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Large-Scale Inverse Methods and Design of Acoustic Metamaterials in Sierra-SD

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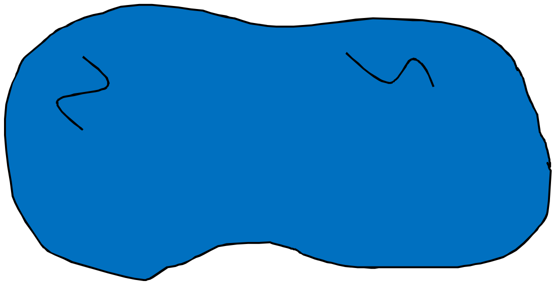
Outline

- Inverse problems in computational mechanics
- Discussion of inverse methods in Sierra-SD
- Design of acoustic metamaterials

Inverse Problems: Observing the Unobservable

Suppose we have a “black box” system in the *as-manufactured* state that has only partially known parameters

Question: can we *non-destructively* interrogate the system to “see what is inside”?



Typical quantities of interest:

- Material properties
- Loads
- Boundary conditions
- Residual stresses
- Size/shape/location of inclusions (e.g. composite materials)

Example applications:

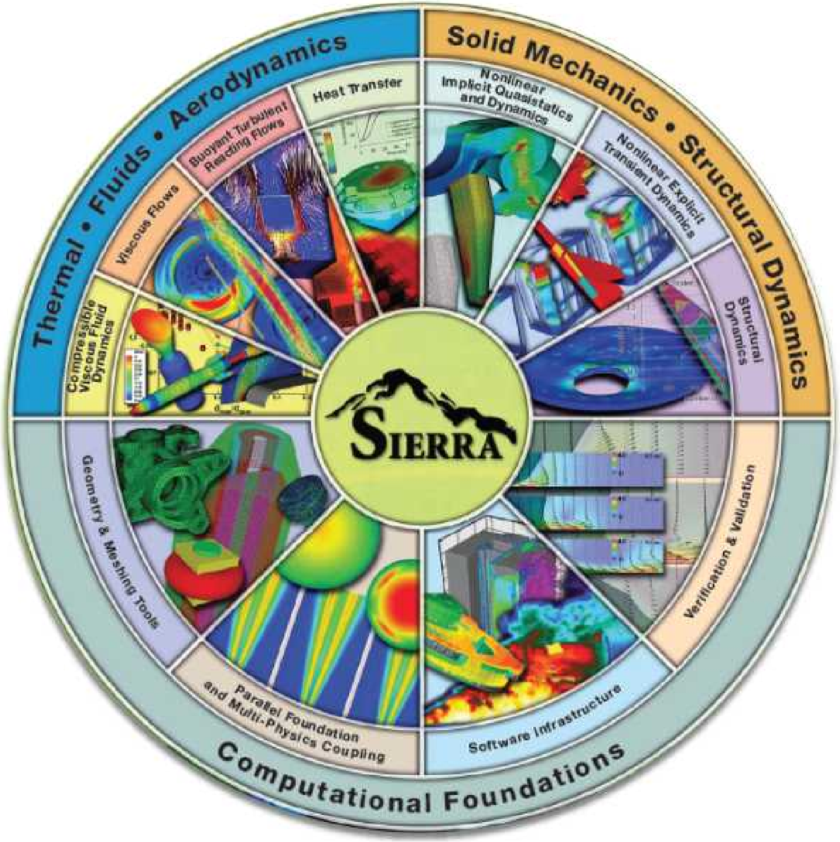
- Seismic imaging
- Medical imaging
- Non-destructive evaluation

Categories of Inverse Problems

- Imaging
  - Medical ultrasound
  - Seismic exploration
- Calibration of material models
  - Structural material properties, circuits, thermal properties, etc.
- Force reconstruction
  - Sub-structuring for mechanical testing of components
- Optimal Experimental Design
  - Best placement of sensors, test fixture setups
- Shape reconstruction
  - E.g. inverse scattering

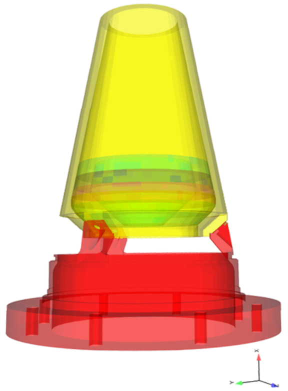
Inverse Problems - Motivation

Sierra Mechanics provides a massively parallel framework for physics simulations, but requires knowing all model parameters



The model could be lacking information:

- material properties?
- boundary conditions?
- loading conditions?
- Internal flaws from aging?
- Preloading effects?



The missing link:
Experimental measurements + solution of inverse problem

PDE-Constrained Optimization Formulation

Abstract optimization formulation

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$

Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$

PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}^T\boldsymbol{g}$$

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \boldsymbol{g}_u^T\boldsymbol{w} \\ J_p + \boldsymbol{g}_p^T\boldsymbol{w} \\ \boldsymbol{g} \end{Bmatrix} = \{0\}$$

First order optimality conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \boldsymbol{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \boldsymbol{g}_p^T \\ \boldsymbol{g}_u & \boldsymbol{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta\boldsymbol{u} \\ \delta\boldsymbol{p} \\ \boldsymbol{w}^\star \end{Bmatrix} = -\begin{Bmatrix} J_u \\ J_p \\ \boldsymbol{g} \end{Bmatrix}$$

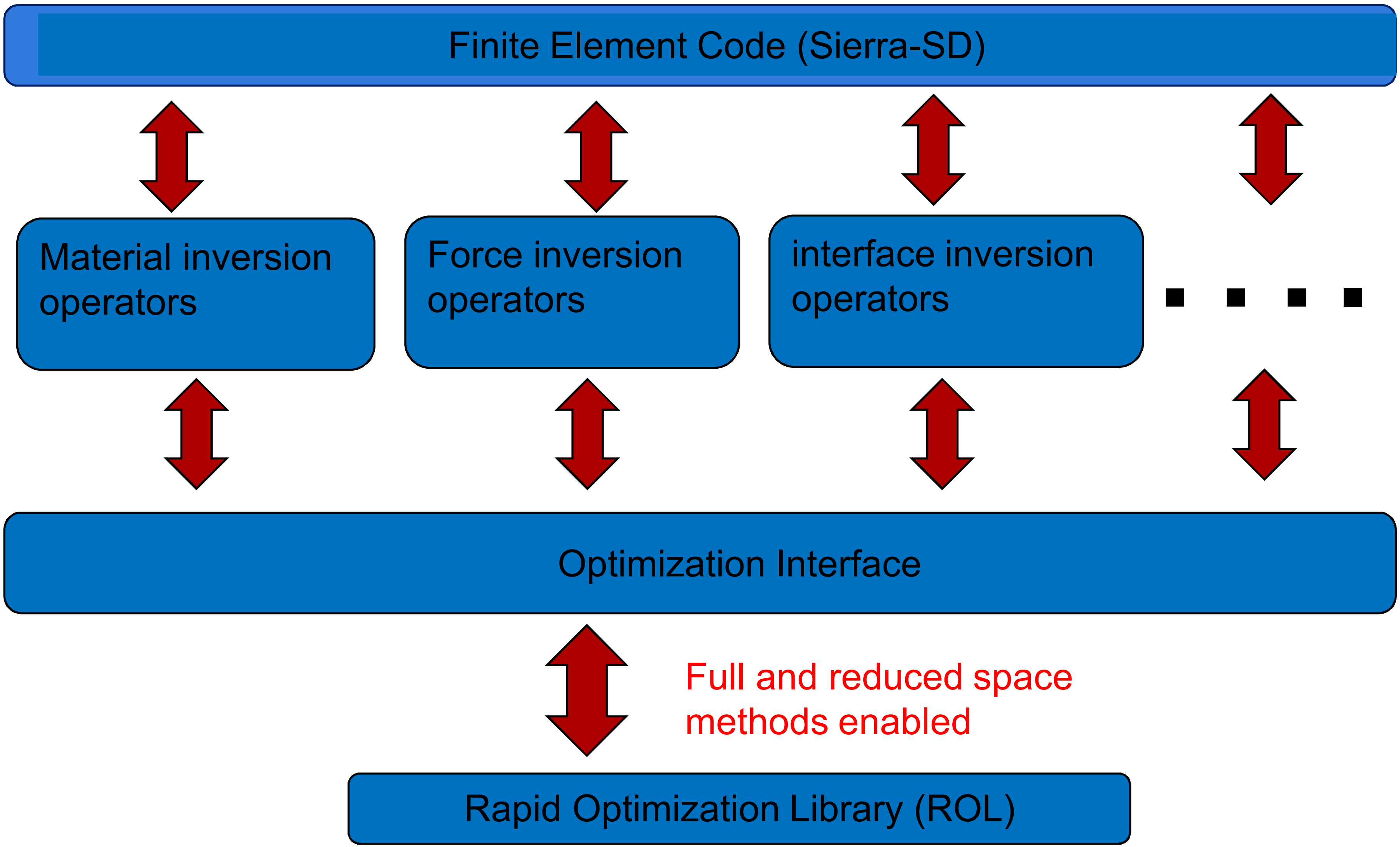
Newton iteration

$$\boldsymbol{W}\Delta\boldsymbol{p} = -\hat{J}',$$

$$\boldsymbol{W} = \boldsymbol{g}_p^T\boldsymbol{g}_u^{-T}(\mathcal{L}_{uu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p + \mathcal{L}_{pp}$$

