

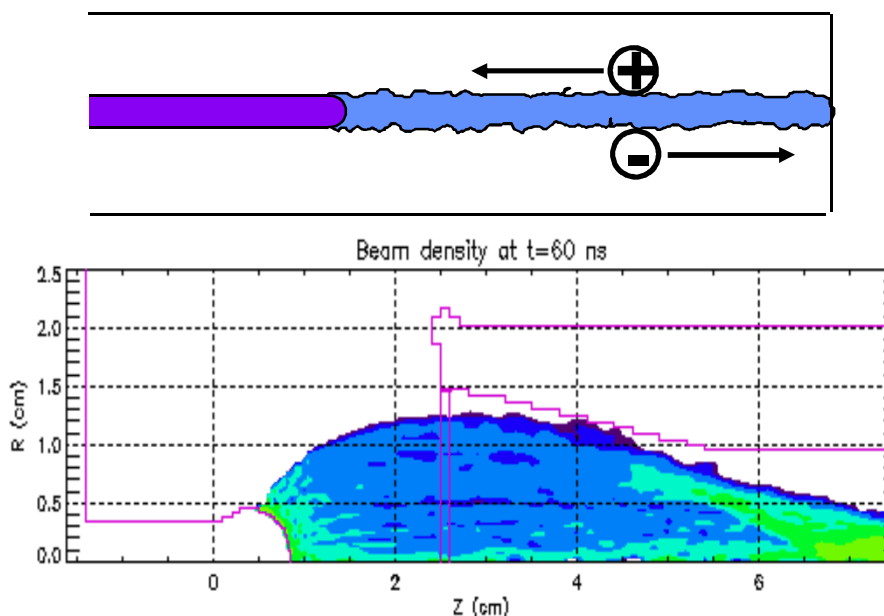
# Intense Charged Particle Beam Physics and Applications

SAND2017-5490C

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International Conference on Plasma Science,  
ICOPS 2017 Mini-Course, Atlantic City NJ  
May 25, 2017



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# Outline

This lecture will present some theoretical constructs of intense charged particle beam generation and transport with occasional examples of present day use of such beams.

## **Introduction to Intense Beams:**

- Description and parameters

- Force free equilibria for a cold e-beam in vacuum

- Beam envelope equation

## **High Power Diodes:**

- Space charge limited emission

- In the presence of background gas/plasma

- With electron backscatter

## **Propagation:**

- e-beams in vacuum

- e-beams in plasma - example, the paraxial diode

- The ion hose instability

## **Magnetically insulated flow – equilibria**

- in vacuum

- with ions

# Introduction to Intense Beams

**Intense charged particle beams have the unique characteristic that their density and velocity are large enough to induce strong electric and magnetic “self-fields” that are sufficient to greatly influence the beams’ dynamics.**

**Advances in pulsed power technology over the last 30 years has led to the production of electron and ion beams with currents  $> 10$  kA and kinetic energies ranging from 100 keV to  $> 10$  MeV, with pulse durations in the 10-100 ns range.**

**The beams are generated in the accelerating gap of a high current diode and can either be accelerated after extraction from the diode to higher energies, transported in vacuum or in background gas/plasma, or they can be focused directly onto a target.**

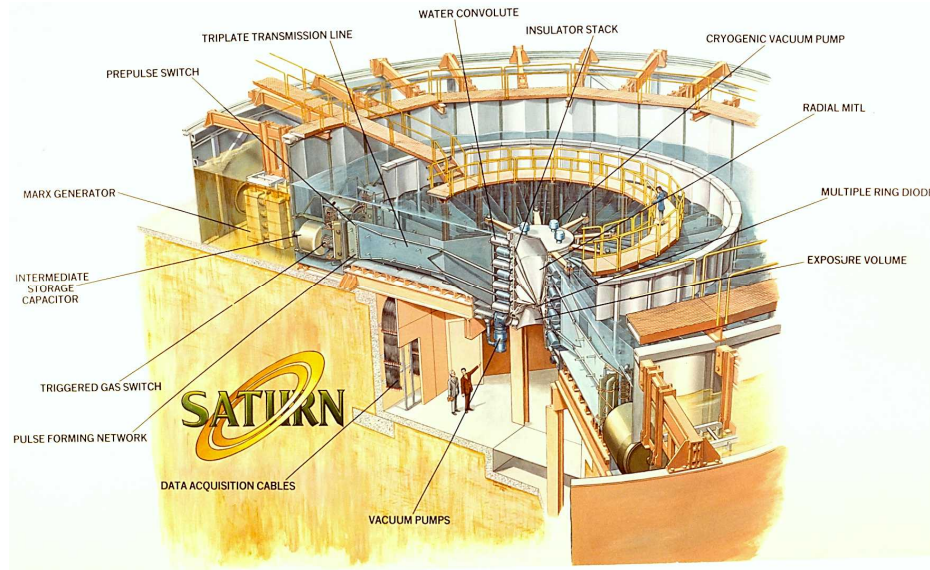
**Applications include materials testing and modification, production of X-rays for lithography, radiography or nuclear weapon effects, high-power microwave generation, or nuclear fusion.**

## **Key Overview References:**

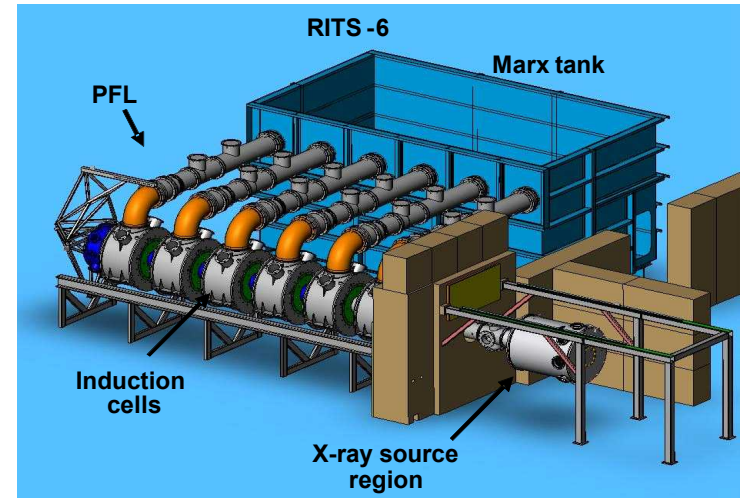
1. J.D. Lawson, *The Physics of Charged particle Beams*, Oxford Univ. Press, 1977
2. R.B. Miller, *Intense Charged Particle Beams*, Plenum Press, New York, 1982
3. S. Humphries, *Principles of Charged Particle Acceleration*, John Wiley and Sons., 1986
4. R.N. Sudan, Ch. 6.3 *Collective Beam-Plasma Interaction*, in Handbook of Plasma Physics Vol 2. Basic Plasma Physics II, edit. A.A. Galeev and R.N. Sudan, North-Holland Pub., 1984
5. R.C. Davidson, Ch 9.1 *Relativistic Electron Beam-Plasma Interaction with Intense Self-fields*, *ibid*

# Introduction to Intense Beams

We will consider High power particle beams in the 10 GW– 10 TW range that are generated from pulsed power accelerators.



**Saturn Accelerator,  
Sandia National Laboratories  
1.6 MeV, 10 MA, 40ns e-beam driver**



**RITS-6 Accelerator,  
Sandia National Laboratories  
10 MeV, 180 kA, 70ns e-beam driver**

The beams are generated in the accelerating gap of a high current diode and can either be propagated in vacuum or in background gas/plasma.

# Typical Beam Parameters

**Energy  $E_b = 0.1\text{-}20$  MeV,**

**Current  $I_b = 0.01\text{-}10$  MA,**

**Pulse length  $\tau_b = 10\text{-}100\text{ns}$ ,**

**Beam radius  $r_b = 0.1\text{-}10$  cm,**

**Velocity  $v_b/c = 0.1\text{-}1$**

**Current density  $j_b = 1\text{-}1000$  kA/cm<sup>2</sup>,**

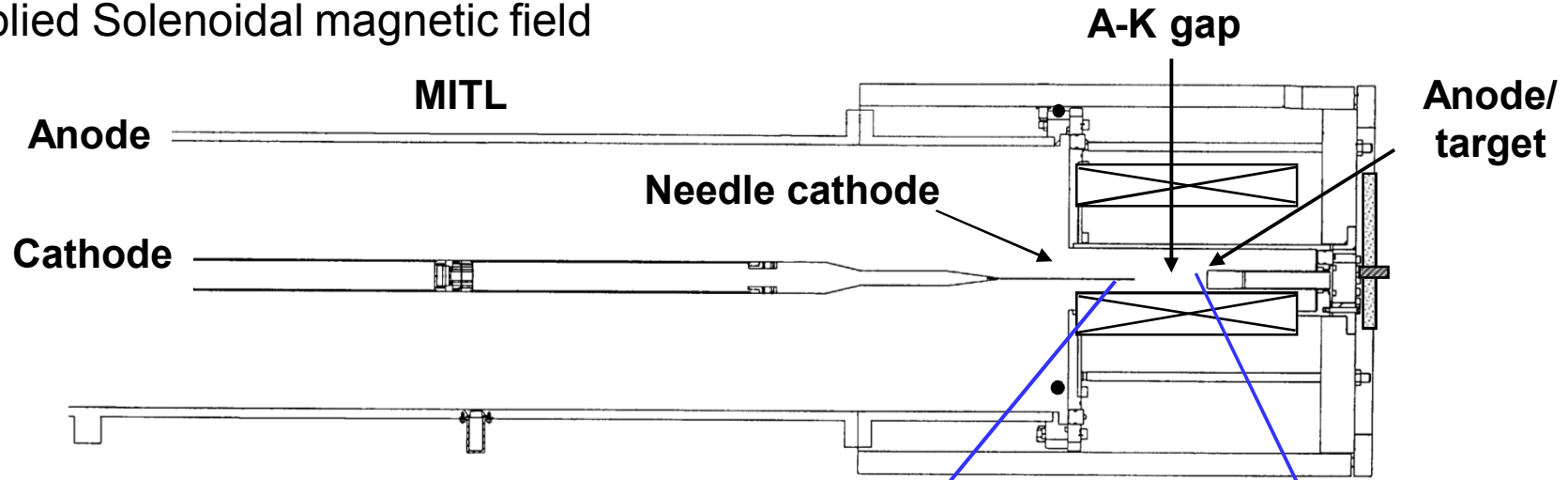
**Beam density  $n_b = 1\text{-}10$  cm<sup>-3</sup>,**

