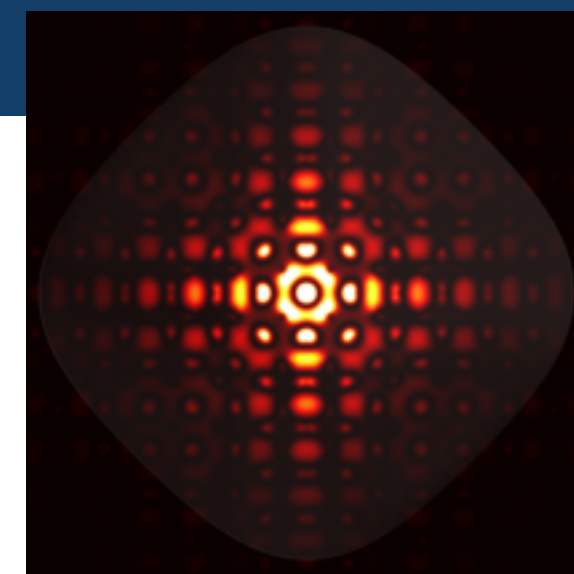
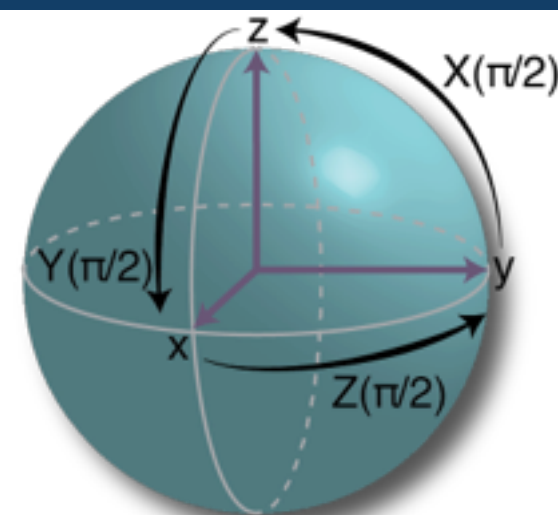
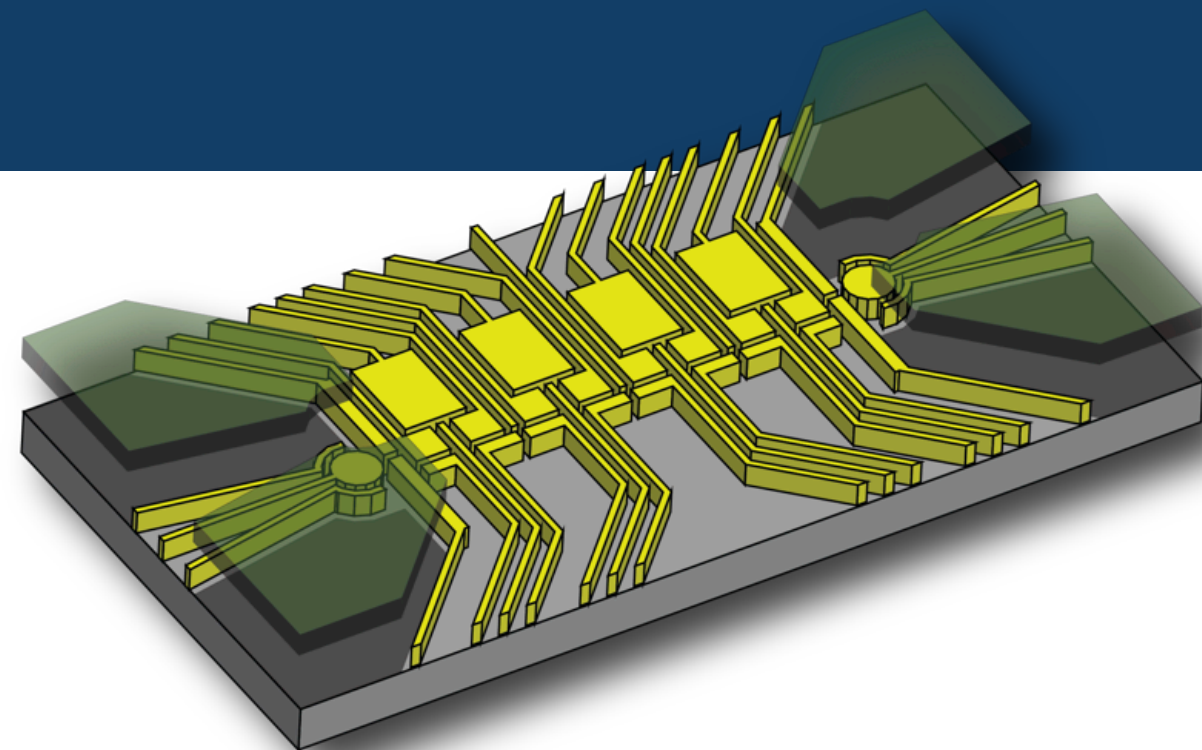


Exceptional service in the national interest



Computational modeling of semiconductor qubits

John King Gamble

Center for Computing Research, Sandia National Laboratories



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Rewind four years...

- PhD from UW in 2013
 - Supervisors: Sue Coppersmith and Mark Friesen
- Moved to Sandia National Labs as a Truman Fellow 2013-2016.
- Permanent staff at Sandia 2016-present.

- Toward the end of my PhD, we started digging into the physical details of Si qubits that had been previously swept under the rug.
- I proposed to extend my graduate work in a Truman Fellowship at Sandia.
- This is the story of what happened over the next three years.

 = WI folks we've "convinced" to come to Sandia :)

Sue Mark F. me

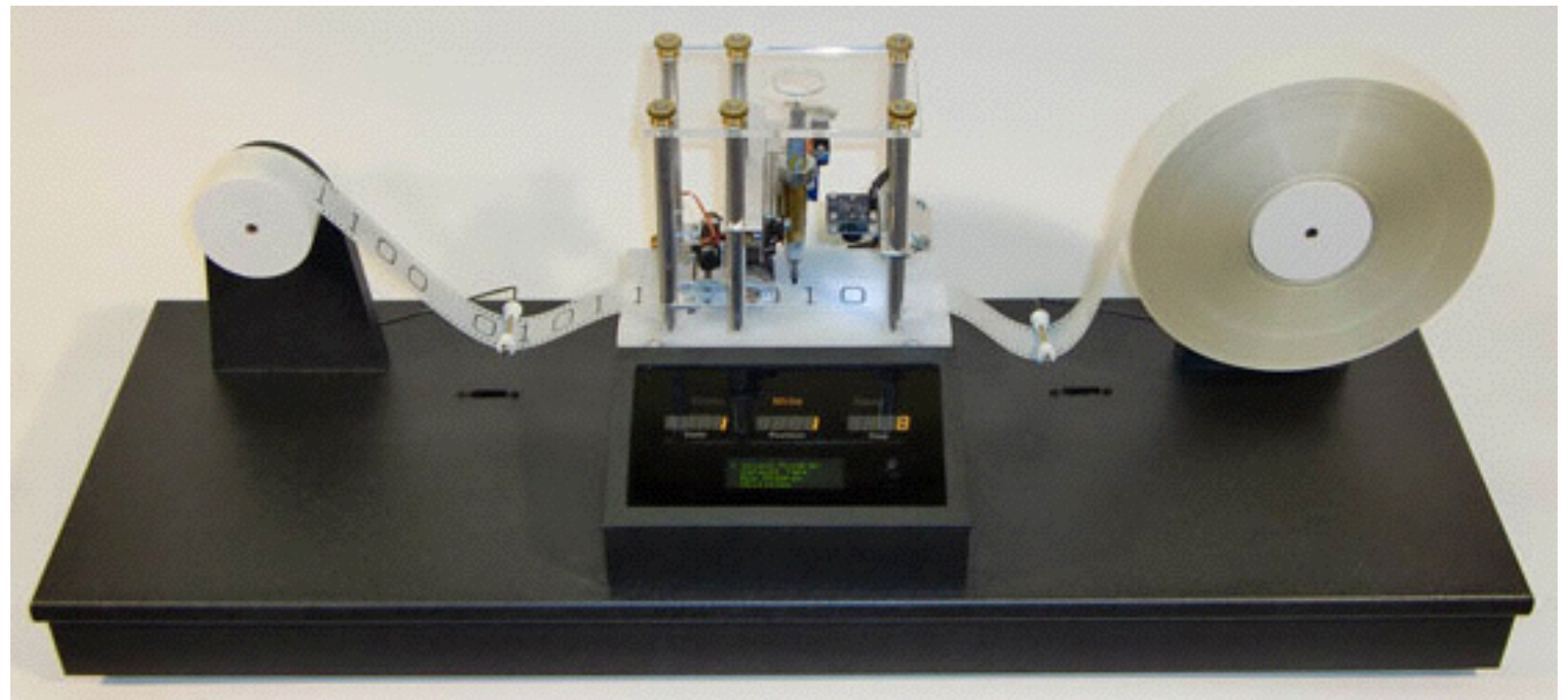


UW Si qubit group circa 2010.

Let's talk about information

- What can you do with information?
 - Acquire
 - Process
 - Transmit
- Where does the information live?
 - Paper
 - Computers
 - Genetic code

- **Information is physical!!**



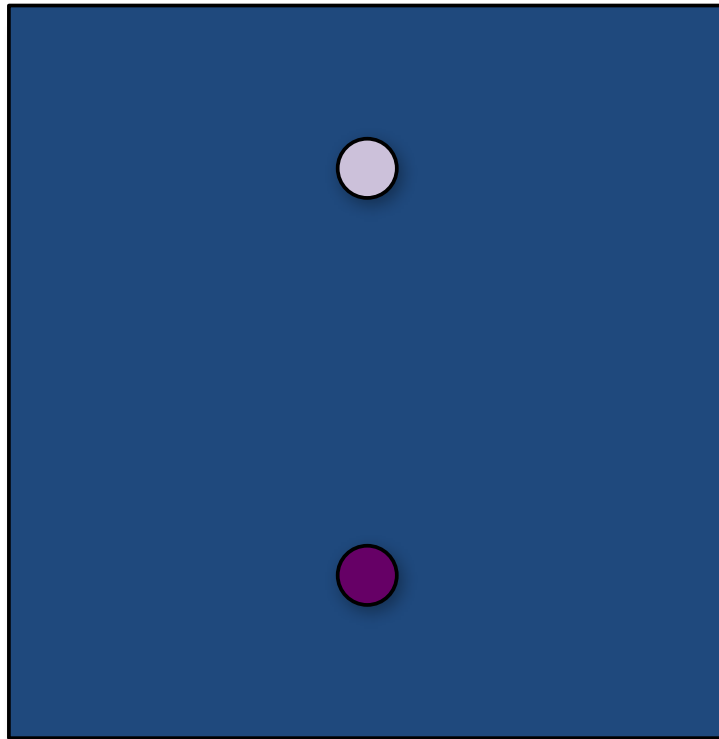
Example: physical Turing machine.

Question: How does my *choice* of physical system in which information is stored affect my ability to *acquire*, *process*, and *transmit* it?

Answer: It can have a profound effect!

How information is stored in a physical system

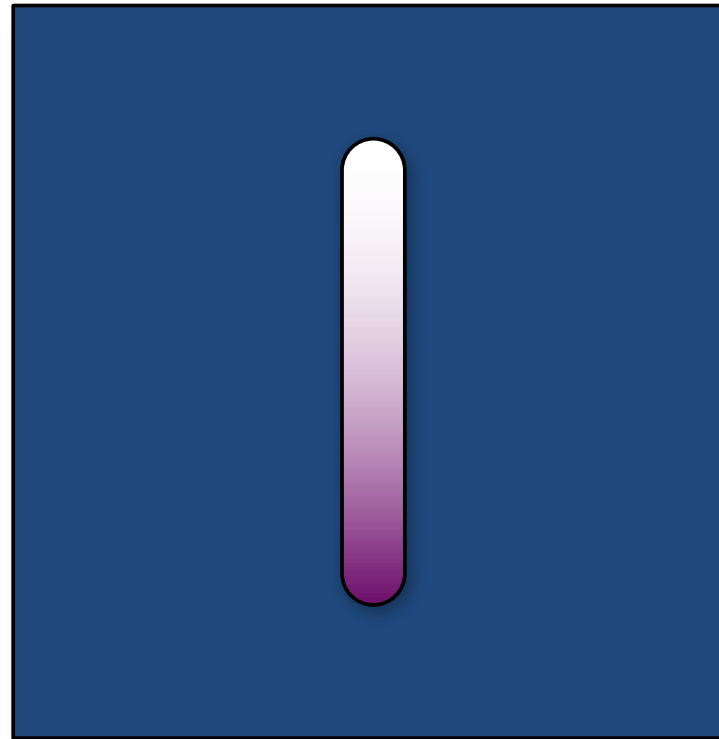
classical, deterministic



Bit:

- “On” or “off.”
- State can be represented by one of two points.

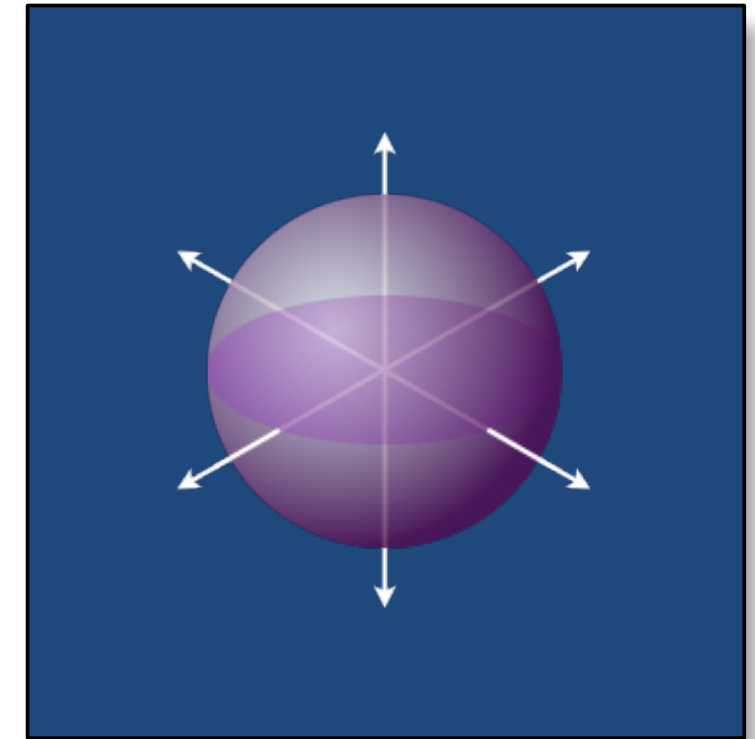
classical, probabilistic



Pbit:

- *Probability* of “on” or “off.”
- Probabilities sum to one.
- State can be represented by a point on a line segment.

quantum



Qubit:

- *Amplitude* of “on” or “off.”
- Probability is square of amplitude.
- Probabilities sum to one.
- State can be represented by a point on sphere.

What would a quantum computer be good for?

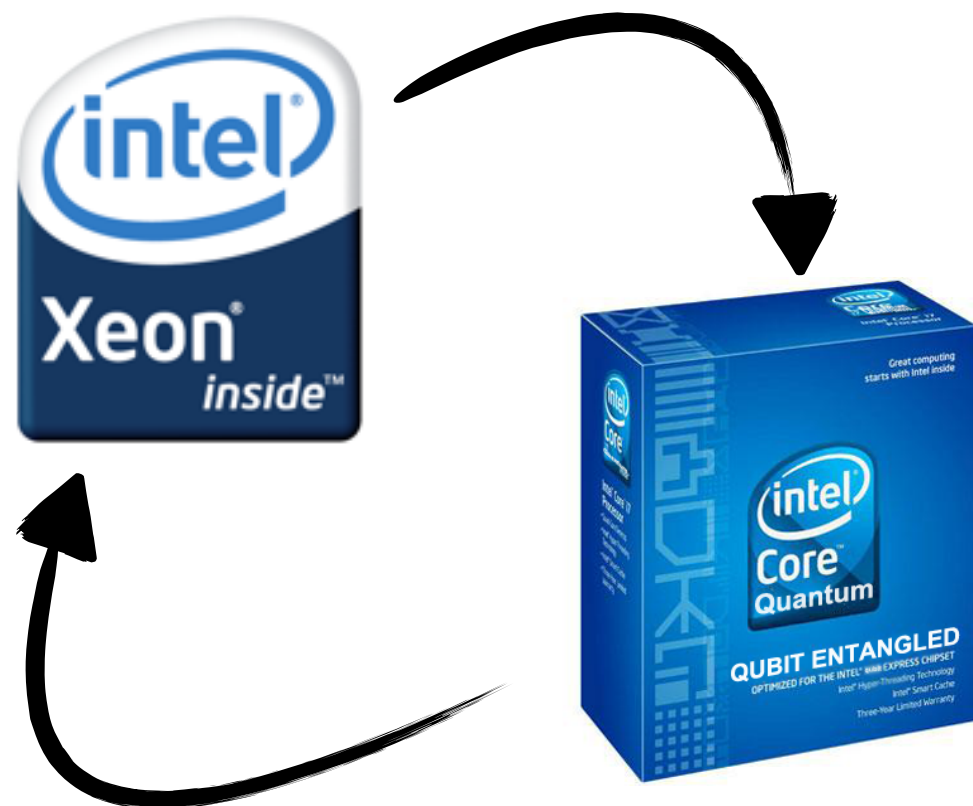


Shor's algorithm: exponential speedup for integer factoring.



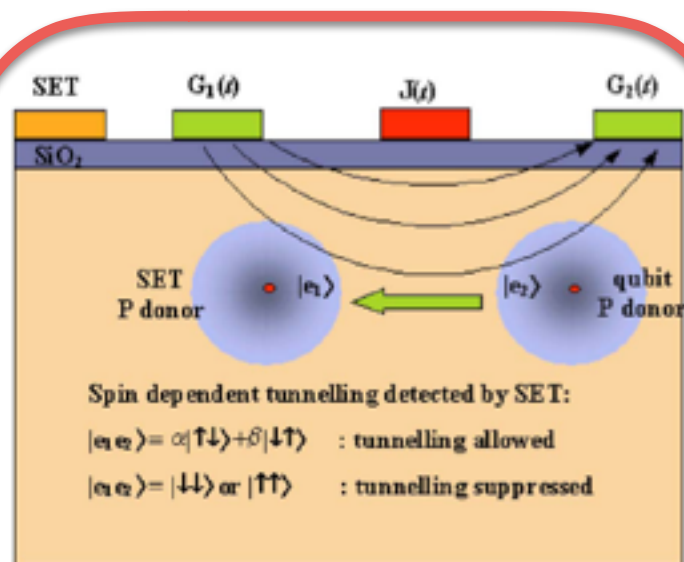
Grover's algorithm: square root speedup for unstructured database search.

Not better for general-purpose computing: quantum co-processor

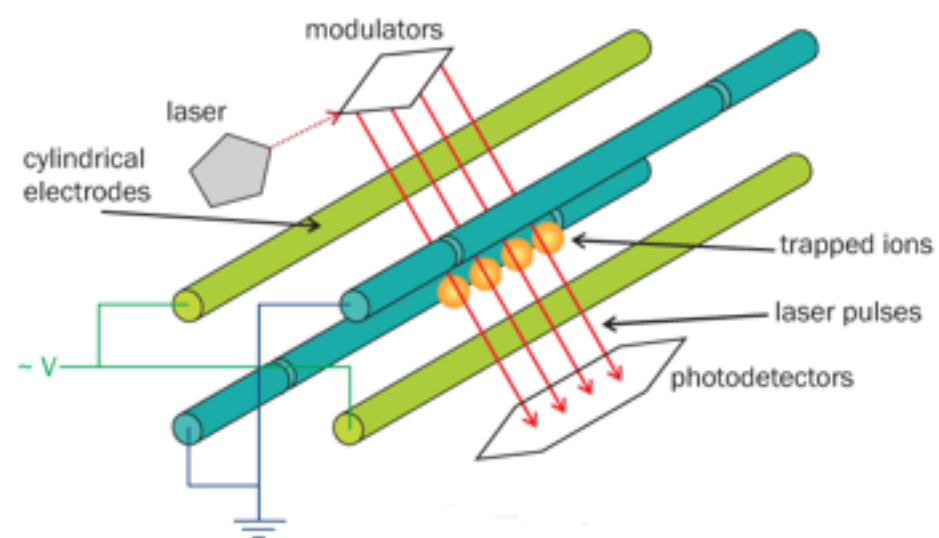


Quantum simulation (Feynman): Using a quantum computer to study physical systems directly (quantum chemistry, particle physics, condensed matter physics, etc.)

