

# Nonlinear Dynamics of Multi-Joint Structures

Brett Robertson

April 29<sup>th</sup>, 2016

M.S. Mechanical Engineering Thesis Defense

Committee:

Chair – Dr. Marc Mignolet

Member – Dr. Matt Brake

Member – Dr. Yongming Liu



## Motivation

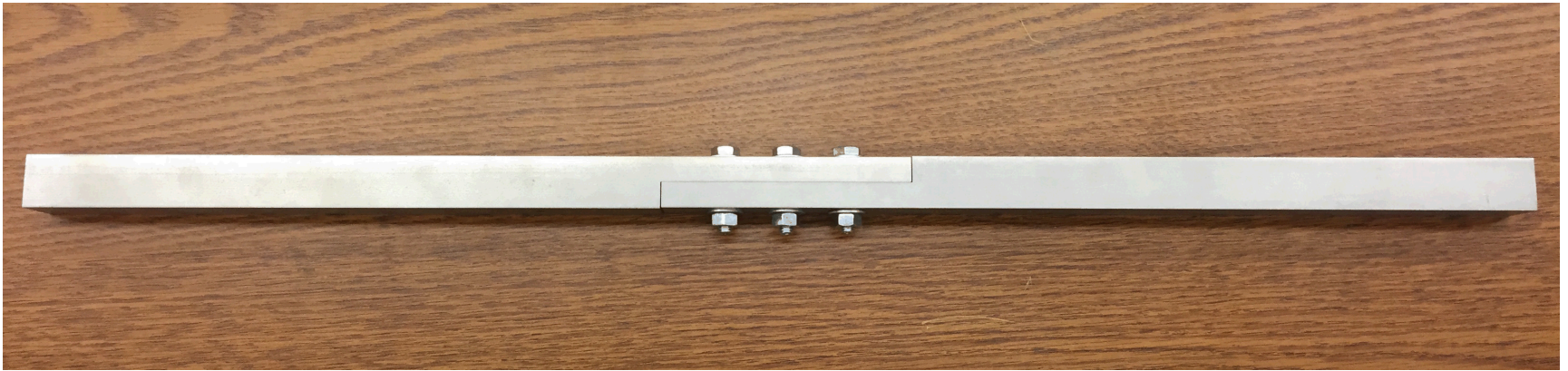


- Nonlinearities in jointed structures can largely change the dynamics of a system
- When structures contain only a few joints, the nonlinearities may not affect computations much, but when they contain hundreds or thousands of joints, the effort increases drastically
- Want to be able to determine when fully nonlinear models are necessary and when some linearization will suffice

## Aleatoric vs. Epistemic Uncertainty

- Joints have intrinsically variable properties, from one nominally identical joint to another (aleatoric uncertainty)
- Joint *models* introduce epistemic uncertainty as they approximate the physics
- It is suggested that if a joint model is somewhat simplified, the increase in epistemic uncertainty can be present, as long as its effects remain small with respect to those of its aleatoric counterpart
- Will explore the simplification of a stochastic multi-joint model to observe if the increased epistemic uncertainty will be noticeable compared to the existing variability in the response

## Core Test Article: Brake-Reuss Beam



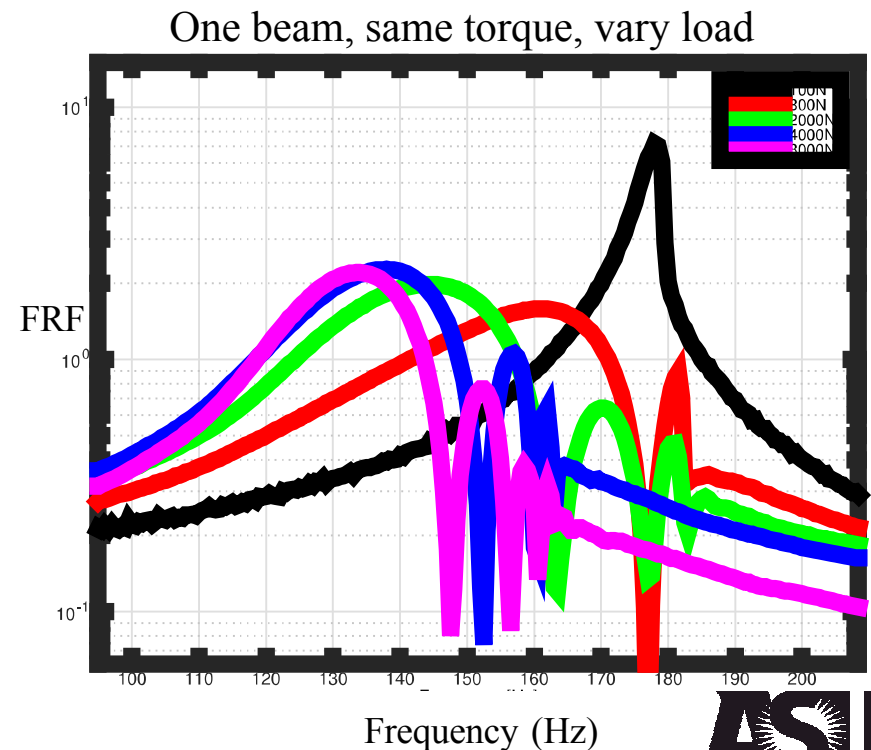
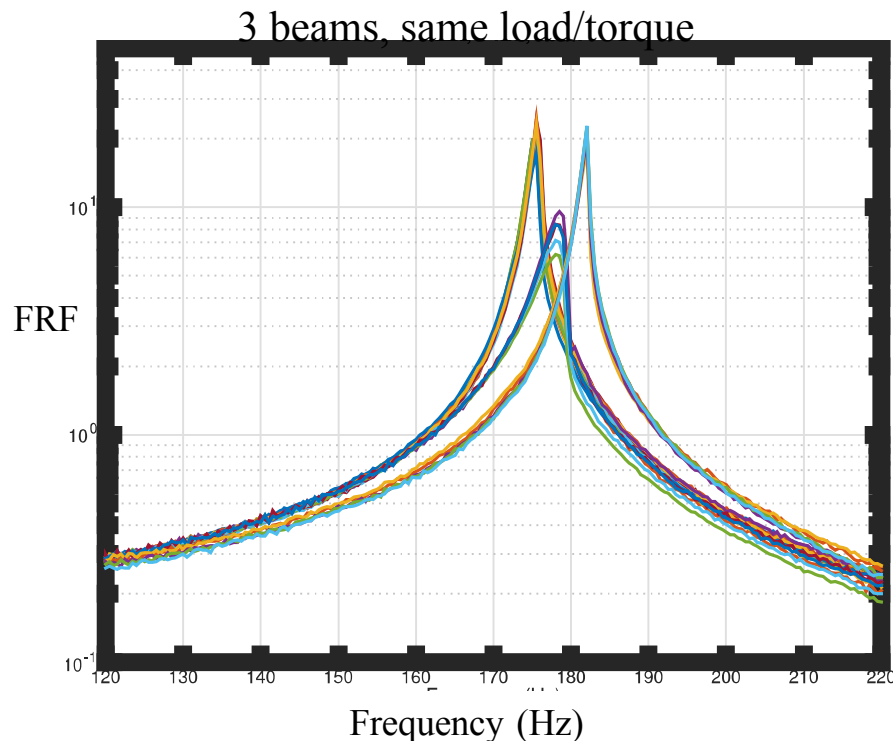
- Brake-Reuss (BR) Beam – 3 Bolt Lap Joint
- Used as baseline model for nonlinear dynamics research
- Impact experiments were performed during Nonlinear Mechanics and Dynamics Institute 2015 (NOMAD)

## Research Goals

- BR Beam:
  - Obtain experimental data
  - Create reduced order model (ROM) of BR beam that allows a nonlinear interface/joint model
  - Find parameters of joint model that replicate frequency shift found experimentally
  - Make correlations between impact level and frequency shift/damping ratio increase
- Multi-Joint Frame:
  - Design frame which contains multiple BR beams, create ROM, and implement into solver (Romulis)
  - Relate impact level/location and joint activity
  - Determine if certain joints can be modeled linearly when aleatoric uncertainty is present

