

Verification of Coupled Codes (Fluid/Structure and Thermal/Mechanical)

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Outline

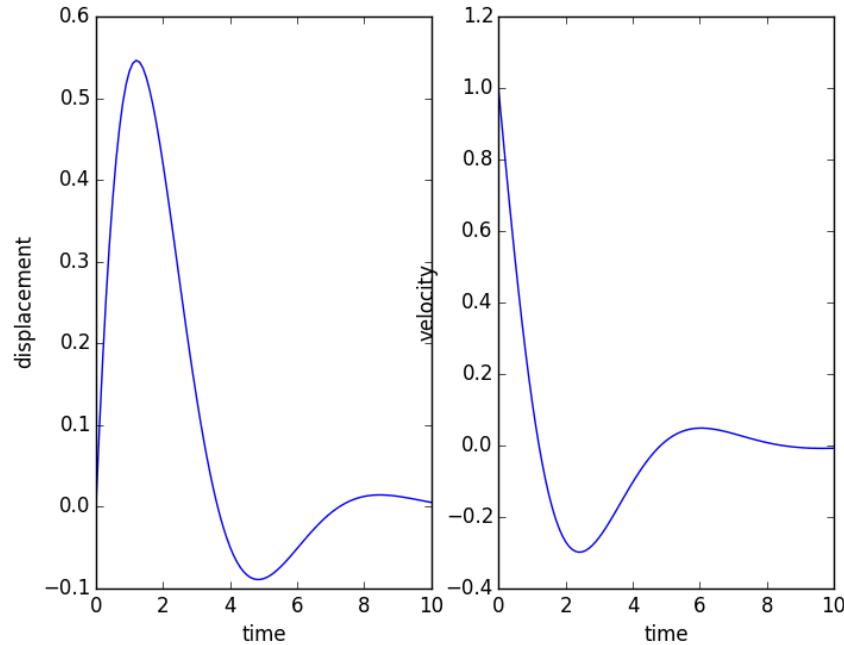
- Motivation
- First Attempt at Piston and Questions
- What Finally Worked
- Extensions to Panel Flutter
- Lessons Learned

Motivation

- We develop multiple single physics codes (Sierra Mechanics)
 - thermal, mechanical, structural, aerodynamics
- Each code has extensive verification and regression test suites
- Coupled code verification is much more limited and difficult
- Some common couplings we consider
 - thermal/mechanical
 - fluid/structural or fluid/thermal
 - mechanical/structural (pre-load)
- Verification of two-way coupling is required to demonstrate accuracy of a coupling scheme:
 - Example: CSS (first order) and GSS (second order) for fluid/structure

First Attempt at Piston

- Problem is a spring-mass piston attached to a semi-infinite fluid column
- Easy to derive the damped piston solution
- These questions arose:
 - What happens in the fluid?
 - How to keep the problem linear?
 - Can the codes actually solve the problem?
 - Is the problem going to show second order convergence?
- What isn't in the test
 - Complex interfaces – only a single face
 - Non-matching meshed interfaces



$$m \ddot{x} + k x = F_s(t)$$



The Fluid Part of the Solution

- The surface **velocity** is input from the piston to the fluid
- The 1D linearized version of the Euler equations: acoustic wave propagation
- At the surface fluid velocity and pressure are proportional
- The solution for semi-infinite domain is waves propagating rightward from the piston surface ($x=0$)
 - can solve completely using characteristics
- The resulting **pressure** provides the boundary condition back to the piston

$$\begin{aligned}\rho \dot{v} + v_x &= 0 \\ \dot{p} + \rho c^2 v_x &= 0\end{aligned}$$

$$p(0, t) = \rho c v(0, t), \quad t \geq 0$$

$$\begin{aligned}v(x, t) &= \begin{cases} v_p(0, t - x/c), & x/c < t, \\ v_0(x), & x/c \geq t \end{cases} \\ p(x, t) &= \begin{cases} \rho c v_p(0, t - x/c), & x/c < t, \\ p_0(x), & x/c \geq t \end{cases}\end{aligned}$$

The Coupled Solution

- The damping force is proportional to piston velocity $m \ddot{x} + k x = F_s(t)$

$$F_s = -p A = -\rho c A v_p = -\rho c A \dot{x} = -c_d \dot{x}$$

- Initial conditions:
 - fluid at rest (zero velocity)
 - piston has zero displacement and nonzero velocity

$$x(t) = (v_0/\omega_d) \exp(-\xi\omega_0 t) \sin(\omega_d t)$$

$$\omega_0 = \sqrt{k/m}, \quad \xi = c/(2m\omega_0), \quad \omega_d = \sqrt{(1 - \xi^2)}\omega_0$$

- The piston will damp to zero for large enough damping coefficient
- The fluid transports whatever data is at the piston boundary at the sound speed
 - there is no smoothing of this data

Nonlinearities Are Avoided to Emphasize Coupling Aspect

- The fluid code actually solves the full Euler equations
 - Small pressure/velocity perturbations in the fluid will reduce the Euler equations to acoustics
- The fluid solver handles finite mesh motion
 - We restrict to small displacements at the piston boundary
- We did not address a fully nonlinear piston but this could be done with MMS
- Similar issues arose with thermal/mechanical verification
 - Solid mechanics code is Lagrangian with finite deformations

