



SAND2017-5824C

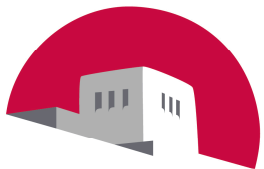


Spin Squeezing on Nanophotonic Waveguides

Xiaodong Qi,¹ David Melchior,² Poul Jessen,²
Jongmin Lee,³ Yuan-Yu Jau,³ and Ivan H. Deutsch¹

2017-06-09

[1]



UNM

[2]



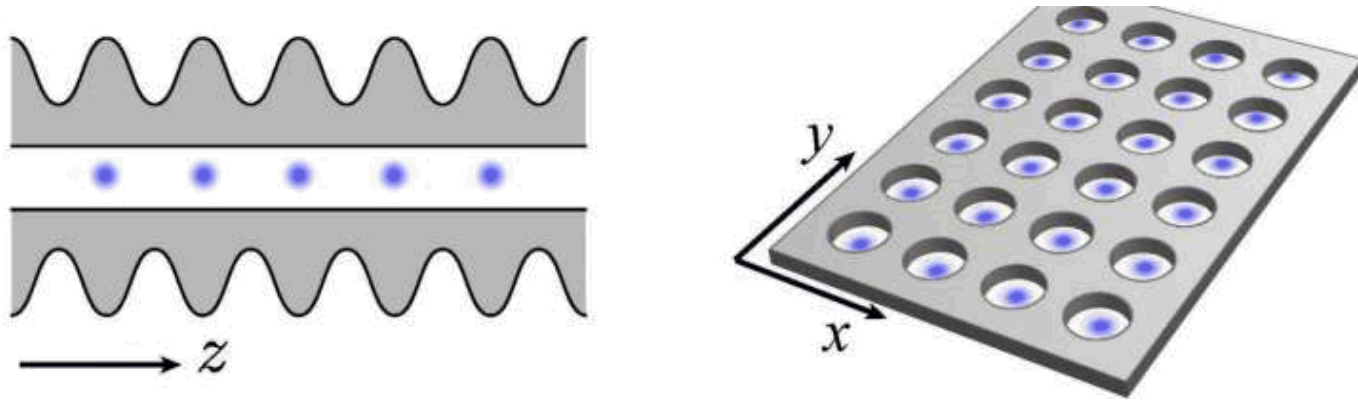
THE UNIVERSITY
OF ARIZONA®

[3]

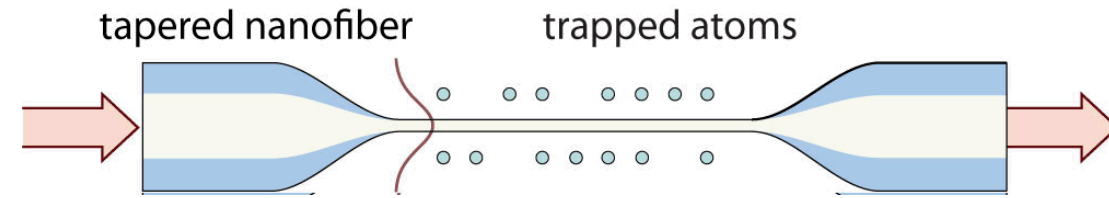


Sandia
National
Laboratories

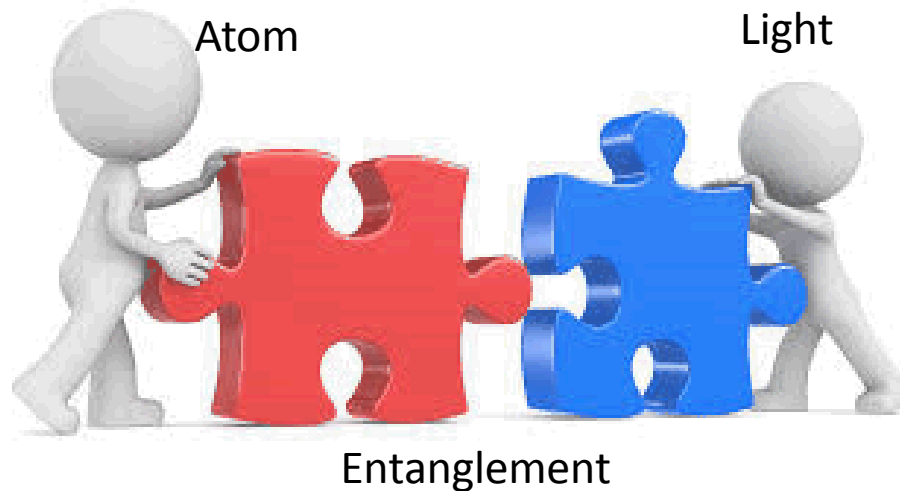
Quantum Atom-Light Interfaces with Nanophotonics



Proc Natl Acad Sci U S A. 113 (2016)



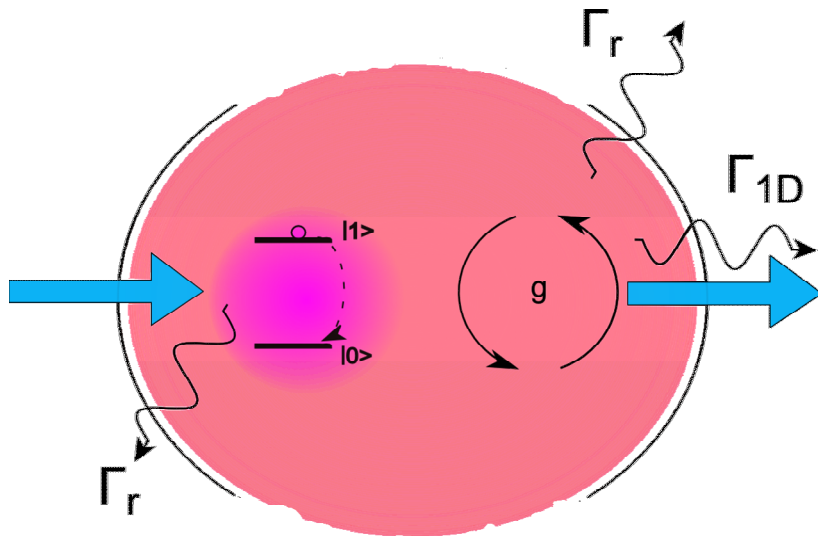
Phy Rev A 93, 023817 (2016)



Strong Atom-Photon Coupling in Photonics

Cooperativity:

$$C \equiv \mathcal{F} \frac{\Gamma_{1D}}{\Gamma_0} = \mathcal{F} \frac{\sigma_0}{A_{\text{eff}}}$$



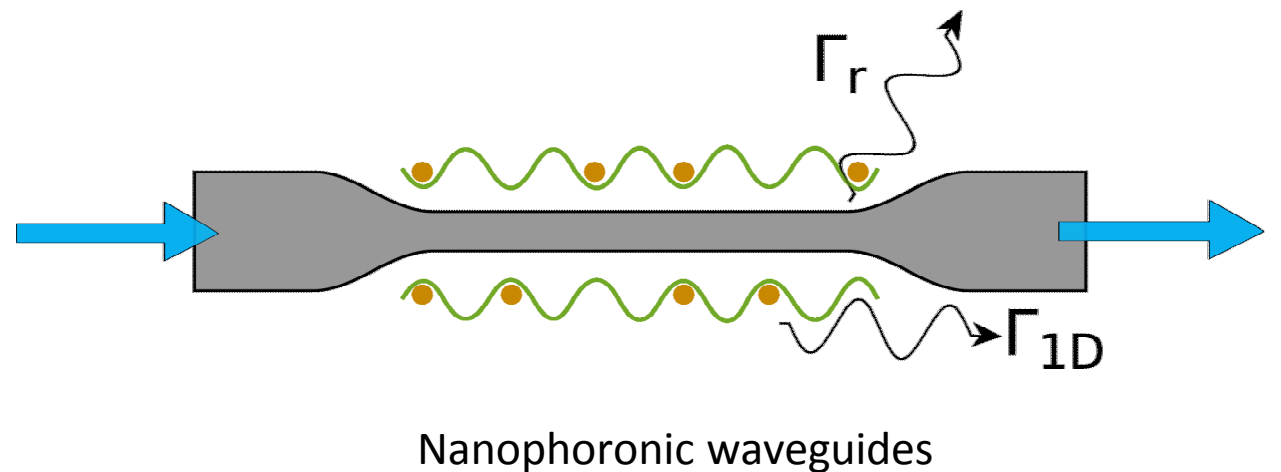
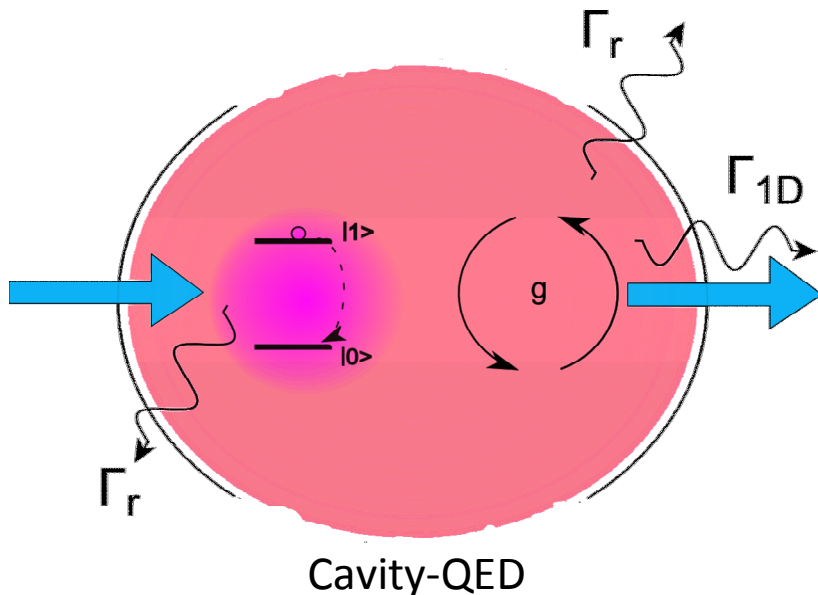
Cavity-QED