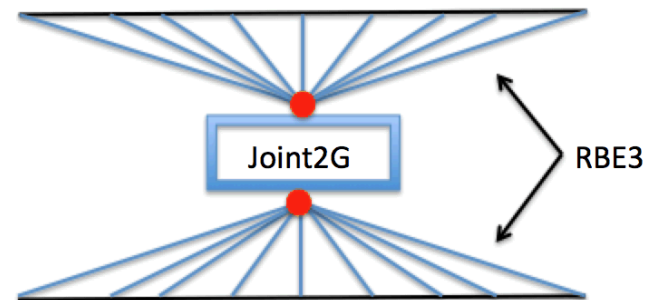
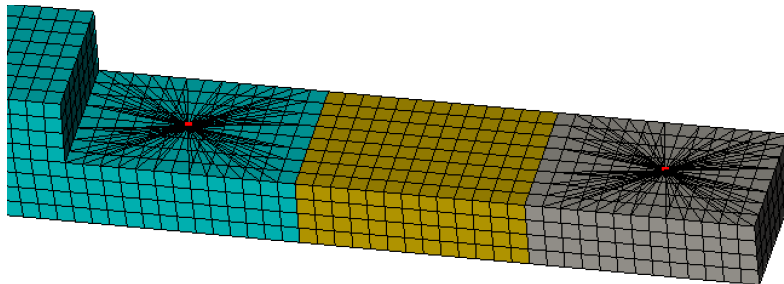
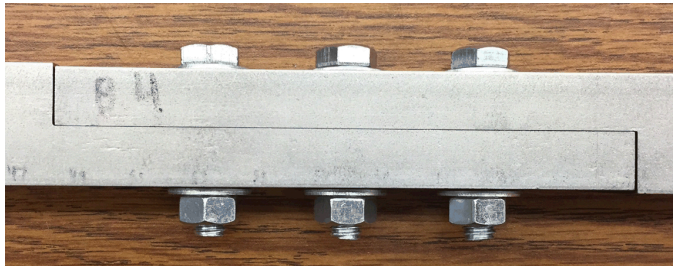
## Impact Hammer Experiments – BR Beam

- Support beam by bungee cords (free-free)
- Vary interface, torque, load level
- Large data set was collected



## Modeling the BR Beam

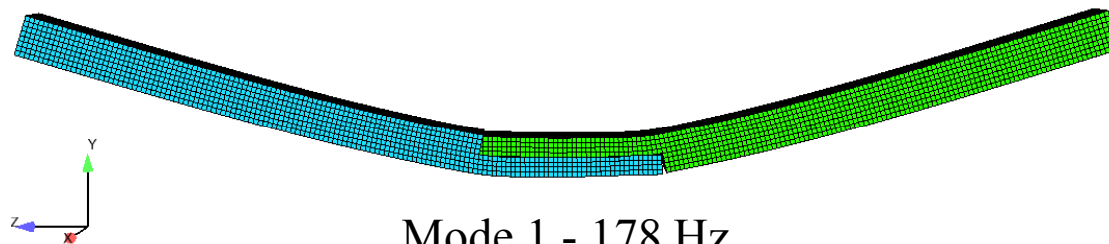
- After discussion with modeling group at NOMAD (Stuttgart), the interface was cut into three equal sections
- The middle section completely tied, due to tightness of interface
- Outer sections are rigidly connected to single nodes, act as interface



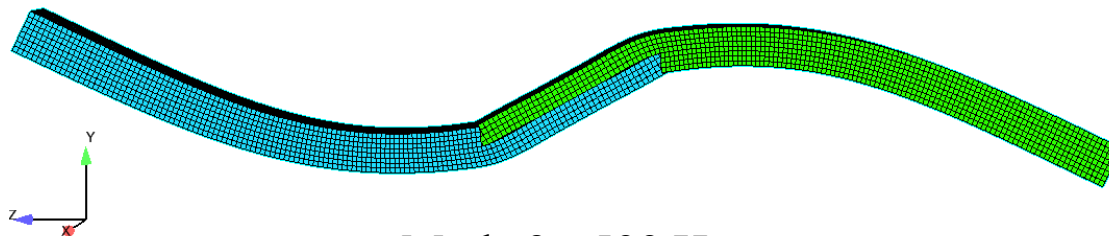


## In-Plane Modes of BR Beam

- Able to match linear natural frequencies of first two in-plane modes using springs as interface elements



Mode 1 - 178 Hz

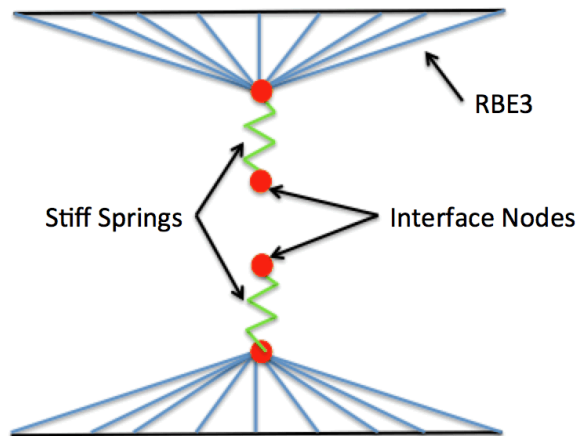


Mode 2 - 599 Hz



## Craig-Bampton Reduced Order Model (ROM)

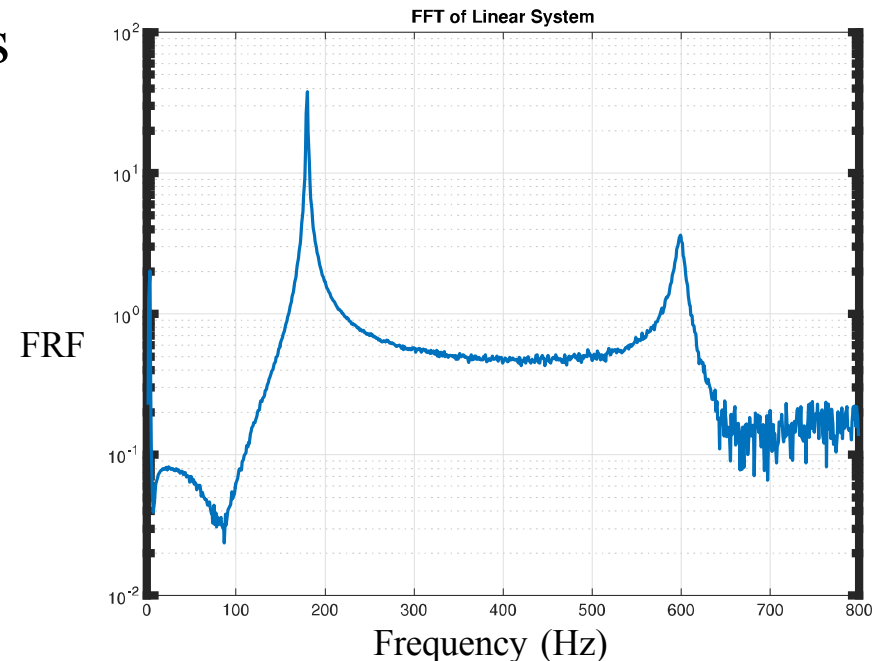
- Reduced number of degrees of freedom (d.o.f) is highly desirable for computational efficiency
- ROM is created by Craig-Bampton in Sierra by specifying the interface nodes and number of fixed interface modes
- 4 interface nodes (3 d.o.f each), 13 fixed interface modes = **25 d.o.f total**



- Issues arise with direct RBE3 connections, solved with stiff springs

## Linear Analysis of BR Beam

- Using Romulis, an impact hammer simulation is performed on the beam
- Linear interface elements are used to verify ROM is performing correctly
- Frequencies of first two modes confirmed – 178 Hz, 599 Hz



