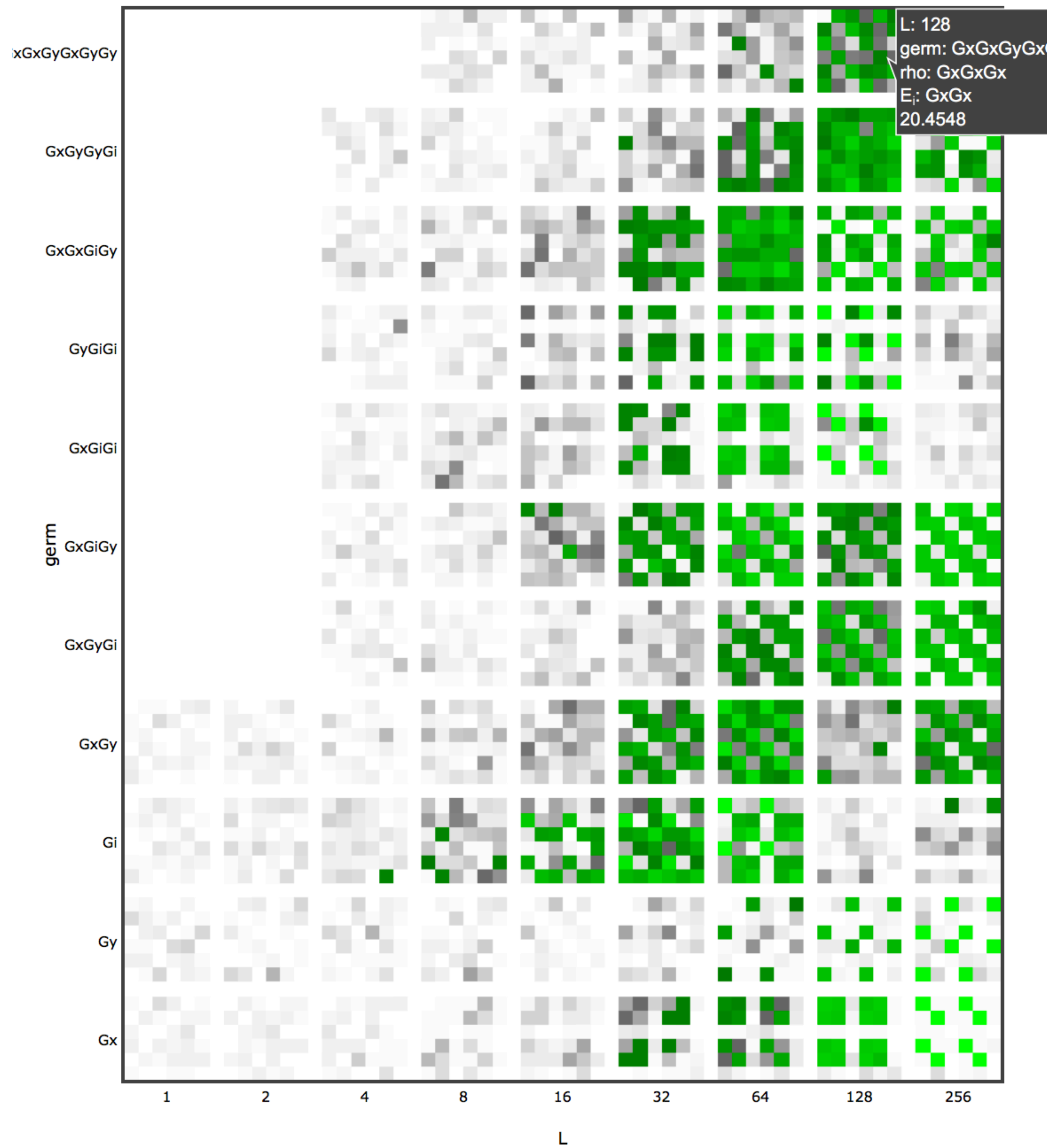
- What we need is a systematic way to *test* (validate) them.
  - Where the model came from isn't our concern
  - Whether it works (predicts results accurately) *is*.

TESTING & VALIDATING ERROR MODELS



TESTING & VALIDATING ERROR MODELS

- Run test suite of circuits.
- Record counts.
- Predict probabilities w/model.
- Compare data to predictions.
- Are there any statistically significant differences?
- YES: quantify "extra" error.
- NO: model validated; establish upper bound on possible undetected extra error.



WHY STUDY EXOTIC ERRORS?

1. Because when we test the process matrix model (this is intrinsic to GST with PyGSTi), it's <u>not</u> generally valid!

2. Because theorists and qubit designers *expect* other errors.

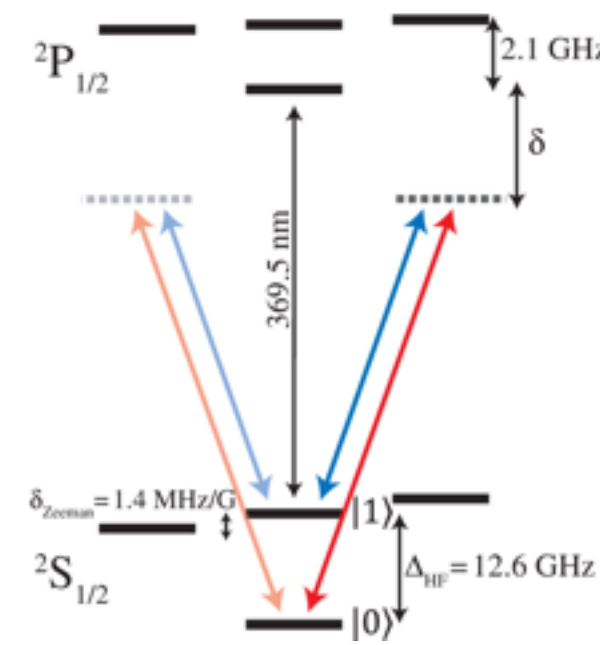
3. Because QEC theorists are {very/slightly} worried about them.

4. Because sometimes these things crop up totally unexpectedly (e.g., context-dependent gates), and we have no idea in advance what's going on or whether it's going to be significant

5. Because our goal is to completely understand — and predict — the observed behavior of quantum processors.

LEAKAGE

$^{171}Yb^{+}$ Energy Levels



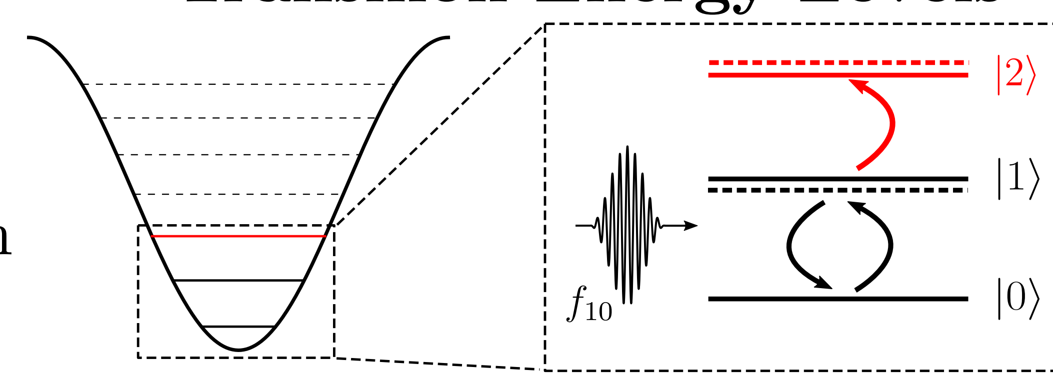
- Leakage: Basically, your qubit isn't actually a qubit — there are other states, and for whatever reason it ends up there.