Strong Atom-Photon Coupling in Photonics

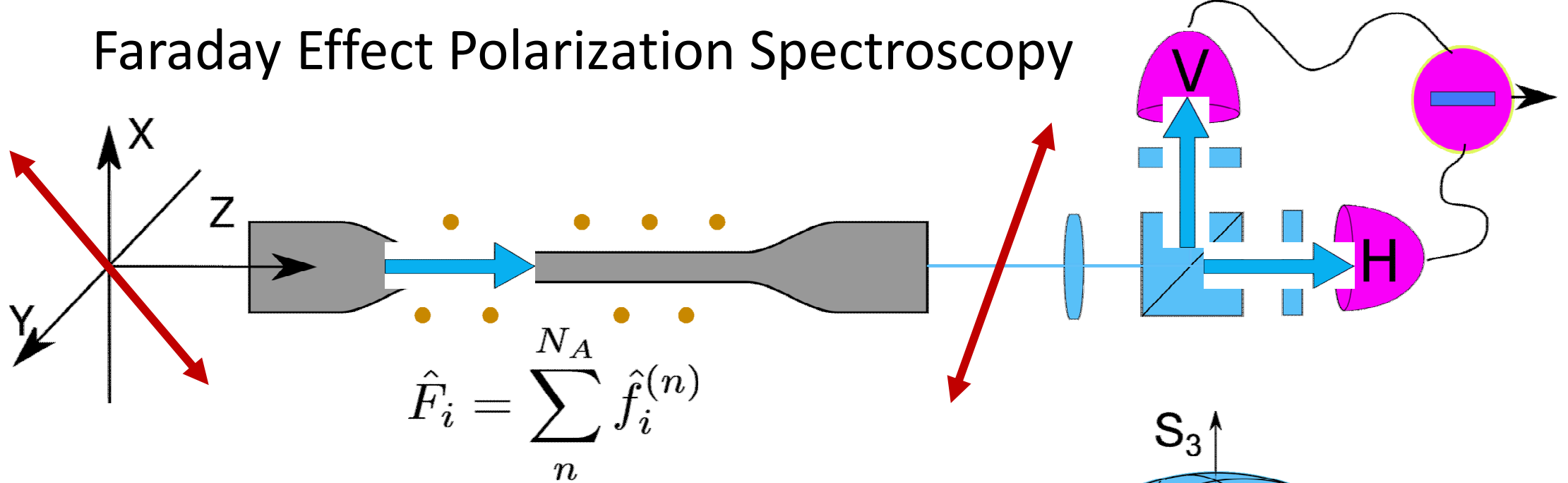
Cooperativity:

$$C \equiv \mathcal{F} \frac{\Gamma_{1D}}{\Gamma_0} = \mathcal{F} \frac{\sigma_0}{A_{\text{eff}}}$$

$$C = N_A \frac{\Gamma_{1D}}{\Gamma_0} = N_A \frac{\sigma_0}{A_{\text{eff}}} = N_A \frac{OD_{\text{eff}}}{N_A}$$



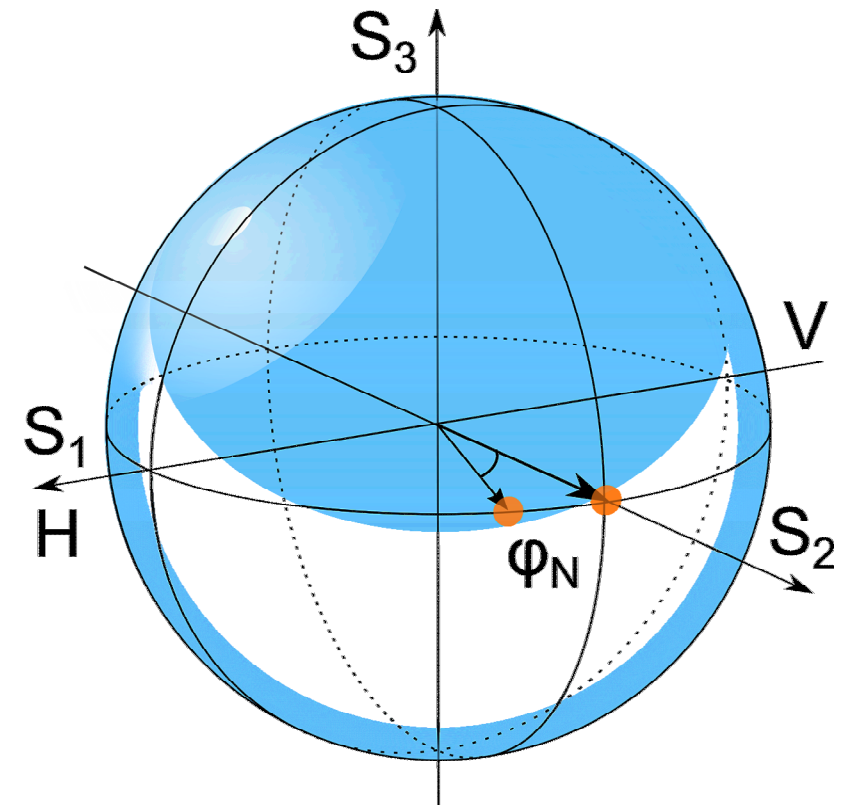
Faraday Effect Polarization Spectroscopy



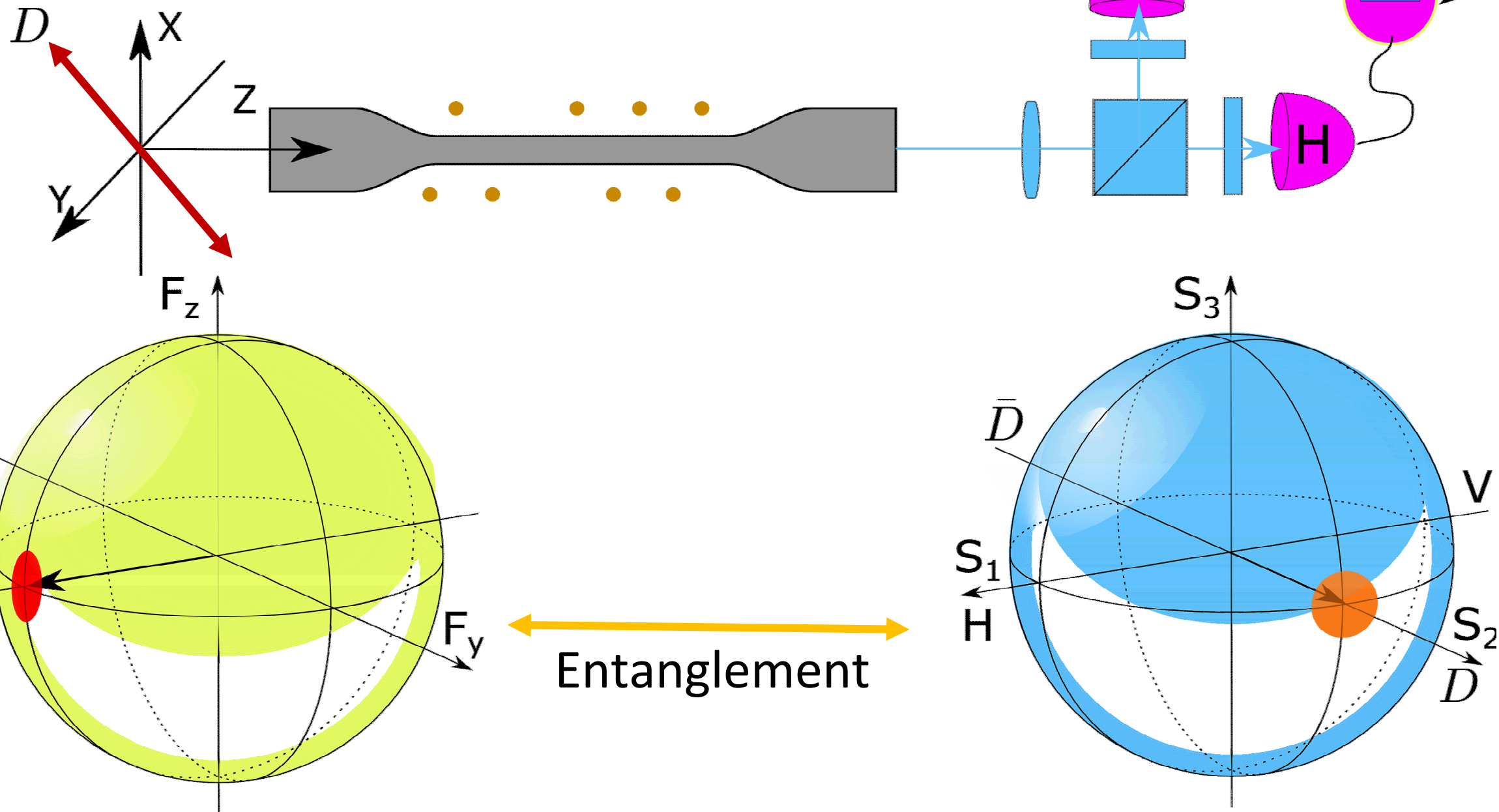
Light-atom interaction Hamiltonian

$$\begin{aligned} \hat{H}_{LS} &= - \sum_{n=1}^{N_A} \hat{\mathbf{E}}^{(-)}(\mathbf{r}'_n) \cdot \hat{\boldsymbol{\alpha}}^{(n)} \cdot \hat{\mathbf{E}}^{(+)}(\mathbf{r}'_n) \\ &= \hbar \sum_{i,j} \chi_{ij} \hat{F}_i \hat{S}_j \\ &\rightarrow \hbar \chi_{33} \hat{F}_z \hat{S}_3 \end{aligned}$$

