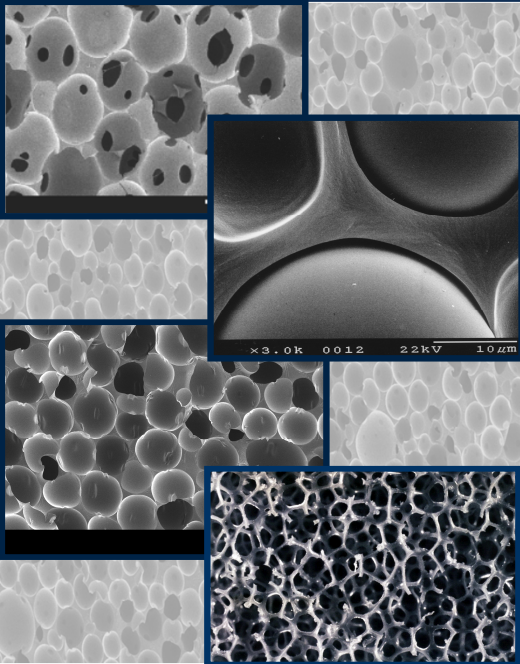


# Flex Foam Model for Rigid and Flexible Foams

Mike Neilsen, Wei-Yang Lu,  
Brian Werner, David Lo

Sandia National Laboratories

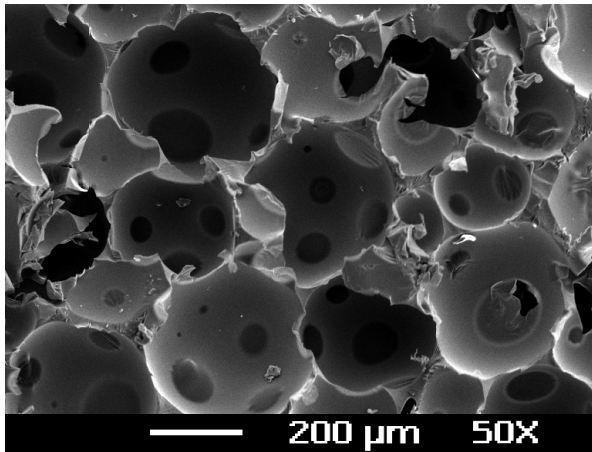


# Outline

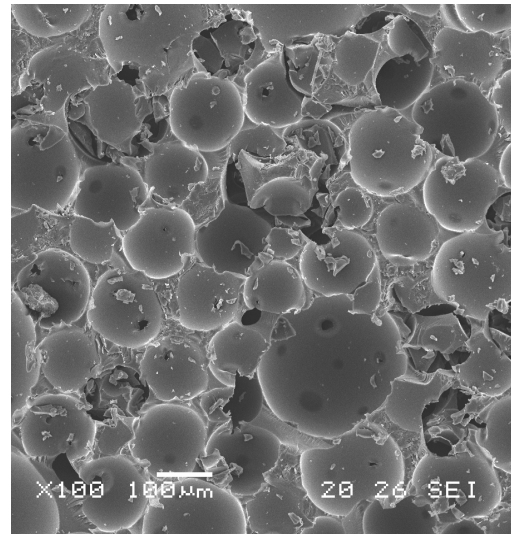
---

- ❑ Behavior of Foams
- ❑ Foam Micromechanics (cell level)
- ❑ Unified Creep Plasticity Damage (UCPD) Model for Rigid Foams
- ❑ Flex Foam Model for Flexible and Rigid Foams
- ❑ Simulations
- ❑ Summary and Future Work

