

Intense Charged Particle Beam Physics and Applications

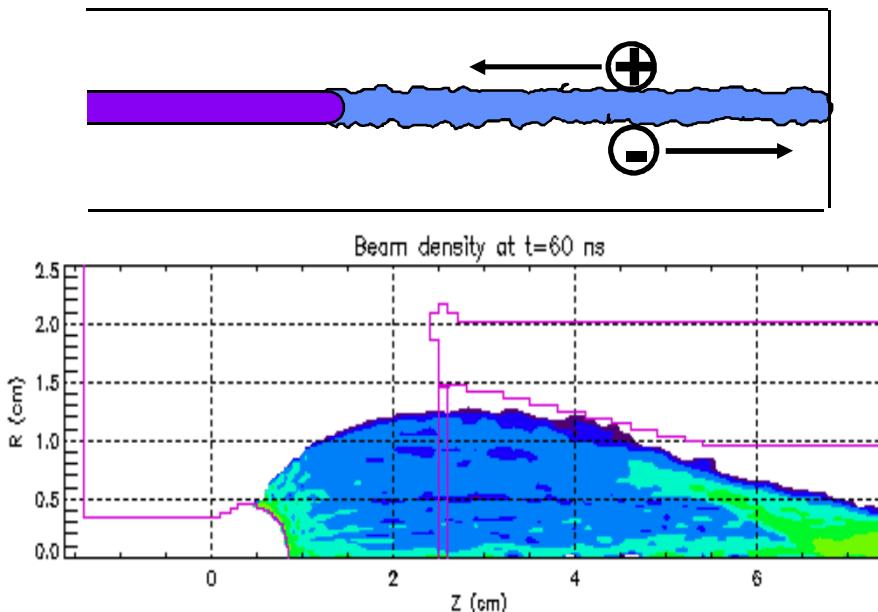
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Outline

This lecture will present some theoretical constructs of intense charged particle beam generation and transport with occasional examples of present day use of such beams.

Introduction to Intense Beams:

- Description and parameters
- Force free equilibria for a cold e-beam in vacuum
- Beam envelope equation

High Power Diodes:

- Space charge limited emission
- In the presence of background gas/plasma
- With electron backscatter

Propagation:

- e-beams in vacuum
- e-beams in plasma - example, the paraxial diode
- The ion hose instability

Magnetically insulated flow – equilibria

- in vacuum
- with ions

Introduction to Intense Beams

Intense charged particle beams have the unique characteristic that their density and velocity are large enough to induce strong electric and magnetic “self-fields” that are sufficient to greatly influence the beams’ dynamics.

Advances in pulsed power technology over the last 30 years has led to the production of electron and ion beams with currents > 10 kA and kinetic energies ranging from 100 keV to > 10 MeV, with pulse durations in the 10-100 ns range.

The beams are generated in the accelerating gap of a high current diode and can either be accelerated after extraction from the diode to higher energies, transported in vacuum or in background gas/plasma, or they can be focused directly onto a target.

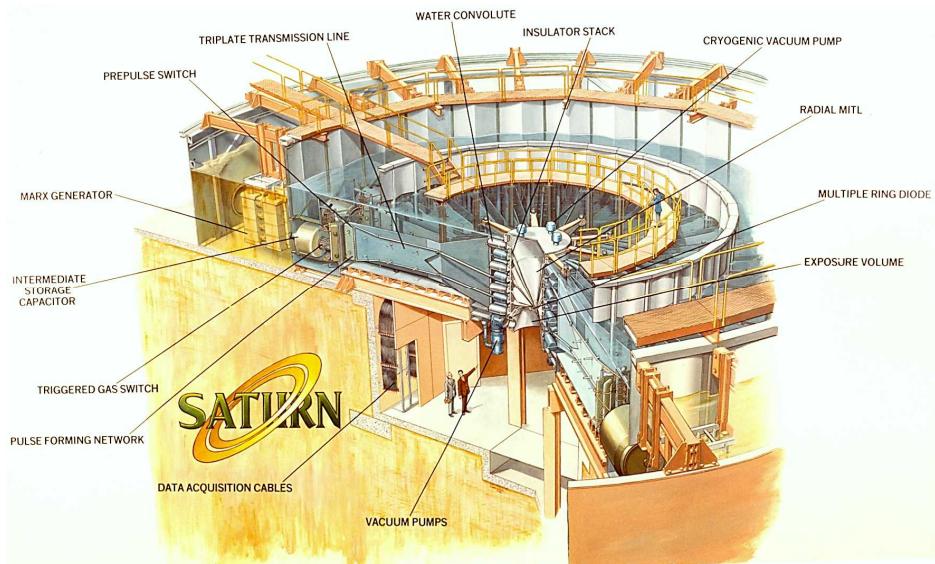
Applications include materials testing and modification, production of X-rays for lithography, radiography or nuclear weapon effects, high-power microwave generation, or nuclear fusion.

Key Overview References:

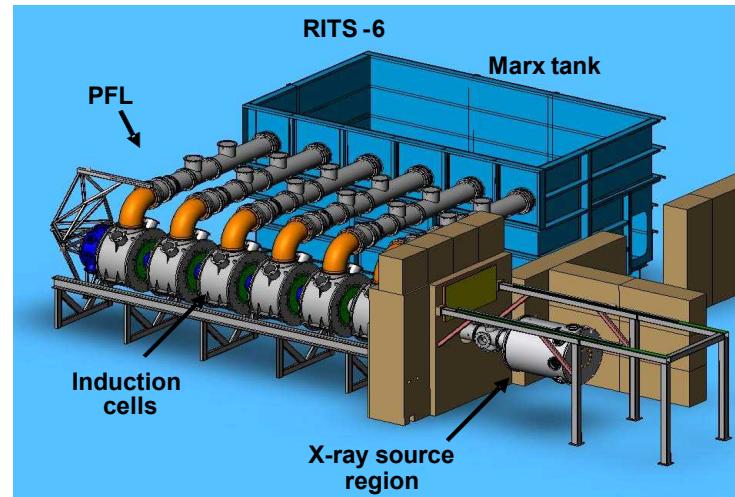
1. J.D. Lawson, *The Physics of Charged particle Beams*, Oxford Univ. Press, 1977
2. R.B. Miller, *Intense Charged Particle Beams*, Plenum Press, New York, 1982
3. S. Humphries, *Principles of Charged Particle Acceleration*, John Wiley and Sons., 1986
4. R.N. Sudan, Ch. 6.3 *Collective Beam-Plasma Interaction*, in *Handbook of Plasma Physics Vol 2. Basic Plasma Physics II*, edit. A.A. Galeev and R.N. Sudan, North-Holland Pub., 1984
5. R.C. Davidson, Ch 9.1 *Relativistic Electron Beam-Plasma Interaction with Intense Self-fields*, ibid

Introduction to Intense Beams

We will consider High power particle beams in the 10 GW– 10 TW range that are generated from pulsed power accelerators.



**Saturn Accelerator,
Sandia National Laboratories**
1.6 MeV, 10 MA, 40ns e-beam driver



**RITS-6 Accelerator,
Sandia National Laboratories**
10 MeV, 180 kA, 70ns e-beam driver

The beams are generated in the accelerating gap of a high current diode and can either be propagated in vacuum or in background gas/plasma.