Next 3 slides: aside on thermal/mechanical coupled code verification

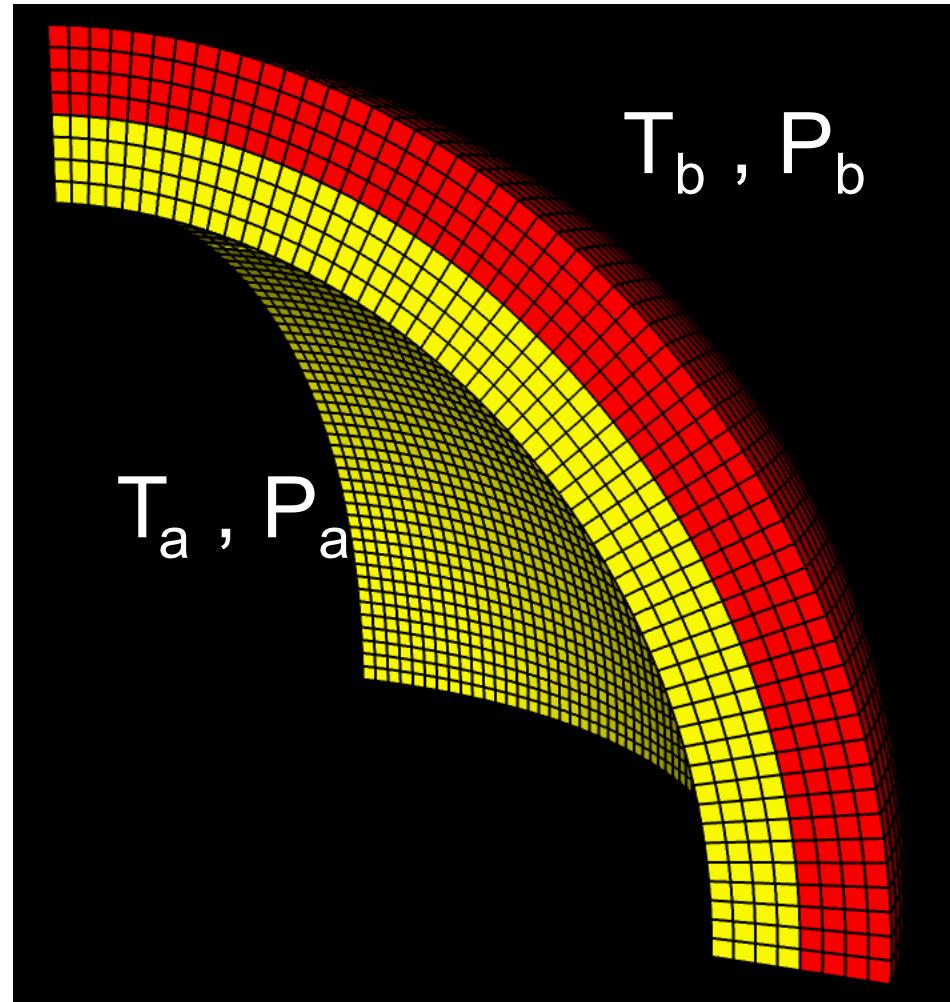
Thermal-Mechanical Coupling: A Pressurized Sphere

Features tested

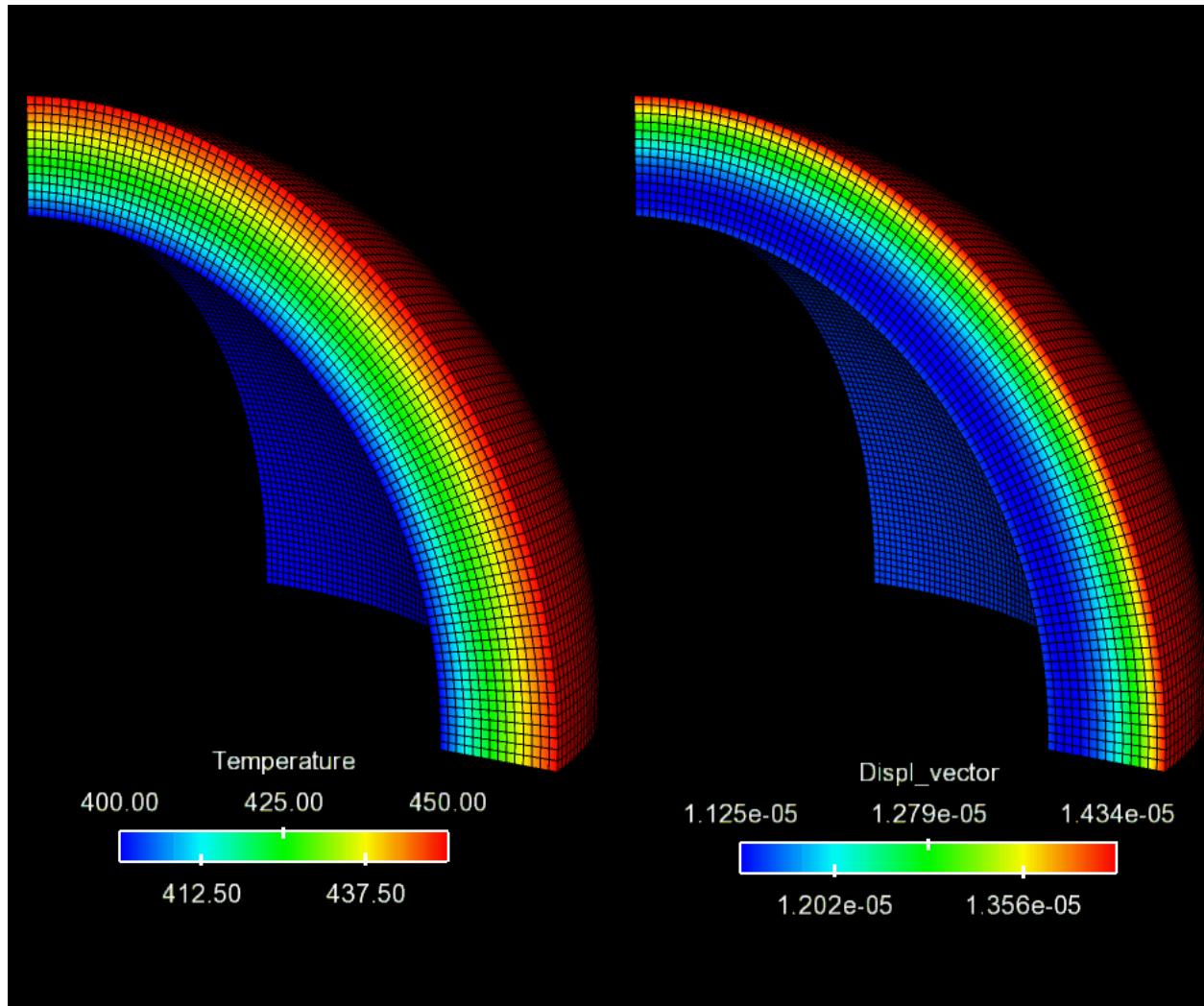
- Thermal expansion
- Pressurization
- Contact