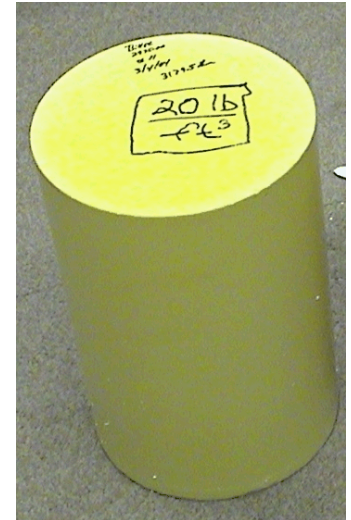
# Polyurethane Foam



176 kg/m<sup>3</sup> (11 pcf)  
polyurethane foam

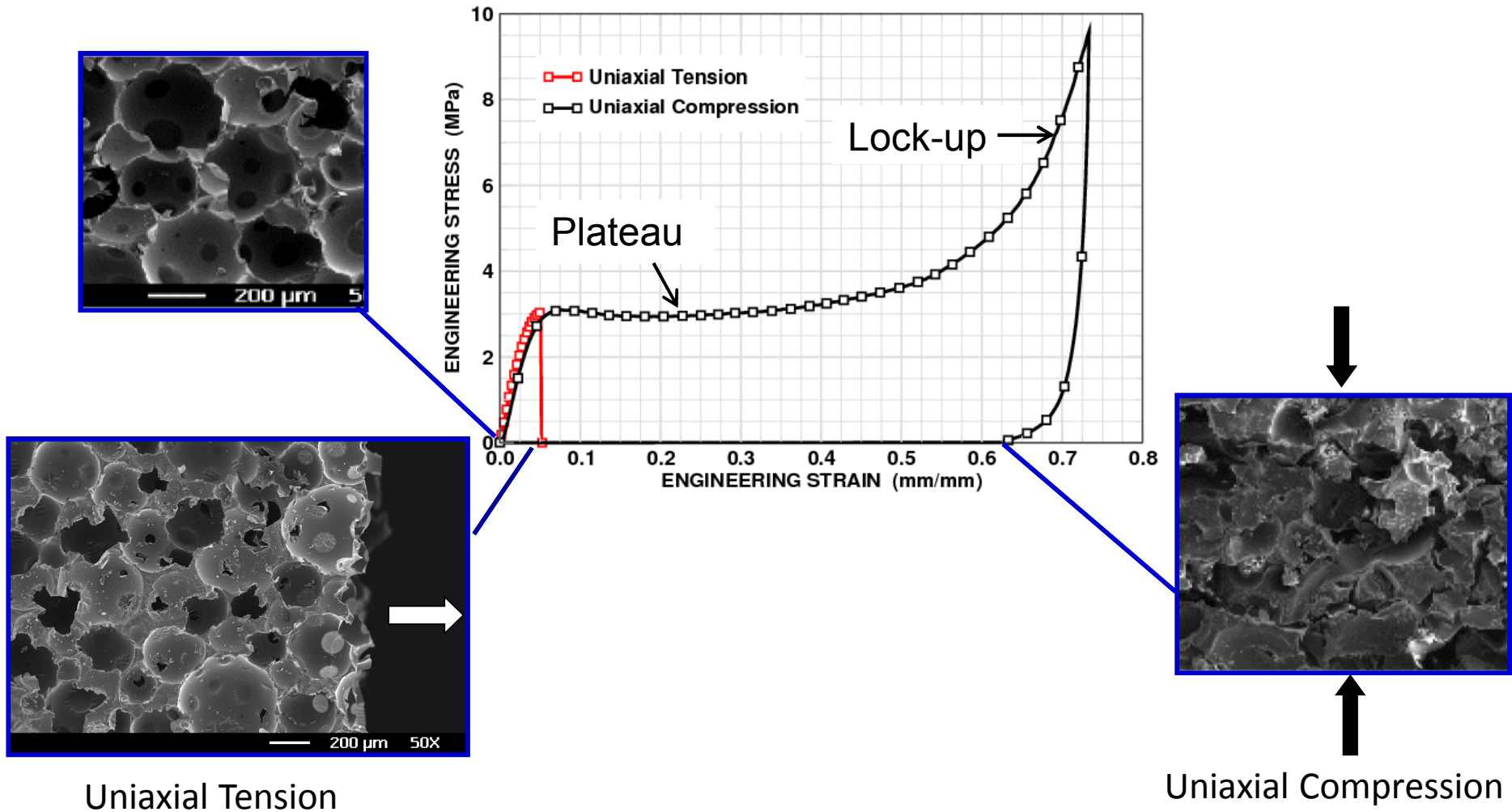


320 kg/m<sup>3</sup> (20 pcf) rigid  
polyurethane foam



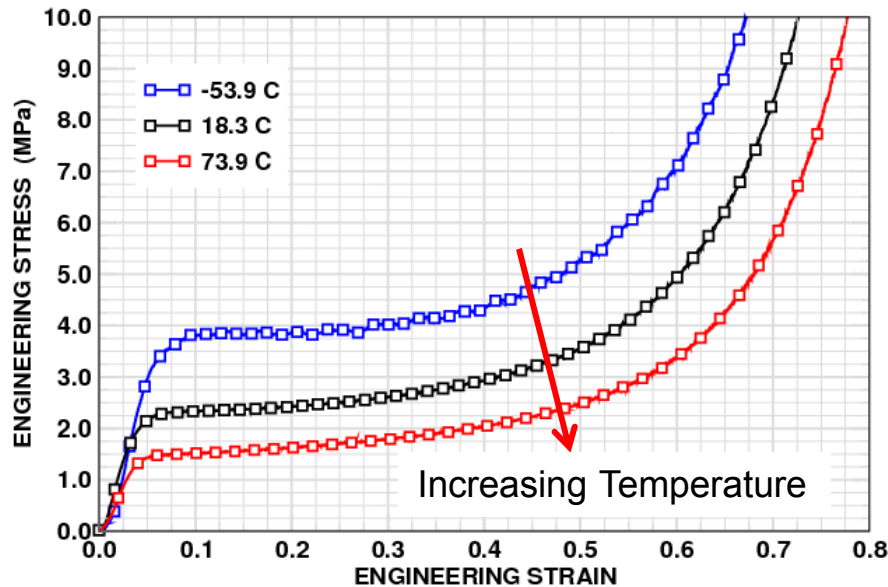
30.5 cm (12 in.) tall billet

# Experiments on 176 kg/m<sup>3</sup> (11 pcf) Foam



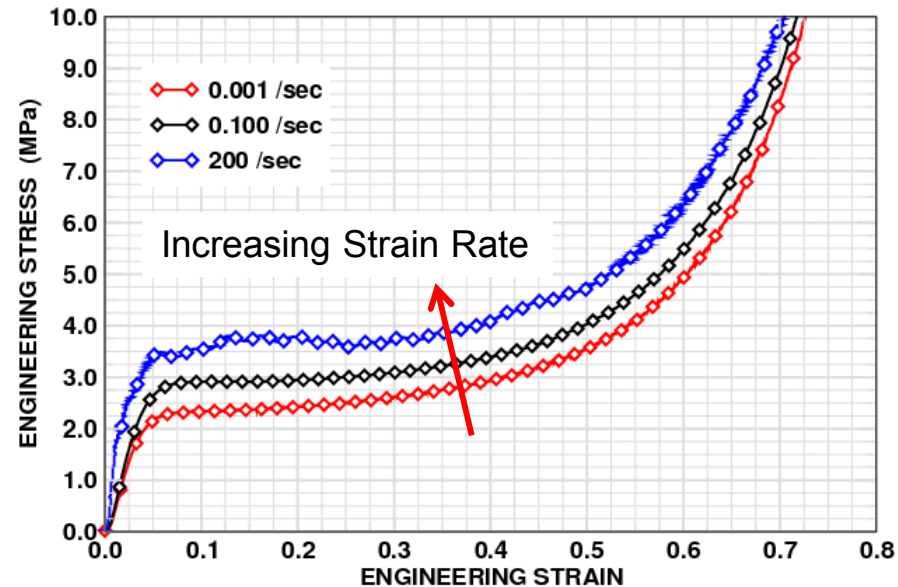


# Experiments on 192 kg/m<sup>3</sup> (12 pcf) Foam



Temperature Effects

