SAND2018-1933J

Short Communication

Relationship of Compressive Stress-Strain Response of Engineering Materials Obtained at Constant Engineering and True Strain Rates

Bo Song*, Brett Sanborn

Sandia National Laboratories, Albuquerque, NM 87185

Abstract

In this study, a Johnson-Cook model was used as an example to analyze the relationship of compressive stress-strain response of engineering materials experimentally obtained at constant engineering and true strain rates. There was a minimal deviation between the stress-strain curves obtained at the same constant engineering and true strain rates. The stress-strain curves obtained at either constant engineering or true strain rates could be converted from one to the other, which both represented the intrinsic material response. There is no need to specify the testing requirement of constant engineering or true strain rates for material property characterization, provided that either constant engineering or constant true strain rate is attained during the experiment.

Keywords: Johnson-Cook model, constant true strain rate, constant engineering strain rate, stress-strain curve, material response

Introduction

In engineering practice, the mechanical properties of materials are usually described in engineering stress-strain curves. However, in constitutive modeling and numerical simulation, material properties are commonly described with true stress-strain curves. Engineering stress-strain curves are directly measured with experiments at various constant engineering strain rates which are used to develop a strain-rate-dependent stress-strain constitutive relationship. Prior to determination and calibration of material model constants, the engineering measurements must be converted into true measurements. Under uniaxial compression, the true strain, ε, is calculated with

$$\varepsilon = -\ln(1-e) \tag{1}$$

*Corresponding author, bsong@sandia.gov; +1-505-844-4285.

where e represents engineering strain. Assuming an incompressible solid, the true stress, σ, is calculated as

$$\sigma = S \cdot (1 - e) \tag{2}$$

where S is engineering stress. Both stress and strain are taken to be positive in compression. Therefore, the measured engineering stress-strain response can be simply converted to true stress-strain response. However, the converted true stress-strain response is specific to a constant engineering strain rate facilitated in the experiment. This may generate an awkward situation in model development where the stress and strain are described in true measurements, whereas the strain rate is described in engineering measurements.

Differentiating Eq. (1) yields true strain rate, $\dot{\varepsilon}$,

$$\dot{\varepsilon} = \frac{\dot{e}}{1 - e} \tag{3}$$

where $\dot{e}$ is engineering strain rate. Equation (3) indicates that when a constant engineering strain rate is facilitated in an experiment, the true strain rate will not remain constant. Instead, the true strain rate increases with increasing engineering strain in compression, which has been discussed in the literature [1-3]. Consequently, constant true strain rate experiments have been proposed and recommended for unifying variants (true stress, true strain, and true strain rate) in constitutive modeling [4-7]. A post-processing technique developed by Sofuoglu [8] extracted constant true strain rate response from the data originally obtained at a constant cross-head speed (nearly constant engineering strain rate) in an MTS test. Nowadays, closed-loop servohydraulic systems have been significantly improved in commercial material testing frames such that materials can be characterized at constant true strain rates, but only within quasi-static (or low) strain rate regime.