Exact solution assumes linear mechanics

- Derivation of the exact solution is fairly simple
- Makes use of radial symmetry and linearity

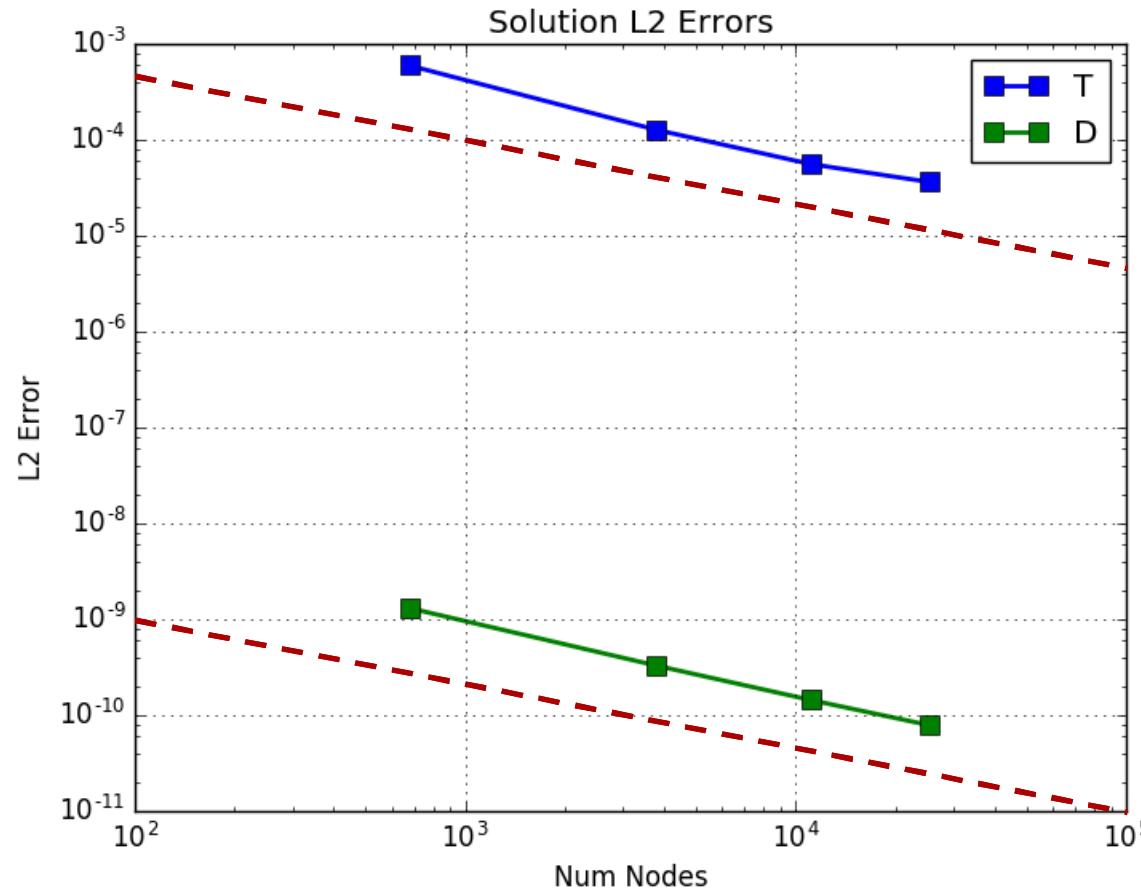


Solution Maintains Small Strains and Displacements



Coupled Solution Demonstrates Second Order Accuracy

For both temperatures (T) and displacement (D)



Back to FSI: Can the Codes Solve the Verification Problem?

- The updated exact solution requires more from the structural solvers
 - Forcing function from the piston
 - Non-zero initial conditions (displacement, velocity)
- The time integrator (Newmark beta) could not handle nonzero initial displacements
 - Accuracy was reduced to first order
- Forcing functions could be specified
- We updated further the exact solution
 - Zero initial conditions for the piston
 - Now the solution does not damp out to zero, but the fluid solution is smoother
- Our verification approach: divide and conquer:
 - first develop single-physics tests to resolve errors
 - then test the coupled code version to confirm accuracy

Final Form of the Solution

- The piston can propagate a discontinuity into the fluid
- Convergence less than first order
- We needed a problem with a smoother solution in the fluid
- Solution: zero initial conditions for piston with a source term
- Also zero initial acceleration!