## Nonlinear Joint Model: 4-Parameter Iwan

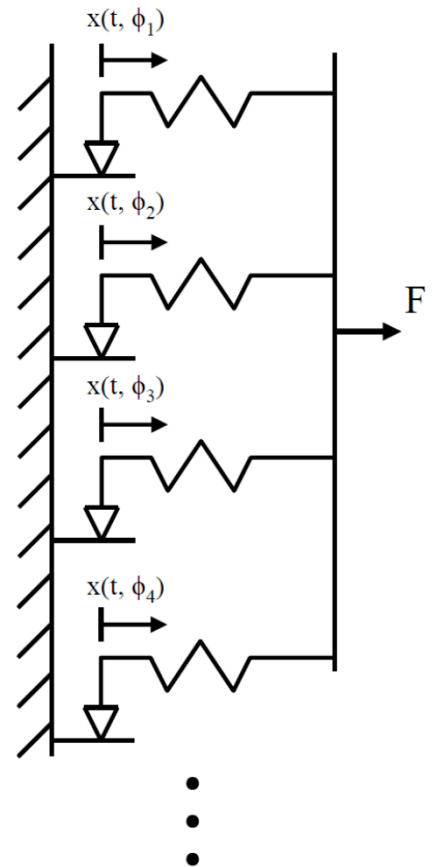
- Phenomenological model with likely epistemic uncertainty
- The Iwan is a distribution of Jenkins elements, allowing microslip/macroslip across an interface
- Depends on 4 parameters:

$F_S$  : force at which macroslip occurs

$K_T$  : tangential stiffness at no slip

$\chi$  : strength of singularity at 0 (dimensionless)

$\beta$  : ratio of joint stiffness (dimensionless)

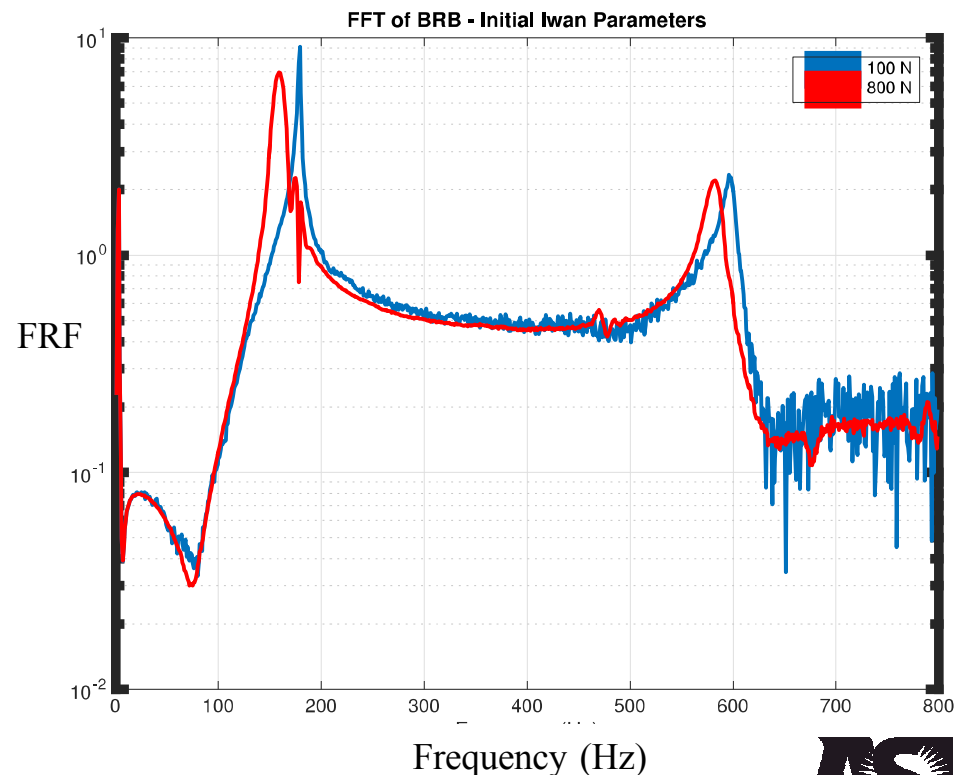


## Identification of Iwan Parameters

- Attempting to match the mode 1 frequency shift between 100 N and 800 N impacts, Iwan parameters were approximated as:

$$F_S = 200, \quad K_T = 1e10, \quad \chi = -0.5, \quad \text{and} \quad \beta = 0.5$$

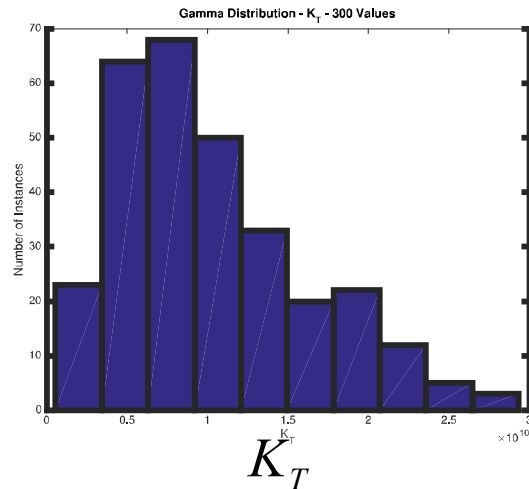
Mode	Frequency- 100 N	Frequency- 800 N
1	178 Hz	160 Hz



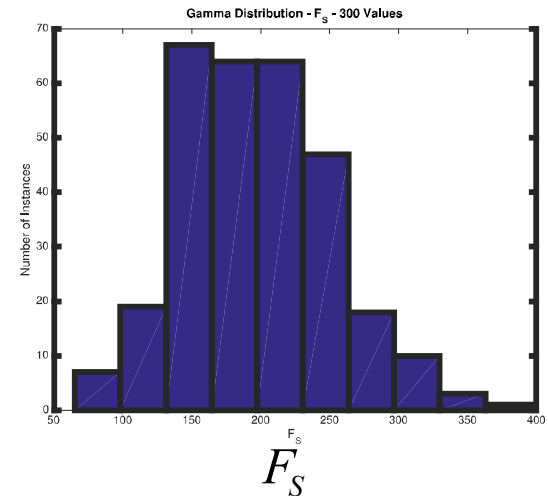
## Introducing Uncertainty into Iwan Model

- Gamma distributions created to maximize entropy

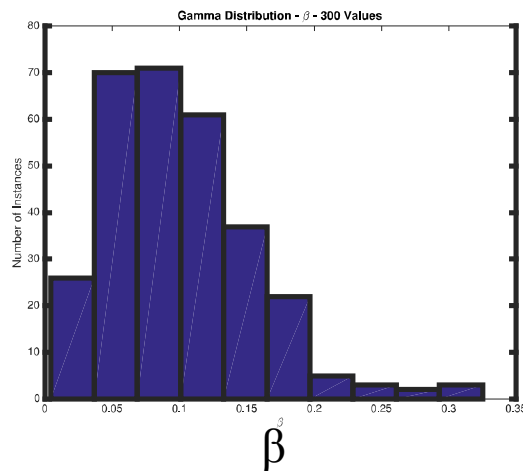
$$K_T = \bar{K}_T X_1$$



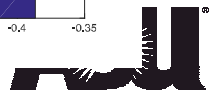
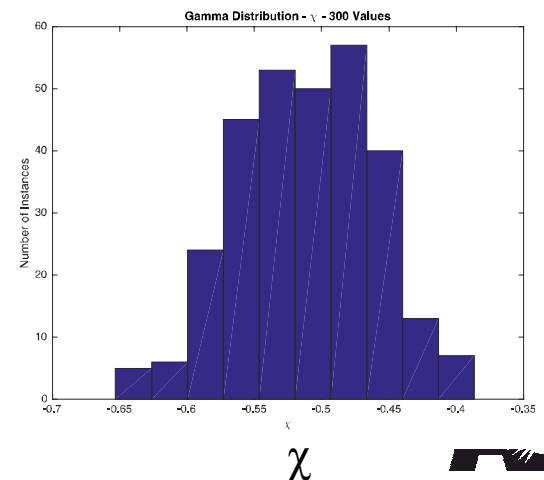
$$F_S = \bar{F}_S X_2$$