Kolsky bar, also called split Hopkinson bar, has become a common apparatus for material-property characterization at dynamic (or high) strain rates since 1949 [9]. In a conventional Kolsky bar test, the specimen, in most cases, is subjected to neither constant engineering strain rate nor constant true strain rate. Since the 1970s, pulse shaping techniques have been developed and implemented in Kolsky bar experiments to facilitate constant engineering strain rates [10]. Similar to quasi-static tests, the constant engineering strain rates result in increasing true strain rates with increasing specimen strain in dynamic compression tests. Conversely, a constant true strain results in decreasing engineering strain rate. Ramesh and Narasimhan [11] found that a conventional Kolsky bar test without pulse shaping generated a more nearly constant true strain rate rather than engineering strain rate, especially for work-hardening materials. However, this is not generally applicable to all materials. Some authors have suggested that it may be desirable to develop a general experimental procedure to conduct Kolsky bar tests with constant true strain rates. Methods exist that utilize variable impedance strikers to achieve constant true strain rates in the sample [12]. The downside of these techniques is that the variable impedance shape of the striker was experimentally iterated upon several times to fine-tune both the value and constancy of the true strain rate history. Arriving at a striker shape inevitably included manufacturing time between each set of experiments. This process also required that the material response was known prior to manufacture of the striker. This made the process particularly difficult since the high-rate material response was the primary goal of study. When a striker design was reached, the striker could only be used for a single material at a single value of constant true strain rate. Since families of stress-strain curves at different strain rates are needed for model calibration, this process can quickly become expensive and time-consuming. Recently, Lim et al. [13] studied 304 stainless steel under constant true strain rate in compression. Since 304 exhibits significant work hardening, constant true strain rate was easily achieved with minimal pulse shaping. They also investigated the difference in the experimental compressive stress-strain response of 304 stainless steel obtained at constant engineering strain rates and true engineering strain rates. Very little difference in the experimental results was observed when strain rates were below 3000 s^{-1}. However, at a higher strain rate of 7000 s^{-1}, the experimental flow stress at constant engineering strain rate was higher than that obtained at comparable constant true strain rate [13].

There exist substantial challenges in both pulse shaper and/or variable impedance striker design to facilitate a constant true strain rate compared to a constant engineering strain rate in Kolsky bar experiments, especially for non-work-hardening materials. This raises some questions –

1) Is it worth or necessary to perform specific Kolsky bar experiments at constant true strain rates?
2) Are current experimental stress-strain data based on constant engineering strain rates sufficient to describe true material response for strain-rate-dependent model development?
3) Is there any relationship of stress-strain response obtained at constant engineering strain rates and constant true strain rates?

In this paper, we used the Johnson-Cook model, based on the data in Ref. [13] as an example to answer these questions, particularly to demonstrate the relationship of stress-strain response obtained at constant engineering strain rates and constant true strain rates.

Johnson-Cook Model Based on Constant True or Engineering Strain Rates

When temperature effects are not considered, the general form of the Johnson-Cook model has the form [14],

$$\sigma = \left(A + B\varepsilon^{n}\right)\cdot\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) \tag{4}$$

where σ is true stress; ε represents the plastic strain; A, B, C, and n are material constants; $\dot{\varepsilon}$ is plastic true strain rate; and $\dot{\varepsilon}_0$ is reference strain rate. In Eq. (4), the first term on the right represents the true stress-strain response at the reference true strain rate, $\dot{\varepsilon}_0$, while the second term simply represents a scaling of the strain rate effect.

As mentioned earlier, the stress-strain response of materials is usually obtained at constant engineering strain rates rather than constant true strain rates. This makes it difficult to directly determine the materials constants in Eq. (4) from experimental data. Here, we consider the Johnson-Cook relation with engineering strain rate,

$$\sigma = \left(A_1 + B_1\varepsilon^{n_1}\right)\cdot\left(1 + C_1 \ln\frac{\dot{e}}{\dot{e}_0}\right) \tag{5}$$

where A_1, B_1, C_1, and n_1 are material constants; $\dot{e}$ is actual engineering strain rate; $\dot{e}_0$ is a constant reference engineering strain rate, $\dot{e}_0 = R_0$. Now we use Eq. (5) to calculate the true stress-strain response at a constant true strain rate. As indicated with Eqs. (1) and (3), a constant true strain rate represents an exponentially declining true strain rate history with increasing true plastic strain,

$$\dot{e} = (1-e)\dot{\varepsilon} = \dot{\varepsilon}\exp(-\varepsilon) \tag{6}$$

Equation (5) thus becomes

$$\sigma = \left(A_1 + B_1\varepsilon^{n_1}\right)\cdot\left(1 - C_1\varepsilon + C_1 \ln\frac{\dot{\varepsilon}}{R_0}\right) \tag{7}$$

Equation (7) gives the true stress-strain response description based on constant true strain rate. In the case when a constant true strain rate has the same value as the reference engineering strain rate, $\dot{\varepsilon} = R_0$, Equation (7) becomes

$$\sigma = \left(A_1 + B_1\varepsilon^{n_1}\right)\cdot\left(1 - C_1\varepsilon\right) \tag{8}$$