Constant Strain Rate = 0.001 / second



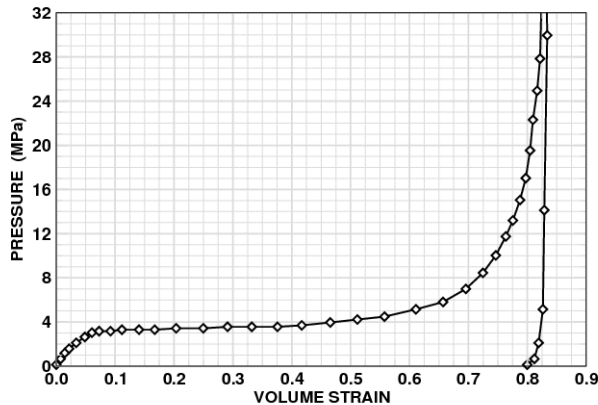
Strain-Rate Effects

Constant Temperature = 18.3 °C

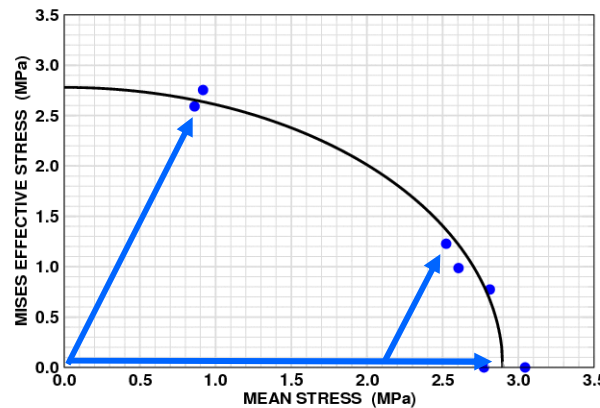
Uniaxial Compression of FR3712

# Experiments on 192 kg/m<sup>3</sup> (12 pcf) Foam

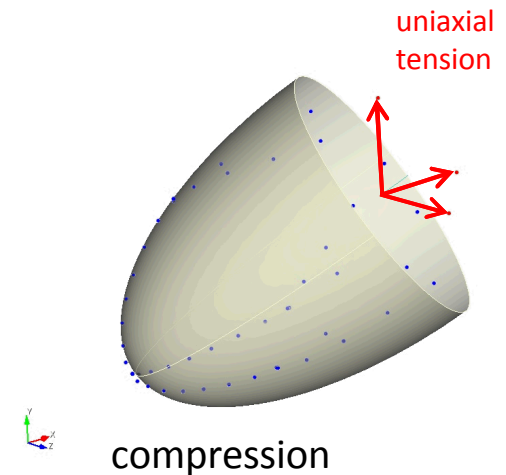
Unlike metals, foams exhibit large permanent volume changes – volumetric plasticity