- Leakage turns out to be surprisingly tricky *conceptually*. Why?
  1. It can be either *visible* (SC qubits) or *invisible* (ions).
  2. Everybody tends to think about it from a *modeling perspective*, rather than a *phenomenological* perspective. <u>We can't see Hilbert space!</u>

- Our best thinking: The most reliable way to detect and define leakage is **"The qubit has leaked iff my gates stop working [as well as they did before it leaked]."**

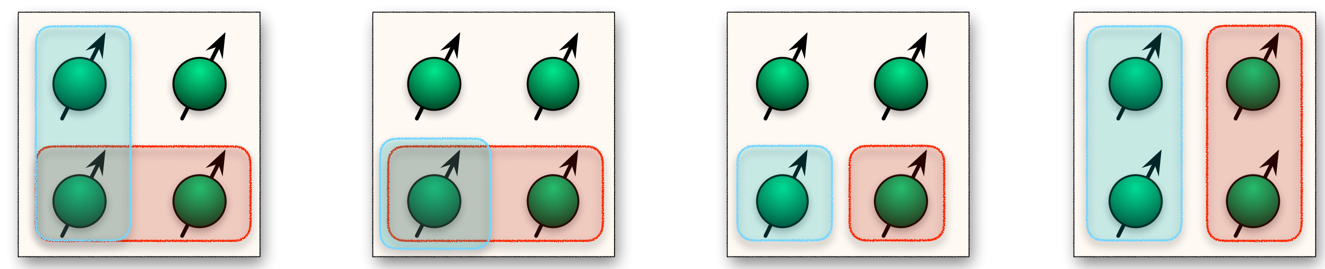
- This leads to pretty deep ambiguity between "leakage" and "non-Markovian decoherence".

Transmon Energy Levels



CROSS-TALK

- Everybody knows what crosstalk is — except it can mean several different things.



  - *Operation* crosstalk
    - Operations on *target qubits* cause action on *spectator qubits.*
    - Could be dependent on target state (entangling crosstalk).
    - Includes intermediate measurement (not just gate) operations.
  - *Detection* crosstalk
    - Result of measuring *target qubit* influenced by *spectator qubit.*
  - *Idle* crosstalk
    - Unwanted always-on entangling interaction.
- General: **The results of an operation depend on a variable or context that they should not depend on.**

INTERMEDIATE MEASUREMENTS

- This isn't technically "an error" — the point is simply that measurements are going to happen *in the middle of circuits*, not just at the end, as occurs in the traditional "SPAM" setup.

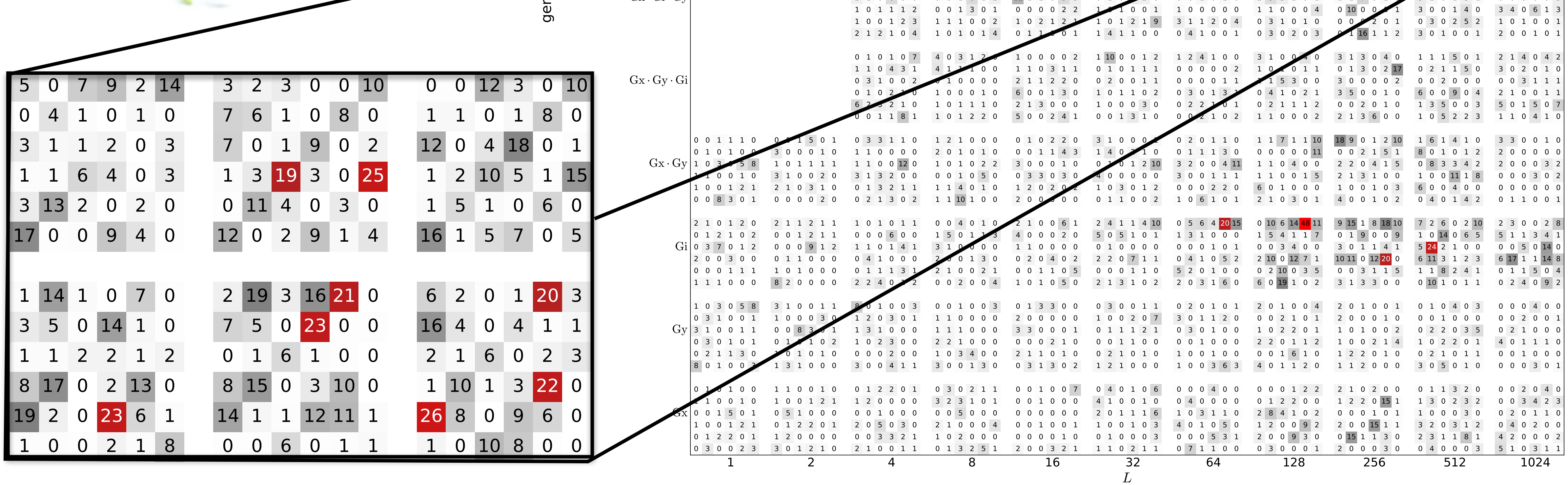
- Intermediate measurements aren't described by POVMs — that only tells you the *outcome probabilities*, not the *post-measurement state*.

- Instead, describe measurements by *Completely Positive Trace-Reducing (CPTR) Map-Valued Measures* (CPTRMVMs).
  ==> not a set of *effects* (positive operators) that sum to identity
  ==> instead, a set of *conditional channels* that sum to a CPTP map.

- Now we can do tomography (GST) on those channels.

DRIFT

Why do we see GST data inconsistent with *any* process matrices?



DRIFT

- Why do we see data inconsistent with *any* process matrices?
- Lots of potential *non-Markovian* effects could cause it…
…but we suspect the main cause is plain ol' boring *drift*.
  - ☛ Gates *are* representable by process matrices — but the process matrix changes (slowly) over time.
  - ☛ Mostly Hamiltonian parameters (coherent errors) drift over time, but incoherent parameters can drift too (T_1).
- We can try to adjust GST to deal with this… but it would be better to confront drift head-on and characterize it *directly*.
- Drifting parameters may drift slowly — but they should *not* be correlated with gates, states, etc (that's crosstalk)

CONTEXT-DEPENDENCE

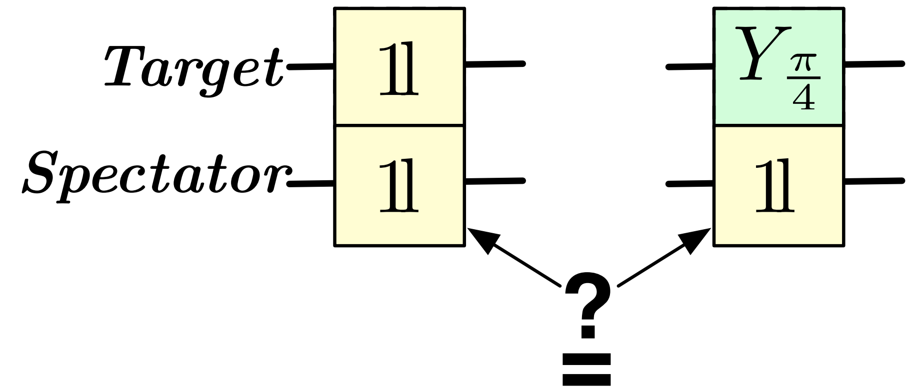
- The other main type of non-Markovianity we've seen is *context-dependent gates*.