Hessian calculation

Operator-Based Inverse Problems



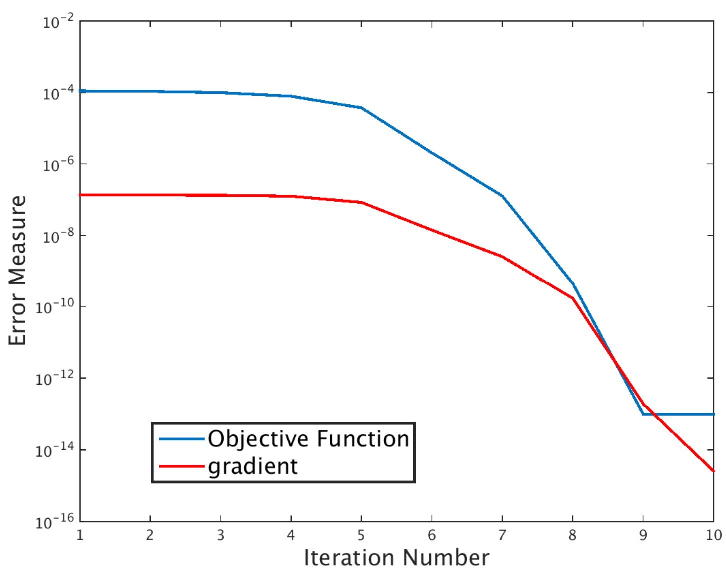
Examples

- Material characterization
- Delamination detection
- Residual stress characterization
- Force (source) reconstruction

- Acoustic metamaterial design
  - Transient shock isolation
  - Notch filter design
  - Steady-state vibration mitigation
  - Acoustic cloaking

Frequency-Domain Material Inversion

- Dashpot/foam calibration on mass-mock
- Full Newton with adjoint-based Hessians
- Measured displacements on foam block
- Stiffness parameters from previous slide
- Initial guess: zero damping



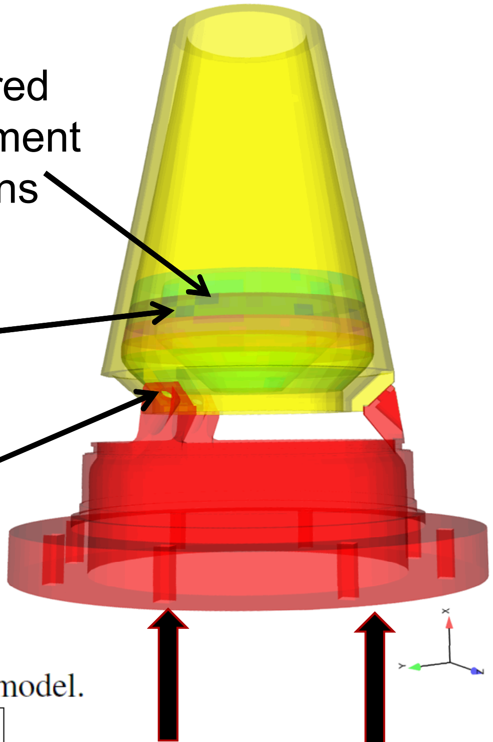


Table 3. Dashpot parameters for mass-mock model.

	cx	cy	cz
exact	1000.0	N/A	N/A
computed	1000.0	N/A	N/A
initial guess	0	N/A	N/A

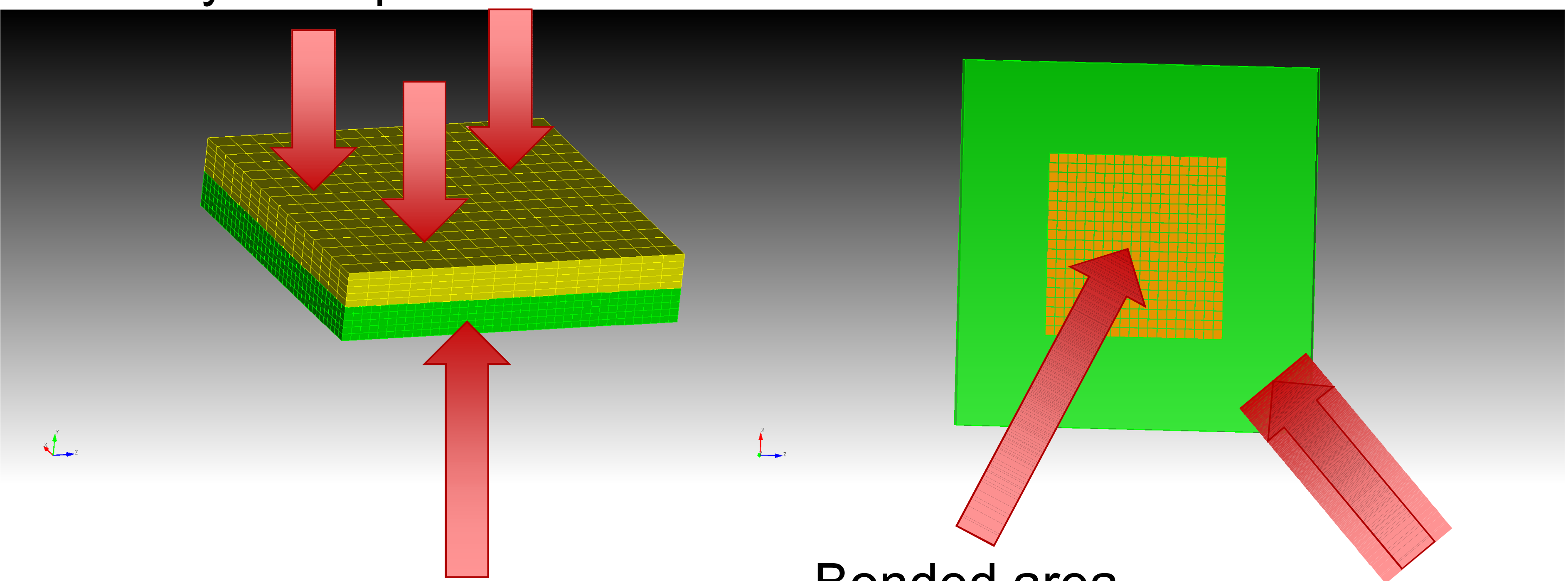
Table 4. Viscoelastic foam parameters for mass-mock model.

	Imaginary part of G	Imaginary part of K
exact	362.4	785.2
computed	362.3	785.0
initial guess	0	0

Delamination Detection

Partially-bonded plates – can we invert for the bonded/debonded regions?

Steady-state pressure load at 2000Hz



Simply supported on bottom

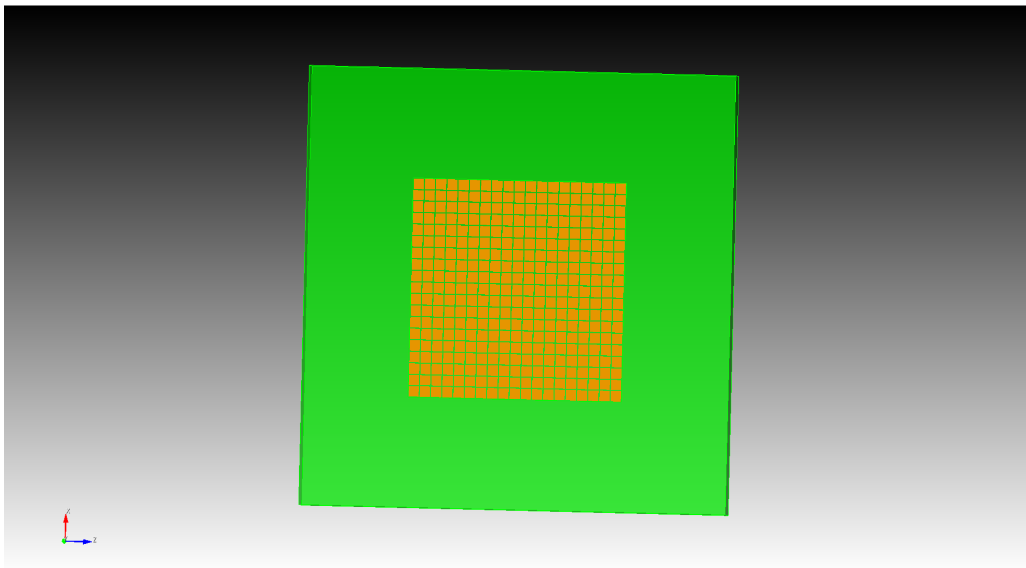
Bonded area

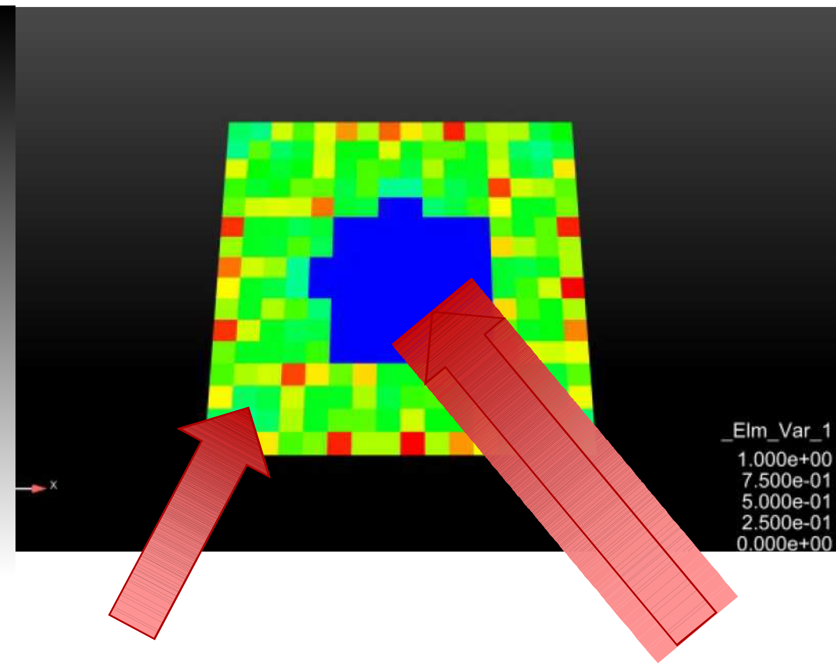
De-bonded area

Delamination Example

Partially-bonded plates – can we invert for the bonded/debonded regions?