**Total charge  $Q_b = 5\text{-}500$  mC**

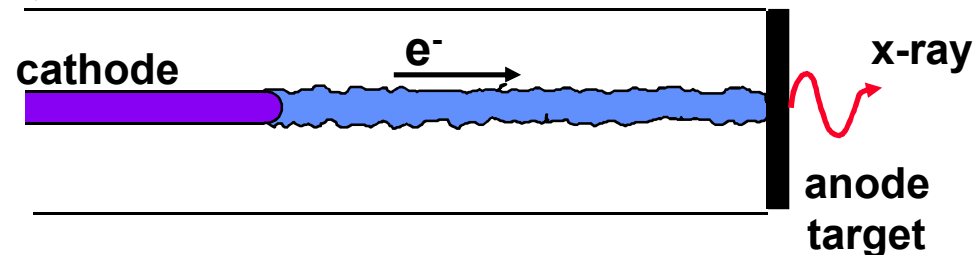
**Relativistic factor  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1\text{-}40$**

# Example: Pulsed-power driven e-beam diode for x-ray radiography applications

**The Immersed  $B_z$  diode<sup>1</sup>:** the electron beam is created in the accelerating gap of a high current diode and guided in vacuum to an anode/target via an applied Solenoidal magnetic field



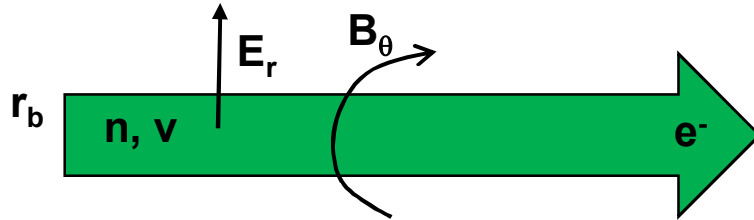
Bremsstrahlung x-rays are created when the e-beam is stopped in a high atomic number converter.



**Energy  $E_b = 2\text{-}10$  MeV, Current  $I_b = 20\text{-}150$  kA, Pulse length  $\tau_b = 50\text{-}100$  ns**

# Beam Self-field Forces

Consider the 1-D radial forces on a cold (zero emittance) uniform density electron beam drifting in vacuum.



The self-fields at the beam edge  $E_r = 2\pi en r_b$   
 $B_\theta = 2\pi en r_b (v/c)$

Gaussian units are used  
 $c = 3 \times 10^{10}$  cm/s

The radial Lorentz force acting on the beam

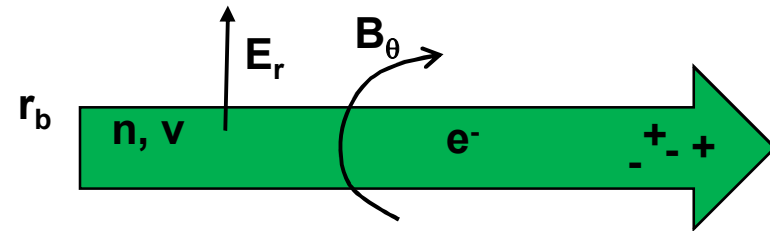
$$F_r = e(E_r + v \times B_\theta)$$
$$= 2\pi e^2 n r_b (1 - (v/c)^2) \equiv 2\pi e^2 n r_b / \gamma^2$$

The beam will only be in force balance ( $F=0$ ) when  $v=c$  ( $\gamma \rightarrow \infty$ ). Hence all beams will expand under their self-electric field unless their space-charge ( $en$ ) is partially neutralized.

# The Beam Envelope Equation

For relativistic beams propagating in vacuum with finite emittance  $\varepsilon$  and axial velocity  $v_z$ , the force balance equation for the beam radial edge  $r_b$  (envelope) is

$$\frac{d^2 r_b}{dz^2} = \frac{2v}{\beta^2 r_b} [1 - f_e - (1 - f_b) \beta^2] + \frac{\varepsilon^2}{r_b^3}$$



Where the conditions  $v_z \gg v_r, v_\theta$  such that  $d/dt \cong v_z d/dz$ , and  $\beta = v_z/c$ ,  $\gamma = 1/\sqrt{1 - \beta^2}$

$\varepsilon$  = beam emittance,

$$v \equiv \frac{eI_b}{\gamma\beta mc^3} \equiv \frac{I_b}{I_A} \quad I_A = \gamma\beta 17 \text{ (kA)}, \text{ is the Alfvén current}$$

$$\varepsilon = 4\sqrt{\langle r^2 \rangle \langle r'^2 \rangle - \langle rr' \rangle^2}$$

$$f_e = \frac{n_p}{n_b} \quad \text{is the charge neutralization fraction}$$

$$f_b = \frac{I_p}{I_b} \quad \text{is the current neutralization fraction}$$



# Intense Beam Generation

## Diodes

# Space-charge limited diodes<sup>1</sup>.

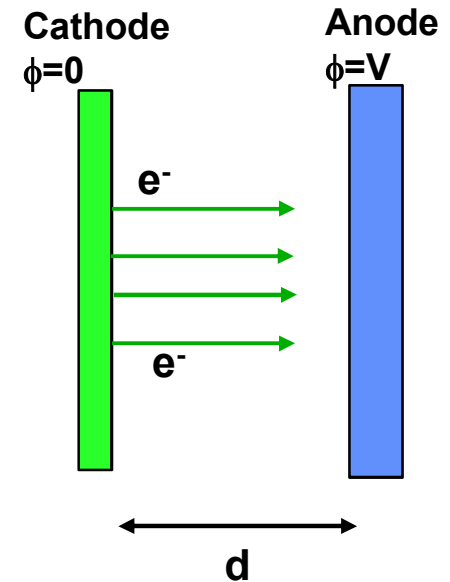
SCL diode: e.g. 1-D, non-relativistic large area

$$\frac{d^2\phi}{dz^2} = en_e$$

$$j = en_e v = \text{const.}$$

$$\frac{1}{2}mv^2 = e\phi$$

$$\rightarrow j \propto \frac{1}{d^2} \sqrt{\frac{2e}{m}} V^{3/2}$$



If there is some space charge neutralization, than

$$en_e \rightarrow en_e(1-f) \quad f=n_i/n_e$$

$$\rightarrow j \propto \frac{1}{(1-f)d^2} \sqrt{\frac{2e}{m}} V^{3/2}$$

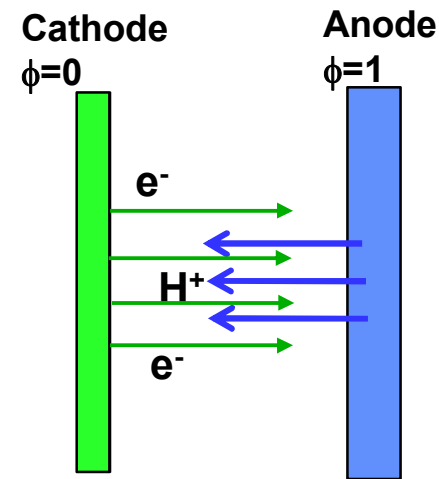
1. I. Langmiur and K. Blodgett, Phys. Rev. **22**, 347 (1923); C.B. Wheeler, J. Phys. A **10**, 631 (1977)

# Bi-polar (electron + ion) space charge limited and self-magnetic field limited diodes.

SCL diodes<sup>1</sup>: e.g. 1-D, non-relativistic large area

$$I = \alpha I_{cl}, \quad 0.5 < \alpha < 5 \quad (\alpha = 1.86 \text{ for planar bi-polar diode})$$

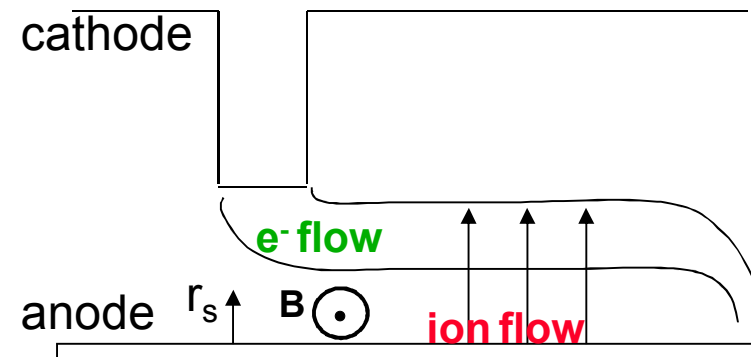
$$I_{cl} = \frac{1}{9\pi} \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2} \text{ A},$$



Self-limited diodes: e.g. rod-pinch<sup>2</sup>

$$I = \alpha I_{crit}, \quad 2.0 < \alpha < 2.6$$

$$I_{crit} = 8.5 \frac{\sqrt{\gamma^2 - 1}}{\ln(r_c / r_a)} \text{ kA}, \quad \gamma = 1 + eV/mc^2$$



**The factor  $\alpha$ , is dependent on geometry, voltage, and ion distribution**

1. I. Langmiur and K. Blodgett, Phys. Rev. **22**, 347 (1923); C.B. Wheeler, J. Phys. A **10**, 631 (1977)
2. G. Cooperstein et al. Phys. Plasmas, **8**, 4618 (2001)

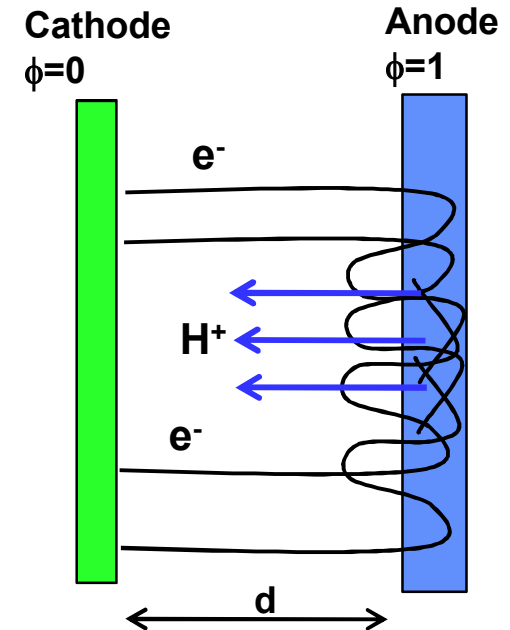
# The effect of electron backscatter at the anode<sup>1</sup>