- Context dependence is pretty general — e.g., time is an example of a context (on which gates shouldn't depend).

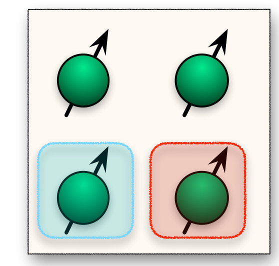
- But we mean specifically: When the effect of a gate depends on what gate occurred immediately before it.

- Examples we've observed:
  - $\boldsymbol{X}_{\pi/2}$ gates consistently under-rotate after a 2-qubit gate.
  - ***Idle*** gates are small Z rotations iff they follow an $\boldsymbol{X}_{\pi/2}$ gate.
  - Every gate is terrible whenever it follows a *different* gate.

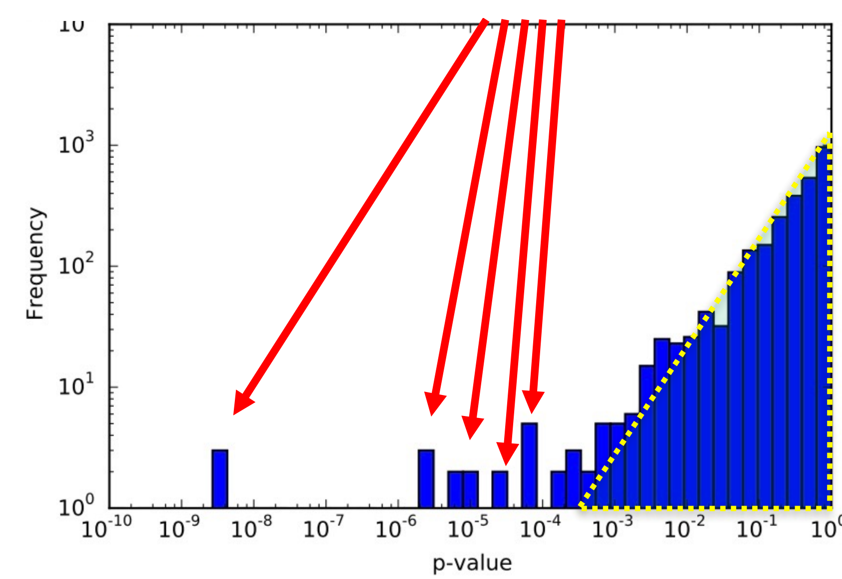
TESTS FOR CROSS-TALK



- Focus: *operation crosstalk.*
- Basic idea:
  - **Q**: Does effect of a gate on ***Spectator*** vary depending on whether we do some gate to ***Target*** at the same time?
  - **A**: Study behavior of ***Spectator*** *with* & *without* "driving" ***Target***.



- IBM's *simultaneous RB* is an example of this kind of protocol.
- We do *cross-GST* experiments to tomograph gates in both contexts
- But instead of tomographic reconstruction, we do simple statistical tests to ask: "Are all of these experiment pairs consistent with each other?



TESTS FOR CONTEXT-DEPENDENCE

- We can *test* for context dependence the same way as any other non-Markovianity — do GST, look for model violation.

- To distinguish it from other effects, we enlarge our model by letting the gates depend on context, and see if that fits better.

- **Example**: Gateset $\{G_I, G_X, G_Y\}$. We suspect G_I is context-dependent.
  1. Introduce a new gate G_{I2}. ==> gateset $\{G_I, G_{I2}, G_X, G_Y\}$.
  2. For each circuit run, rename all G_I after a G_X or a G_Y to "G_{I2}".

$$\boxed{G_I - G_X - G_I - G_I - G_Y - G_I - G_I} \implies \boxed{G_I - G_X - G_{I2} - G_I - G_Y - G_{I2} - G_I}$$

  3. Rerun GST analysis to fit G_I and G_{I2}.
  4. If the fit is significantly better, we've identified evidence for context dependence — *and* have an estimate of how G_I depends on context.

TESTS FOR DRIFT

- L*ots* of ways to study time-correlated noise (drift). But we want to integrate our analysis as closely as possible with RB, tomography, QEC, etc.

  ==> same kind of data (circuits —> counts).

- Standard practice: *binned* counts

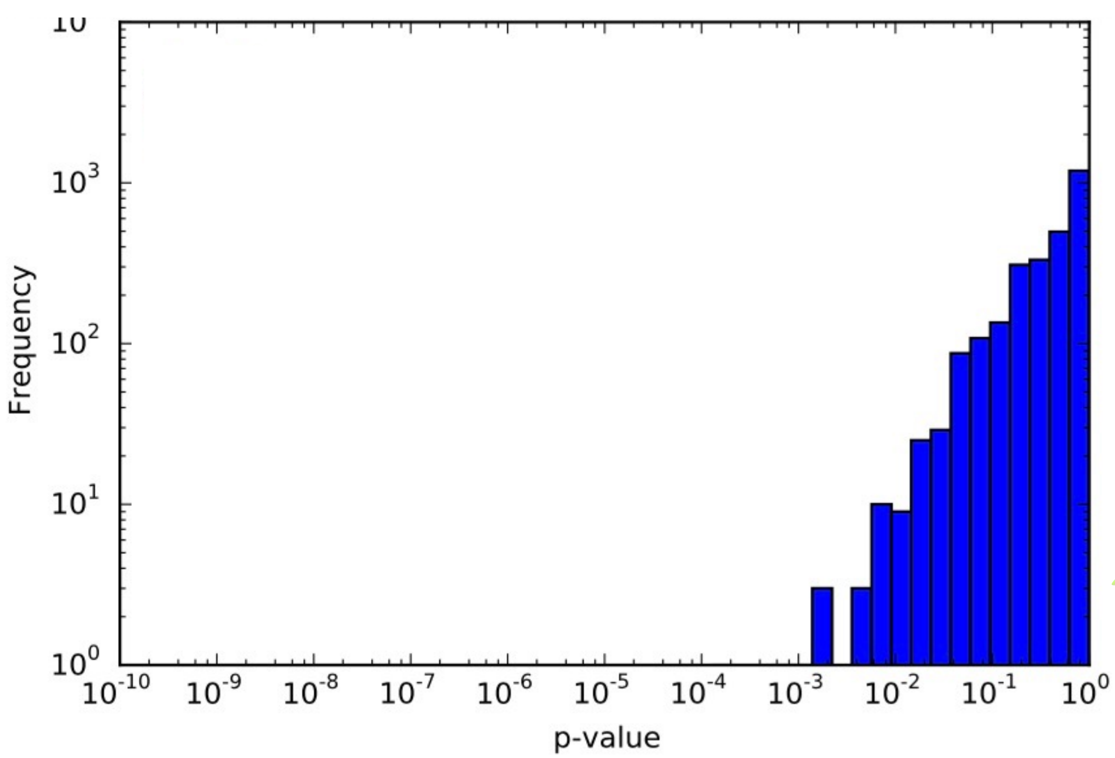
  ==> N repeats of each circuit yield just $\{n,\ N\text{-}n\}$.