Exact bonded/de-bonded areas





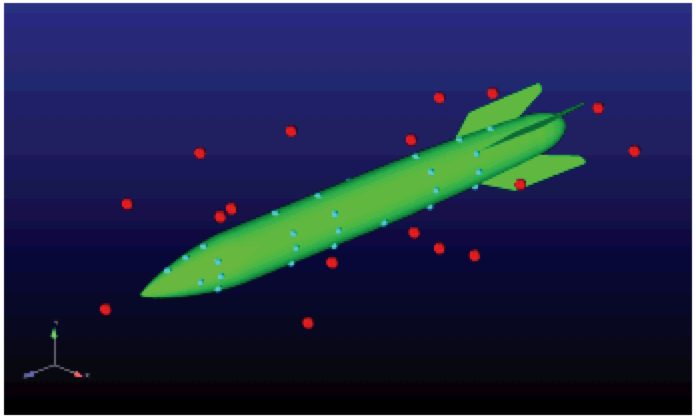
Initial guess for optimization: Completely de-bonded

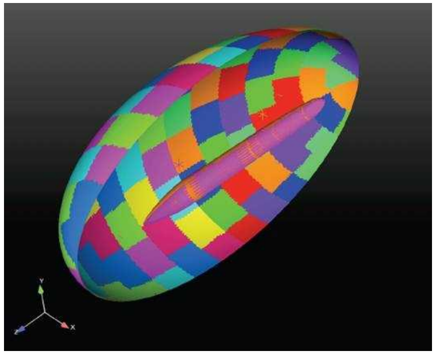
Bonded area (penalty=1)

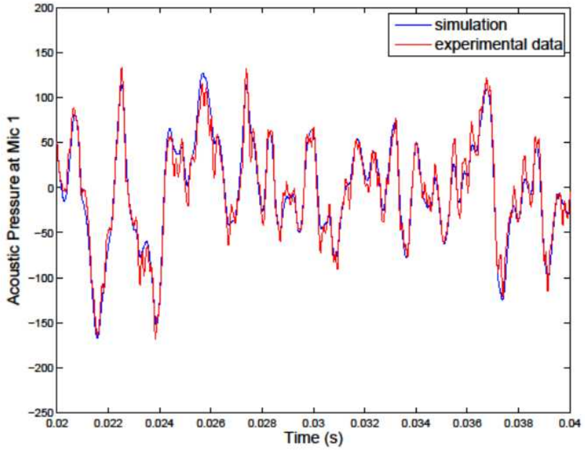
De-bonded Area (penalty=0)

Source Inversion in Sierra-SD

- Goal: reconstruct acoustic field using inverse problem to obtain acoustic patch inputs that produce the given microphone measurements



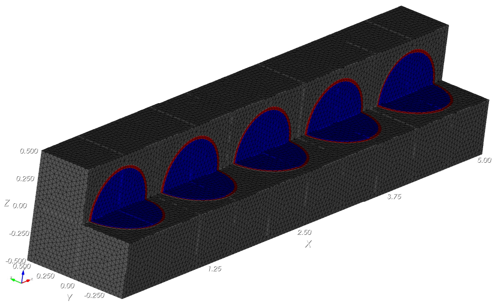




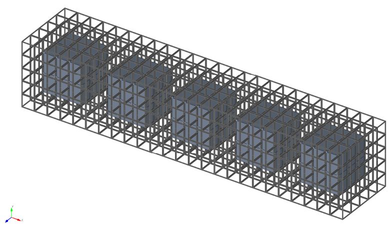
- Additional research on-going
- How to regularize the inverse problem – gradient regularization (penalize jumps across neighboring patches)
- How to place microphones

A Revolution in Acoustic Metamaterials

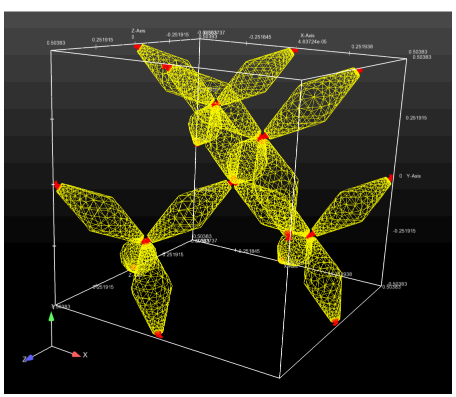
Breakthrough technology could allow us to mitigate harsh vibration environments



Multiphase composite



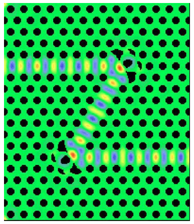
Lattice with embedded masses

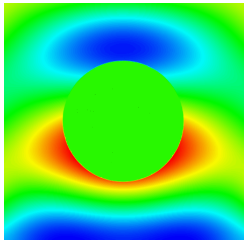


Pentamode lattice

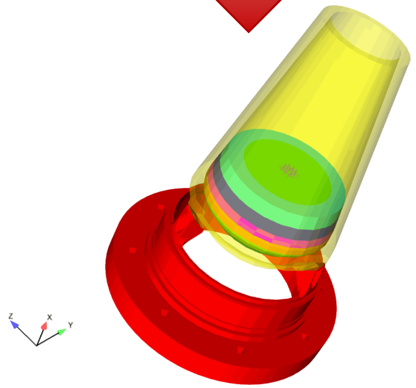
Transformative technology

Re-directed load paths





cloaking

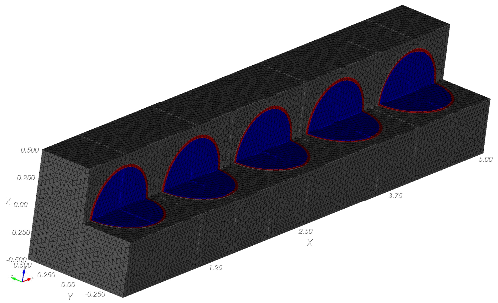


Vibration isolation

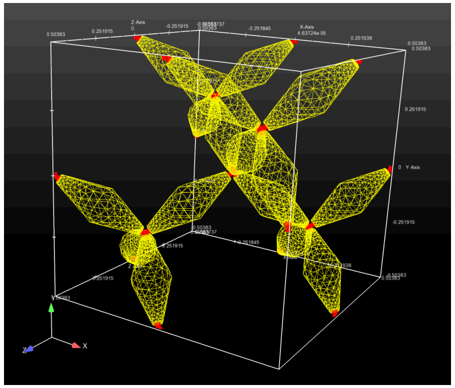
What is an acoustic metamaterial

- Acoustic metamaterials: designed to produce dynamic material properties not found in individual materials themselves
  - Negative moduli, negative density, negative refractive index, imaginary speed of sound!!! (not possible in traditional materials)
- First demonstrated in 2000 by Liu et al, *Science*

Multiphase composite



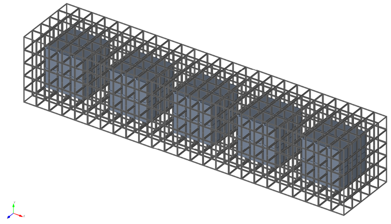
Pentamode lattice



How do metamaterials work?

$$v = \sqrt{\frac{B}{\rho}} \quad \text{where} \quad B = \text{bulk modulus} = \frac{\Delta P}{-\Delta V / V} = -V\frac{dP}{dV}$$

$$\rho = \text{density}$$



Lattice with embedded masses

Exotic Material Properties

Speed of sound in a material:

$$v = \sqrt{\frac{B}{\rho}} \quad \text{where} \quad B = \begin{matrix}\text{bulk}\\ \text{modulus}\end{matrix} = \frac{\Delta P}{-\Delta V / V} = -V\frac{dP}{dV}$$

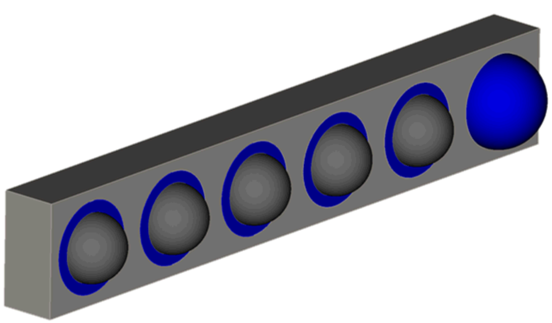
$$\rho = \text{density}$$

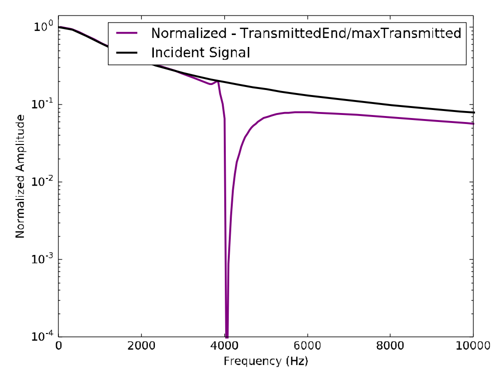
- Typically metamaterials exhibit "effective" properties such as:
  - Negative moduli, B, imaginary sound speed (no wave propagation)
  - Negative density, ρ, imaginary sound speed (no wave propagation)
  - Negative moduli and density, real sound speed (waves propagate again!)
  - We can design a mechanical filter by alternating the sign of B and ρ!