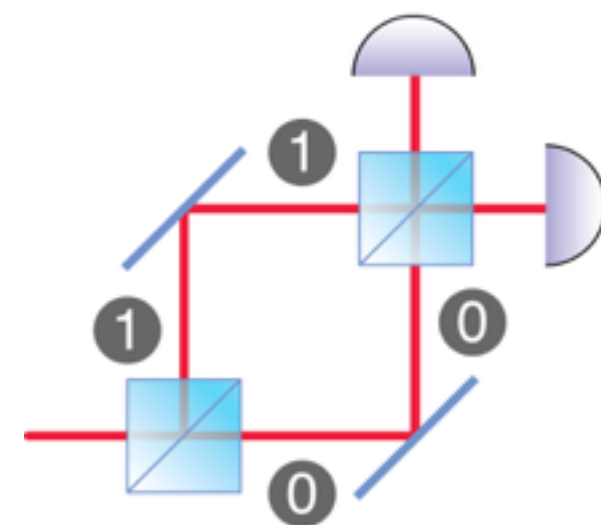
Examples of qubits



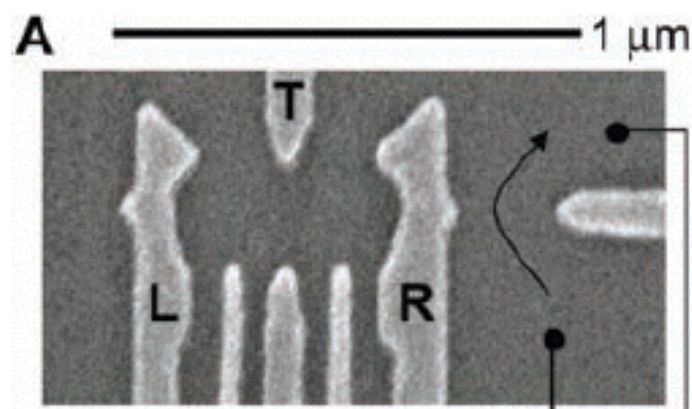
Semiconductor Donors



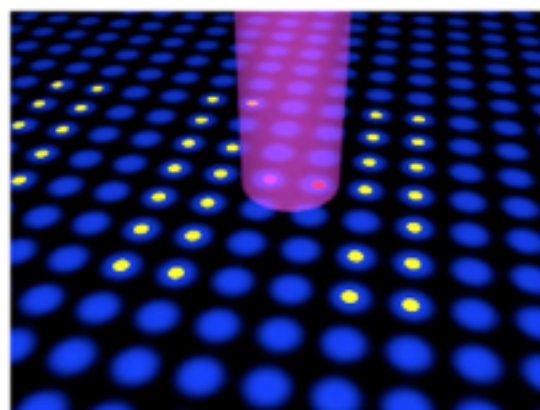
Trapped Ions



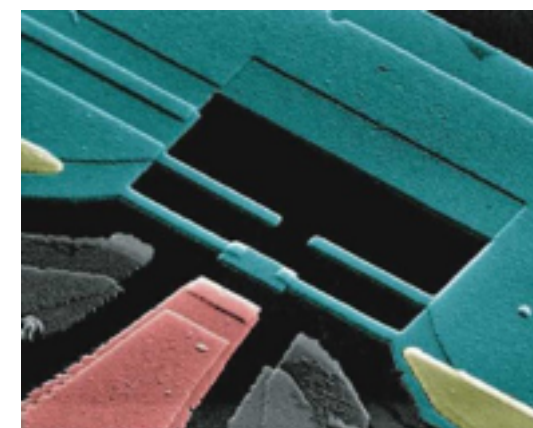
Optical



Semiconductor Dots



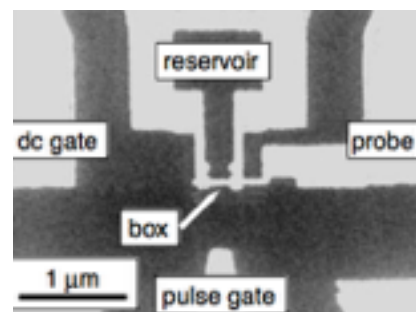
Trapped Atoms



Superconducting

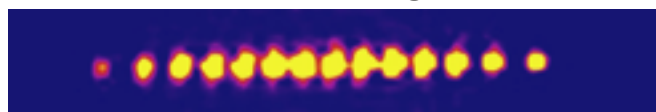
Qubit timeline (not comprehensive)

cooper-pair box



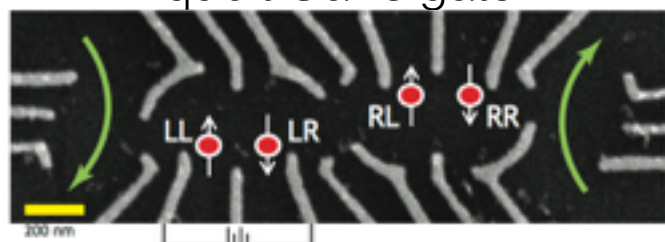
Y. Nakamura, *et al.*
Nature (1999).

14-qubit entanglement



T. Monz, *et al.* PRL (2011).

2-qubit GaAs gate



K. Norwak, *et al.* Science (2011).

R. Brunner, *et al.* PRL (2011).

M. Shulman, *et al.* Science (2012).

5-qubit web platform



IBM (2016).

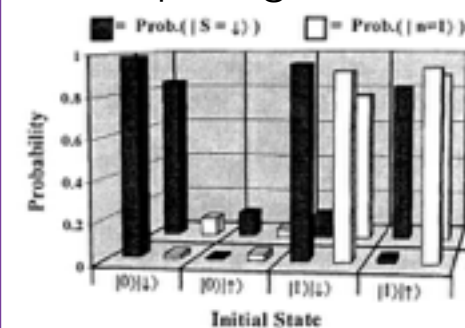
semiconductors
superconductors
trapped ions

1990

2000

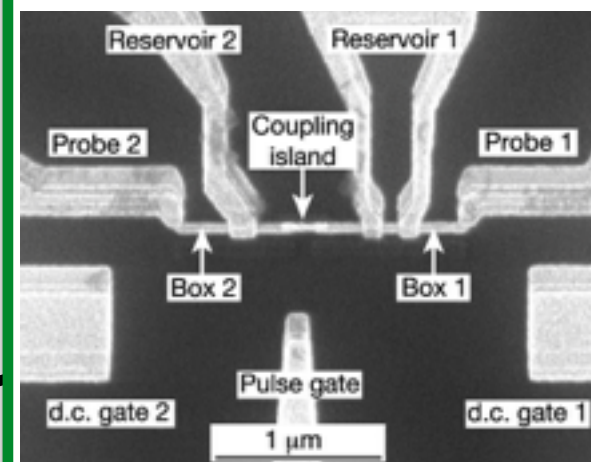
2010

2-qubit gates



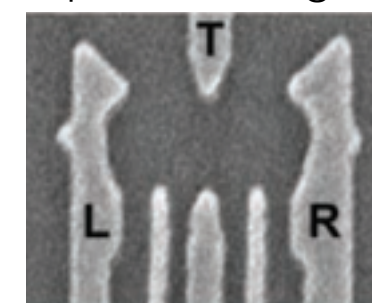
C. Monroe, *et al.*
PRL (1995).

2-qubit gates



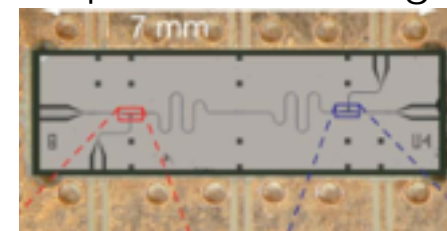
Y. Pashkin, *et al.*,
Nature (2002).

1-qubit GaAs gates



J. Petta, *et al.*
Science (2005)

two-qubit transmon gates



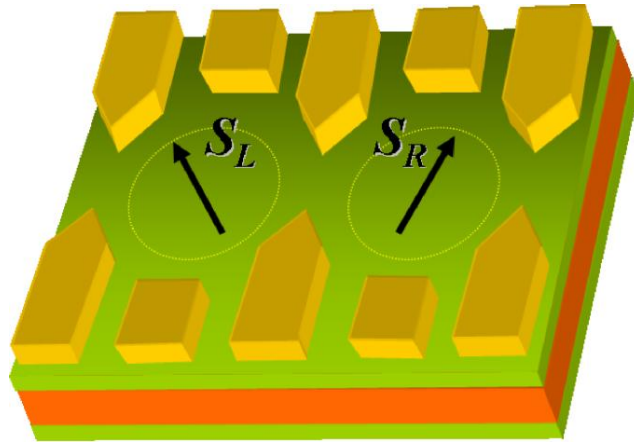
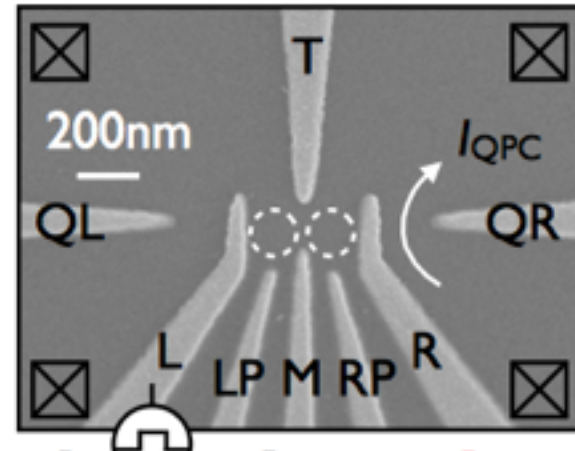
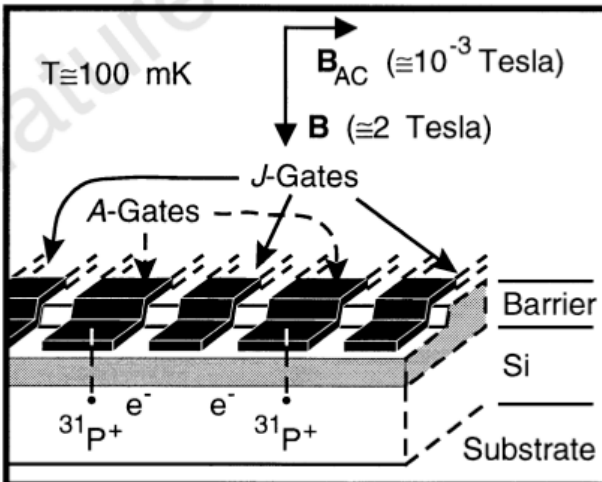
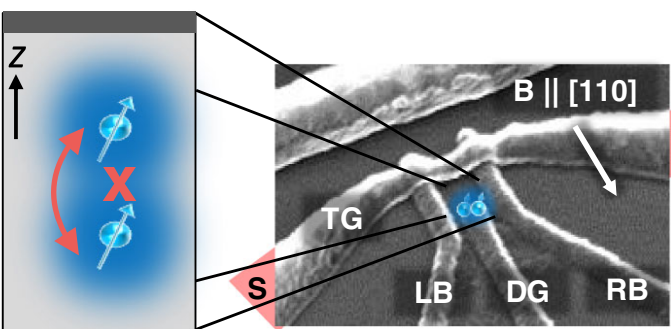
L. DiCarlo, *et al.*
Nature (2009).

2-qubit silicon gate



M. Veldhorst, *et al.*
Nature (2015).

Qubits in semiconductors: “artificial atoms”

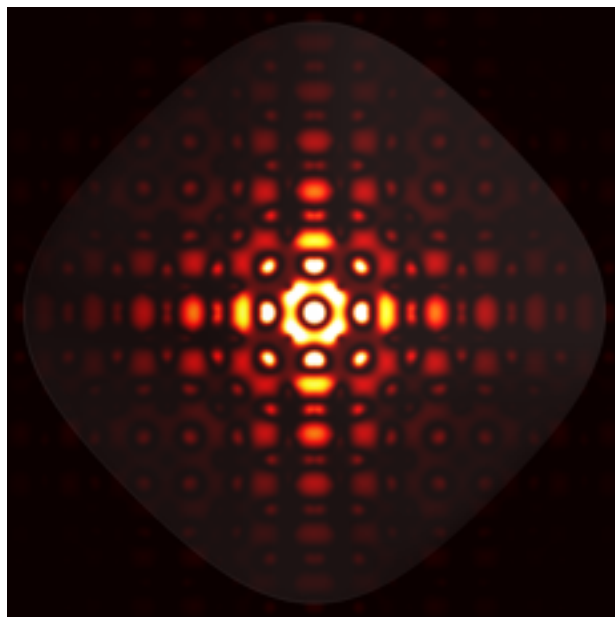
| | Theoretical proposal | Example realization | Remarks |
|----------------------------|--|---|---|
| Electrostatic Quantum dots |  <p>Loss & DiVencenzo, PRA (1998).</p> |  <p>[Kim, et al. Nature 511, 70 (2014)]</p> | <ul style="list-style-type: none"> Extremely flexible due to user-defined nanostructure. Can achieve long-time coherence through materials engineering. Risk: every dot is different. |
| Donor impurities |  <p>Bruce Kane, Nature (1998).</p> |  <p>[Dehollain, et al. PRL 112, 236801 (2014)]</p> | <ul style="list-style-type: none"> Two qubits for the price of one (nuclear spin + bound electron spin)! Nuclear spins have the highest fidelity of any known solid state qubit - every donor is (about) the same! Risk: every donor-donor <i>coupling</i> is different. |

Question: Donor qubits seem ideal from the standpoint of single-qubit operations, but can they be coupled reliably?

Answer: Not directly, but perhaps via dot-donor hybrid qubits. To be able to make predictions about this system, we need a theoretical framework that can handle *both* dots and donors!

Part 1:

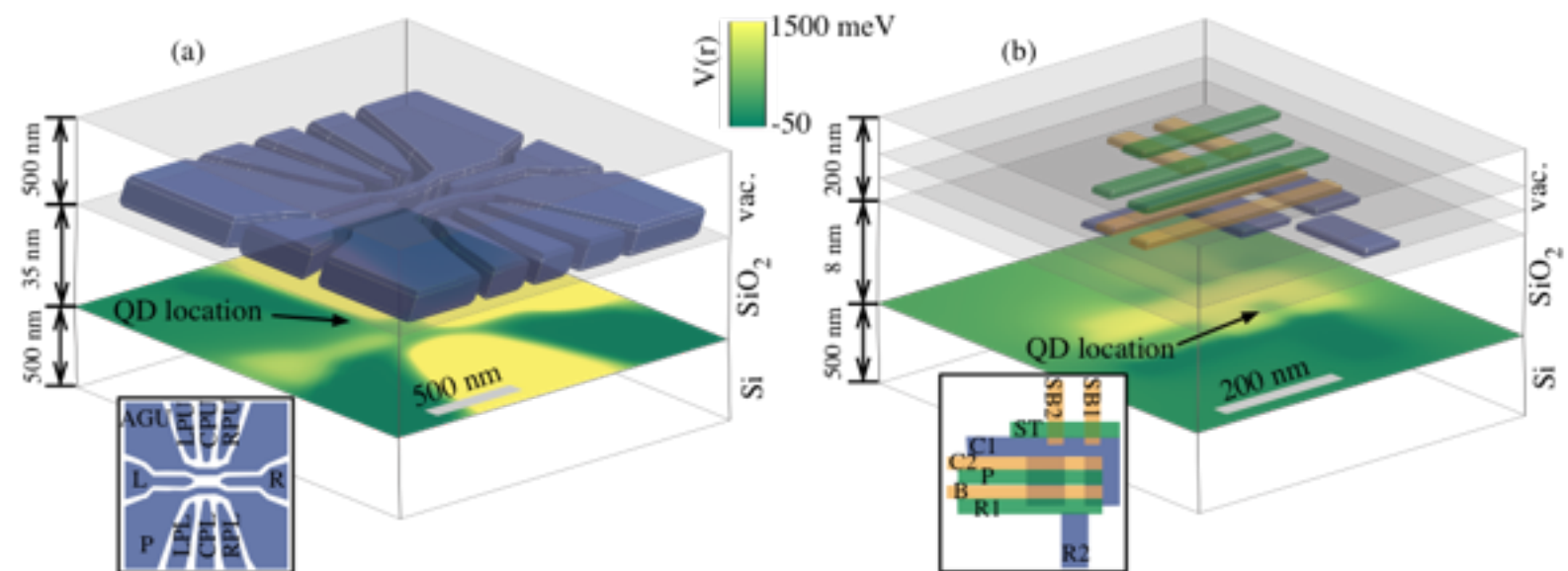
donors are hard to couple directly



[Gamble & Jacobson, *et al.*, PRB **91**, 235318 (2015)]

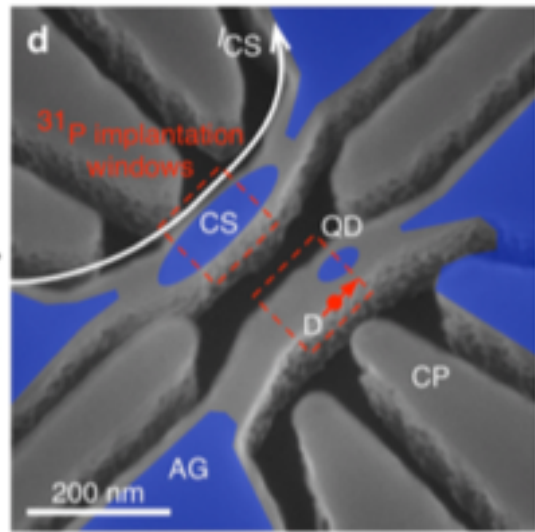
Part 2:

the same theory we used for the donors works well for dots



[Gamble, *et al.*, APL (2016)]

Combining both species of artificial atoms

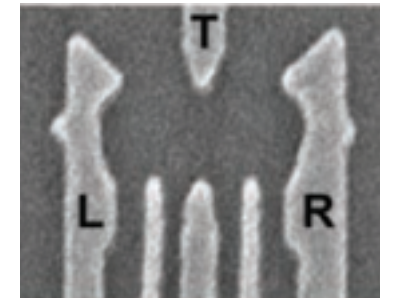


One cell has been demonstrated in experiment...

... and has been reproduced in M. Rudolph, *et al.* IEDM (2016).

Quantum dots can be reliably coupled!

1-qubit GaAs gates

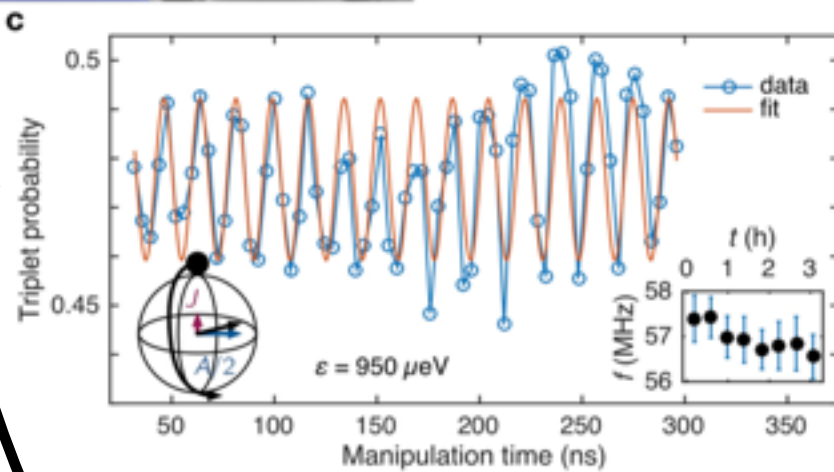
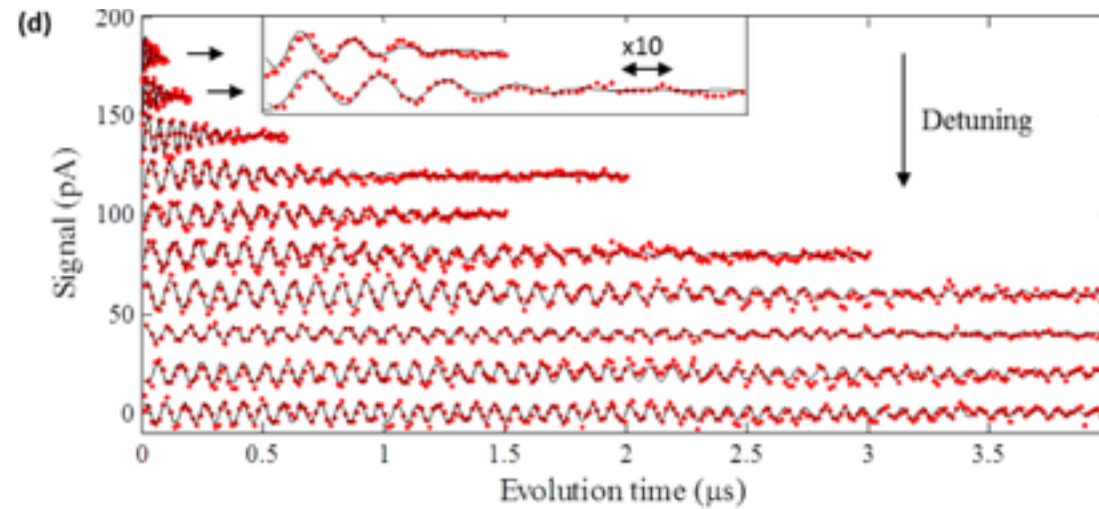


J. Petta, *et al.* Science (2010)

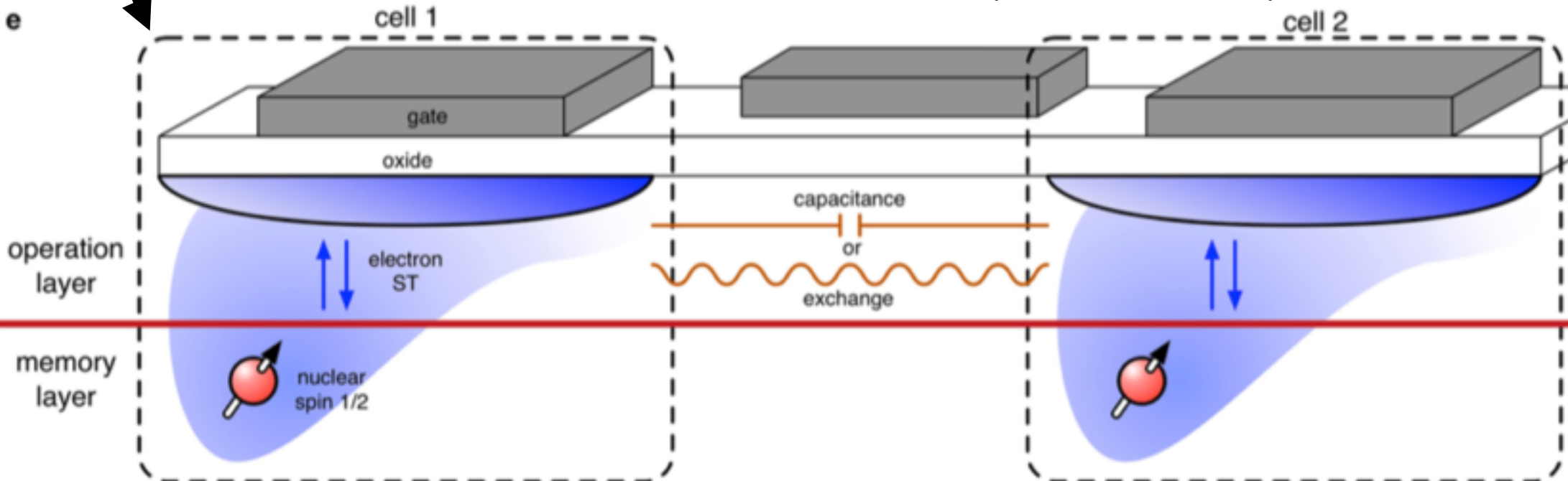
2-qubit silicon gate



M. Veldhorst, *et al.* Nature (2015).



Use *both* impurities and quantum dots!



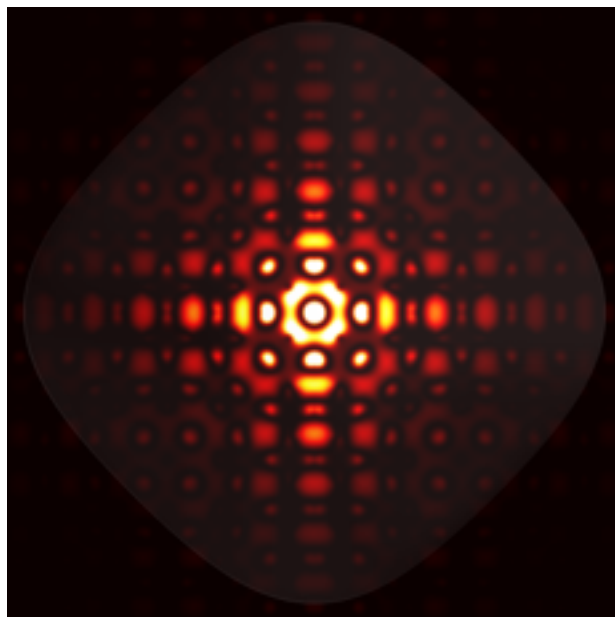
P. Harvey-Collard, *et al.*, arXiv:1512.01606.