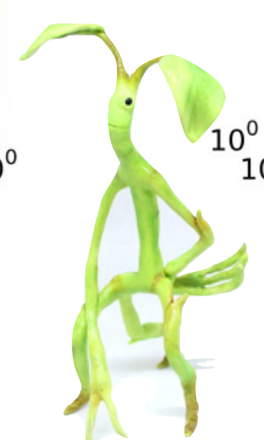
- For drift detection, we've considered two variants:
  - *Chunked* data: $n/KN \Longrightarrow \{n_1/N,\ n_2/N,\ \ldots n_K/N\}$

    ==> (same analysis as for crosstalk)
  - *Rastered* data: $n/N \Longrightarrow \{0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0 \ldots\}$

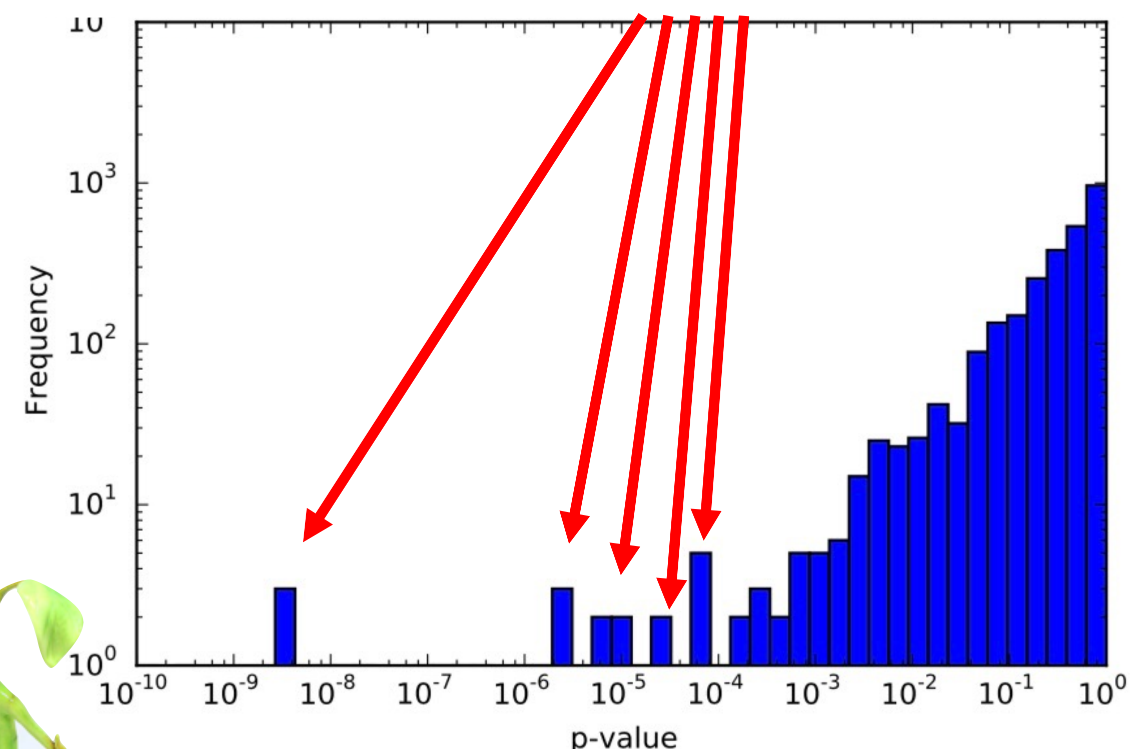
TESTS FOR DRIFT
CHUNKED DATA

Simulated GST data
Two data sets, no drift





Experimental GST data
Two data sets, signs of drift



TESTS FOR DRIFT

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TESTS FOR DRIFT
RASTERED DATA

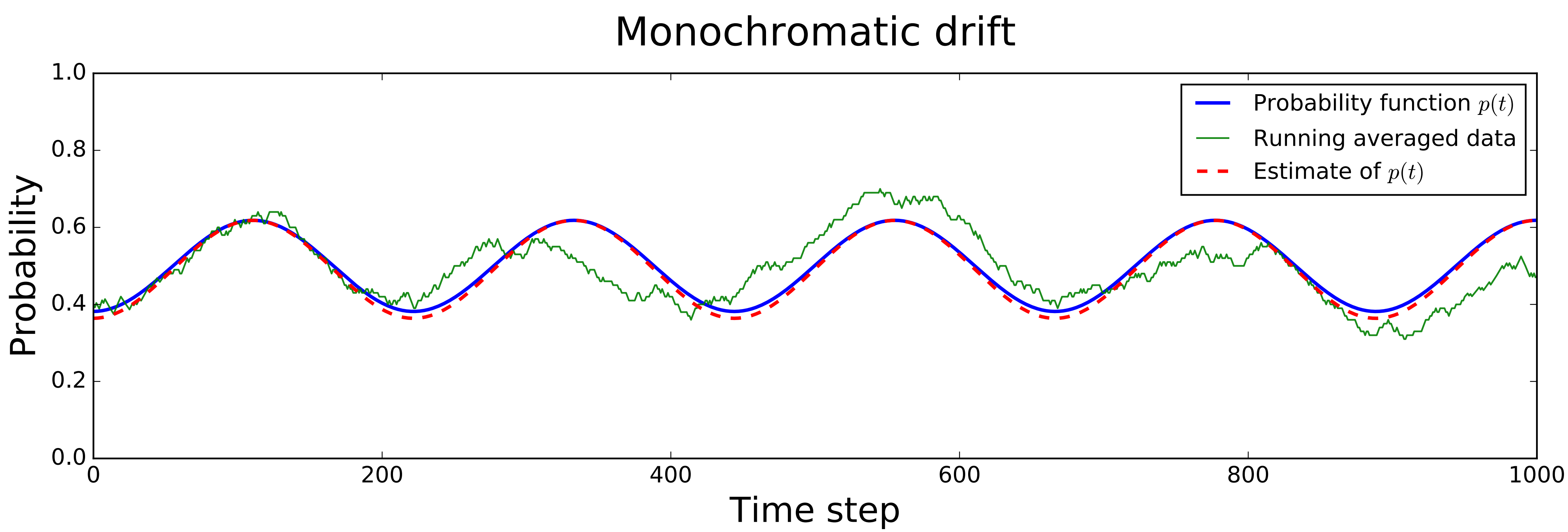
- For rastered/time-stamped data, we can use model selection to see whether a simple model for $Pr(t)$ fits the counts "surprisingly well".