Typical Beam Parameters

Energy E_b = 0.1-20 MeV,

Velocity v_b/c = 0.1-1

Current I_b = 0.01-10 MA,

Current density j_b = 1-1000 kA/cm²,

Pulse length τ_b = 10-100ns,

Beam density n_b = 1-10 cm⁻³,

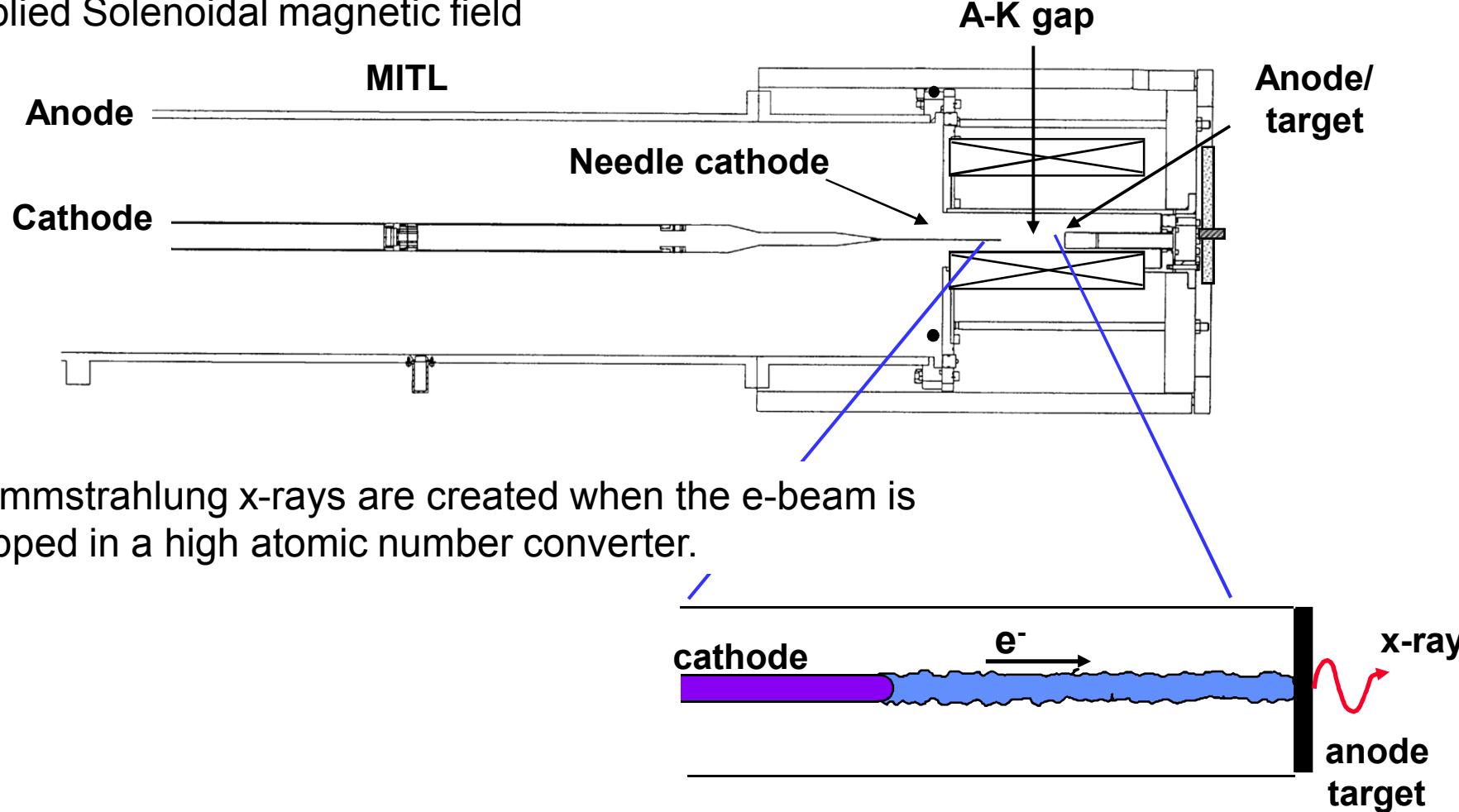
Beam radius r_b = 0.1-10 cm,

Total charge Q_b = 5-500 mC

Relativistic factor $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1-40$

Example: Pulsed-power driven e-beam diode for x-ray radiography applications

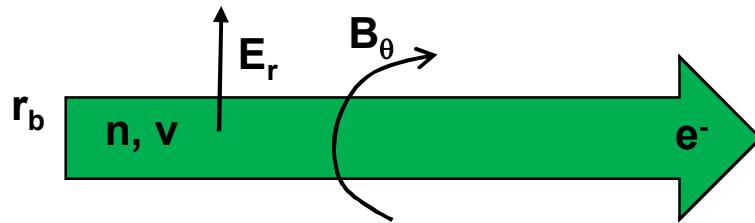
The Immersed B_z diode¹: the electron beam is created in the accelerating gap of a high current diode and guided in vacuum to an anode/target via an applied Solenoidal magnetic field



Energy $E_b = 2-10$ MeV, Current $I_b = 20-150$ kA, Pulse length $\tau_b = 50-100$ ns

Beam Self-field Forces

Consider the 1-D radial forces on a cold (zero emittance) uniform density electron beam drifting in vacuum.



The self-fields at the beam edge $E_r = 2\pi enr_b$
 $B_\theta = 2\pi enr_b(v/c)$

Gaussian units are used
 $c = 3 \times 10^{10} \text{ cm/s}$

The radial Lorentz force acting on the beam

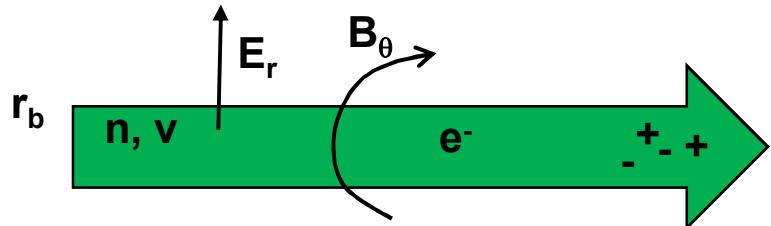
$$\begin{aligned} F_r &= e(E_r + v \times B_\theta) \\ &= 2\pi e^2 nr_b (1 - (v/c)^2) \equiv 2\pi e^2 nr_b / \gamma^2 \end{aligned}$$

The beam will only be in force balance ($F=0$) when $v=c$ ($\gamma \rightarrow \infty$). Hence all beams will expand under their self-electric field unless their space-charge (en) is partially neutralized.

The Beam Envelope Equation

For relativistic beams propagating in vacuum with finite emittance ϵ and axial velocity v_z , the force balance equation for the beam radial edge r_b (envelope) is

$$\frac{d^2 r_b}{dz^2} = \frac{2v}{\beta^2 r_b} [1 - f_e - (1 - f_b) \beta^2] + \frac{\epsilon^2}{r_b^3}$$



Where the conditions $v_z \gg v_p, v_\theta$ such that $d/dt \approx v_z d/dz$, and $\beta = v_z/c$, $\gamma = 1/\sqrt{1 - \beta^2}$

ϵ = beam emittance,

$$v \equiv \frac{eI_b}{\gamma\beta mc^3} \equiv \frac{I_b}{I_A} \quad I_A = \gamma\beta 17 \text{ (kA)}, \quad \text{is the Alven current}$$

$$\epsilon = 4\sqrt{\langle r^2 \rangle \langle r'^2 \rangle - \langle rr' \rangle^2}$$

$$f_e = \frac{n_p}{n_b} \quad \text{is the charge neutralization fraction}$$

$$f_b = \frac{I_p}{I_b} \quad \text{is the current neutralization fraction}$$

Intense Beam Generation

Diodes

Space-charge limited diodes¹.

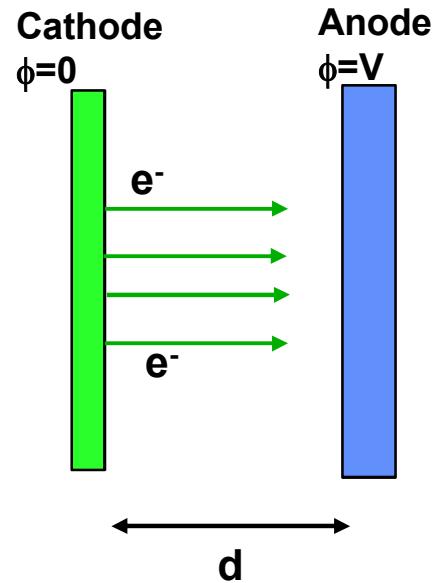
SCL diode: e.g. 1-D, non-relativistic large area

$$\frac{d^2\varphi}{dz^2} = en_e$$

$$j = en_e v = \text{const.}$$

$$\frac{1}{2}mv^2 = e\varphi$$

$$\rightarrow j \propto \frac{1}{d^2} \sqrt{\frac{2e}{m}} V^{3/2}$$



If there is some space charge neutralization, then

$$en_e \rightarrow en_e(1-f) \quad f=n_i/n_e$$

$$\rightarrow j \propto \frac{1}{(1-f)d^2} \sqrt{\frac{2e}{m}} V^{3/2}$$

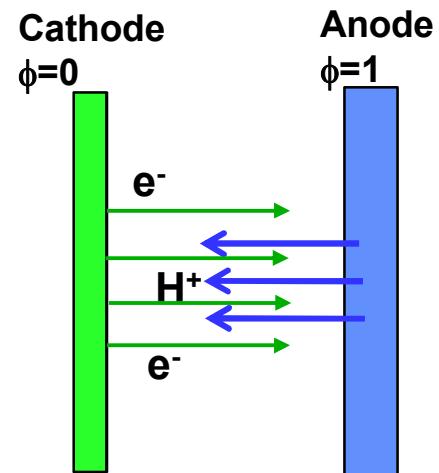
1. I. Langmuir and K. Blodgett, Phys. Rev. **22**, 347 (1923); C.B. Wheeler, J. Phys. A **10**, 631 (1977)

Bi-polar (electron + ion) space charge limited and self-magnetic field limited diodes.

SCL diodes¹: e.g. 1-D, non-relativistic large area

$$I = \alpha I_{cl}, \quad 0.5 < \alpha < 5 \quad (\alpha = 1.86 \text{ for planar bi-polar diode})$$

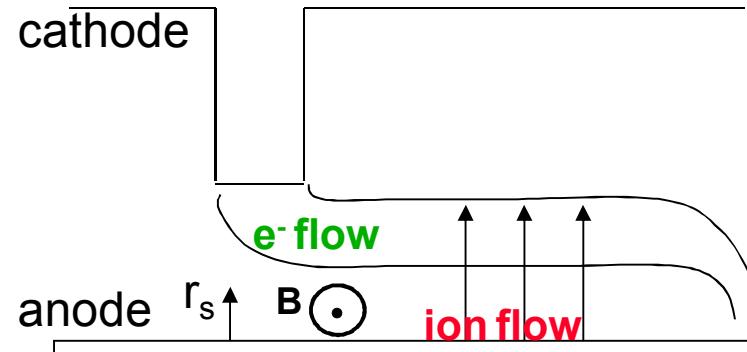
$$I_{cl} = \frac{1}{9\pi} \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2} \text{ A},$$



Self-limited diodes: e.g. rod-pinch²

$$I = \alpha I_{crit}, \quad 2.0 < \alpha < 2.6$$

$$I_{crit} = 8.5 \frac{\sqrt{\gamma^2 - 1}}{\ln(r_c/r_a)} \text{ kA}, \quad \gamma = 1 + eV/mc^2$$



The factor α , is dependent on geometry, voltage, and ion distribution

1. I. Langmuir and K. Blodgett, Phys. Rev. **22**, 347 (1923); C.B. Wheeler, J. Phys. A **10**, 631 (1977)
2. G. Cooperstein et al. Phys. Plasmas, **8**, 4618 (2001)