$$\beta = \bar{\beta} X_3$$

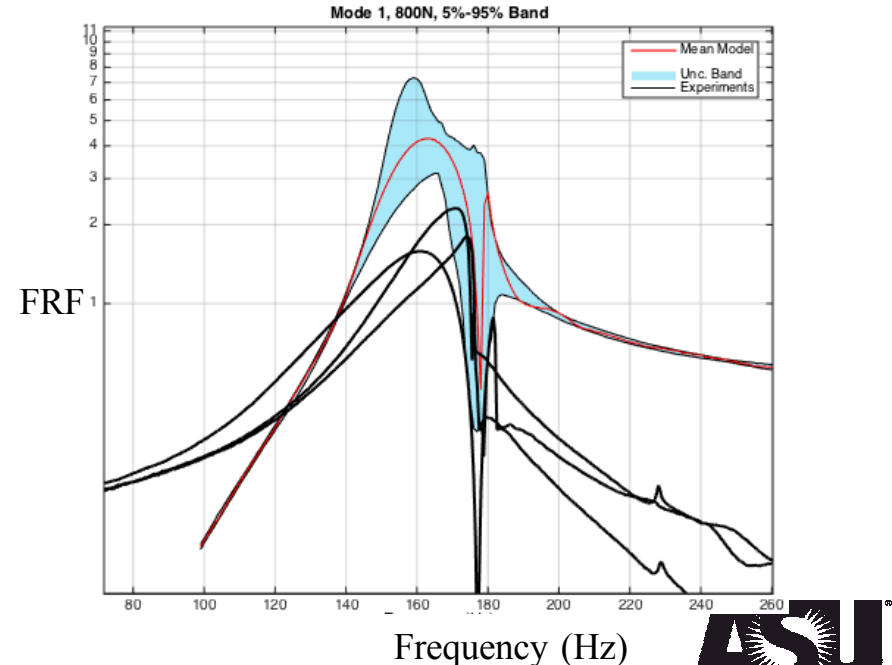
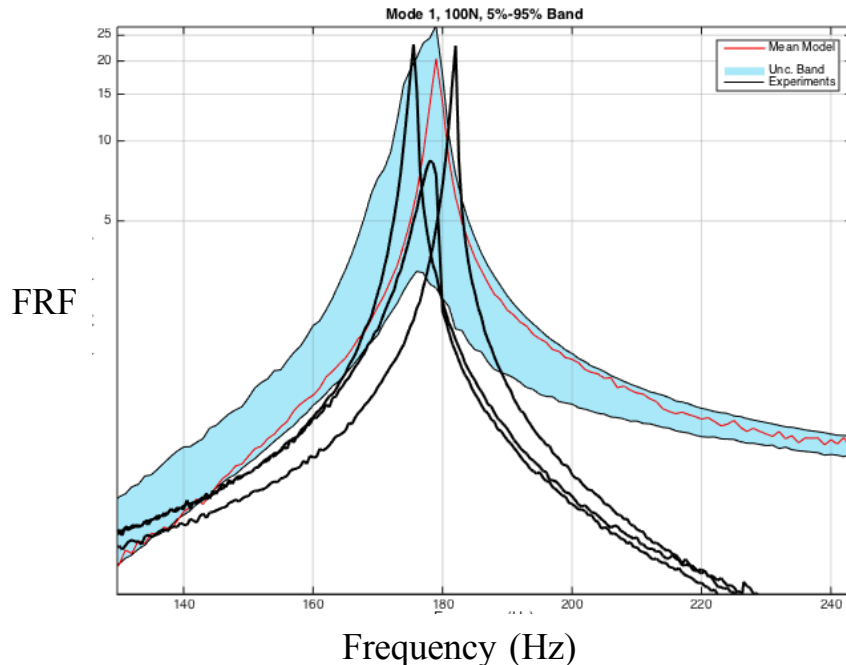


$$\chi = -\frac{1}{1 + \bar{\chi} X_4}$$



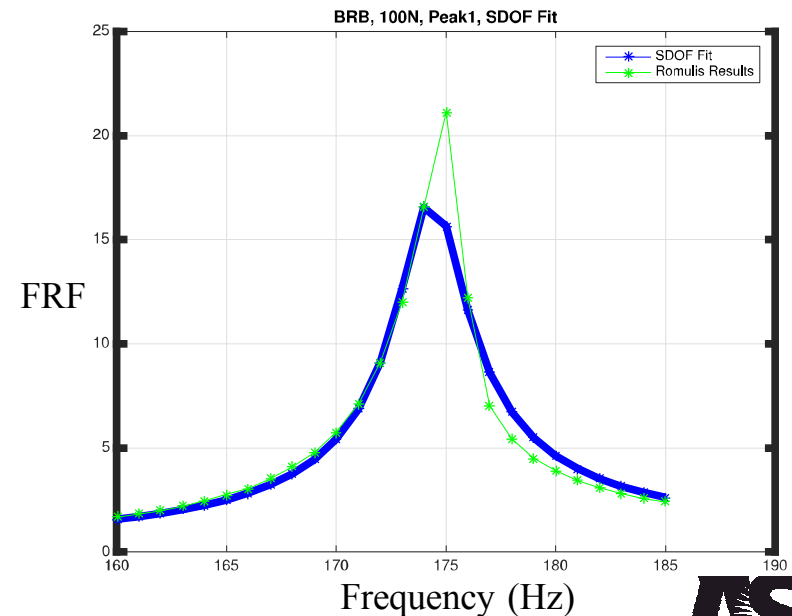
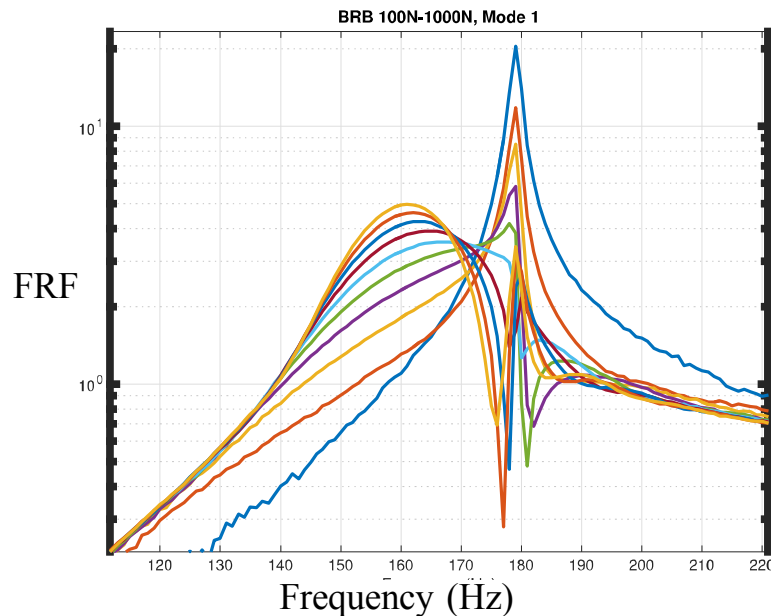
## Uncertainty Propagation – BR Beam

- Monte Carlo – 300 tests, each with different Iwan parameters
- Uncertainty Bands – 5<sup>th</sup> and 95<sup>th</sup> percentile
- 100 N bands fit somewhat nicely, but 800 N bands do not capture the same amount of dissipation as experiments