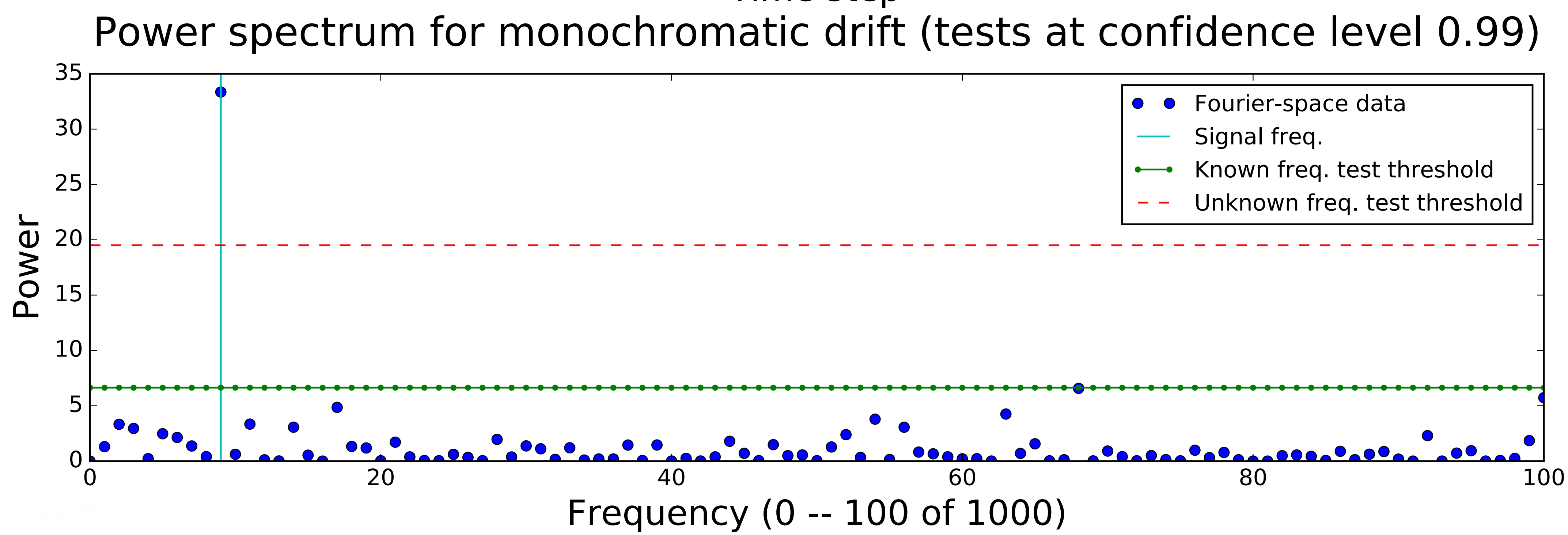
- Various simple models — with few parameters — are possible.
  - (low-frequency, piecewise constant, 60Hz, etc...)

- However, a nice <u>generic</u> model for many drifts (esp. samples from stationary stochastic processes) is *Fourier-sparse functions.*
  - Slow drifts <==> low-frequency ==> sparse
  - 60Hz can be "fast", but is still Fourier-sparse

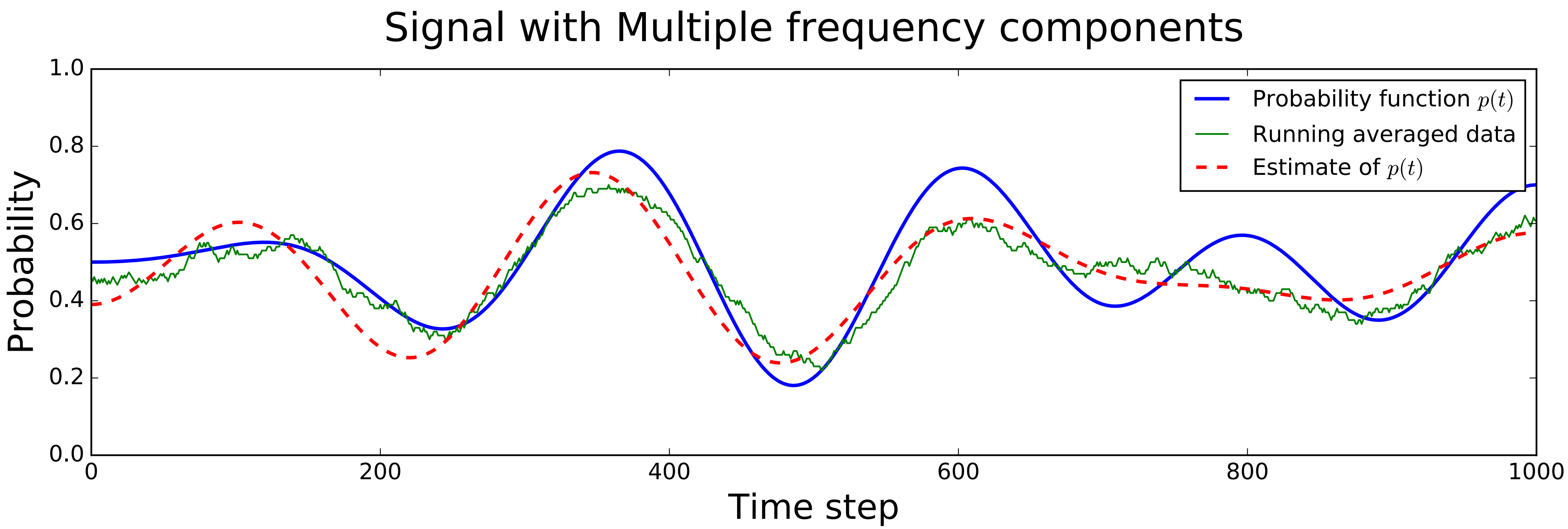
- **Analysis method:** (1) Fourier transform the count data *directly*.
  (2) Throw out low-power components that are statistically consistent with constant $Pr(t)$.