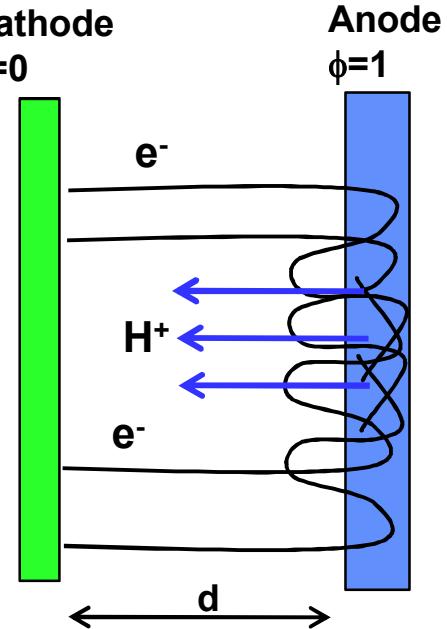
The effect of electron backscatter at the anode¹

Bi-polar Child-Langmuir currents are modified by the presence of the extra electron space charge scattered into the A-K gap

This enhances the ion space charge emission and changes the impedance

$$\left(\frac{d\phi}{dx} \right)^2 = \frac{16}{9} \frac{j_e}{j_{cl}} g(\phi)$$

$$g(\phi) = \sqrt{\phi + \frac{eV}{2mc^2} \phi^2} - \left(\frac{M}{m} \right)^{1/2} \frac{j_i}{j_e} \left(1 - \sqrt{1 - \phi} \right) +$$



$$j_{cl} = \frac{1}{9\pi} \sqrt{\frac{2eV}{m}} \frac{V}{d^2}$$

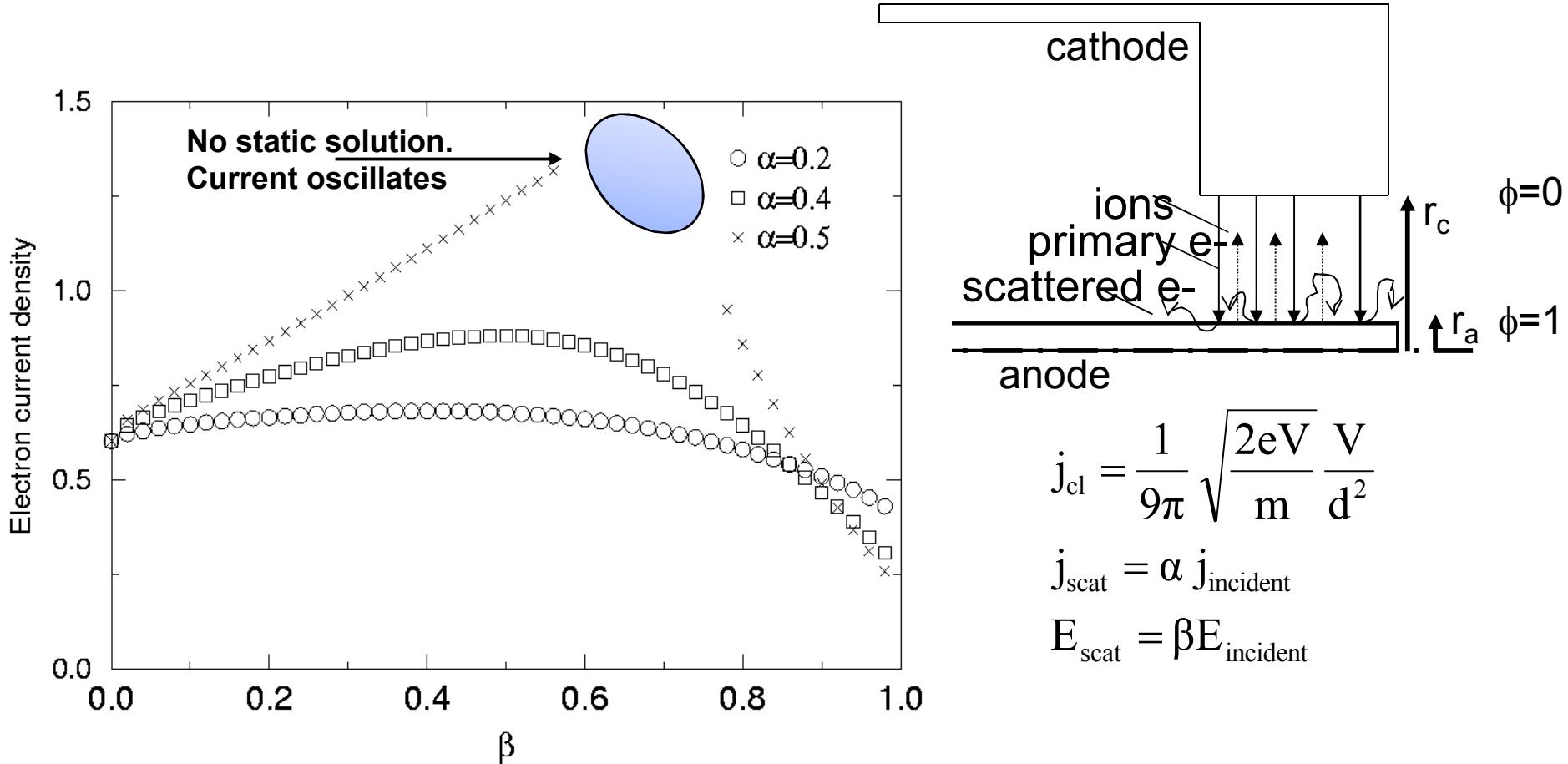
$$j_{scat} = \alpha j_{incident}$$

$$E_{scat} = \beta E_{incident}$$

Extra terms due to scattering

1. N.R. Pereira, JAP **54**, 6307 (1986)
- D. Mosher, G. Cooperstein et al, Proc. 11th Intl. Beams Conf. (1996)
- V. Engelko, V. Kusnetsov et al. JAP **88**, 3879 (2000)
- B.V. Oliver, T.C. Genoni et al., JAP **90**, 4951 (2001)

Electron backscatter can be significant in cylindrical diodes, results in decreased but stable impedance!



As the fraction of reflected beam current goes up, so does the total current. However, there is a maximum and the current is stable.

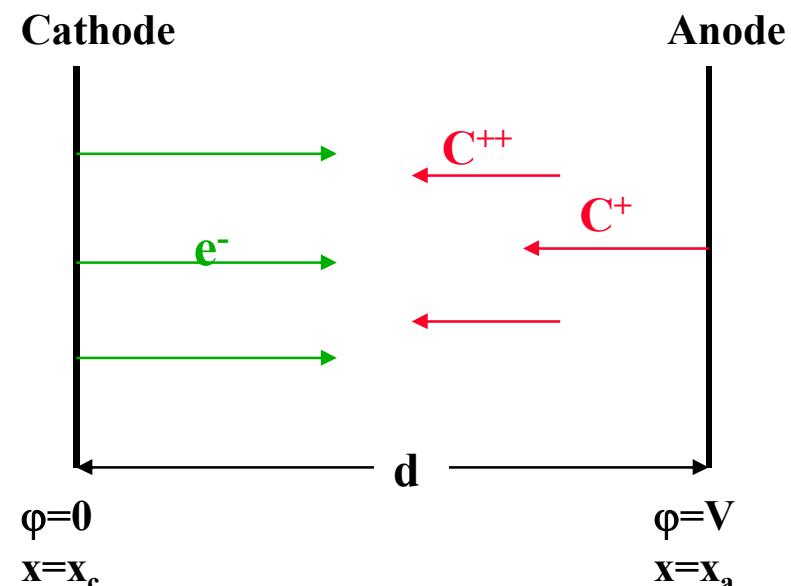
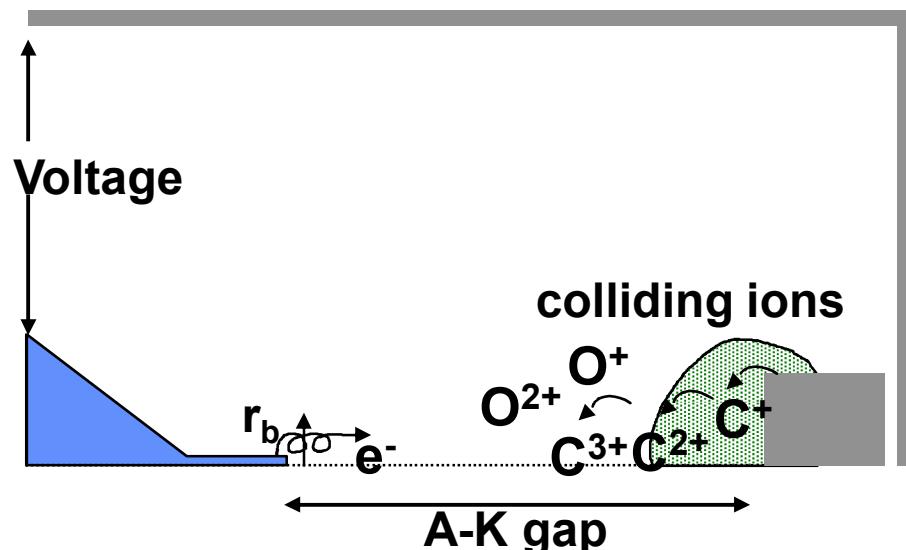
Charge stripping of ions can increase electron emission and thus current¹

Non-protonic ions (e.g. C,O) from the anode collide and charge-strip while traversing the A-K gap.

The electron current I_e increases when the excess ion charge reaches the cathode. Feedback causes impedance collapse.