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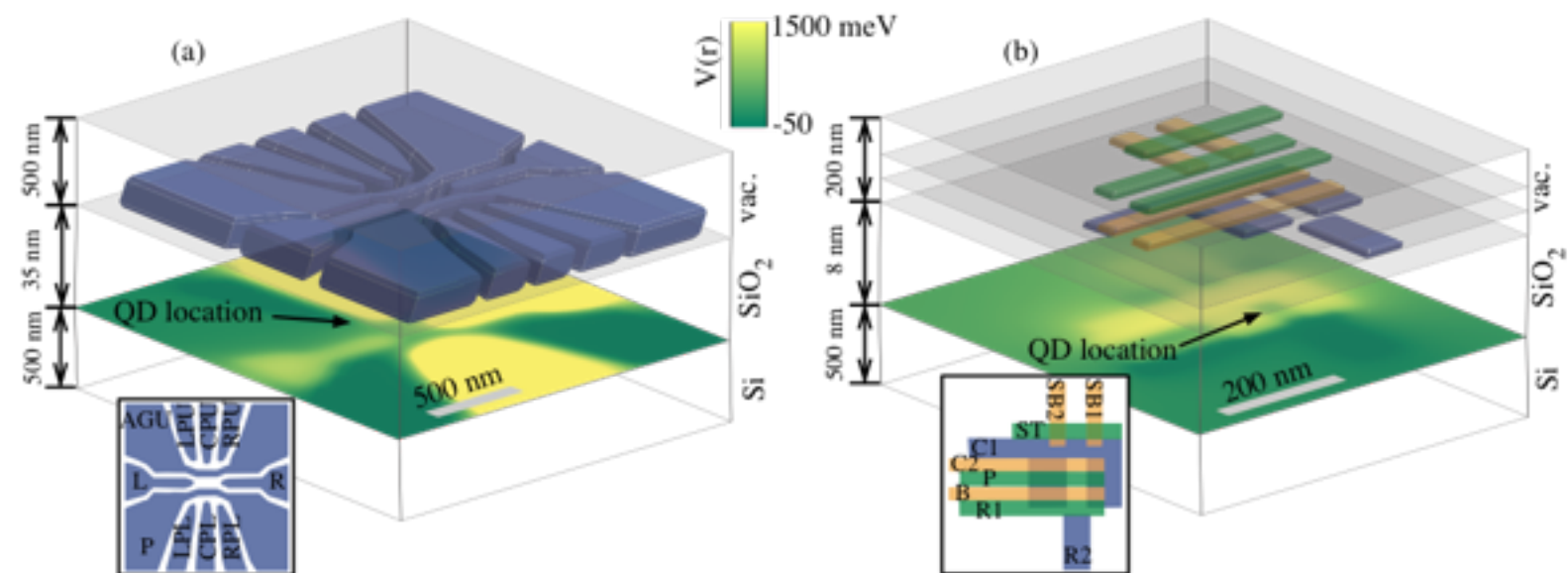
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[Gamble & Jacobson, *et al.*, PRB **91**, 235318 (2015)]

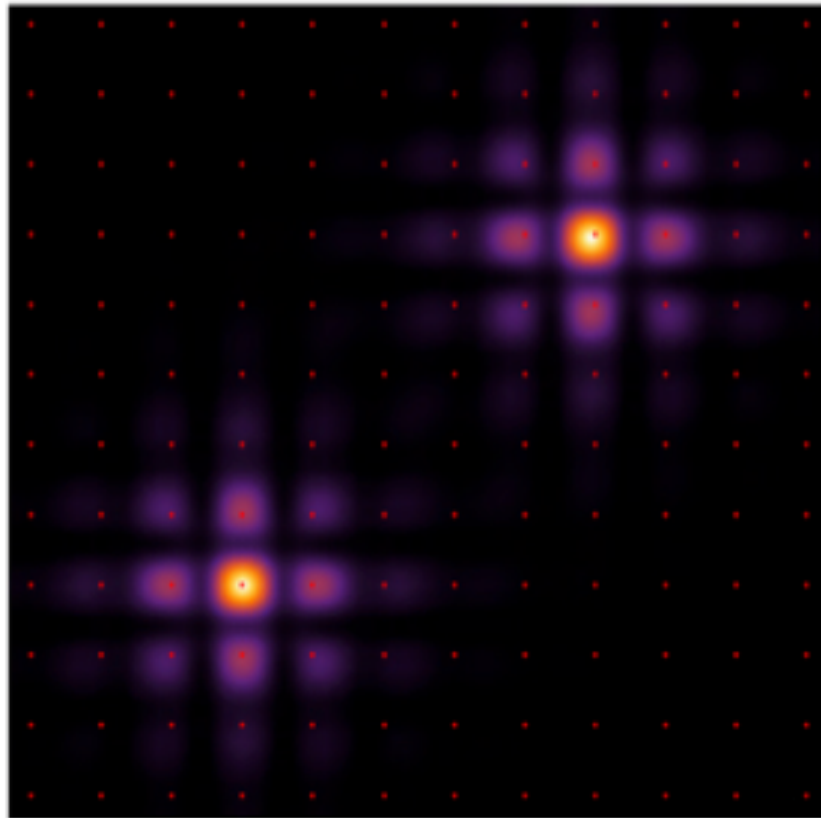
Part 2:

the same theory we used for the donors works well for dots



[Gamble, *et al.*, APL (2016)]

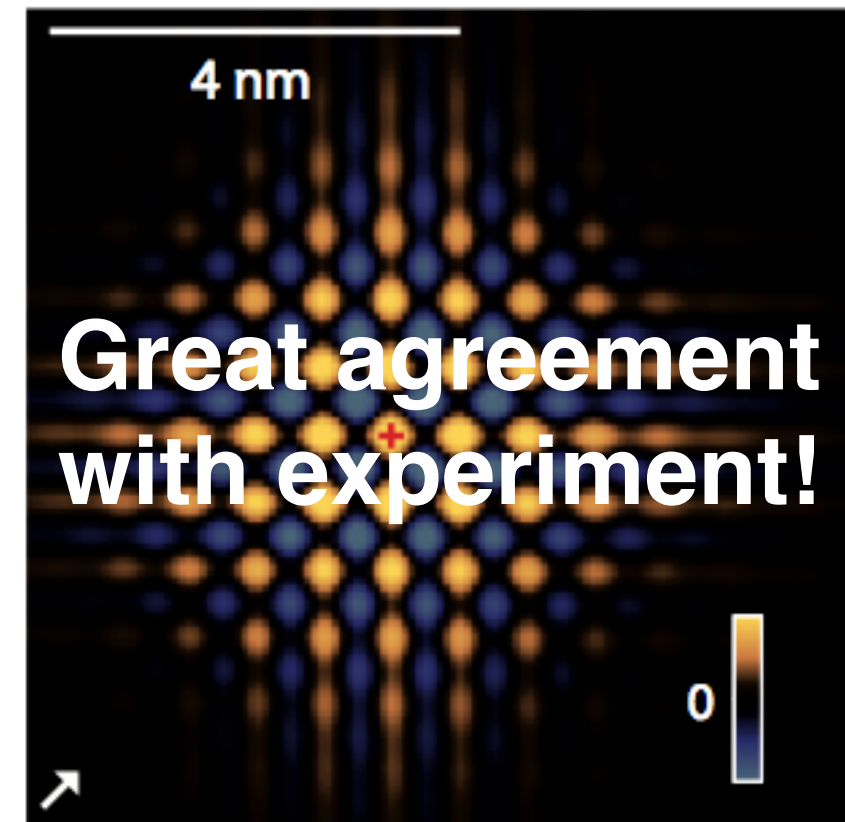
Effective mass



[Pica, *et al.* Phys. Rev. B 89, 235306 (2014)]

Efficient, but only qualitatively reliable. Requires careful application of many approximations.

Atomistic tight-binding

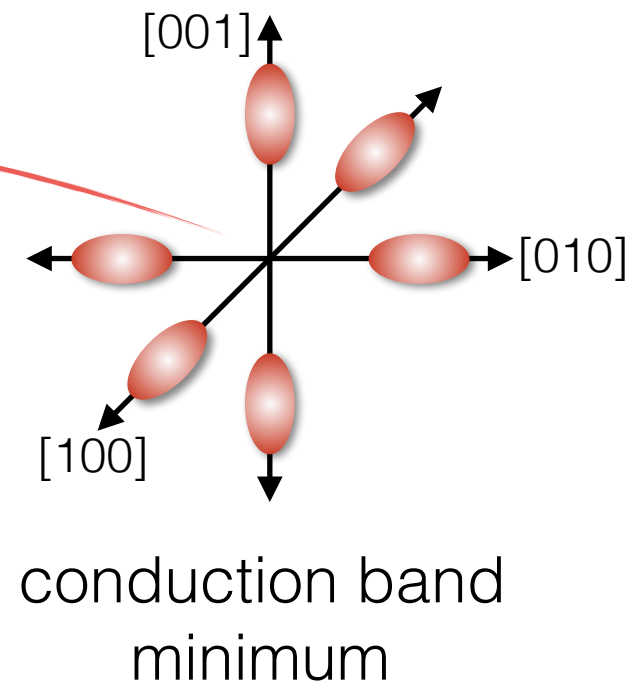
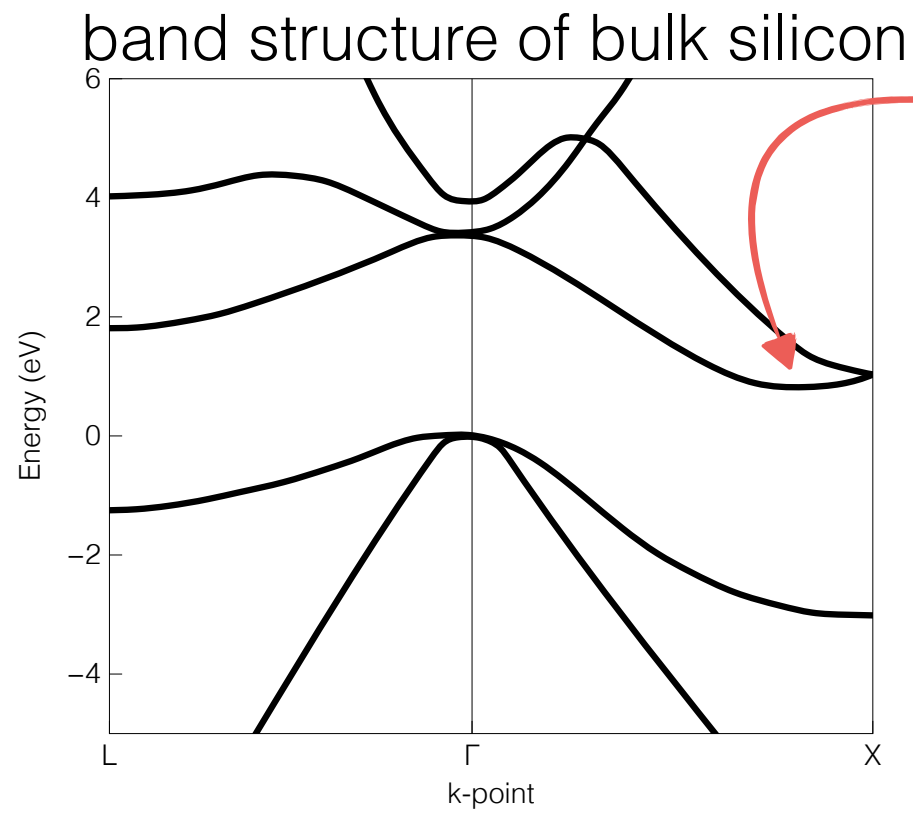


[Salfi, *et al.* Nat. Mat. 13, 605-610 (2014)]

Robust, but computationally intensive.

We want to develop a **quantitatively accurate** effective mass theory - use tight-binding to check for consistency.

Electronic structure of donors in Si



A quantum 'princess and the pea'

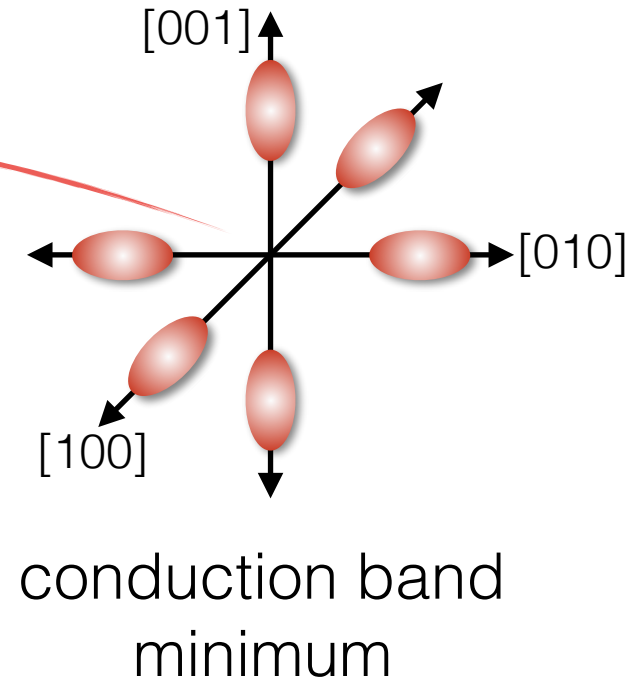
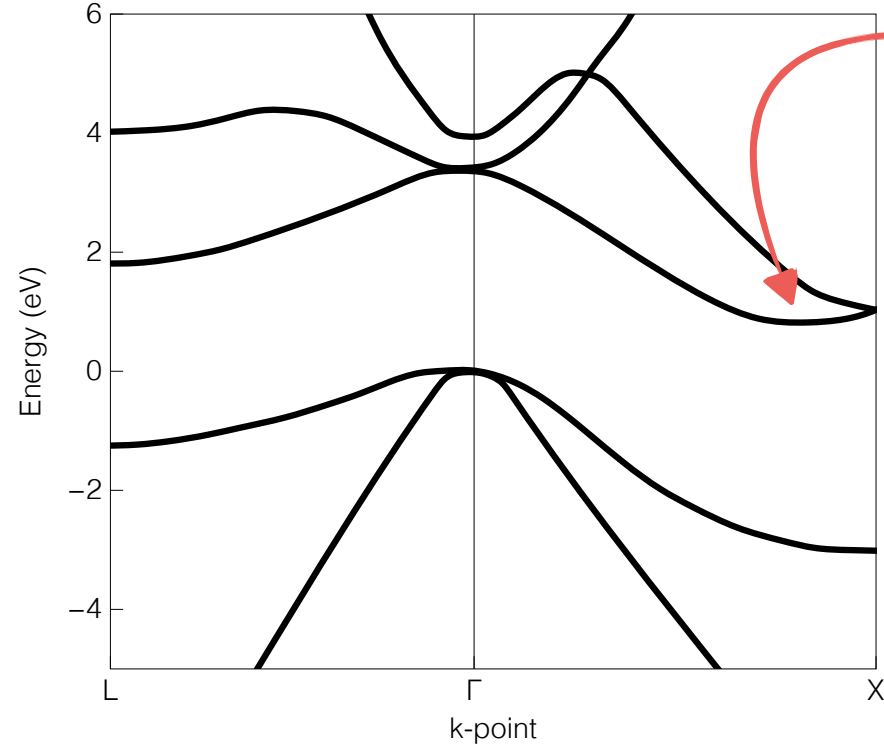
- Long-wavelength (10^{-6} m) electrostatics significantly impacted by small, atomistic features (10^{-10} m)
- Full atomistic modeling of the entire device not feasible: too big
- We designed a multiscale model that can incorporate both atomistic disorder and micron-scale devices.



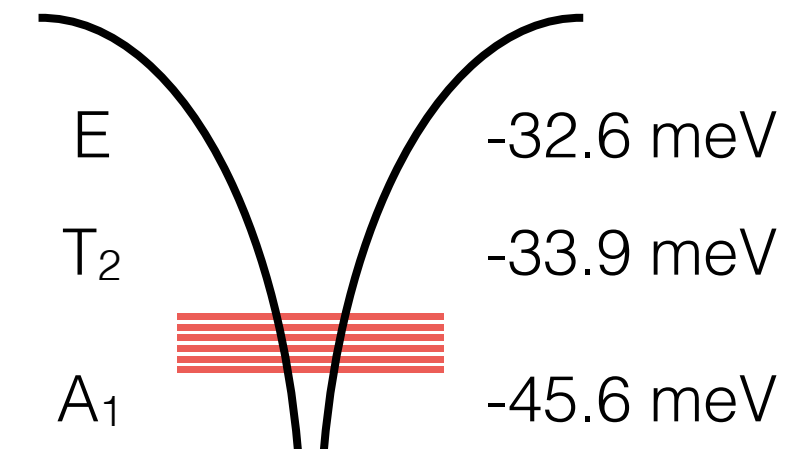
Illustration by Edmund Dulac, 1911.

Electronic structure of donors in Si

band structure of bulk silicon



phosphorous donor



$$\psi(\mathbf{r}) = \sum_j F_j(\mathbf{r}) e^{i\mathbf{k}_0^j \cdot \mathbf{r}} u_{\mathbf{k}_0^j}(\mathbf{r})$$

$$u_{\mathbf{k}_0^j}(\mathbf{r}) = \sum_{\mathbf{G}} A_{j,\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}}$$

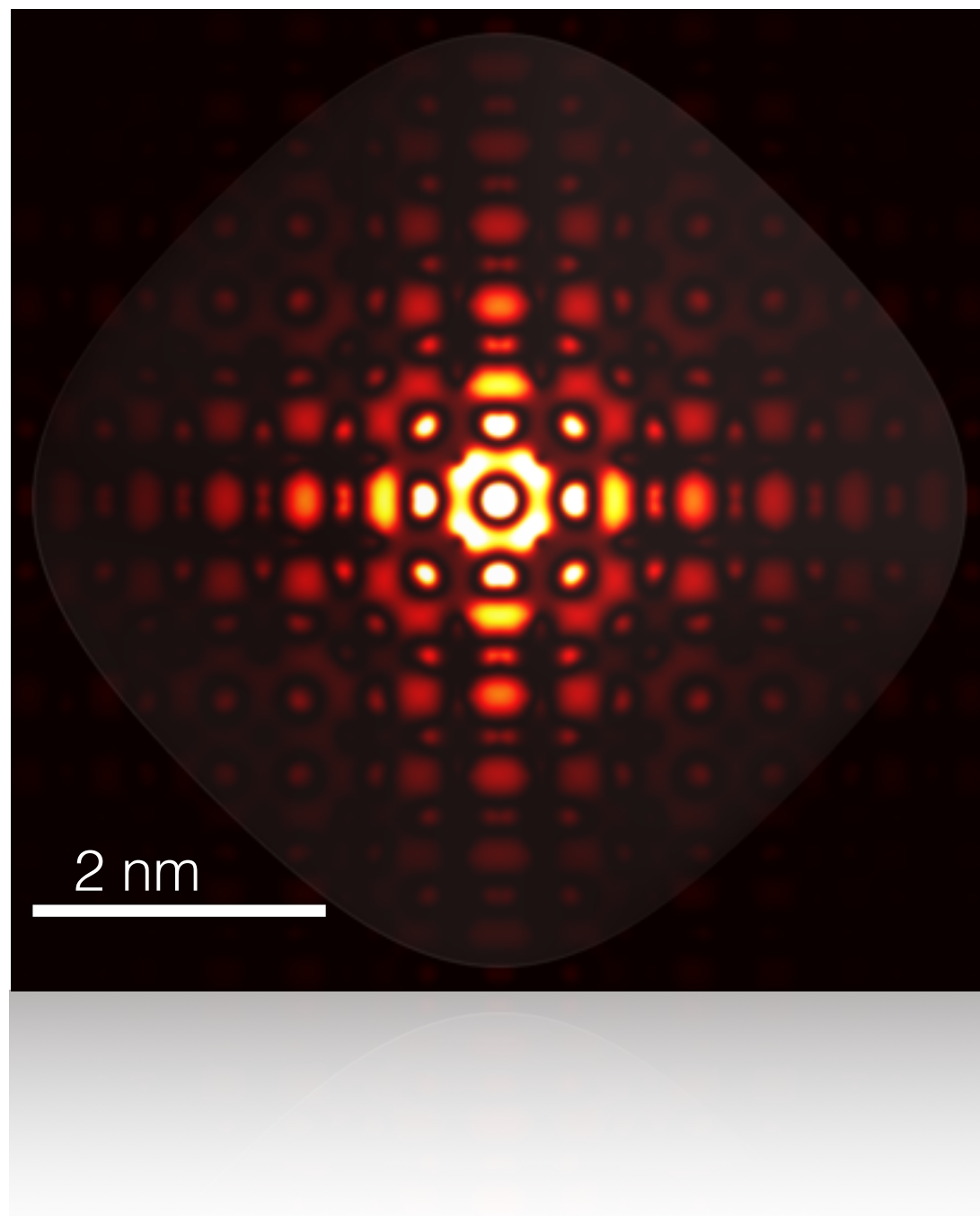
$$EF_l(\mathbf{r}) = \left(\hat{T}_l + U(\mathbf{r}) \right) F_l(\mathbf{r}) + \sum_j V_{lj}^{VO}(\mathbf{r}) F_j(\mathbf{r})$$

$$V_{lj}^{VO}(\mathbf{r}) = \sum_{\mathbf{G}, \mathbf{G}'} (1 - \delta_{\mathbf{G}, \mathbf{G}'} \delta_{j,l}) A_{l,\mathbf{G}'}^* A_{j,\mathbf{G}} e^{i\mathbf{r} \cdot (\mathbf{G} - \mathbf{G}' + \mathbf{k}_0^j - \mathbf{k}_0^l)} U(\mathbf{r})$$

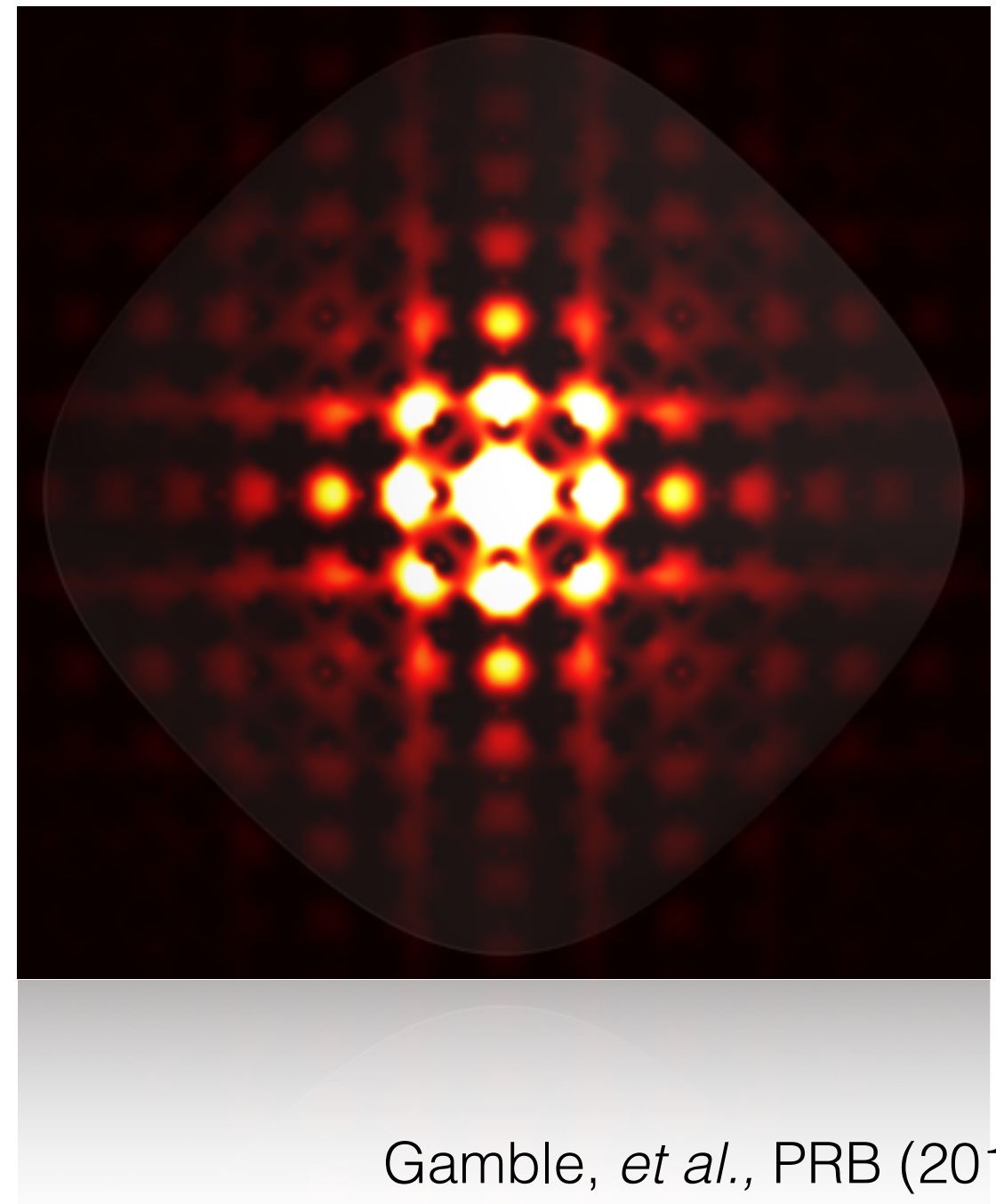
Gamble, *et al.*, PRB (2015).

