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Uncertainty Quantification of a Genetic Algorithm for Neutron Energy Spectrum Adjustment

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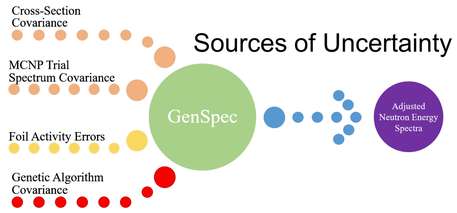
The development of a neutron energy spectrum adjustment code has prompted a further investigation into the uncertainty of the produced spectra.

Sources of Uncertainty



Cholesky Decomposition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{bmatrix}$$

For Cholesky decomposition, assume matrix A has the form:

$$A = LL^T$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{bmatrix} = \begin{bmatrix} l_{1,1} & 0 & \dots & 0 \\ l_{2,1} & l_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{n,1} & l_{n,2} & \dots & l_{n,n} \end{bmatrix} \begin{bmatrix} l_{1,1}^T & l_{1,2}^T & \dots & l_{1,n}^T \\ 0 & l_{2,2}^T & \dots & l_{2,n}^T \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & l_{n,n}^T \end{bmatrix}$$

Neutron Activation Analysis

Activation Reaction	Half-Life	Activity (Bq/atom-isotope)	Counting Uncertainty (%)
^{58}Ni(n,p)^{58}Co - Reference	70.83 d	9.7808E-18	2.9
^{24}Mg(n,p)^{24}Na	14.957 h *	1.2890E-17	2.3
^{27}Al(n,α)^{24}Na	14.957 h *	6.1585E-18	2.3
^{32}S(n,p)^{32}P Cf-equ	14.284 d	7.0703E+14 n/cm^2	3.6
^{46}Ti(n,p)^{46}Sc	83.788 d	7.8515E-19	2.3
^{47}Ti(n,p)^{47}Sc	3.349 d	3.8055E-17	3.1
^{48}Ti(n,p)^{48}Sc	43.67 h	8.8424E-19	1.4
^{55}Mn(n,2n)^{54}Mn	312.3 d	--	--
^{54}Fe(n,p)^{54}Mn	312.3 d	1.6954E-18	2.2
^{56}Fe(n,p)^{56}Mn	2.579 h *	5.8136E-17	1.8
^{59}Co(n,p)^{59}Fe	44.495 d	1.7965E-19	5.8
^{59}Co(n,2n)^{58}Co	70.83 d	--	--
^{58}Ni(n,2n)^{57}Ni	35.9 h	--	--
^{60}Ni(n,p)^{60}Co	1925.27 d	6.3203E-21	5.3
^{63}Cu(n,α)^{60}Co	1925.27 d	--	--
^{64}Zn(n,p)^{64}Cu	12.701 h *	4.7963E-16	3.1
^{90}Zr(n,2n)^{89}Zr	78.41 h	1.7102E-19	5.1
^{93}Nb(n,2n)^{92m}Nb	10.15 d	2.5526E-19	2.0
^{115}In(n,n')^{115m}In	4.486 h *	--	--
^{23}Na(n,γ)^{24}Na	14.957 h *	1.8587E-15	2.2
^{45}Sc(n,γ)^{46}Sc	83.788 d	7.3161E-16	2.2
^{55}Mn(n, γ)^{56}Mn	2.579 h *	3.4622E-13	1.8
^{58}Fe(n,γ)^{59}Fe	44.495 d	8.3270E-17	2.2
^{59}Co(n,γ)^{60}Co	1925.27 d	5.6707E-17	1.4
^{63}Cu(n,γ)^{64}Cu	12.701 h *	--	--
^{98}Mo(n,γ)^{99}Mo	2.748 d	2.4853E-15	1.7
^{109}Ag(n,γ)^{110m}Ag	249.78 d	8.7524E-17	0.81
^{186}W(n,γ)^{187}W	23.72 h *	2.2104E-13	1.5
^{197}Au(n,γ)^{198}Au	2.694 d	5.6538E-13	3.2
^{23}Na(n,γ)^{24}Na - Cd	14.957 h *	4.2940E-16	2.2
^{45}Sc(n,γ)^{46}Sc - Cd	83.788 d	1.1446E-16	2.2
^{55}Mn(n, γ)^{56}Mn - Cd	2.579 h *	9.1556E-14	1.9
^{58}Fe(n,γ)^{59}Fe - Cd	44.495 d	2.5208E-17	1.8
^{59}Co(n,γ)^{60}Co - Cd	1925.27 d	1.8968E-17	1.4
^{63}Cu(n,γ)^{64}Cu - Cd	12.701 h *	--	--
^{98}Mo(n,γ)^{99}Mo - Cd	2.748 d	2.3502E-15	1.6
^{109}Ag(n,γ)^{110m}Ag	249.78 d	5.7839E-17	0.8
^{197}Au(n,γ)^{198}Au - Cd	2.694 d	4.7342E-13	3.2
^{235}U(n,f)FP - BB	^{140}Ba - 12.752 d	3.1399E-09 #fis	3.5
^{238}U(n,f)FP - BB	^{140}Ba - 12.752 d	2.8531E-10 #fis	3.5
^{239}Pu(n,f)FP - BB	^{140}Ba - 12.752 d	3.1557E-09 #fis	3.5
^{237}Np(n,f)FP - BB	^{140}Ba - 12.752 d	1.5815E-09 #fis	3.5

