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Two-way Coupling of Fracture and Fluid Flow Using a Phase-field Model

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Motivation

Fluid-driven crack propagation is present in many physical applications (CO_2 sequestration, groundwater contamination, fluid induced fault activation)

- Long term simulations (10,000 years+) are required

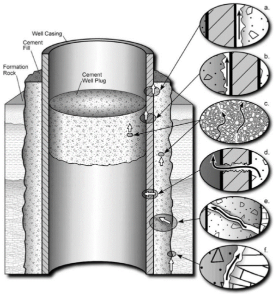

Image courtesy of National Energy Technology Lab

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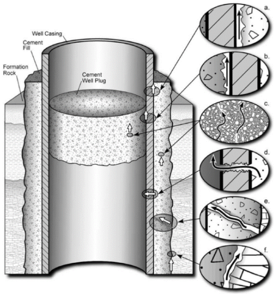
- Long term simulations (10,000 years+) are required
- The coupling of fracture and fluid flow is important: pore pressure induces fracture, which causes leaks



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Cracks in a material represent:

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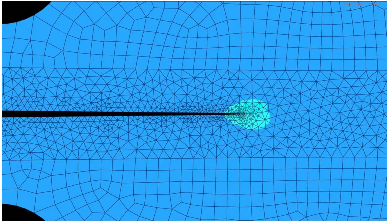
  - Mesh-based methods risk mesh dependency; cracks follow mesh edges, not physical path

Common Computational Fracture Models

Traditional methods: (XFEM, peridynamics, gradient damage, cohesive zone method)

- Dependence on adaptive remeshing after each time-step



XFEM representation of a fracture, courtesy of Spear et al, 2011

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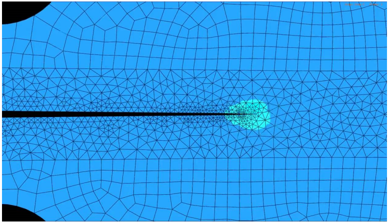

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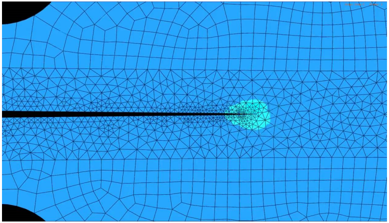
- Sharp discontinuties: difficult to track



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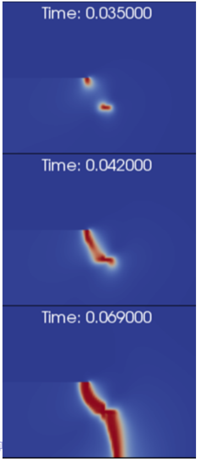
Phase-Field Model of Fracture

- Introduce a scalar phase-field variable to track damage
- Advantages:



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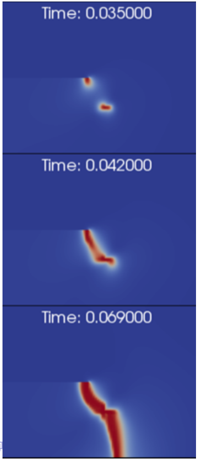

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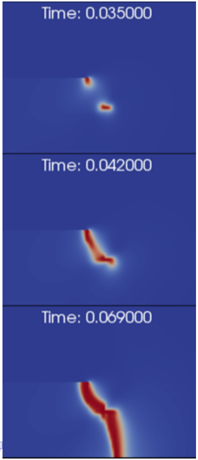

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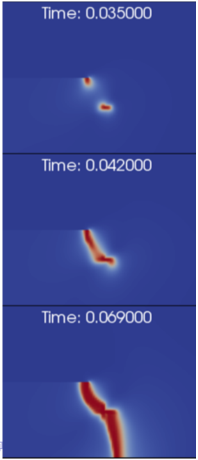
  - As lengthscale $\rightarrow 0$, model converges to Griffith's linear elastic fracture mechanics



Assumptions and Boundary Conditions

Linear-elastic, Isotropic: $\psi_e(\epsilon) = \frac{1}{2}\lambda\epsilon_{ii}\epsilon_{jj} + \mu\epsilon_{ij}\epsilon_{ij}$