The donor wave function is sharply peaked and oscillatory

Effective mass
calculation



Atomistic calculation
(NEMO 3D)



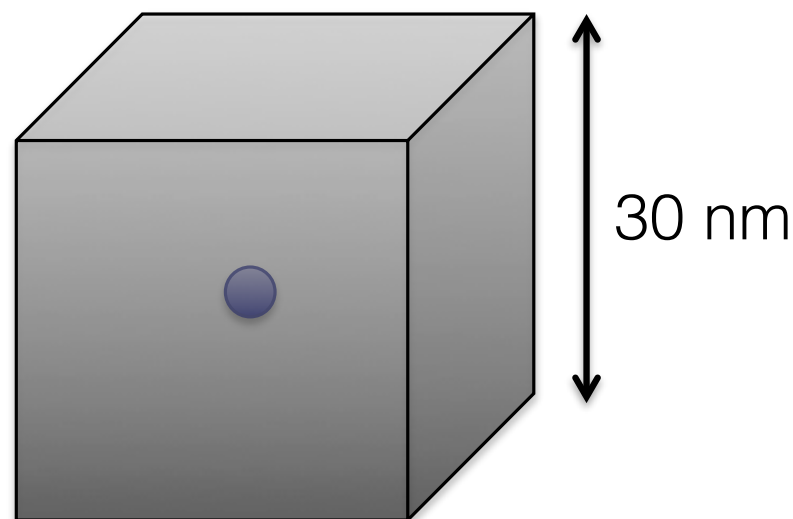
Gamble, *et al.*, PRB (2015).

Use effective mass to probe tunnel coupling: a critical figure of merit

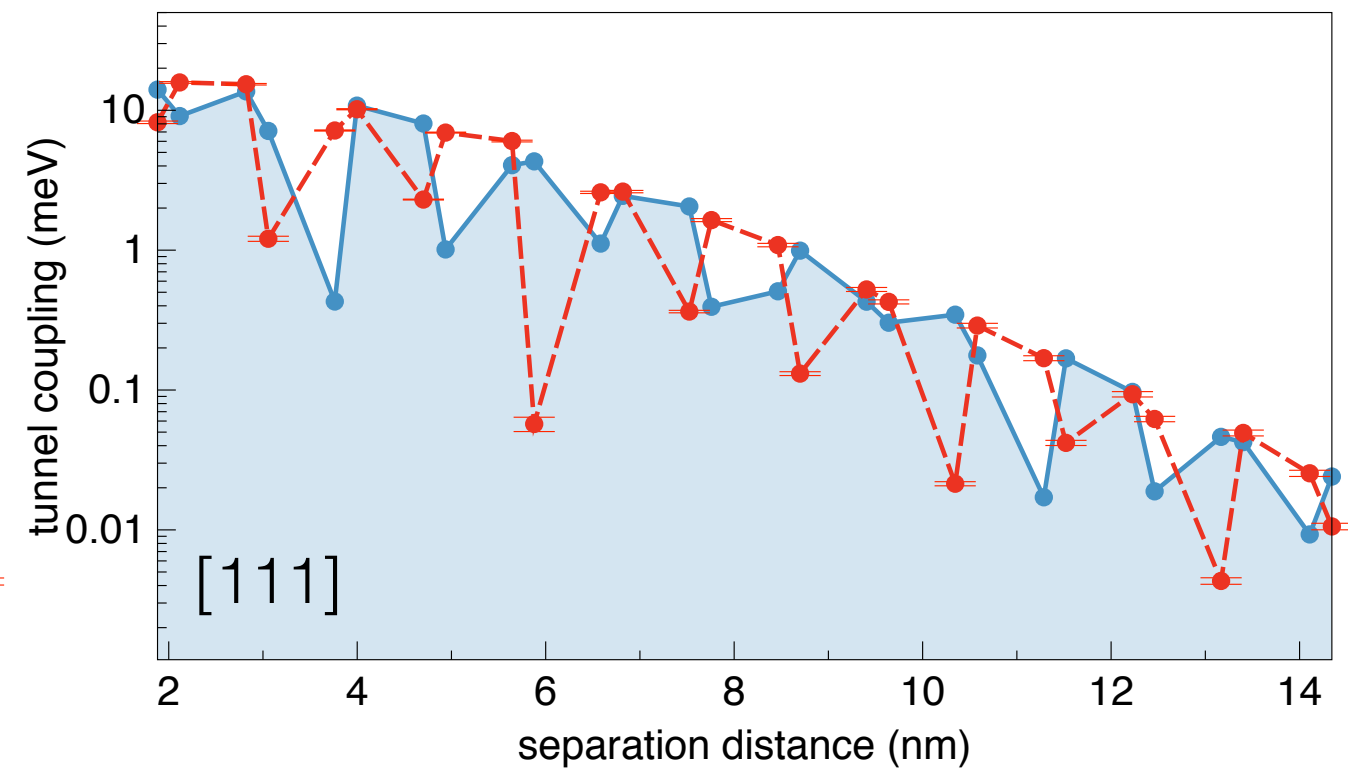
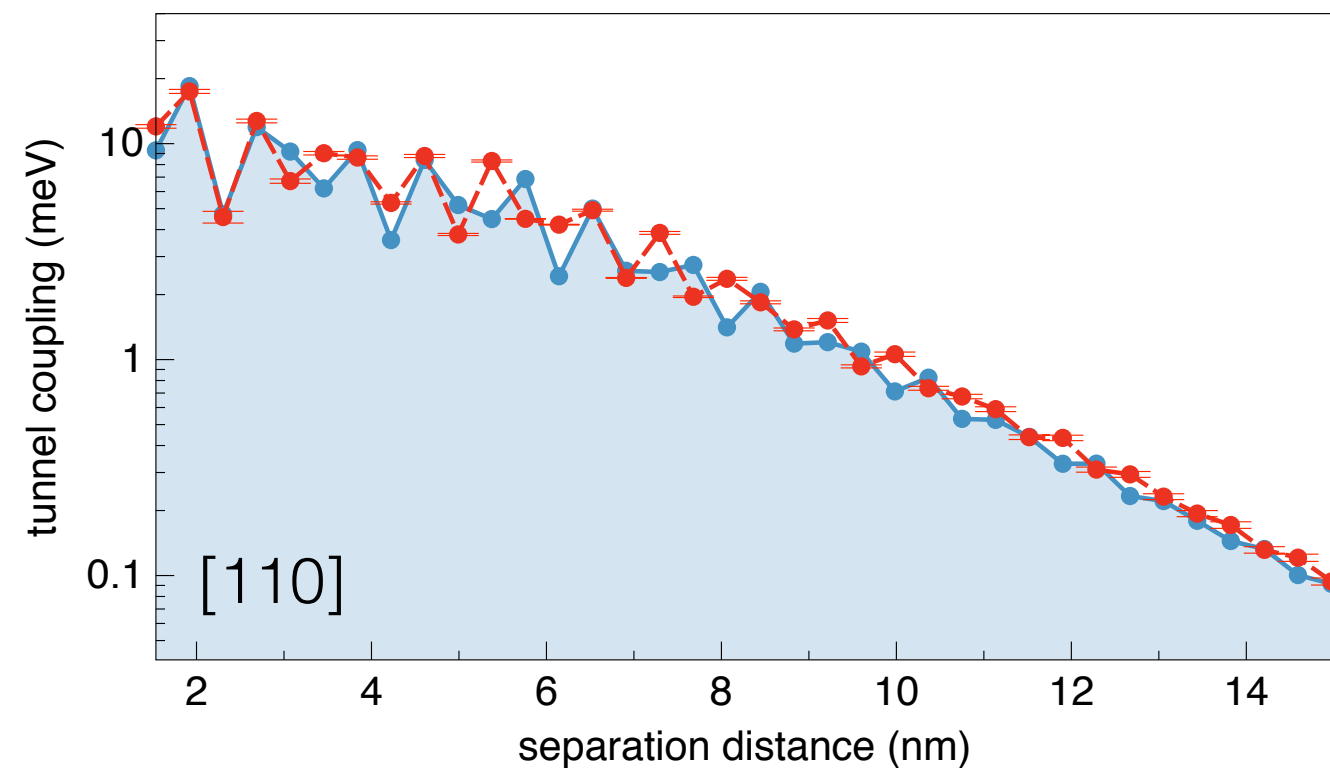
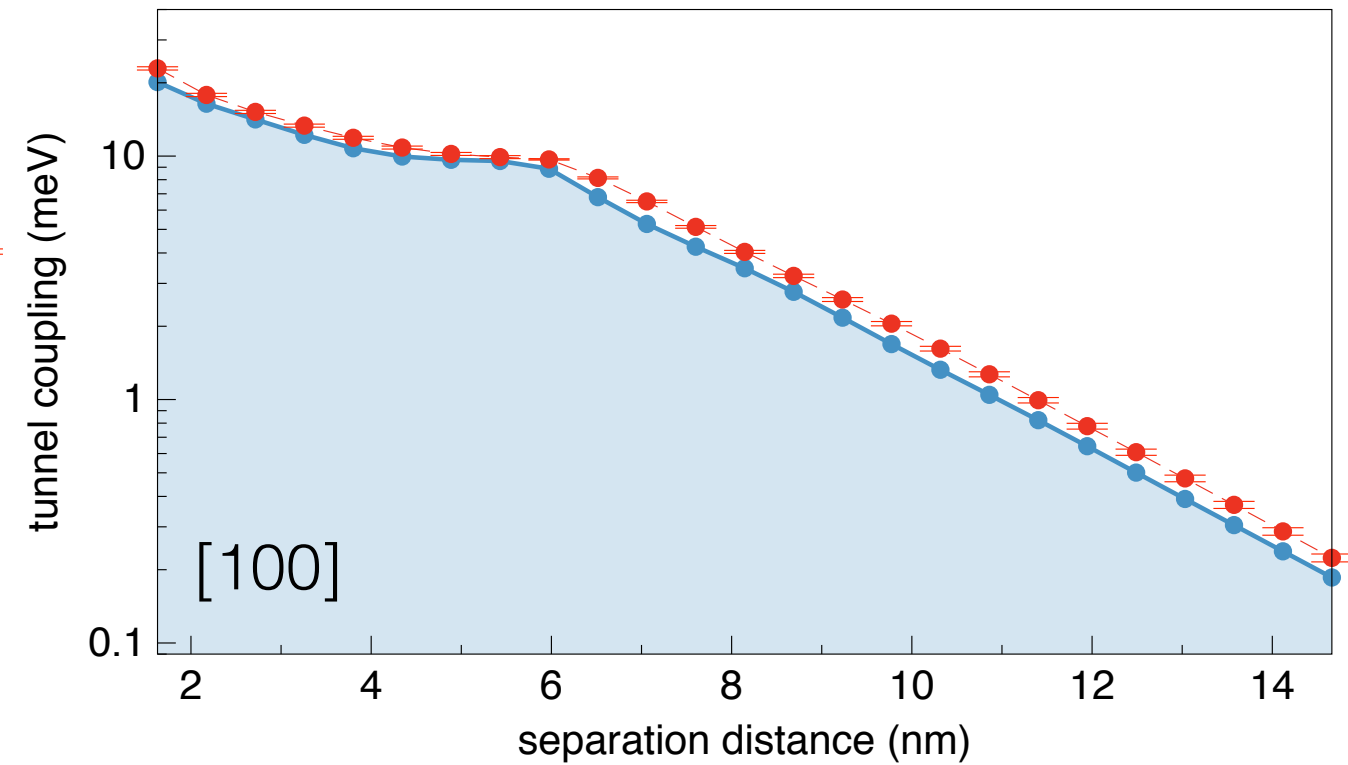
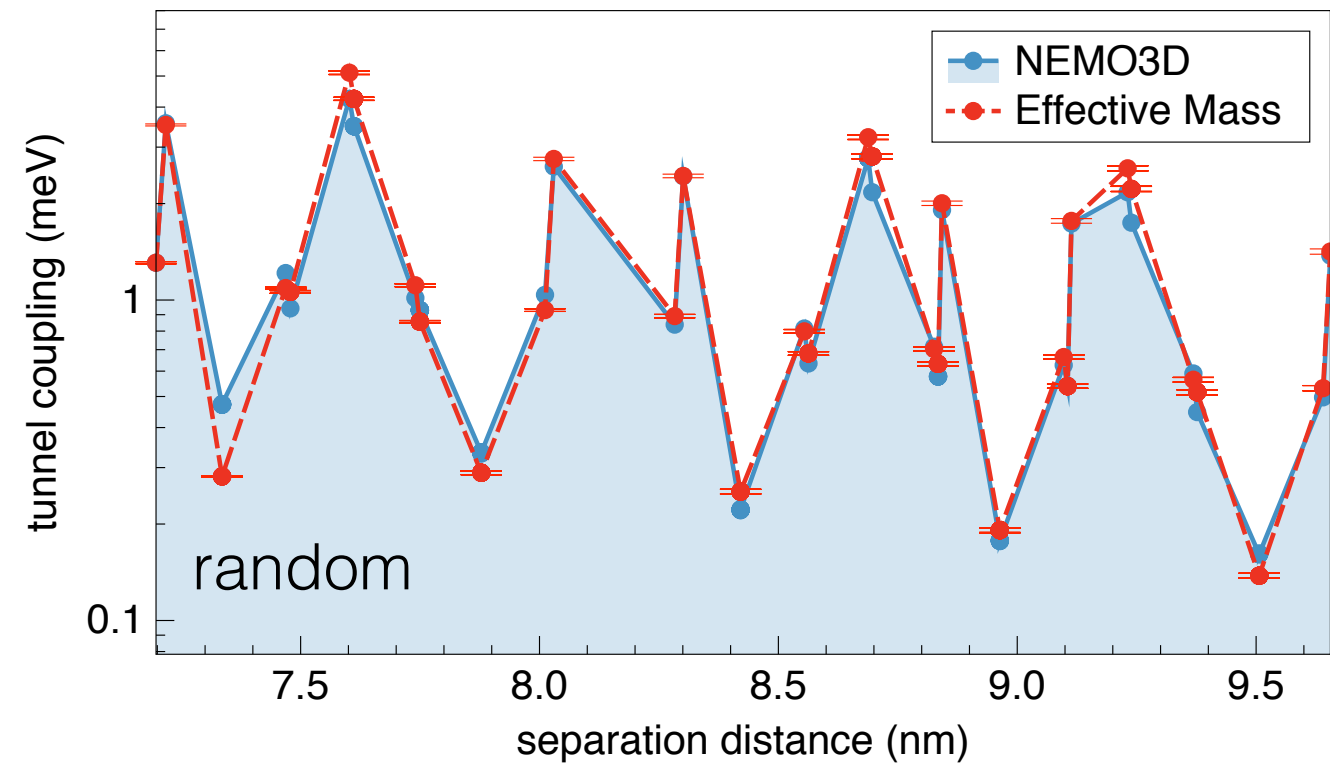
- **Tunnel coupling** is the difference between the lowest two eigenstates of a two-donor problem:

$$t = E_1 - E_0$$

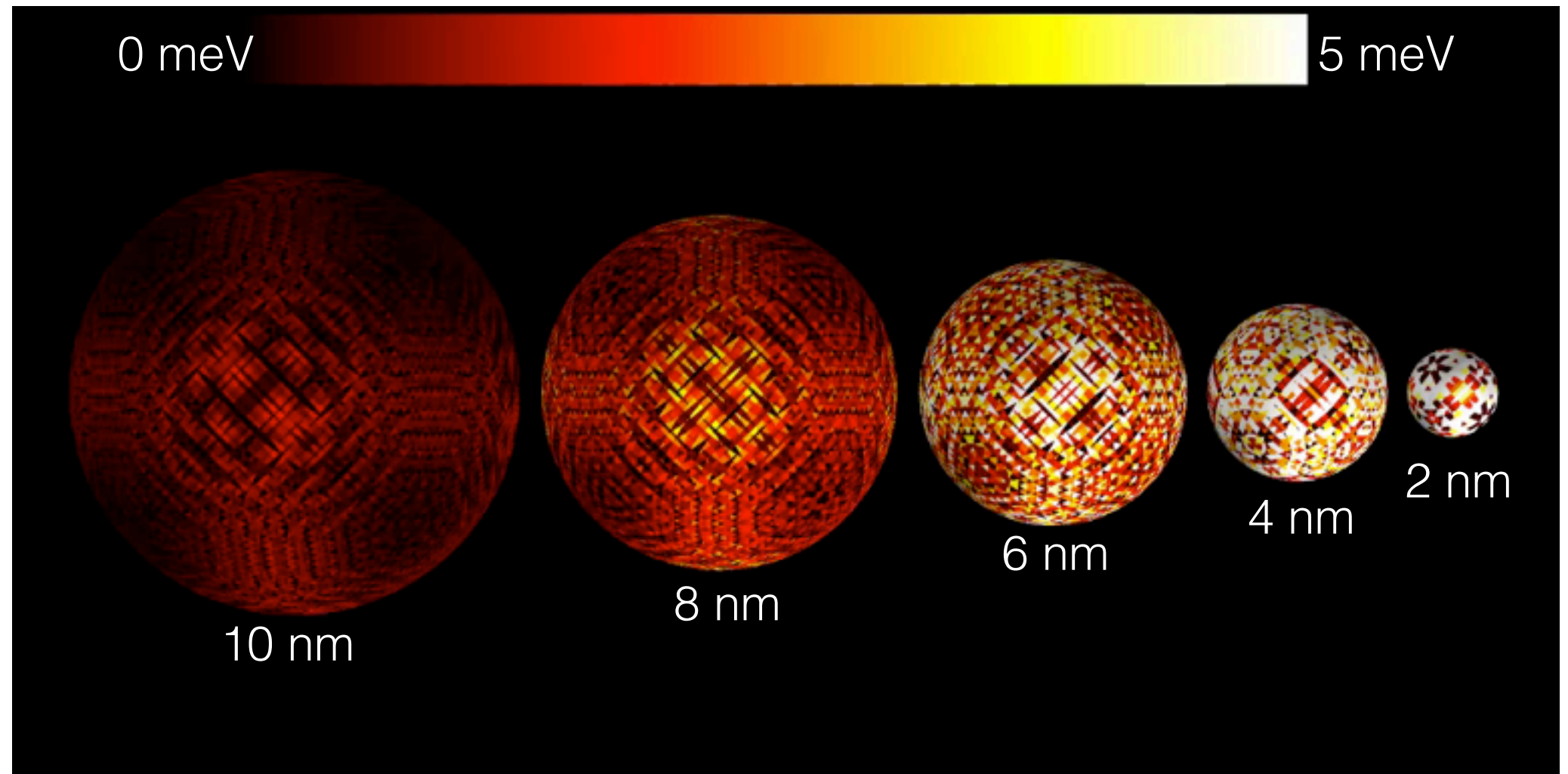
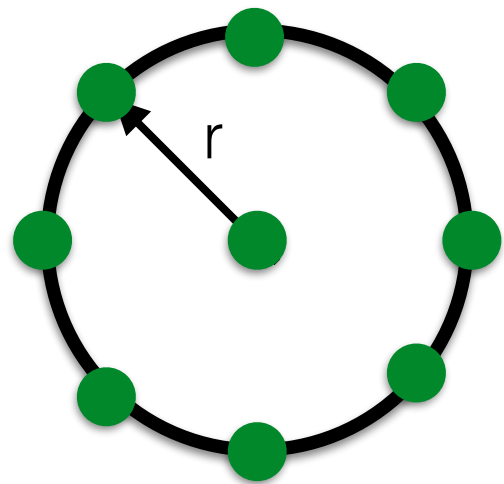
- Tells us about the coupling strength: important for electron-electron interactions and electron transport.
- We compute the tunnel coupling between one donor at the origin and the second at **all possible locations** in a 30 nm cube (~ 1.3 million locations).
- Effective mass code took about **1.5 hours per shot on one core** = $\sim 150k$ compute-hours total (after using symmetry).



Effective mass and tight-binding agree: tunnel coupling is very sensitive to position



Probing the entire range of tunnel couplings, there are no stable regions



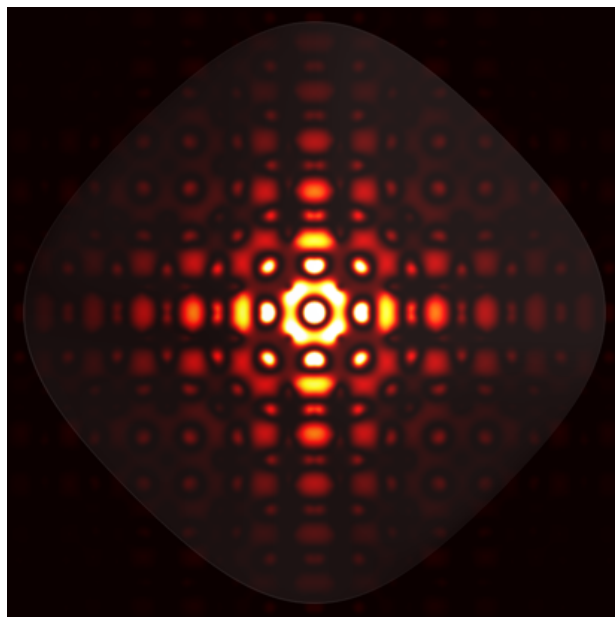
Gamble, *et al.*, PRB (2015).