Equation (8) shows the relationship of true stress-strain curves obtained at the same reference engineering and true strain rate with a conversion factor of $(1 - C_1\varepsilon)$. To obtain the absolute deviation, Δ, between the stress-strain curves obtained at the same reference constant engineering and true strain rates, $\dot{e}_0 = \dot{\varepsilon}_0 = R_0$, using Eq. (8), we have

$$\Delta = \left(A_1 + B_1\varepsilon^{n_1}\right)\cdot C_1\varepsilon \tag{9}$$

The relative deviation is

$$\delta = \frac{\Delta}{\left(A_1 + B_1\varepsilon^{n_1}\right)} = C_1\varepsilon \tag{10}$$

Equation (10) indicates that the stress deviation increases with increasing strain. At a certain strain, the deviation of stresses obtained at constant engineering and true strain rates depends on strain-rate sensitivity. Higher strain-rate-sensitivity results in a larger deviation.

The Johnson-Cook material constants based on constant engineering and true strain rates determined by Lim et al. [13] are listed in Table 1. Lim et al. [13] characterized 304 stainless steel at similar reference engineering (0.01 s^{-1}) and true (0.008 s^{-1}) strain rates. Experimentally obtained true stress-strain behavior at constant engineering strain rate (dotted line) and constant true strain rate (dashed line) are shown in Fig. 1. Using Eq. (8) and the material constants in Table 1, the stress-strain curve at the same constant true strain rate of 0.01 s^{-1} is calculated and shown (solid line) in Fig. 1. Thus, the *experimental* stress-strain curve obtained under *constant true strain rate* conditions (dashed line) is compared to the *calculated* stress-strain curve at *constant true strain rate* (solid line) that originated from a *constant engineering strain rate* experiment. The two stress strain curves obtained by experiment and calculation show reasonable agreement. The relative stress deviation was also calculated using Eq. (10). The results show that the relative stress deviation is only 1% at a specimen strain of 0.36, which is negligible, although the absolute deviation increases with specimen plastic strain (Fig. 1).

Equation (7) also suggests that the absolute deviation of stress-strain curves at the same true and engineering strain rates is independent of strain rate. However, the relative deviation (Eq. (10)) decreases with increasing strain rate, due to strain-rate sensitivity,

$$\delta\left(\dot{\varepsilon}\right) = \frac{C_1 \varepsilon}{1 + C_1 \ln \frac{\dot{\varepsilon}}{R_0}} \tag{11}$$

Table 1. Johnson-Cook material model constants for 304 stainless steel [13]

Constant True Strain Rate Testing	A (MPa)	B (MPa)	n	C	$\dot{\varepsilon}_0$ (s^{-1})
	566	832.8	0.5647	0.02876	0.008
Constant Engineering Strain Rate Testing	A_1 (MPa)	B_1 (MPa)	n_1	C_1	$\dot{\varepsilon}_0$ (s^{-1})
	546	851	0.5075	0.0275	0.01