Hydrostatic Compression  
Pressurization Rate 0.1 MPa/sec



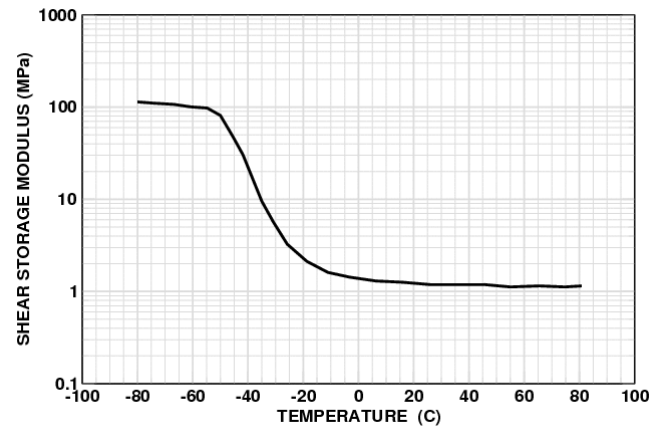
Triaxial Compression  
W.A. Olsson, 2006



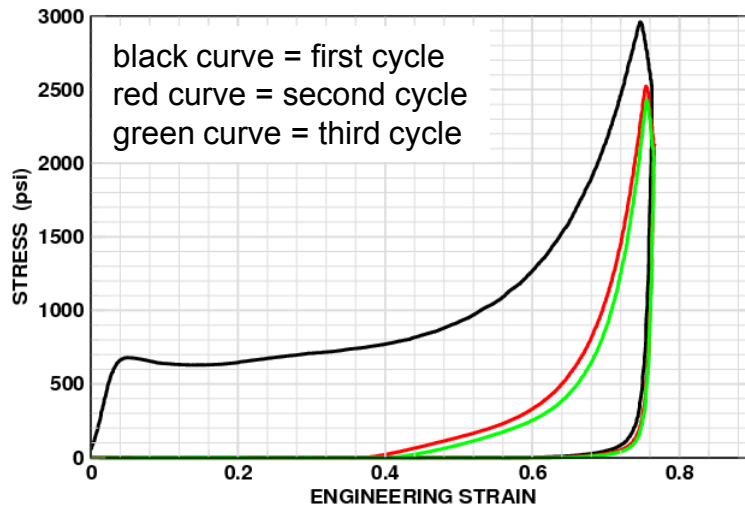
Yield Surface in  
Principal Stress Space

Reference: M.K. Nielsen, W-Y. Lu, W.A. Olsson, A.M. Kraynik, W.M. Scherzinger, 'Foam Constitutive Models from Complementary Experiments and Cell-Level Simulations,' 15<sup>th</sup> UCTAM, Univ. of Colorado at Boulder, 2006.

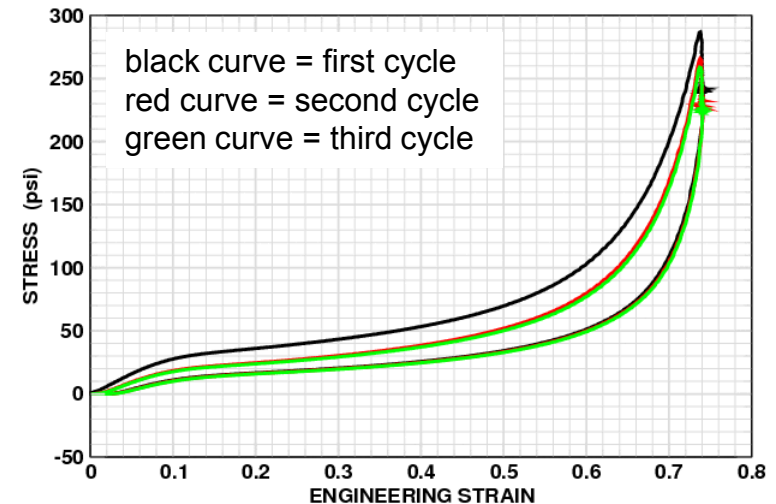
# Experiments on Flexible Foam



DMA shows  $T_g \sim -35^\circ\text{C}$



Rigid at  $-53.9^\circ\text{C}$

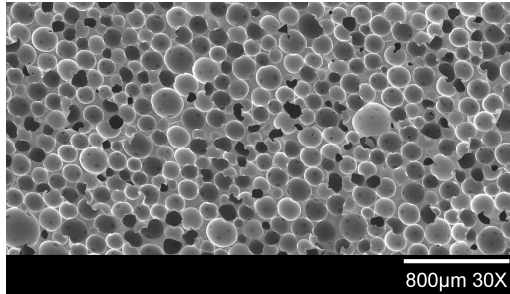


Flexible at  $21.1^\circ\text{C}$

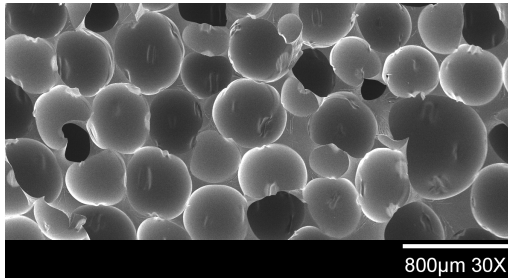
Cyclic stress-strain curves for 15pcf shipping container foam (Brian Werner, 8343 and April Nissen, 8344)