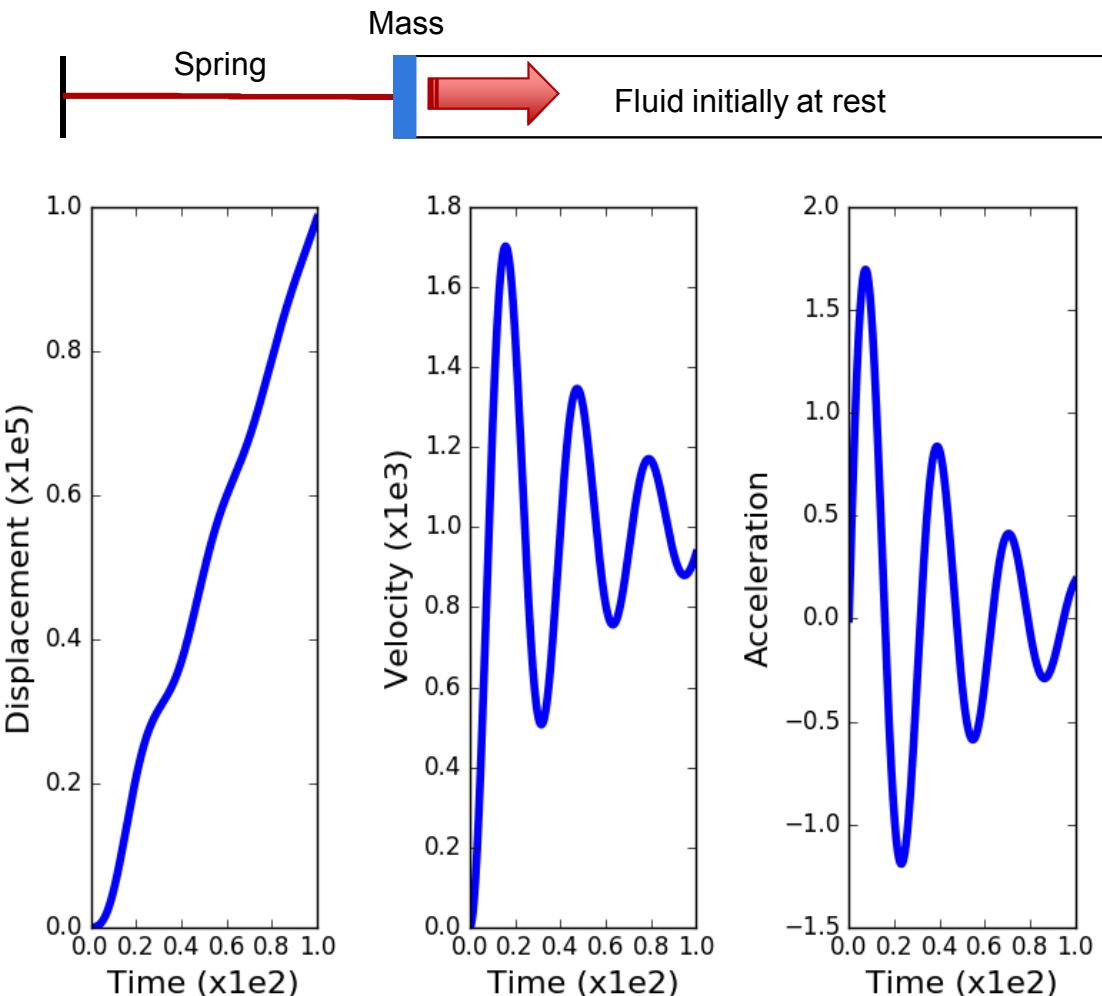
$$d = \frac{c}{2m}, \omega = \sqrt{\frac{k}{m} - d^2}$$

$$\beta = \frac{c}{k}, a = \bar{v}\beta, b = \frac{(ad - \bar{v})}{\beta}$$

$$ma + cv + ku = \bar{v}kt$$

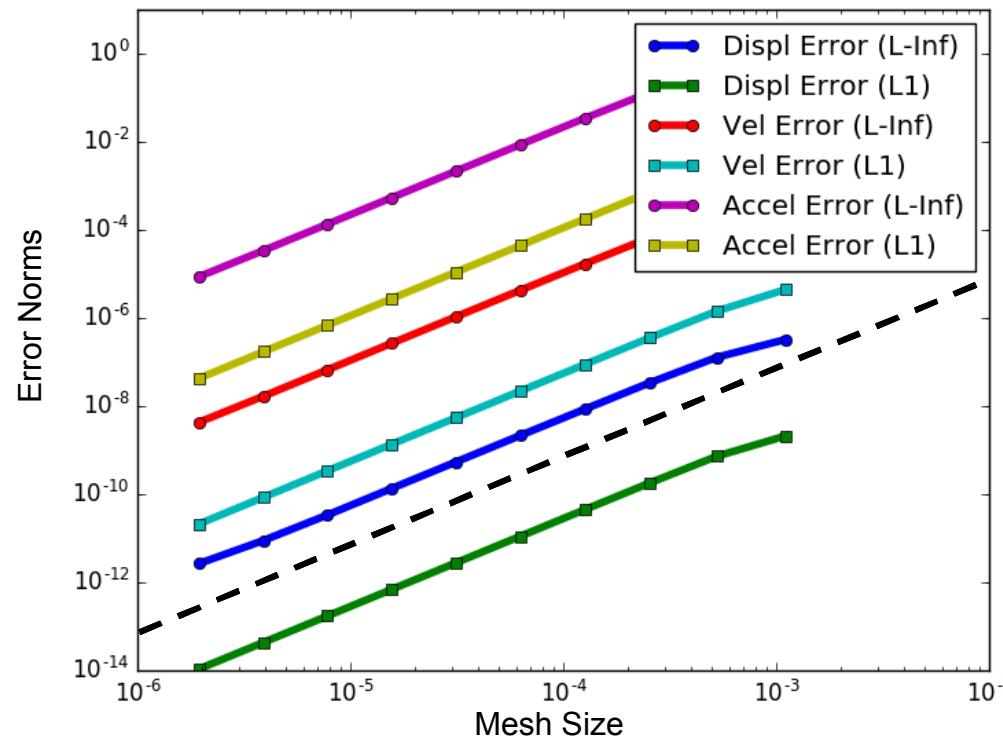
$$v(0) = u(0) = 0$$

$$u(t) = e^{-dt} (a \cos(\omega t) + b \sin(\omega t)) + \bar{v}(t - \beta)$$



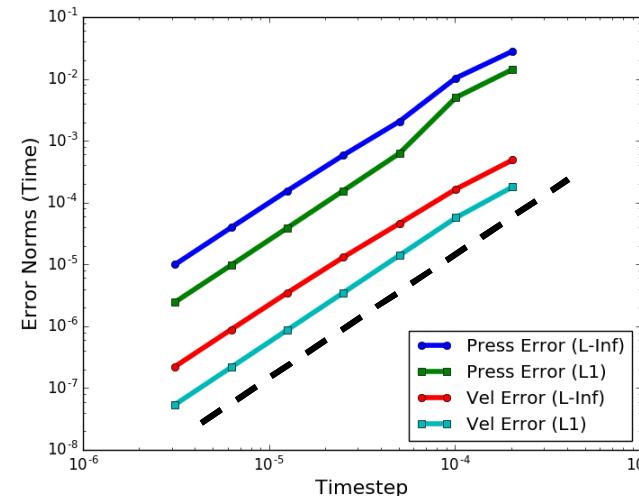
Structural Only is Second Order

- One shell element connected to one spring for each node
- Damping applied based on constant damping coefficient
- Body force using time dependent load function on the shell
- Newmark time integrator
- All variables converge second order