Metamaterials - Mechanical Filter Design

Uniqueness of metamaterials – allow for frequency-selective designs!

- Broadband – goal is to eliminate vibration in wide (or entire) frequency band
- Band-stop – stop energy in a specified frequency band - negative stiffness or negative density
- Band-pass – allow only a band of frequencies to propagate
- Notch – only filter at one particular frequency

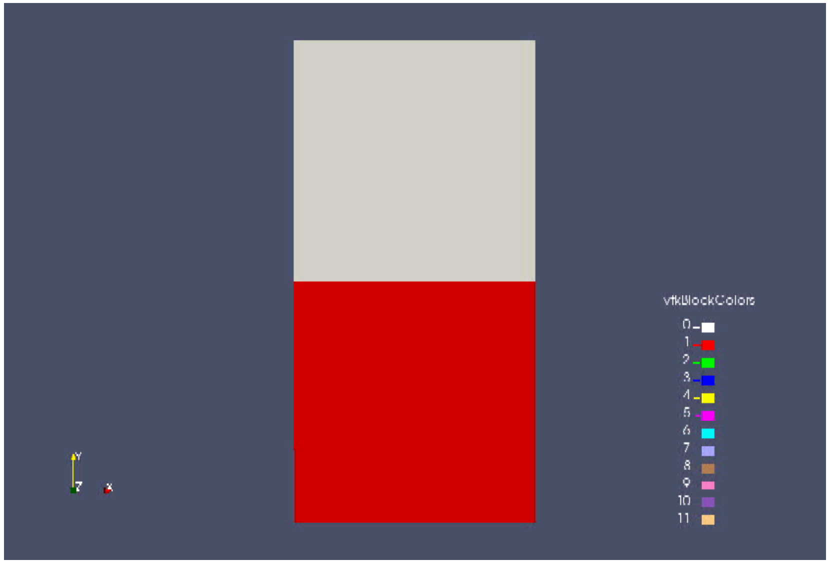




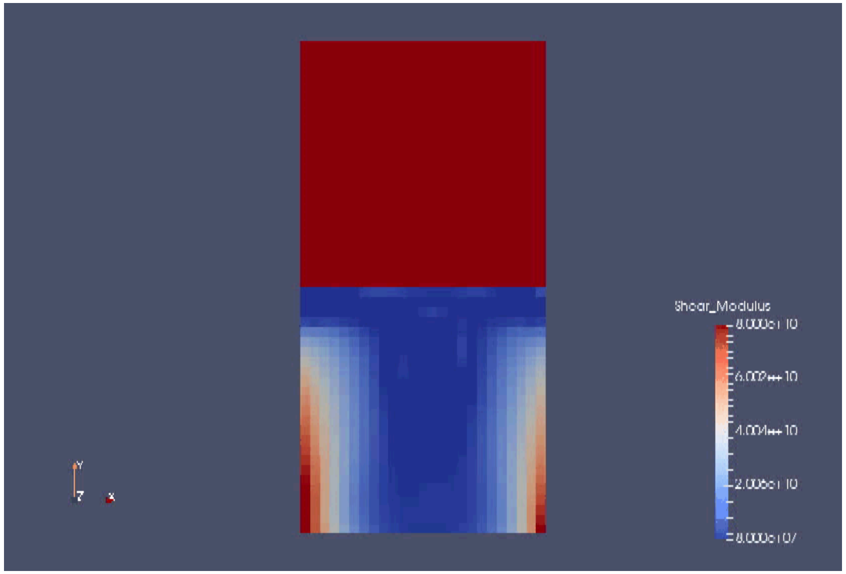
Transient Shock Isolation

Goal: Design the bottom material such that the top block does not move

Initial guess



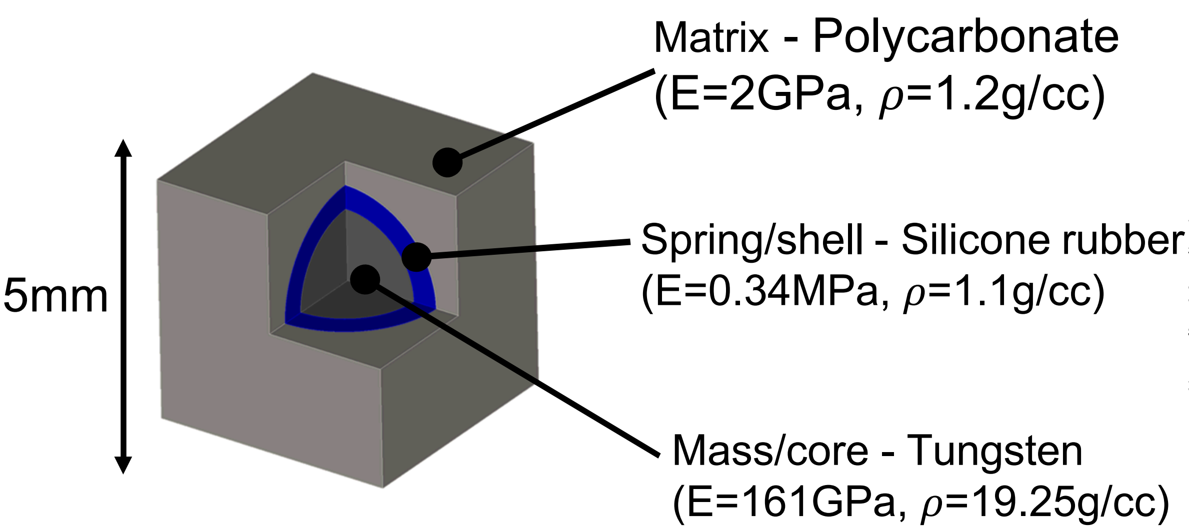
optimized

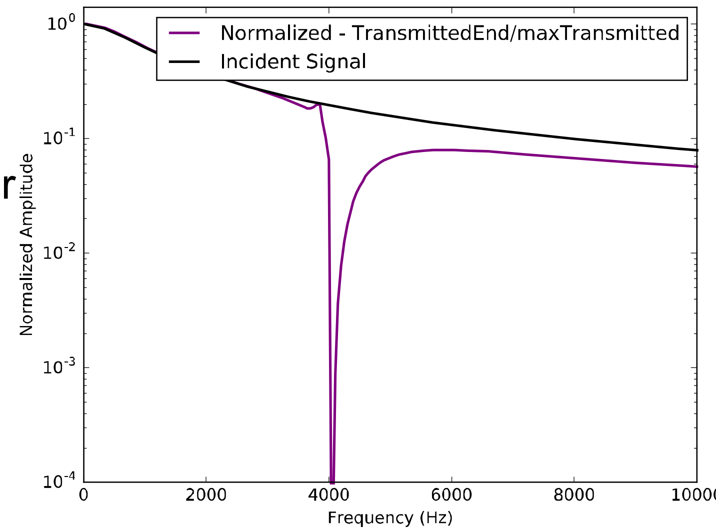


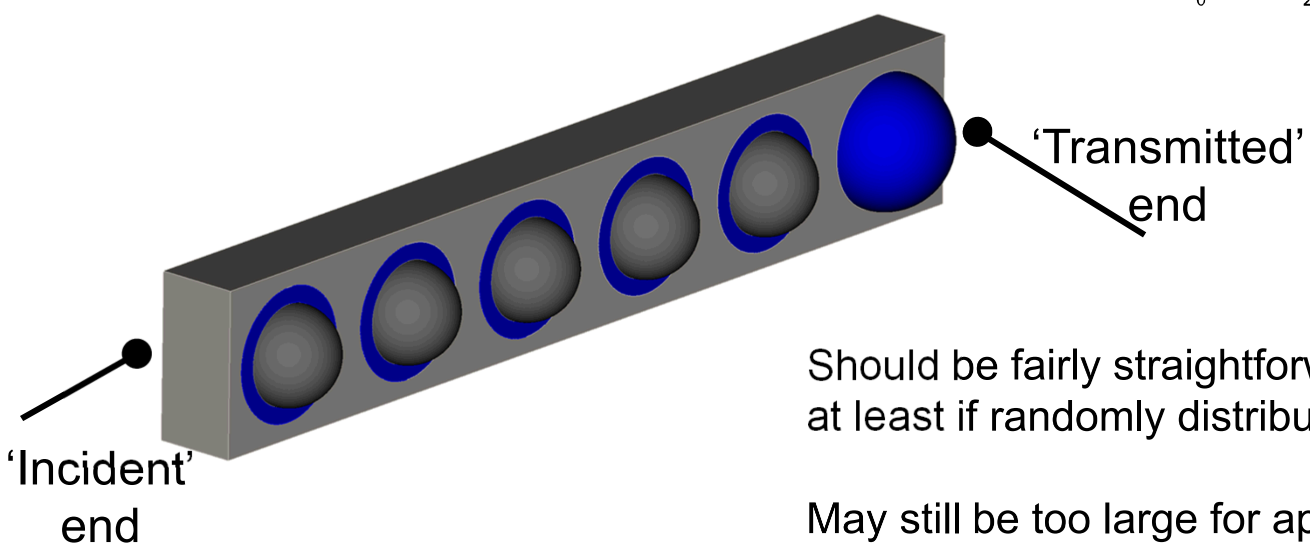
Top block: steel