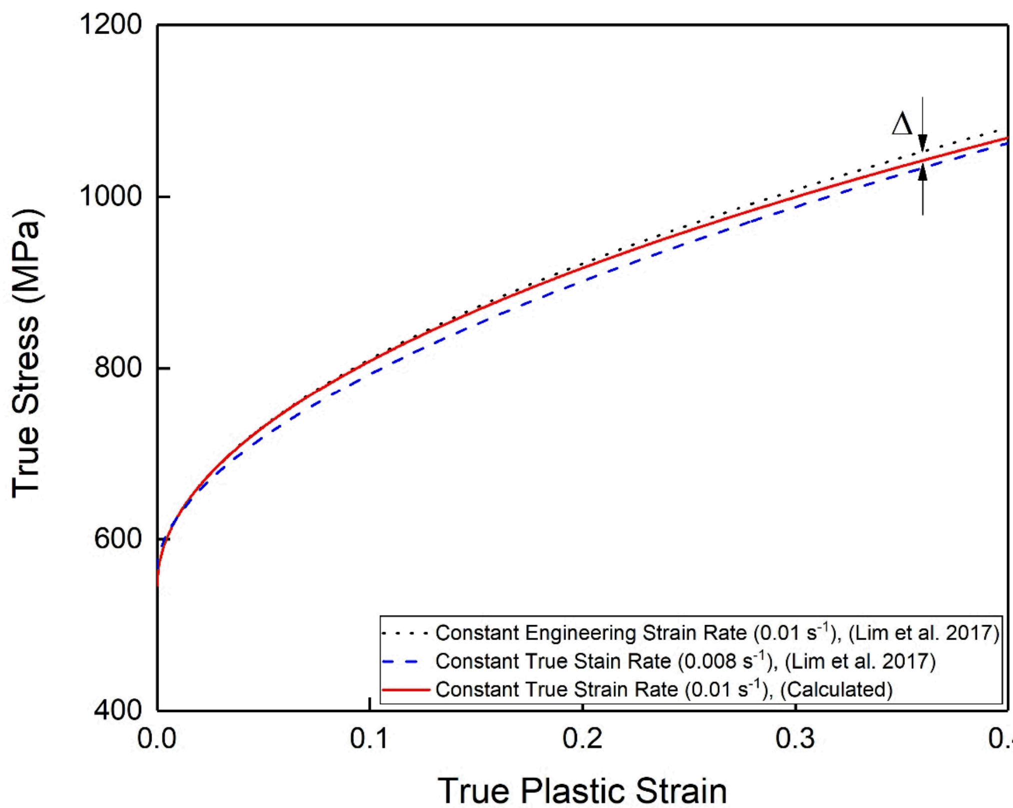

Figure 1. Comparison of compressive stress-strain curves of 304 stainless steel obtained at the same constant true and engineering strain rates.

Now we use the material constants (Table 1) determined at constant engineering strain rates by Lim et al. [13] to predict the stress-strain curves at constant true strain rates and compare with the material model at constant true strain rates which was also presented in [13]. The comparison results are shown in Fig. 2. As shown in Fig. 2, the dotted lines represent the stress-strain curves obtained at constant engineering strain rates; the solid lines are the calculated stress-strain curves with Eq. (7) from the corresponding dotted lines; and the dashed lines are the stress-strain curves obtained at constant true strain rates by Lim et al. [13]. As shown in Fig. 2, although little difference among these three curves is observed, the calculated stress-strain curves with Eq. (7) are observed to be very consistent with the stress-strain curves obtained at constant true strain rates by Lim et al [13]. This means Eq. (7) is sufficiently accurate to convert the stress-strain curves obtained at constant engineering strain rates to the stress-strain response at constant true strain rates. Figure 2 also shows a little larger deviation at low strain rates (0.001 and 0.01 s^{-1}). This is possibly due to inconsistent values of material constant A in the Johnson-Cook model for constant true and engineering strain rates. The material constant A represents the yield strength which should be the same whether the tests were performed at constant true- or engineering strain rates.