# Foam Micromechanics, Cell Geometry – Andy Kraynik

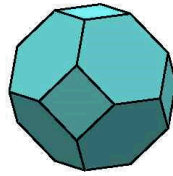
Need spatially-periodic representative volume



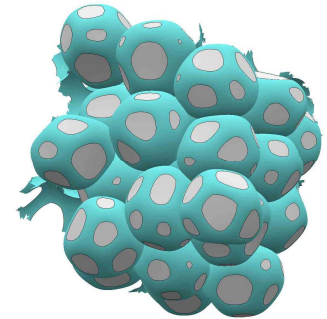
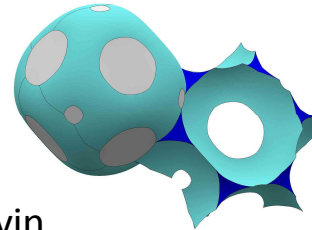
PMDI-20 (Wei-Yang Lu, SNL)



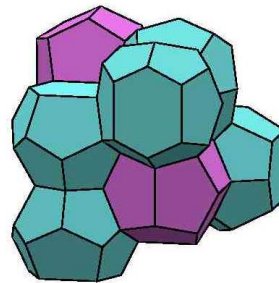
TufFoam (L. Whinnery, SNL)



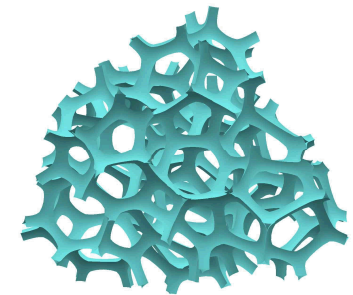
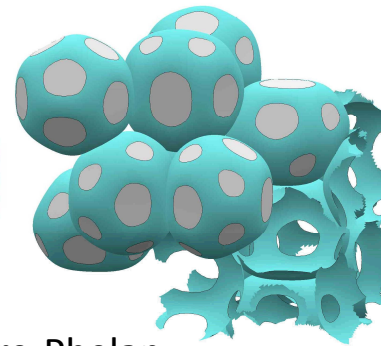
Kelvin



Random 27 bubbles



Weaire-Phelan



Plateau borders

A.M. Kraynik, "Foam structure: From soap froth to solid foams." *MRS Bulletin* 28.04 (2003): 275-278.

W. Thomson, Lord Kelvin, "On the division of space with minimum partitional area", *Phil. Mag.* vol. 24 (1887), 503.

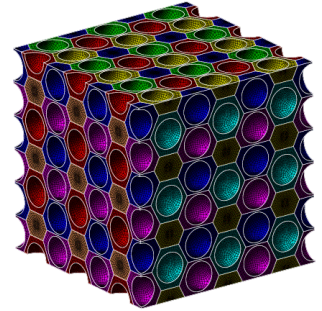
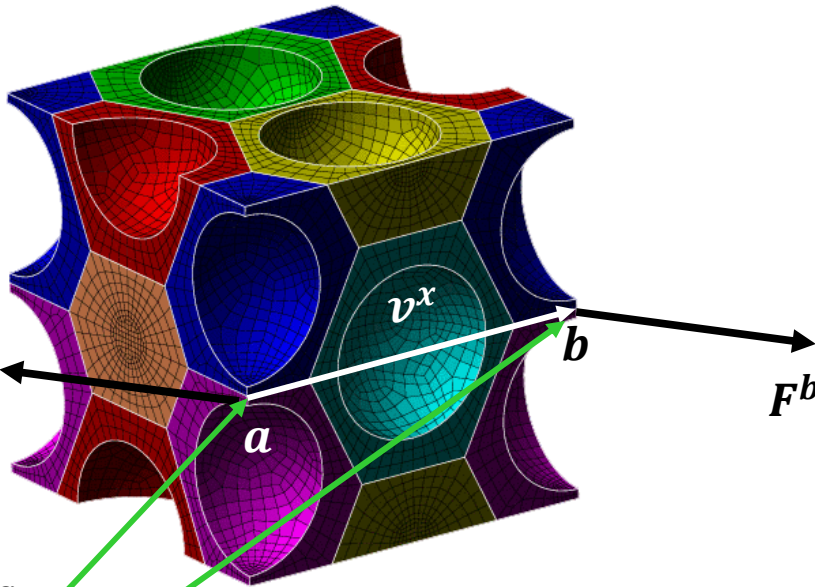
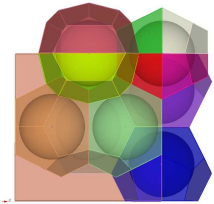
D. Weaire and R. Phelan, "A counterexample to Kelvin's conjecture on minimal surfaces", *Phil. Mag. Lett.* **69** (1994), 107.