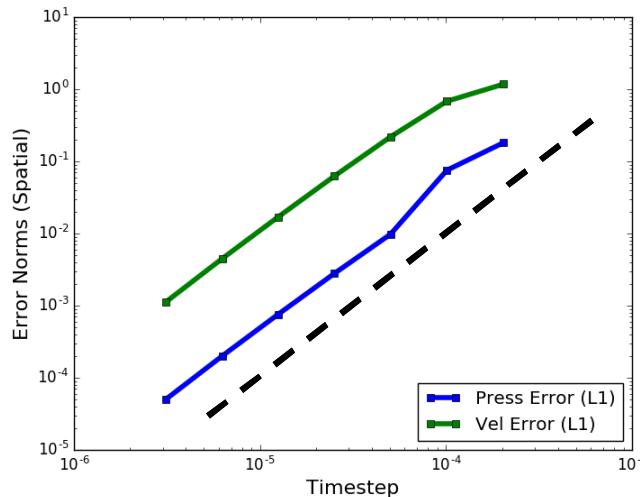


Fluids Only is Second Order

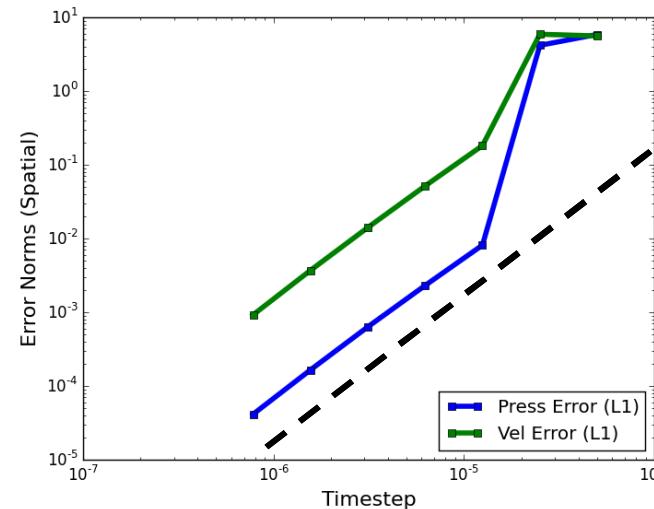
- Piston solution applied as displacement BC on fluid
 - Time integrators: BDF2 and AB2
 - Meshes with 50-3200 elements
- Verify fluid pressure, velocity, and displacement at piston surface
- Verify fluid pressure/velocity using spatial norms



Temporal error at piston surface (BDF2)



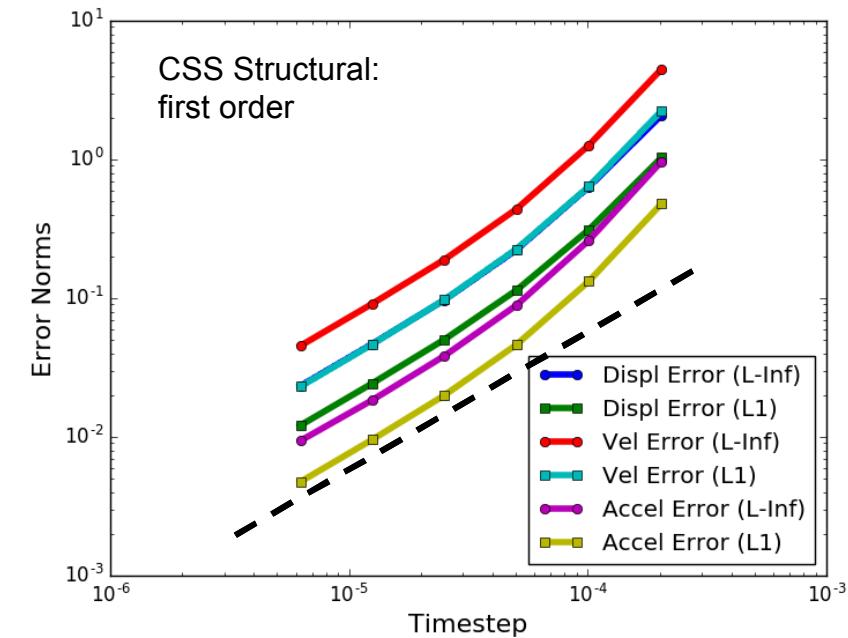
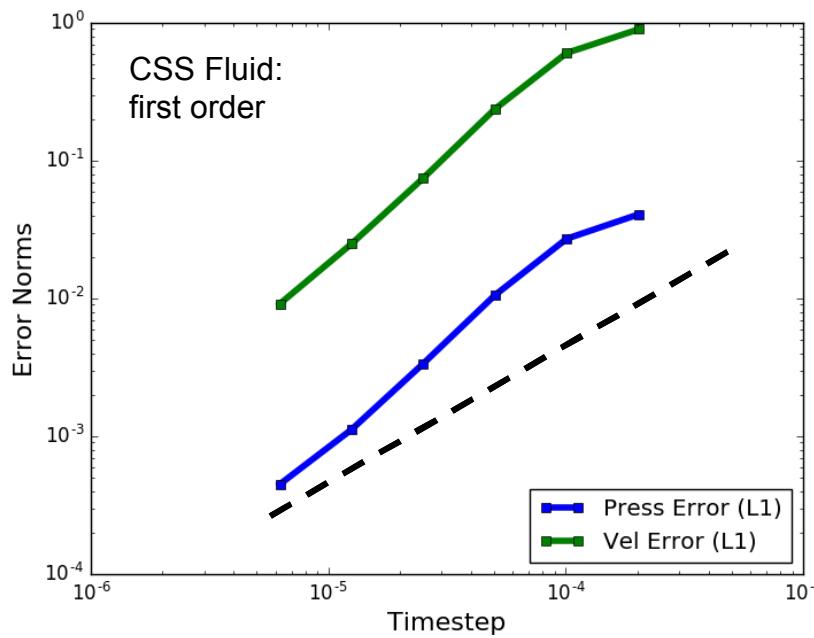
L1 spatial error (BDF2)