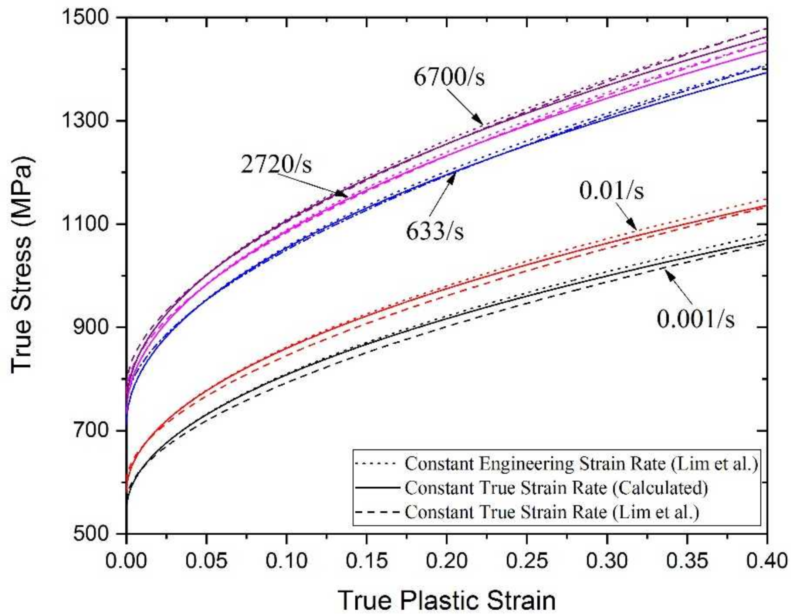

Figure 2. Comparison of stress-strain curves calculated with Eq. (7)

and experimentally obtained by Lim et al. [13]

As mentioned earlier, the Johnson-Cook equation (Eq. (5)) indicates that the true stress is a function of true strain and engineering strain rate. We used the material constants (Table 1) presented by Lim et al. [13] and plot the true stress surface, as shown in Fig. 3. It is noted that in Fig. 3, the true stress-strain response is presented with engineering strain rate. If the true stress-strain response needs to be presented with true strain rate, only a transformation is needed. In fact, the stress surface presented in Fig. 3 represents a full map in terms of the stress-strain response at true strain rates. As shown in Fig. 3, the stress-strain curve at a constant engineering strain rate, i.e., 1000 s^{-1} in this case, is represented with the intersection of the stress surface and the constant engineering strain rate surface (in green); however, the stress-strain curve at the same constant (1000 s^{-1}) but true strain rate is represented by the intersection of the stress surface and the blue surface which represents the constant true strain rate of 1000 s^{-1}. Looking at both blue and green surfaces, they begin with the same value (1000 s^{-1}) at zero true plastic strain because there is no difference between true and engineering strain and strain rate when plastic strain is zero. Then both surfaces deviate with increasing true plastic strain. The blue surface represents a constant true strain rate, while engineering strain rate decreases with increasing true plastic strain. Furthermore, the true stress-strain curve at the constant true strain rate is lower than that obtained at the engineering strain rate with the same constant, as shown in Fig. 3. In summary, as long as the stress-strain data are fully obtained at either true or engineering strain rates, the intrinsic material response is obtained. If the stress-strain response is obtained at constant engineering strain rates, the stress-strain response at constant true strain rates can be calculated with pre-determined material models, i.e., Johnson-Cook model in this study, or vice versa. Depending on experimental capabilities and challenges, experimentalists may decide the most convenient and realistic way in terms of constant engineering or true strain rate testing to determine the material properties.