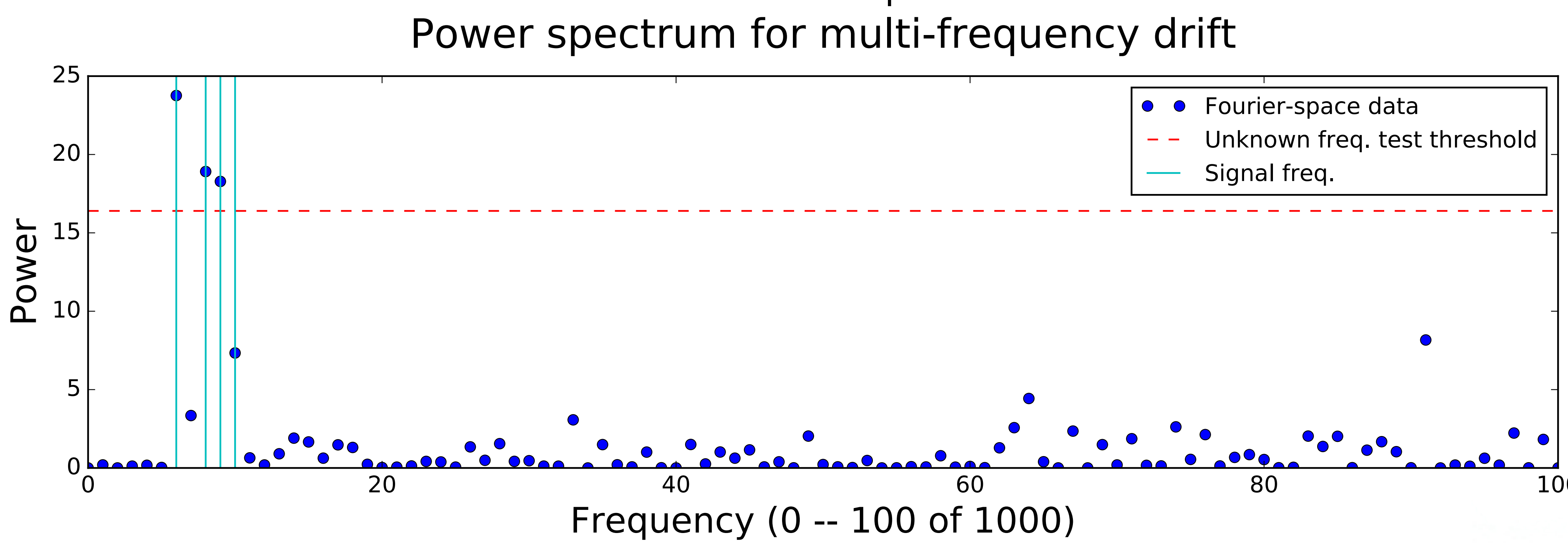
DRIFT EXAMPLE 1: A SINGLE FREQUENCY





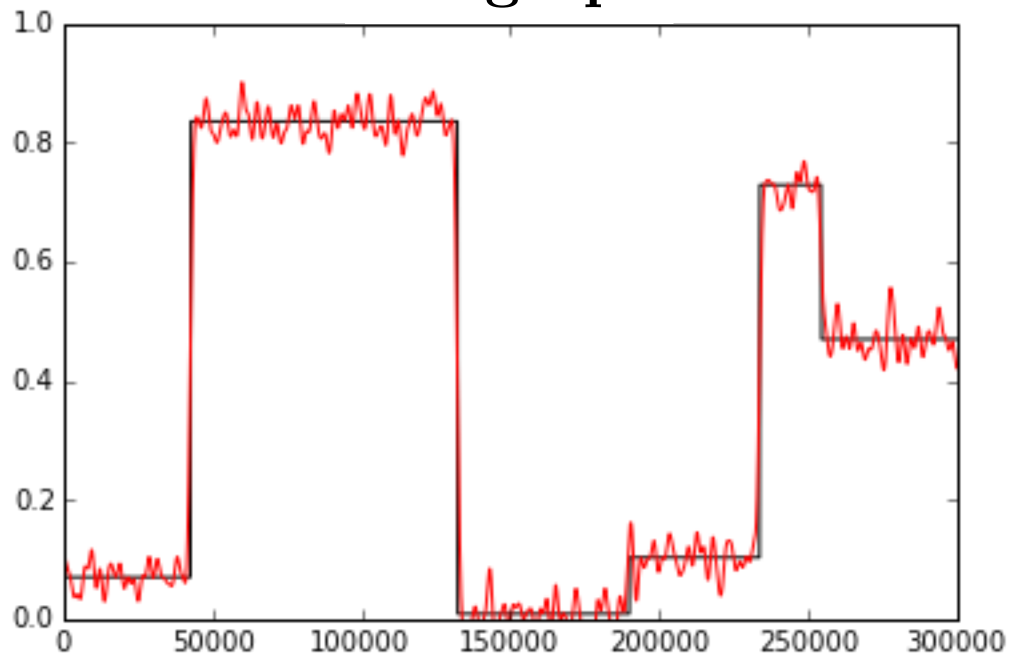
DRIFT EXAMPLE 2: MULTIPLE FREQUENCIES



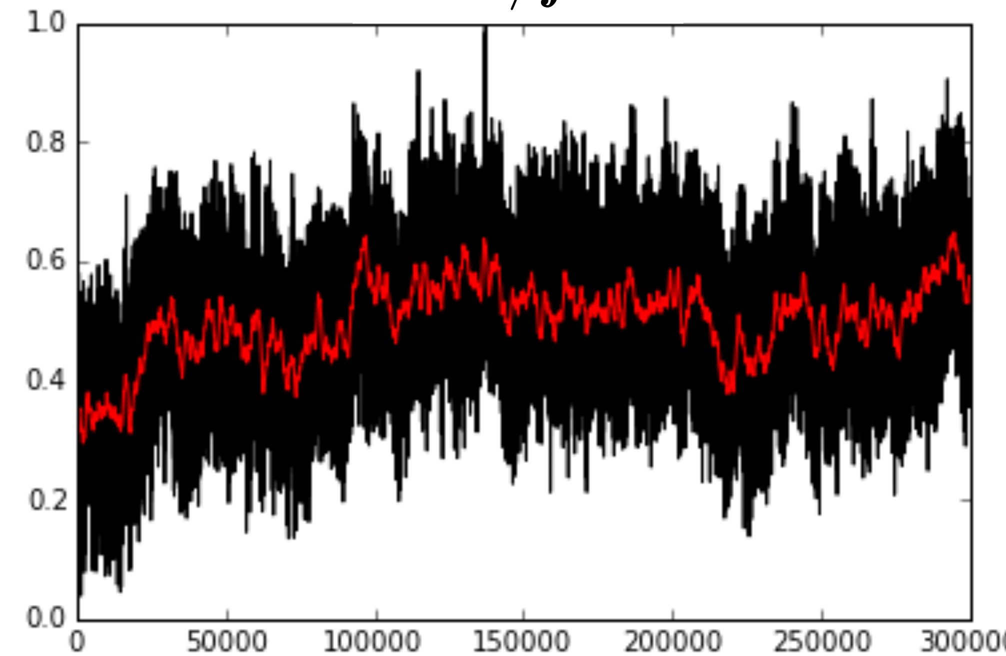


DRIFT EXAMPLE 3:
SIMULATED NOISE PROCESSES

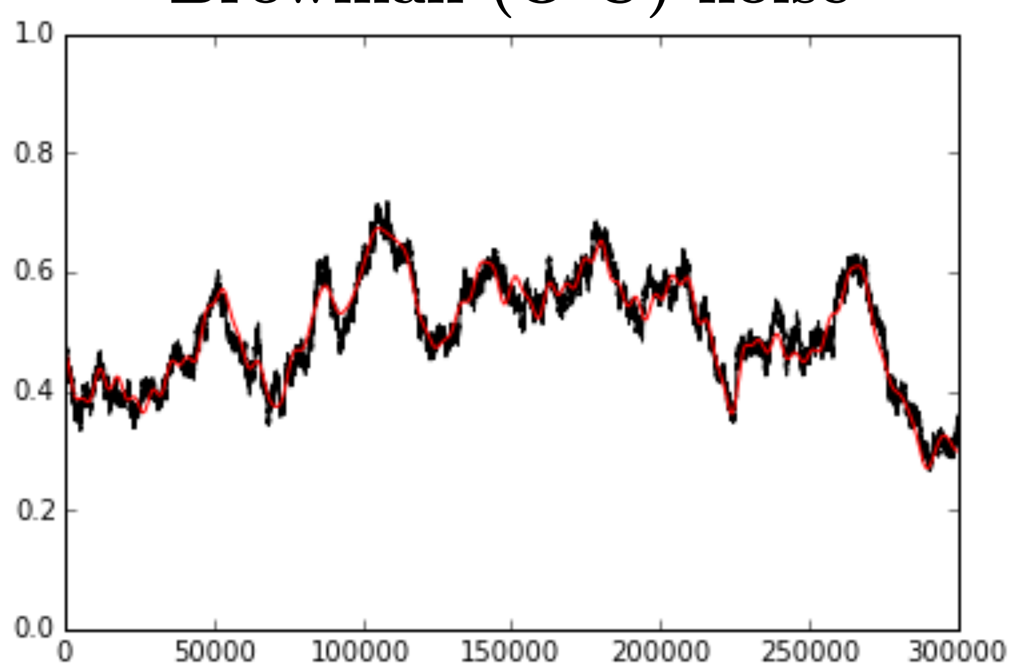
Telegraph