Multivariate Normal Random Sampling

$$X \sim \mathcal{N}(\mu, \Sigma) \qquad \Sigma = E[(X - \vec{\mu})(X - \vec{\mu}^T)] = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \dots & \sigma_{1,d} \\ \sigma_{2,1} & \sigma_{2,2} & \dots & \sigma_{2,d} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{d,1} & \sigma_{d,2} & \dots & \sigma_{d,d} \end{bmatrix}$$

$$\sigma_{i,j} = E[(\vec{x_i} - \vec{\mu_i})(\vec{x_j} - \vec{\mu_j})]$$

$$\Sigma = LL^T \qquad \vec{z} = (z_1, ..., z_d)^T, \quad Z_i \sim \mathcal{N}(0, 1)$$

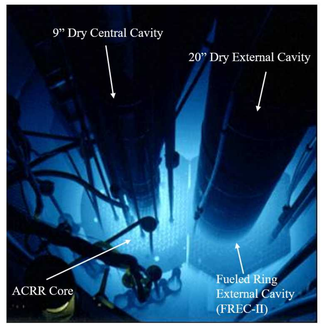
$$x = \mu + LZ$$

$$Z \sim \mathcal{N}(0, 1). \quad \Sigma(Z) = E[ZZ^T] = I$$

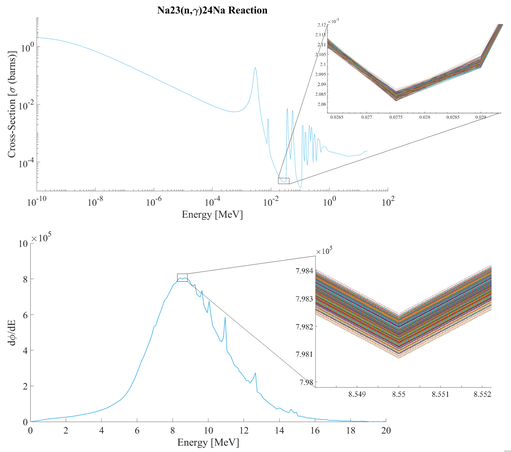
$$E[XX^T] = E[LZ(LZ)^T] = E[LZZ^TL^T]$$

$$\therefore E[XX^T] = LE[ZZ^T] = LL^T = \Sigma(X)$$

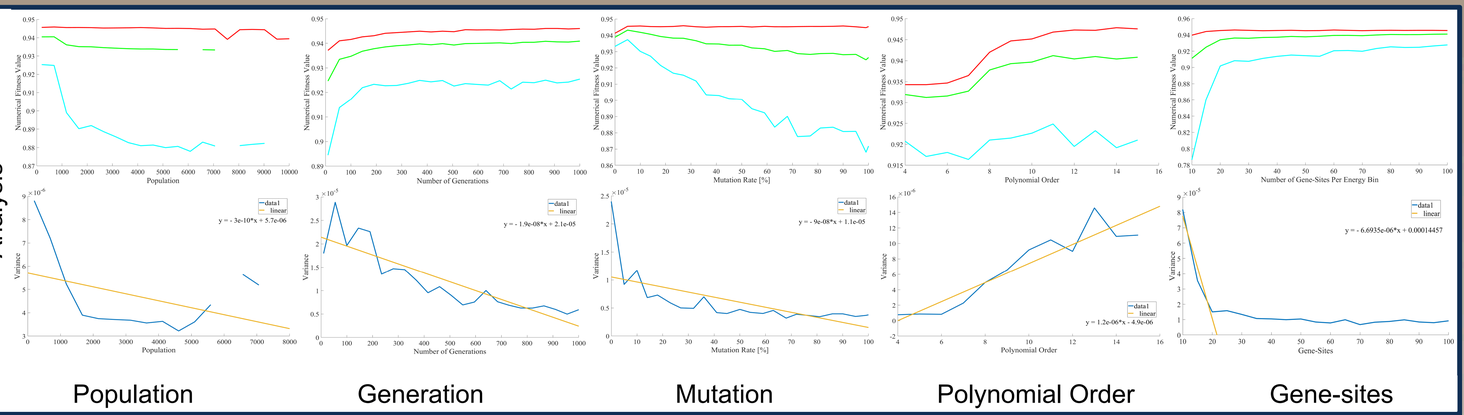
Annular Core Research Reactor



Distribution Realizations (5X mag)

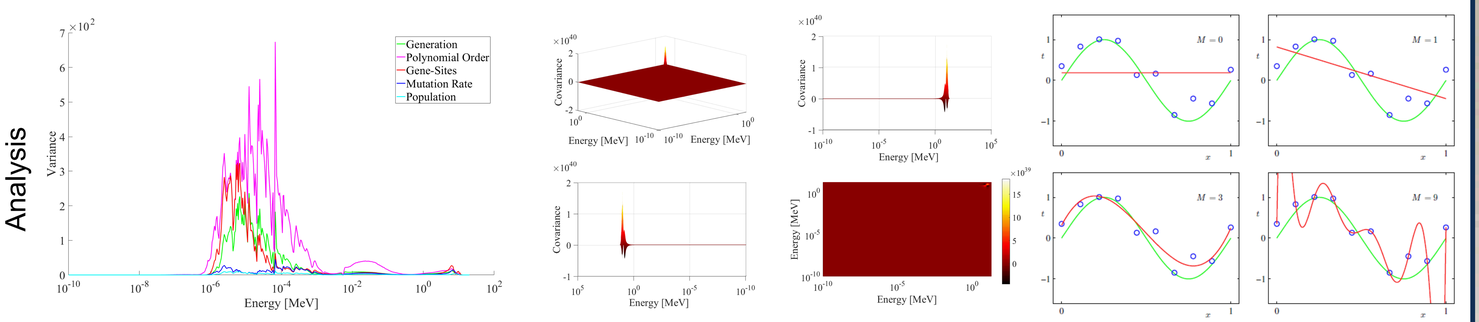


Local Parametric Sensitivity Analysis



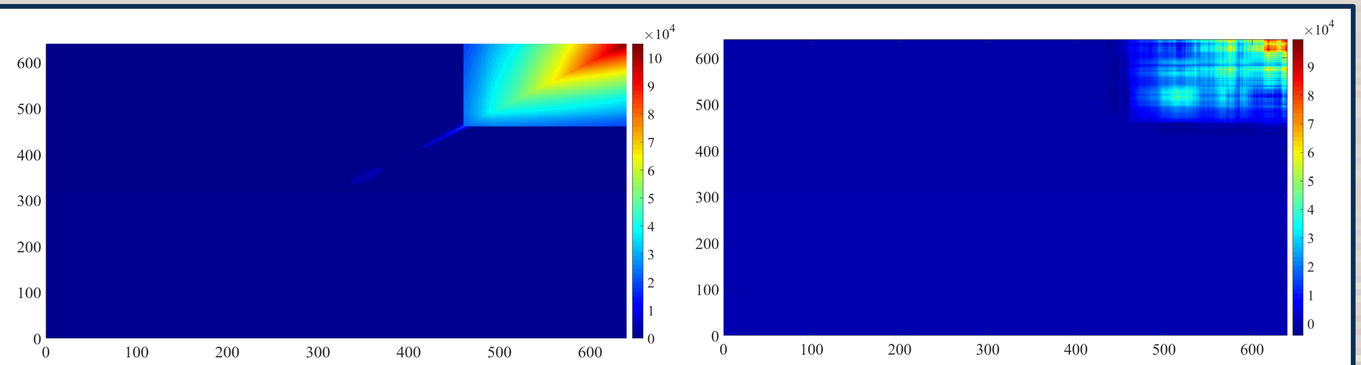
Population | Generation | Mutation | Polynomial Order | Gene-sites

Global Parametric Sensitivity Analysis



Error Contribution | Genetic Algorithm Covariance | Polynomial Order Error

Random Perturbation Analysis



Trial Spectrum Covariance | GenSpec Covariance