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Development of Large-Scale Inverse Methods in Sierra-SD

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Outline

- Inverse problems in computational mechanics
- Discussion of inverse methods in Sierra-SD
- Connections with design of materials and acoustic metamaterials

What is an Inverse Problem?

What is an Inverse Problem?

Inverse problems arise when we have partial information and indirect observations of a system and need to infer (hidden) quantities of interest of the system.

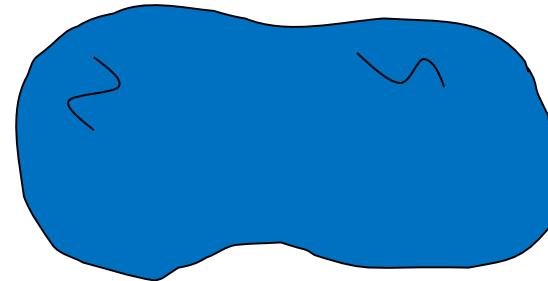
An inverse problem can be viewed as a quest for information that is not directly available from observations or measurements.

The pursuit of a solution to an inverse problem calls for a balance synergy between analysis and experimentation.

Inverse Problems: Observing the Unobservable

Suppose we have a “black box” system in the *as-manufactured* state that has only partially known parameters

Question: can we *non-destructively* interrogate the system to “see what is inside”?



Typical unknown parameters:

- Material properties
- Loads
- Boundary conditions
- Residual stresses
- Size/shape/location of inclusions (e.g. composite materials)

Example applications:

- Seismic imaging
- Medical imaging
- Non-destructive evaluation

Challenges

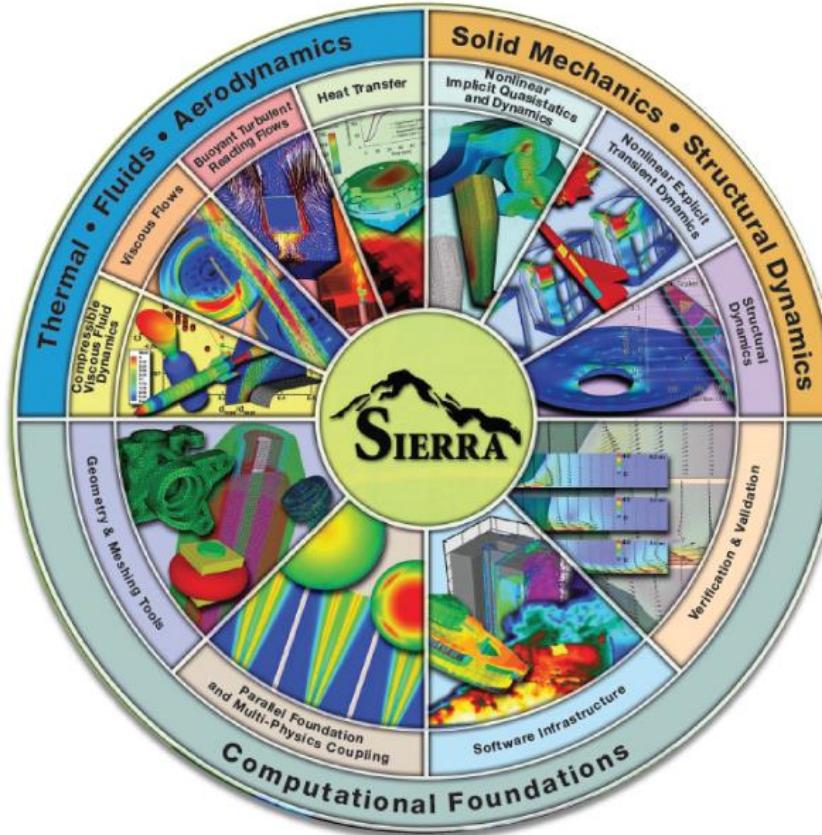
- Inverse problems can be ill-posed.
 - Solution may not exist.
 - Solution may not be unique.
 - Solution may be unstable. That is, it may be sensitive to small changes in the input data.
- Can be very computationally demanding.
- But... regularization can be used to mitigate these issues
- Or we can re-formulate the problem (different objective)

Categories of Inverse Problems

- Imaging
 - Ultrasound medical
 - seismic
- Calibration of material models
 - Structural material properties, circuits, thermal properties, etc.
- Optimal Experimental Design
 - Best placement of sensors, test fixture setups
- Shape reconstruction
 - E.g. inverse scattering
- Design of materials
 - Design material microstructure to achieve desired properties
 - E.g. Cloaking, camouflage, noise suppression, etc

Inverse Problems - Motivation

For long-term monitoring of structural systems, parallel multiphysics forward solvers (Sierra Mechanics) are not enough



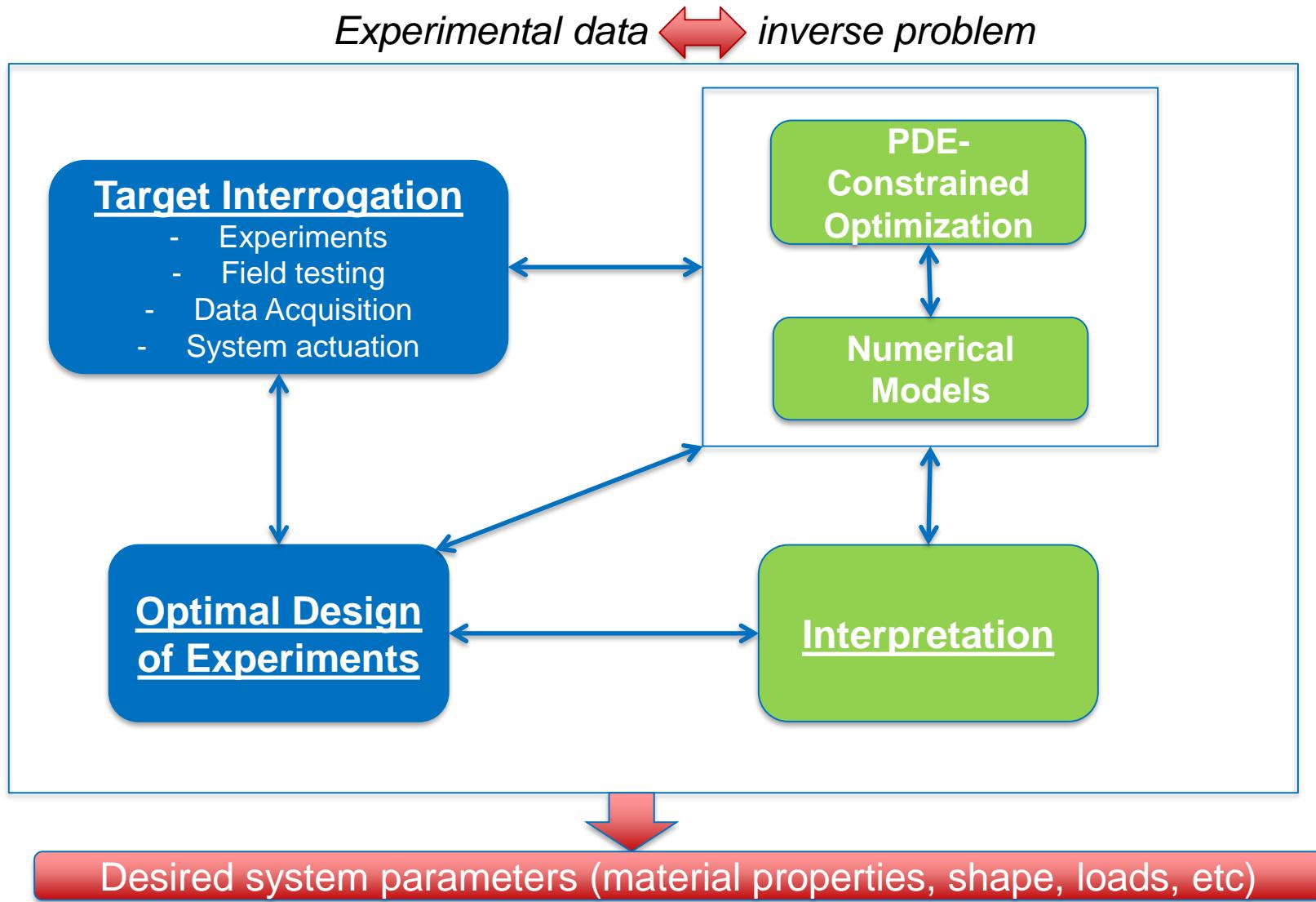
Partially known information:

- material properties?
- boundary conditions?
- loading conditions?
- Internal flaws from aging?
- Preloading effects?