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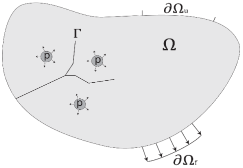
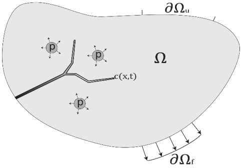
$$(BC)\begin{cases} u_i &= g_i \\ \sigma_{ij} n_j &= t_i \\ \frac{\partial c}{\partial x_i} n_i &= 0 \end{cases}$$

$$(IC)\begin{cases} u(\boldsymbol{x},0) &= \boldsymbol{u}_0(\boldsymbol{x}) \\ \dot{u}(\boldsymbol{x},0) &= \boldsymbol{v}_0(\boldsymbol{x}) \\ c(\boldsymbol{x},0) &= c_0(\boldsymbol{x}) \end{cases}$$





Phase-Field Equations

The phase-field formulation, based on a variational statement of brittle fracture, gives the total potential energy:

$$\Psi\left(\epsilon, \Gamma\right) = \int_{\Omega} \Psi_e(\epsilon) d\Omega + \int_{\Gamma} G_c d\Gamma$$

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Converting the second integral to a volume integral and introducing scalar c, length scale l_0:

$$\Gamma_{l_0}(c) = \int_{\Omega} \frac{1}{4 l_0} [(c-1)^2 + 4 l_0 |\nabla c|^2] d\Omega$$

Tightly Coupled System

Considering the Lagrangian and applying the Euler-Lagrange equations, we arrive at the strong form:

$$\left(\frac{4 l_0 c \mathcal{H}}{G_c} + 1\right) c - 4 l_0^2 \frac{\partial^2 c}{\partial x_i^2} = 1$$

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \ddot{u}_i$$

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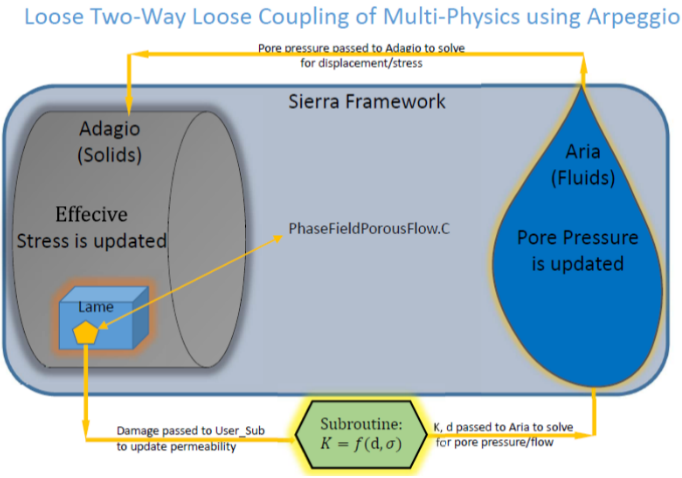
Degrade Only the Tensile Portion of Strain Energy

$$\tilde{\psi}(\epsilon, c) = c^2 \psi_e^+(\epsilon) + \psi_e^-(\epsilon)$$

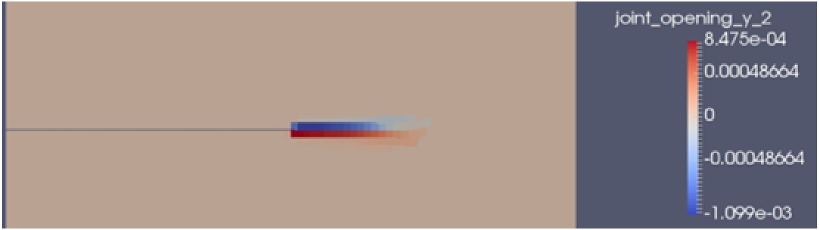
Overview of Two-Way Coupling



Joint Opening

Calculate joint opening:

$$\lambda_{\perp}^{2} = \frac{\nabla c \cdot \nabla c}{\nabla c \cdot \boldsymbol{C}^{-1} \cdot \nabla c}$$



Stress is Degraded as Function of Fluid Pressure, Damage

Modify stress by hydrostatic contribution:

$$\boldsymbol{\sigma} = c^2 \tilde{\boldsymbol{\sigma}}_{eff} - b\boldsymbol{p}$$

- Total stress is decayed by both damage and pore pressure
- Accounts for fluid pressures that are introduced

Permeability Modified as Function of Joint-Opening

Damage Alters Fluid Pressure

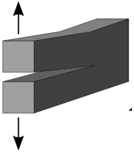
- Onset of Poiseuille flow– the second half of the coupling
- Consider cubic law (Lubrication Theory) to alter permeability

$$K_F = \alpha \frac{w^3}{12s}$$

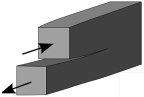
$$K_{eff} = K_m \boldsymbol{I} + K_F(I - \boldsymbol{nn}),$$

Material Properties and BCs

Parameter	Symbol	Value
Young's Modulus	E	10 GPa
Density	ρ	2560 $\frac{kg}{m^3}$
Critical Fracture Energy	G_c	100 $\frac{J}{m^3}$
Fracture Length Scale	l_0	0.02 m.
Poisson Ratio	ν	0.155
Biot's Coefficient	b	1
Edge Length of Hex Element	h	0.01 m.

