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A NEW METHOD TO COMPUTE THE RADIANT CORRECTION OF BARE-WIRE THERMOCOUPLES

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Abstract
The proper consideration of the radiation correction for bare-wire thermocouple measurements requires consideration of the convective and radiative heat transfer of the thermocouple with its surroundings, as well as conductive heat transfer between the thermocouple bead and the connecting thermocouple wires. This has rarely been considered in the past, and to do so has involved complex simulation of the complete thermocouple energy balance. This paper discusses a new, easy-to-implement approach for calculating the radiant correction for thermocouples that considers the full complexity of the thermocouple energy balance. Examples of the radiation correction computed with this new method as a function of temperature and thermocouple bead and wire size are given, and are compared with traditional simplified estimates of the radiant correction that only consider heat transfer around either the thermocouple wires or an idealized thermocouple bead. Under most investigated conditions, the radiant correction is shown to be substantially larger than that given by an energy balance that ignores conduction from the thermocouple bead to its connecting wires. Further, the analysis of thermal conduction to the connecting thermocouple wires shows that the measured temperature is sensitive to variations in gas temperature over a spatial dimension that is much larger than the thermocouple bead.

Introduction
Knowledge of temperature in chemically reacting flows is essential for accurate understanding of chemical reaction rates and heat transfer. Similarly, predictions of chemical species production and destruction rates and heat transfer can only be meaningfully compared to experiments if the actual temperature is known within reasonable accuracy. As an indicator of the importance of knowing temperature in chemically reacting flows, a large number of laser diagnostic techniques have been developed for measuring temperature. Many of these techniques offer outstanding temporal and/or spatial response characteristics, while avoiding physical perturbation of the flow. However, these techniques generally require high-cost laser diagnostic systems and detailed knowledge of the associated diagnostic technique in order to apply them accurately, as well as suitable optical access to the process of interest. Consequently, thermocouples continue to be widely used in experimental combustion research as well as in industrial combustion applications, such as furnaces and boilers. In addition, calibration and validation of these laser-based temperature measurements frequently relies on comparison against thermocouple measurements [1], further bolstering the need for accurate correction of thermocouple measurements in combustion environments.

At high temperatures (greater than approximately 800 K), the readout from a thermocouple may underestimate the actual gas temperature because of radiant loss of energy from the thermocouple to cooler radiant surroundings. To estimate the radiant correction to the measured thermocouple temperature for bare-wire thermocouples, a simple radiation-convection energy balance, written around the thermocouple bead, is typically invoked, as shown in Eq. 1:

$$h(T_g - T_{tc}) = \varepsilon_{tc}\sigma(T_{tc}^4 - T_w^4) \quad (1)$$

where *h* is the convective heat transfer coefficient, the subscript *g* refers to gas, *tc* refers to the thermocouple (specifically the thermocouple bead), and *w* refers to the characteristic radiant surroundings, which are often dominated by a wall. From Eq. 1, one can easily solve for the radiation correction, as shown in Eq. 2:

$$T_g = T_{tc} + \varepsilon_{tc}\sigma(T_{tc}^4 - T_w^4)\frac{d}{kNu} \quad (2)$$

where the convective coefficient has been replaced by the definition of the Nusselt number:

$$Nu \equiv {}^{hd}/_{k} \quad (3)$$

where k is the thermal conductivity of the gas. The literature has many correlations for Nusselt number as a function of Reynolds and Prandtl numbers for flow over various shaped objects.

There are innumerable examples of the application of Eq. 2 to correct thermocouple measurements in the combustion literature. Unfortunately, this approach is fundamentally incorrect for the typical case in which a thermocouple bead is larger in diameter than the attached wires, as it fails to account for the effect of rapid conductive heat transfer between the bead and the wires, combined with the different convective and radiative heat transfer over the bead and the wires. Rather, in nearly all cases of practical importance, the energy exchange between the bead and the attached wires is sufficiently fast that the thermocouple bead temperature is lower than expected, because the cylindrical thermocouple wires experience reduced convective heat transfer than the approximately round bead.