## Linear Fit of BR Beam

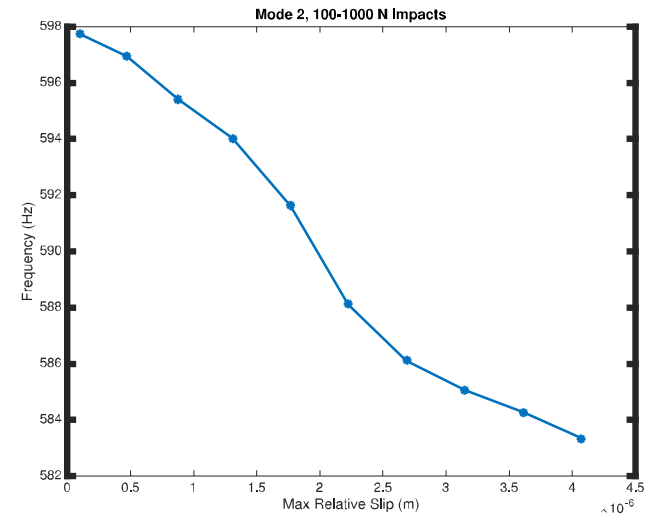
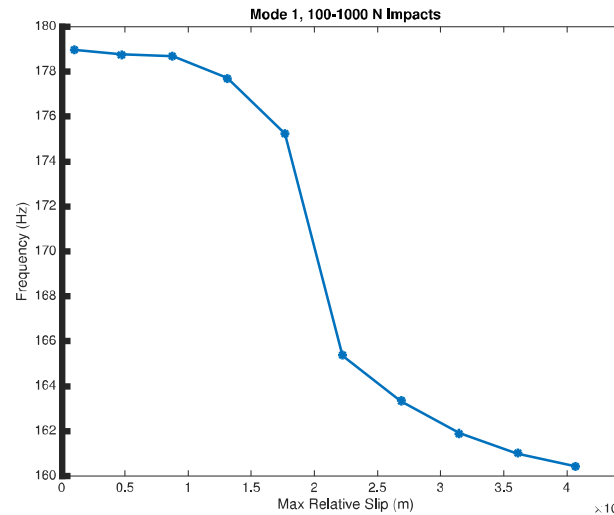
- Mean model analyzed for impacts of 100 to 1000 N to observe relationships between impact level, frequency, and damping ratio
- Fit frequency and damping ratio to linear, single degree of freedom (SDOF) system



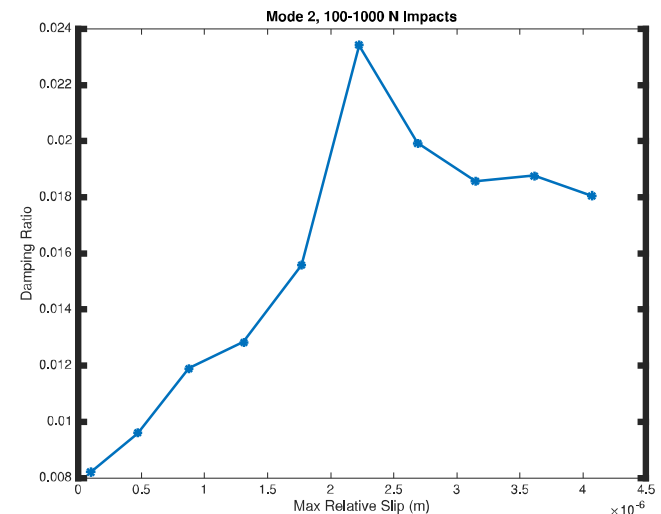
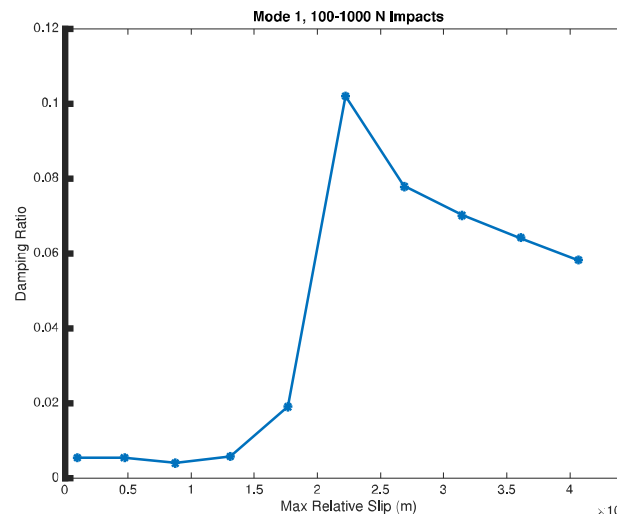


## Linear Correlations

- Frequency (Hz)  
vs.  
Relative Slip (m)

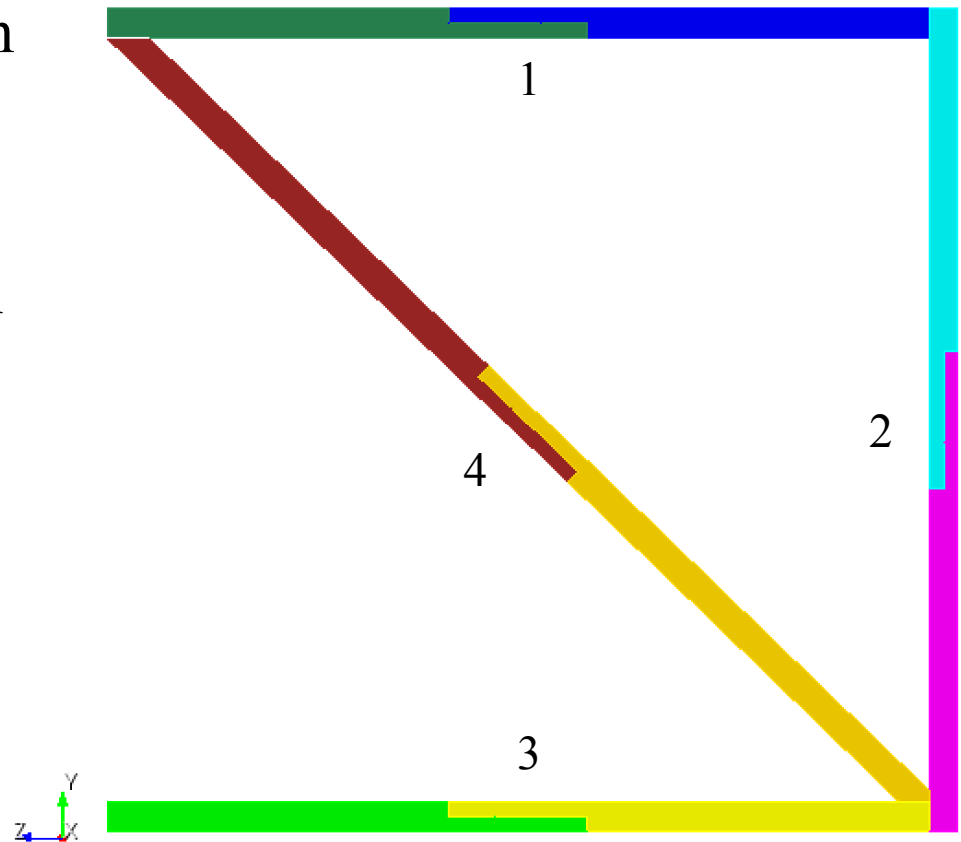


- Damping Ratio  
vs.  
Relative Slip (m)



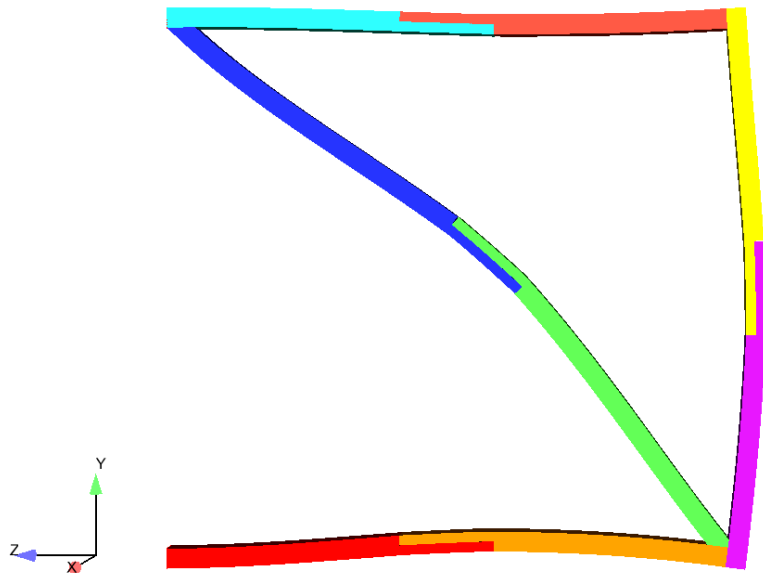
## Multi-Joint Frame

- Introduce multiple joints in one structure
- Designed frame with 4 BR beams
- Same interface modeling techniques as BR beam