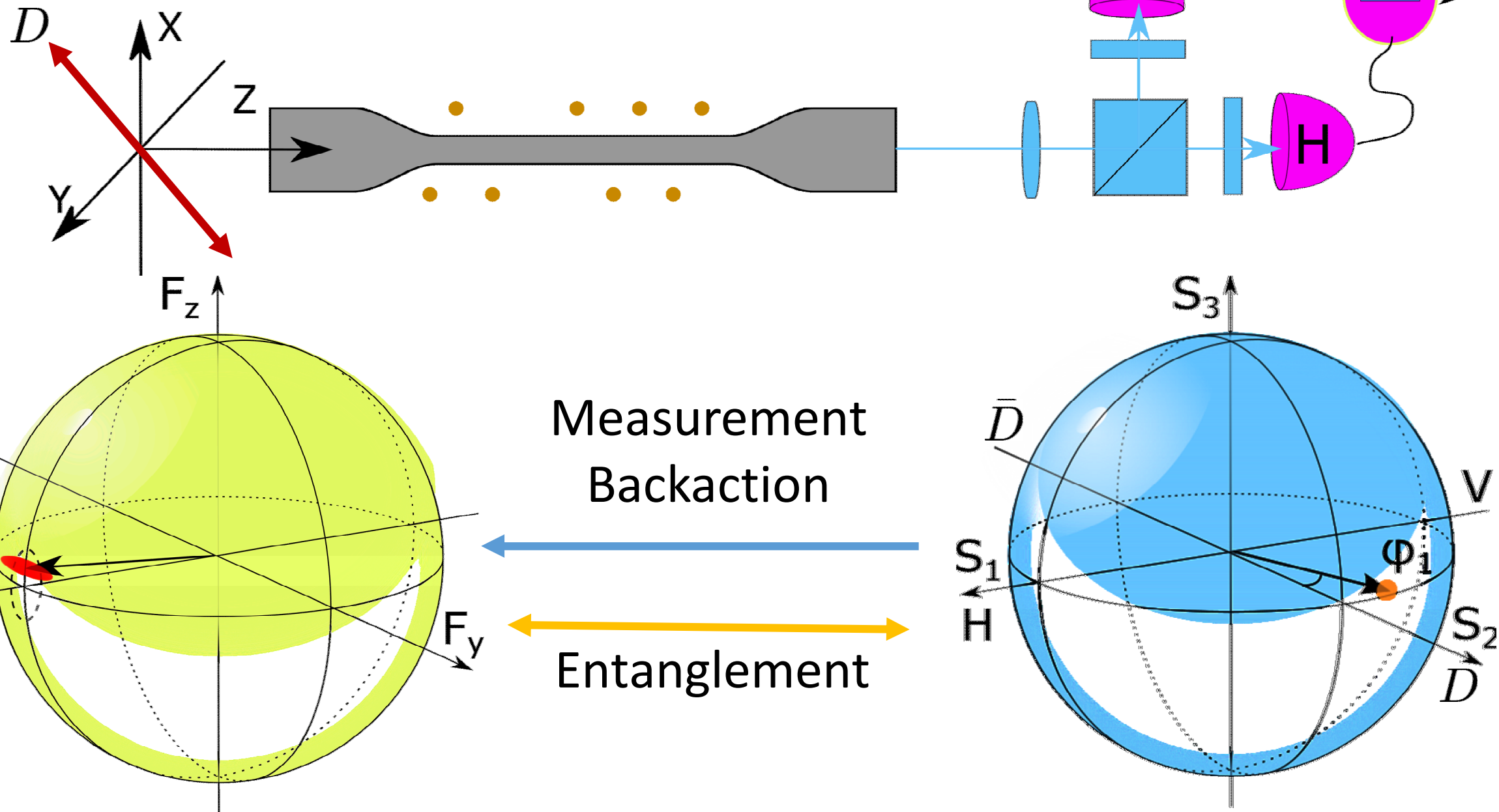
$\hat{S}_i$ are the Stokes vector operators.



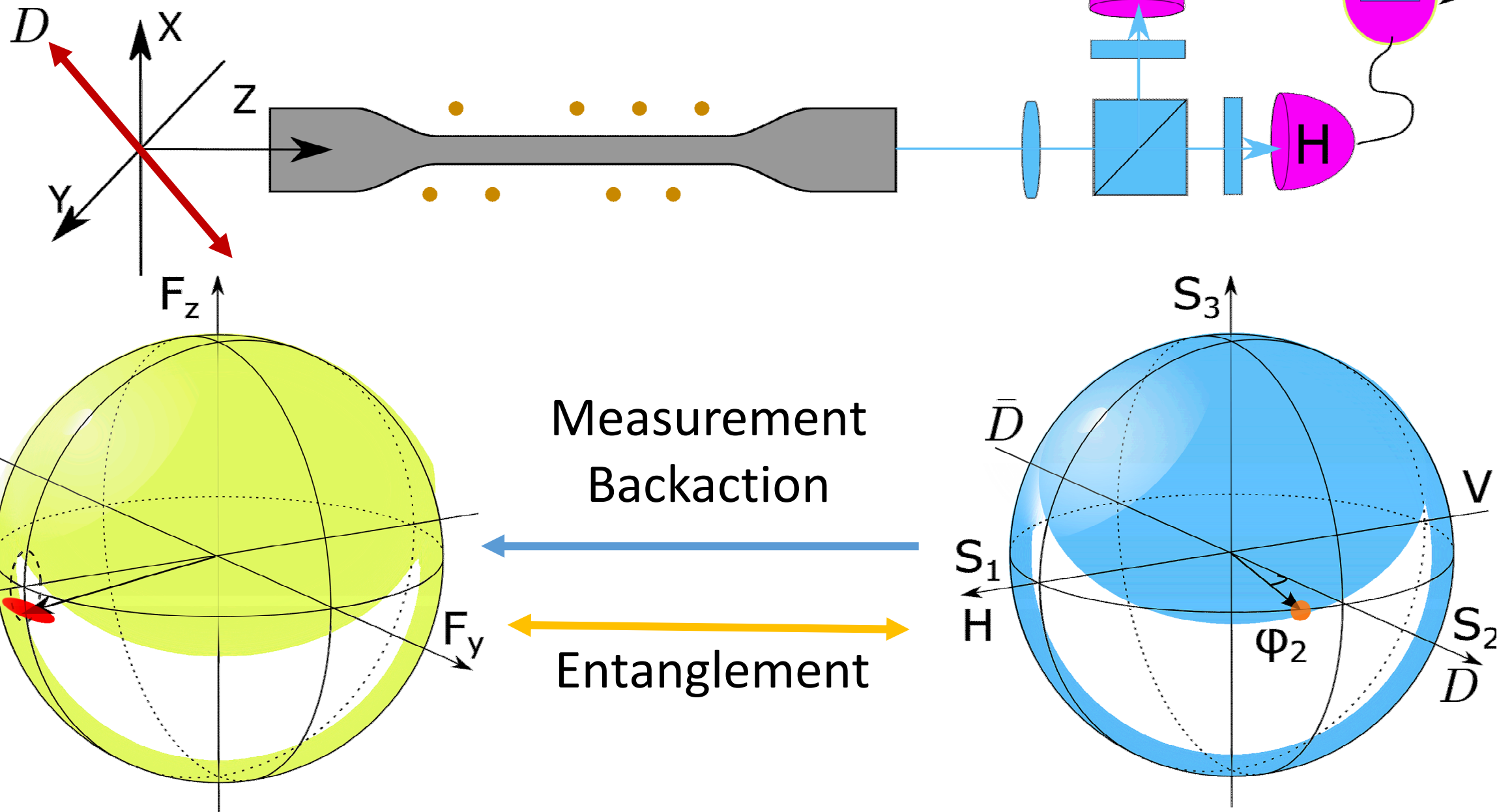
Squeezing via QND Measurement



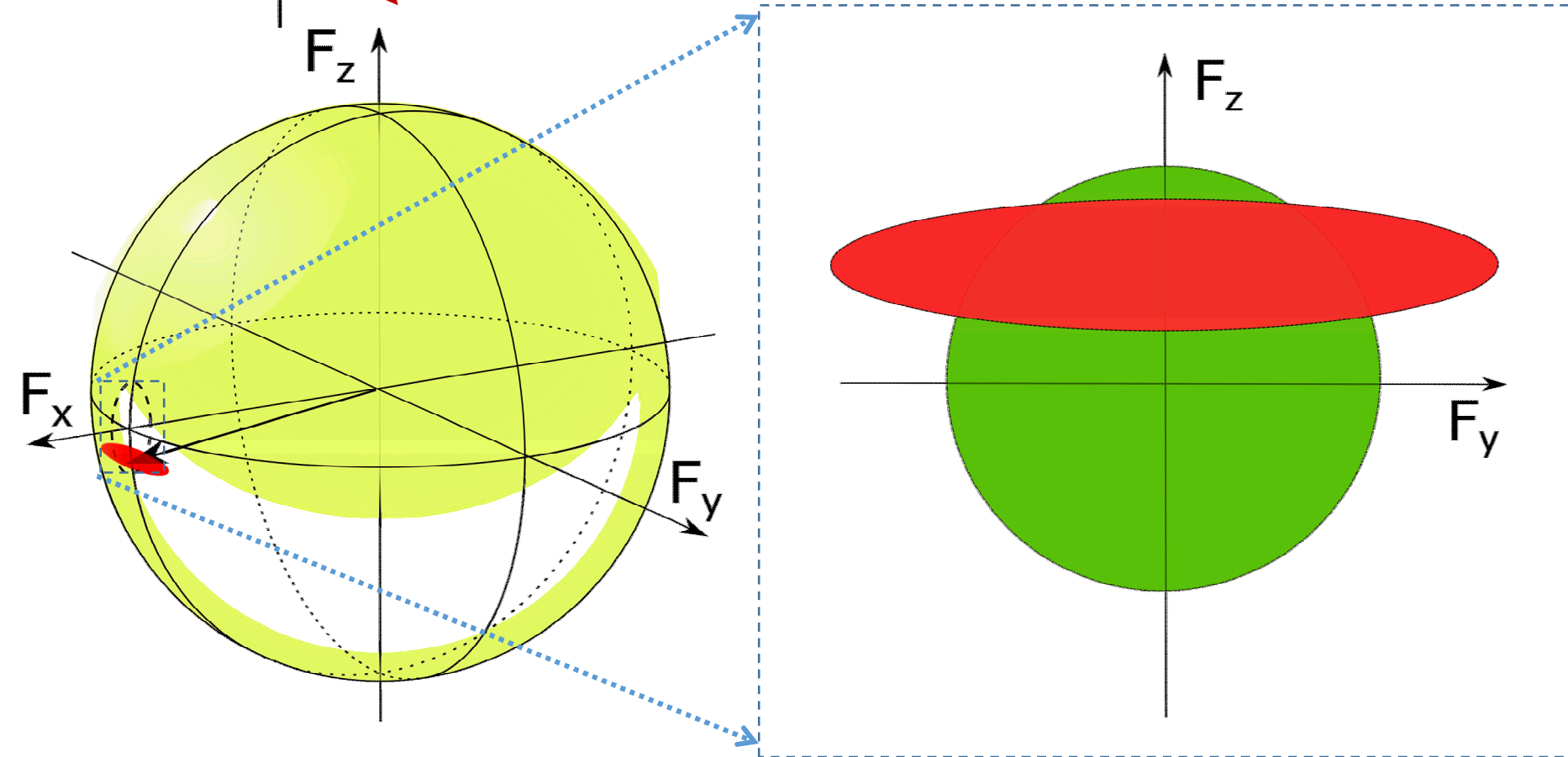
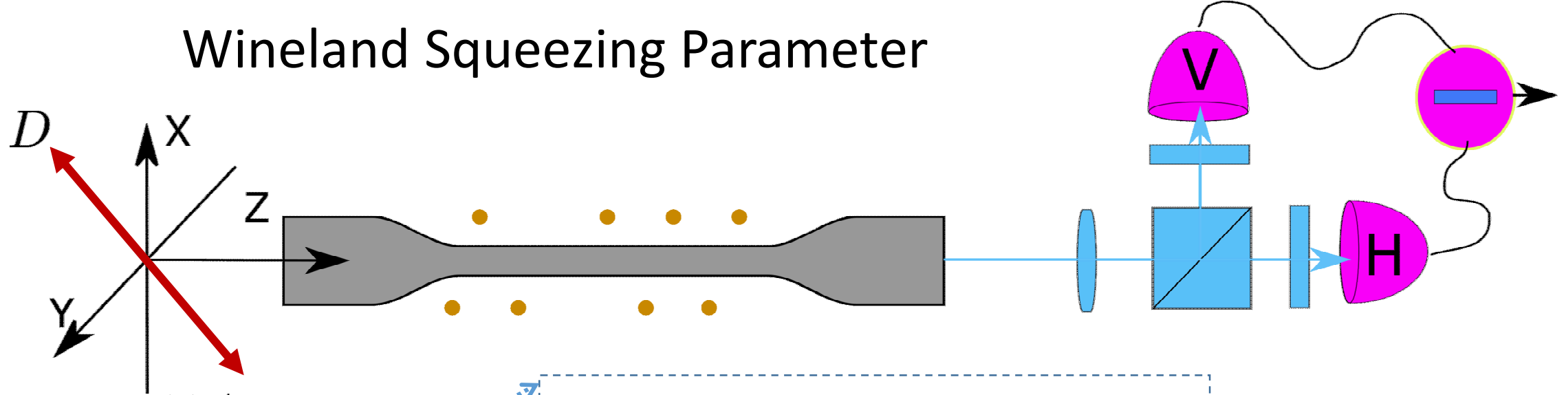
Squeezing via QND Measurement