In the past, the only way to properly compute this complex conjugate heat transfer problem about the thermocouple bead was to perform a full finite difference calculation for heat transfer about the bead and the attached lead wires [2], which is cumbersome and therefore has rarely been employed. In this paper, a newly developed procedure for easily calculating the conjugate heat transfer of convection and radiation for a thermocouple bead connected to thermocouple lead wires is reported.

Method of Calculation

Figure 1 shows a backlit photograph of a typical commercially produced bare-wire thermocouple. The larger diameter of the bead, relative to the connecting wires, is clearly evident, as is the generally spherical shape of the bead.

Figure 1. Backlit photograph of a commercially produced type-R thermocouple with a wire diameter of 51 μm, as observed under a microscope.

Figure 2 shows an idealized configuration of a bare wire thermocouple. For steady conditions, relative to the timescales of heat transfer around the thermocouple, the energy balance about the thermocouple bead itself can be expressed as

$$h_{sph}\left(SA_{sph} - 2A_{cyl}\right)\left(T_g - T_{TC}\right) = \epsilon\sigma SA_{sph}(T_{TC}^4 - T_w^4) + 2A_{cyl}\gamma\frac{dT}{dx} \tag{4}$$

where *SA* refers to surface area, *A* is cross-sectional area, *dT/dx* is evaluated where the lead wires connect to the bead and is positive for heat flow away from the bead, and γ is the thermal conductivity of the thermocouple lead metals. Eq. 4 can be rearranged to yield the desired gas temperature value:

$$T_g = T_{TC} + \left(\epsilon\sigma SA_{sph}(T_{TC}^4 - T_w^4) + 2A_{cyl}\gamma\frac{dT}{dx}\right) \Big/ h_{sph}\left(SA_{sph} - 2A_{cyl}\right) \tag{5}$$

where the expression to the right of the thermocouple temperature represents the proper radiant correction. The principal challenge in applying Eq. 5 is to determine the conduction loss from the bead to the connecting lead wires.

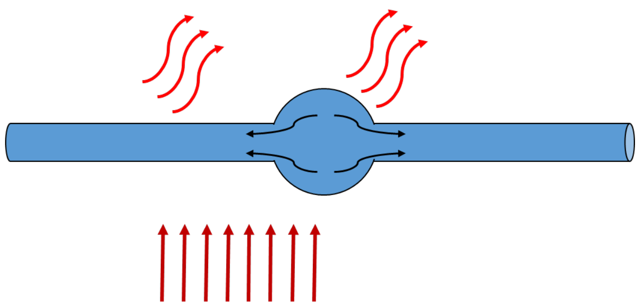

Figure 2. Schematic illustration of a bare wire thermocouple bead, the attached lead wires, and the dominant modes of heat transfer influencing the radiant heat transfer correction. The straight red lines indicate convective heat transfer from the gas to the wire and bead; the wavy red lines indicate radiant heat transfer with the surroundings; and the black lines indicate the thermal conductivity between the bead and its attached lead wires.

For a properly designed thermocouple, the thermocouple wires will have sufficient length that the bead temperature is not influenced by heat loss to cooler lead wire supports. In this case, the thermocouple wires can be assumed to reach a canonical steady-state temperature sufficiently far away from the bead, based on the balance of convective heating and radiant cooling of the cylindrical wires. In the vicinity of the bead, the temperature profile along the lead wires can be computed by assuming the temperature is nearly uniform across the wire cross-section, such that the solution for linear heat flow along a long rod (defined mathematically as a rod for which $l\sqrt{(2h/kR)} \gg 1$) is applicable. For a long rod with a fixed temperature at an end, T_0, and differential surface loss given by $H(T - T_{amb})pdx$, where p is the rod perimeter, the solution of the equation of steady flow is given by Eq. 6 [3]:

$$T(x) = T_0 exp\left(-x\sqrt{Hp/\gamma A}\right) \tag{6}$$

where x is the distance along the rod. In our case, the heat loss at the surface of the rod is combined heat gain from convection and heat loss from radiation, so the loss term can be expressed as

$$\epsilon\sigma(T_{rod}^4 - T_w^4) - h(T_{rod} - T_g) \tag{7}$$

which can be rearranged as

$$\left[{\epsilon\sigma(T_{rod}^4 - T_w^4)}/{(T_{rod} - T_g)} - h\right](T_{rod} - T_g) \tag{8}$$

from which

$$H = \left[{\epsilon\sigma(T_{rod}^4 - T_w^4)}/{(T_{rod} - T_g)} - h\right] \tag{9}$$