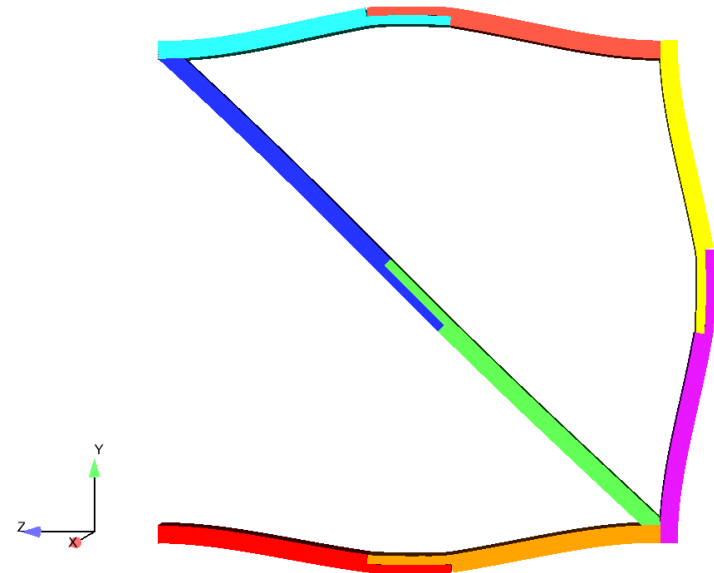


## Modal Joint Activity

- Only interested in in-plane modes
- Asymmetry allows certain modes to have some joints active and some joints not as active



Mode 1

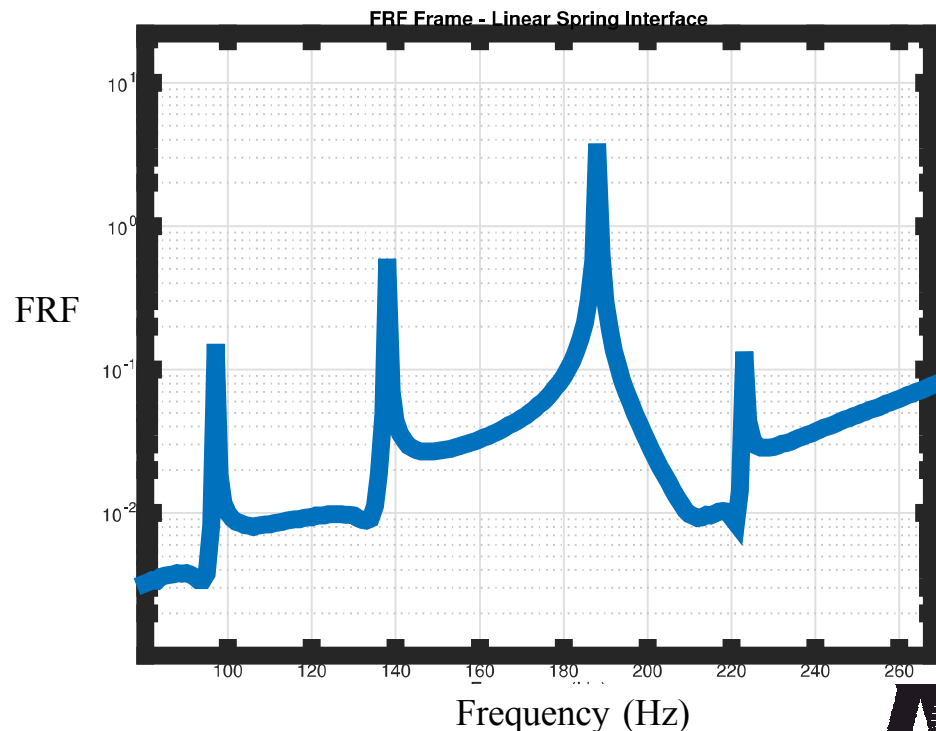


Mode 4

## Impact Hammer Simulation

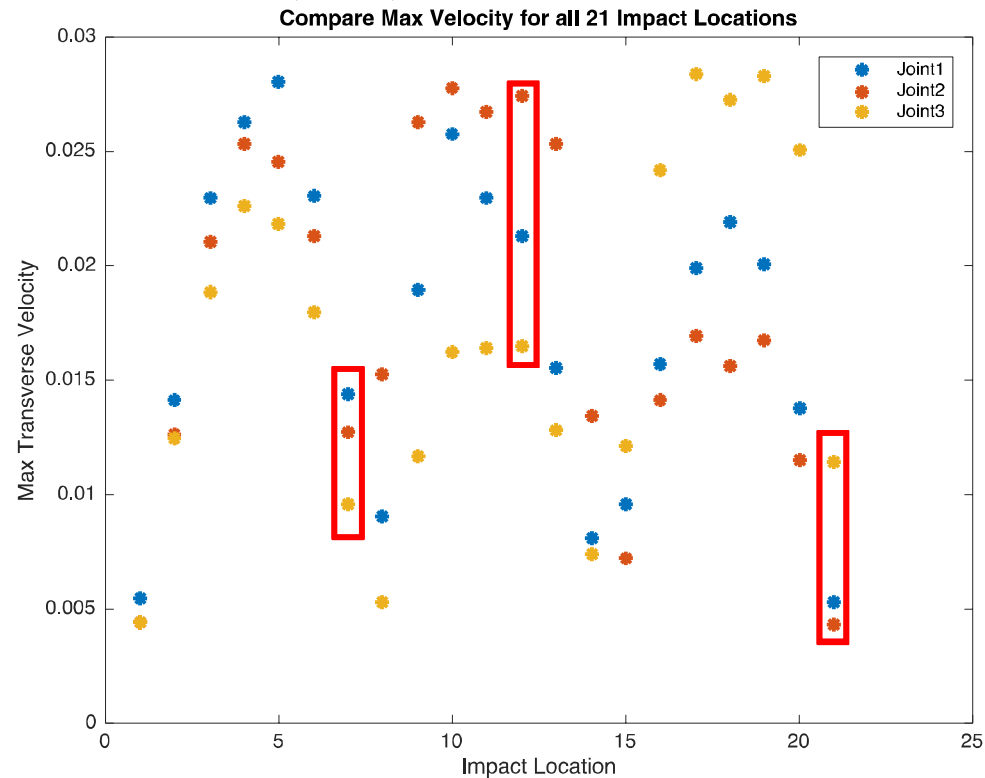
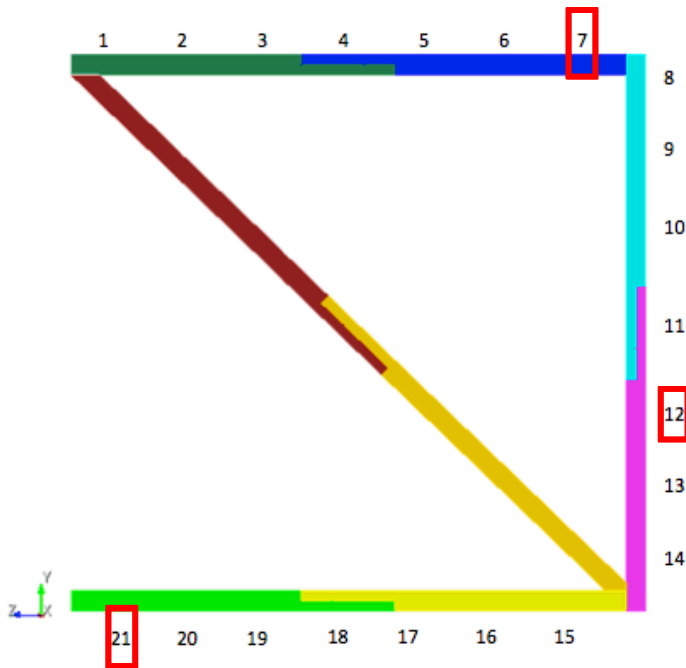
- Create Craig-Bampton ROM with same methods as BR beam
- Test setup created for Romulis to simulate impact hammer test
- ROM is verified with linear interface models first