The missing link:

**Experimental measurements and
solution of inverse problem**

Inverse Problems - The Interaction of Experiments and Simulation



PDE-Constrained Optimization Formulation

Abstract optimization formulation

$$\underset{\mathbf{u}, \mathbf{p}}{\text{minimize}} \quad J(\mathbf{u}, \mathbf{p})$$

$$\text{subject to} \quad \mathbf{g}(\mathbf{u}, \mathbf{p}) = \mathbf{0}$$

$$\mathcal{L}(\mathbf{u}, \mathbf{p}, \mathbf{w}) := J + \mathbf{w}^T \mathbf{g}$$

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \mathbf{g}_u^T \mathbf{w} \\ J_p + \mathbf{g}_p^T \mathbf{w} \\ \mathbf{g} \end{Bmatrix} = \{\mathbf{0}\}$$

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \mathbf{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \mathbf{g}_p^T \\ \mathbf{g}_u & \mathbf{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta \mathbf{u} \\ \delta \mathbf{p} \\ \mathbf{w}^* \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \mathbf{g} \end{Bmatrix}$$

$$\mathbf{W} \Delta \mathbf{p} = -\hat{\mathbf{J}}',$$

$$\mathbf{W} = \mathbf{g}_p^T \mathbf{g}_u^{-T} (\mathcal{L}_{uu} \mathbf{g}_u^{-1} \mathbf{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} \mathbf{g}_u^{-1} \mathbf{g}_p + \mathcal{L}_{pp}$$