In our case, there is some dependence of H along the rod, because of the variation in the wire temperature as one moves away from the bead in combination with the T^4 dependence of the radiation term in Eq. 9. To first order, we assume Eq. 6 still holds, except we evaluate it at each position x with H calculated at that position. Another consideration is that the steady-state solution for the wire temperature far away from the bead is not the ambient gas temperature, as presumed in deriving Eq. 6, but rather is the equilibrium wire temperature with radiant loss, solved by applying Eq. 2 to the wire. Then, the application of Eq. 6 to the thermocouple lead wires becomes

$$T(x) = T_{wire} + (T_{bead} - T_{wire})exp\left(-x\sqrt{4H/\gamma d}\right) \tag{10}$$

where T_{wire} refers to the thermocouple lead wire temperature far away from the bead. From Eq. 10, the temperature gradient where the lead wire contacts the bead can be readily determined as

$$\left.{dT}/{dx}\right]_{x=0} = -(T_{bead} - T_{wire})\sqrt{4H/\gamma d} \tag{11}$$

where H is determined from Eq. 9 with T_{rod} replaced by T_{bead}.

Nusselt Number Correlations

Application of these equations for the thermocouple bead and wire temperatures requires the use of Nusselt number correlations (per Eq. 3) to determine the convective coefficients, h, for heat transfer to a sphere and to a cylinder in cross-flow. These correlations are used to describe the effects of gas properties and of increasing flow velocities on heat transfer to/from bodies immersed in gaseous flow. As the gas velocity increases, the gas that initially flows uniformly around the body first separates from the downstream edge of the body and then the separation point moves further upstream along the body as the flow continues to increase, influencing the overall rate of heat transfer. Once the separation point is sufficiently far upstream, vortices are periodically shed from the boundary layer, for example as the well-known sequence of vortices shed alternately from the top and bottom of a cylinder in cross-flow (known as a von Karman vortex sheet). These flow dynamics influence the heat transfer to immersed bodies in complex ways, so their net effect is expressed via Nusselt number correlations that are applicable over certain ranges of Reynolds and Prandtl numbers in which they have been characterized. Correlations that have been developed for application over a wide range of conditions can be less accurate over certain ranges of conditions than correlations that have been developed for application over more limited ranges of conditions. Because of the small sizes of thermocouples that are typically employed, the characteristic Reynolds number of the flow around the thermocouples is fairly low. Table 1 shows calculated Reynolds numbers for 1 m/s flow of N_2 over cylinders and spheres corresponding to the range of thermocouple bead and wire sizes commonly employed for bare-wire thermocouples. As the Reynolds number is proportional to the flow velocity, the values in Table 1 can be easily translated to other flow regimes by scaling according to the flow velocity. It can be seen from Table 1 that for bare-wire thermocouple sizes and flow velocities typically encountered flow conditions in flames, the Reynolds number of the thermocouple wire usually is less than 10 (corresponding to a 10 m/s flow of N_2 over a 250 μm wire at 1600 K) and for smaller wire sizes and/or flow velocities is often well under 1. The Reynolds number for the thermocouple bead is typically a factor of 2-3 larger than that of the wire, based on the size of the bead relative to the wire.

Table 1. Reynolds number for 1 m/s N_2 flow over a thermocouple

Geometry	Size (μm)[1]	Temperature		
		1200 K	1600 K	2000 K
cylinder	25	0.16	0.10	0.07
	127	0.81	0.51	0.35
	254	1.63	1.01	0.70
	508	3.25	2.02	1.40
sphere	76	0.49	0.30	0.21
	127	0.81	0.51	0.35
	254	1.63	1.01	0.70
	508	3.25	2.02	1.40
	1016	6.50	4.04	2.80

[1] The sizes shown here correspond to units of “mil” or thousands of an inch, which is the unit by which thermocouples are sold in the United States.