Bi-polar Child-Langmuir currents are modified by the presence of the extra electron space charge scattered into the A-K gap

This enhances the ion space charge emission and changes the impedance

$$\left(\frac{d\phi}{dx}\right)^2 = \frac{16}{9} \frac{j_e}{j_{cl}} g(\phi)$$

$$g(\phi) = \sqrt{\phi + \frac{eV}{2mc^2} \phi^2} - \left(\frac{M}{m}\right)^{1/2} \frac{j_i}{j_e} (1 - \sqrt{1 - \phi}) +$$



$$j_{cl} = \frac{1}{9\pi} \sqrt{\frac{2eV}{m}} \frac{V}{d^2}$$

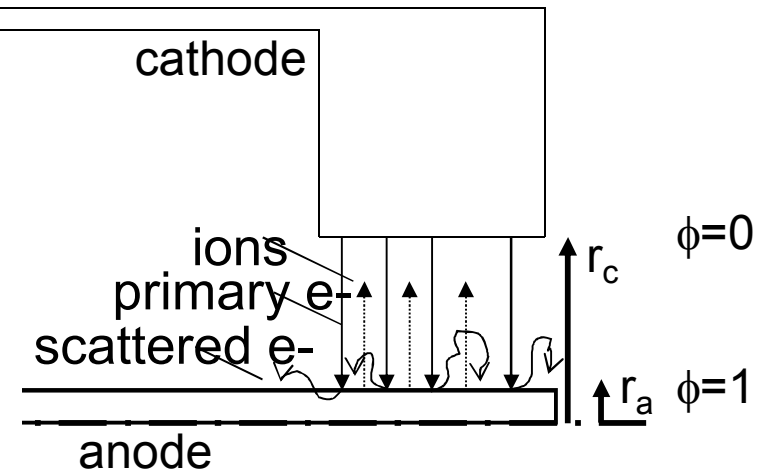
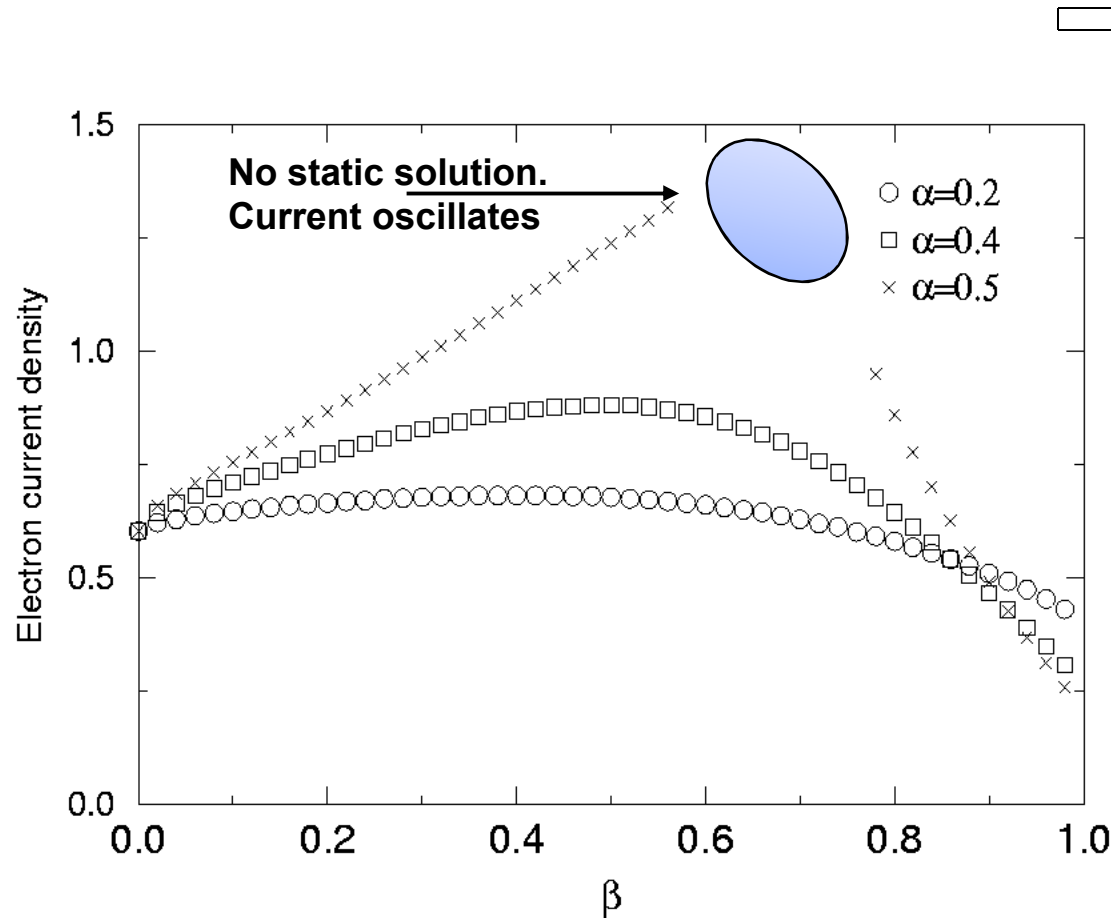
$$j_{scat} = \alpha j_{incident}$$

$$E_{scat} = \beta E_{incident}$$

Extra terms due to scattering

1. N.R. Pereira, JAP **54**, 6307 (1986)
- D. Mosher, G. Cooperstein et al, Proc. 11<sup>th</sup> Intl. Beams Conf. (1996)
- V. Engelko, V. Kusnetsov et al. JAP **88**, 3879 (2000)
- B.V. Oliver, T.C. Genoni et al., JAP **90**, 4951 (2001)

# Electron backscatter can be significant in cylindrical diodes, results in decreased but stable impedance!



$$j_{cl} = \frac{1}{9\pi} \sqrt{\frac{2eV}{m}} \frac{V}{d^2}$$

$$j_{scat} = \alpha j_{incident}$$

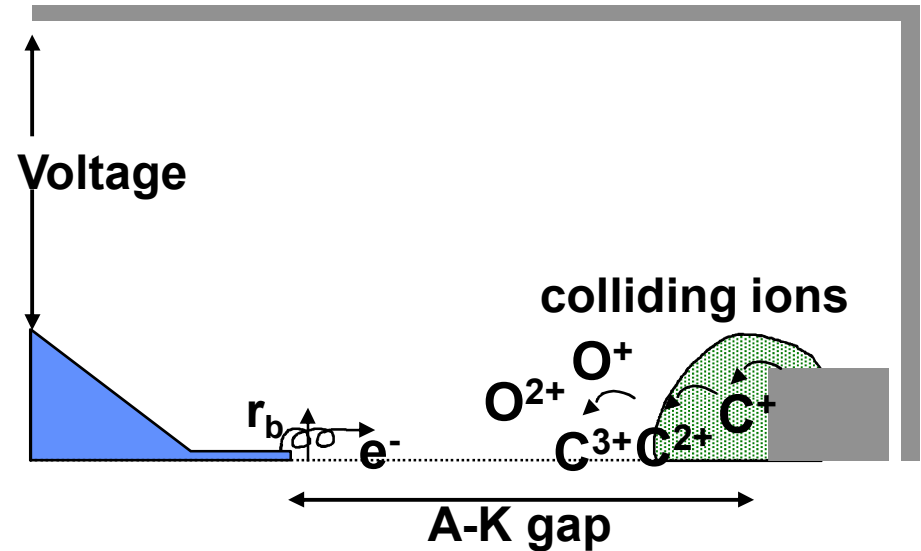
$$E_{scat} = \beta E_{incident}$$

**As the fraction of reflected beam current goes up, so does the total current. However, there is a maximum and the current is stable.**

# Charge stripping of ions can increase electron emission and thus current<sup>1</sup>

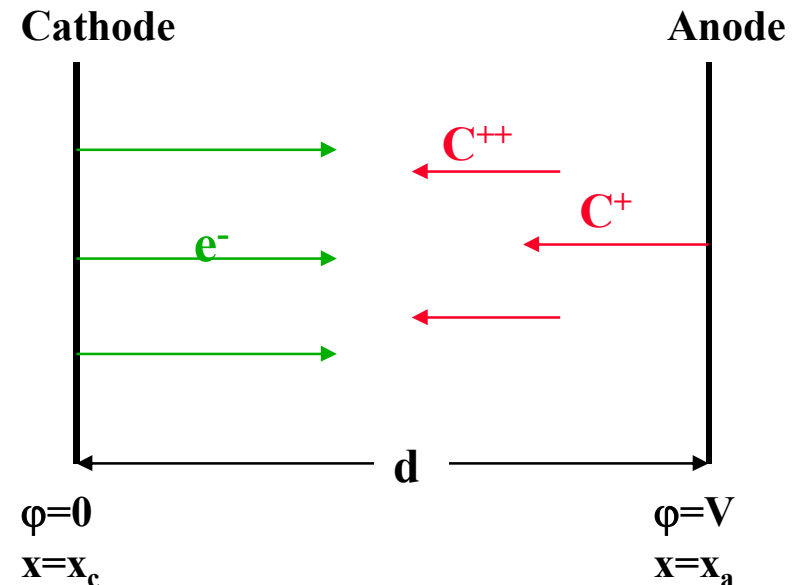
Non-protonic ions (e.g. C, O) from the anode collide and charge-strip while traversing the A-K gap.

The electron current  $I_e$  increases when the excess ion charge reaches the cathode. Feedback causes impedance collapse.