Mode	Frequency
1	97.19 Hz
2	138.12 Hz
3	188.27 Hz
4	223.21 Hz



## Impact Location Determination

- 21 impacts performed on linear system – 100N
- Max velocity at joints 1, 2, and 3 recorded
- Three interesting cases chosen – 7, 12, 21



## Linear Correlation to BR Beam

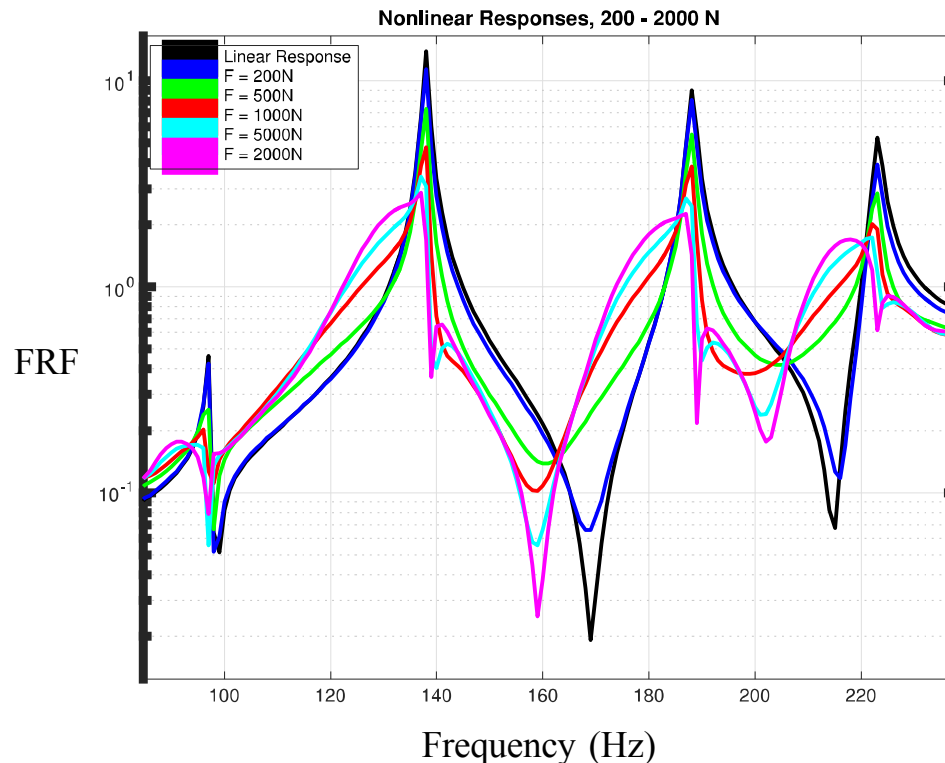
- Max relative displacements between interface nodes are recorded for 100 N and compared to the same metric of the single beam

100 N Impact	Max Relative Disp. (m)	Ratio to BR Beam
BR Beam	3.660e-8	1.00
Frame – Location 7	1.616e-8	0.4415
Frame – Location 12	2.520e-8	0.6885
Frame – Location 21	1.230e-8	0.3361

Impact Location	100 N Equiv.	200 N Equiv.	300 N Equiv.	400 N Equiv.
7	225	450	675	900
12	150	300	450	600
21	300	600	900	1200

## Nonlinear Interface Models

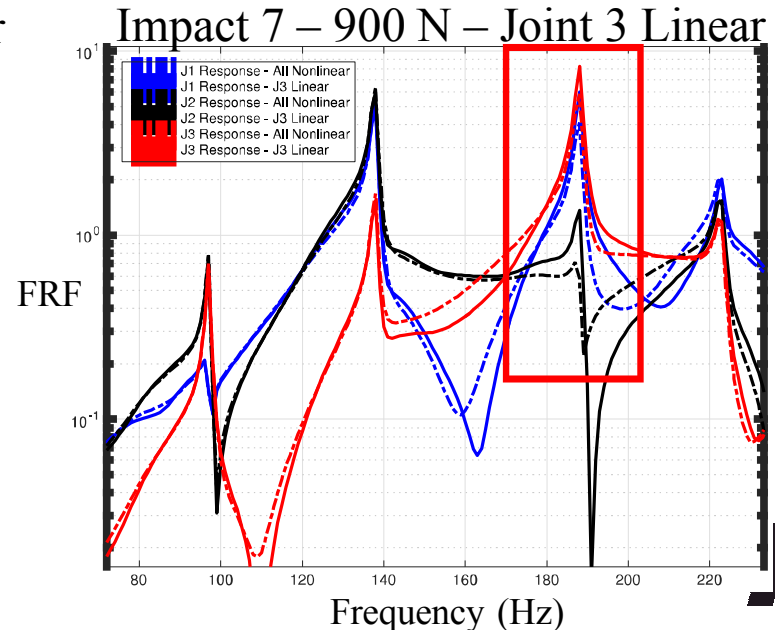
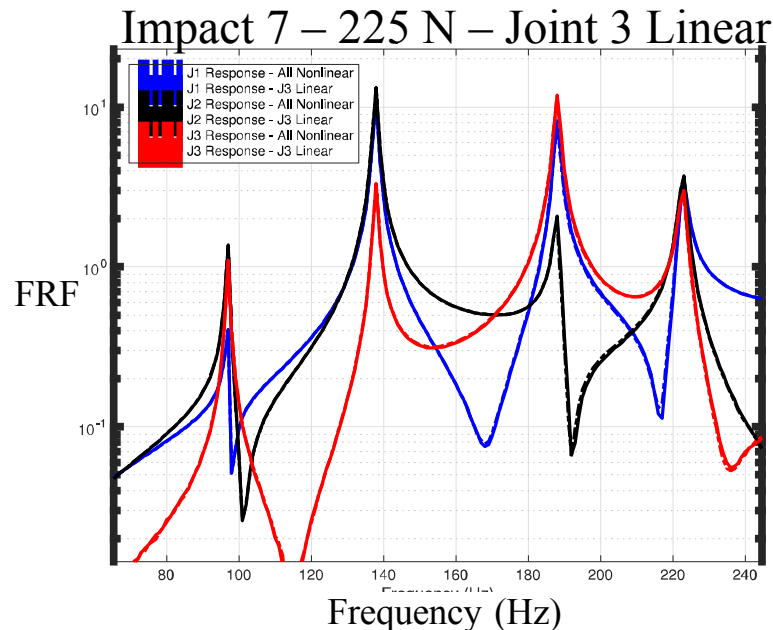
- Iwan elements are introduced at the joints (along beam direction only) with same mean parameters used for the beam
- Several impact simulations are performed at different levels to observe how mean model responds