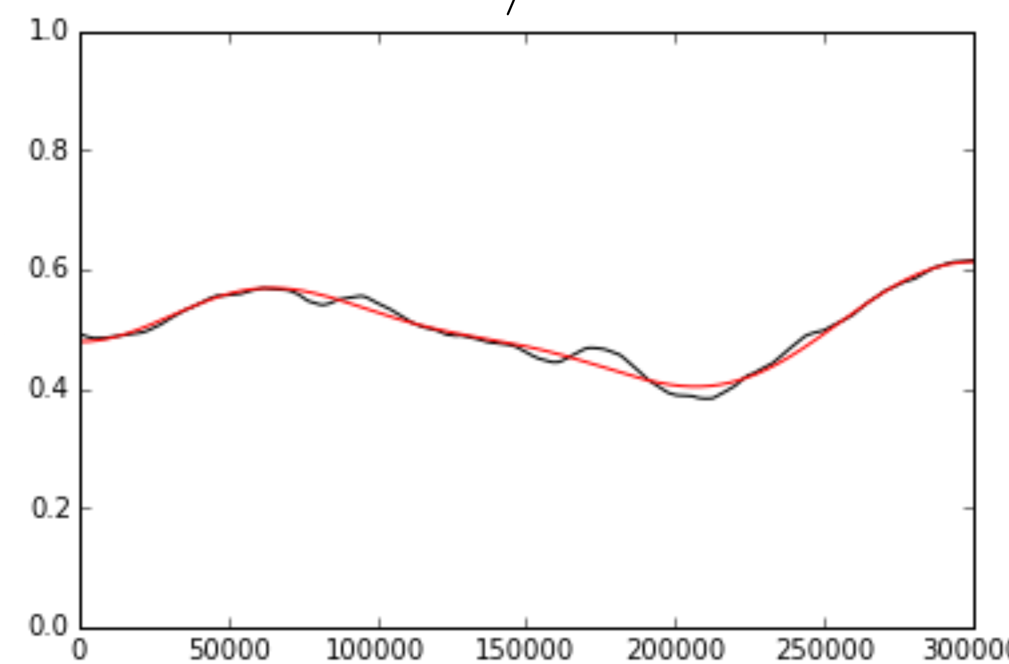
$1/f$



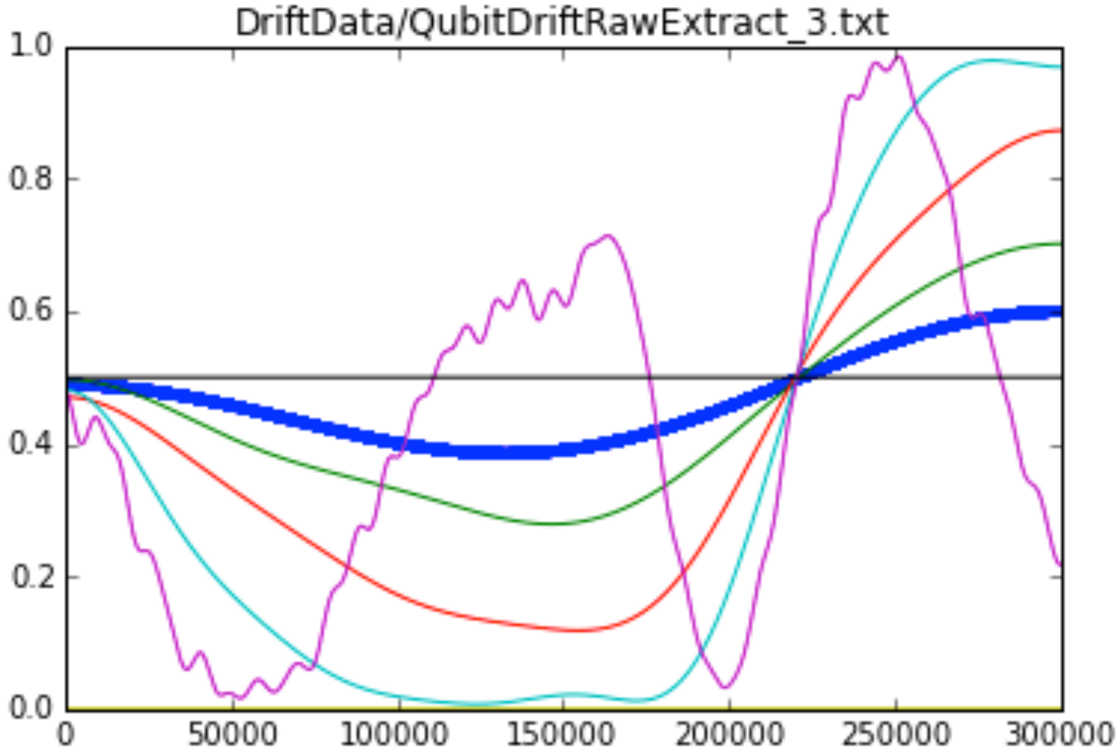
Brownian (O-U) noise

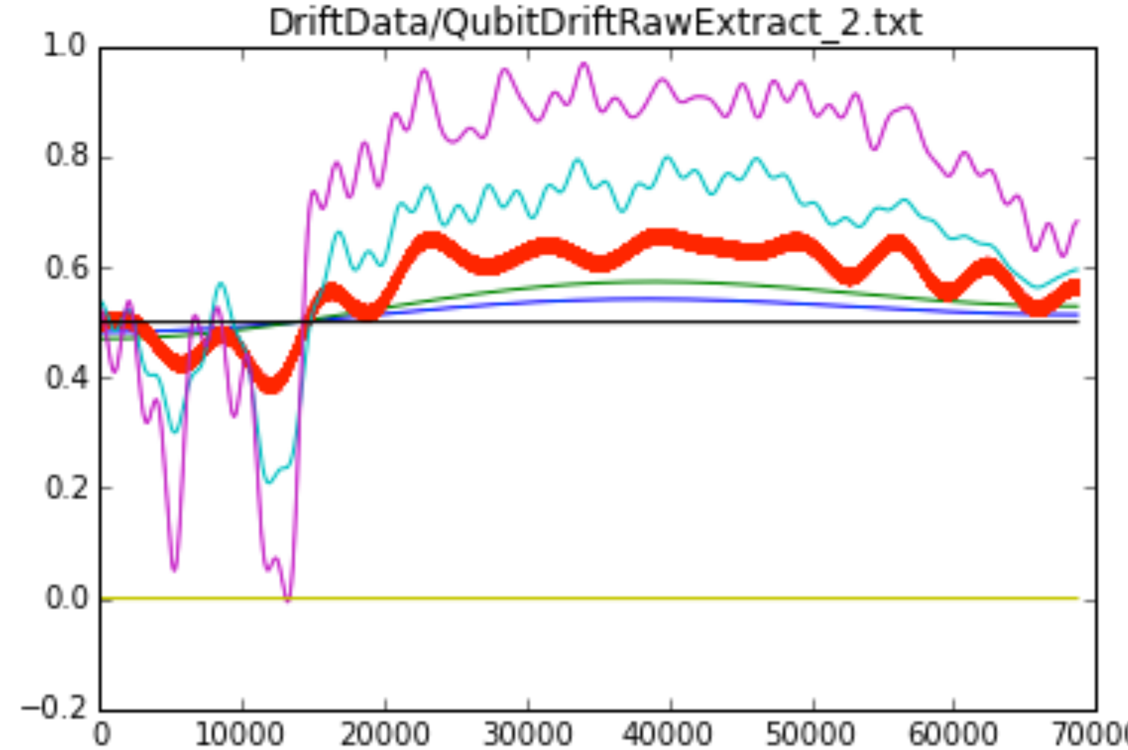


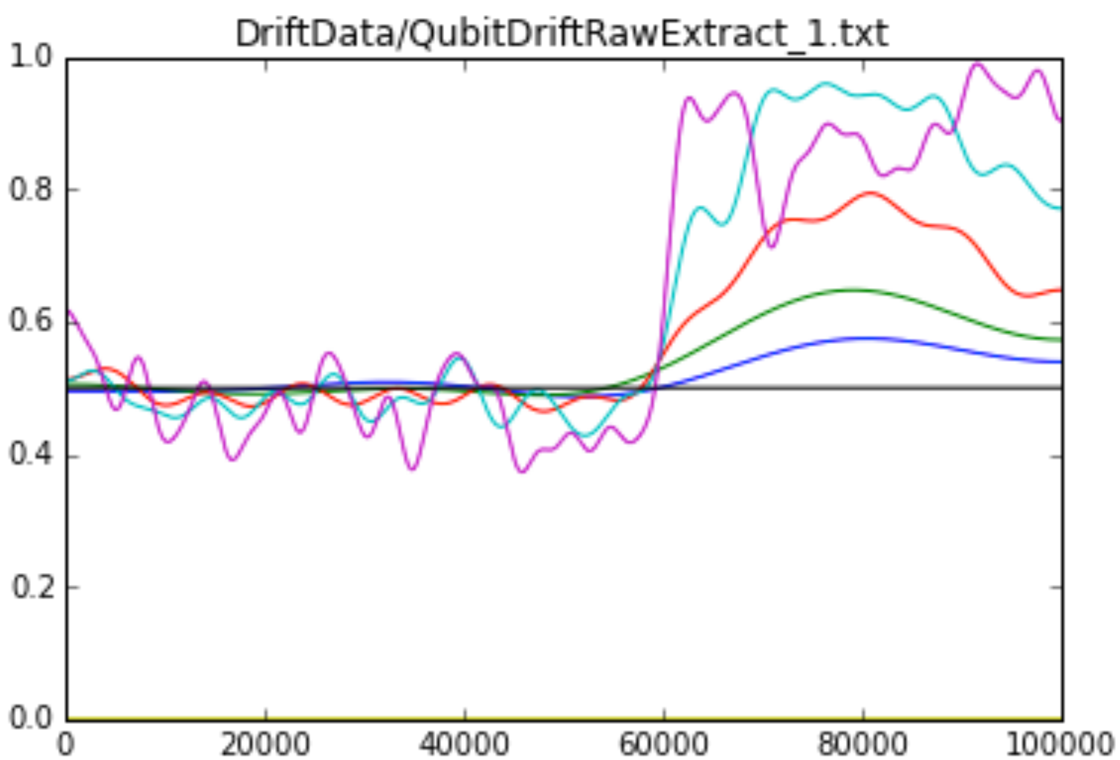
$1/f^4$

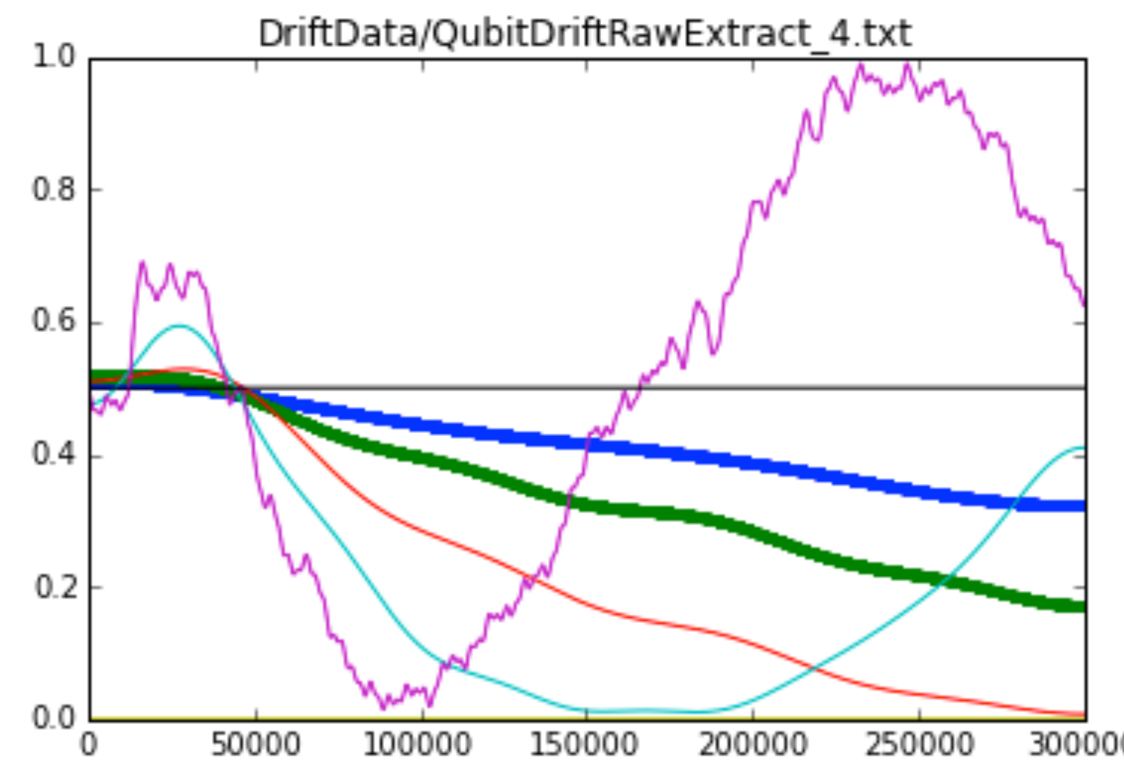


DRIFT EXAMPLE 4: EXPERIMENTAL DATA