Objective function

PDE constraint

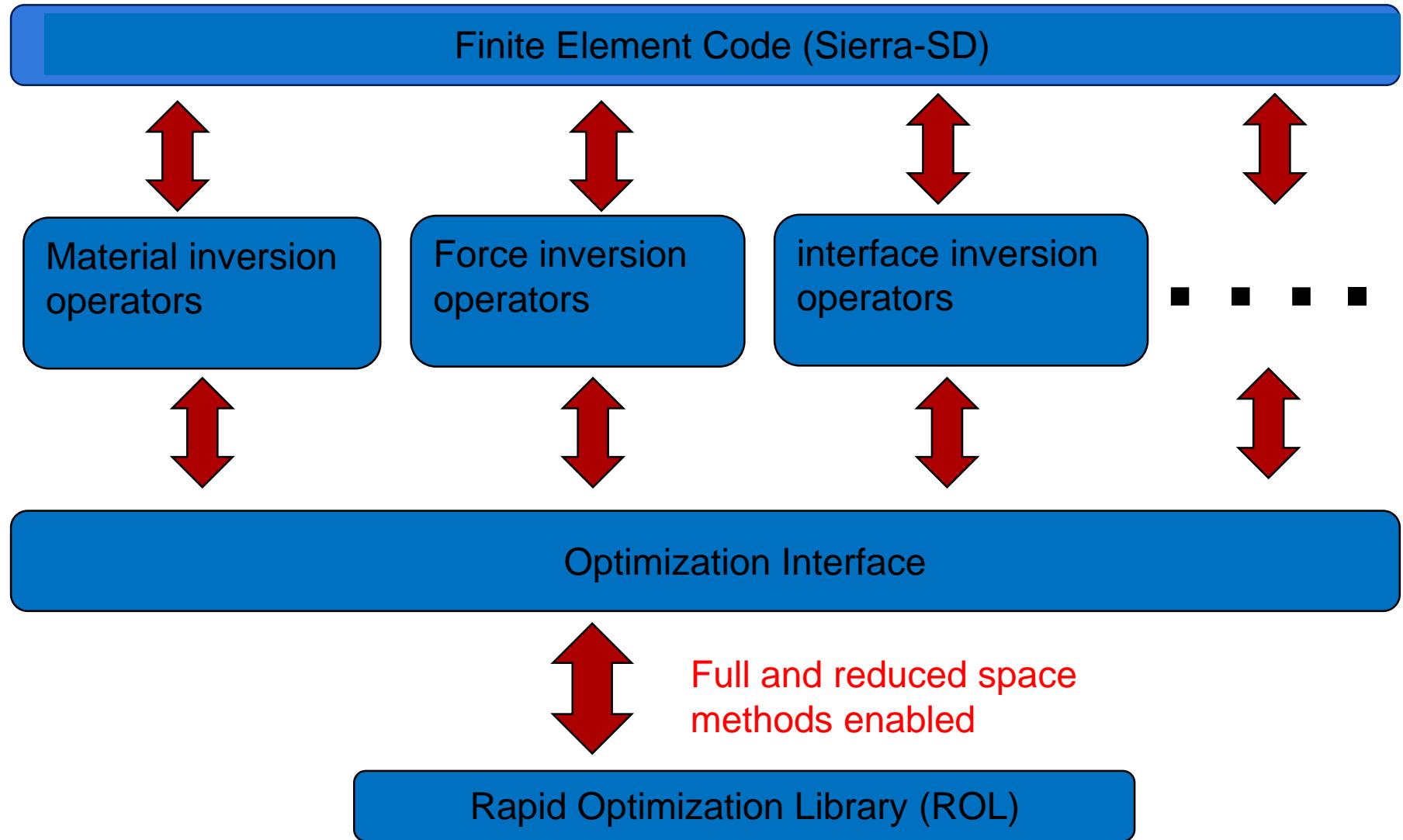
Lagrangian

First order optimality conditions

Newton iteration

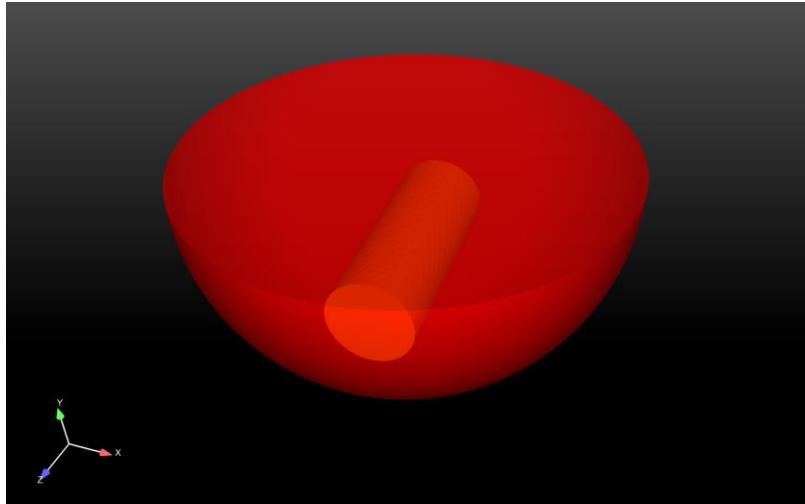
Hessian calculation

Operator-Based Inverse Problems

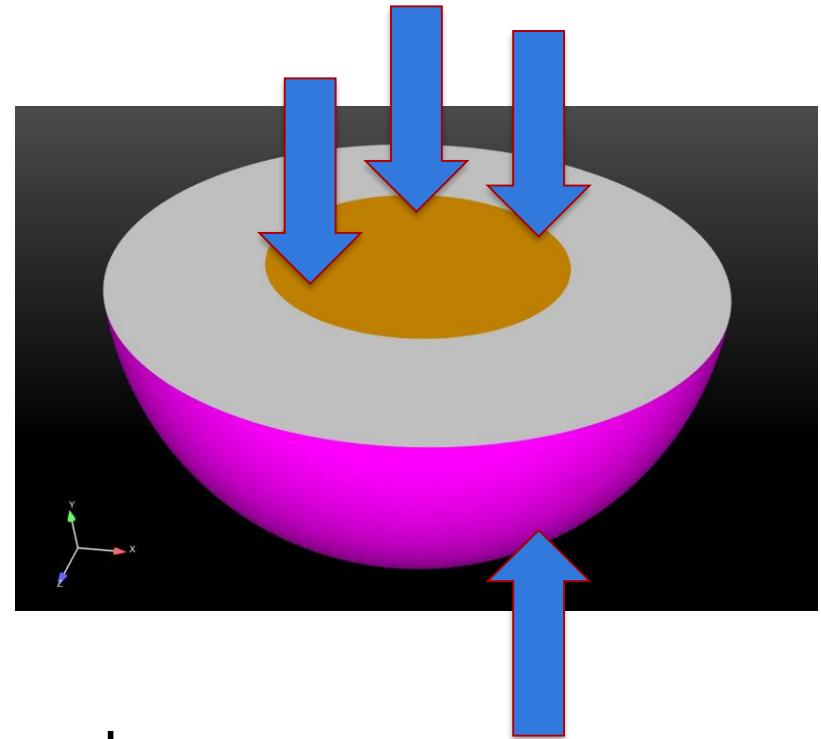


Buried Tunnel Model – MECE vs Least Squares Objectives

Buried inclusion model



Applied pressure

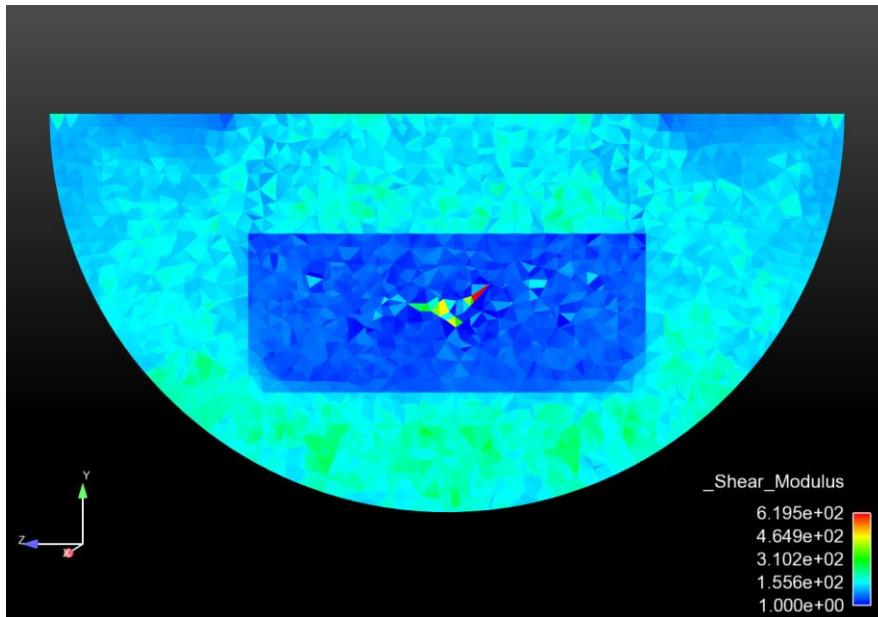


Goal: locate buried inclusion and surrounding material properties

Fixed boundary

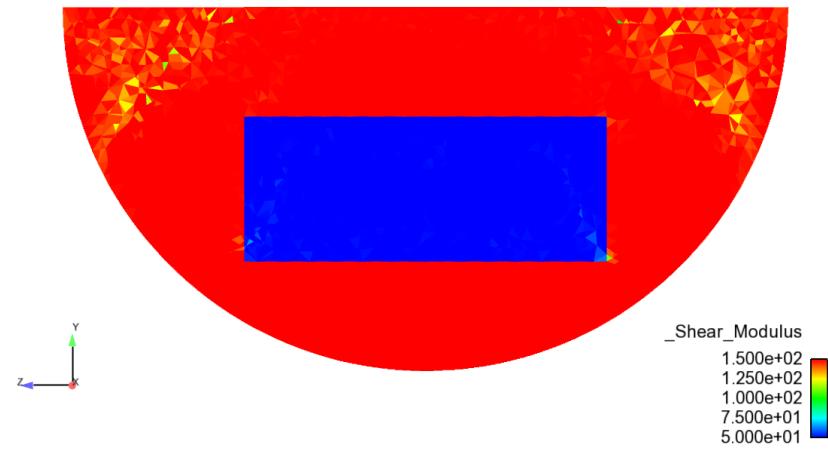
Shear Modulus – MECE vs Least Squares Objectives

Least Squares



1000 iterations

MECE



30 iterations

Exact bulk moduli: 50 (matrix), 150 (inclusion)

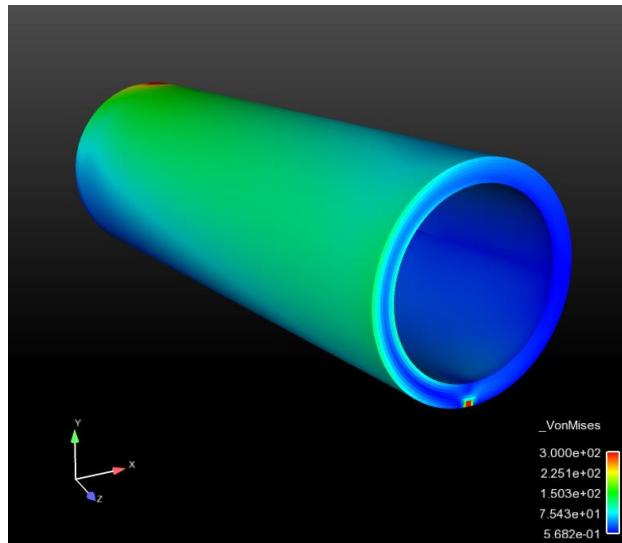
Residual Stress Inversion

Equilibrium constraint: $\nabla \cdot \sigma = 0$

Approach: source inversion for tractions on cut-plane

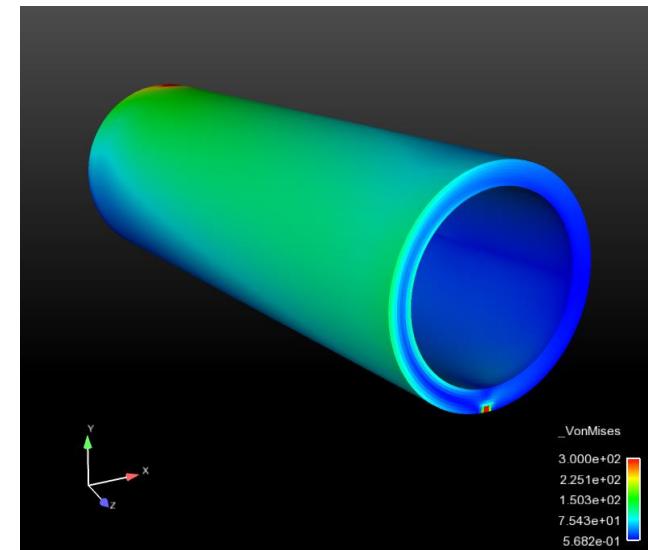
Prestressed Tube Example

Exact stress field (from forward solution)



Stress concentrated around fixed points

Stress field from inverted tractions

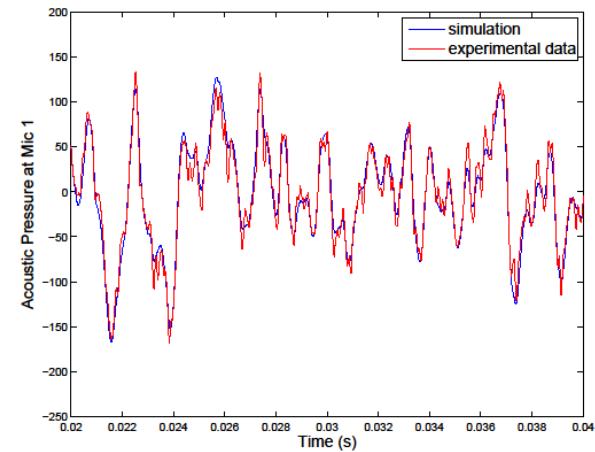
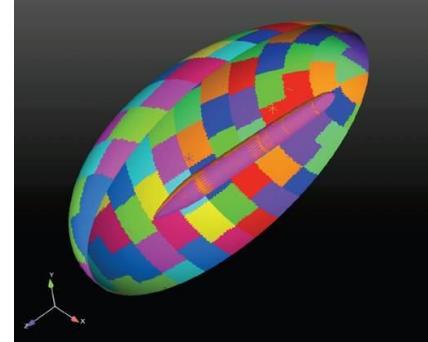
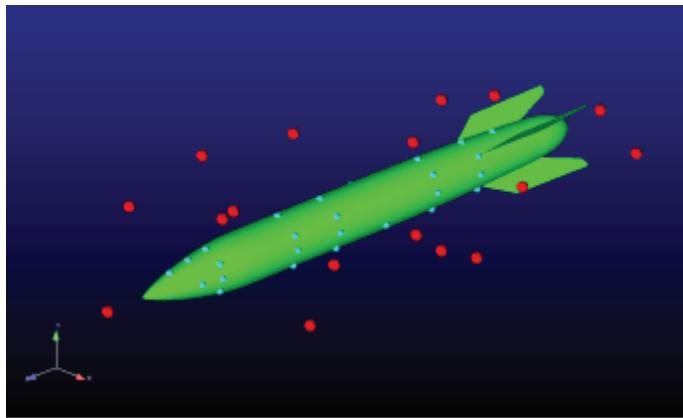