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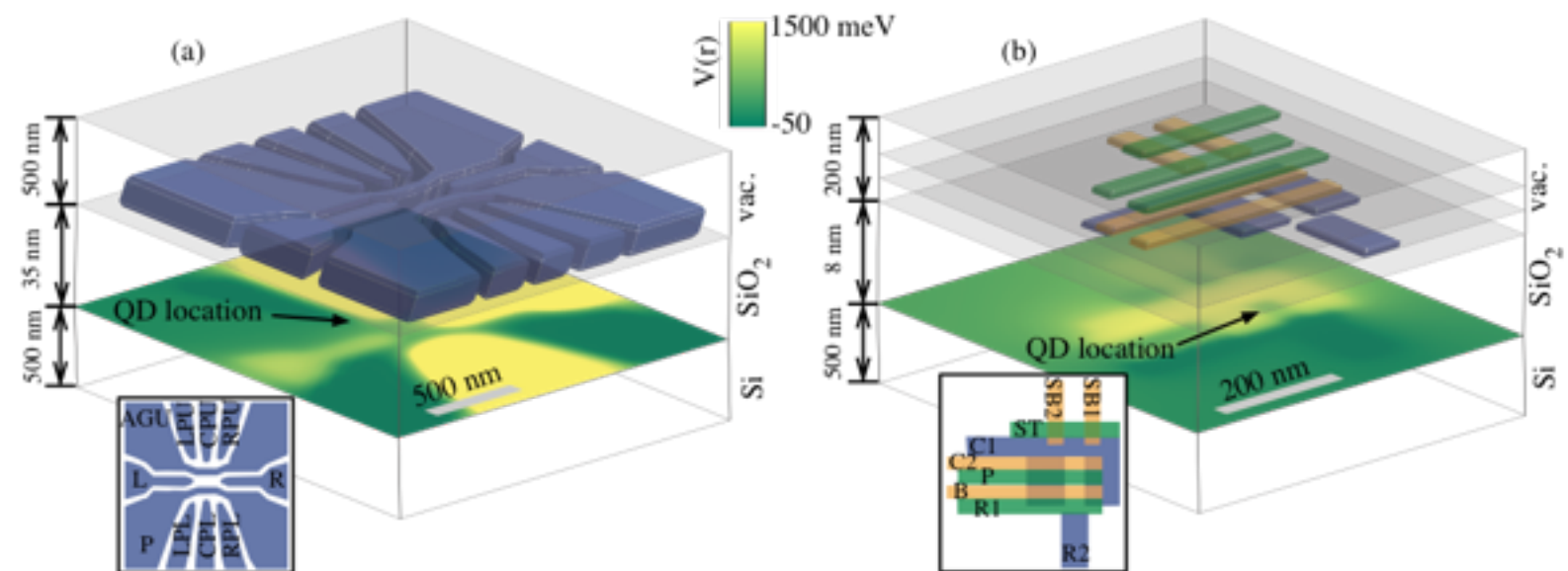
donors are hard to couple directly



[Gamble & Jacobson, *et al.*, PRB **91**, 235318 (2015)]

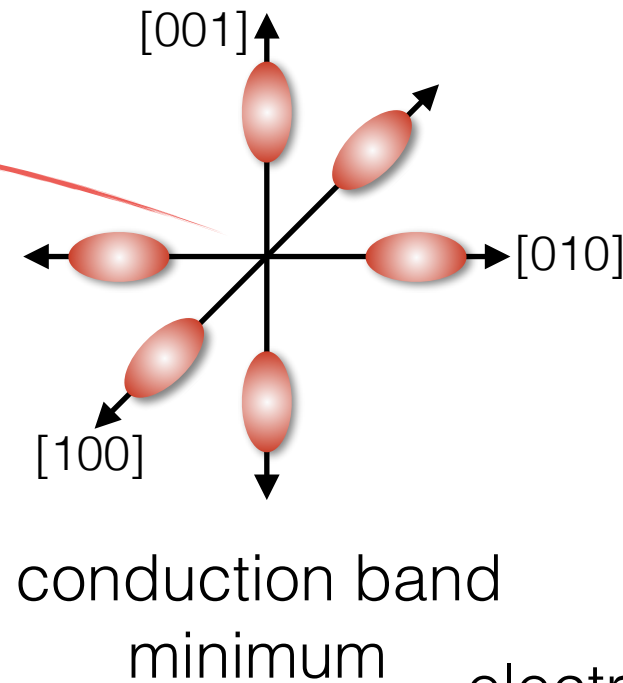
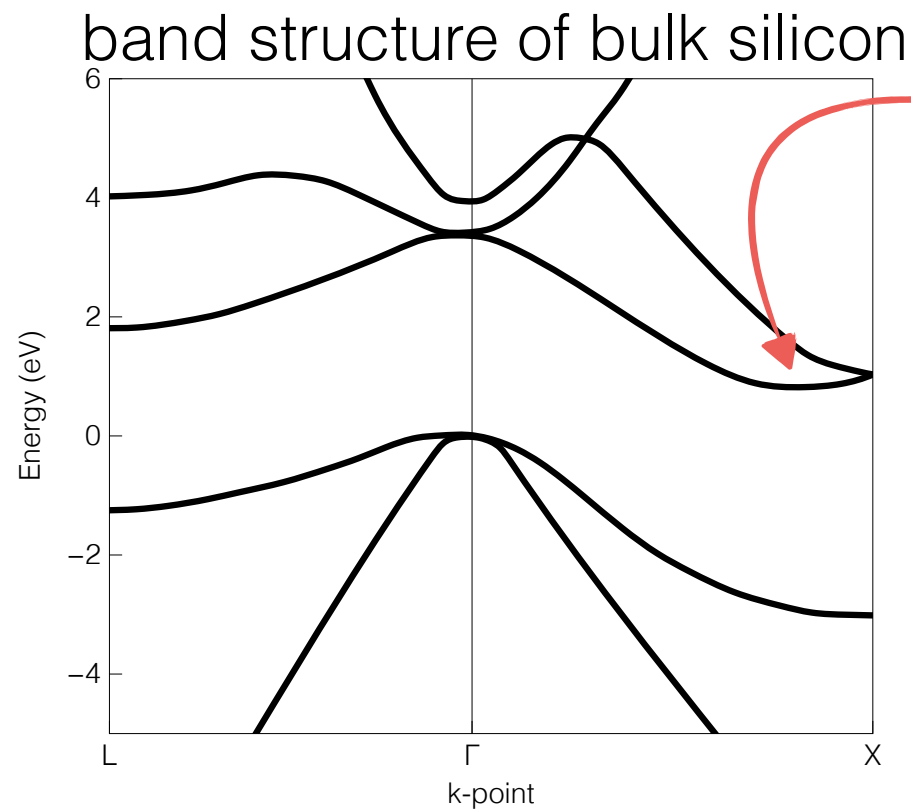
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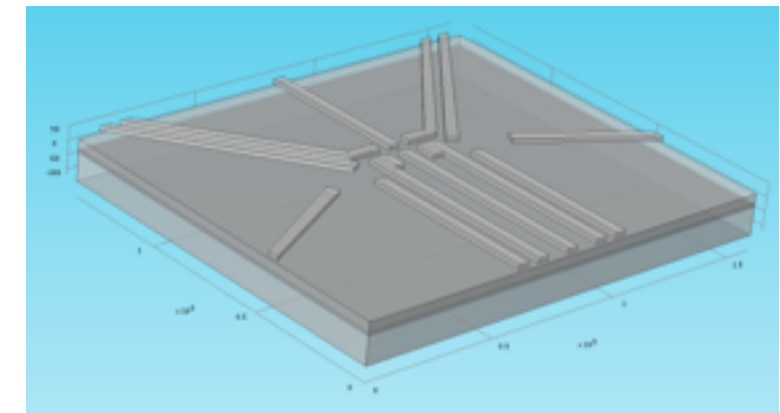


[Gamble, *et al.*, APL (2016)]

Electronic structure of quantum dot electrons in Si



Finite element model



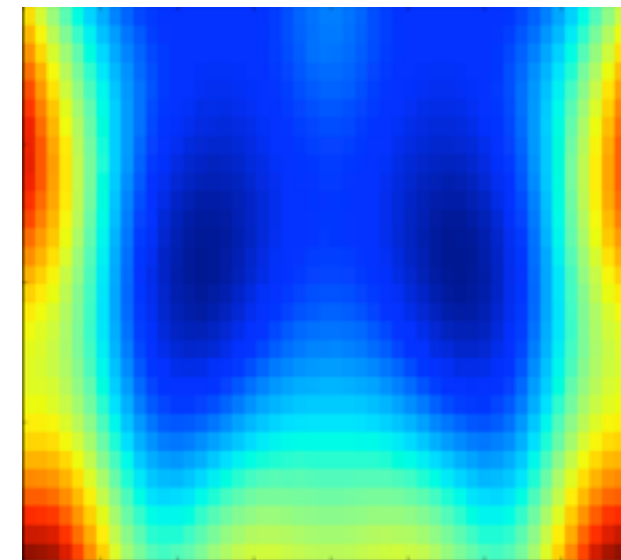
$$\psi(\mathbf{r}) = \sum_j F_j(\mathbf{r}) e^{i\mathbf{k}_0^j \cdot \mathbf{r}} u_{\mathbf{k}_0^j}(\mathbf{r})$$

$$u_{\mathbf{k}_0^j}(\mathbf{r}) = \sum_{\mathbf{G}} A_{j,\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}}$$

$$EF_l(\mathbf{r}) = \left(\hat{T}_l + U(\mathbf{r}) \right) F_l(\mathbf{r}) + \sum_j V_{lj}^{VO}(\mathbf{r}) F_j(\mathbf{r})$$

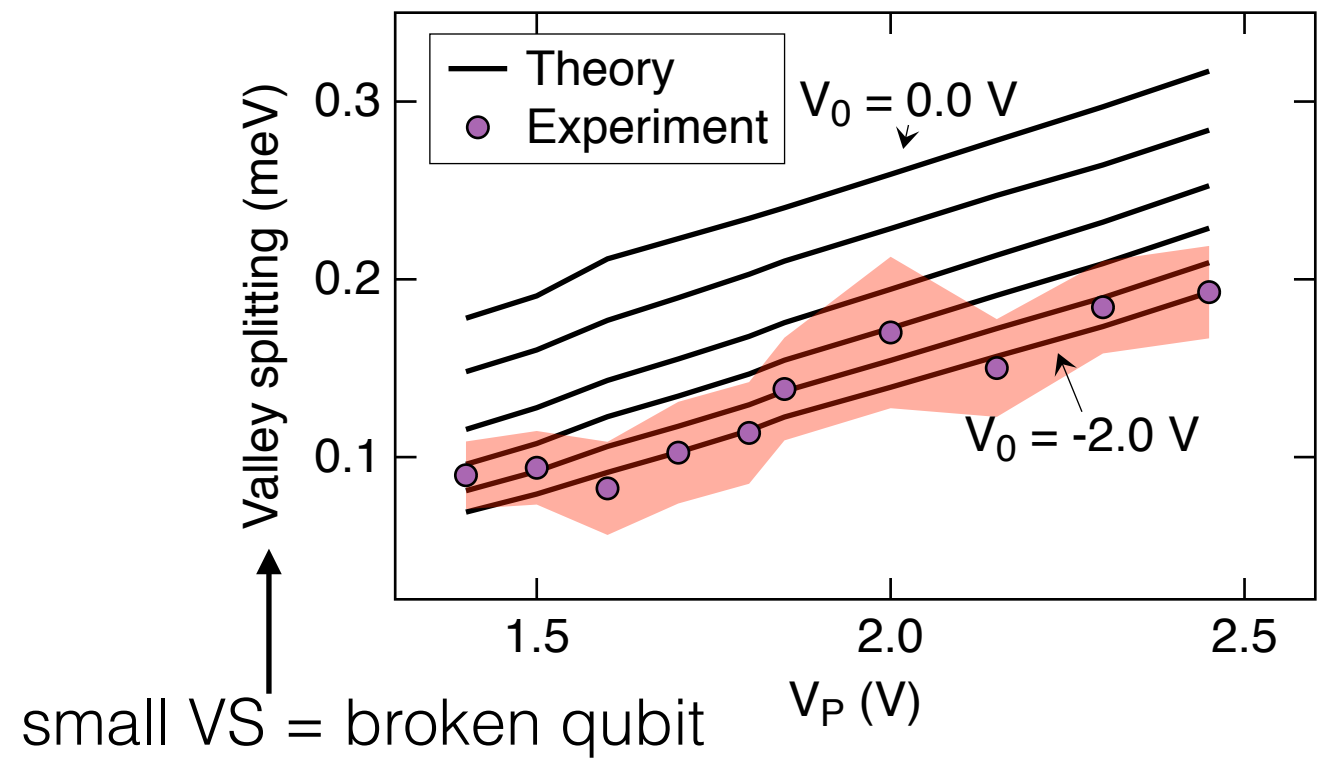
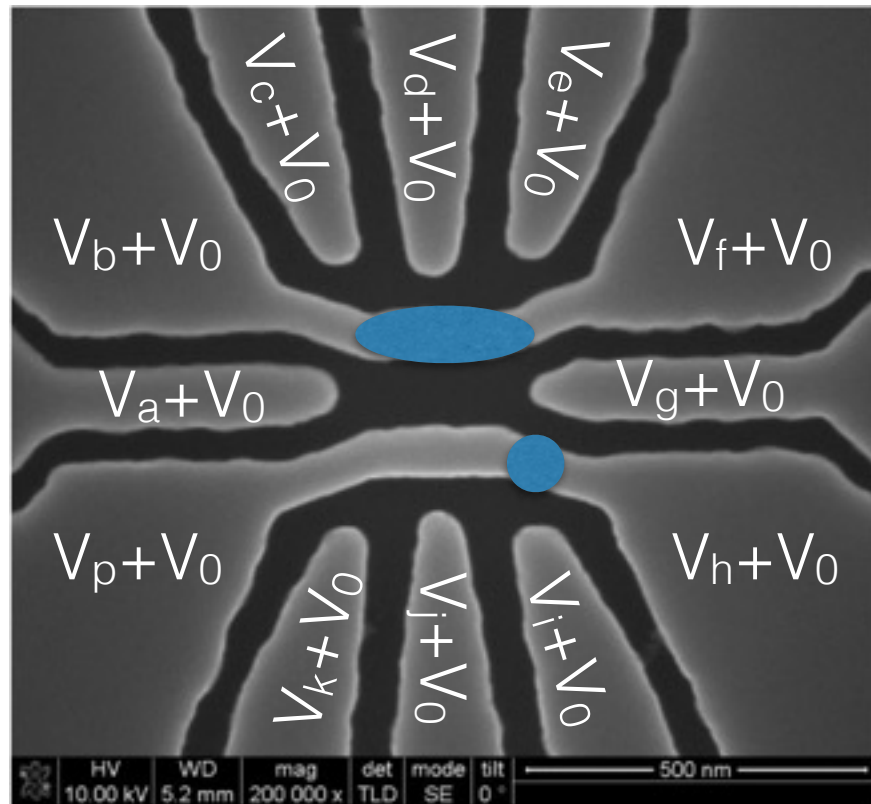
$$V_{lj}^{VO}(\mathbf{r}) = \sum_{\mathbf{G}, \mathbf{G}'} (1 - \delta_{\mathbf{G}, \mathbf{G}'} \delta_{j,l}) A_{l,\mathbf{G}'}^* A_{j,\mathbf{G}} e^{i\mathbf{r} \cdot (\mathbf{G} - \mathbf{G}' + \mathbf{k}_0^j - \mathbf{k}_0^l)} U(\mathbf{r})$$

electrostatic landscape

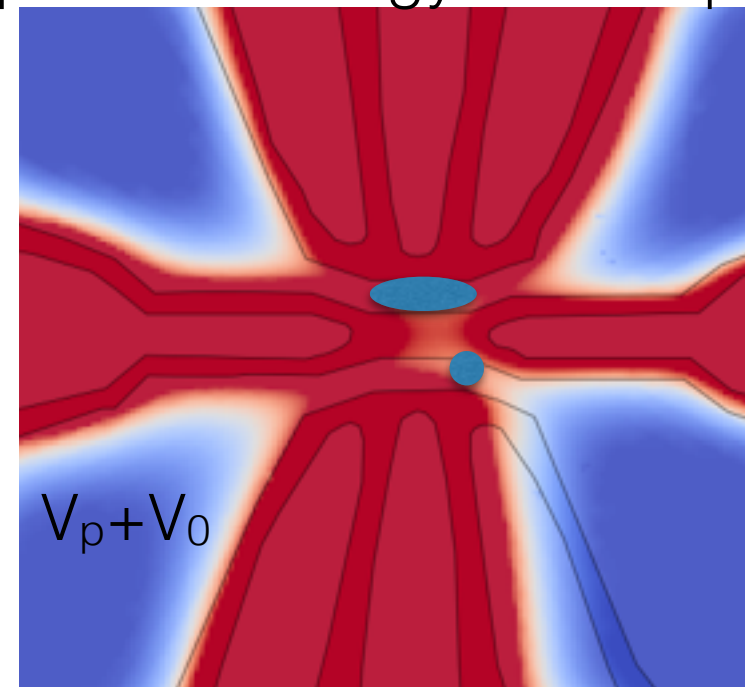
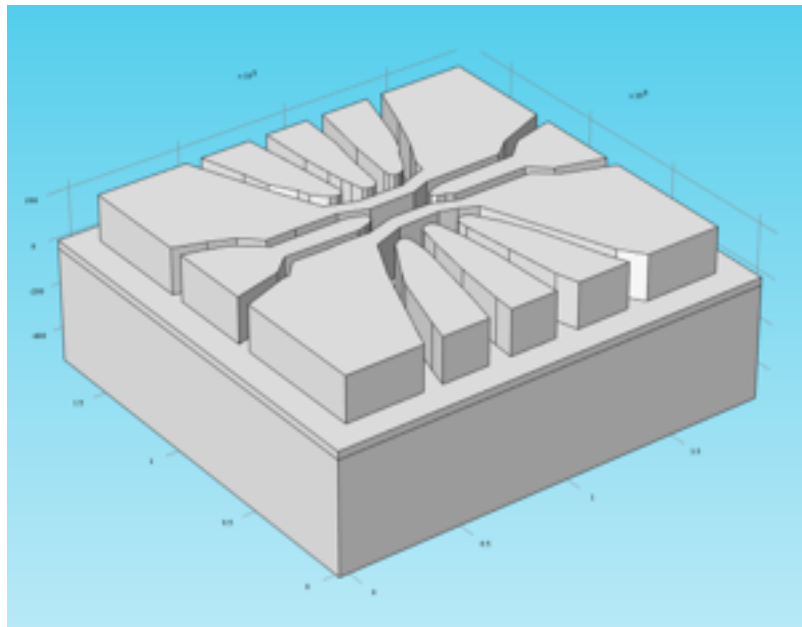


Gamble, *et al.*, APL (2016).

Case study: valley splitting directly from a device design and experimental voltages.



potential energy landscape



To confirm the results, we modeled a second system.