Bottom block: single phase fixed, two-phase, multi-phase

Notch Filter Design





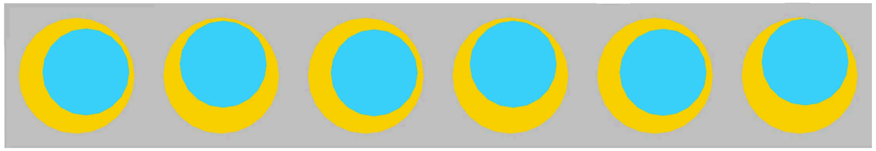


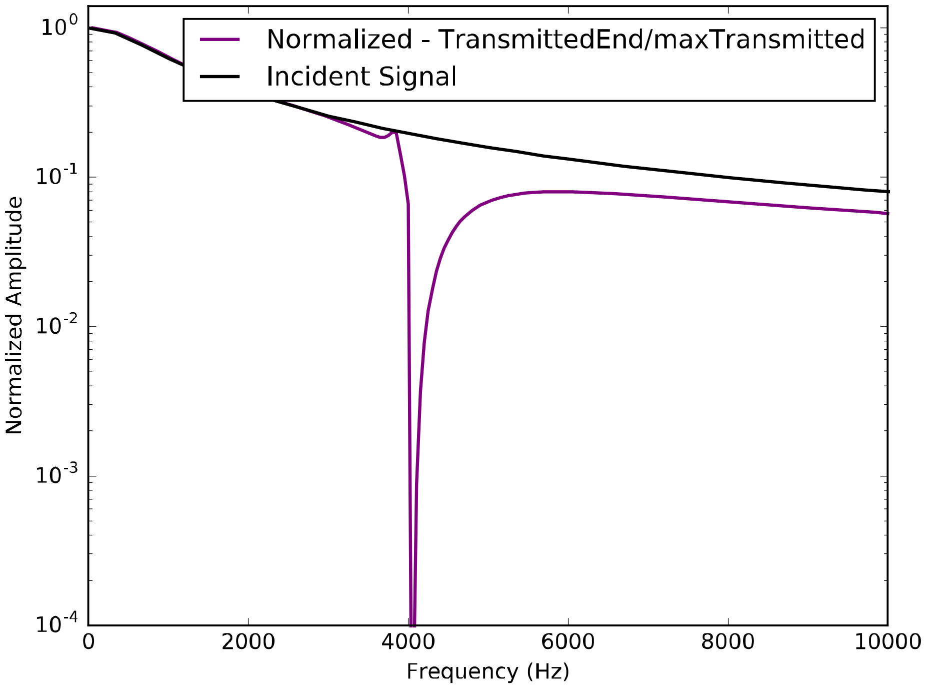
Should be fairly straightforward to manufacture, at least if randomly distributed.

May still be too large for application.

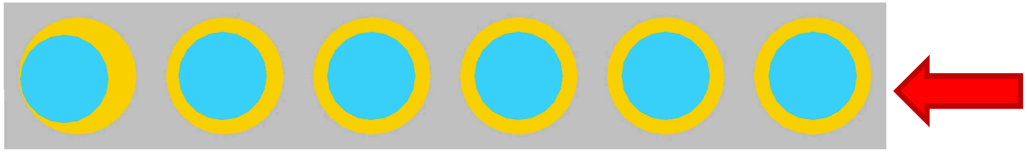
Sphere-in-Shell Oscillators

4000Hz



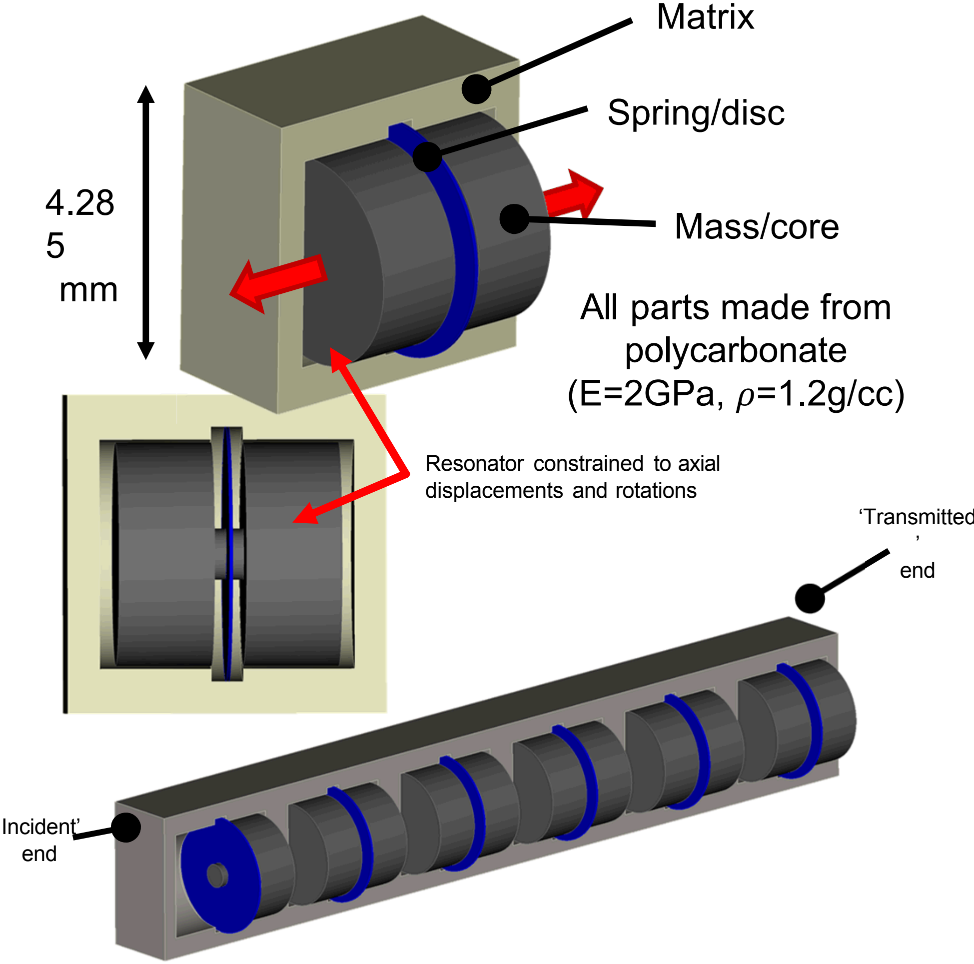


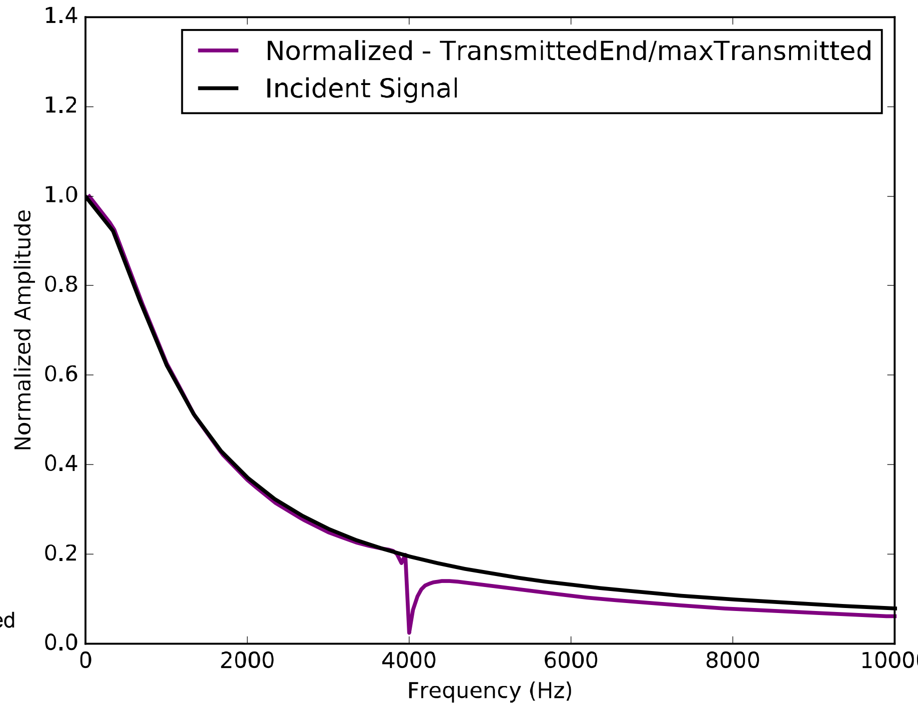
4050Hz



Reduced motion at end

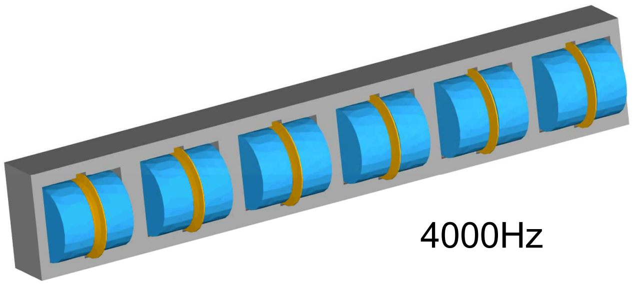
‘Dumbbell’ Resonators



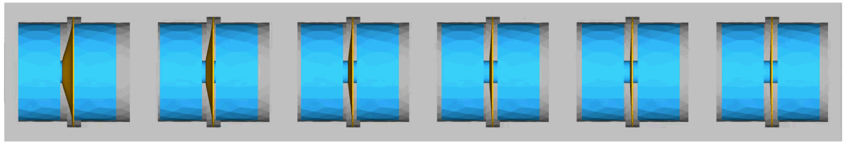


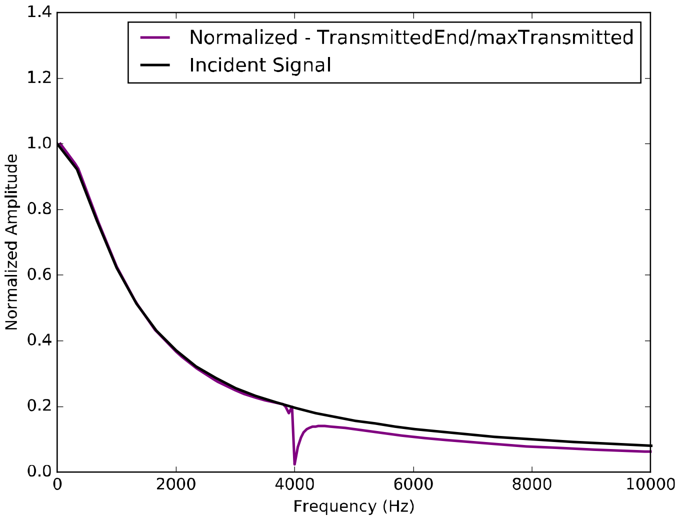
Can be 3D printed from a single material, will need to modify matrix geometry to remove material from empty spaces.