Inverted stresses

*Joint work with Joe Bishop (1554)

Source Inversion in Sierra-SD

- Goal: reconstruct acoustic field using inverse problem to obtain acoustic patch inputs that produce the given microphone measurements

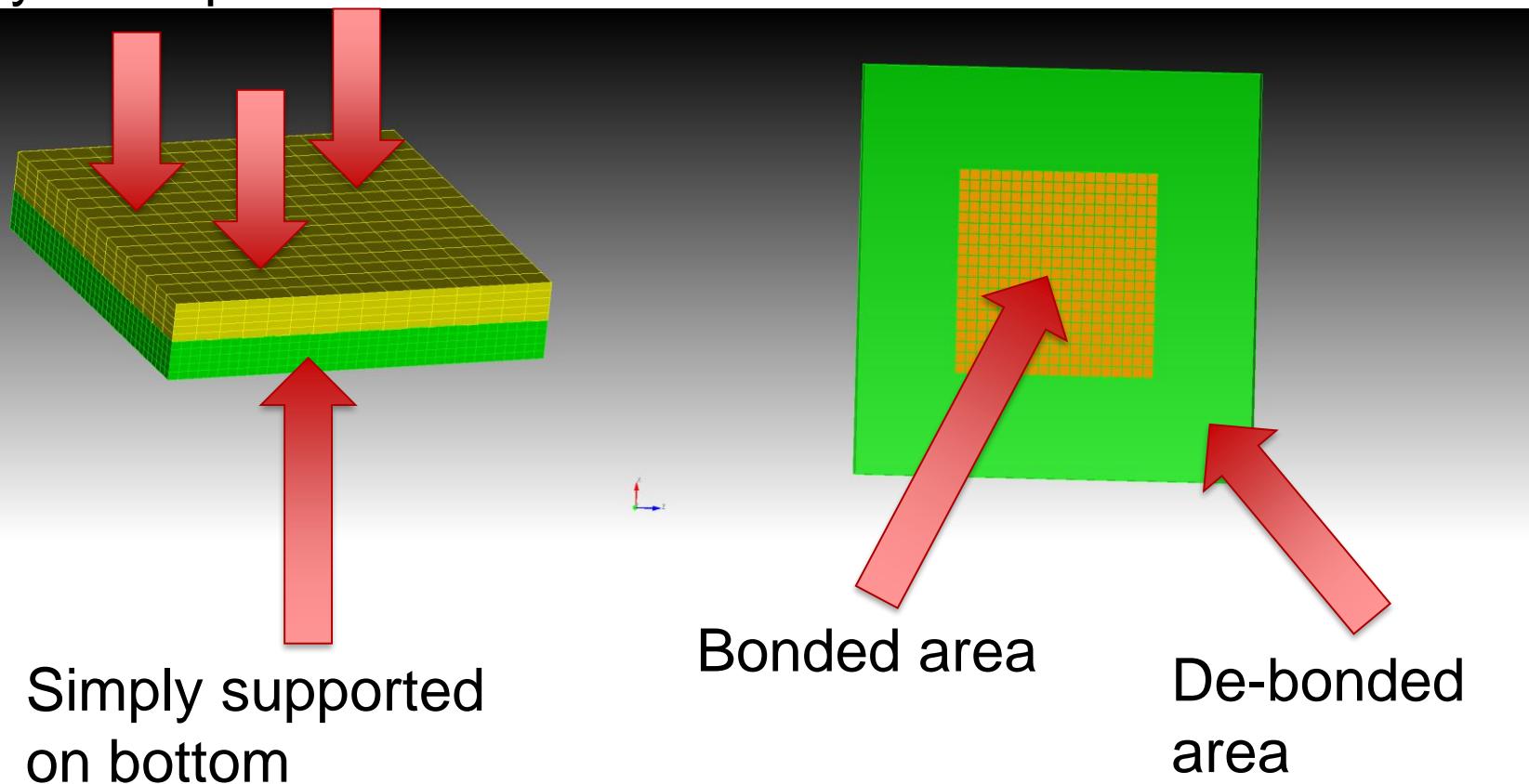


- Additional research on-going
- How to regularize the inverse problem – gradient regularization (penalize jumps across neighboring patches)
- How to place microphones

Delamination Detection

Partially-bonded plates – can we invert for the bonded/debonded regions?

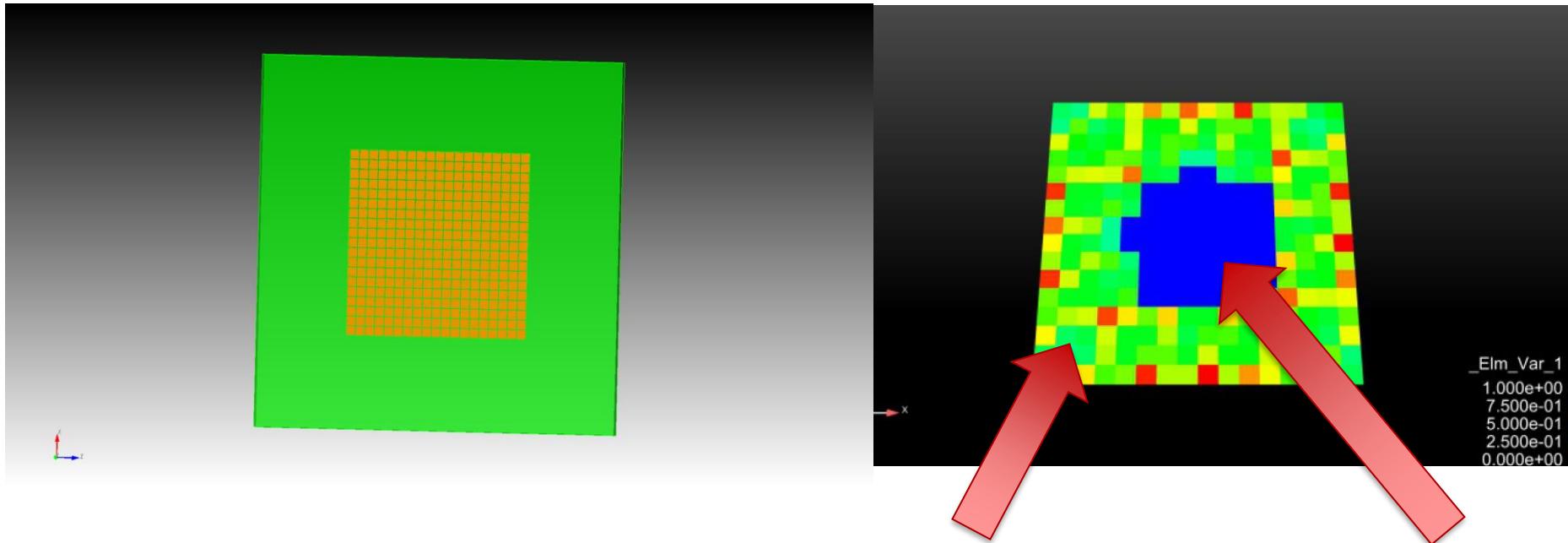
Steady-state pressure load at 2000Hz



Delamination Example

Partially-bonded plates – can we invert for the bonded/debonded regions?