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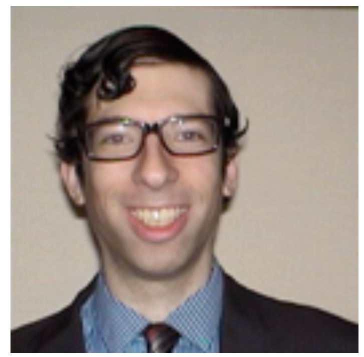
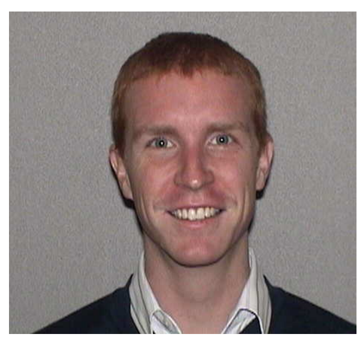
Tim Proctor

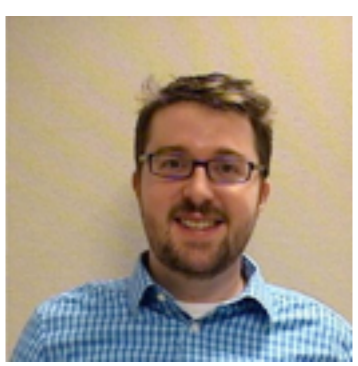

Travis Scholten


Kenny Rudinger




Erik Nielsen


Kevin Young


John Gamble


Peter Maunz


Mohan Sarovar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