Squeezing via QND Measurement



Wineland Squeezing Parameter



$$\xi^2 = \frac{2N_A}{f} \frac{\Delta F_z^2}{|\langle \hat{F}_x \rangle|^2}$$

Wineland squeezing parameter

Modeling Spin Squeezing Dynamics

- Spin squeezing parameter

$$\xi^2 = \frac{2N_A}{f} \frac{\Delta F_z^2}{|\langle \hat{F}_x \rangle|^2}$$

- One-body and two-body operators that matter

$$\Delta F_z^2 = \underbrace{N_A (\Delta \hat{f}_z^{(1)})^2}_{\text{one-body}} + \underbrace{N_A (N_A - 1) \langle \Delta \hat{f}_z^{(1)} \Delta \hat{f}_z^{(2)} \rangle}_{\text{two-body}}$$
$$\langle \hat{F}_x \rangle = \underbrace{N_A \langle \hat{f}_x^{(1)} \rangle}_{\text{one-body}}$$

Dynamics of QND Measurement with Decoherence

- Stochastic master equation

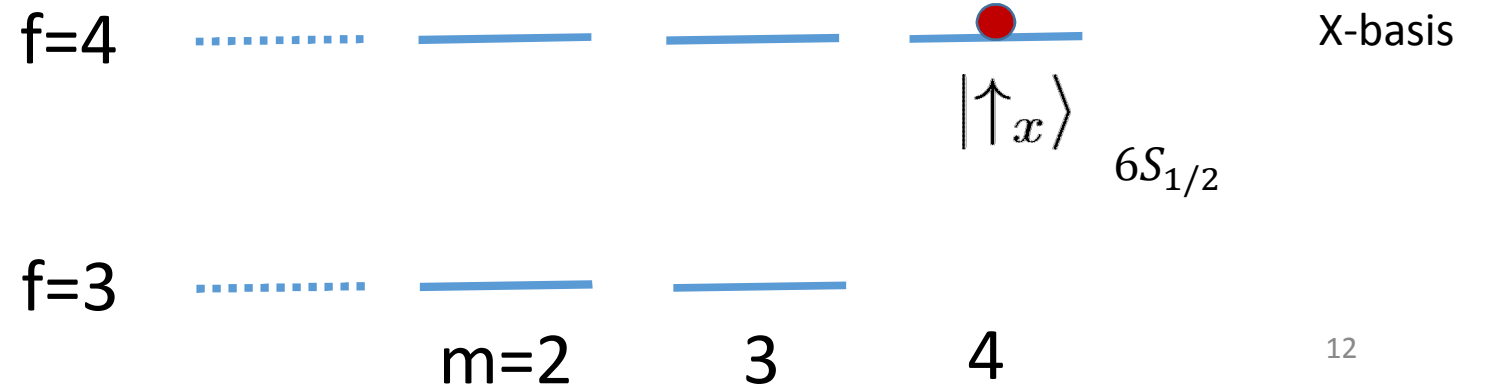
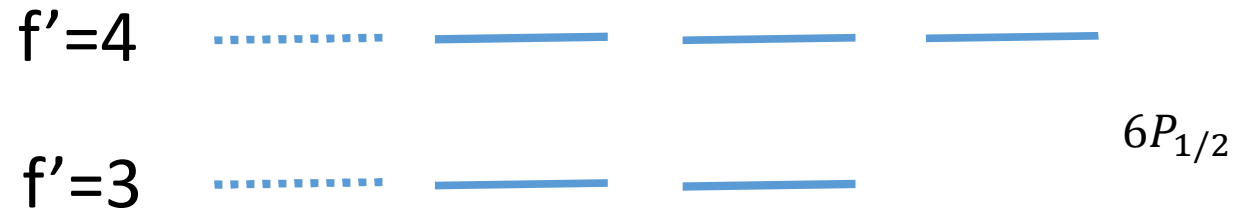
$$\mathcal{H}[\hat{\rho}] = \hat{F}_z \hat{\rho} + \hat{\rho} \hat{F}_z - 2\langle \hat{F}_z \rangle \hat{\rho}, \quad \mathcal{L}[\hat{\rho}] = -\frac{1}{2}(\hat{\rho} \hat{F}_z^2 + \hat{F}_z^2 \hat{\rho}) + \hat{F}_z \hat{\rho} \hat{F}_z.$$

$$d\hat{\rho} = \underbrace{s \sqrt{\frac{\kappa}{4}} \mathcal{H}[\hat{\rho}] dW + \frac{\kappa}{4} \mathcal{L}[\hat{\rho}] dt}_{\text{collective measurement}} + \underbrace{\gamma_s \sum_n \mathcal{D}_n[\hat{\rho}] dt}_{\text{optical pumping}},$$

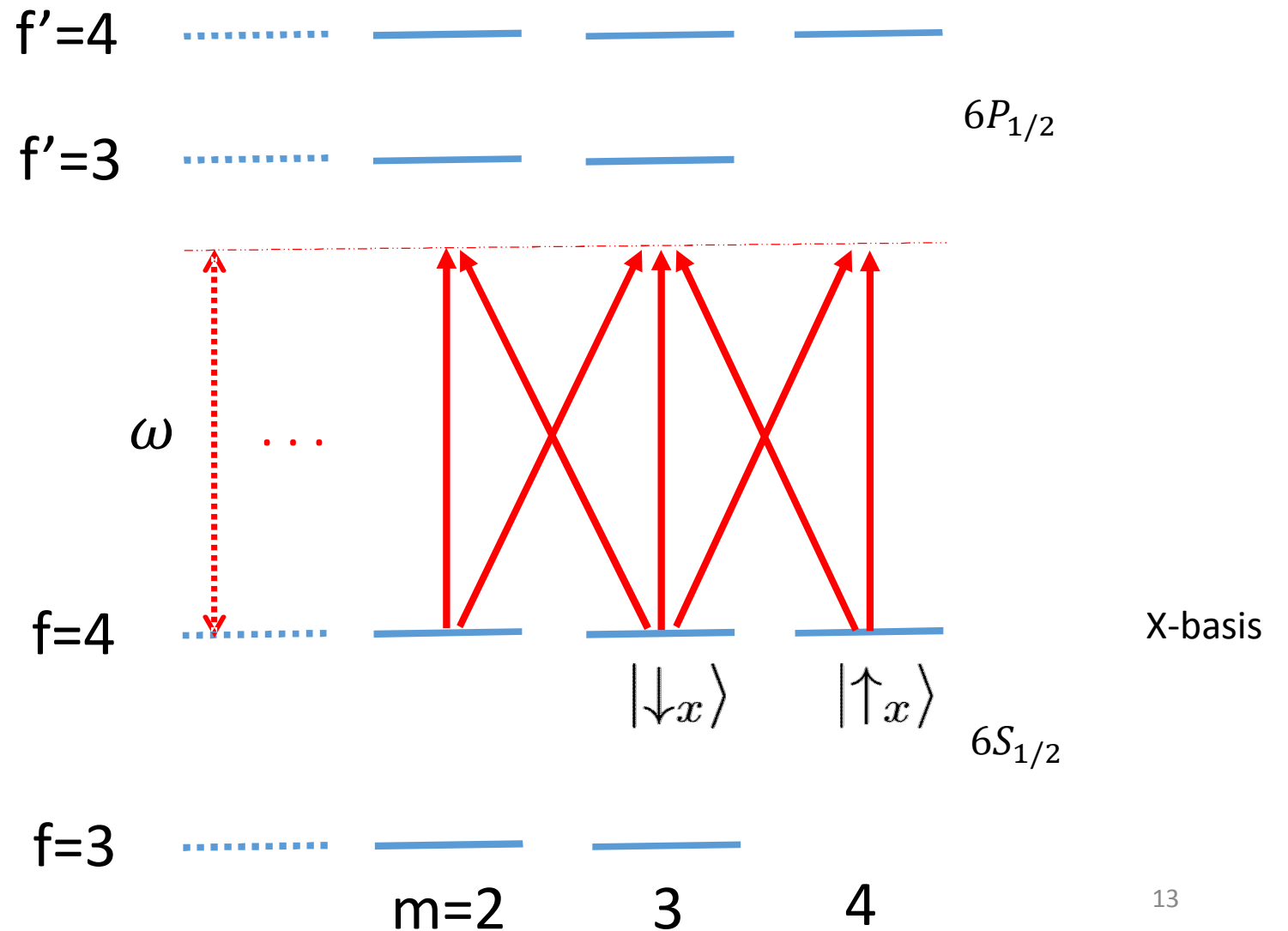
where $s = \text{sign}(\chi_{33})$ and $\hat{\rho}$ is the collective atomic state.