Exact bonded/de-bonded areas



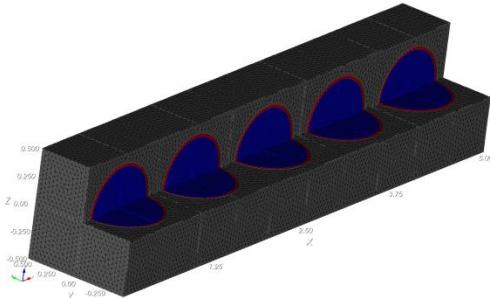
Initial guess for optimization:
Completely de-bonded

Bonded area
(penalty=1)

De-bonded
Area (penalty=0)

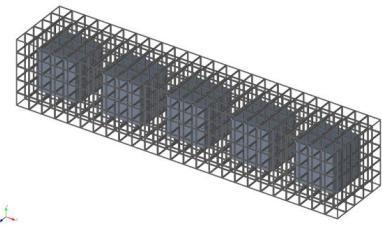
A Revolution in Acoustic Metamaterials

Breakthrough technology could allow us to **mitigate harsh vibration environments**

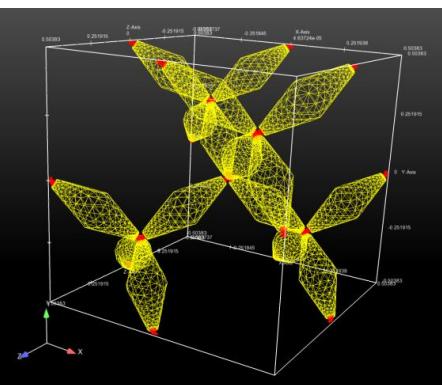


Multiphase composite

Transformative technology

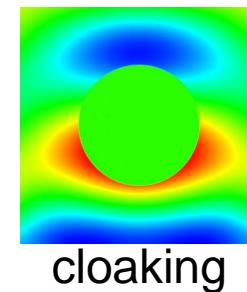
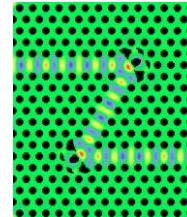


Lattice with embedded masses

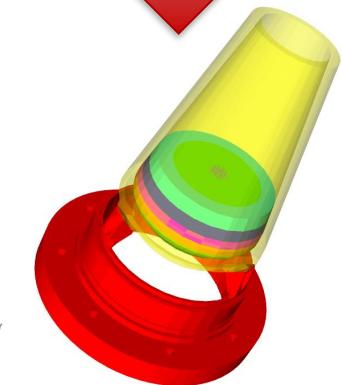


Pentamode lattice

Re-directed load paths



cloaking



Vibration isolation

Vibration Control with Acoustic Metamaterials

Problem: Harsh vibration environments pose serious threat to sensitive components in structural systems.

Challenge:

- Existing damping materials only provide limited vibration protection and cannot re-direct energy

As a Result:

- Sensitive components are exposed to potentially damaging vibration profiles.
 - Electronics packages, accelerometers, etc

Proposal: Large-scale optimization and multi-material additive manufacturing to design **acoustic metamaterials** for vibration control.

What is an acoustic metamaterial

- Acoustic metamaterials: Multiphase composite materials designed to produce dynamic material properties not found in individual materials themselves
 - Negative moduli, negative density, negative refractive index, imaginary speed of sound!!! (not possible in traditional composite materials)
- First demonstrated in 2000 by Liu et al, *Science*

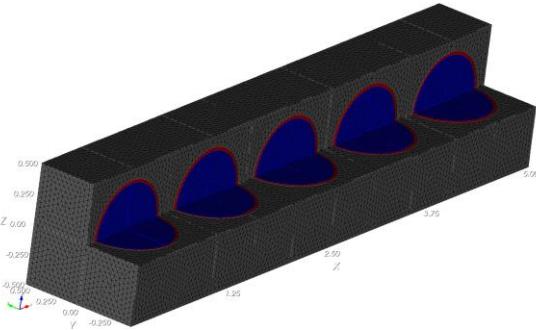
Mechanical Filter Design

Uniqueness of metamaterials – allow for frequency-selective designs!

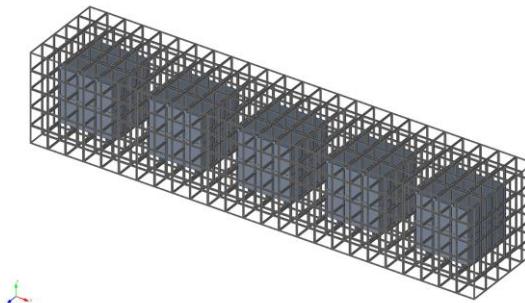
- Broadband – goal is to eliminate vibration in wide (or entire) frequency band - foam pads work well for this
- Band-stop – negative stiffness or negative density
- Band-pass – negative stiffness AND negative density (see paper on sharepoint)
- Notch – only filter at one particular frequency

Candidate Metamaterial Designs

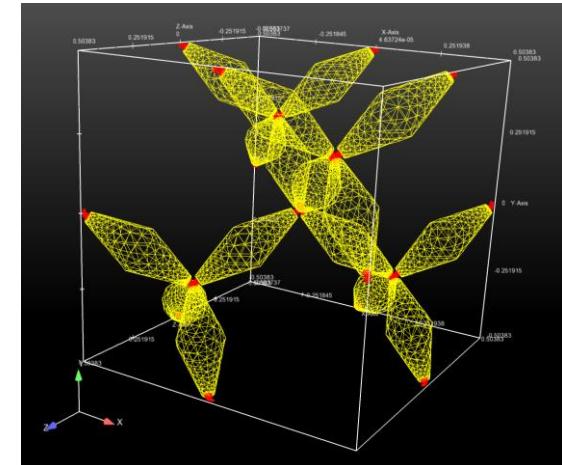
Finite element models have been developed and analysis is on-going for 3 classes of metamaterial designs. In all cases, local resonating elements are embedded in a host material.