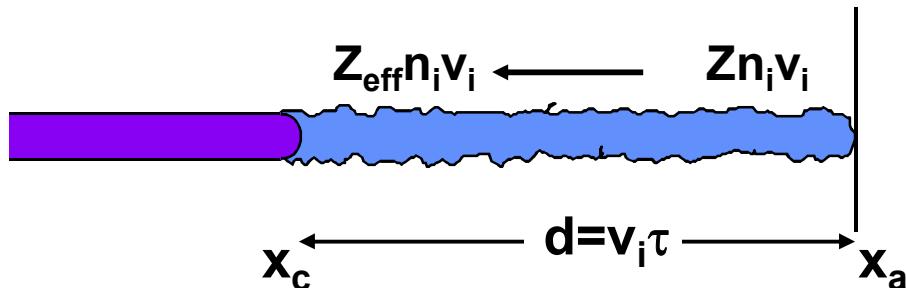
$$j_i = j_e \sqrt{\frac{Zm}{M}} (1 + eV/2mc^2)$$

But $j_i(x_c, t) \neq j_i(x_a, t)$ because of charge stripping during ion transit across the gap!



The ion current density increases during transit across the gap

Ion current density increases in proportion to the ionization fraction.



$$Z_{\text{eff}} = Z + v_{\text{ion}} \tau, \quad \tau = \frac{d}{v_i}$$

$$j_i(x_c, t) = \frac{Z_{\text{eff}}}{Z} j_i(x_a, t - \tau)$$

The ionization frequency v_{ion} is proportional to the ion current density at the anode and the ionization cross-section σ_{ion} :

$$v_{\text{ion}} = n_i \langle v_i \sigma_{\text{ion}} \rangle \approx \frac{j_i(x_a, t - \tau)}{Z e} \sigma_{\text{ion}}$$

Non-linear response occurs because Z_{eff} is proportional to j_i through v_{ion}

Non-linear (explosive) electron current time-dependence results¹

The time dependent solutions for the electron current density are nonlinear because of the relation of J_i to J_e and Z_{eff} .

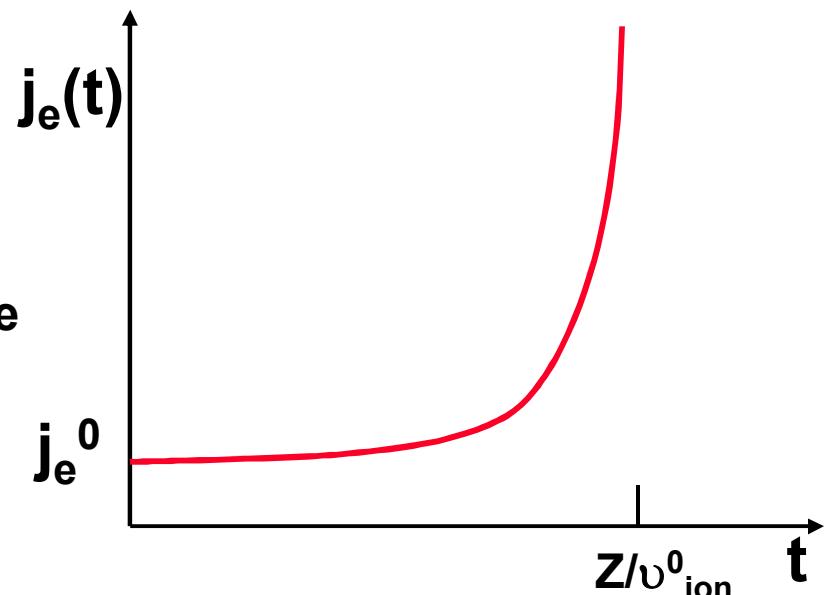
$$\left(\frac{Z_{\text{eff}}}{Z} - 1\right) = \frac{v_{\text{ion}}\tau}{Z} = \frac{j_i(x_a, t-\tau)\tau}{Z^2 e} \sigma_{\text{ion}} \equiv \sqrt{\frac{Zm}{M}(1+eV/mc^2)} \frac{j_e(x_a, t-\tau)\tau}{Z^2 e} \sigma_{\text{ion}}$$

$$j_e(x_a, t) = j_e^0 \frac{1}{1-\alpha t}$$

Solutions result in explosive behavior on time scale $1/\alpha \sim Z/v_{\text{ion}}^0$

$$\alpha = \frac{j_e^0}{Z^2 e} \sqrt{\frac{Zm}{M}(1+eV/2mc^2)} \sigma_{\text{ion}}$$

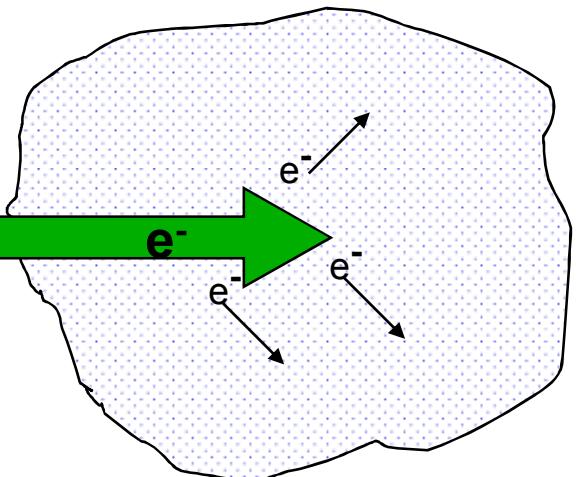
~ 10 ns



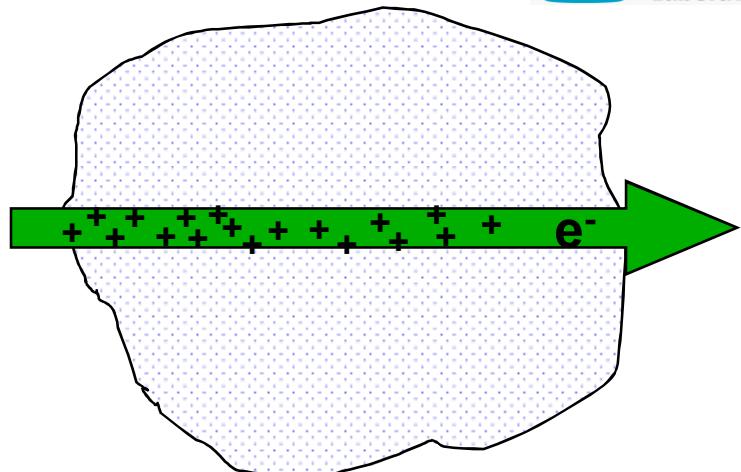
Beam Propagation

In vacuum and gas/plasma

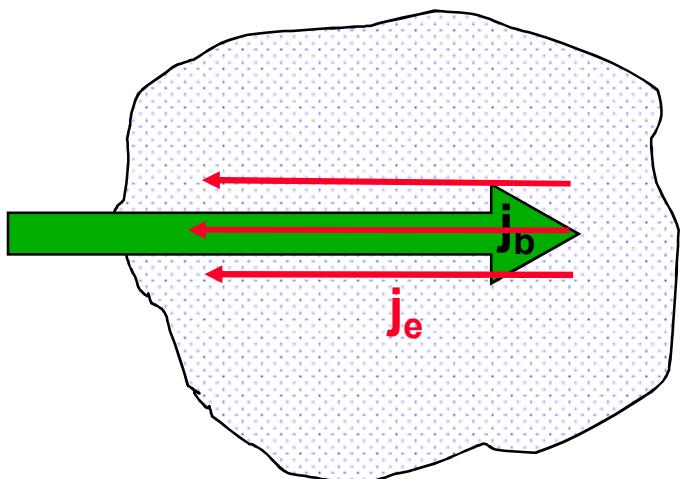
Electron beam neutralization in plasma



e-beam expels plasma electrons



Creates a plasma ion channel so beam + plasma is quasi-neutral: $n_i - n_e - n_b \approx 0$



Plasma return current driven by inductive electric field and determined by generalized Ohm's law

$$\text{Ohm's law} \quad j_e = \sigma(E + \frac{V_e}{c} \times B); \quad \sigma = \text{conductivity}$$

$$\nabla \times B = \frac{4\pi}{c} (j_e + j_b); \quad j_i \approx 0$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t};$$

$$\frac{\partial B}{\partial t} = -\nabla \times \frac{c}{4\pi \sigma n_e} \left(\nabla \times B - \frac{4\pi}{c} j_b \right) \times B + \nabla \times \frac{c^2}{4\pi \sigma} \left(\nabla \times B - \frac{4\pi}{c} j_b \right)$$

Paraxial diode: a classic beam propagation problem in overdense plasma $n_b/n_e \ll 1$. Gas-cell acts as a $\frac{1}{4}$ betatron focusing lens¹

Gas breakdown sufficient for complete charge neutralization but incomplete current neutralization.

$$\frac{d^2 r_b}{dz^2} \approx -\frac{1}{r_b} \frac{2I_{\text{net}}}{I_A} + \frac{\varepsilon^2}{r_b^3}, \quad I_{\text{net}} = I_b + I_{\text{plasma}}$$