$$\langle \hat{O} \rangle = \text{tr} [\hat{\rho} \hat{O}].$$

Coherent coupling and optical pumping



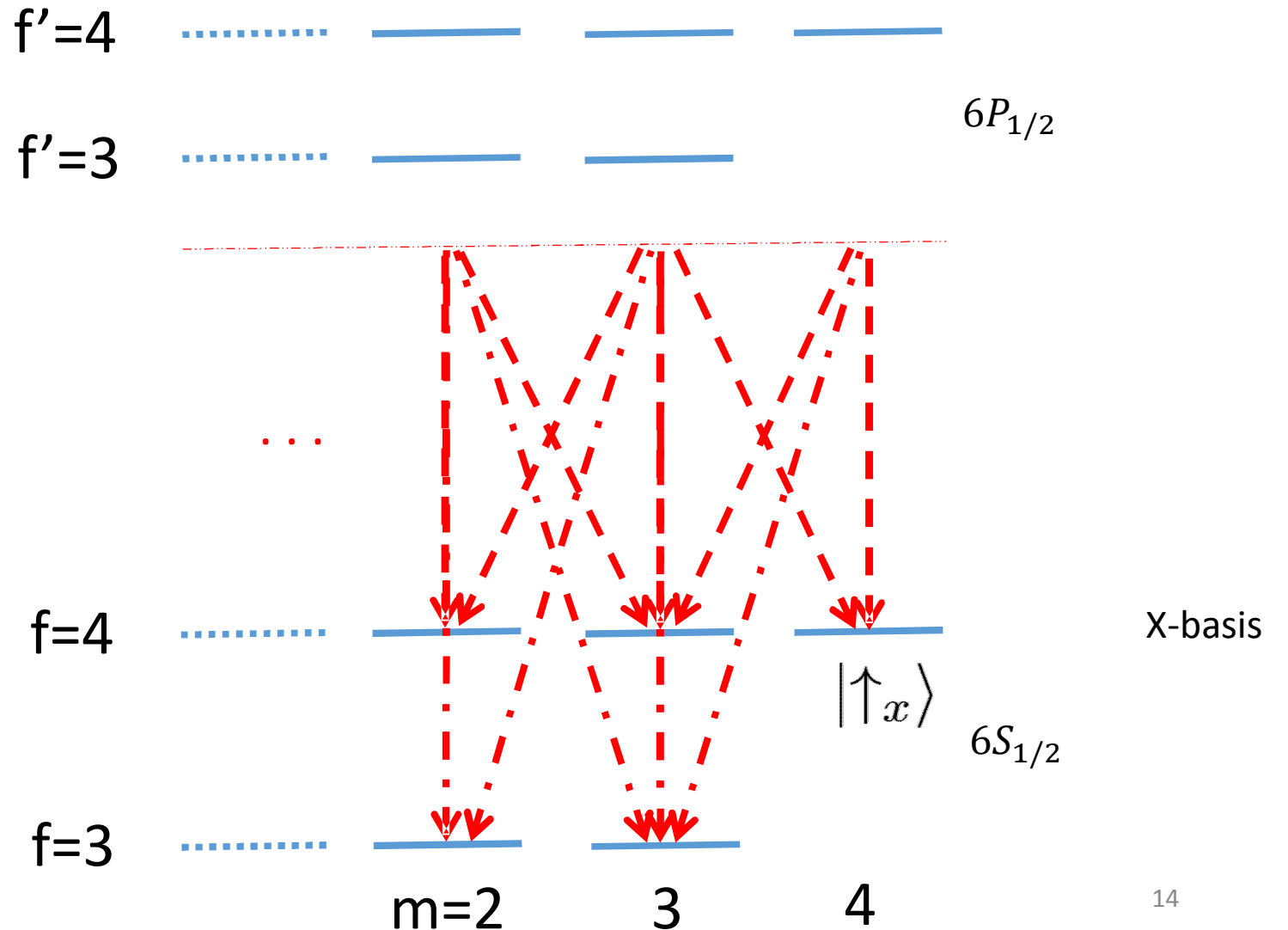
Coherent coupling and optical pumping



Coherent coupling and optical pumping

$$\begin{aligned}\gamma_s \mathcal{D}[\rho] &= -\frac{i}{\hbar} [\hat{H}_{\text{eff}} \hat{\rho} - \hat{\rho} \hat{H}_{\text{eff}}^\dagger] \\ &\quad + \sum_{f_b, f_a, q} \hat{W}_q^{f_b f_a} \hat{\rho} \hat{W}_q^{f_b f_a \dagger} \\ &= \sum_{a, b} \gamma_{ba} |b\rangle \langle a|.\end{aligned}$$

$$\gamma_s \equiv \frac{\Gamma \Omega^2}{4 \Delta_{\text{eff}}^2} = \frac{\sigma_0}{A_0} \frac{\Gamma^2}{4 \Delta_{\text{eff}}^2} \dot{N}_L$$



Modeling Spin Squeezing Dynamics

- Equations of spin dynamics (Gaussian approximation)

$$d\langle \hat{\sigma}_{ba} \rangle = d\langle \hat{\sigma}_{ba} \rangle|_{op} + d\langle \hat{\sigma}_{ba} \rangle|_{\mathcal{H}} + d\langle \hat{\sigma}_{ba} \rangle|_{\mathcal{L}} \quad (1)$$

$$d\langle \Delta\sigma_{ba}^{(1)} \Delta\sigma_{dc}^{(2)} \rangle_s = d\langle \Delta\sigma_{ba}^{(1)} \Delta\sigma_{dc}^{(2)} \rangle_s|_{op} + d\langle \Delta\sigma_{ba}^{(1)} \Delta\sigma_{dc}^{(2)} \rangle_s|_{\mathcal{H}} + d\langle \Delta\sigma_{ba}^{(1)} \Delta\sigma_{dc}^{(2)} \rangle_s|_{\mathcal{L}}. \quad (2)$$

where $\hat{\sigma}_{ba} = |b\rangle\langle a|$.

$$\hat{f} = \sum_{b,a} \langle b|\hat{f}|a\rangle \hat{\sigma}_{ba}.$$

Physical Essence of Spin Squeezing with Faraday interaction

- Measurement strength



$$\kappa \sim |\chi_{33}|^2 \sim \text{Re} [(\mathbf{u}_H^*(\mathbf{r}') \times \mathbf{u}_V(\mathbf{r}')) \cdot \mathbf{e}_{\tilde{z}}] \sim \frac{1}{A_{\text{int}}^2}$$

$$A_{\text{int}}^2 \propto \frac{1}{n_g \text{Re} [\mathbf{u}_H^* \times \mathbf{u}_V] \cdot \hat{e}_{\tilde{z}}} = \frac{1}{n_g \text{Re} [\mathbf{u}_D^* \times \mathbf{u}_{\bar{D}}] \cdot \hat{e}_{\tilde{z}}}$$

Characteristic photo scattering rate

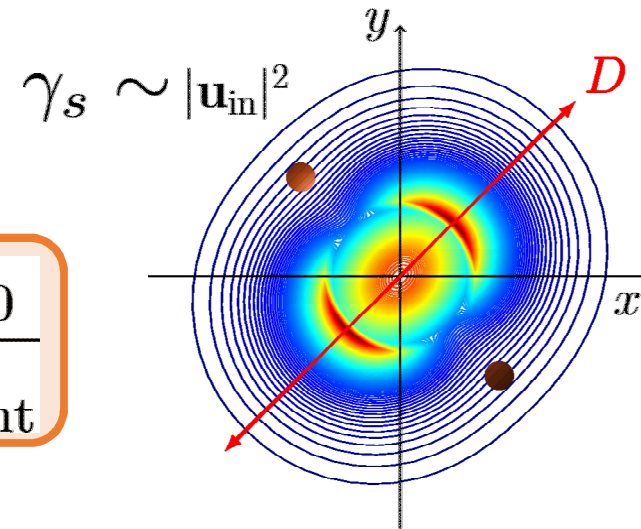


$$\gamma_s \sim |\mathbf{u}_D(\mathbf{r}')|^2 \sim \frac{1}{A_0}$$

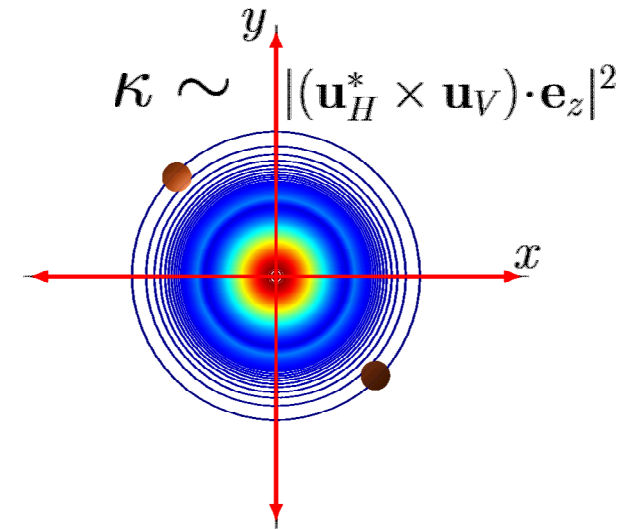
Optimal Geometry

- Figure of merit--Optical Depth per atom:

$$\frac{\text{OD}_{\text{eff}}}{N_A} = \frac{\kappa}{\gamma_s} = \sigma_0 \frac{A_0}{A_{\text{int}}^2}$$