Multiphase composite



Lattice with
embedded
masses



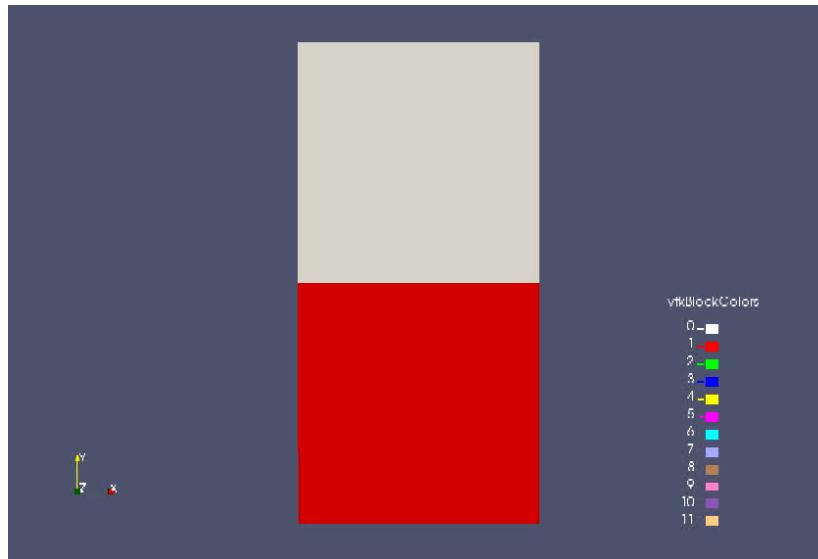
Pentamode lattice

Inverse homogenization – to achieve spatially distributed material properties

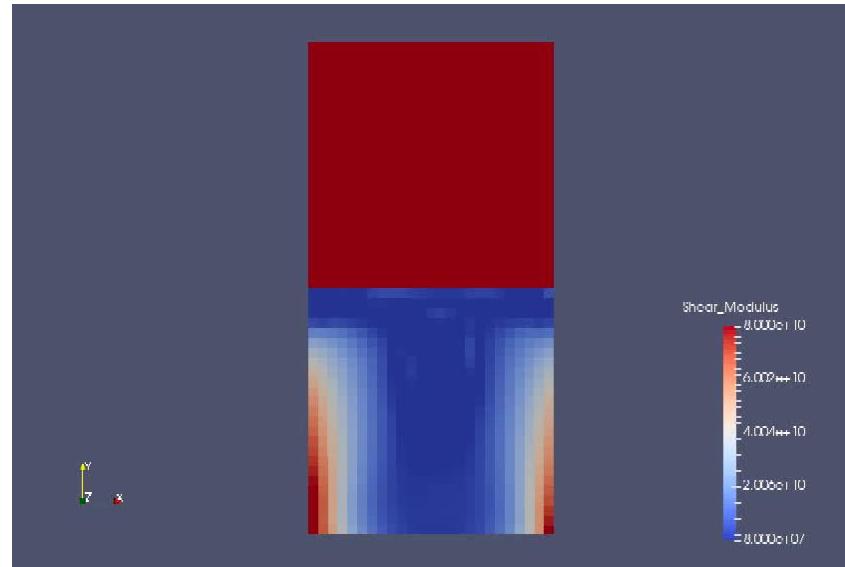
Transient Shock Isolation

Goal: design the bottom material such that the top block does not move

Initial guess



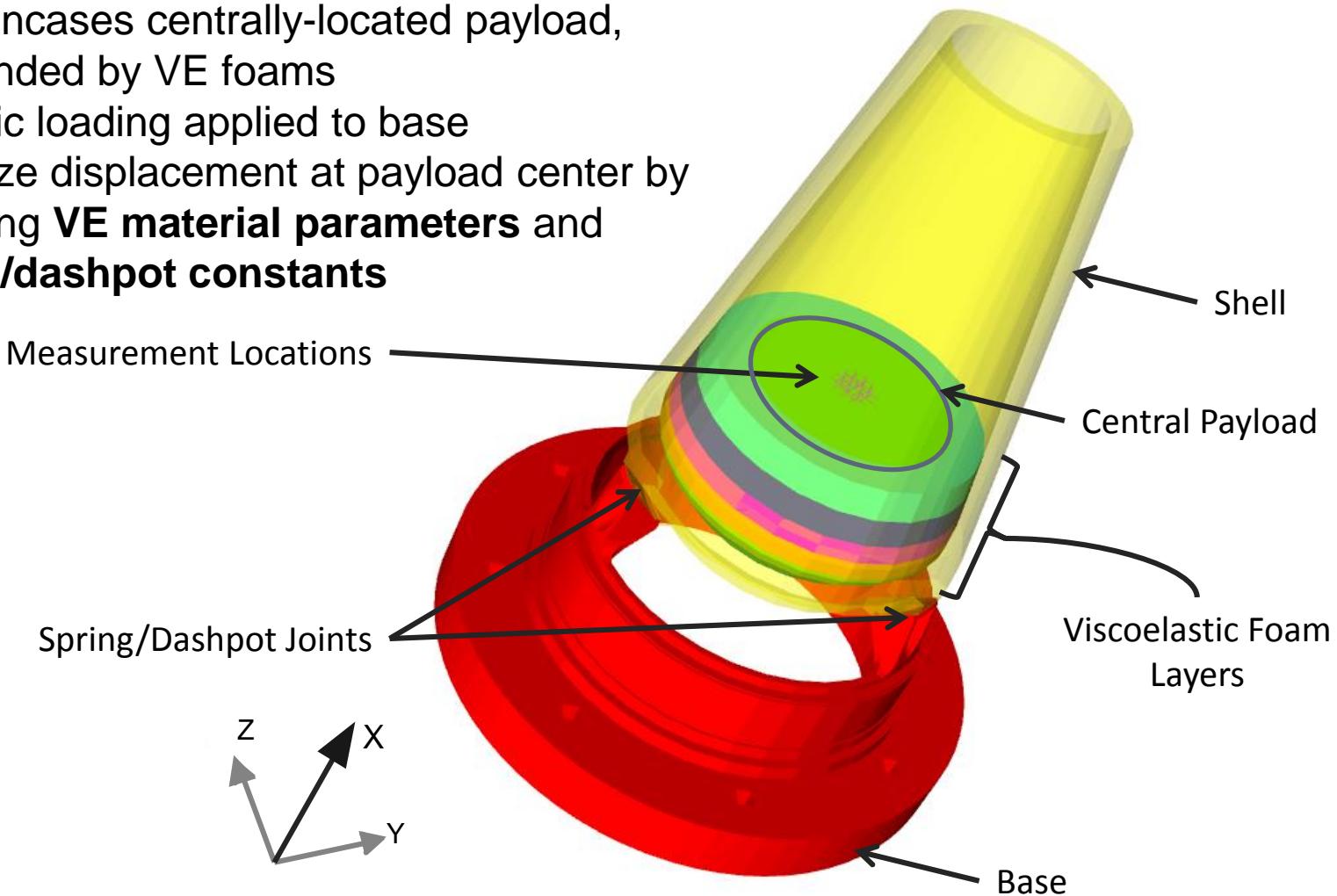
optimized



Top block: steel
Bottom block: single phase fixed, two-phase,
multi-phase

Inverse Problems: Mechanical Vibration Reduction

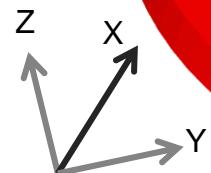
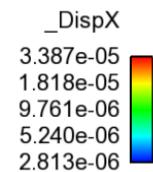
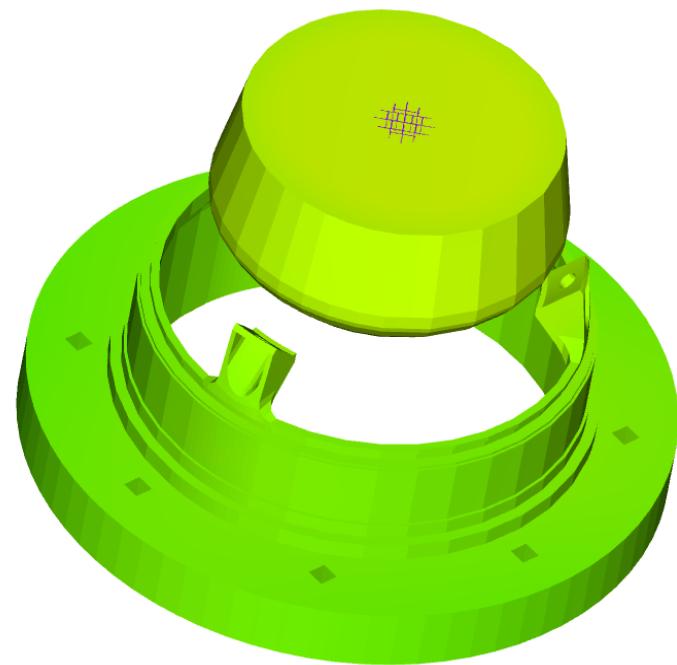
- Shell encases centrally-located payload, surrounded by VE foams
- Periodic loading applied to base
- Minimize displacement at payload center by adjusting **VE material parameters** and **spring/dashpot constants**



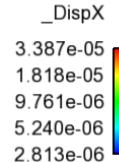
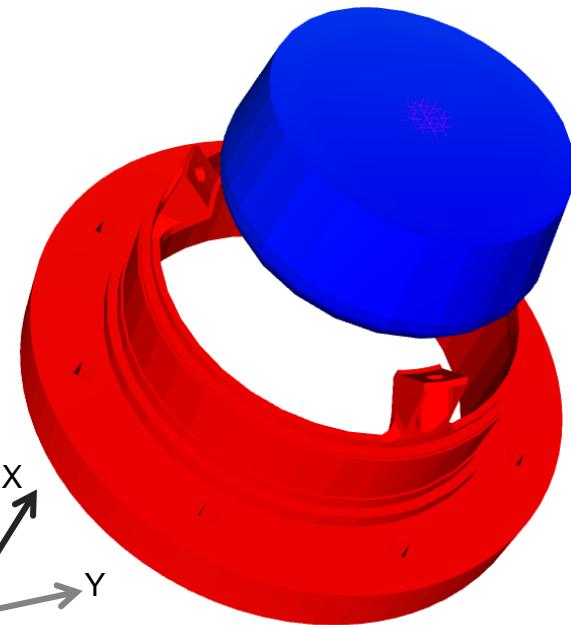
Case Study 1: Mechanical Vibration Reduction

- Displacement at measurement locations minimized (dependent on frequency)