Prof. K. Brakke, Susquehanna University, **Surface Evolver**, [www.susqu.edu/brakke/evolver](http://www.susqu.edu/brakke/evolver)

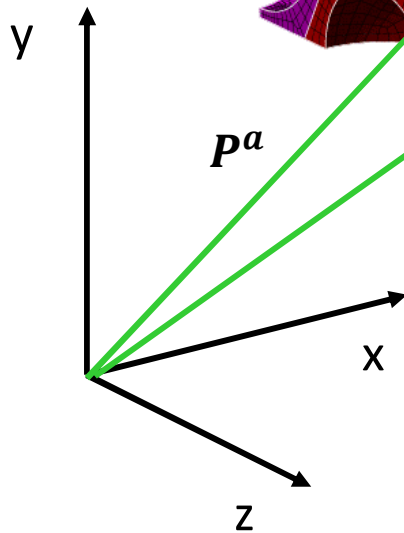
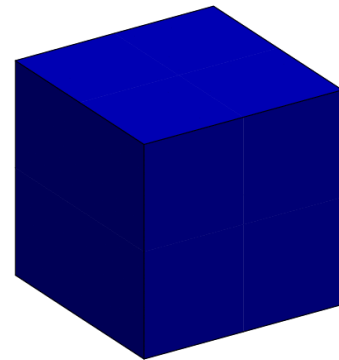
# Cell-Level to Macro Connection

Weaire-Phelan Unit Cell

Periodic b.c. :  $\mathbf{p}^b = \mathbf{p}^a + \mathbf{v}^x$



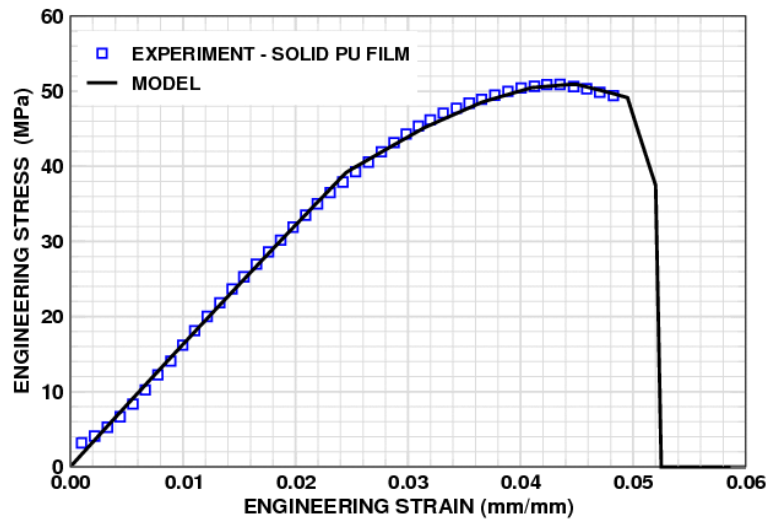
“Equivalent”  
Continuum



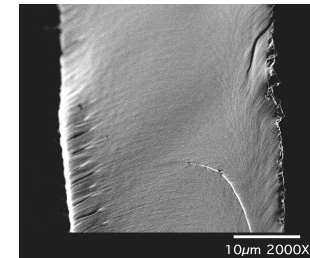
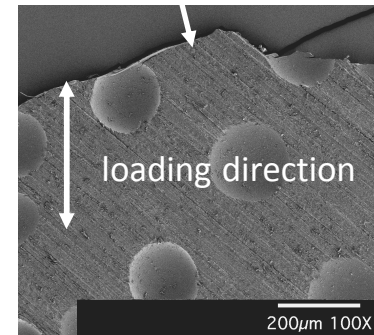
$$\bar{\sigma}_{ij} = \frac{1}{\text{volume}} \int \sigma_{ij} dv = \frac{1}{\text{volume}} \sum_{n=\text{periodic nodes}} \mathbf{P}_i^n \mathbf{F}_j^n$$