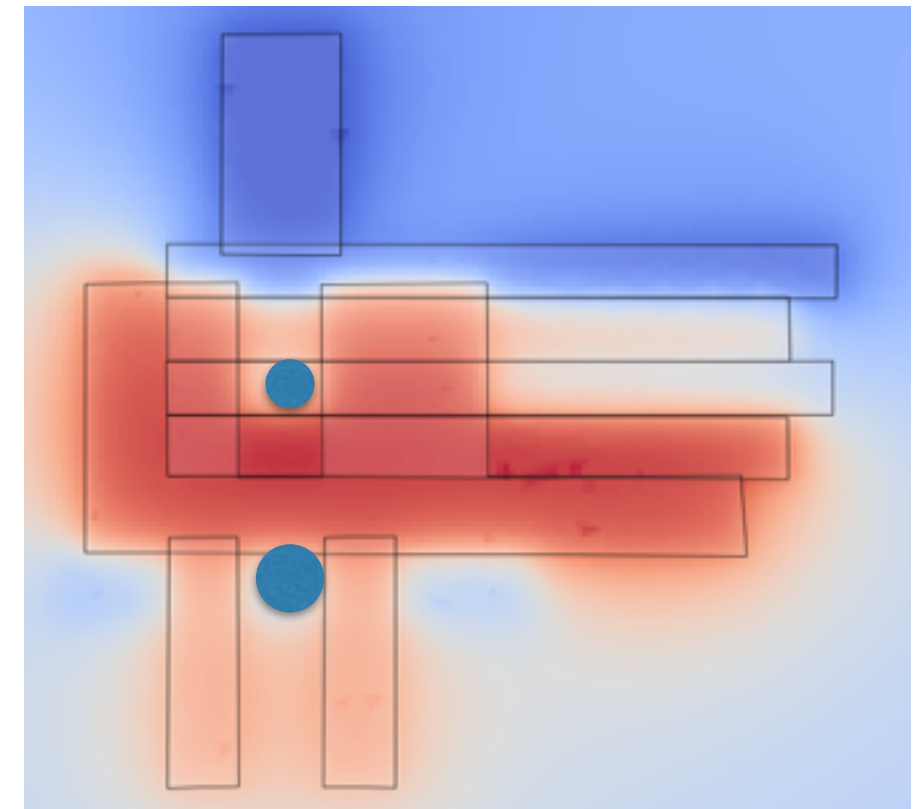
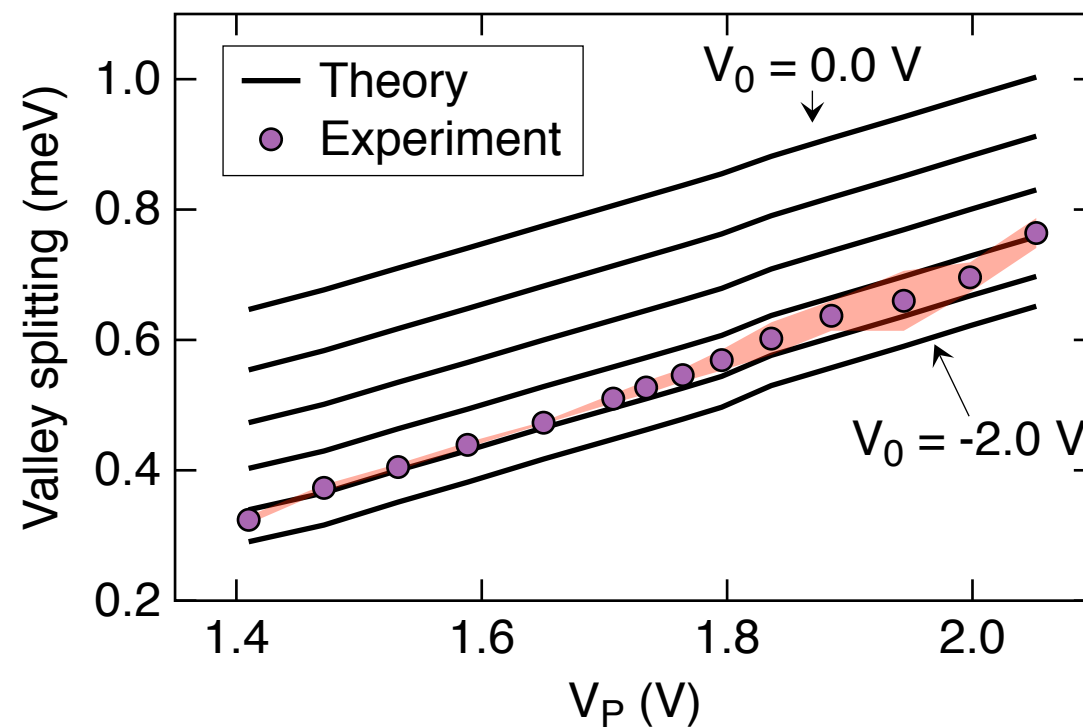
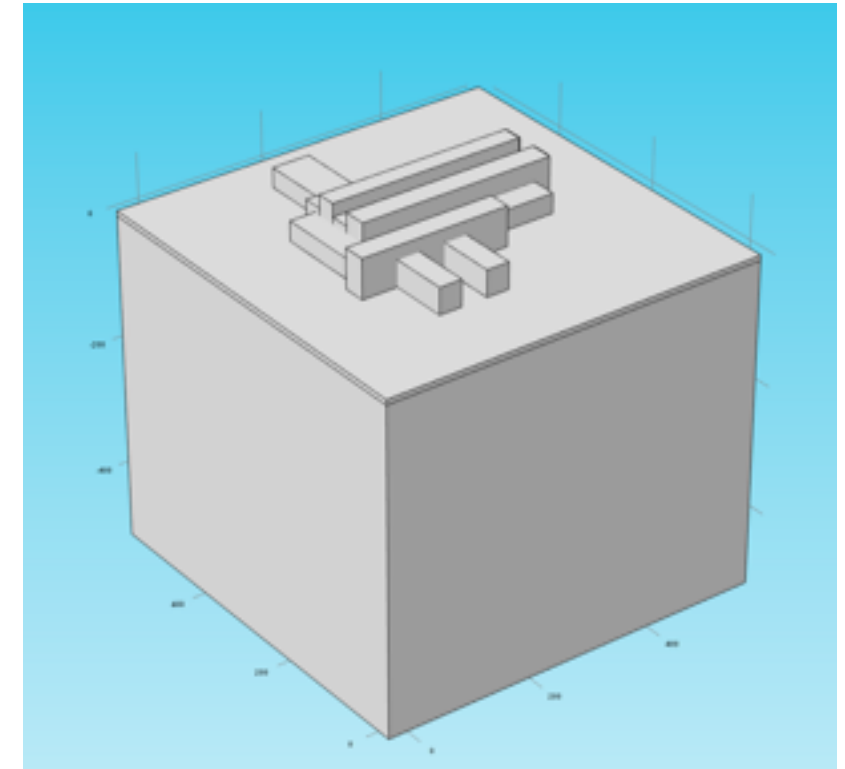
ARTICLE

Received 11 Jan 2013 | Accepted 27 May 2013 | Published 27 Jun 2013

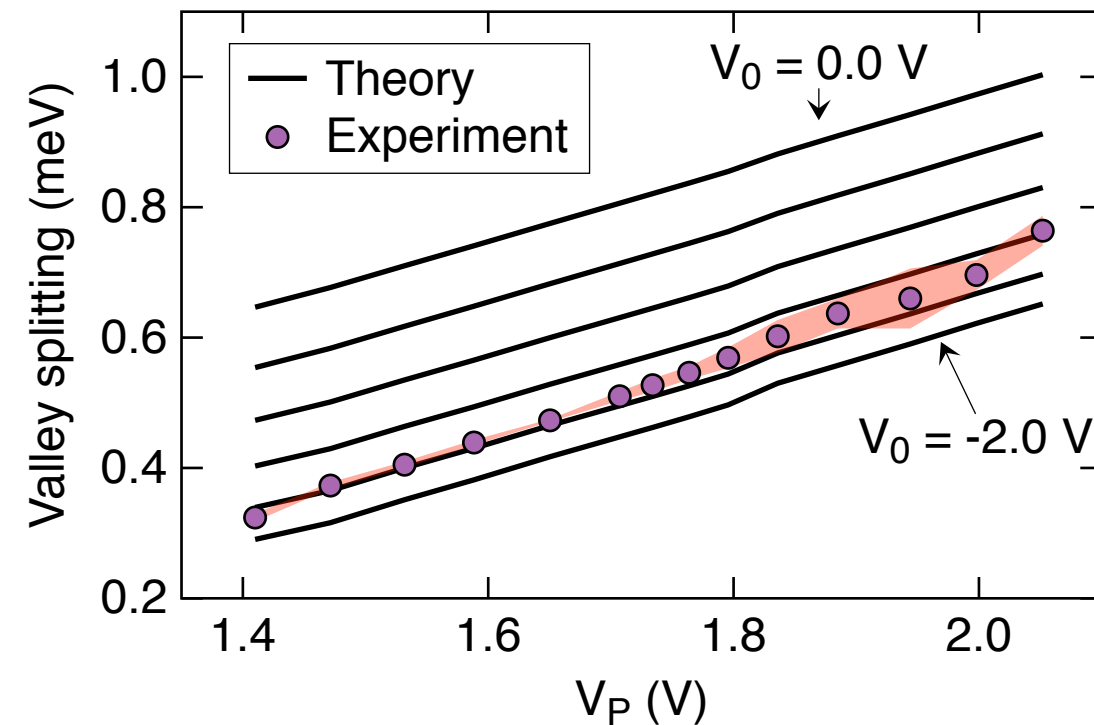
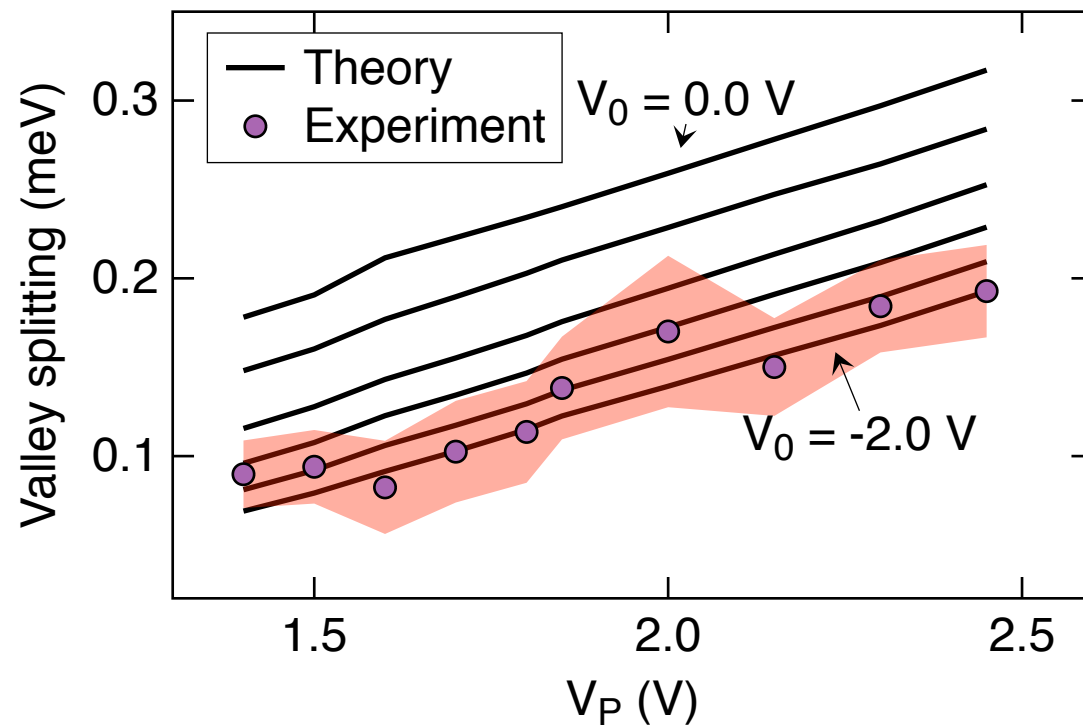
DOI: 10.1038/ncomms3069

Spin-valley lifetimes in a silicon quantum dot with tunable valley splitting

C.H. Yang¹, A. Rossi¹, R. Ruskov², N.S. Lai¹, F.A. Mohiyaddin¹, S. Lee³, C. Tahan², G. Klimeck³, A. Morello¹ & A.S. Dzurak¹



In both cases, the theoretical predictions are consistent with experiment

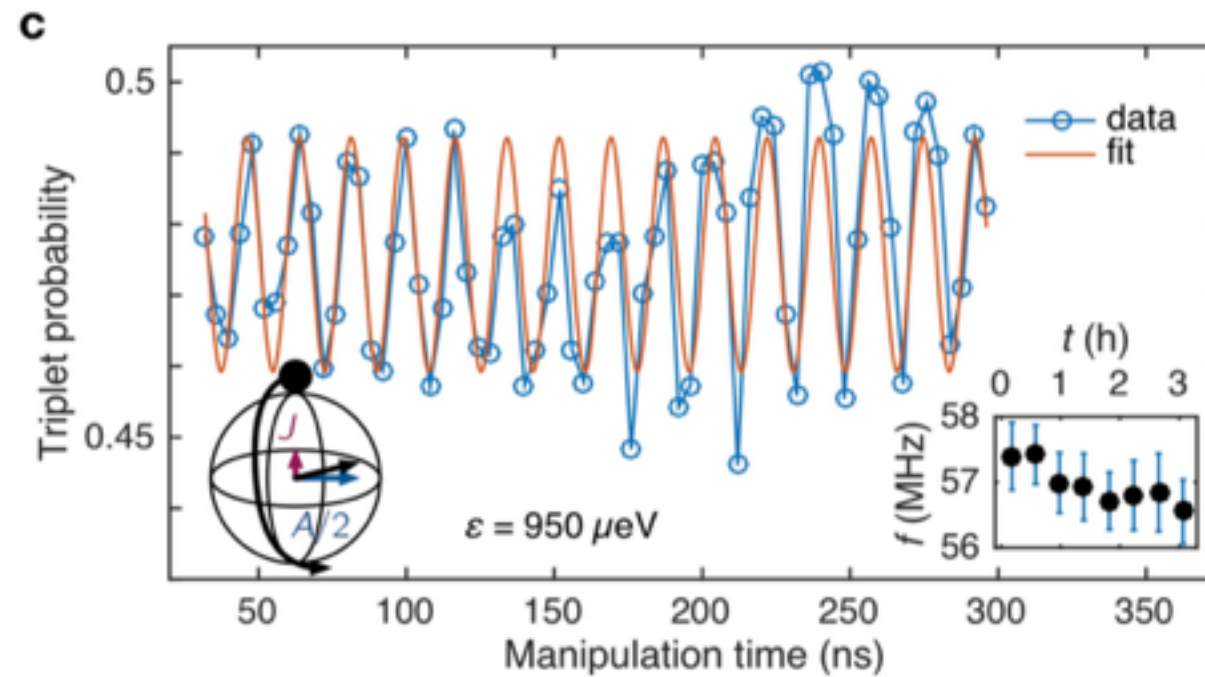
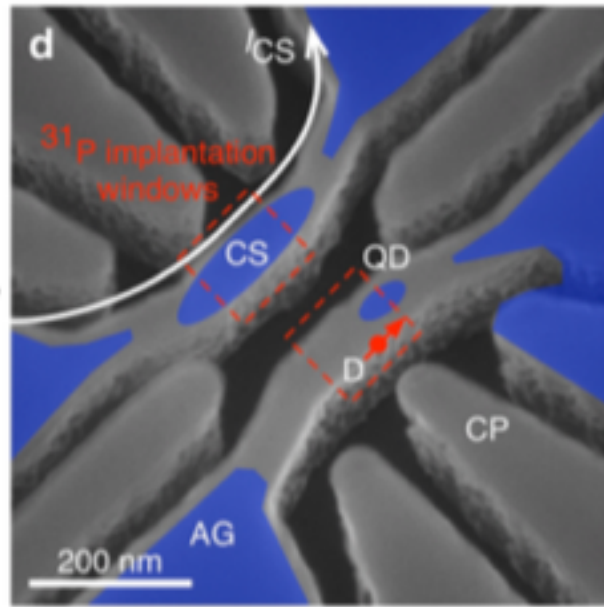


- Very different fabrication processes and device geometries.
- Since small VS leads to broken qubits, we need to be able to accurately model it to design qubit structures.
- The theoretical predictions are consistent between devices, despite many differences in fabrication (and large spacetime separation).
- In practice, threshold voltages tend to be smaller magnitude than 1.8 V - this discrepancy is consistent with detailed statistical disorder study.

Gamble, *et al.*, APL (2016).

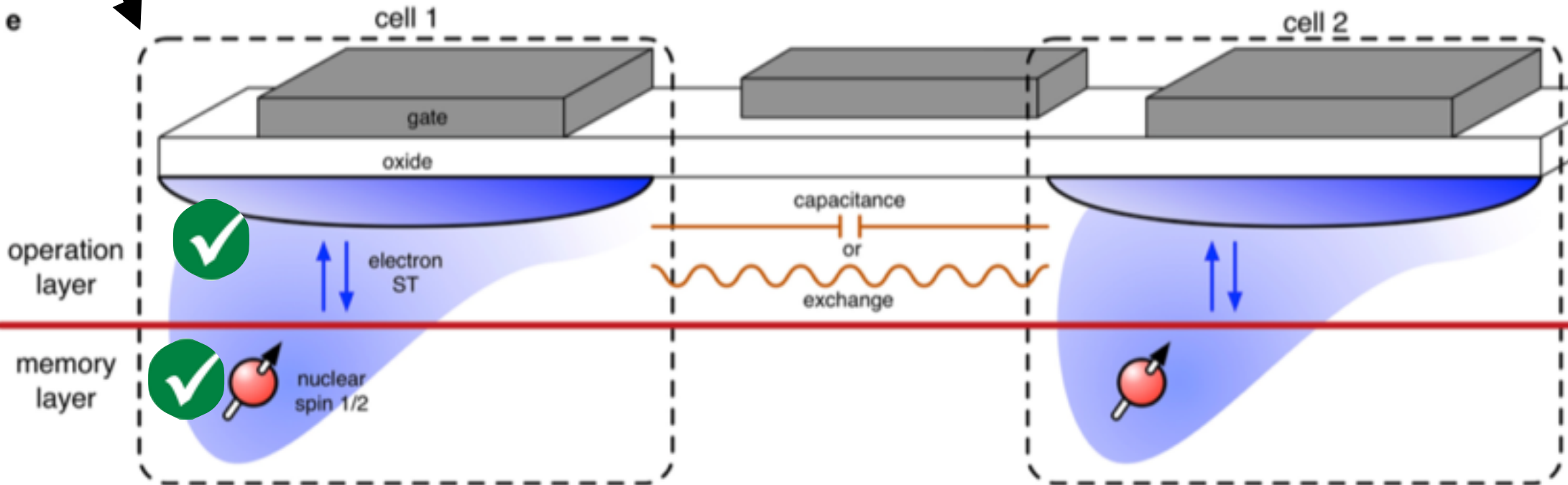
Combining both species of artificial atoms

Quantum dots can be reliably coupled!



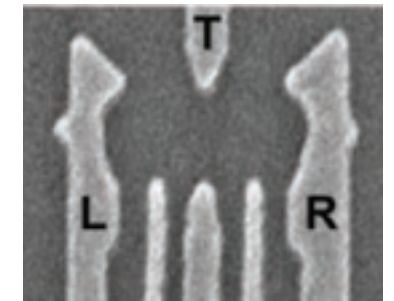
One cell has been demonstrated in experiment.

Use *both* impurities and quantum dots!



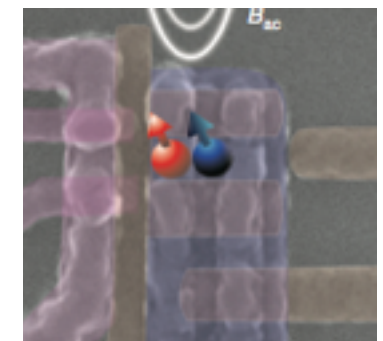
P. Harvey-Collard, *et al.*, arXiv:1512.01606.

1-qubit GaAs gates



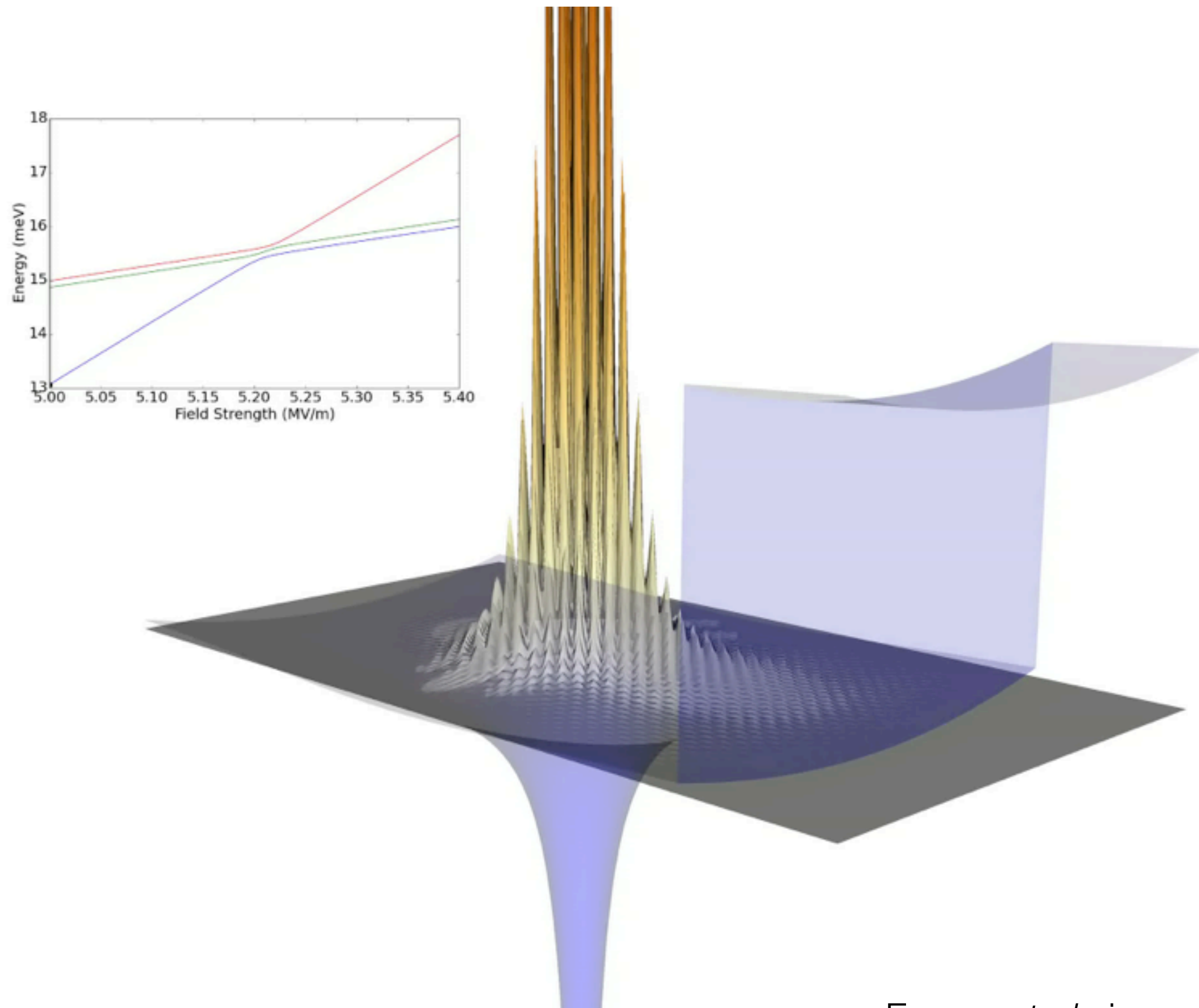
J. Petta, *et al.*
Science (2010)

2-qubit silicon gate



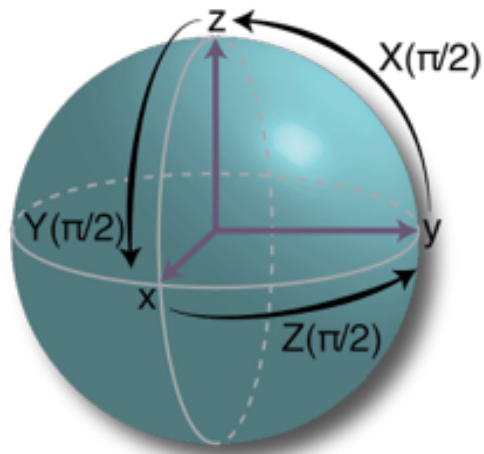
M. Veldhorst, *et al.*
Nature (2015).

Putting the pieces together: impurity + interface dot

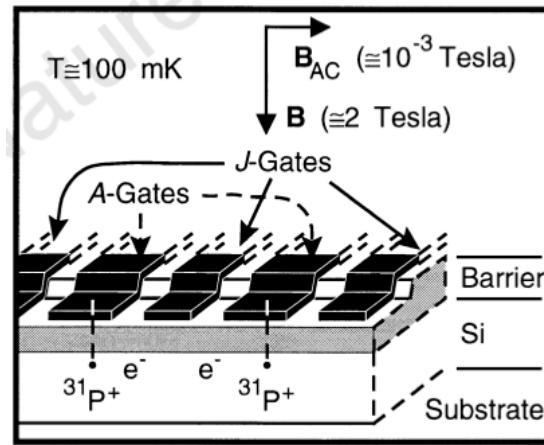


Frees, *et al.*, in preparation.