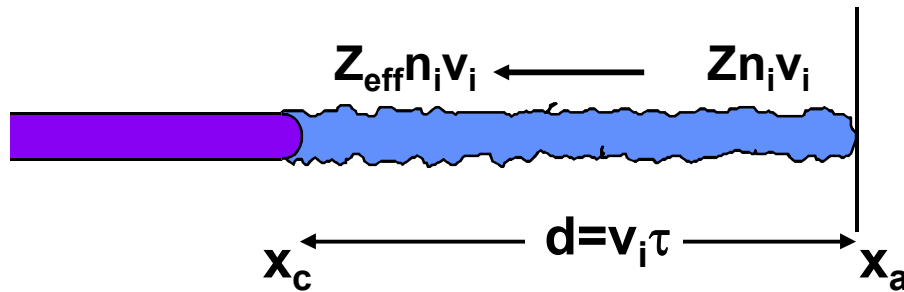
$$j_i = j_e \sqrt{\frac{Zm}{M} (1 + eV/2mc^2)}$$

But  $j_i(x_c, t) \neq j_i(x_a, t)$  because of charge stripping during ion transit across the gap!



# The ion current density increases during transit across the gap

Ion current density increases in proportion to the ionization fraction.



$$Z_{\text{eff}} = Z + v_{\text{ion}} \tau, \quad \tau = \frac{d}{v_i}$$

$$j_i(x_c, t) = \frac{Z_{\text{eff}}}{Z} j_i(x_a, t - \tau)$$

The ionization frequency  $v_{\text{ion}}$  is proportional to the ion current density at the anode and the ionization cross-section  $\sigma_{\text{ion}}$ :

$$v_{\text{ion}} = n_i \langle v_i \sigma_{\text{ion}} \rangle \approx \frac{j_i(x_a, t - \tau)}{Ze} \sigma_{\text{ion}}$$

**Non-linear response occurs because  $Z_{\text{eff}}$  is proportional to  $j_i$  through  $v_{\text{ion}}$**

# Non-linear (explosive) electron current time-dependence results<sup>1</sup>

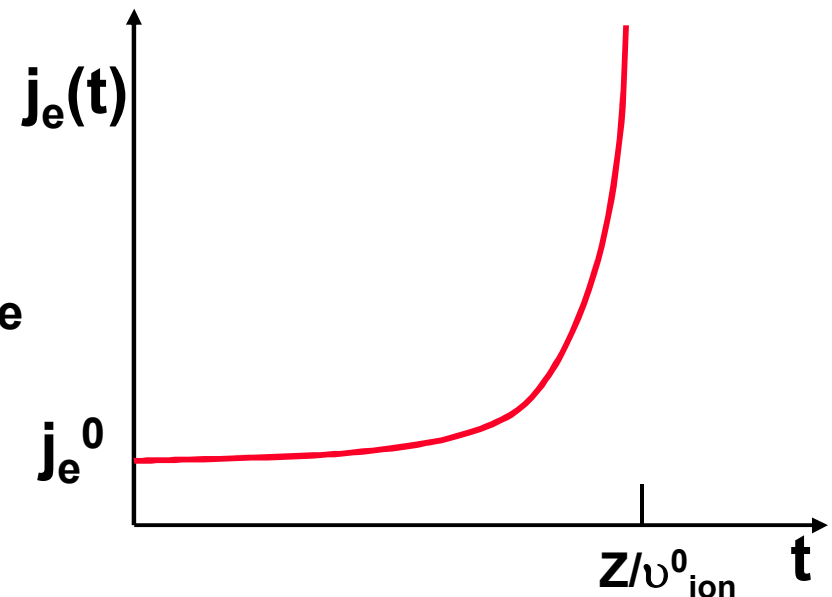
The time dependent solutions for the electron current density are nonlinear because of the relation of  $J_i$  to  $J_e$  and  $Z_{\text{eff}}$ .

$$\left(\frac{Z_{\text{eff}}}{Z} - 1\right) = \frac{v_{\text{ion}} \tau}{Z} = \frac{j_i(x_a, t-\tau) \tau}{Z^2 e} \sigma_{\text{ion}} \equiv \sqrt{\frac{Z_m}{M} (1 + eV/mc^2)} \frac{j_e(x_a, t-\tau) \tau}{Z^2 e} \sigma_{\text{ion}}$$

$$j_e(x_a, t) = j_e^0 \frac{1}{1 - \alpha t}$$

Solutions result in explosive behavior on time scale  $1/\alpha \sim Z/v_{\text{ion}}^0$

$$\alpha = \frac{j_e^0}{Z^2 e} \sqrt{\frac{Z_m}{M} (1 + eV/2mc^2)} \sigma_{\text{ion}} \\ \sim 10 \text{ ns}$$

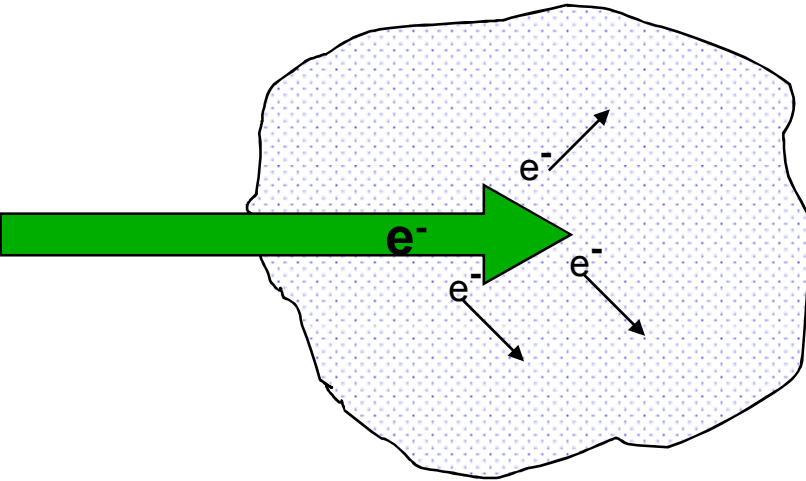




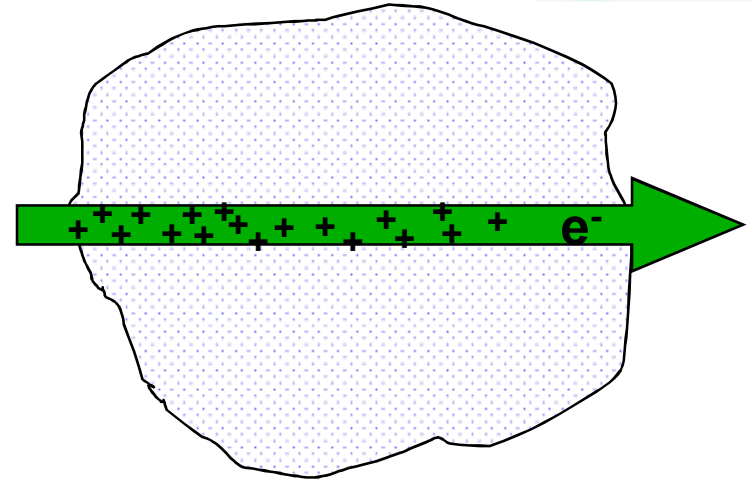
# Beam Propagation

**In vacuum and gas/plasma**

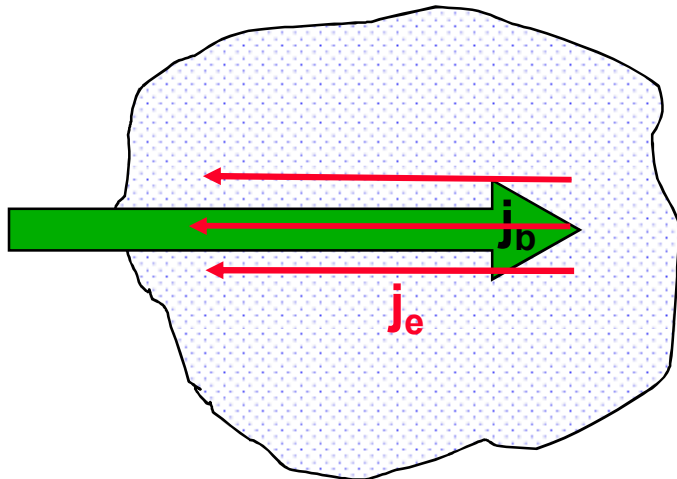
# Electron beam neutralization in plasma



e-beam expels plasma electrons



Creates a plasma ion channel so beam + plasma is quasi-neutral:  $n_i - n_e - n_b \cong 0$



Ohm's law  $j_e = \sigma(E + \frac{v_e}{c} \times B); \quad \sigma = \text{conductivity}$

$$\nabla \times B = \frac{4\pi}{c} (j_e + j_b); \quad j_i \cong 0$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t};$$

Plasma return current driven by inductive electric field and determined by generalized Ohm's law

$$\frac{\partial B}{\partial t} = -\nabla \times \frac{c}{4\pi en_e} \left( \nabla \times B - \frac{4\pi}{c} j_b \right) \times B + \nabla \times \frac{c^2}{4\pi \sigma} \left( \nabla \times B - \frac{4\pi}{c} j_b \right)$$

# Paraxial diode: a classic beam propagation problem in overdense plasma $n_b/n_e \ll 1$ . Gas-cell acts as a $\frac{1}{4}$ betatron focusing lens<sup>1</sup>

Gas breakdown sufficient for complete charge neutralization but incomplete current neutralization.

$$\frac{d^2 r_b}{dz^2} \cong -\frac{1}{r_b} \frac{2I_{\text{net}}}{I_A} + \frac{\varepsilon^2}{r_b^3}, \quad I_{\text{net}} = I_b + I_{\text{plasma}}$$

For  $\varepsilon^2 \ll 2R^2 I_{\text{net}}/I_A$

$$F \cong \frac{R}{2} \sqrt{\frac{\pi I_A}{I_{\text{net}}}},$$

$$\propto \sqrt{\frac{\gamma}{I_{\text{net}}}}$$

$$I_A = \gamma\beta \, 17 \text{ (kA)},$$

$$\varepsilon = 4\sqrt{\langle r^2 \rangle \langle r'^2 \rangle - \langle rr' \rangle^2}$$