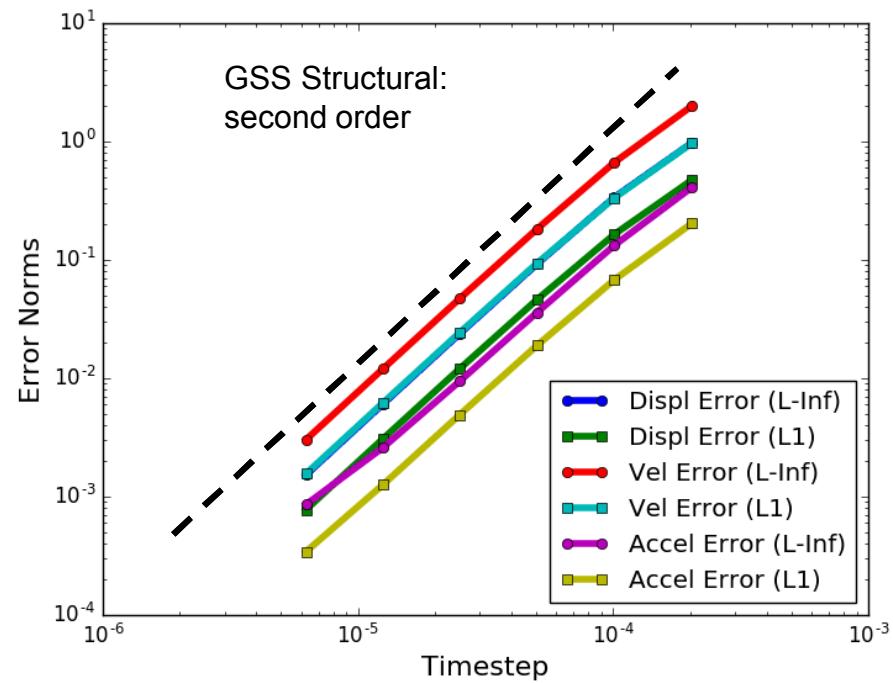
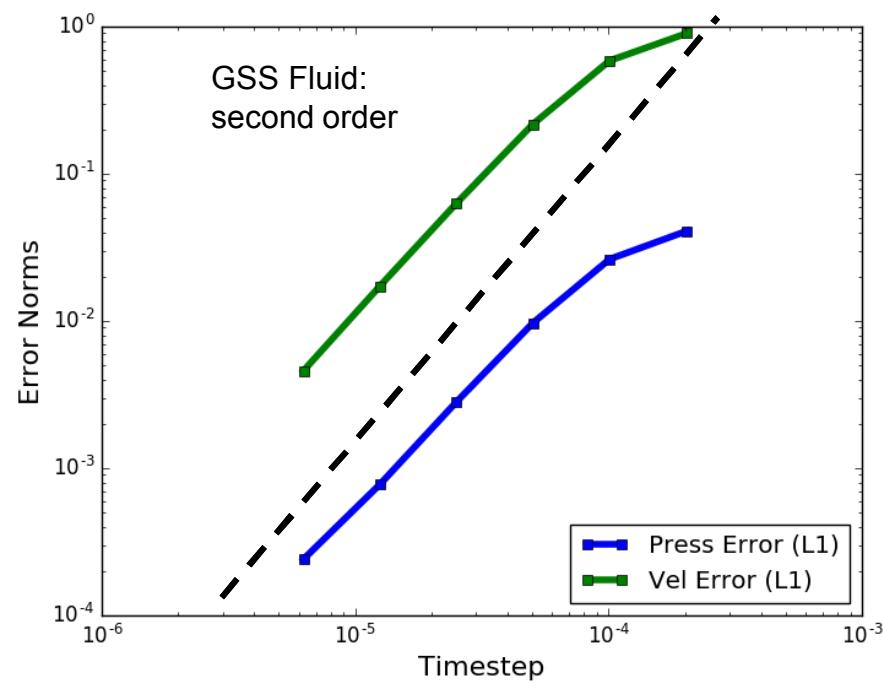
L1 spatial error (AB2)

Coupled Convergence: CSS Scheme is First Order

- Test now exercises full two-way coupling
- Verified second order uncoupled for each physics
- We seek to verify coupling accuracy for two schemes (CSS and GSS)
 - CSS should be first order
 - GSS should be second order