The values of the Reynolds numbers shown in Table 1 are on the lower end of those typically reported in the literature for correlations of Nusselt numbers, so one should be careful to use available correlations that are appropriate for these flow conditions. In particular, flow over a cylinder begins periodic vortex shedding for a Reynolds number of 49 [4], which leads to changes in the heat transfer rate, so only Nusselt number correlations for Re < 49 should be used for analyzing heat transfer to thermocouple wires. Of the available correlations for steady forced flow over a cylinder at low Reynolds numbers, the one published by Collis and Williams [5] has proven to be the most consistently tested and verified and is shown in Eq. 12:

$$Nu_{d,cyl} = \left(0.24 + 0.56Re_d^{0.45}\right)\left(T_m/T_\infty\right)^{0.17} \qquad (12)$$

This correlation was obtained for 0.02 < Re < 44, with the Reynolds number evaluated at the so-called film temperature, T_m, equal to the mean of the surface and the freestream temperature, T_∞. The Collis and Williams correlation has been extensively validated with data from hot-wire anemometry [6,7] and compensated thermocouples [8]. Numerical simulations of flow past cylindrical rods have also confirmed the predictions of the Collis and Williams correlation [9-11], including the applicability of the film temperature to characterize the heat transfer rate [11].

The original Collis and Williams experiments and similar experimental and numerical determinations of the Nusselt number correlations have used heated wires, as is experimentally convenient, such that the gas viscosity decreases (with decreasing temperature) in the boundary layer when moving away from the wire towards the freestream. In contrast, for a thermocouple in hot gases amidst colder radiative boundary conditions (as typical in combustion applications), the thermocouple is cooler than the surrounding gases and the gas viscosity *increases* when moving away from the thermocouple. This raises concerns regarding the suitability of the Nusselt number correlations for application to the radiation correction of thermocouples, particularly if using relatively large thermocouples where a significant temperature difference exists between the thermocouple and the freestream gas. However, the compensated thermocouple study of Miles and Gouldin [8] supports the applicability of the Collis and Williams correlation both when the thermocouple is heating and cooling, and the only available study in which the Nusselt number was measured for a *cooled* cylinder supported in a flow of gases [12] specifically supports the use of the film temperature and the temperature dependence exponent in the Collis and Williams correlation. Thus, with the information available to-date, it appears that the Collis and Williams correlation is quite accurate for application to thermocouples in hot gases and is used in the calculations presented here for the thermocouple wires.

For flow over spheres, the Nusselt number asymptotes to a value of 2.0 for zero flow velocity and this value of Nu = 2 is often used in radiation corrections of thermocouples for simplicity and because of limited knowledge of the local gas velocity as needed for a more accurate determination of Nu. The most widely used correlation over low Re is that of Ranz and Marshall [13], which was derived for evaporating drops over a Reynolds number of 0 to 200 and is shown in Eq. 13:

$$Nu_{d,sph} = \left(2.0 + 0.60 Re_d^{1/2} Pr^{1/3}\right) \quad (13)$$

Abramzon and Sirignano [14] recommended the use of a correlation for Nu that was given by Clift et al. [15] which has the following form:

$$Nu_{d,sph} = 1 + (1 + Re_d Pr)^{1/3} Re_d^{0.077} \quad (14)$$

for $Re_d < 400$, with a further correction for $Re_d < 1$ where the final Re term is replaced by 1. This correlation gives somewhat smaller Nu values than the Ranz and Marshall correlation for typical thermocouple conditions. The Clift correlation was adopted here for the calculations of the complete thermocouple radiation correction, Eq. 5, although most of the comparisons of the results of applying Eq. 5 are made against the more simplistic and widely used assumption of Nu = 2.

Computed Results and Discussion

For the practical situation in which one has measured a temperature with a thermocouple and would like to know the proper radiation correction, the solution procedure is to first make a guess as to what the actual gas temperature might be and then apply Eq. 2 to iteratively solve for the wire temperature far away from the bead, for that postulated gas temperature. Once the steady-state wire temperature has been determined, Eq. 11 is applied to determine *dT/dx* at the bead and then Eq. 5 is employed to compute a new T_{gas}. Based on the difference between the computed T_{gas} and the original guess for T_{gas}, a new guess is made and the solution process is performed iteratively until convergence on the true gas temperature is reached.