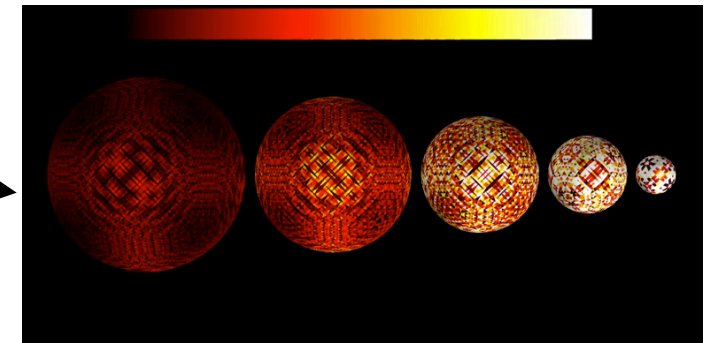
Summary



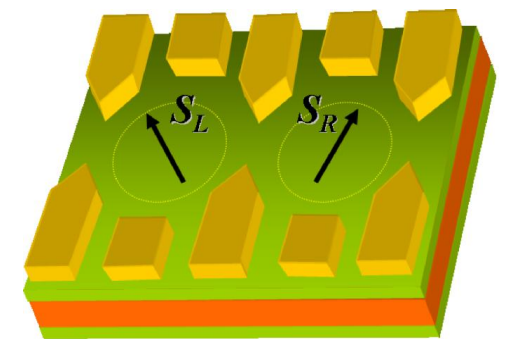
Want to encode qubits in a convenient physical system.



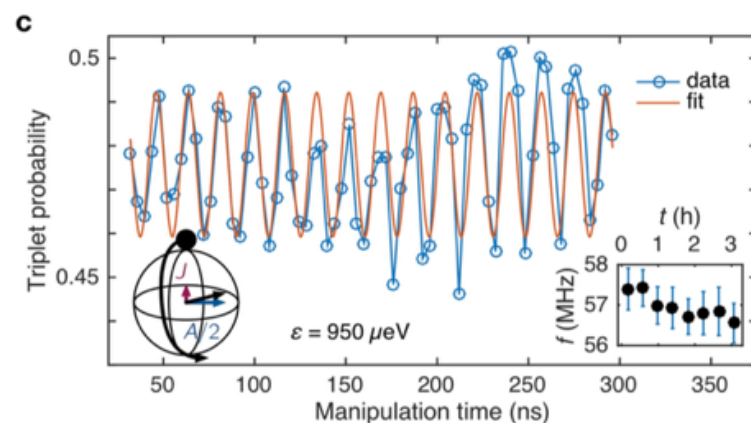
Donors in silicon seem ideal...



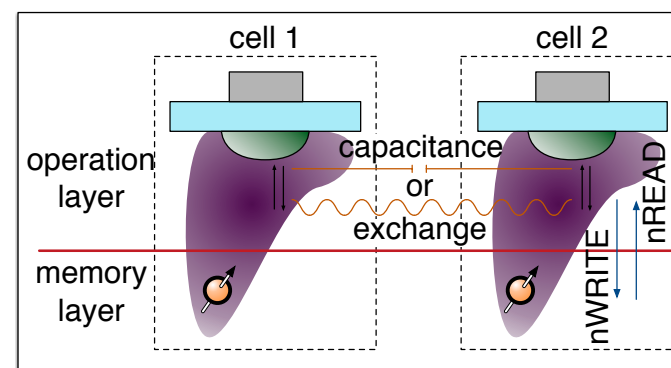
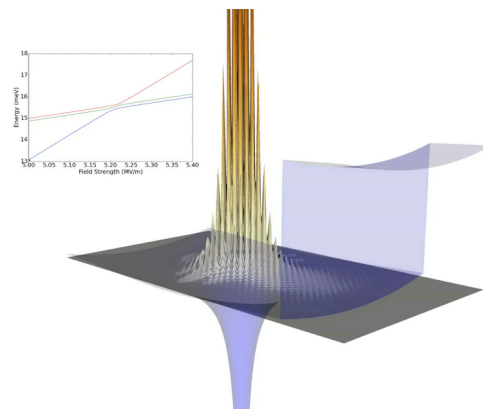
...but they can't be reliably coupled!



Quantum dots can be coupled more easily.



Experimental evidence + unified multiscale device model.



Mediate donor-donor interactions through dots.

SNL Theory collaborators

- Andrew D. Baczewski
- N. Tobias Jacobson
- Inès Montaña
- Jonathan E. Moussa
- Richard P. Muller
- Erik Nielsen
- Leon Maurer

SNL experiment collaborators

- Malcolm Carroll
- Patrick Harvey-Collard
- Martin Rudolph

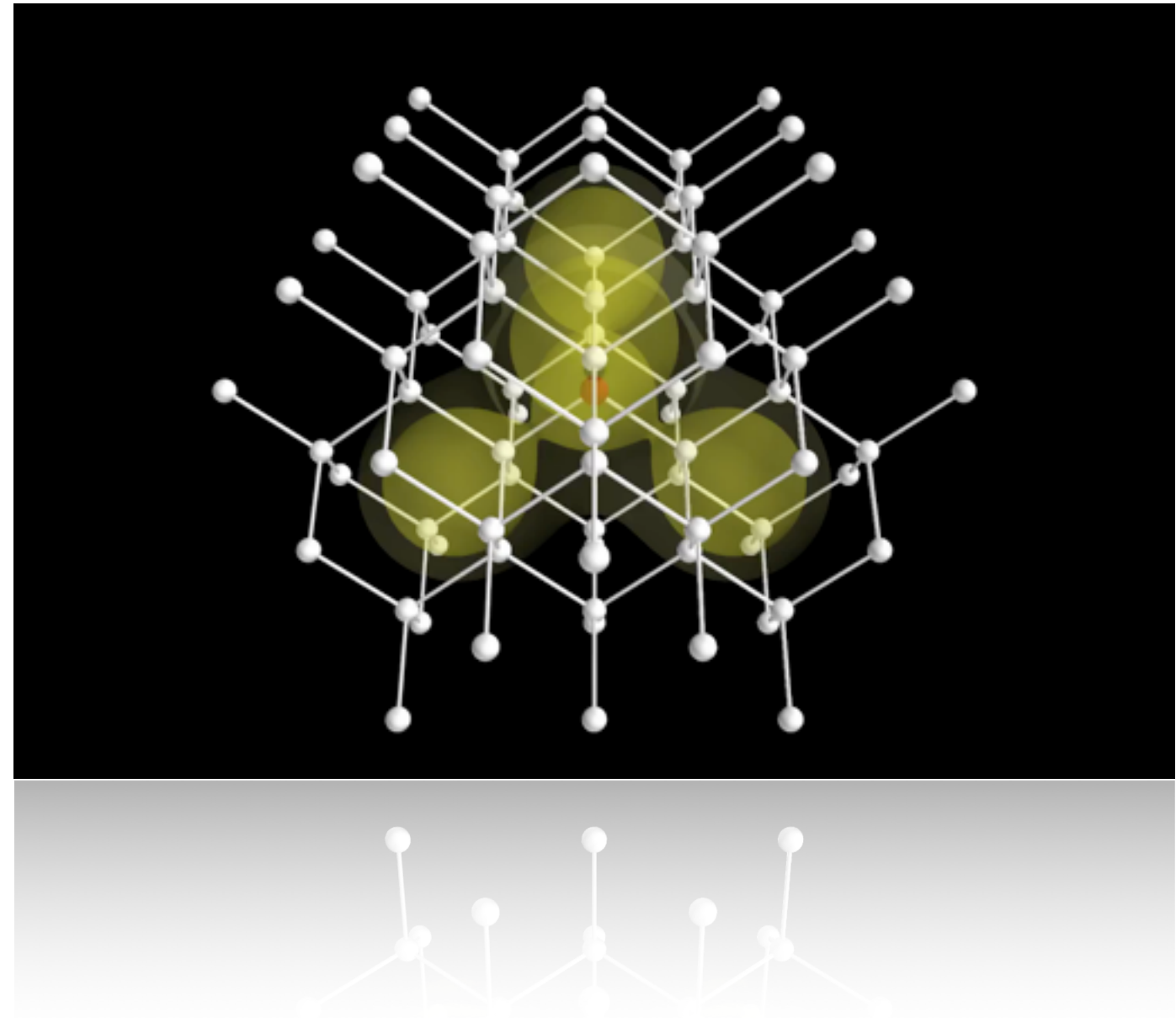
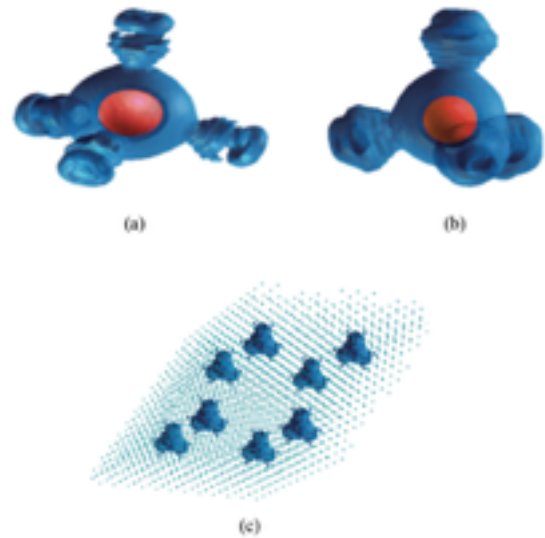
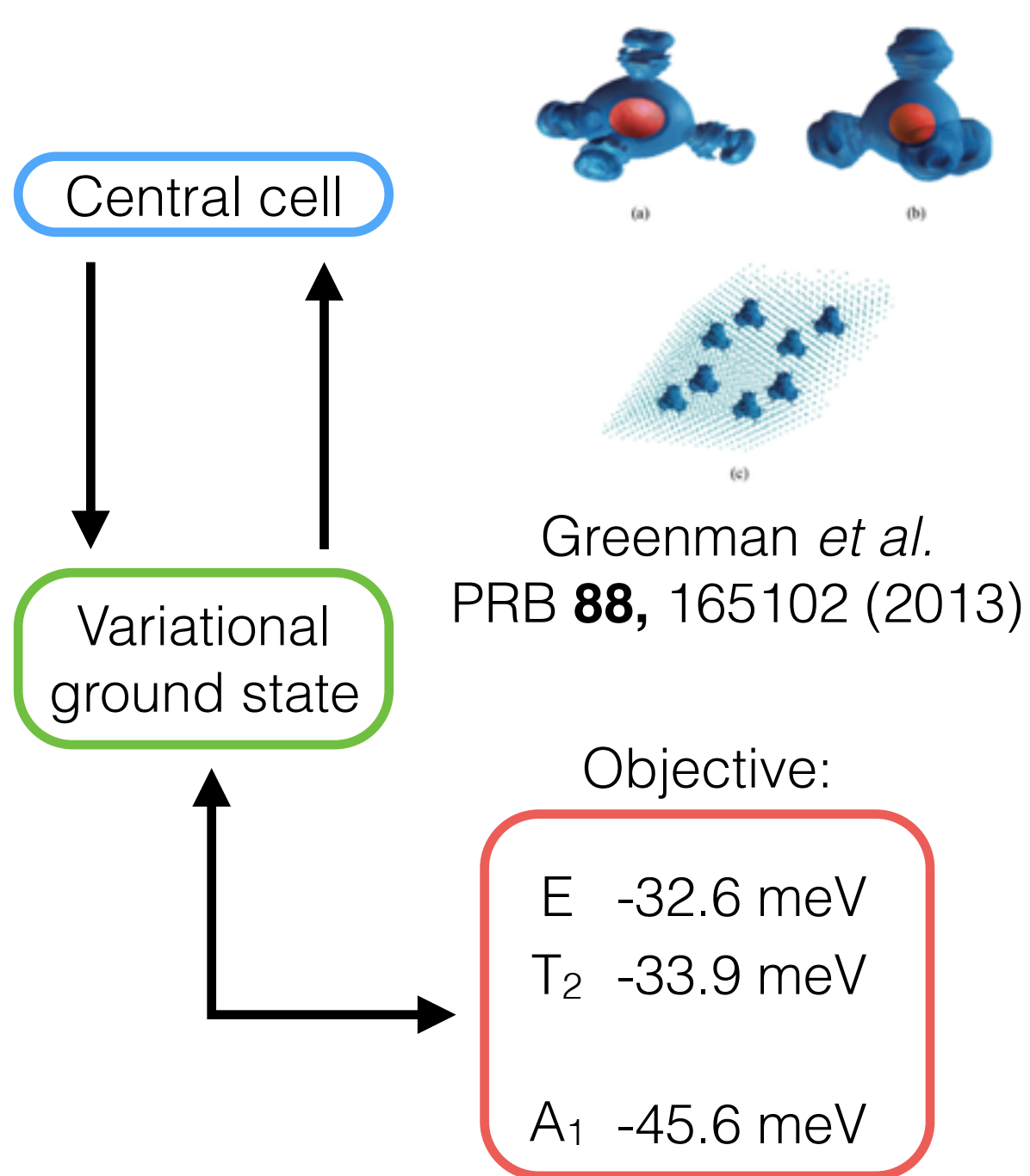
External collaborators

- A. S. Dzurak (UNSW)
- Adam Frees (U-Wisconsin)
- A. Rossi (Cambridge)
- C. H. Yang (UNSW)

Funding through the **Truman Fellowship program** (LDRD).

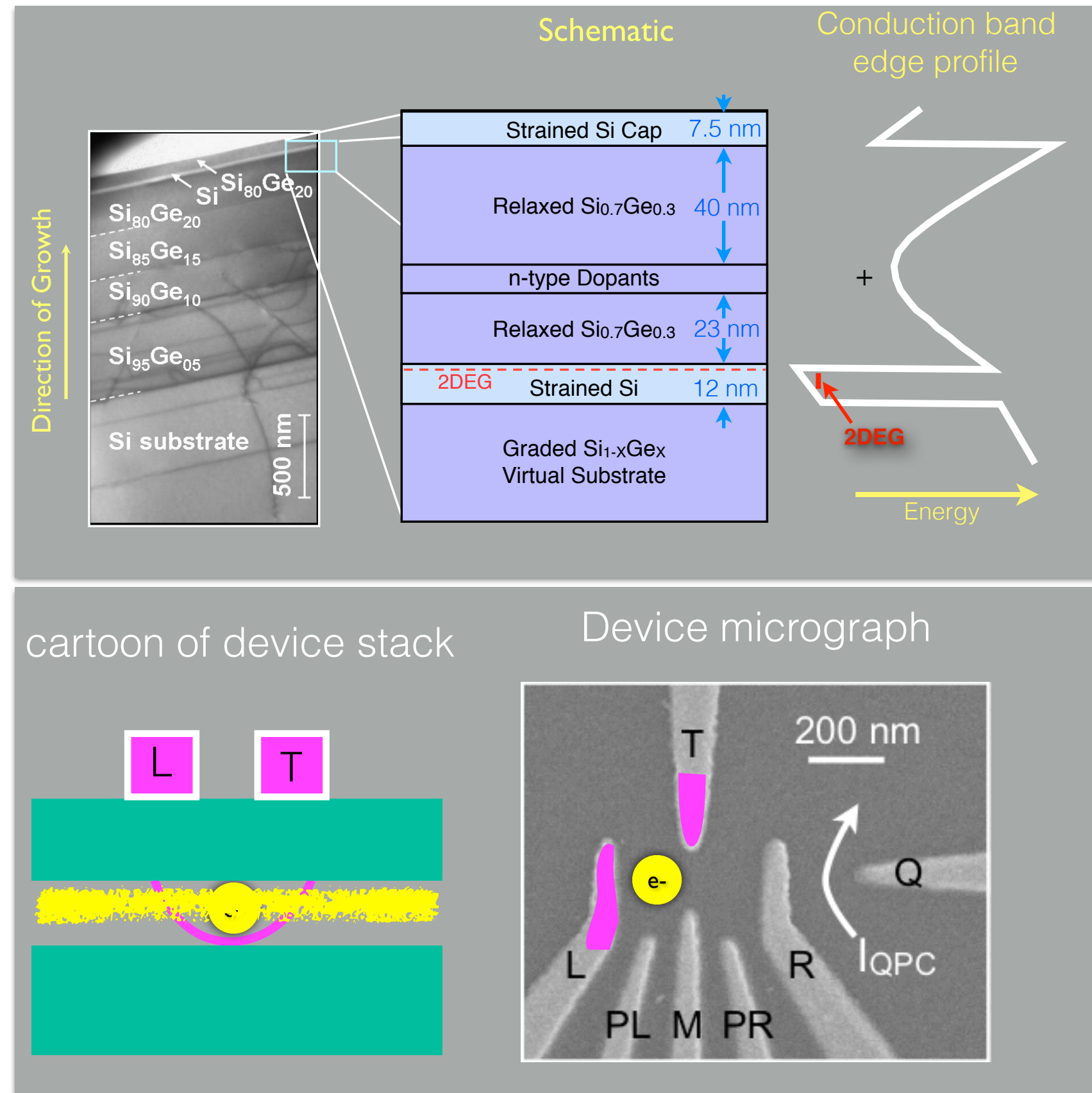
Find a good central cell and donor wave function by two-stage optimization

The bulk-screened coulomb potential is too weak to explain observed binding energies on its own. We need to modify it to get the energies right.



Quantum dot electron spin qubits

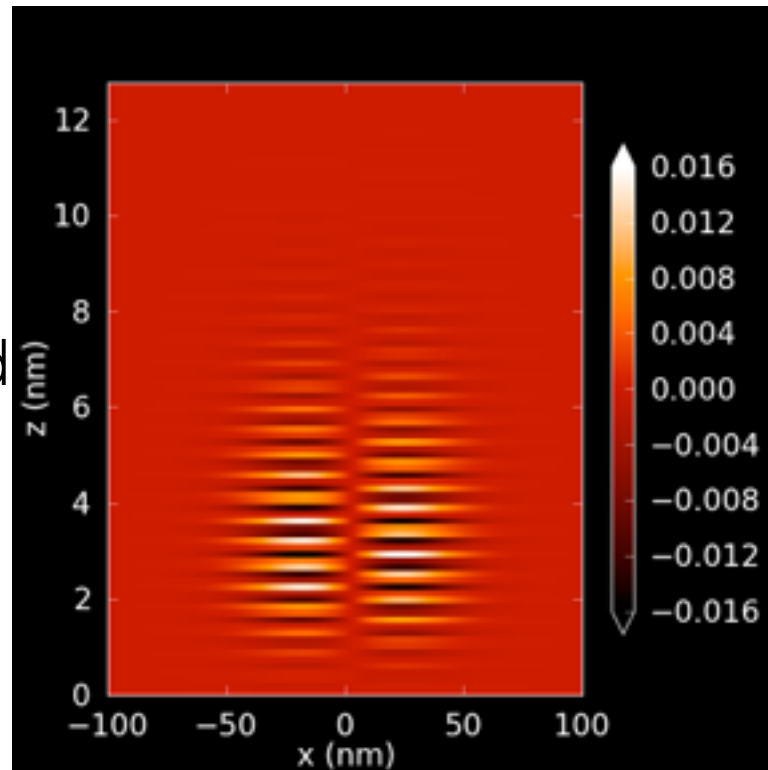
- One example is electrostatically defined, Si/SiGe quantum dots
- Electrons trapped below surface in a two-dimensional electron gas (2DEG) formed in a quantum well of a semiconductor heterostructure
- Once confined in the well, further confined to a quantum dot with electrostatic depletion gates patterned on the top of the sample
- Electron number is controlled by electrically tuning the depletion gates



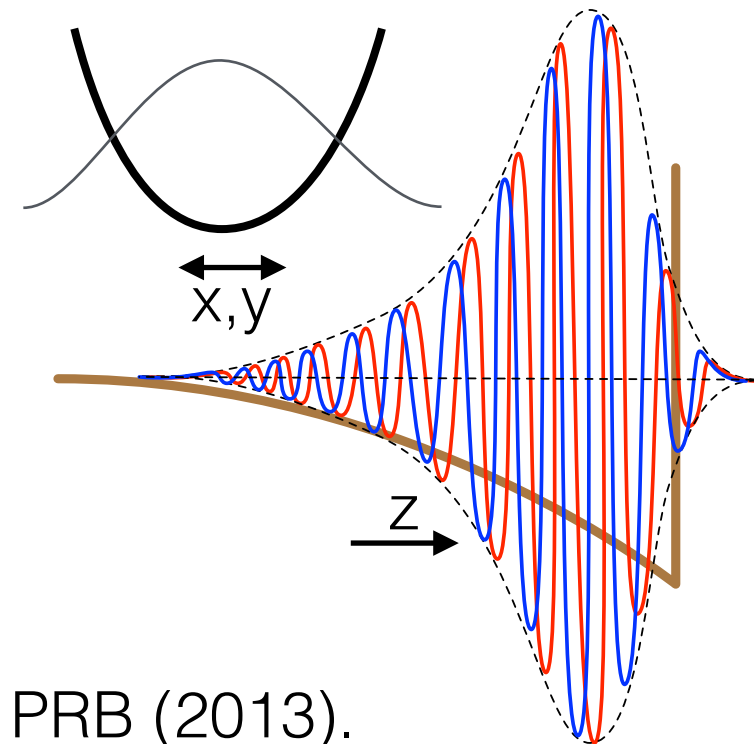
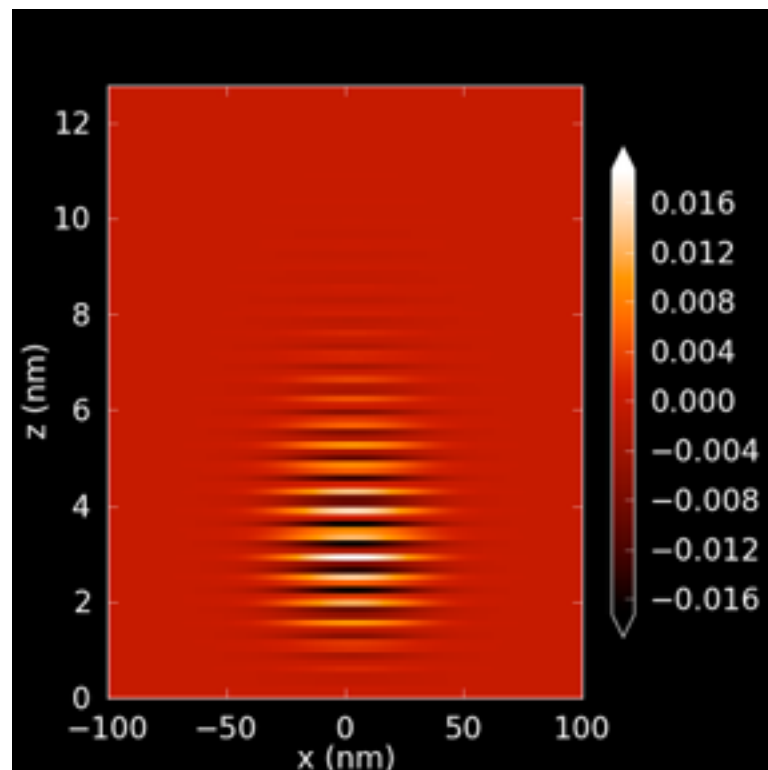
As with donors, the “valley physics” of dots is complicated and multiscale

- Complicated interference patterns develop between different conduction band valleys.
- In real space, the electronic wave function exhibits fast oscillations along the direction of the semiconductor stack (z), modulated by a slow envelope.
- Since we want a two level system, we can't have extra degrees of freedom laying around!