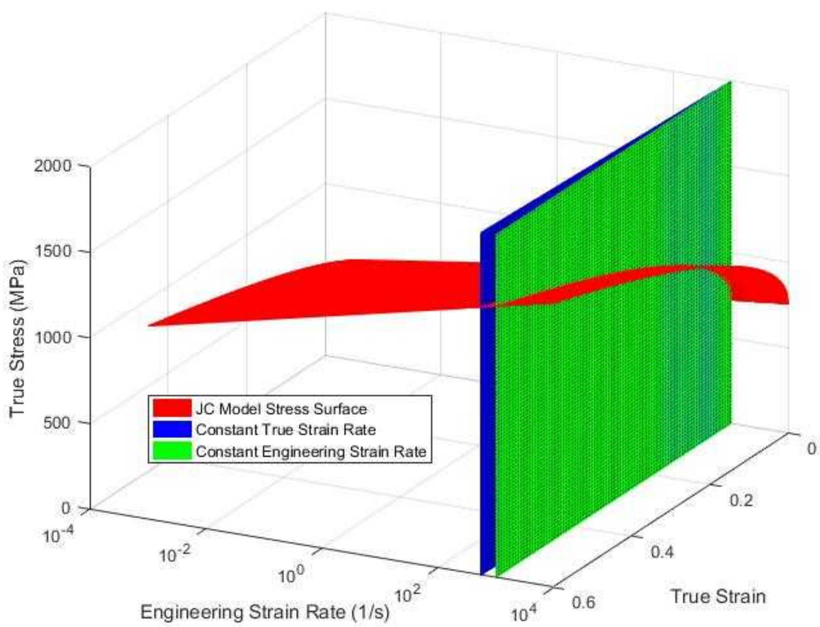

Figure 3. True stress surface for material response extrusion

at constant true and engineering strain rates

Concluding Remarks

A Johnson-Cook model was used to demonstrate the relationship of the stress-strain response based on constant true or engineering strain rates. Using 304 stainless steel as an example, the true stress-strain curve obtained at a constant engineering strain rate is slightly different from that obtained at the same true strain rate. The difference is however very minimal, i.e., less than 1%, which is negligible. This conclusion is consistent with the following work by Forrestal et al. [15].

Compared to the work by Forrestal et al. [15], this study provides more quantitative and in-depth analysis on the stress-strain difference obtained at constant true and engineering strain rates. Mechanical testing data based on either constant true strain rate or constant engineering strain rate constructs the intrinsic rate-dependent material response, just in different presenting forms. As an intrinsic material response, the stress-strain behavior in terms of engineering strain rate can be converted into true strain rate regime, or vice versa, through proper coordination transformation. It is not necessary or mandatory to specify mechanical testing at constant engineering or true strain rates. Depending on experimental capability and challenges, the most convenient and practical way for either constant true or engineering strain rate testing is recommended, instead of one way or the other.

Acknowledgement

The authors would like to thank Drs. Edmundo Corona and William Scherzinger for the valuable discussion on this topic and manuscript.

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

References

[1] Fitzsimons G, Kuhn HA, A device for control of high speed compression testing at constant true strain rates. J Phys E: Sci Instum 1982; 15:508-510.

[2] Fitzsimons G, Kuhn HA, DeArdo AJ, Semple V. High-speed compression testing at constant true strain rates for hot working studies. J Testing Evaluation 1984; 12:27-32.

[3] Rohm H, Jaros D, Benedikt J. Constant strain rate compression of biopolymer gels. J Text Studies 1995; 26:665-674.

[4] Baudelet B, Suery M. Constant stress creep and constant true strain-rate tensile tests of the superplastic alloy PbSn. J Mater Sci 1972; 7:512-516.

[5] Hartley CS, Jenkins DA. Tensile testing at constant true plastic strain rate. J Metals 1980; 32:23-28.

[6] Lautenschlager, EP, Brittain JO. Constant true strain rate apparatus. Rev Sci Instru 1968; 39:1563-1565.

[7] Balasundar I, Ravi KR, Raghu T. On the high temperature deformation behavior of titanium alloy BT3-1. Mater Sci Eng 2017; A684:135-145.

[8] Sofuoglu H. A new technique used in obtaining true stress-strain curves for constant strain-rates. Exp Tech 2003; 27:35-37.

[9] Kolsky H. An investigation of the mechanical properties of materials at very high rates of loading. Proc Phys Soc London 1949; B62:676-700.

[10] Chen W, Song B. Split Hopkinson (Kolsky) Bar Design, Testing and Applications. 2011; Springer, New York.

[11] Ramesh KT, Narasimhan S. Finite deformations and the dynamic measurement of radial strains in compression Kolsky bar experiments. Int J Solids Structures 1996; 33:3723-3738.

[12] Casem, DT, Hopkinson bar pulse-shaping with variable impedance projectiles – an inverse approach to projectile design. ARL-TR-5246, US Army Research Laboratory, Aberdeen Proving Ground, MD, 2010.

[13] Lim BH, Liao H, Chen WW, Forrestal MJ. Effects of constant engineering and true strain rates on the mechanical behavior of 304 stainless steel. J. Dynamic Behavior Mater 2017; 3:76-82.

[14] Johnson GR, Cook WH. A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures. In: Proceedings of the 7th International Symposium on Ballistics, 1983. The Hague, The Netherlands, pp541-547.

[15] Forrestal MJ, Lim BH, Chen WW. A viscoplastic constitutive equation from Kolsky bar data for two steels. J. Dynamic Behavior Mater 2017; (DOI 10.1007/s40870-017-0131-5).