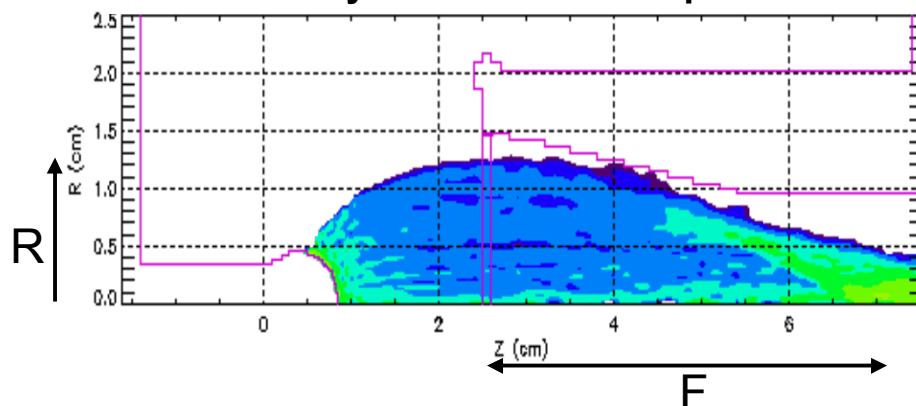
Net current (beam + plasma)  $I_{\text{net}} = crB_\theta/2$

$$\frac{\partial B}{\partial t} = -\nabla \times \frac{c}{4\pi n_e} \left( \nabla \times B - \frac{4\pi}{c} j_b \right) \times B + \nabla \times \frac{c^2}{4\pi\sigma} \left( \nabla \times B - \frac{4\pi}{c} j_b \right)$$

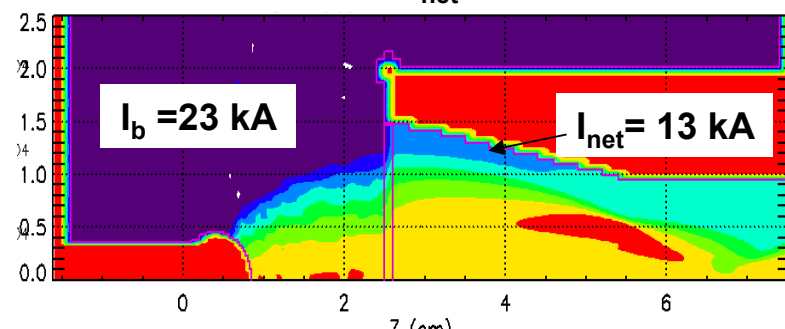
Hall current advection

Resistive diffusion

Beam density contours from Lsp simulations



Contours of  $I_{\text{net}}$

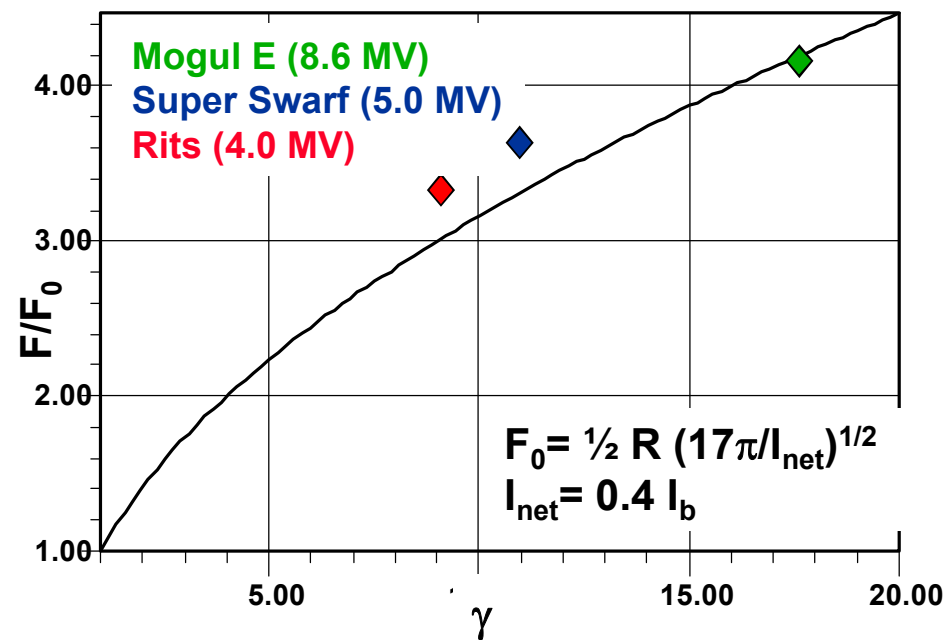


Data suggests average net current in cell  $\sim 0.3\text{-}0.4 I_b$

Envelope oscillation wavelength  $\lambda \approx 2\pi a(I_A/I_{\text{net}})^{1/2}$

Equilibrium radius  $a \approx \varepsilon(I_A/2I_{\text{net}})^{1/2}$

### Focal length vs. $\gamma$ ,



### Beam radius vs. drift length

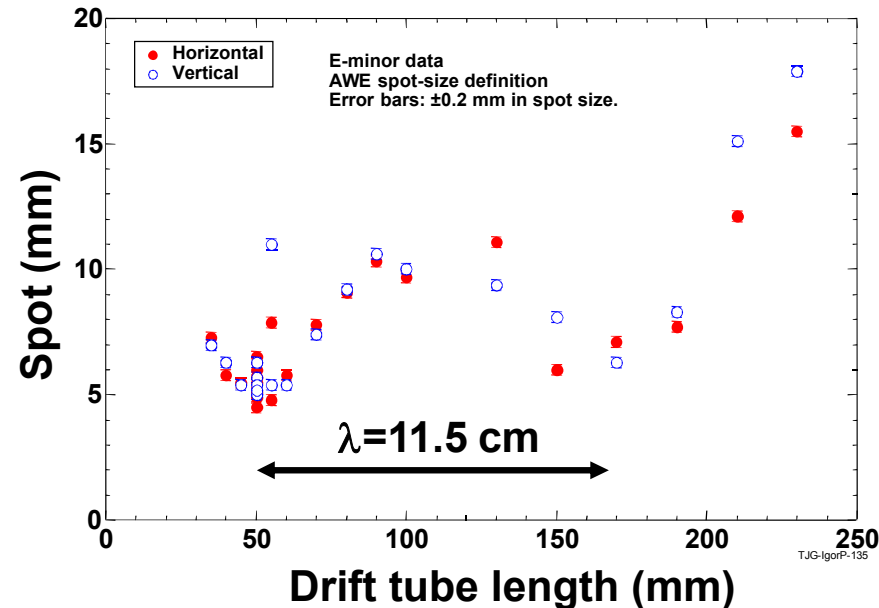


Fig. courtesy of A. Birrell<sup>1</sup>

Data from AWE<sup>1,2</sup> (5.3 MV,  $I_b = 33 \text{ kA}$ )

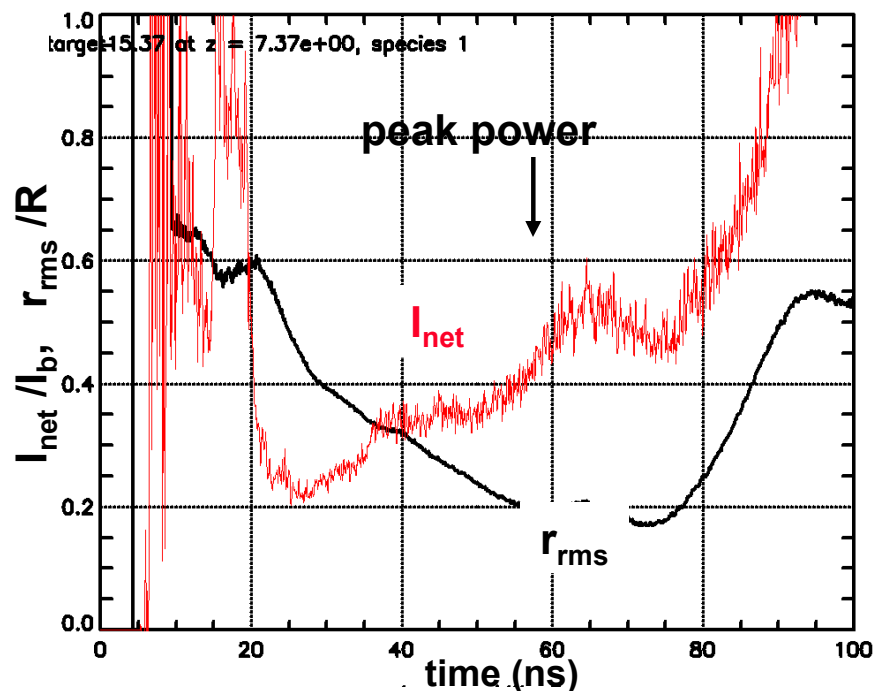
$\lambda = 11.5 \text{ cm}$  implies a time average net current  $I_{\text{net}}/I_b \sim 0.4$

$a = 0.5 \text{ cm}$  implies emittance  $\varepsilon \sim 0.18 \text{ cm-radian}$

1. A. Birrell et al. IEEE Trans Plasma Sci. 28, 1660 (2000)
2. B.V. Oliver and D. Welch Proc. Intl Conf. Intense Beams, Russia, 2004