1st excited state



ground state



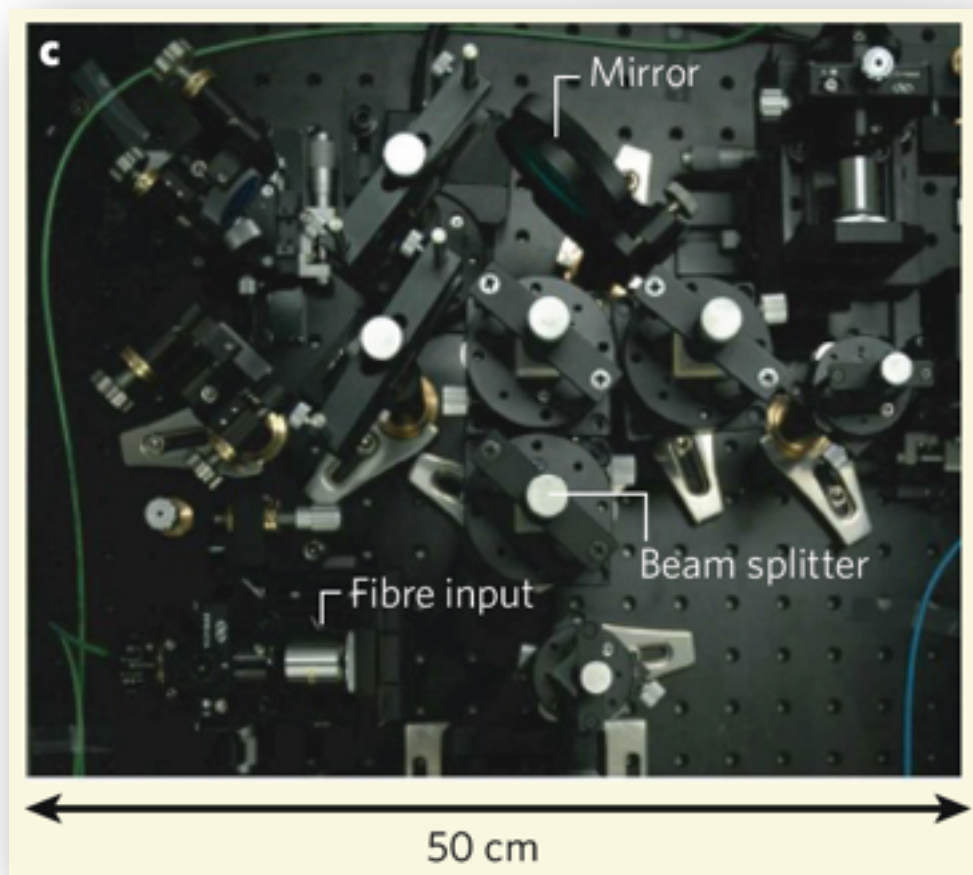
Gamble, *et al.* PRB (2013).

What does this actually look like in a lab?

PMT



APD



Optical Table



Dilution
Refrigerator

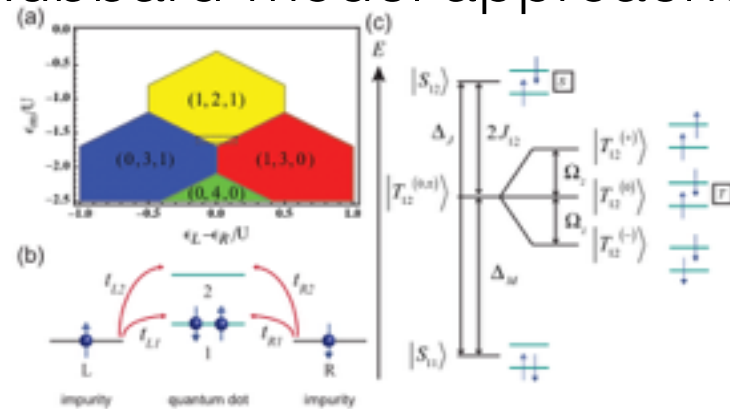


Low T Superconducting
Apparatus

Semiconductor qubits live in here
close to absolute zero

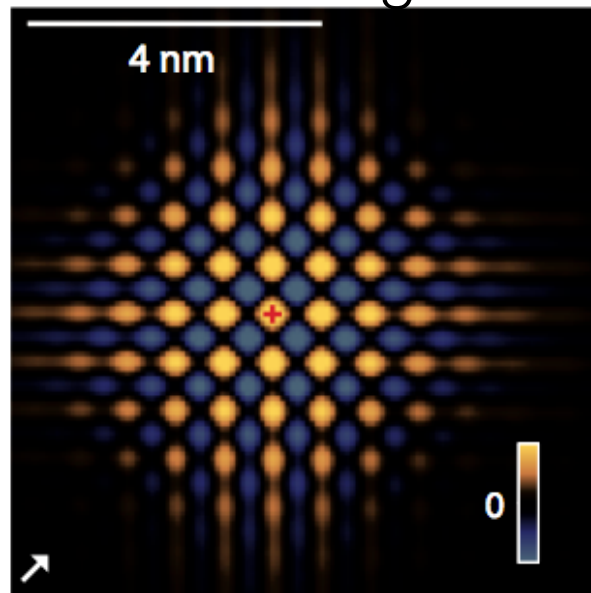
Currently, many different techniques see use in modeling semiconductor QIP systems.

Hubbard-model approaches



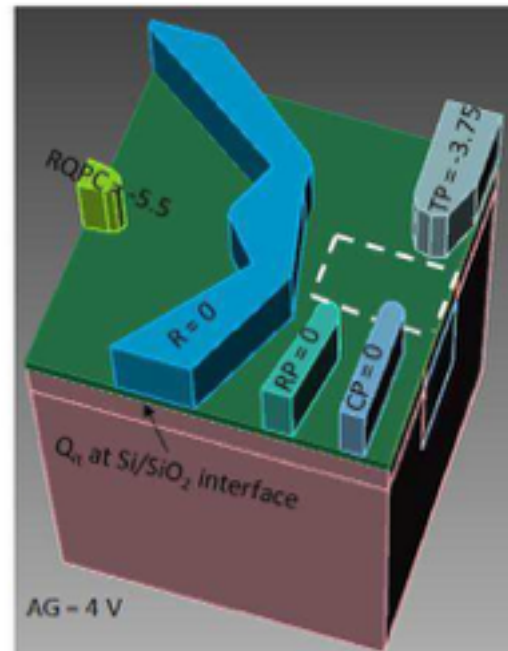
[Srinivasa, et al.
arXiv:1312.1711 (2014)]

Atomistic tight-binding



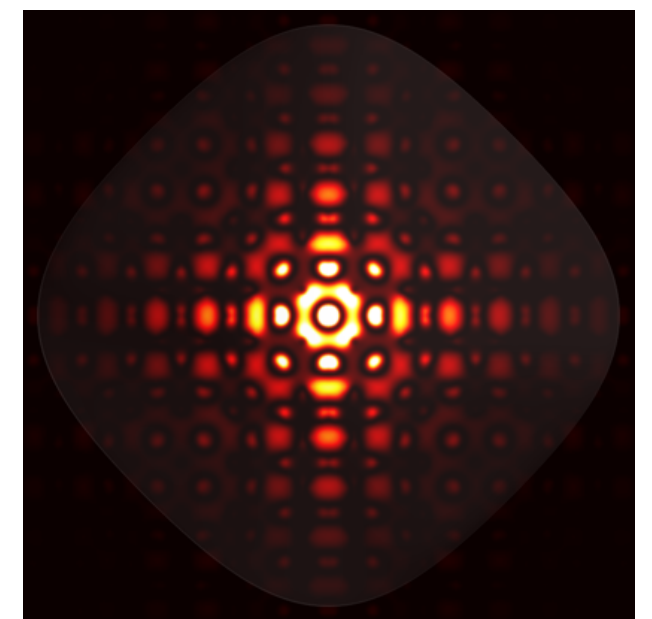
[Salfi, et al. Nat. Mat. 13,
605-610 (2014)]

Device-scale methods



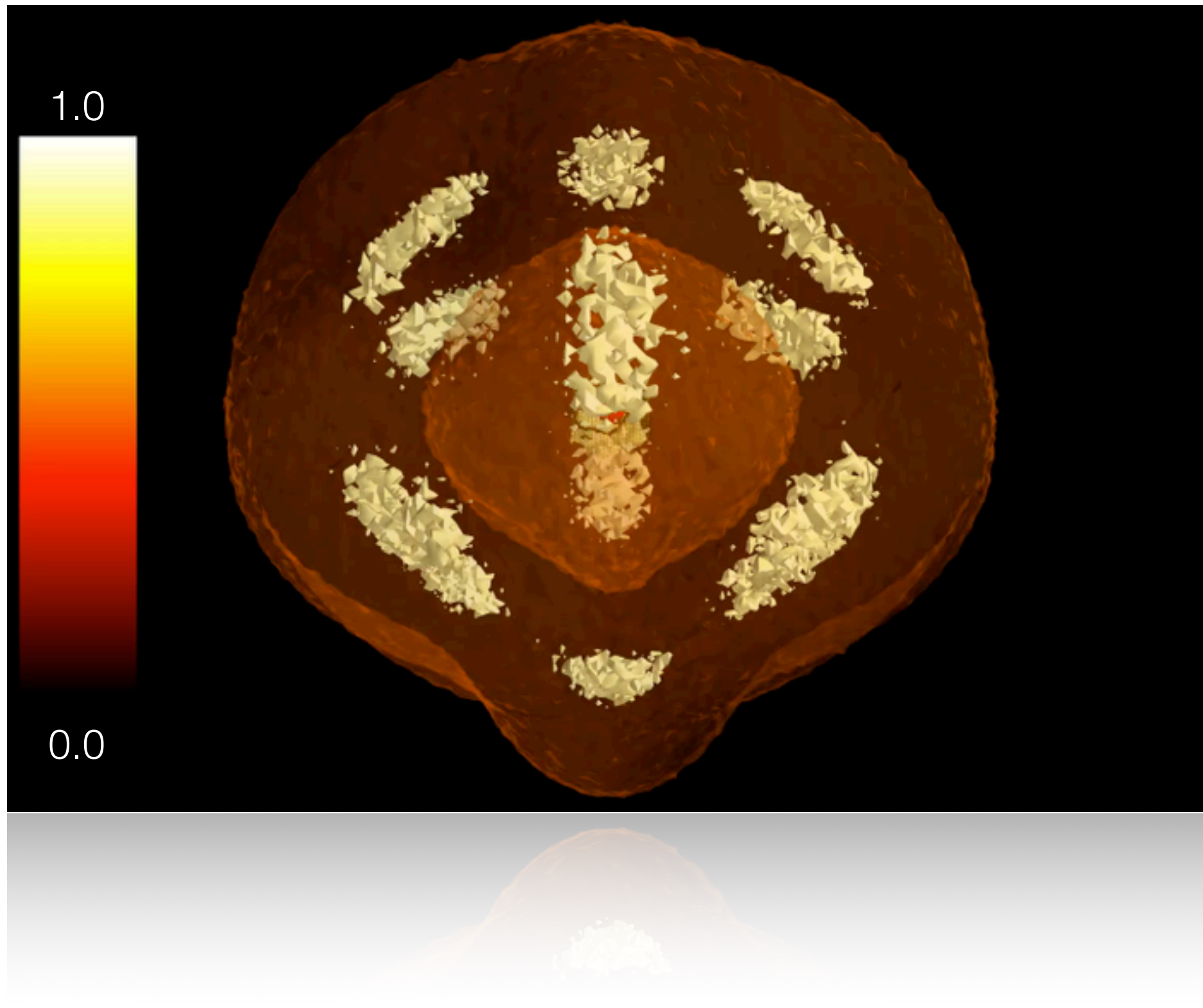
[Gao, et al. J. Appl. Phys. 114,
164302 (2013)]

Effective mass theory



[Gamble & Jacobson, et al.
PRB **91**, 235318 (2015)]

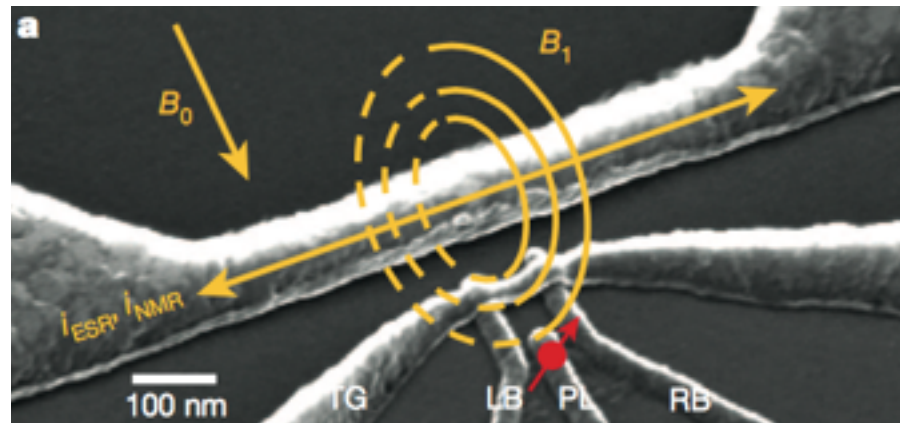
We identify “hot spots” with statistically stable tunnel coupling



Accept:
 $0.1 \text{ meV} < t < 2 \text{ meV}$

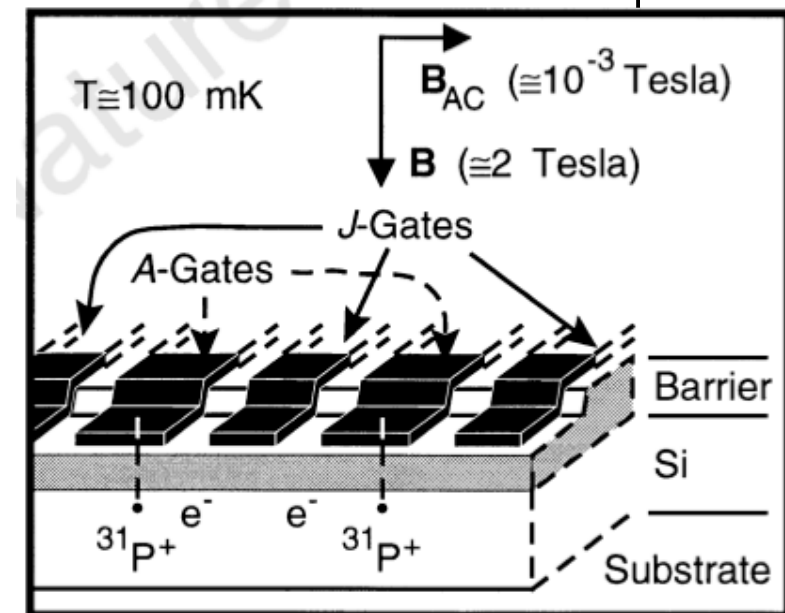
Max prob.:
0.93 at 10 nm along [110]

Straggle:
1 nm isotropic gaussian



J. Pla, *et al.*, Nature (2013).

Electrons bound to impurities



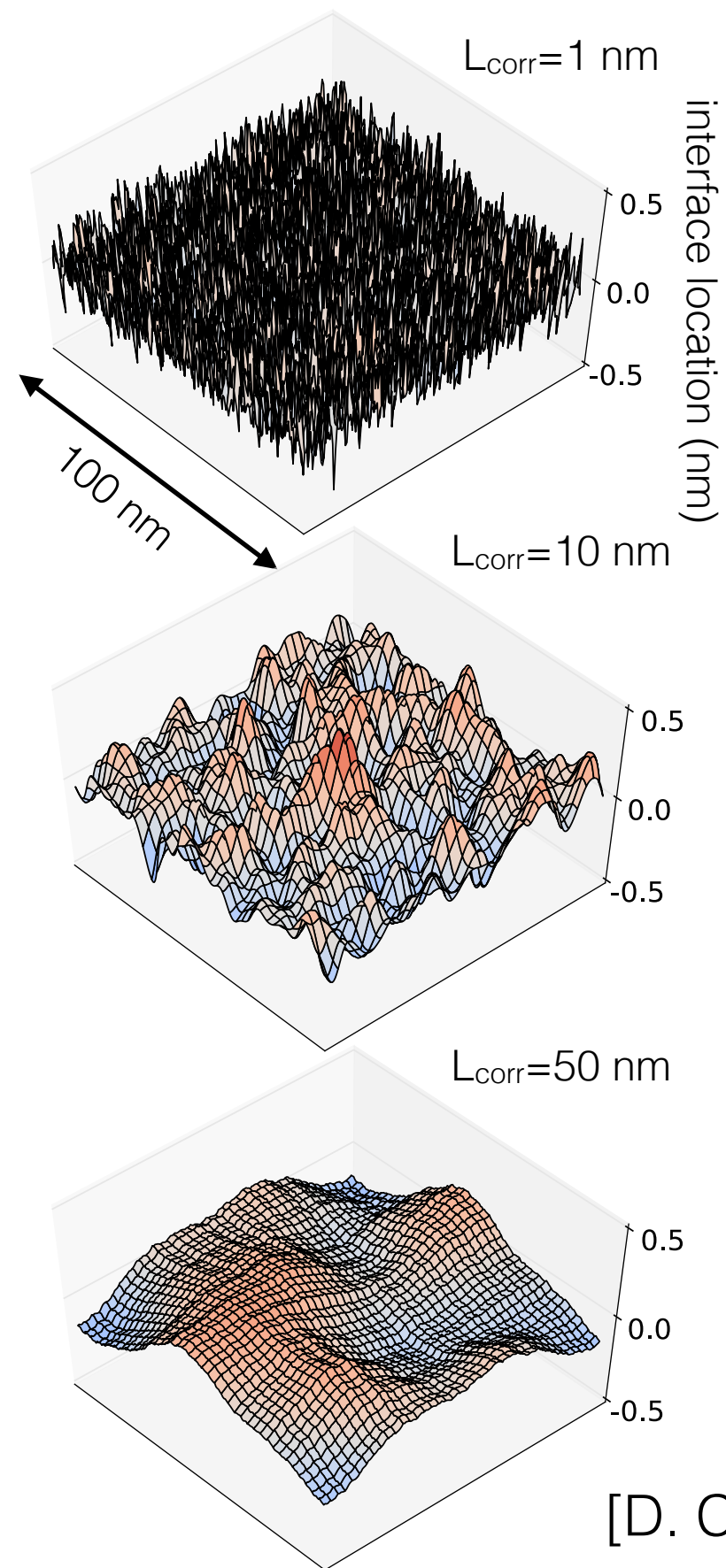
Bruce Kane, Nature (1998).

- Kane quantum computer uses phosphorus donors in silicon as qubits.
- Like hydrogen atoms, except fixed in place in the crystal.
- Experimentally demonstrated!
- But the electronic structure of silicon is complicated...

Question: Can we couple two donor qubits together?

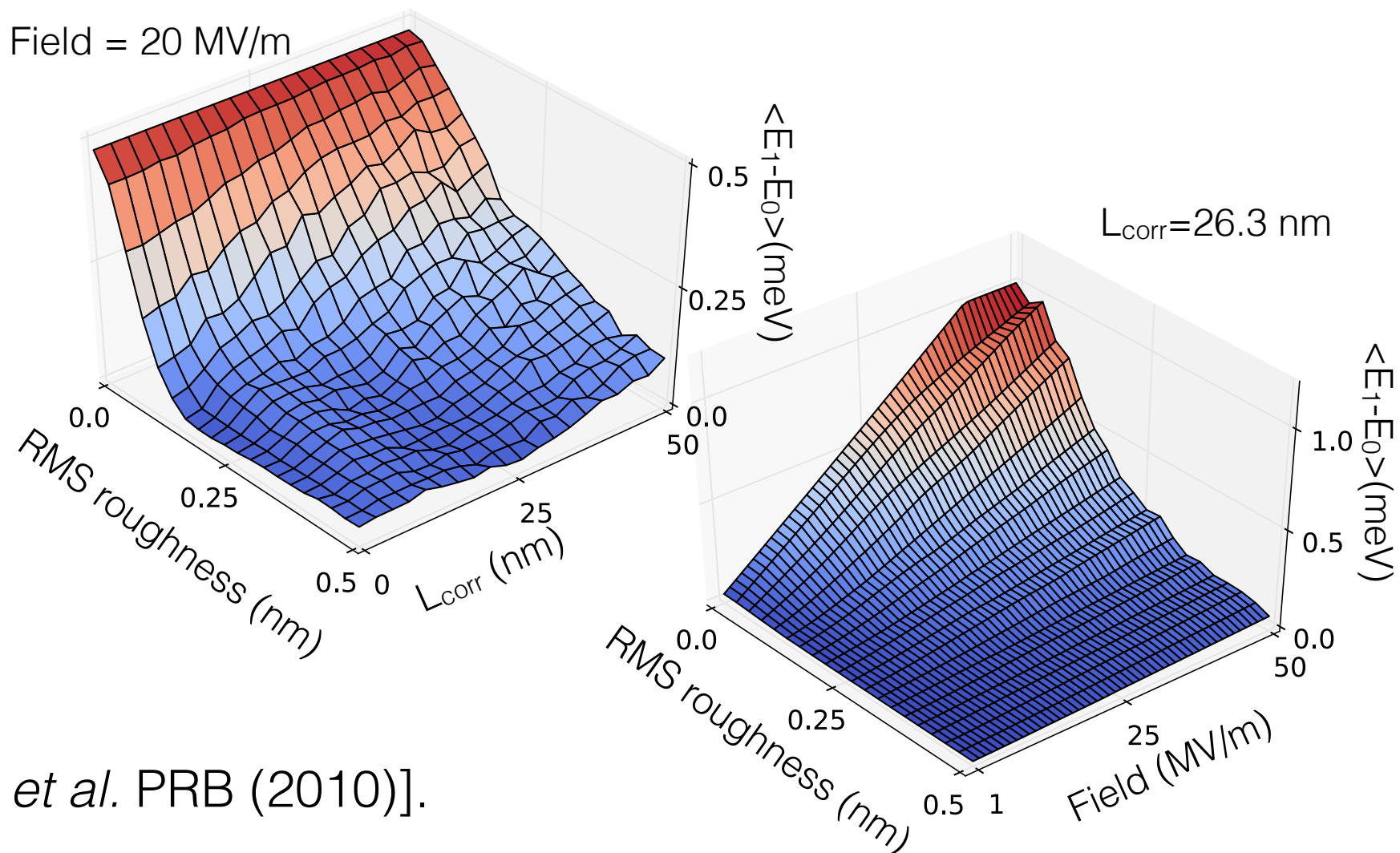
Answer: Probably not, at least not with high probability.

Interface roughness strongly affects valley splitting.



- We simulate rough interfaces by sampling random cases parameterized by RMS roughness and correlation length.
- We have:
 - 20x20 grid of roughness parameters
 - 65 disorder realizations each
 - ~ 100 voltage + offset configurations
- = 2.6 million **non-perturbative** calculations.

Field = 20 MV/m



[D. Culcer *et al.* PRB (2010)].

Realistic interface roughness is consistent with observed valley splitting.