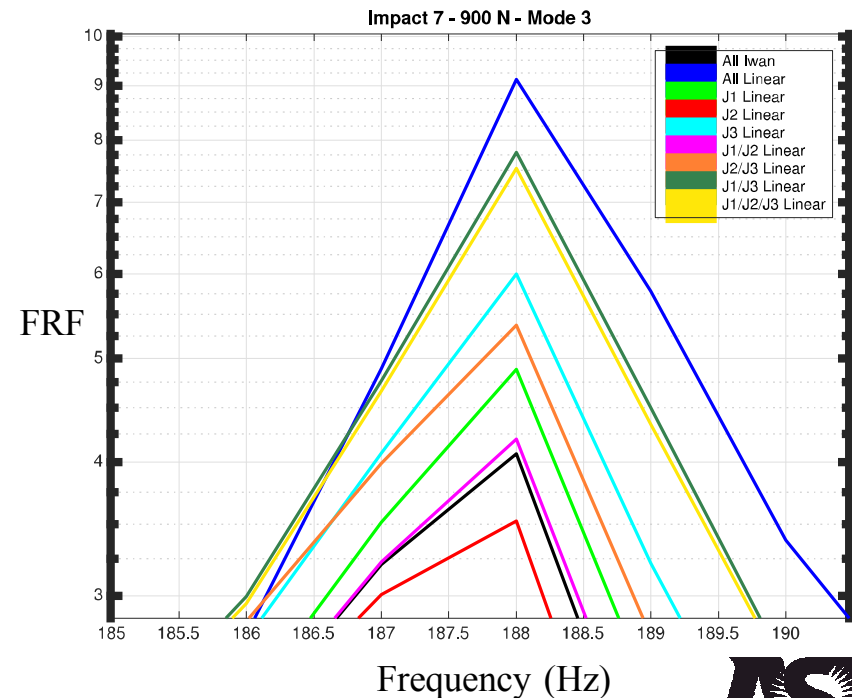
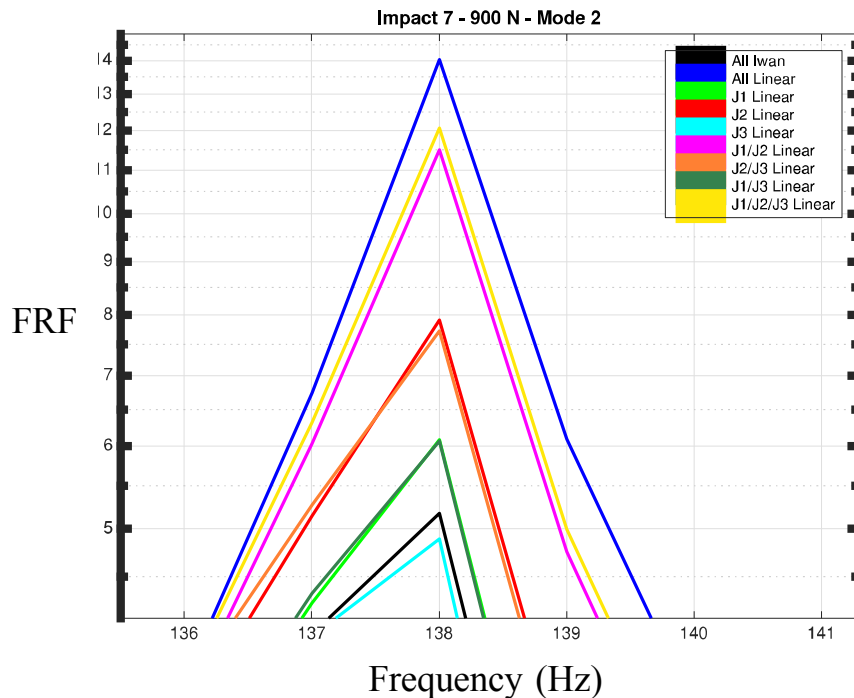
## Approximate Nonlinear Models

- Initial thought is that given a certain impact location, the least active joint (lowest velocity) can be linearized without affecting the dissipation
- At low levels, this is true, but at higher levels when more slip becomes apparent, the approximate models do not capture the correct damping



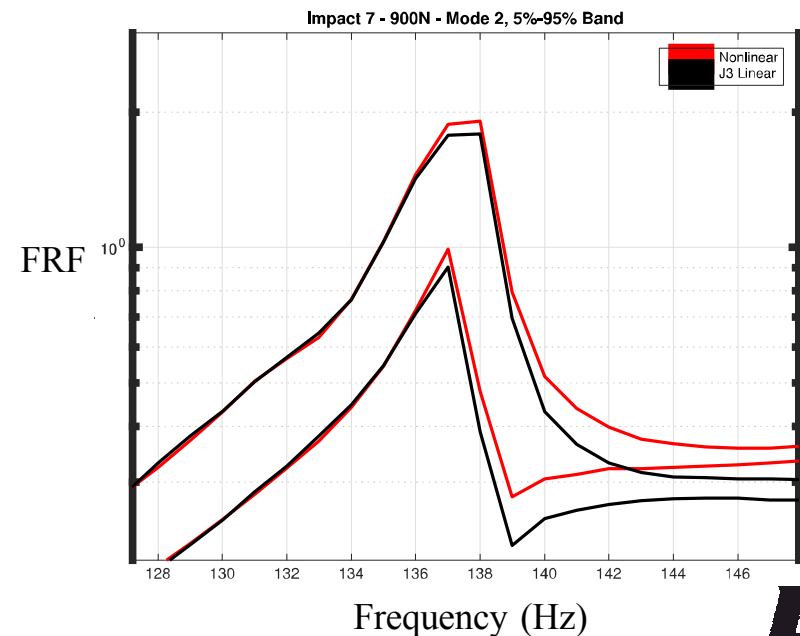
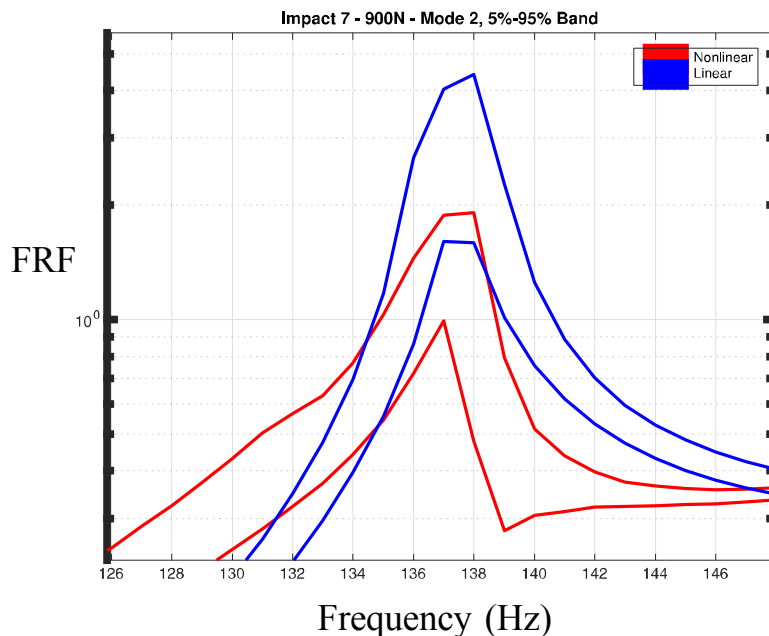
## Approximate Nonlinear Models

- It becomes more apparent, the joint activity is much more dependent on the mode than impact location
- For mode 2, joint 3 is the least active
- For mode 3, joint 2 is the least active



## Introduce Uncertainty into Frame

- Same distributions as previously used with the BR beam
- 300 sample Monte Carlo test is performed
- Simplification of model does not outweigh the variability



## Summary

- Single joint beam was tested, analyzed, and modeled
- Multi-joint frame was designed and analyzed using correlations from the single joint beam
- Comparisons were made for the multi-joint frame when using interface models that are nonlinear, linear, and combinations of both nonlinear and linear
- The computational model adopted for the frame is only required to provide a good estimate of an uncertainty band
- A relaxation of the model complexity (epistemic uncertainty) can be performed as long as it does not affect significantly the band of predictions
- For our model, it is found that the model simplification is often acceptable at lower excitation levels where microslip dominates

## Acknowledgments

- Dr. Mignolet and Dr. Liu – Arizona State University
- Matt Brake – Sandia National Labs
  - Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporations, for the U.S. Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

- Thank you for your attention
  - Questions??