Geometry of the Faraday protocol

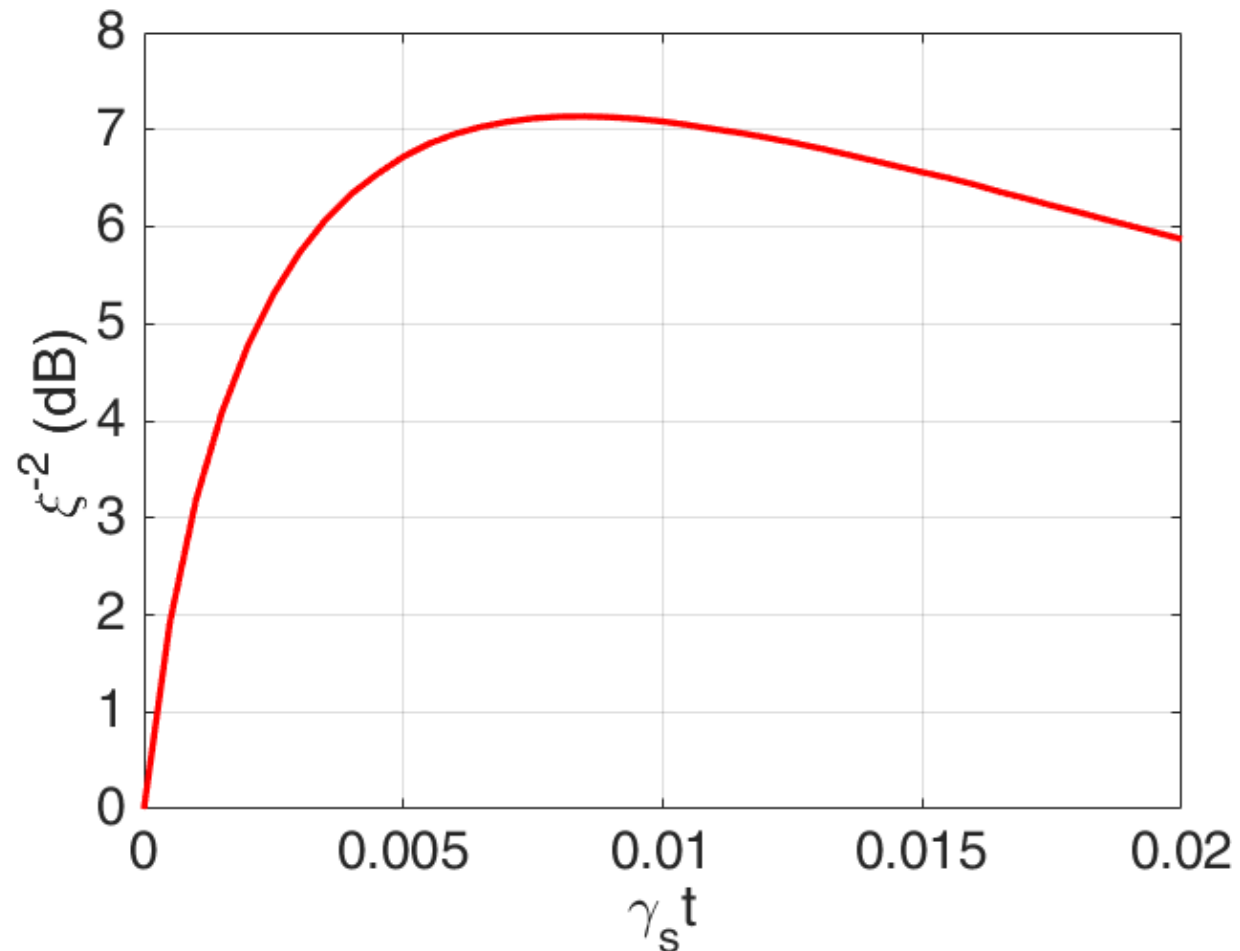


Spin Squeezing Using Faraday Interaction

$$N_A = 2500$$

$$a = 225\text{nm}$$

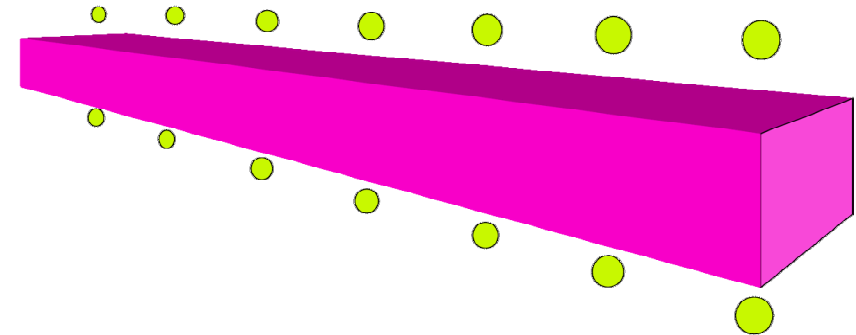
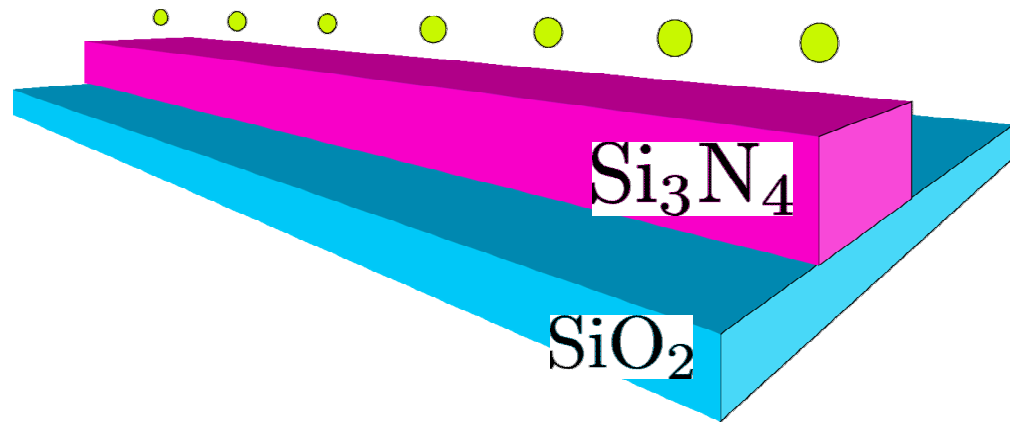
$$\lambda = 895\text{nm}$$



Radial position of atoms: $r'_\perp = 1.8a$ (180nm from the surface)

Spin Squeezing with Nanophotonic Waveguides

- Geometry



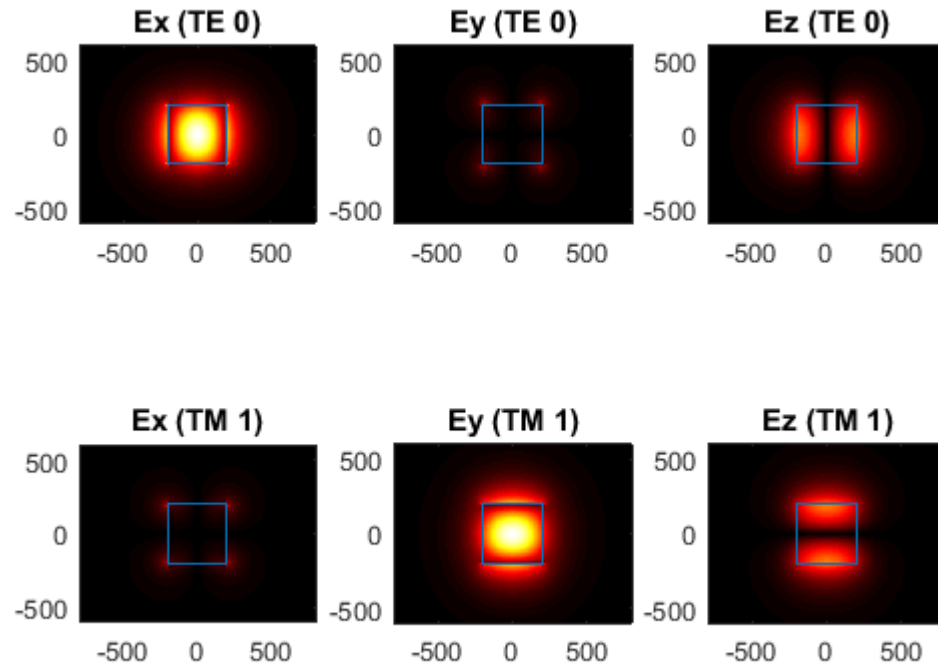
Current focus: square waveguides
 $a \rightarrow 300\text{nm}$