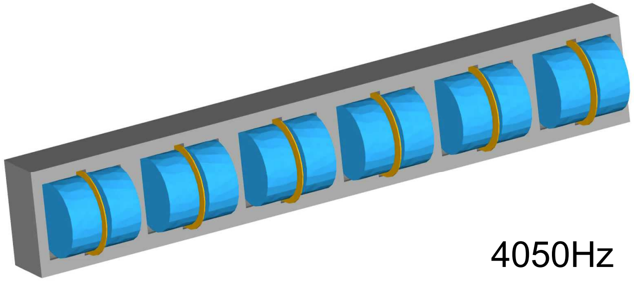
'Dumbbell' Resonators



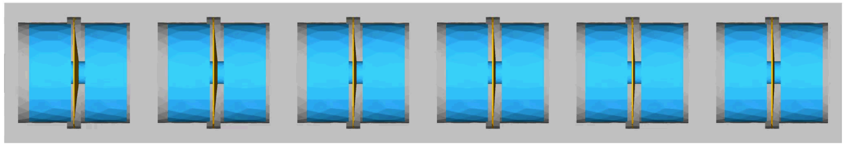
4000Hz







4050Hz



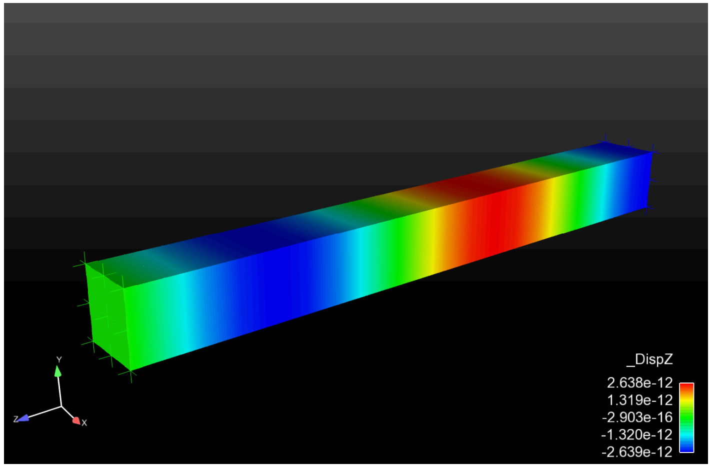
Wave Propagation in Waveguide

- $u(x) = e^{ikx}$
- $k = \frac{\omega}{c}$
- If c is purely imaginary, $u(x) = e^{\frac{-\omega x}{c}}$
- Thus a propagating wave becomes an evanescent wave when stiffness is negative

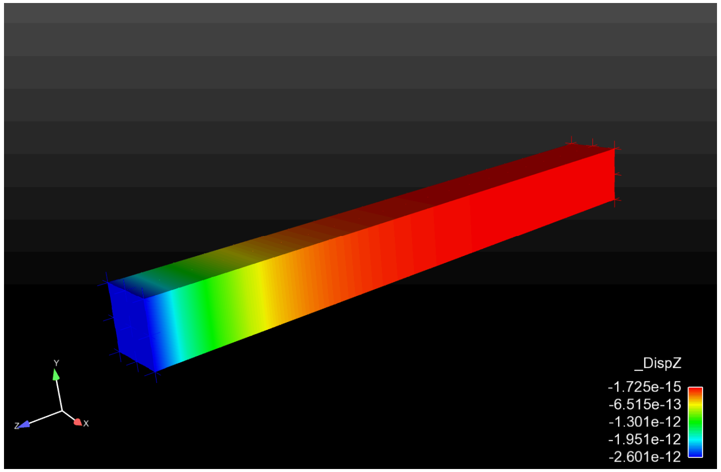
Wave Propagation in Waveguide

Steady-state wave propagation in steel bar – positive vs negative stiffness

1. Negative properties: wave propagation -> evanescent (decaying) waves
2. Makes linear Helmholtz solve easier
3. 3 order of magnitude reduction in wave amplitude in later case
4. Implications for homogenization of the metamaterial