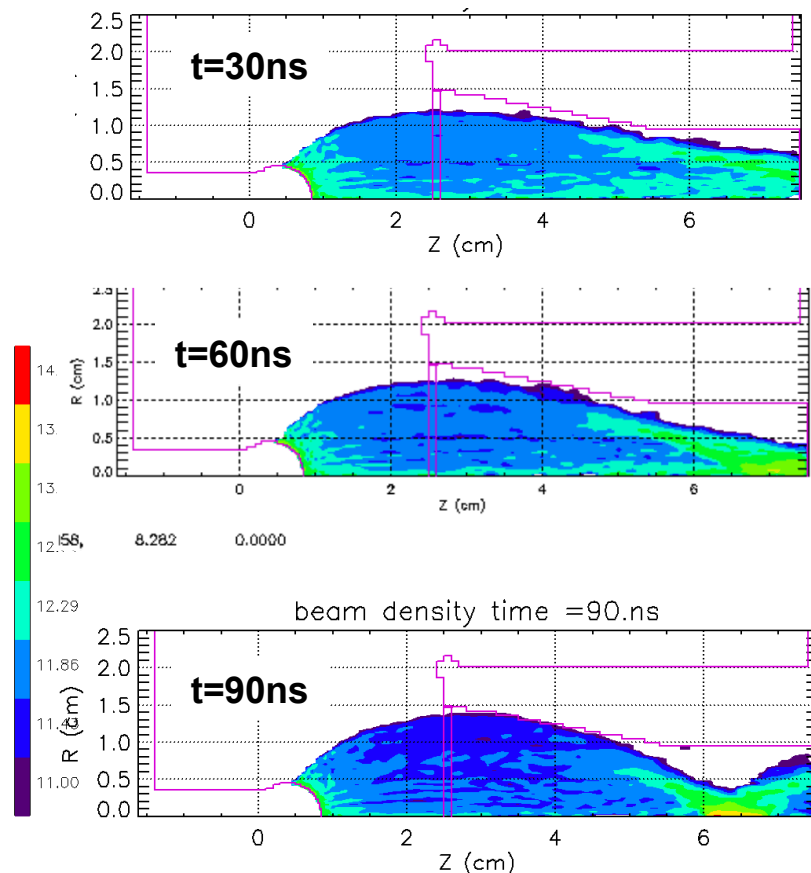
# Time dependence of $I_{\text{net}}$ causes a sweeping focus!

$$F(t) \cong \frac{R}{2} \sqrt{\frac{\pi I_A}{I_{\text{net}}(t)}}$$



Beam rms radius and  $I_{\text{net}}$  vs time. At peak power,  $I_{\text{net}} = 9 \text{ kA}$ ,  $r_{\text{rms}} = 0.2 \text{ cm}$

## Contours of beam density



Focal sweeping is a primary contributor to larger than desired time integrated spots.

# The ion-hose instability

## IFR ion hose instability<sup>[1]</sup>:

Backstreaming ion channel causes space-charge attraction of e-beam. With finite ion mass  $\rightarrow$  ions move  $\rightarrow$  convective instability.

Growth rate  $\sim$  ion inertial time

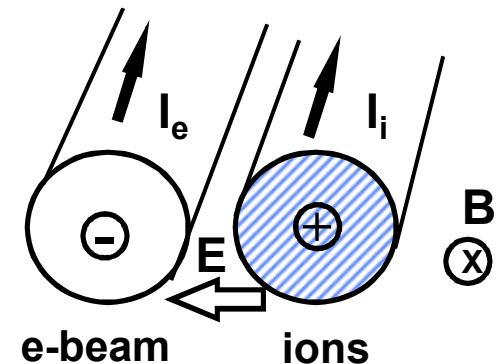
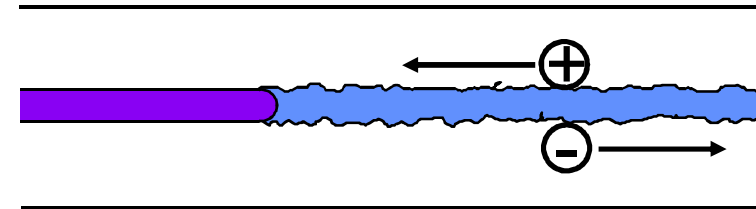
$$\Gamma \approx \omega_{pi} = (4\pi e^2 n_b / M)^{1/2} \sim 5 \times 10^9 \text{ s}^{-1}$$

Convective length

$$\lambda \approx (c/\omega_p) = (\gamma m c^2 / 4\pi e^2 n_i)^{1/2}$$

In the presence of a solenoidal field, beam rotates and growth rate is modified in limit  $(\Omega_e/\omega_p) \gg 1$  [2]

$$\lambda_{ce} \approx \lambda (\Omega_e/\omega_p) \sim \begin{array}{ll} 1 \text{ cm} & \text{immersed-}B_z \text{ diode} \\ 10 \text{ cm} & \text{linear induction accelerators} \end{array}$$



**Time and length scales imply non-linear evolution and saturation.**

1 H. L. Buchanan et al, Phys. Fluids **30**,1456 (1987); EP. Lee Phys. Fluids **21**, 1327 (1978)

2 D.V. Rose, T.C. Genoni, D.R. Welch, Phys. Plasmas, **11**, 4990 (2004)

# Nonlinear modeling of ion-hose saturation amplitude in good agreement with 3-D LSP simulations.

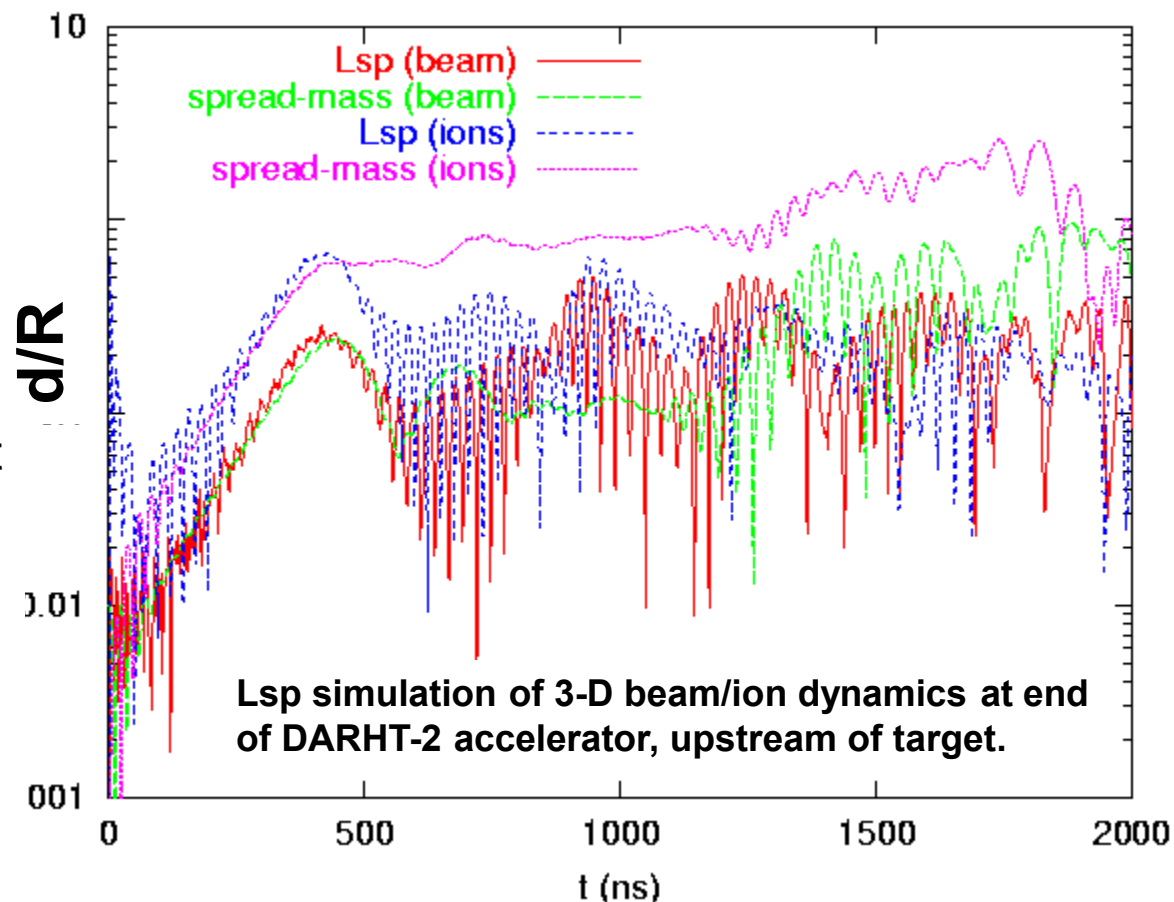
Linear and non-linear theory of ion-hose instability captures primary 3-d dynamics of the beam ion interaction.

Nonlinear evolution occurs within 3-4 growth times.

Spread-mass<sup>1</sup> modeling of ion-hose saturation provides information for scaling to different beam and plasma conditions.

**Saturation amplitude scales with linear growth rate  $1/\lambda_{ce} \propto 1/B$ .**

Beam and ion centroid displacement vs. time  
(normalized by beam radius R)