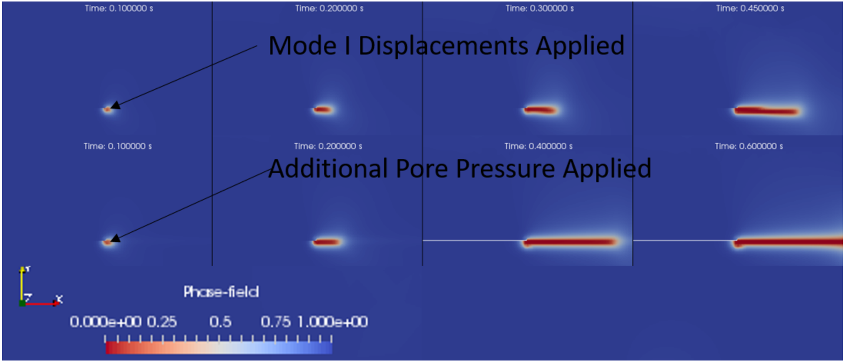


Mode I:
Opening

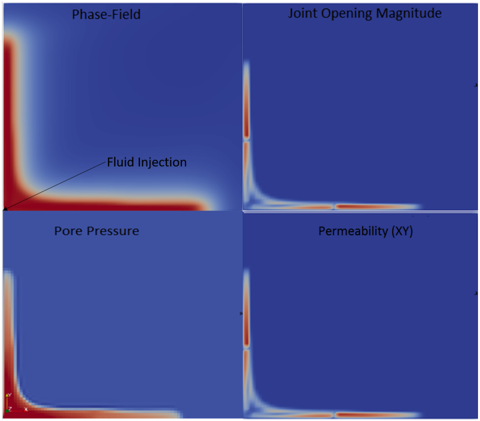


Mode II:
In-plane shear

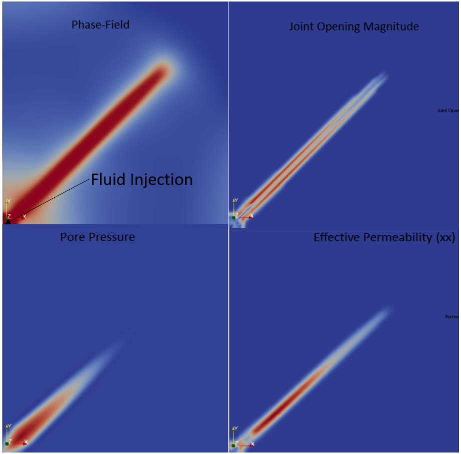
Demonstration of Pore Pressure Contribution to Mode I Failure:



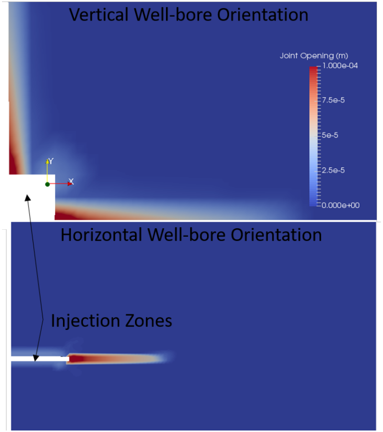
Quarter Symmetry - Fluid Injection

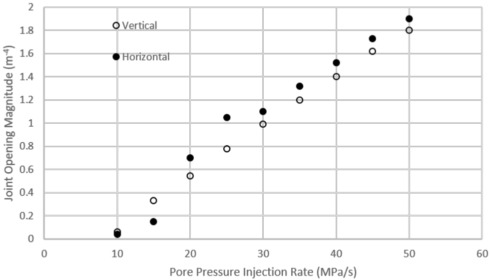


Quarter Symmetry - Alternate BC's



Well-bore Orientation





Conclusions

- Phase-field can approximate fractures in poroelastic materials under different types of stress

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- Fractures stemming from vertical and horizontally oriented well-bores display similar magnitudes of fracture width

Discussion of Future Work

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- Develop a more general, thermodynamically consistent model

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