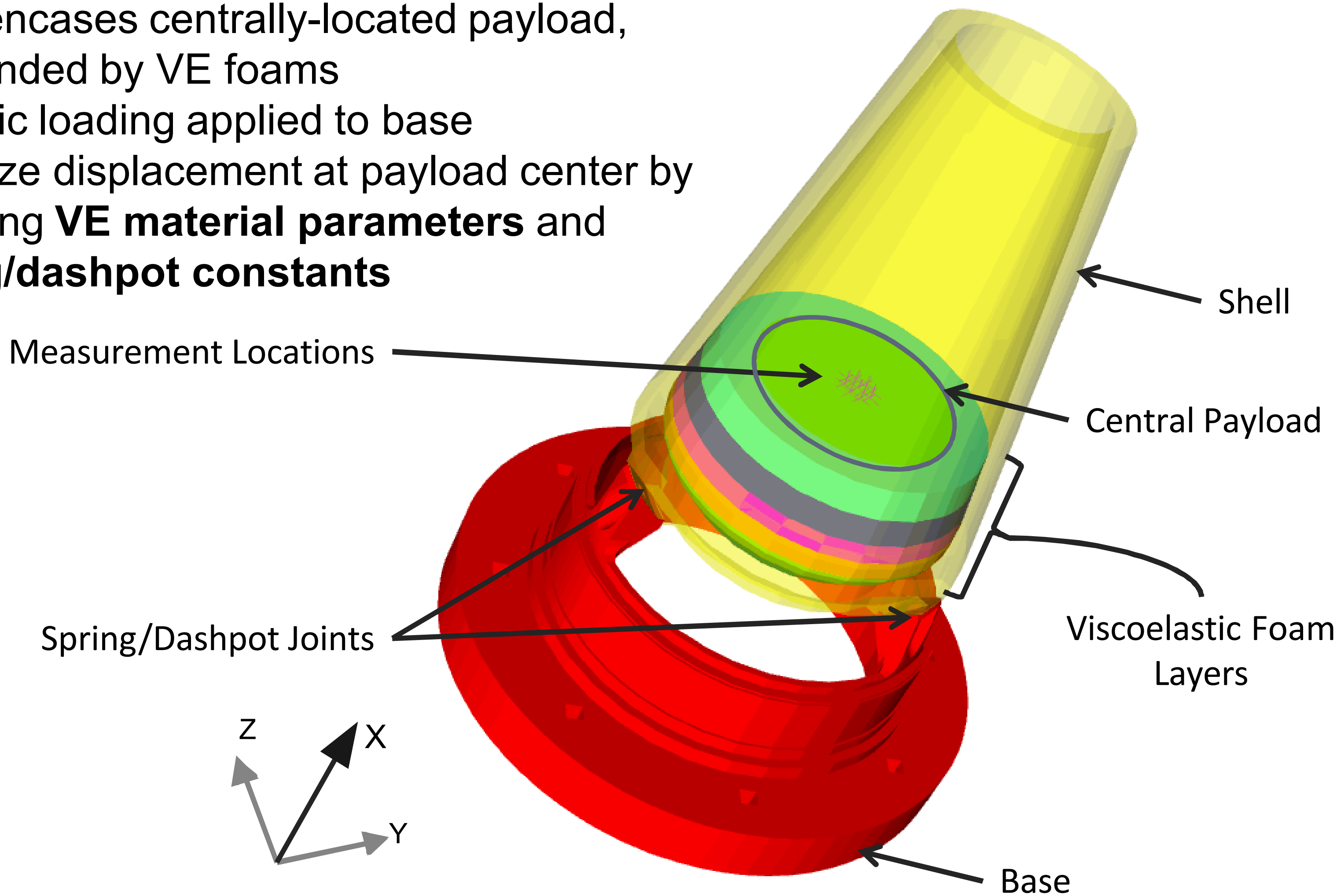
G, K > 0



G, K < 0

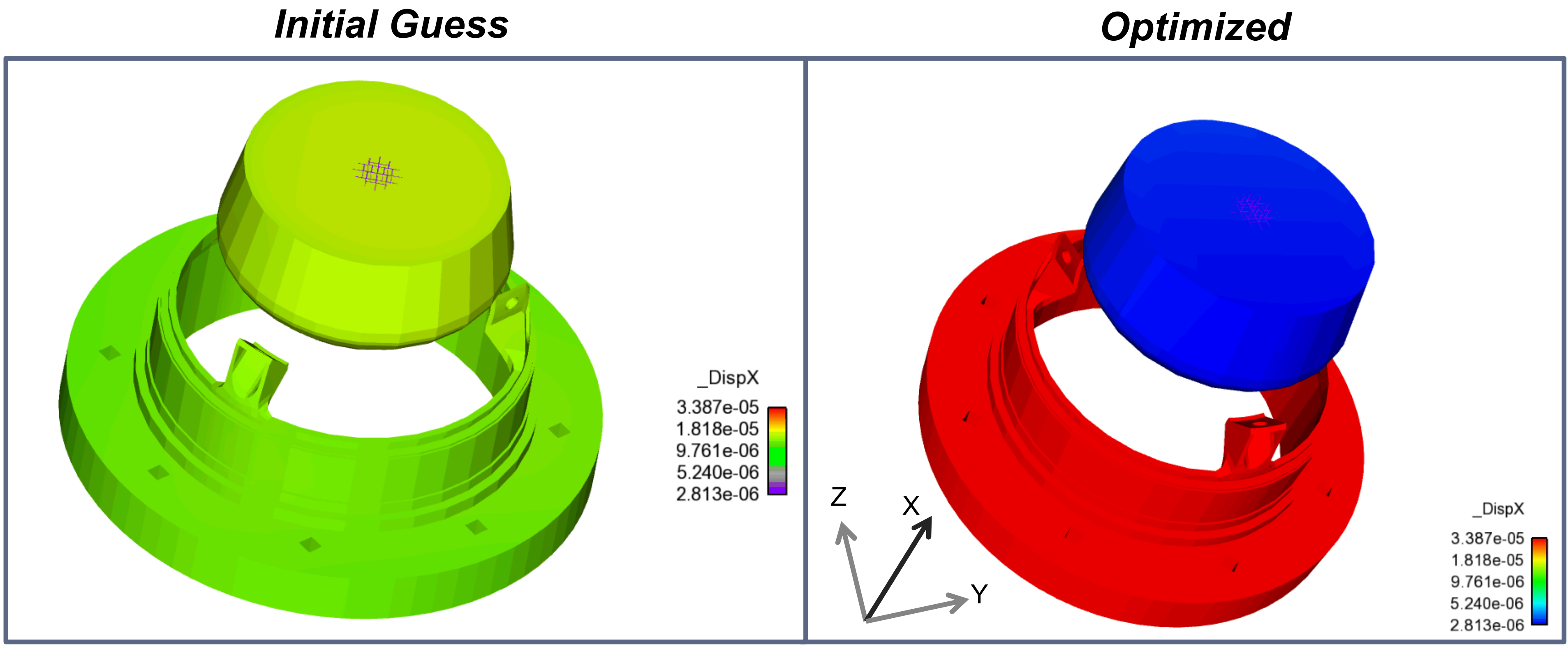
Inverse Problems: *Mechanical Vibration Reduction*

- Shell encases centrally-located payload, surrounded by VE foams
- Periodic loading applied to base
- Minimize displacement at payload center by adjusting **VE material parameters** and **spring/dashpot constants**



Case Study 1: *Mechanical Vibration Reduction*

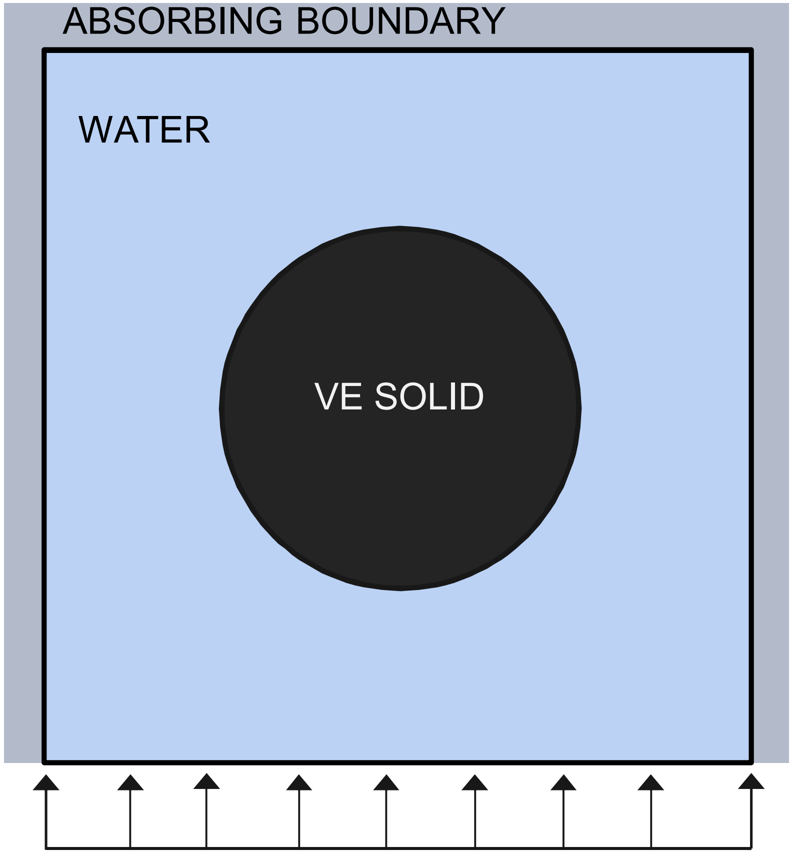
- Displacement at measurement locations minimized (dependent on frequency)

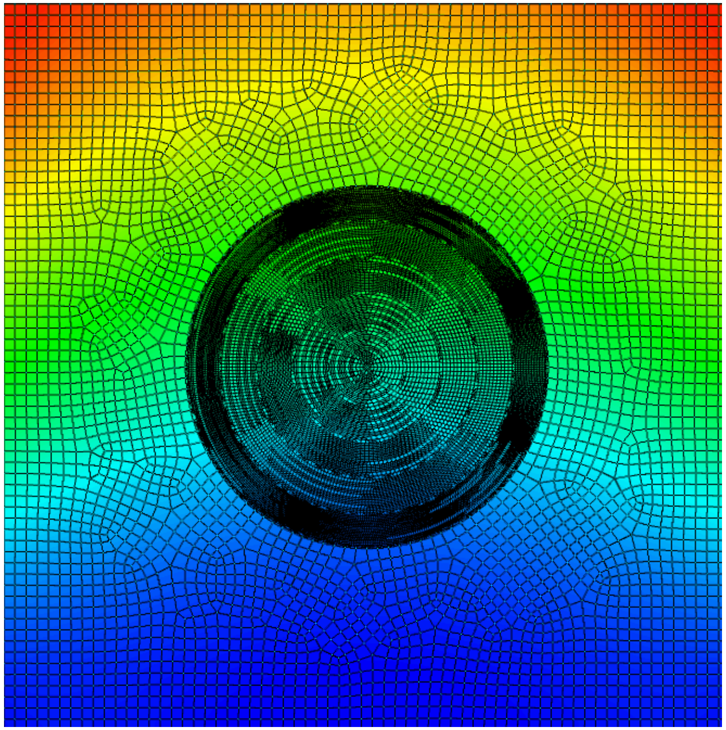


Left: X-displacement in base and payload with initial material guesses, 440 Hz loading;
Right: X-displacement in design

Inverse Problems: *Acoustic Cloaking*

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**

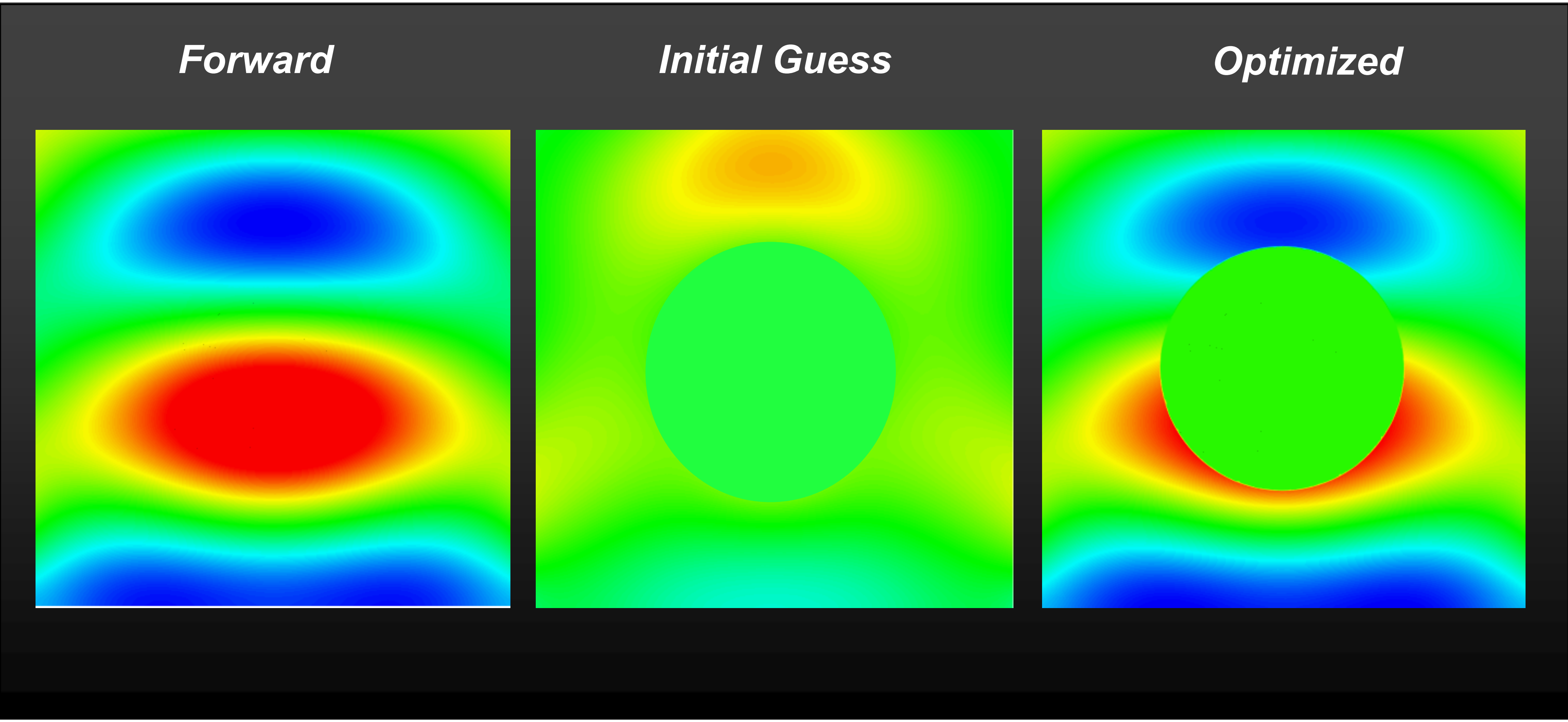




Left: Model Set up
Right: Forward problem pressure distribution (500 Hz loading) in model with 50 layers