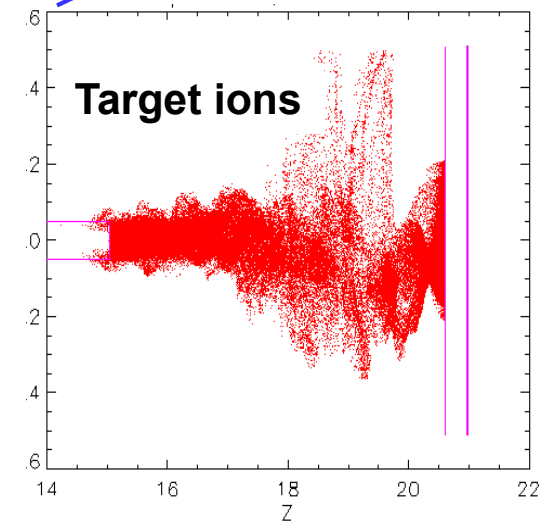
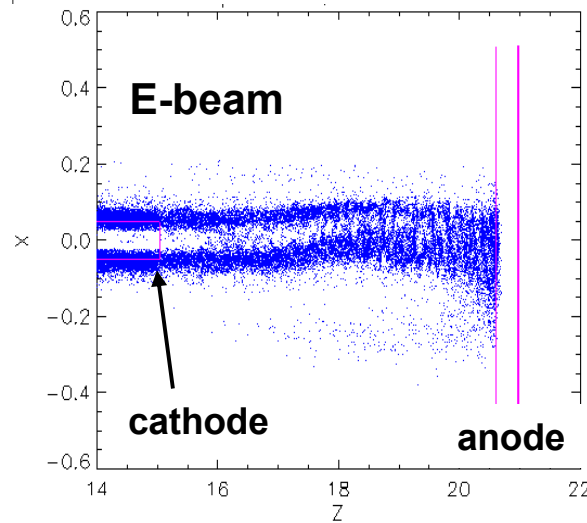
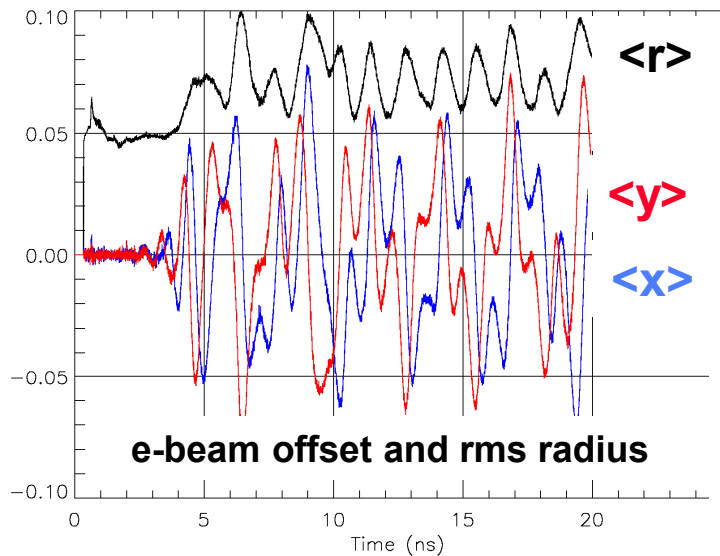
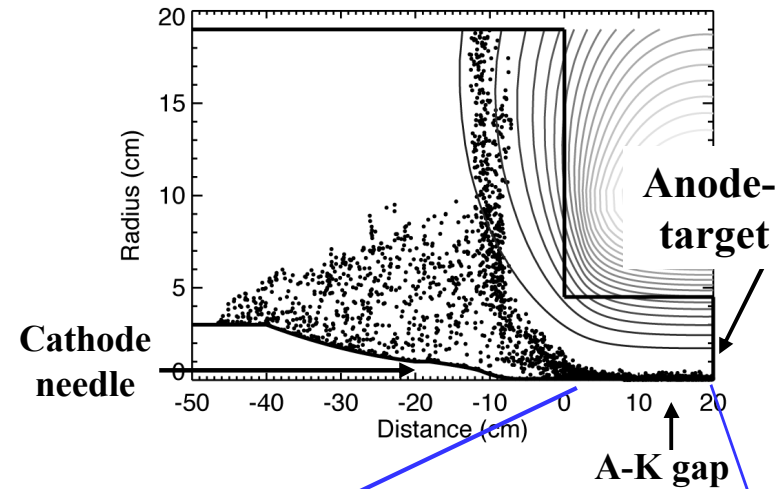
1. E.P. Lee Phys. Fluids 21, 1327 (1978)
2. D.V. Rose, T.C. Genoni, D.R. Welch, Phys. Plasmas, **11**, 4990 (2004):

# The ion-hose instability in the applied-B<sub>z</sub> diode

**Immersed-B<sup>1</sup>:** Diode used for x-ray radiographic applications. Creates high intensity bremsstrahlung radiation.

Beam spot on target determined by ion-hose saturation amplitude

$$\langle r_{\text{sat}} \rangle \approx \frac{c}{\Omega_e} \sqrt{2\gamma \frac{I_b}{I_A}}$$



## 3-D PIC simulations of immersed-B diode electron and ion dynamics

1. M.G. Mazarakis et al. Appl. Phys. Lett **70**, 832 (1997)  
D.R. Welch et al. Laser and Particle Beams **16**, 285, (1998)



# Magnetic Insulation

# The Magnetically Insulated Transmission Line

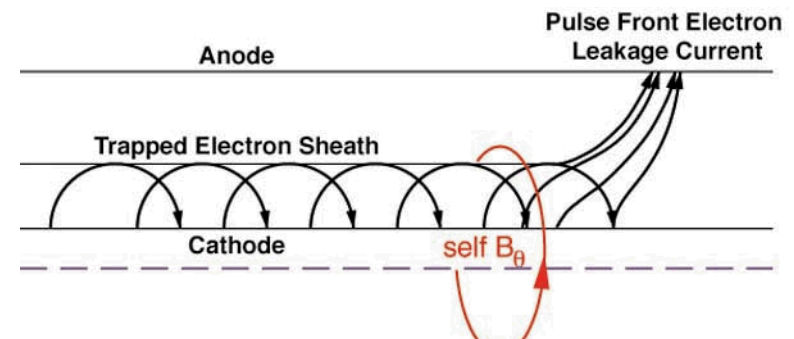
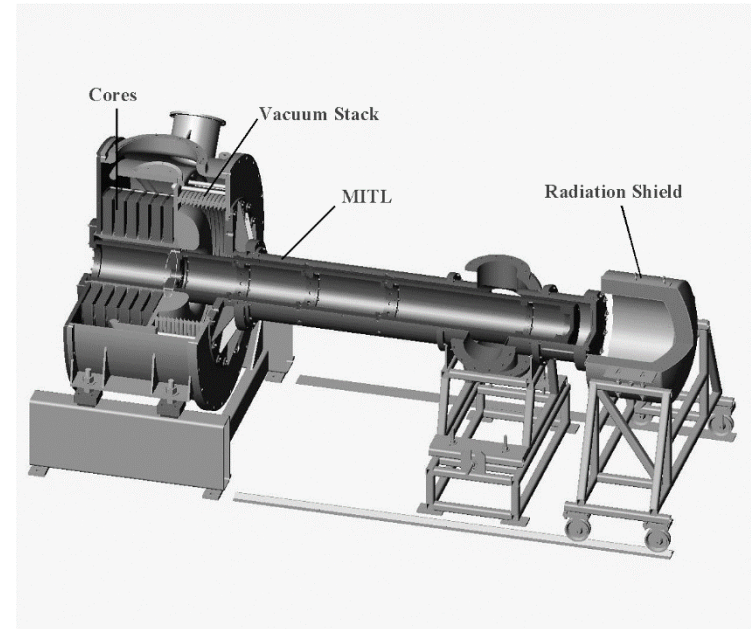
The MITL is a coaxial transmission line (solid metal conductor) that is an integral part of Inductive Voltage Adder (IVA) accelerators:

- they act to connect in series each induction cell
- they allow power transmission downstream

For high voltage systems, the electric field inside the coaxial transmission line exceeds the threshold for electron emission off the cathode surface ( $\sim 200\text{-}250 \text{ keV/cm}$ ). And for high current systems, emitted electrons are magnetically insulated from crossing the A-K gap of the transmission line and flow downstream. Hence the name Magnetically Insulated Transmission Line.

The presence of the emitted electron space-charge and current, inside the line, changes the impedance of the transmission line from its vacuum value. Thus, the MITL's I,V characteristics (operating impedance) are considerably more complicated than a normal vacuum TL. In fact, the MITL impedance is non-linear:

$$Z_{MITL} = \frac{V}{I_{MITL}} = \frac{V}{I(V)}$$

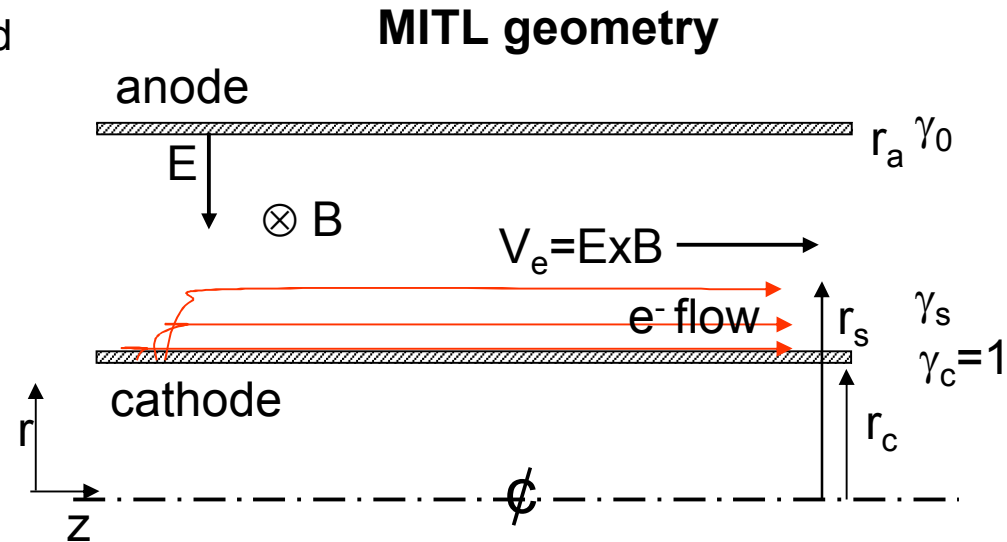


**Illustration of axis-symmetric insulated flow in the MITL**

# 1-D, equilibrium flow models

A lot of good theory was established in the mid 1970s on magnetic insulation which has been used extensively to determine the MITL impedance characteristics<sup>1-3</sup>.