The radiant correction determined by this new procedure as a function of thermocouple temperature is shown in Fig. 3 for a type-R thermocouple with a wire diameter of 101.6 μm and a bead diameter of 200 μm. Most commercially produced thermocouples have a bead diameter that is 2-3x larger than the corresponding wire diameter. Also shown in Fig. 3 are the results corresponding to using the method traditionally employed in the combustion community (employing Eq. 2 for the thermocouple bead, with an assumed Nu = 2) and the method I had recommended earlier [16], which consists of assuming the thermal conduction along the thermocouple wires is so fast that the bead temperature is equal to the steady-state temperature of the wire. Note that the thermal conductivities of the two different wire compositions that make up a type-R thermocouple (Pt and Pt-13% Rh) differ by a factor of two. For the purposes of the calculations here, both lead wires are assumed to have the same thermal conductivity, which is equal to the mean of the two values. One could apply Eqs. 5 and 11 while accounting for distinct thermal conductivities in the two lead wires. However, for the sake of simplicity that is not done here.

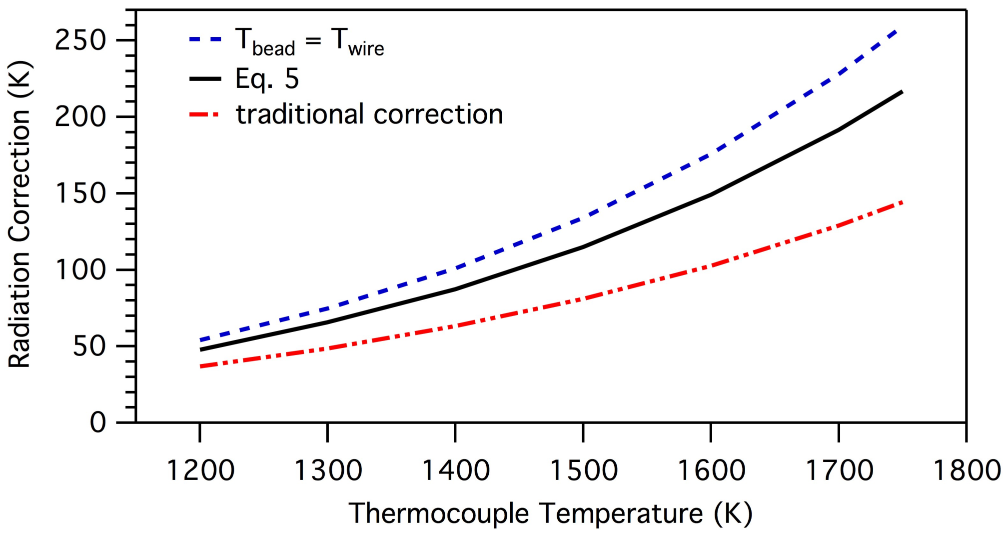

Figure 3. Calculated thermocouple radiation correction as a function of the indicated thermocouple temperature for a type-R thermocouple constructed of 101.6 μm diameter wire, having a bead diameter of 200 μm, placed into the products of a stoichiometric methane-air flame (9.5 vol-% CO_2, 19.0 vol-% H_2O, and 71.5 vol-% N_2) flowing at 1 m/s at 1 atm pressure. The thermocouple is assumed to have an emissivity of 0.25 at all temperatures.

It is clear from Fig. 3, at least under the investigated conditions, that employing the simple assumption that the thermocouple bead temperature is dictated by the wire temperature leads to an overestimation of the actual correction needed. However, employing the traditional method used in the combustion community, in which Eq. 2 is applied to just the thermocouple bead, leads to an even larger underestimation of the true radiation correction and thus an underestimation of the corresponding gas temperature.

Figure 4 shows the influence of the thermocouple bead diameter while keeping the thermocouple wire diameter constant. As expected from Eq. 2, the radiation correction determined using the traditional method is directly proportional to the bead diameter. On the other hand, the strong conductive heat loss from the bead to the connecting lead wires means that the actual radiation correction is essentially immune to the bead diameter up to a size of 250 μm. After this, the increasingly large radiant loss from the bead leads to an increasing dependence of the radiation correction on the bead size. In fact, at a bead size of 300 μm the traditional correction methodology agrees with the proper radiation correction and even overestimates the correction for larger sizes. When calculating the correction for the bead (without accounting for conductance losses to the wires) using the Cliff et al. correlation for Nusselt number, a smaller radiation correction is calculated. At large bead sizes, it can be seen that the correction given by Eq. 5 asymptotes to that calculated by only considering the thermocouple bead, as expected in the limit of a very large bead connected to small wires.