Acoustic Cloaking

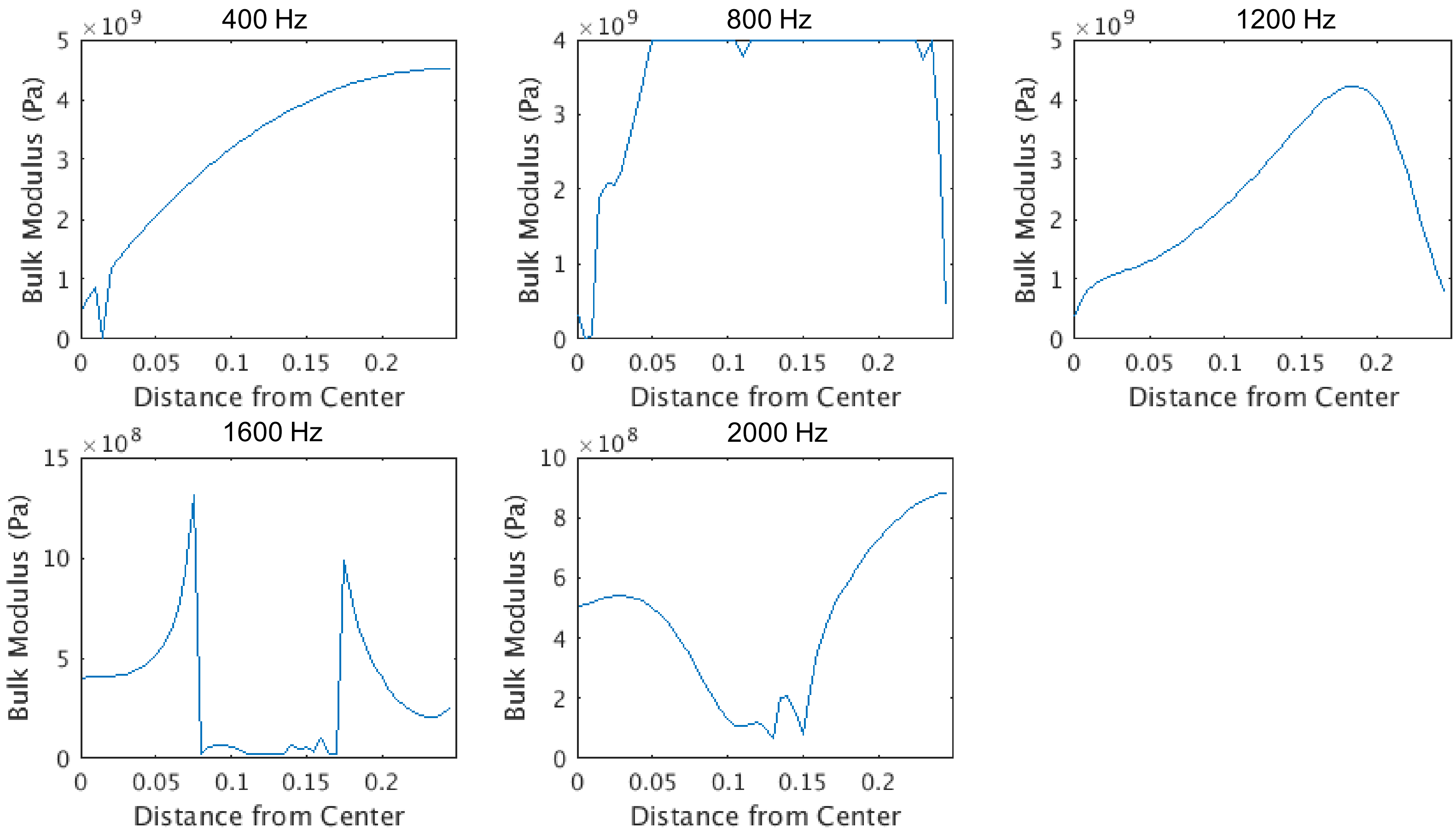
- Optimized VE foams allow recovery of desired pressure distribution



Left: Target acoustic pressure distribution, from forward problem
Center: Acoustic pressure distribution with initial material guess (2000 Hz Loading)
Right: Pressure distribution after convergence to optimized design

Acoustic Cloaking Results: Bulk Modulus

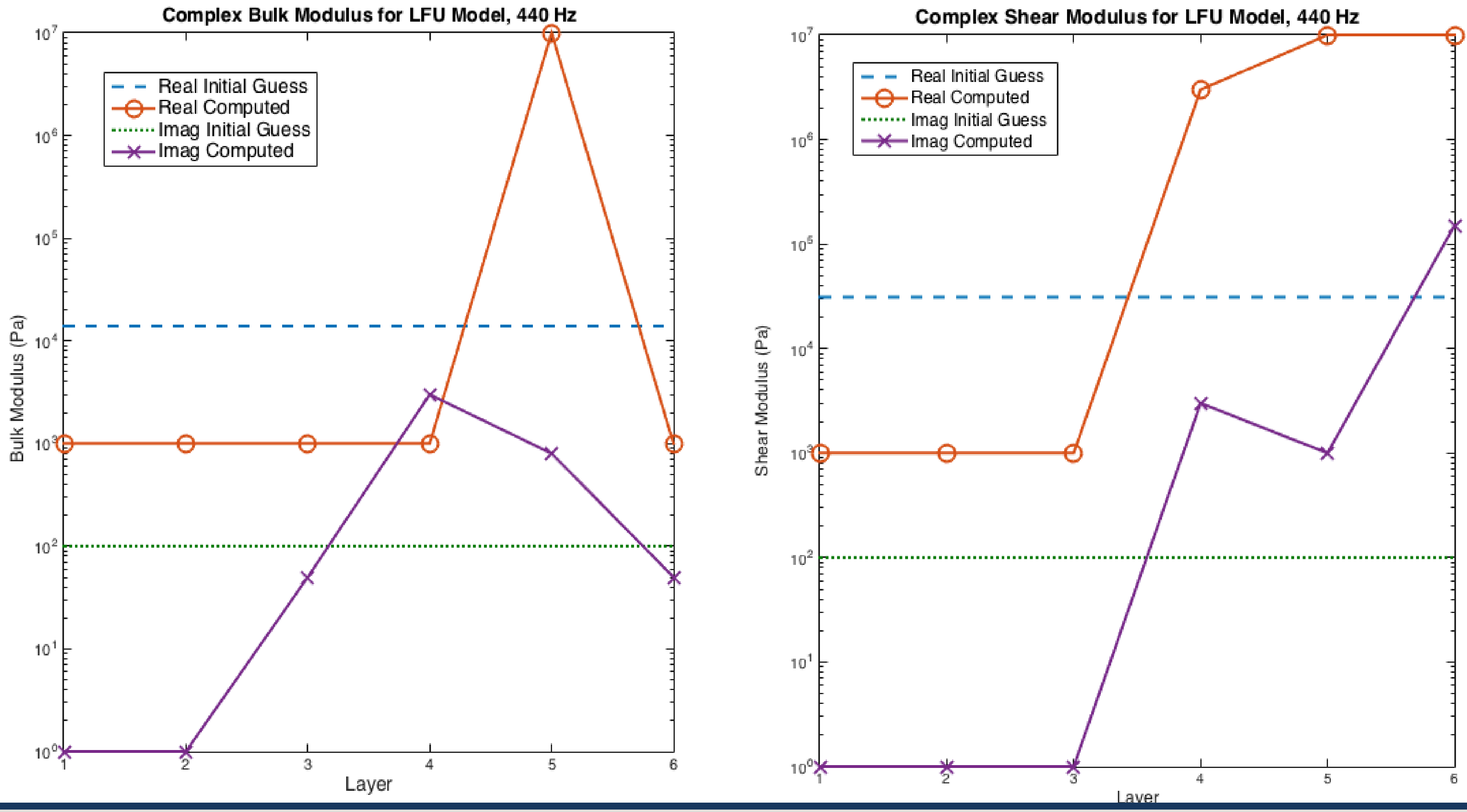
Bulk modulus sensitive to frequency, and varies nontrivially along disk radius



Figures: Real component of bulk modulus along radius, for various frequency

Case Study 1: *Mechanical Vibration Reduction*

Sandia National Laboratories



OBSERVATIONS:

- Elastic Properties: Soft materials selected towards top, stiffer materials near base
- Viscous Properties: Damping is added towards base of viscoelastic region