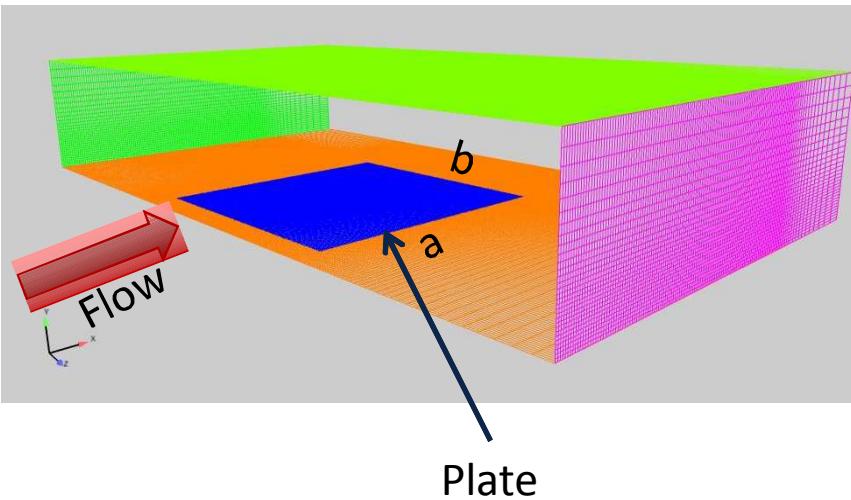


Coupled Convergence: GSS Scheme is Second Order

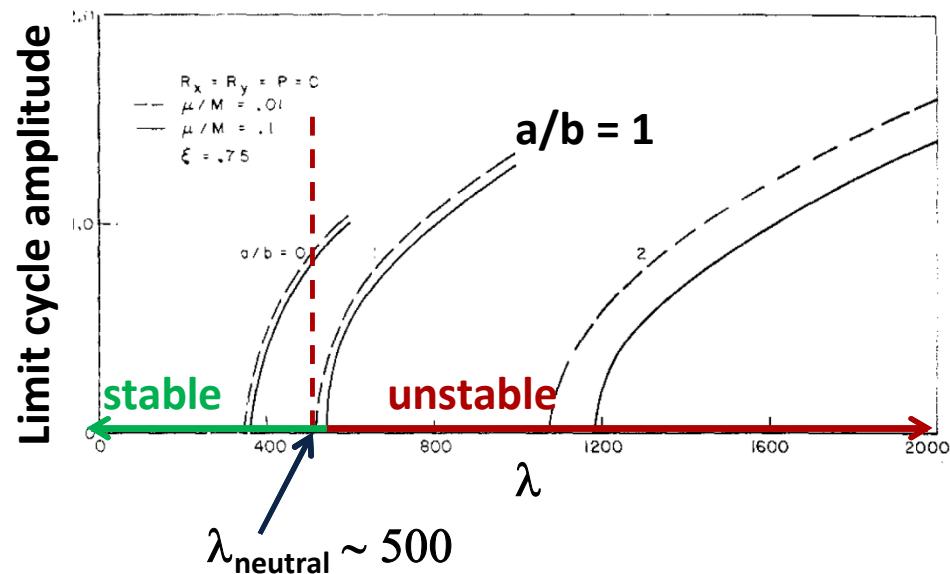


3D Panel Flutter: More Challenging Problem (No Exact Solution)

- 1 meter square thin plate, thickness = 4.6 mm, simply supported edges
- Plate given a sinusoidal initial velocity with maximum magnitude of 0.01 m/s
- Mach number = 2.0, inviscid flow
- Aero-elastic parameter λ ~ ratio of fluid dynamic pressure to plate stiffness

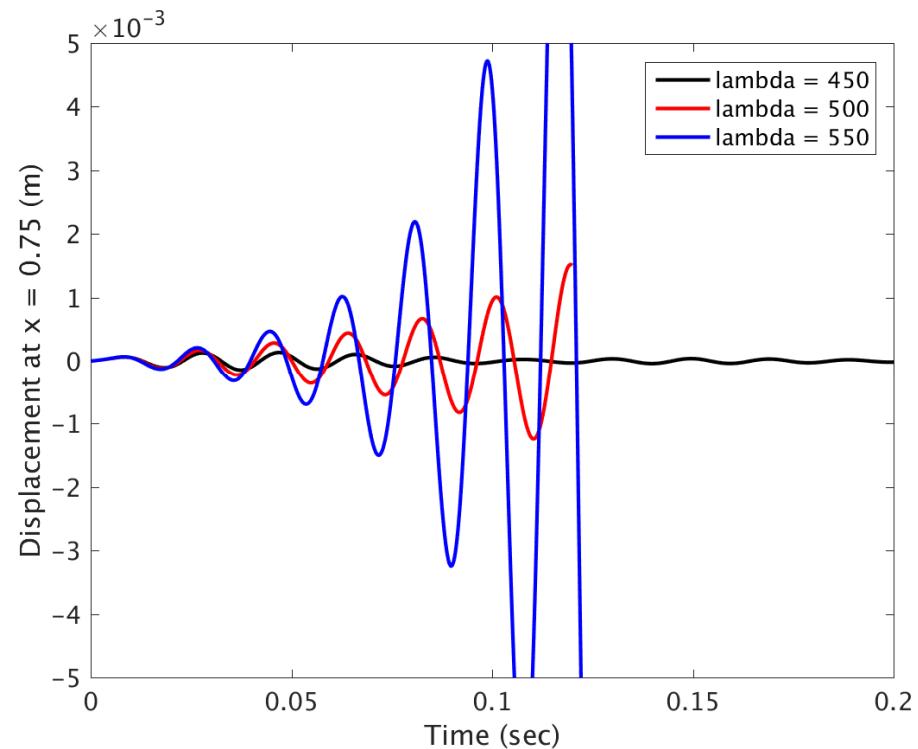
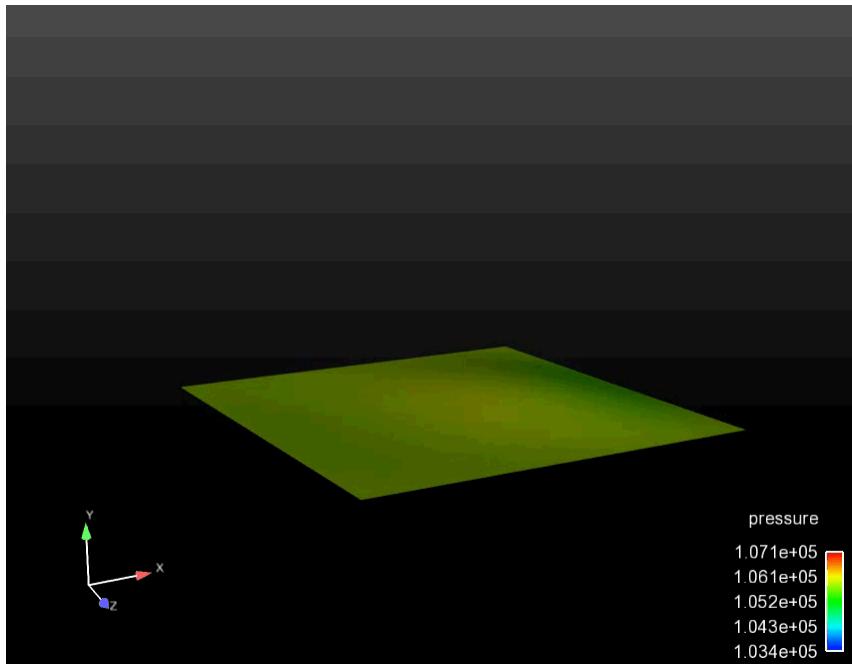