Jongmin Lee
Yuan-Yu Jau

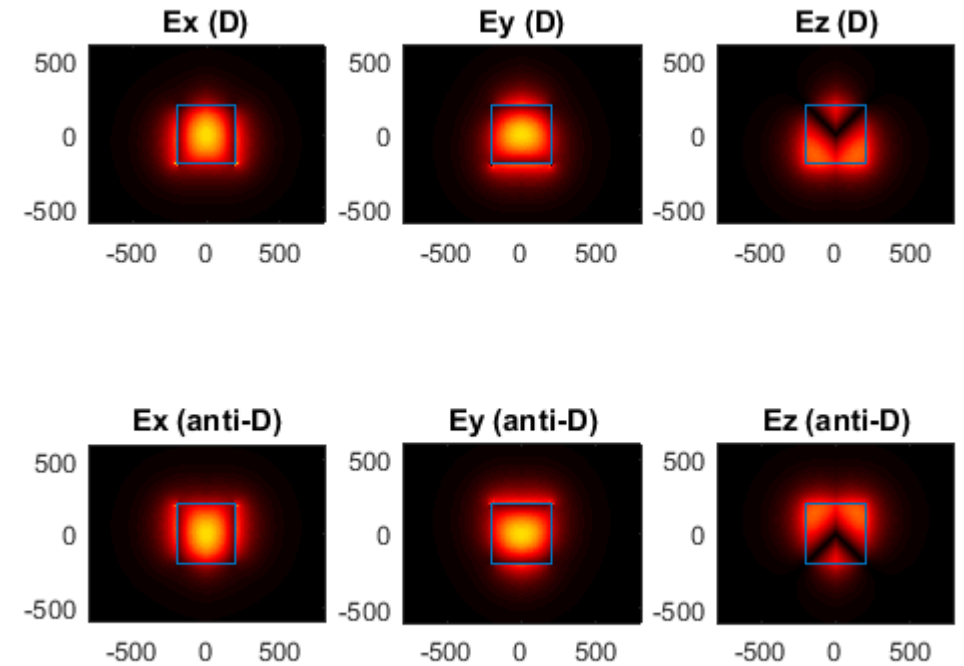


**Sandia
National
Laboratories**

Spin Squeezing on a Square Nano-waveguide



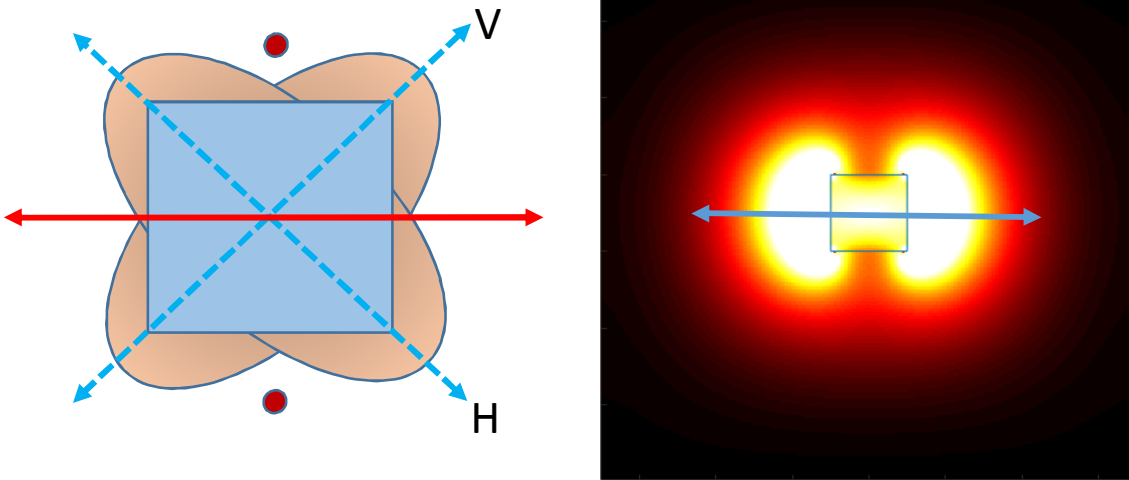
Fundamental TE and TM modes



D and anti-D modes

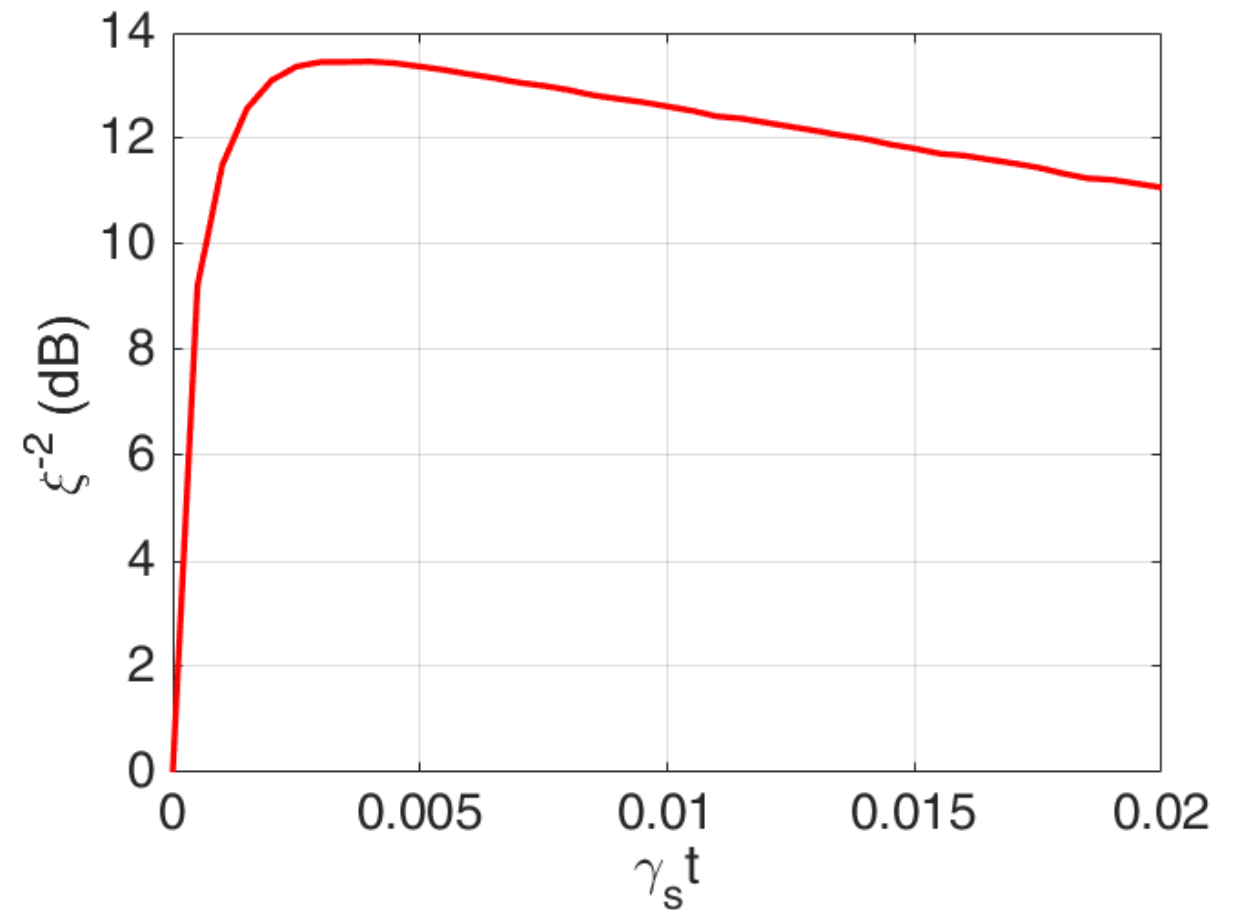
Spin Squeezing on a SWG Platform Using Faraday Interactions

- Protocol geometry:



$d=300\text{nm}$, atoms are 150nm to the surface.

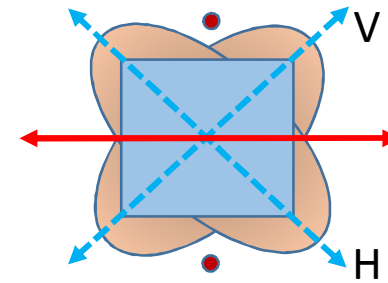
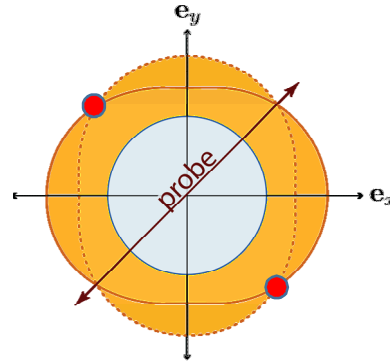
Spin squeezing dynamics (preliminary result)



Summary

- Birefringence and Faraday spin squeezing protocols

$$\frac{\text{OD}_{\text{eff}}}{N_A} = \sigma_0 \frac{A_0}{A_{\text{int}}^2}$$



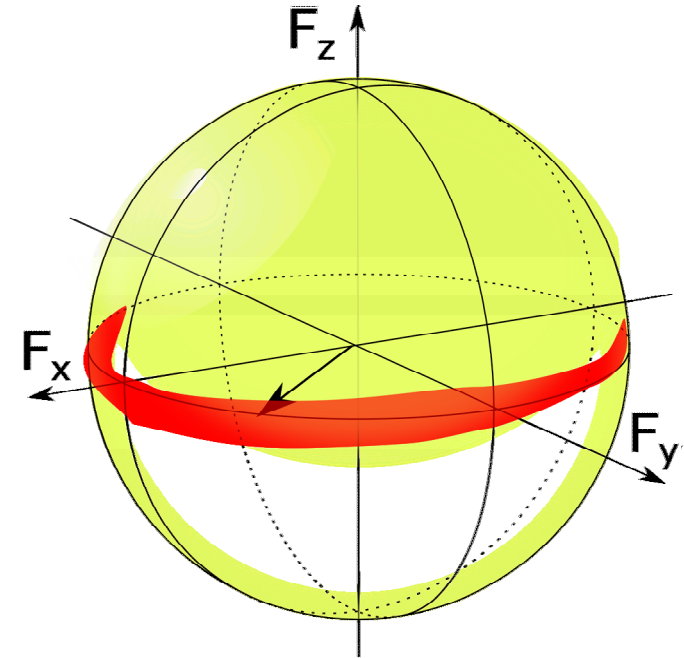
- Typical peak squeezing ($N_A=2500$)

	ξ_{max}^{-2} (dB)
Nanofiber ($r'-a=180\text{nm}$)	7
SWG ($r'-\frac{d}{2}=150\text{nm}$)	14