The relativistic electron flow is treated self-consistently assuming all electrons born at the same potential and the flow is governed by  $E \times B$  drift. Vacuum solutions are matched at the edge of the electron sheath.



$$\nabla \phi \times B = v_e \quad \text{momentum}$$

$$\nabla^2 \phi = n_e \quad \text{Poisson}$$

$$\nabla \times B = -n_e v_e \quad \text{Ampere}$$

$$\gamma - \phi = 1;$$

$$\gamma = \sqrt{1/(1 - v^2)} \quad \text{Cons. of energy}$$

1. John M. Creedon, J. Appl. Phys, **46**, 2946 (1975)
2. M. Reiser, Phys. Fluids, **20**, 477 (1977)
3. A. Ron et al., IEEE Trans. Plasma Sci ps-1,85 (1973)

# 1-D, laminar flow model of Creedon

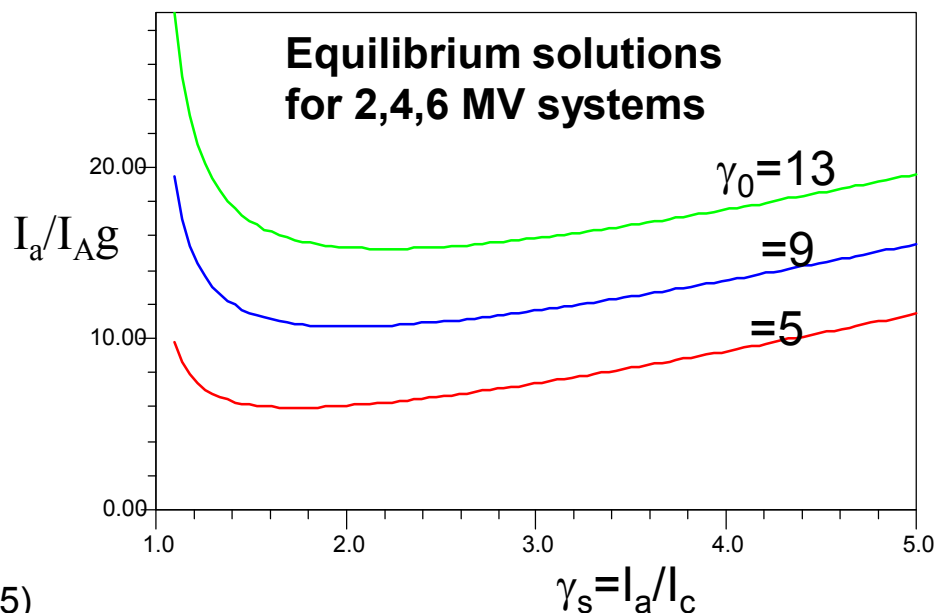
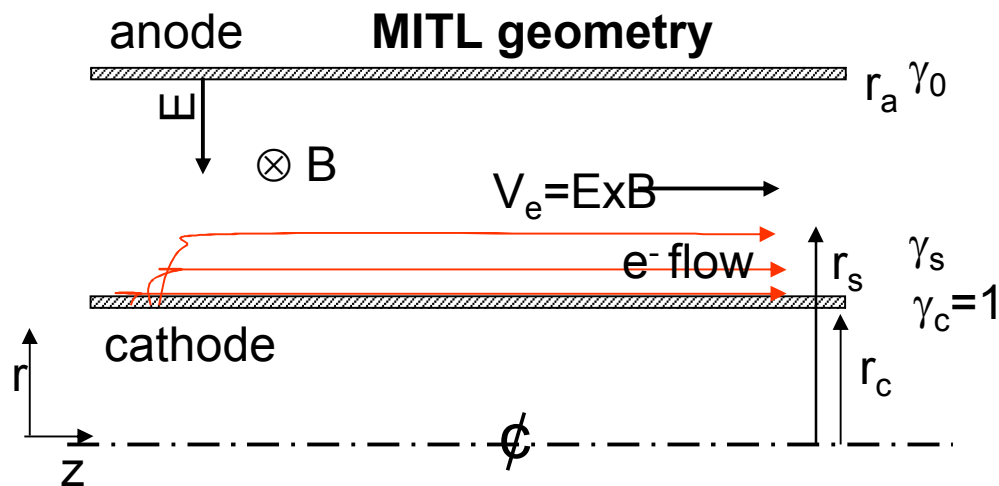
One obtains an infinite number of equilibria relating the MITL current to the voltage for various sheath thickness (energies), e.g. from Creedon<sup>1</sup>:

$$I_a = I_A g \gamma_s \left[ \text{acosh}(\gamma_s) + \frac{\gamma_0 - \gamma_s}{\sqrt{\gamma_s^2 - 1}} \right]$$

$$\gamma_s = \frac{I_a}{I_c}$$

$I_A = 8500$  (kA),  $g = 1/\ln(r_a/r_c)$ ,  
 $\gamma_s = \text{gamma @ electron sheath edge}$ ,  
 $\gamma_0 = 1 + (eV/mc^2) = \text{applied voltage}$

A more generalized model is described by Ottinger<sup>2</sup> et al.

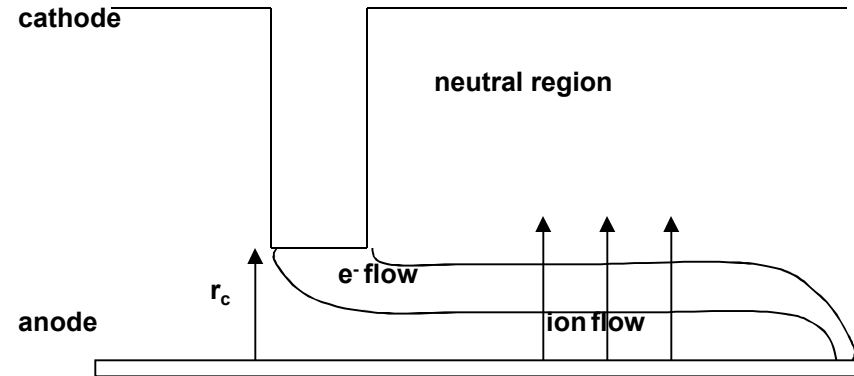


1. John M. Creedon, J. Appl. Phys, **46**, 2946 (1975)
2. Ottinger et al. IEEE Trans. Plasma Sci. vol. 36, pp. 2708- 2008-2721, 2008

# Insulation in the presence of ion emission

- Follows closely Creedon et al.<sup>1</sup> on 1-D radial relativistic equilibrium electron flow
- Includes a background ion space-charge
- Ion current is radial and included via a boundary current. Ion charge and current density are solved self-consistently.
- Considers the region downstream of the cathode blade. It is characterized by being charge and current neutral at radii  $> r_c$

Schematic of insulated flow in the rod-pinch diode



$$\nabla \phi \times \mathbf{B} = \mathbf{v}_e \quad \text{momentum}$$

$$\nabla^2 \phi = n_e \quad \text{Poisson}$$

$$\nabla \times \mathbf{B} = -n_e \mathbf{v}_e \quad \text{Ampere}$$

$$\gamma - \phi = 1;$$

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## Coupled ion response

$$\nabla \cdot \mathbf{J}_i = 0. \Rightarrow r_c J_i = r n_i v_i = \text{const.}$$

$$\frac{1}{2} v_i^2 = \left( \frac{Zm}{M} \right) (\gamma_a - \gamma)$$

$$n_i = \frac{r_c J_i}{r \sqrt{2Zm(\gamma_a - \gamma)/M}}$$

# The Rod-Pinch diode is an example of self-magnetically insulated flow with ions

Diode current well modeled by critical current formulation<sup>1</sup>:

$$I = \alpha I_{\text{crit}}, \quad 2.0 < \alpha < 2.6$$

$$I_{\text{crit}} = 8.5 \frac{\sqrt{\gamma^2 - 1}}{\ln(r_c / r_a)} \text{ kA}, \quad \gamma = 1 + eV/mc^2$$

Operation and  $\alpha$  is described by self-insulated flow theory with the inclusion of ions<sup>2</sup>

Region I

$$\nabla^2 \phi = n_e - n_i,$$

$$\nabla \times B = n_e v_e$$

$$\nabla \phi + v_e \times B = 0,$$

Region II

$$\nabla^2 \phi = -n_i.$$

$$n_i v_i = \frac{r_c}{r} j_c$$

Ions are absolutely necessary for operation!

$$J_i = \frac{4}{9} \frac{(\gamma_a - \gamma_s)^{3/2}}{(r_s - r_a)^2}.$$

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$$I \cong 17 r_s \sqrt{J_i} [\gamma_a - 1]^{1/4} \text{ kA.}$$

rod-pinch diode

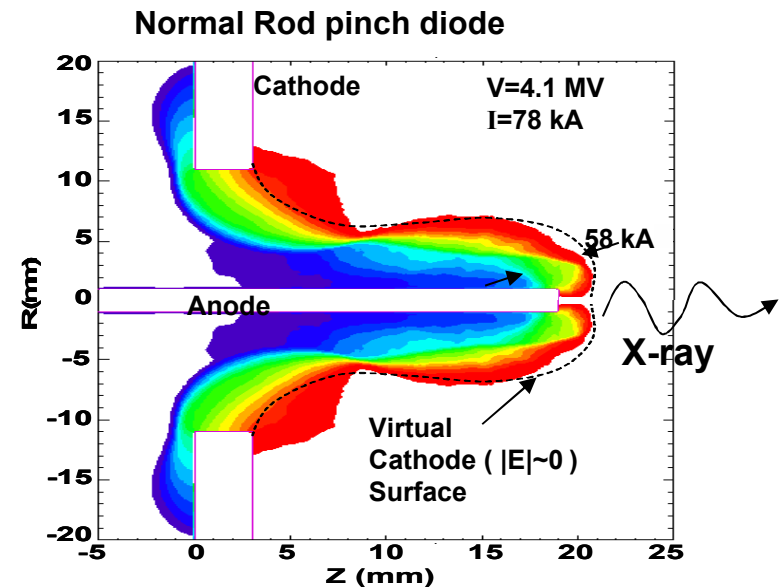
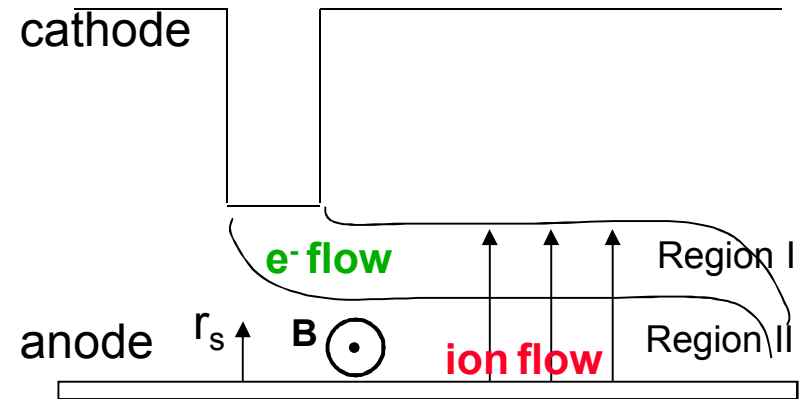


Fig. courtesy of S. Swaneekamp, NRL

1. G. Cooperstein et al. Phys. Plasmas, **8**, 4618 (2001)
2. B.V. Oliver et al. Phys. Plasmas, **11**, (2004);