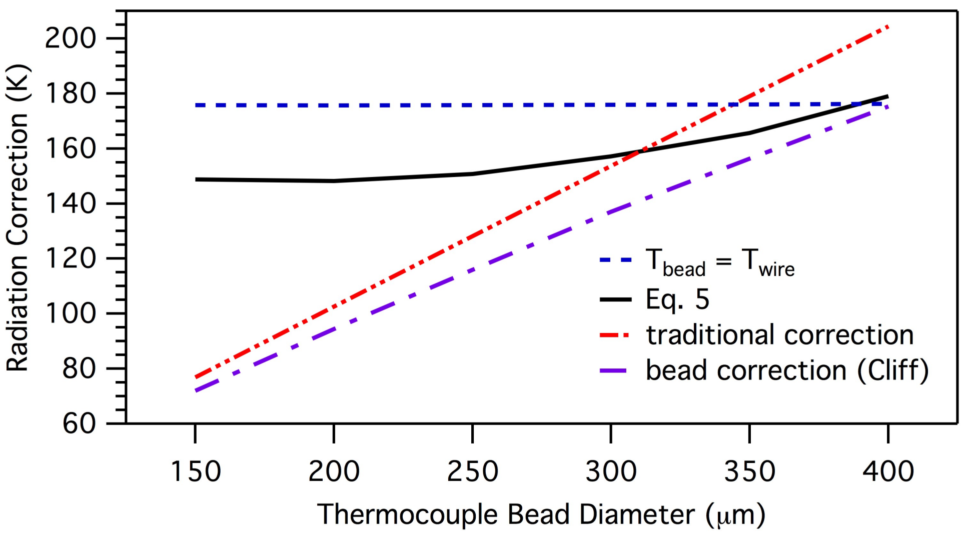

Figure 4. Calculated thermocouple radiation correction as a function of the thermocouple bead diameter, for the same conditions described in the caption of Fig. 3, for an indicated thermocouple temperature of 1600 K. The thermocouple wire diameter is kept constant at 101.6 μm.

Figure 5 shows the influence of the thermocouple wire diameter while maintaining a 2-to-1 ratio of bead diameter to wire diameter, which is a more realistic assumption then the situation presented in Fig. 4. A nearly linear dependence of the radiation correction on thermocouple wire size is evident for all of the correction approaches shown (the traditional correction is directly proportional to bead diameter and so is linear with wire size for the constant ratio of bead to wire size assumed here). For smaller wire sizes, the simple assumption of the bead temperature being dictated by the steady-state wire temperature agrees quite well with the improved result given by Eq. 5. For large wire sizes, the result from Eq. 5 lies approximately midway between the two simpler assumptions. The calculated radiation correction given by ignoring conduction to the thermocouple wires and employing the Cliff correlation for flow over the bead is seen to consistently underpredict the actual radiation correction. In this sense, the simple assumption of assuming Nu = 2 and employing the correction for a bead without accounting for conduction is seen to be an improvement over employing a more accurate determination of the Nusselt number over a sphere.