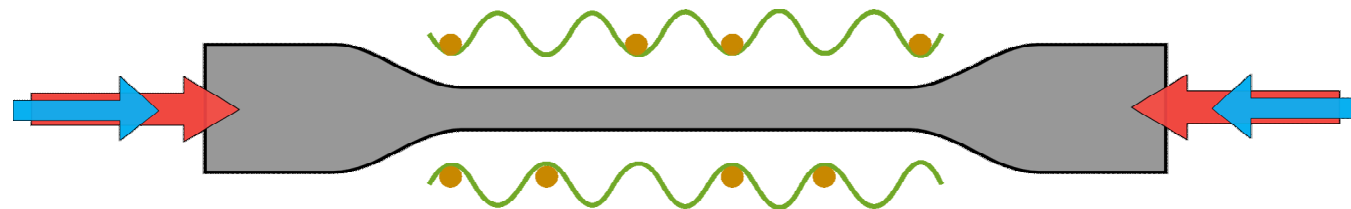
- Toward non-Gaussian states

If $N_A = 2500$, Gaussian limit: $\Delta F_z^2 \leq \langle F_x \rangle$

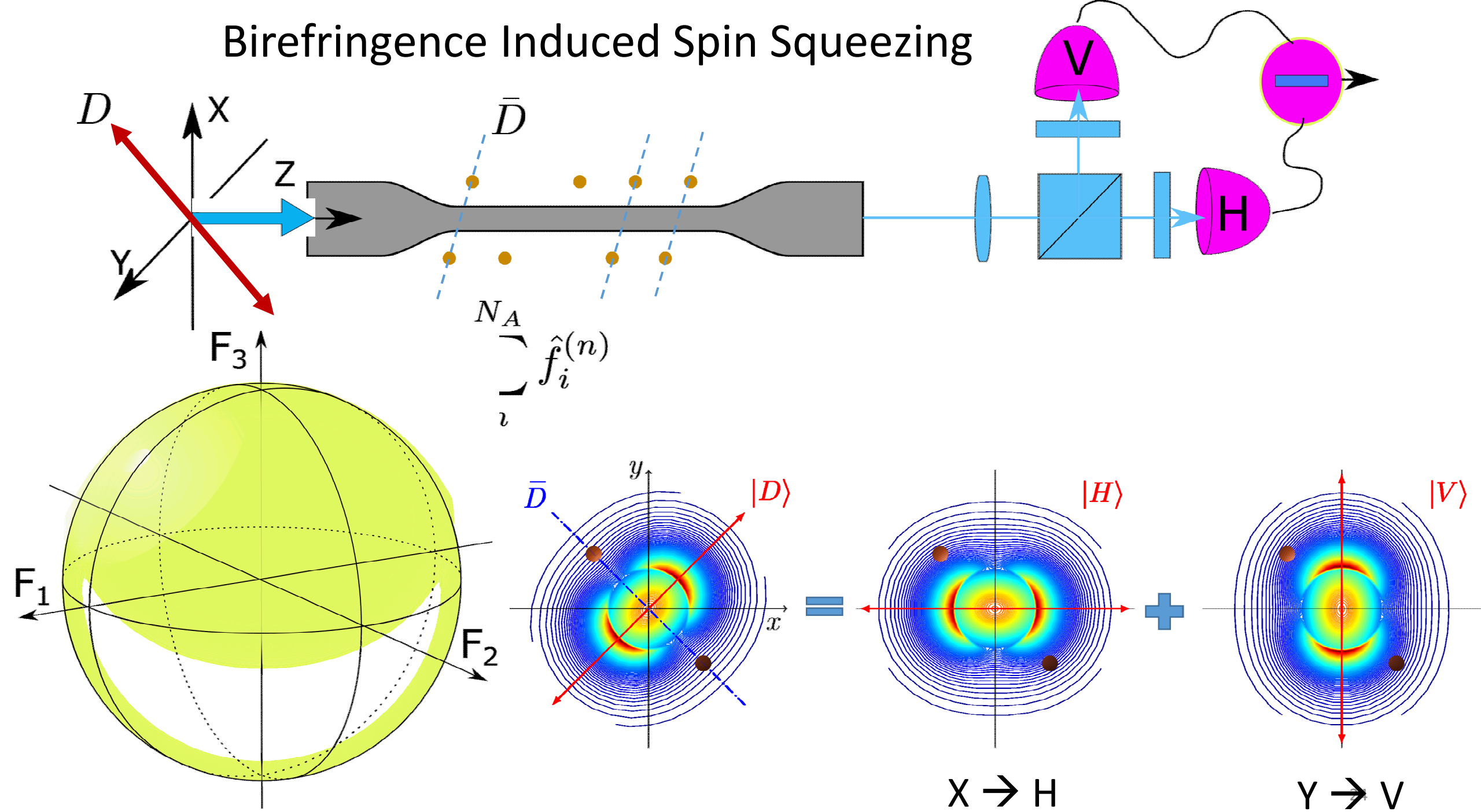
$$\Rightarrow \xi^{-2} \leq \sqrt{2Nf} \approx 20\text{dB}$$



THANK YOU!



Birefringence Induced Spin Squeezing

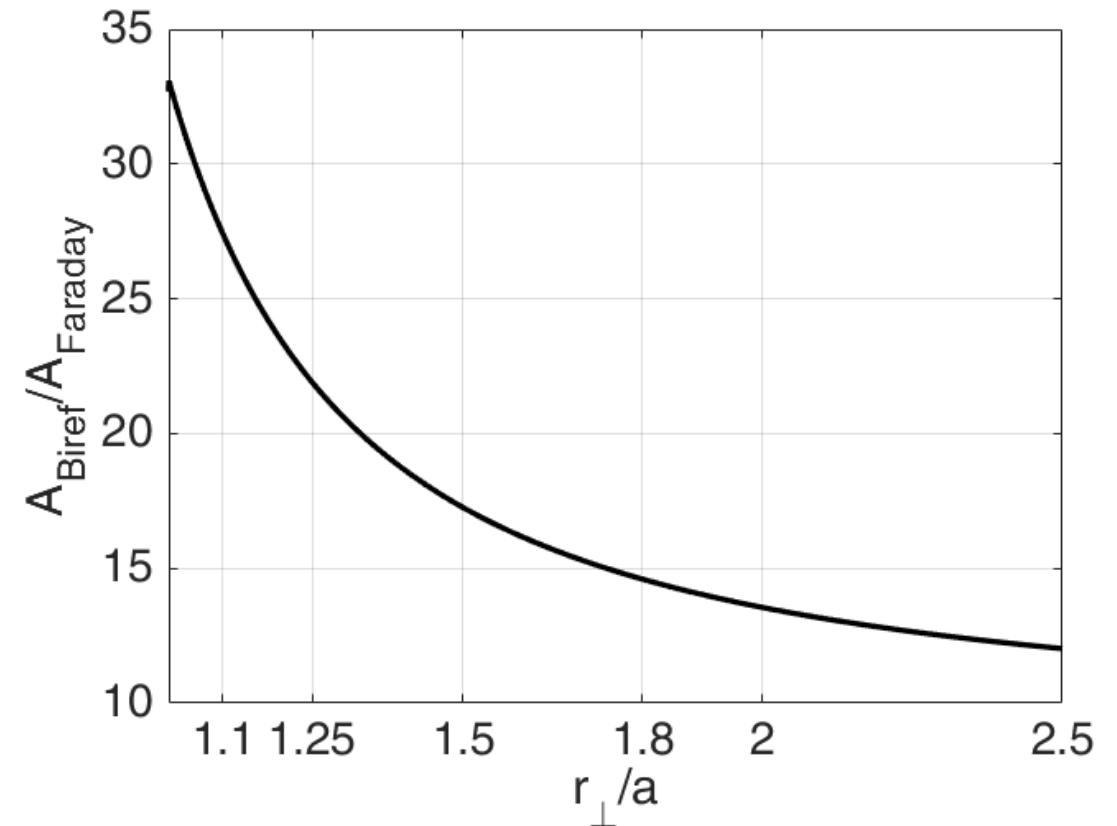


OD per atom (theory and comparison)

Enhancement of the Faraday protocol over the birefringence protocol:

$$\frac{\text{OD}_{biref}}{N_A} = \sigma_0 \frac{A_0}{A_{\text{int}}^2} = \frac{\sigma_0}{A_{biref}}$$
$$\frac{\text{OD}_{Faraday}}{N_A} = \sigma_0 \frac{A_0}{A_{\text{int}}^2} = \frac{\sigma_0}{A_{Faraday}}$$

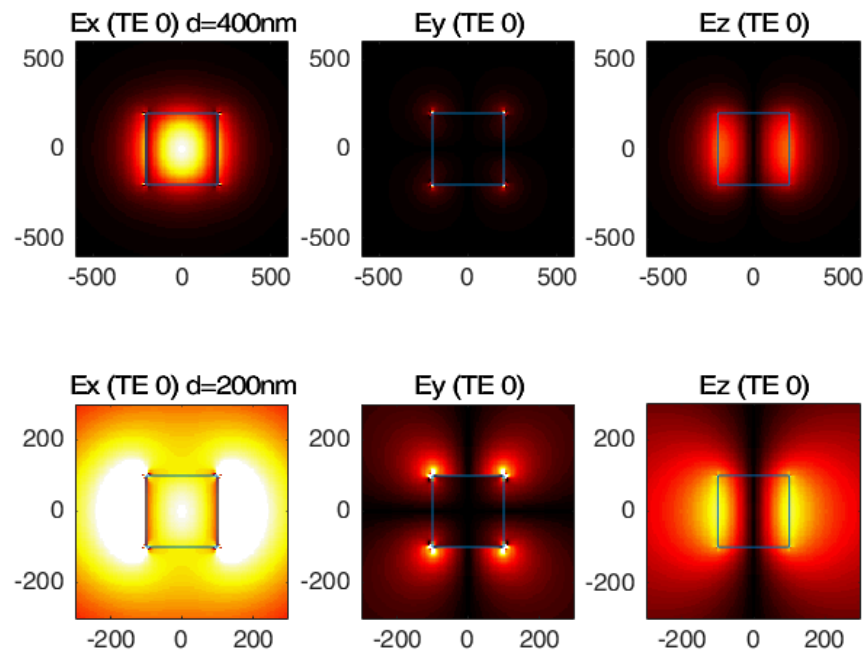
$$A_{biref} = \frac{A_{\text{int}}^2}{A_0}$$
$$A_{Faraday} = \frac{A_{\text{int}}^2}{A_0}$$



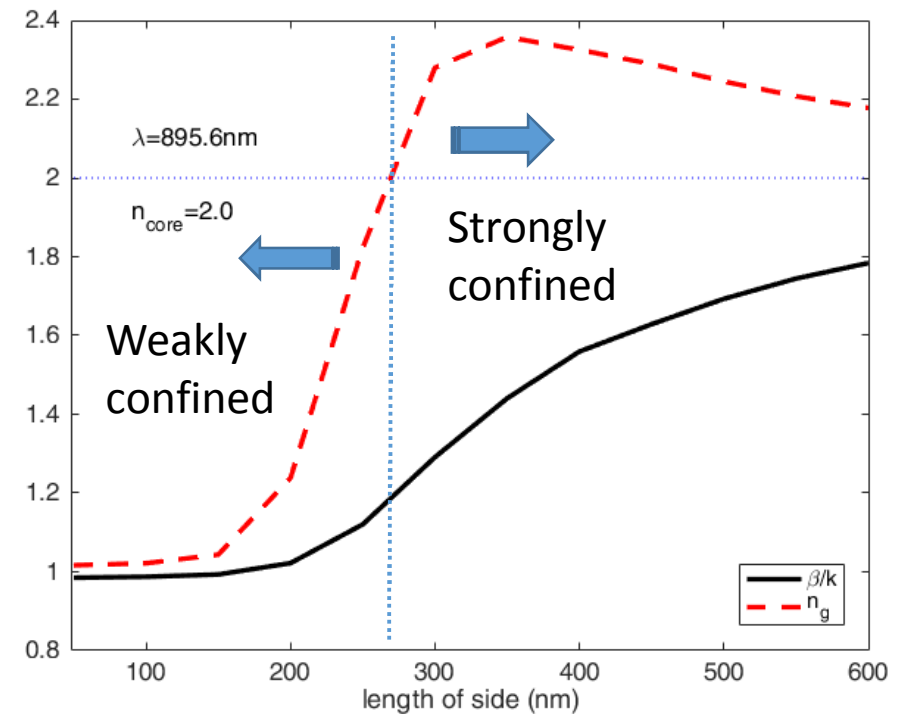
$a=225\text{nm}$

Spin squeezing on a square nano waveguide platform using Faraday interaction

- Square waveguides and their group index of refraction



TE mode components for $d=400\text{nm}$ and 200nm



n_g of fundamental TE and TM modes