Macroscopic Cauchy Stress for “Equivalent” Continuum

# Tension of Solid Polyurethane (PU)



fracture edge

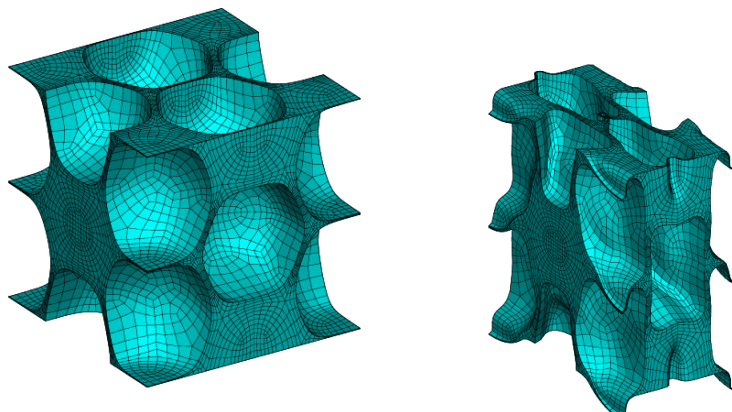


FR3712 Solid Film

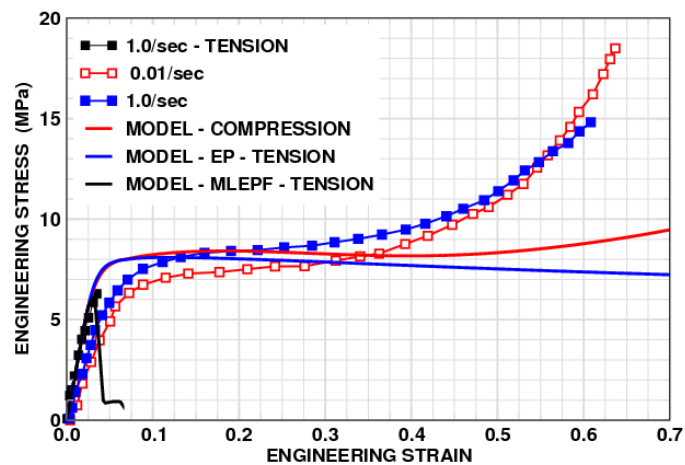
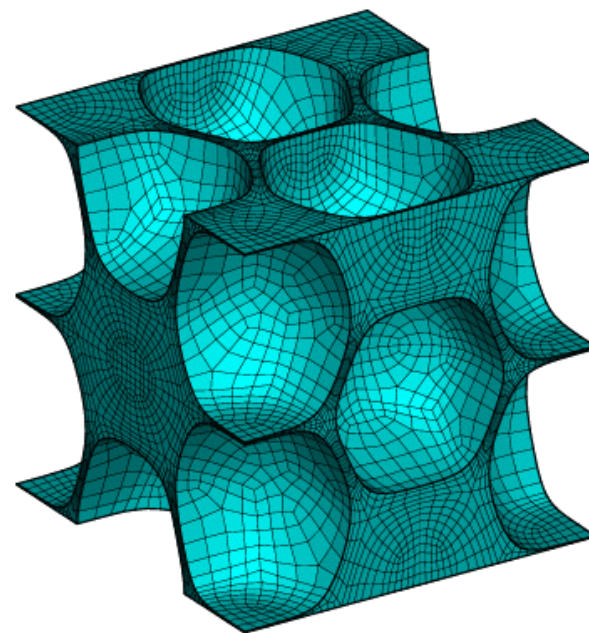
G.W. Wellman, 'A Simple Approach to Modeling Ductile Failure,' SAND2012-1343, Sandia National Labs., June 2012.



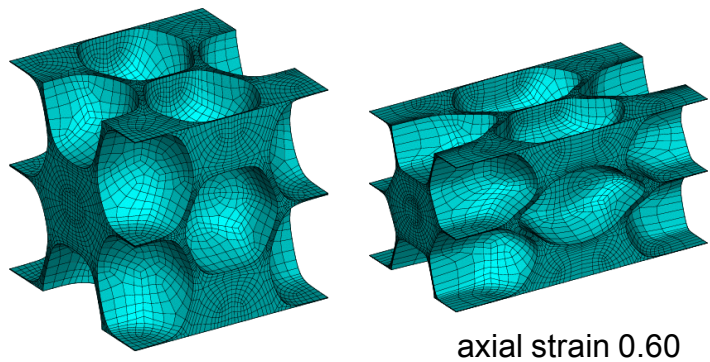
# WP Cell Model Compression – 20 pcf Foam