Initial Guess



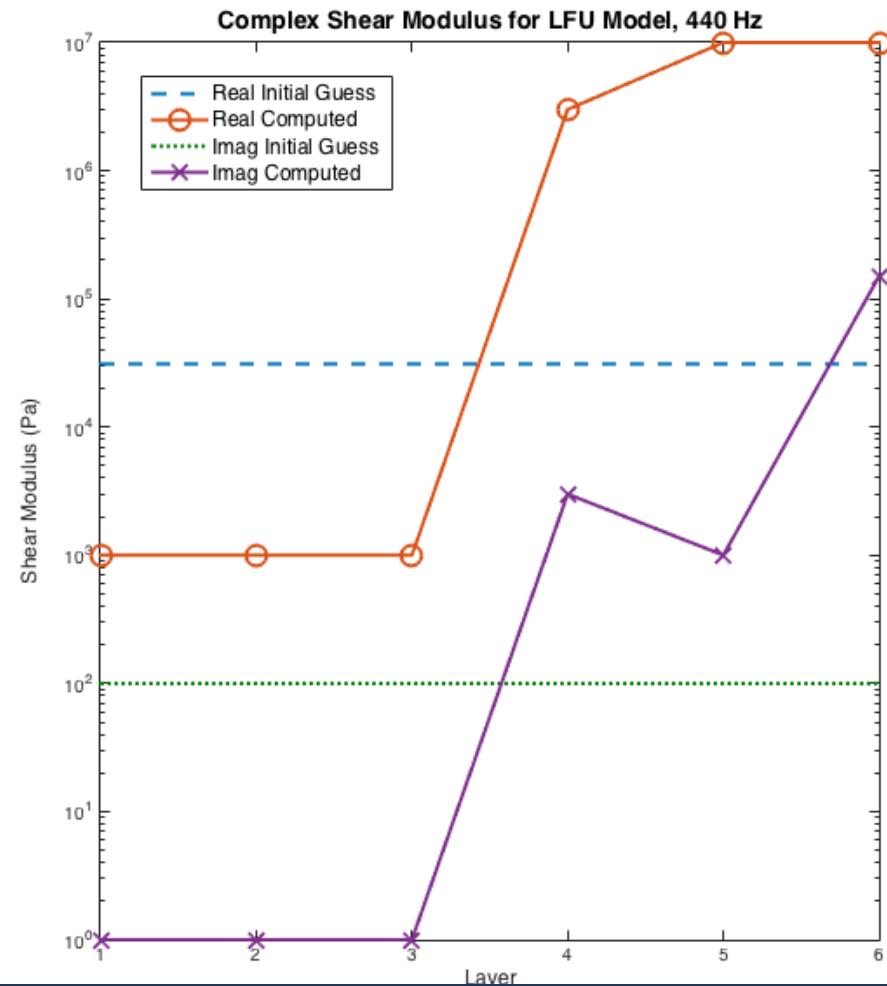
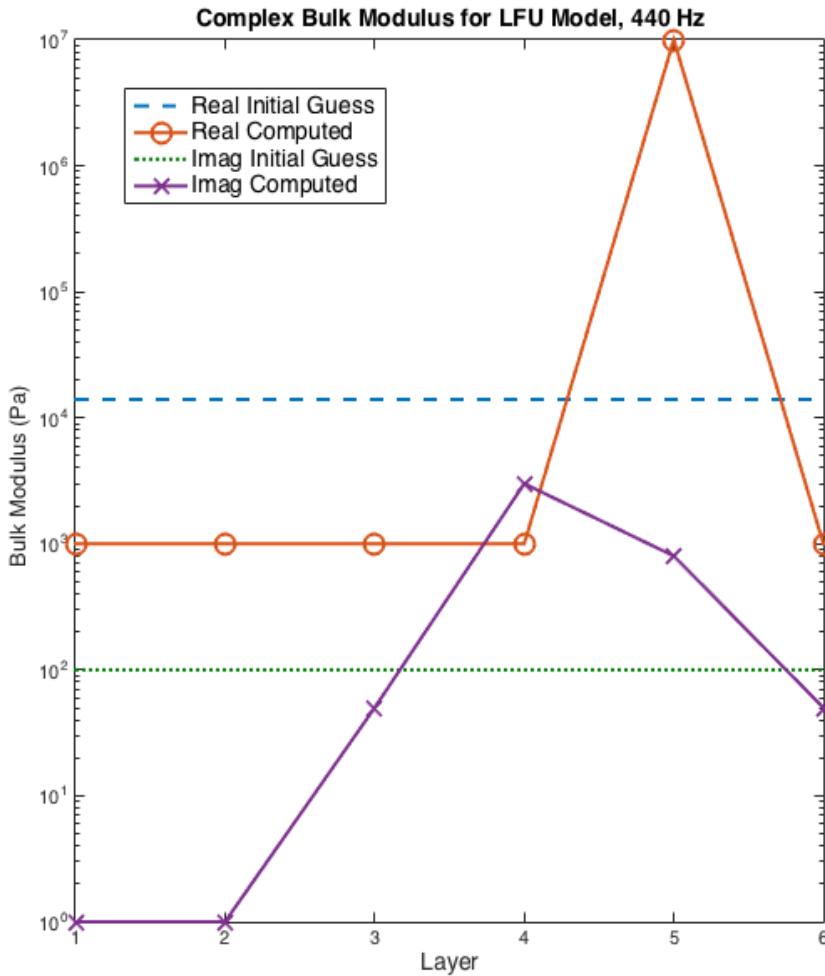
Optimized



Left: X-displacement in base and payload with initial material guesses, 440 Hz loading;

Right: X-displacement in design

Case Study 1: Mechanical Vibration Reduction

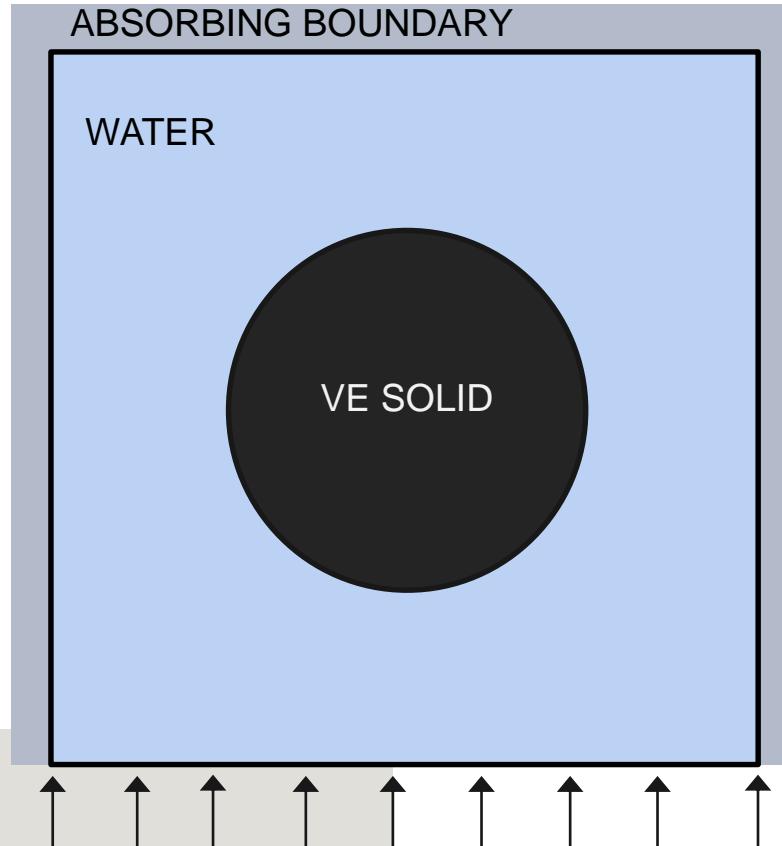


OBSERVATIONS:

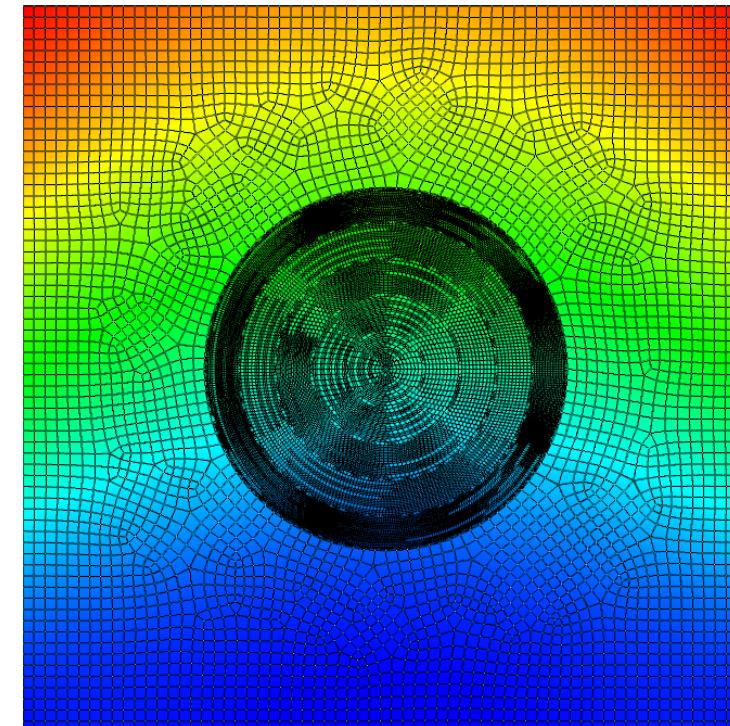
- Elastic Properties: Soft materials selected towards top, stiffer materials near base
- Viscous Properties: Damping is added towards base of viscoelastic region