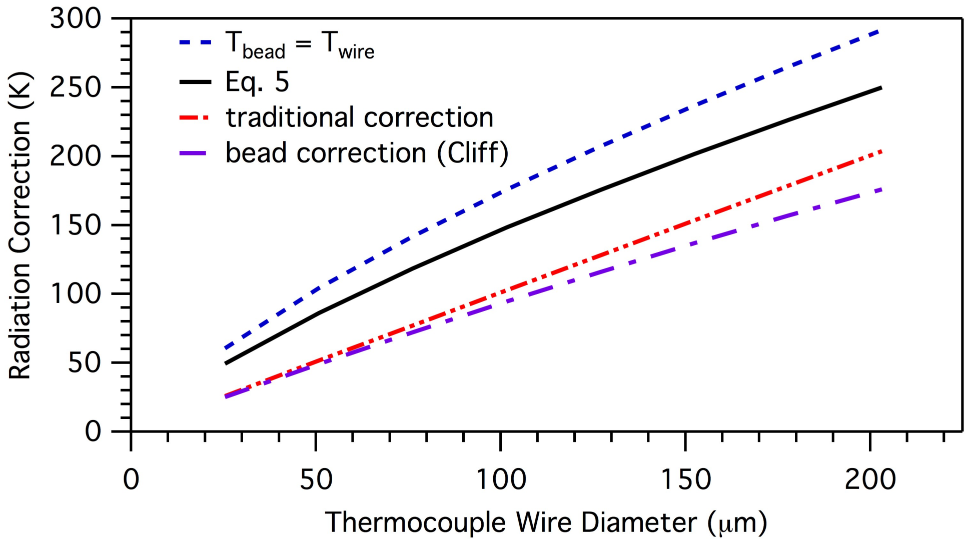

Figure 5. Calculated thermocouple radiation correction as a function of the thermocouple wire diameter when the thermocouple bead diameter is twice the wire diameter, for the same conditions described in the caption of Fig. 3, for an indicated thermocouple temperature of 1600 K.

Figure 6 shows the temperature profile along the thermocouple lead wires as a function of the thermocouple wire diameter while maintaining a 2-to-1 ratio of bead diameter to wire diameter. The larger temperature gradient from the thermocouple bead for smaller wires that is dictated by Eq. 11 is evident. The plot also shows that smaller wires result in a more rapid approach to the steady state wire temperature. It is evident from Fig. 6 that a significant temperature difference over a distance of several mm in the gas phase surrounding a thermocouple will influence the measured temperature by influencing thermal conduction from the bead down the attached wires.

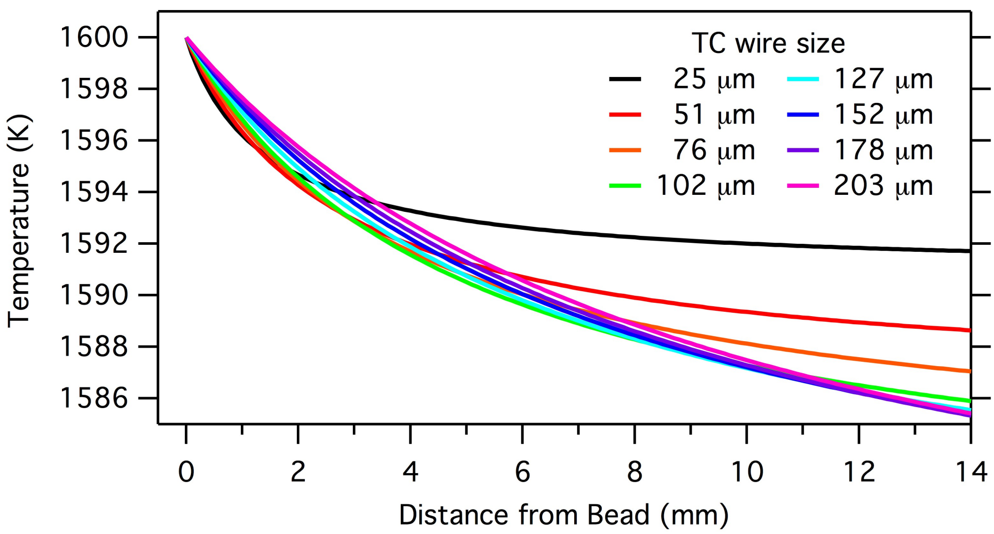

Figure 6. Calculated temperature along the thermocouple lead wires, for the same conditions described in the caption of Fig. 5.

Conclusions

A new, simple procedure for calculating the proper radiant correction to bare-wire thermocouple measurements has been derived, which explicitly accounts for the combined convective and radiant heat flows associated with the thermocouple bead and its attached wires, as well as thermal conductivity between the bead and wires. Under nearly all investigated conditions, the new radiant correction is substantially larger than that predicted from the traditional approach of ignoring conduction between the bead and the connecting wires. Similarly, the new correction is somewhat smaller than predicted by making the simplifying assumption that the thermocouple bead temperature is dictated by the energy balance on the connecting thermocouple wires (i.e. an assumption of infinitely fast thermal conduction in the thermocouple wires).

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