

# Bayesian Methods to Capture Inherent Material Variability in Additively Manufactured Samples

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Additively manufactured samples have demonstrated significant variability in both estimated yield strength and failure stress<sup>1</sup>. This variability is inherent in the material behavior and significantly greater than the measurement uncertainty. Bayesian calibration of material models is an attractive approach in this case because it can quantify the accuracy of estimates of model parameters, but suffers from incorrectly attributing the variability to measurement error rather than aleatoric uncertainty. In this presentation, we explore various approaches to account for material variability within a Bayesian calibration of a J2 plasticity model.

Experimental data is provided in the form of nearly 1000 tensile tests of an additively manufactured specimen from 8 different lots. The Young's modulus may be easily estimated from the data, so we focus on calibrating the yield strength and hardening modulus. We consider several forms of model error, including treating each point along the stress/strain curve as an i.i.d. sample of the response, as well as treating each test curve as a data point with a parameterized error model. While the mean-plus-uncertainty of the calibrated model encompasses the test data, a realization-to-realization comparison suggests the additive noise model does not account for the observed variability in a physically meaningful way. The primary cause is that the variability does not conform to a simple Gaussian description, and further is interpreted within the Bayesian framework as an estimate of the error in the measurement process. As a result, individual realizations from this model cannot be used as a material model in a mechanics code, and as additional data is added, the calibration process becomes increasingly certain of the model parameters even as the variability remains finite.

As an alternative, we consider the Embedded Error Model (EEM)<sup>2</sup> in which the model parameters are represented as polynomial chaos expansions (PCEs) such that the calibration is performed on the coefficients. In this approach, because uncertainty is inherent in the expansion, the coefficients converge to a model representation with finite variance. We will compare the results of the EEM to more traditional Bayesian calibration methods and explore the impact on different measurement error models within this framework.

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<sup>1</sup> Salzbrenner et al., "High-throughput stochastic tensile performance of additively manufactured stainless steel", *J. Mat. Proc. Tech* 241, pp. 1-12 (2017)

<sup>2</sup> Sargsyan, Najm, and Ghanem, "On the statistical calibration of physical models," *Int'l. J. Chemical Kinetics* 47(4), pp. 246-276 (2015).