For $\varepsilon^2 \ll 2R^2 I_{\text{net}}/I_A$

$$F \equiv \frac{R}{2} \sqrt{\frac{\pi I_A}{I_{\text{net}}}}, \quad \propto \sqrt{\frac{\gamma}{I_{\text{net}}}}$$

$$I_A = \gamma \beta 17 \text{ (kA)},$$

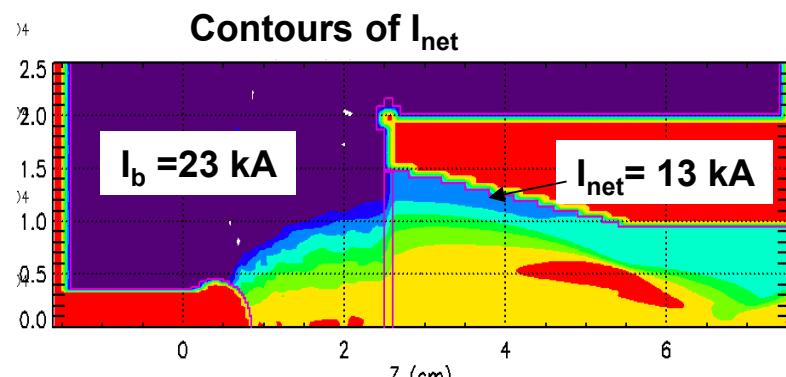
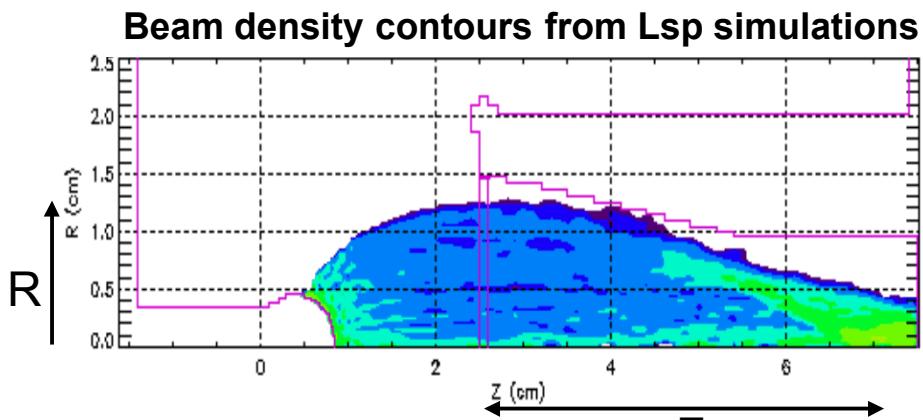
$$\varepsilon = 4\sqrt{\langle r^2 \rangle \langle r'^2 \rangle - \langle rr' \rangle^2}$$

Net current (beam + plasma) $I_{\text{net}} = crB_\theta/2$

$$\frac{\partial B}{\partial t} = -\nabla \times \frac{c}{4\pi e n_e} \left(\nabla \times B - \frac{4\pi}{c} j_b \right) \times B + \nabla \times \frac{c^2}{4\pi \sigma} \left(\nabla \times B - \frac{4\pi}{c} j_b \right)$$

Hall current advection

Resistive diffusion



Data suggests average net current in cell $\sim 0.3\text{-}0.4 I_b$

Envelope oscillation wavelength $\lambda \approx 2\pi a(I_A/I_{\text{net}})^{1/2}$
 Equilibrium radius $a \approx \varepsilon(I_A/2I_{\text{net}})^{1/2}$

Focal length vs. γ ,

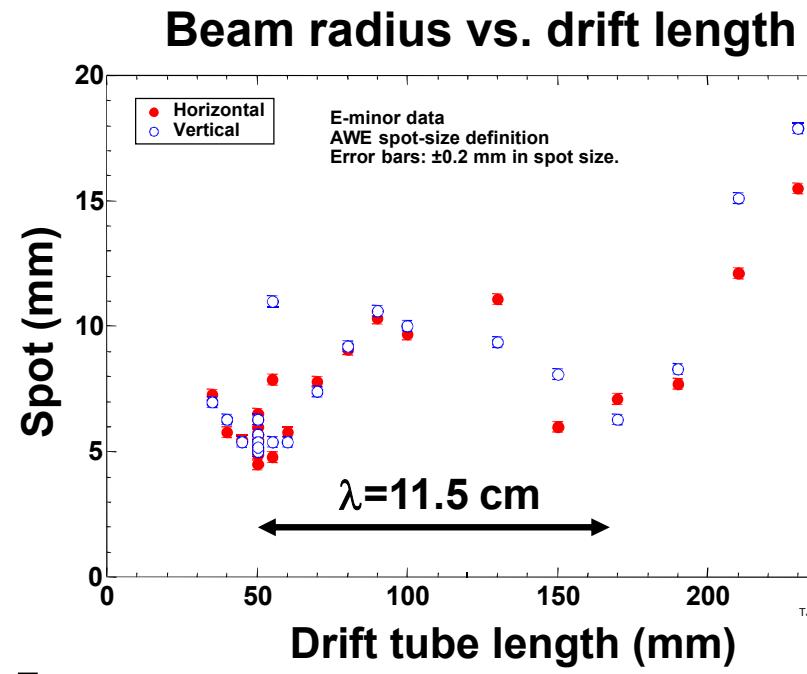
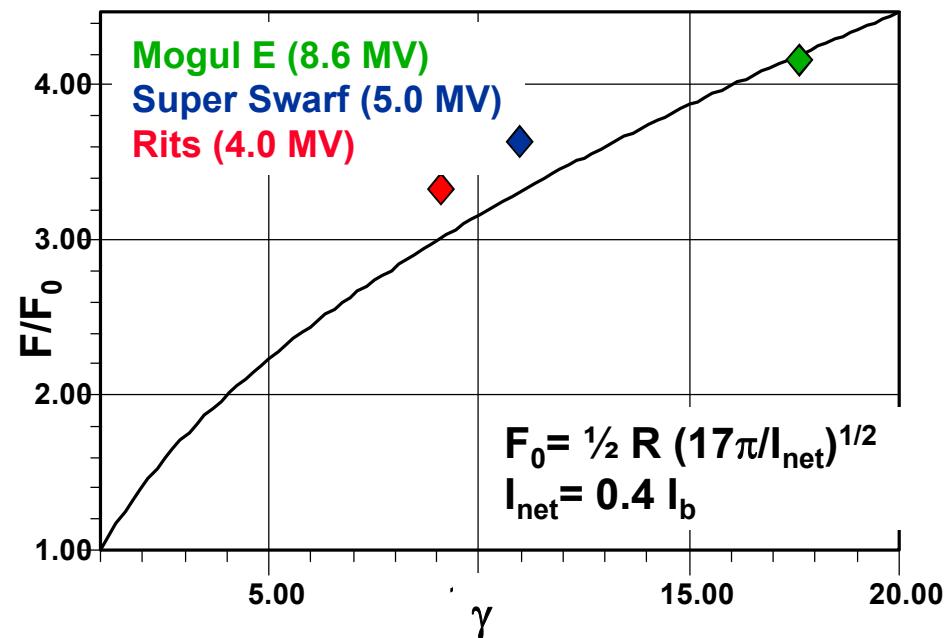


Fig. courtesy of A. Birrell¹

Data from AWE^{1,2} (5.3 MV, $I_b = 33$ kA)

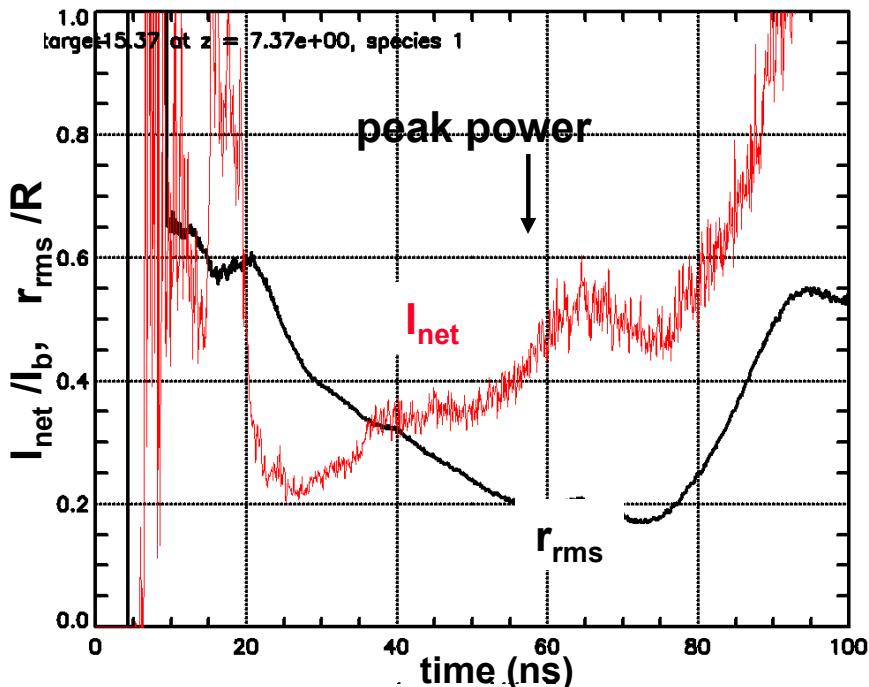
$\lambda = 11.5$ cm implies a time average net current $I_{\text{net}}/I_b \sim 0.4$

$a = 0.5$ cm implies emittance $\varepsilon \sim 0.18$ cm-radian

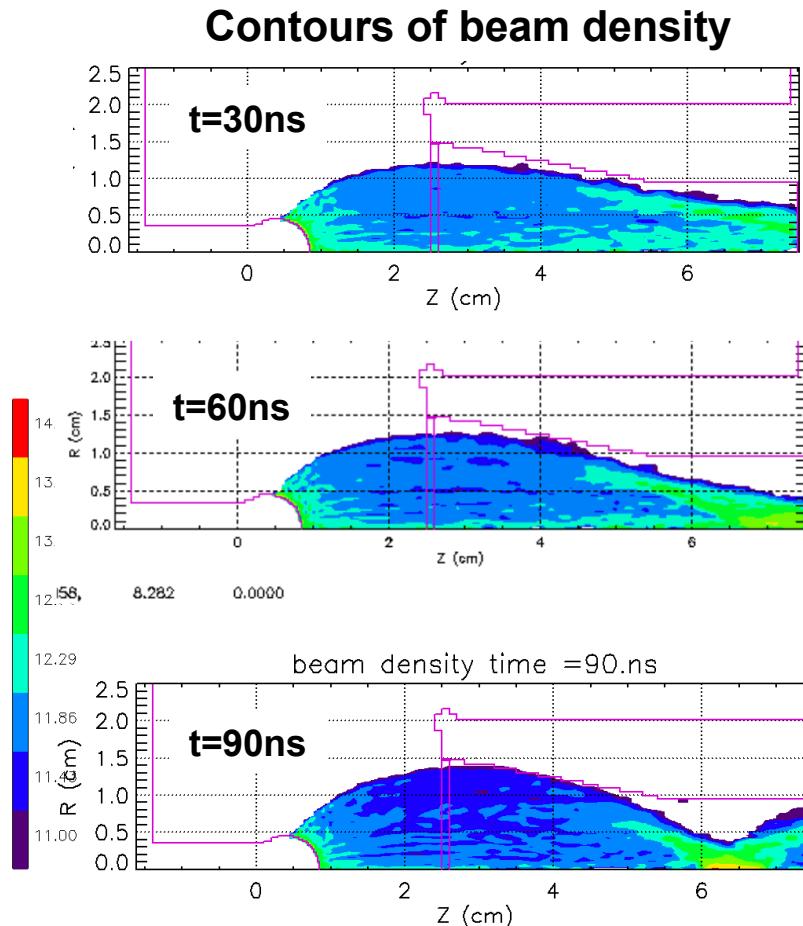
1. A. Birrell et al. IEEE Trans Plasma Sci. 28, 1660 (2000)
2. B.V. Oliver and D. Welch Proc. Intl Conf. Intense Beams, Russia, 2004