Inverse Problems: Acoustic Cloaking

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**



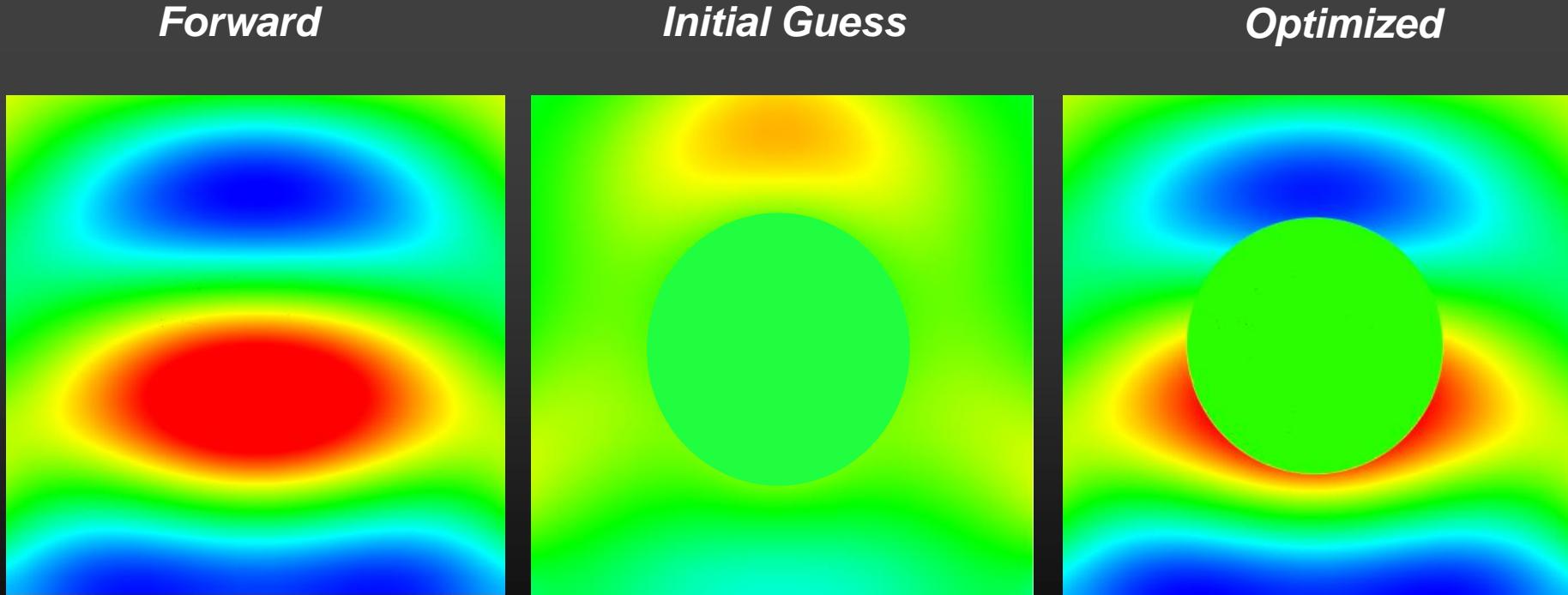
Left: Model Set up



Right: Forward problem pressure distribution (500 Hz loading) in model with 50 layers

Acoustic Cloaking

- Optimized VE foams allow recovery of desired pressure distribution



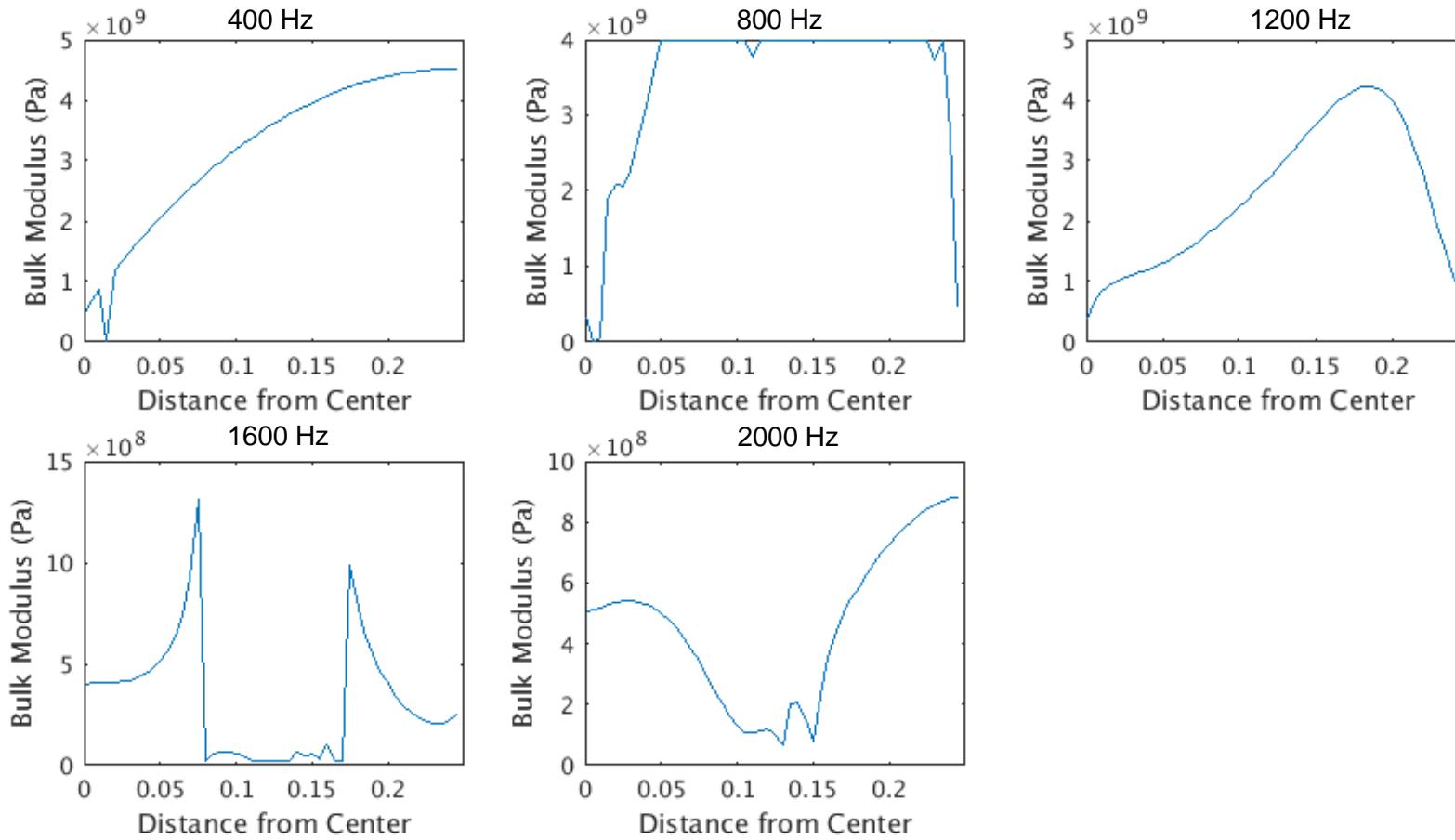
Left: Target acoustic pressure distribution, from forward problem

Center: Acoustic pressure distribution with initial material guess (2000 Hz Loading)

Right: Pressure distribution after convergence to optimized design

Acoustic Cloaking Results: Bulk Modulus

Bulk modulus sensitive to frequency, and varies nontrivially along disk radius



Figures: Real component of bulk modulus along radius, for various frequency