Dowell, *Nonlinear Oscillations of a Fluttering Plate*



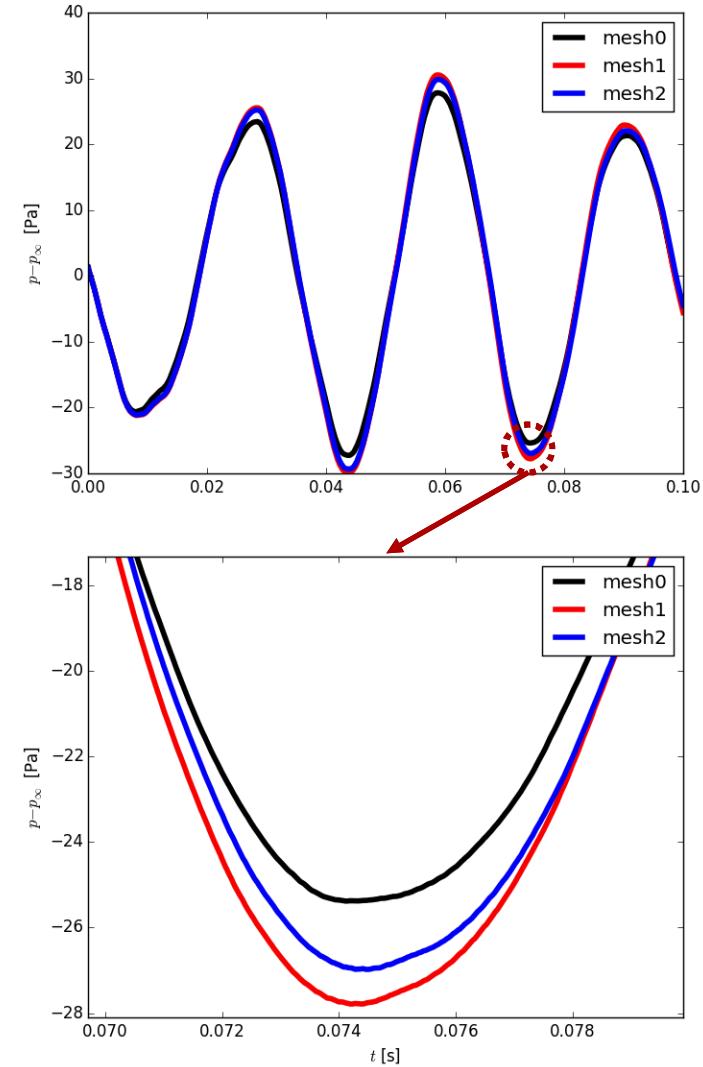
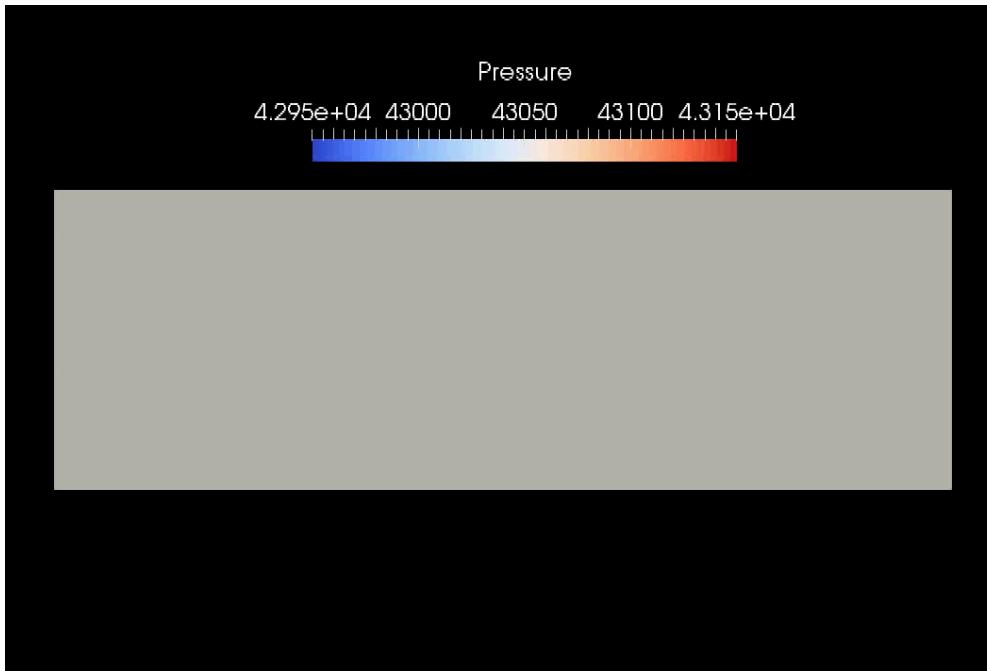
3D Panel Flutter: Stability Limit Consistent With Literature

$$\lambda = 500$$



2D Panel Flutter: Did Not Observe Convergence With Three Meshes

$\lambda = 203$, GSS



Pressure, $x = 0.75$ m

Lessons Learned

- Verification of simple coupled problems (piston) can provide large impact to complex models
 - Panel flutter
 - DNS flow over plate
- Solution nonlinearities may need to be minimized to emphasize code coupling aspects
- The codes need to be able to satisfy all the constraints of the verification problem
- Decoupled versions of the problem are very useful
- Simplified verification problems can result in improvements in solving larger scale, nonlinear coupled problems