Time dependence of I_{net} causes a sweeping focus!

$$F(t) \cong \frac{R}{2} \sqrt{\frac{\pi I_A}{I_{\text{net}}(t)}}$$



Beam rms radius and I_{net} vs time. At peak power, $I_{\text{net}} = 9$ kA, $r_{\text{rms}} = 0.2$ cm



Focal sweeping is a primary contributor to larger than desired time integrated spots.

The ion-hose instability

IFR ion hose instability^[1]:

Backstreaming ion channel causes space-charge attraction of e-beam. With finite ion mass → ions move → convective instability.

Growth rate ~ ion inertial time

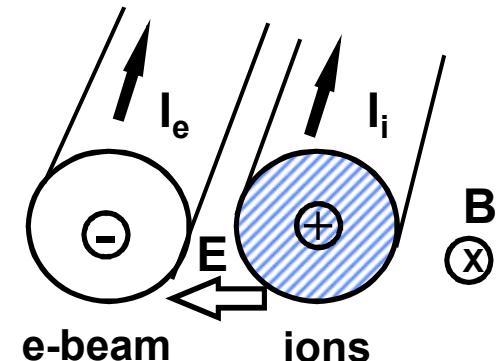
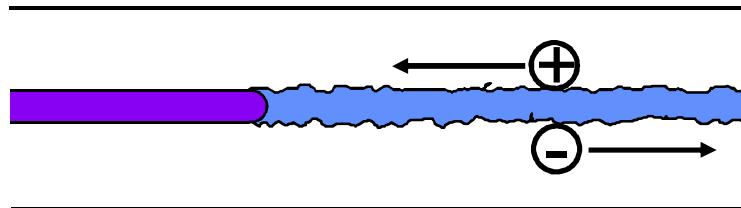
$$\Gamma \approx \omega_{pi} = (4\pi e^2 n_b/M)^{1/2} \sim 5 \times 10^9 \text{ s}^{-1}$$

Convective length

$$\lambda \approx (c/\omega_p) = (\gamma mc^2/4\pi e^2 n_i)^{1/2}$$

In the presence of a solenoidal field, beam rotates and growth rate is modified in limit $(\Omega_e/\omega_p) \gg 1$ ^[2]

$$\lambda_{ce} \approx \lambda (\Omega_e/\omega_p) \sim \begin{array}{ll} 1 \text{ cm} & \text{immersed-}B_z \text{ diode} \\ 10 \text{ cm} & \text{linear induction accelerators} \end{array}$$



Time and length scales imply non-linear evolution and saturation.

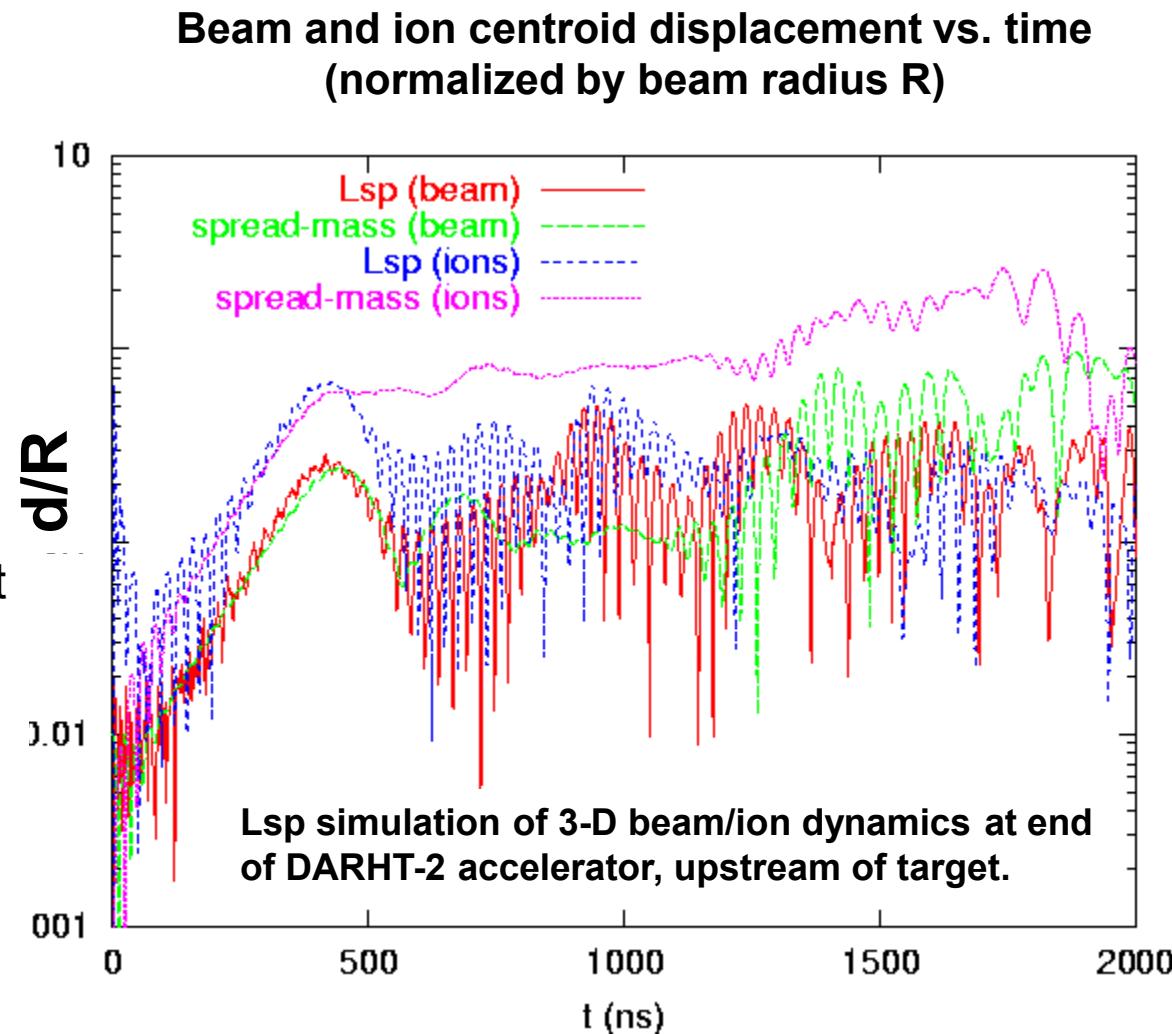
Nonlinear modeling of ion-hose saturation amplitude in good agreement with 3-D LSP simulations.

Linear and non-linear theory of ion-hose instability captures primary 3-d dynamics of the beam ion interaction.

Nonlinear evolution occurs within 3-4 growth times.

Spread-mass¹ modeling of ion-hose saturation provides information for scaling to different beam and plasma conditions.

Saturation amplitude scales with linear growth rate $1/\lambda_{ce} \propto 1/B$.



1. E.P. Lee Phys. Fluids 21, 1327 (1978)

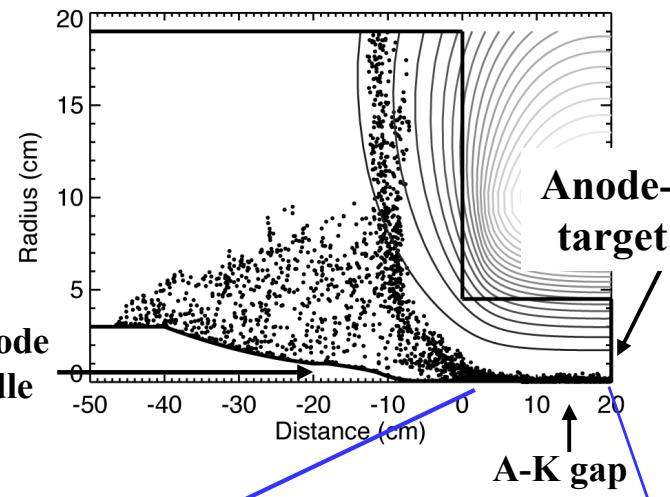
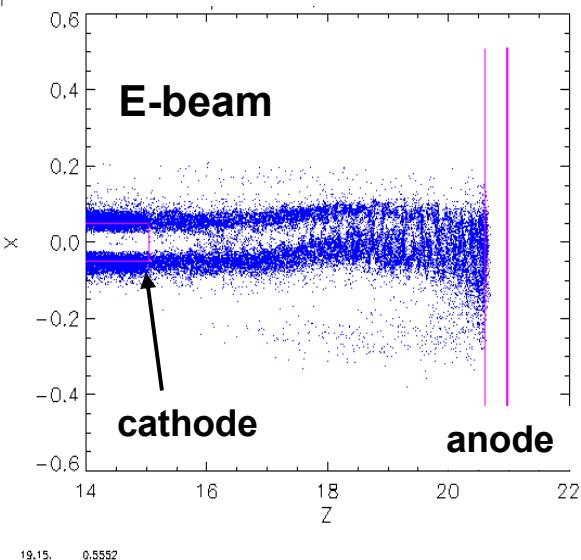
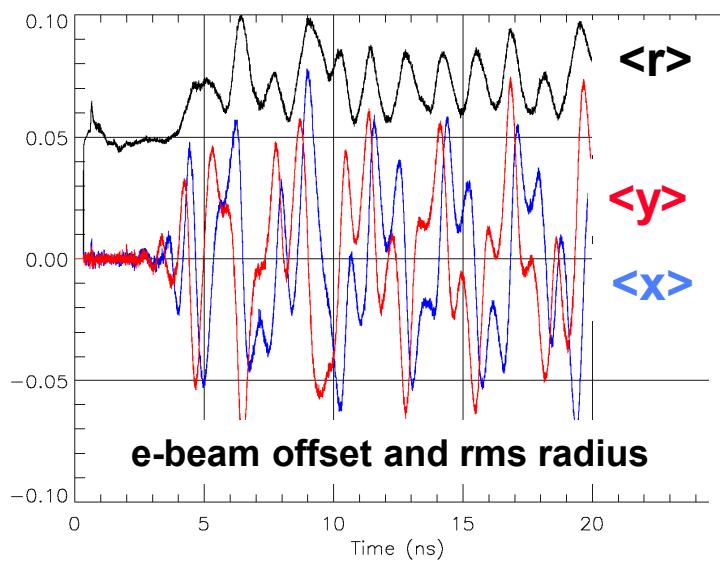
2. D.V. Rose, T.C. Genoni, D.R. Welch, Phys. Plasmas, 11, 4990 (2004):

The ion-hose instability in the applied- B_z diode

Immersed-B¹: Diode used for x-ray radiographic applications. Creates high intensity bremmstrahlung radiation.

Beam spot on target determined by ion-hose saturation amplitude

$$\langle r_{\text{sat}} \rangle \approx \frac{c}{\Omega_e} \sqrt{2\gamma \frac{I_b}{I_A}}$$



3-D PIC simulations of immersed-B diode electron and ion dynamics

1. M.G. Mazarakis et al. Appl. Phys. Lett **70**, 832 (1997)
- D.R. Welch et al. Laser and Particle Beams **16**, 285, (1998)

Magnetic Insulation

The Magnetically Insulated Transmission Line

The MITL is a coaxial transmission line (solid metal conductor) that is an integral part of Inductive Voltage Adder (IVA) accelerators:

- they act to connect in series each induction cell
- they allow power transmission downstream

For high voltage systems, the electric field inside the coaxial transmission line exceeds the threshold for electron emission off the cathode surface (~ 200 - 250 keV/cm). And for high current systems, emitted electrons are magnetically insulated from crossing the A-K gap of the transmission line and flow downstream. Hence the name Magnetically Insulated Transmission Line.

The presence of the emitted electron space-charge and current, inside the line, changes the impedance of the transmission line from its vacuum value. Thus, the MITL's I,V characteristics (operating impedance) are considerably more complicated than a normal vacuum TL. In fact, the MITL impedance is non-linear:

$$Z_{MITL} = \frac{V}{I_{MITL}} = \frac{V}{I(V)}$$

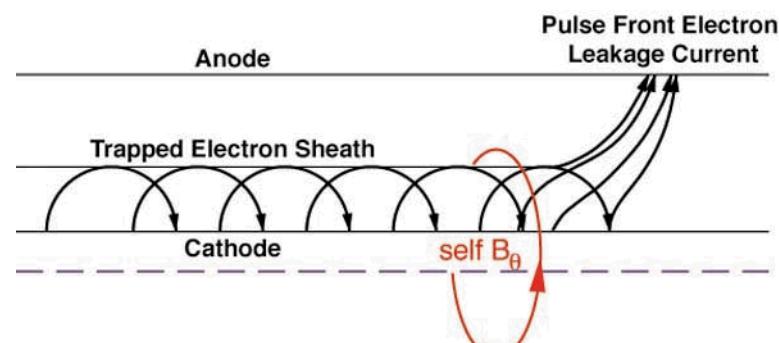
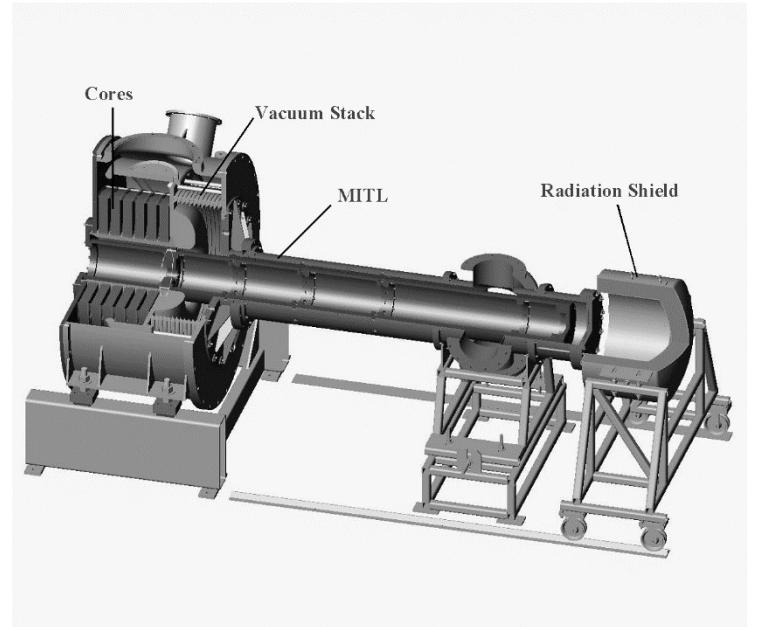


Illustration of axis-symmetric insulated flow in the MITL

1-D, equilibrium flow models

A lot of good theory was established in the mid 1970s on magnetic insulation which has been used extensively to determine the MITL impedance characteristics¹⁻³.

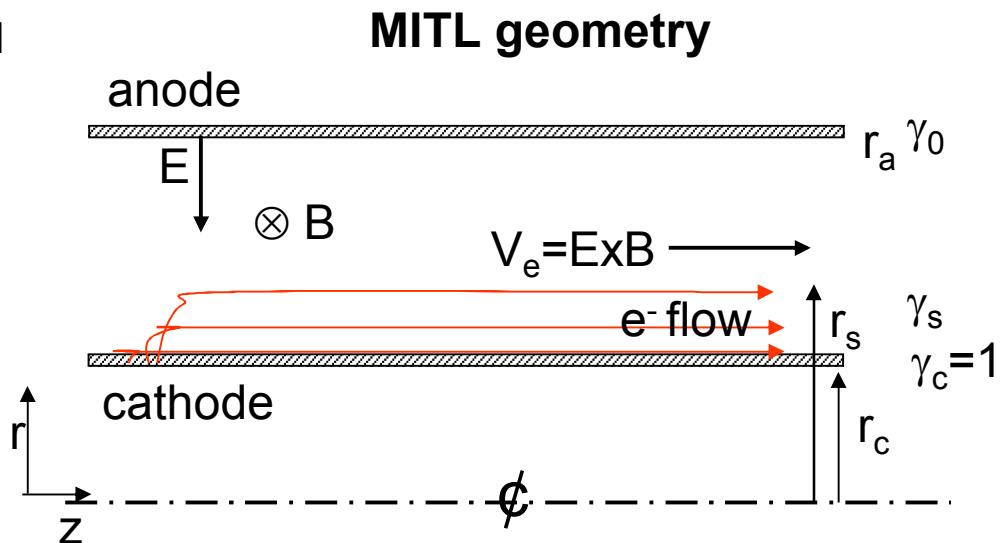
The relativistic electron flow is treated self-consistently assuming all electrons born at the same potential and the flow is governed by $E \times B$ drift. Vacuum solutions are matched at the edge of the electron sheath.

$$\nabla \phi \times B = v_e \quad \text{momentum}$$

$$\nabla^2 \phi = n_e \quad \text{Poisson}$$

$$\nabla \times B = -n_e v_e \quad \text{Ampere}$$
$$\gamma - \phi = 1;$$

$$\gamma = \sqrt{1/(1 - v^2)} \quad \text{Cons. of energy}$$



1. John M. Creedon, J. Appl. Phys, **46**, 2946 (1975)
2. M. Reiser, Phys. Fluids, **20**, 477 (1977)
3. A. Ron et al., IEEE Trans. Plasma Sci-1,85 (1973)

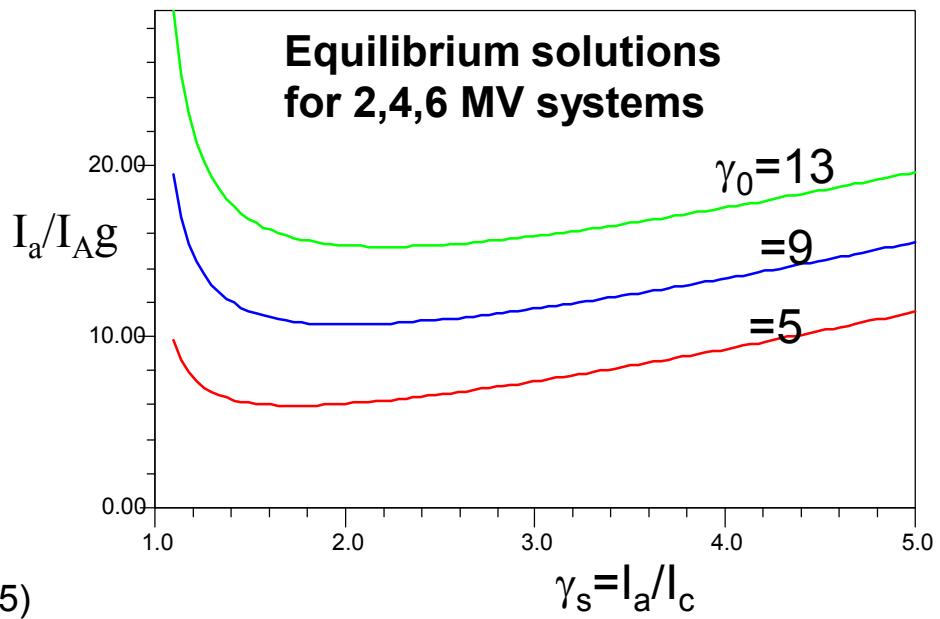
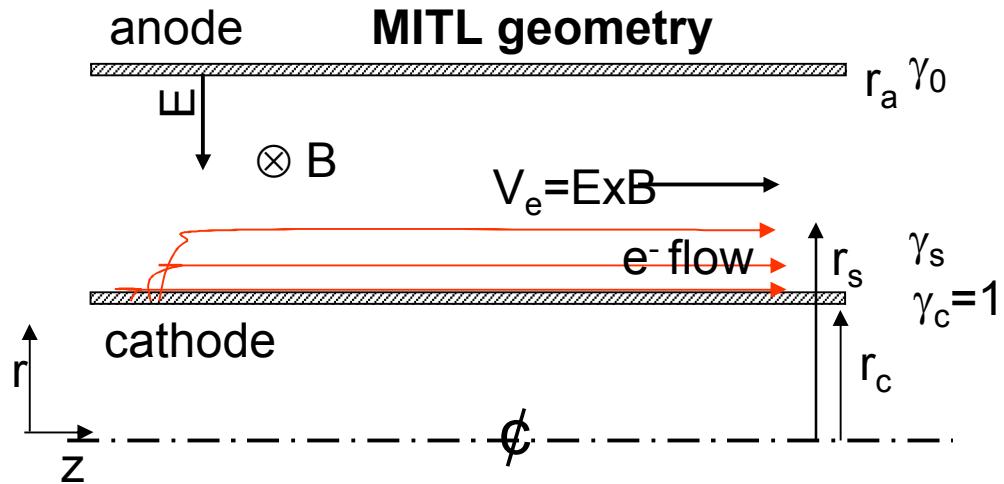
1-D, laminar flow model of Creedon

One obtains an infinite number of equilibria relating the MITL current to the voltage for various sheath thickness (energies), e.g. from Creedon¹:

$$I_a = I_A g \gamma_s \left[\text{acosh}(\gamma_s) + \frac{\gamma_0 - \gamma_s}{\sqrt{\gamma_s^2 - 1}} \right]$$

$$\gamma_s = \frac{I_a}{I_c}$$

$I_A = 8500$ (kA), $g = 1/\ln(r_a/r_c)$,
 γ_s = gamma @ electron sheath edge,
 $\gamma_0 = 1 + (eV/mc^2)$ = applied voltage



A more generalized model is described by Ottlinger² et al.

1. John M. Creedon, J. Appl. Phys, **46**, 2946 (1975)

2. Ottlinger et al. IEEE Trans. Plasma Sci. vol. 36, pp. 2708- 2008-2721, 2008

Insulation in the presence of ion emission

- Follows closely Creedon et al.¹ on 1-D radial relativistic equilibrium electron flow
- Includes a background ion space-charge
- Ion current is radial and included via a boundary current. Ion charge and current density are solved self-consistently.
- Considers the region downstream of the cathode blade. It is characterized by being charge and current neutral at radii $> r_c$

$$\nabla \phi \times B = v_e$$

momentum

$$\nabla^2 \phi = n_e$$

Poisson

$$\nabla \times B = -n_e v_e$$

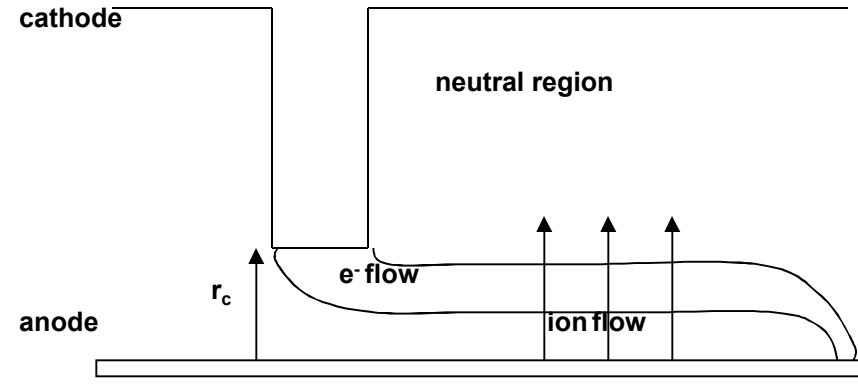
Ampere

$$\gamma - \phi = 1;$$

$$\gamma = \sqrt{1/(1-v^2)}$$

Cons. of energy

Schematic of insulated flow in the rod-pinch diode



Coupled ion response

$$\nabla \bullet J_i = 0. \Rightarrow r_c J_i = r n_i v_i = \text{const.}$$

$$\frac{1}{2} v_i^2 = \left(\frac{Zm}{M} \right) (\gamma_a - \gamma)$$

$$n_i = \frac{r_c J_i}{r \sqrt{2Zm(\gamma_a - \gamma)/M}}$$

The Rod-Pinch diode is an example of self-magnetically insulated flow with ions

Diode current well modeled by critical current formulation¹:

$$I = \alpha I_{\text{crit}}, \quad 2.0 < \alpha < 2.6$$

$$I_{\text{crit}} = 8.5 \frac{\sqrt{\gamma^2 - 1}}{\ln(r_c / r_a)} \text{ kA}, \quad \gamma = 1 + eV/mc^2$$

Operation and α is described by self-insulated flow theory with the inclusion of ions²

Region I

$$\nabla^2 \phi = n_e - n_i, \\ \nabla \times B = n_e v_e \\ \nabla \phi + v_e \times B = 0,$$

Region II

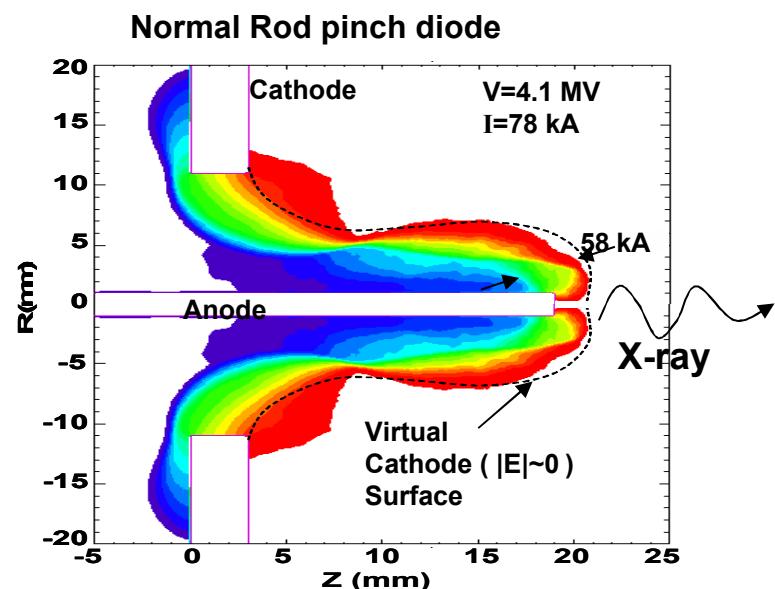
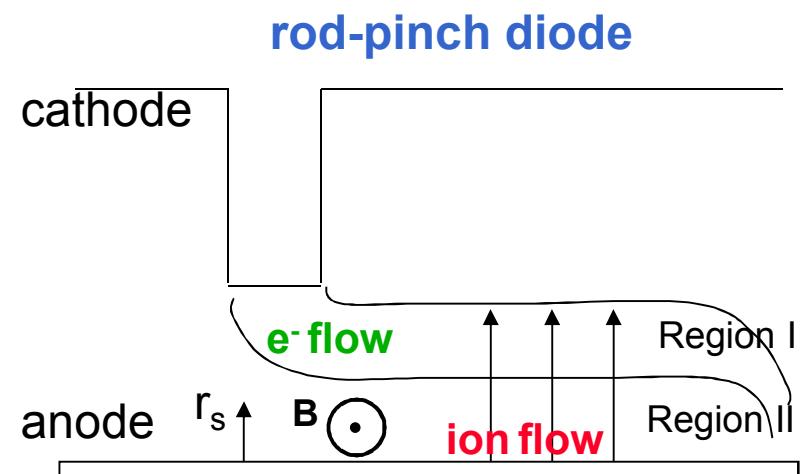
$$\nabla^2 \phi = -n_i. \\ n_i v_i = \frac{r_c}{r} j_c$$

Ions are absolutely necessary for operation!

$$J_i = \frac{4}{9} \frac{(\gamma_a - \gamma_s)^{3/2}}{(r_s - r_a)^2}.$$

$$I \approx 17 r_s \sqrt{J_i} [\gamma_a - 1]^{1/4} \text{ kA.}$$

Child Langmuir like



1. G. Cooperstein et al. Phys. Plasmas, **8**, 4618 (2001)
2. B.V. Oliver et al. Phys. Plasmas, **11**, (2004);

Fig. courtesy of S. Swanekamp, NRL