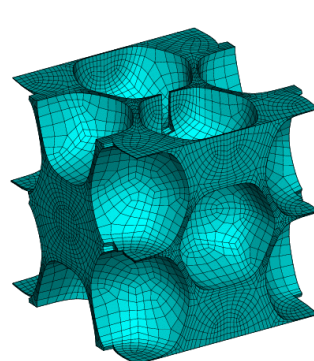
Compression - 100



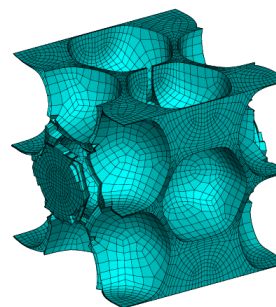
# WP Cell Model - Tension



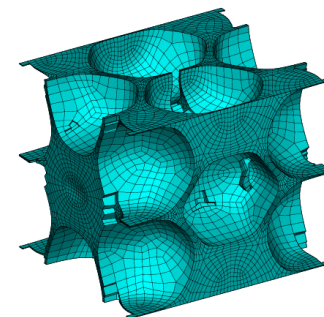
Tension – Elastic Plastic



axial strain 0.03



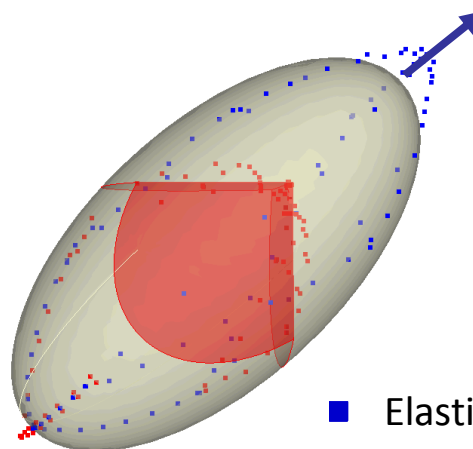
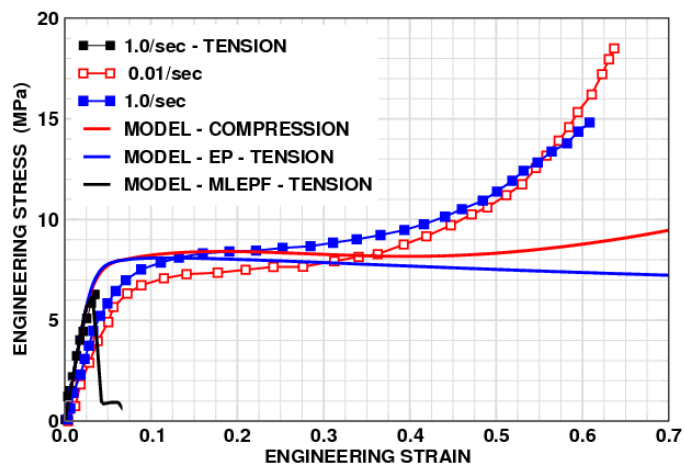
axial strain 0.06



axial strain 0.10

Tension – MLEPF

Tension – MLEPF with lateral compression



Hydrostat

- Elastic-Plastic Polymer
- MLEPF – Tearing Polymer

# Yield and Damage Surfaces for Foams

## Yield /Damage Surfaces Proposed for Rigid Foam

Triantafillou et al. (1989) Stretched ellipsoid w/ buckling cap.

Neilsen et al. (1995). Principal stress criterion.

Puso and Govindjee (1995) Stretched ellipsoid w/ ellipsoidal cap.

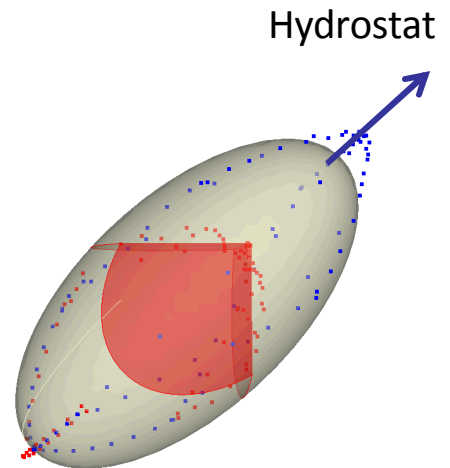
Zhang et al. (1998). Ellipsoid about hydrostat, pressure offset.

Deshpande and Fleck (2000) Ellipsoid about hydrostat – metal foams

Deshpande and Fleck (2001) Ellipsoid capped by max. compressive principal stress surface

Experiments and Cell Level Models indicate we should use

Ellipsoid capped by maximum tensile stress **damage surfaces**



Gibson, L.J., and Ashby, M.F., **Cellular Solids – Structural and Properties**, Pergamon Press, New York, 1988

Neilsen, M. K., Krieg, R. D., and Schreyer, H.L., *Polymer Engineering and Science*, **35**, No. 5, pp. 387-94, 1995

Puso, M.A., and Govindjee, S., ASME MD-Vol. 68/AMD-Vol. 215, **Mechanics of Plastics & Composites**, 1995

Zhang, J., Kikuchi, N., Li, V., Yee, A., and Nusholtz, G., *Intl. J. Impact Engr.*, **21**, No. 5, pp. 369-386, 1998.

Deshpande, V.S., and Fleck, N.A., *J. Mech. Phys. Solids*, **48**, pp. 1253-83, 2000.

Deshpande, V.S., and Fleck, N.A., *Acta. Mater.*, **49**, pp. 1859-1866, 2001.

# UCPD Model for Rigid Foam

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}}^e = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in})$$

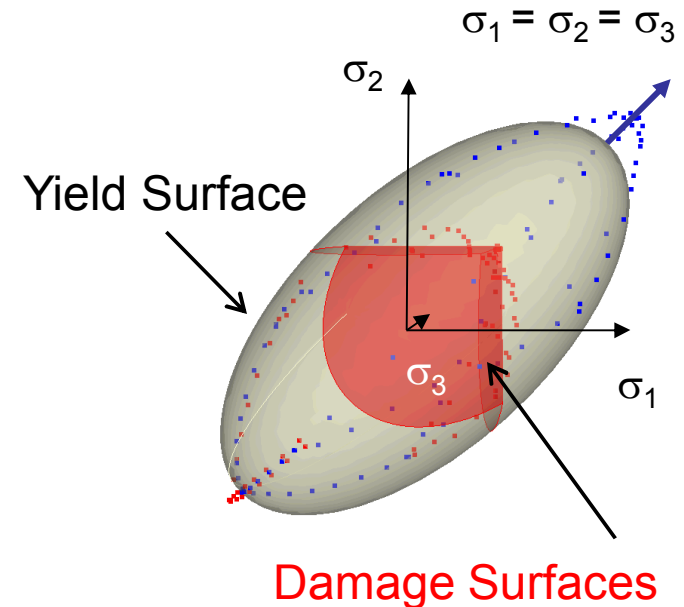
$$\varphi = \frac{\bar{\sigma}^2}{a^2} + \frac{p^2}{b^2} - 1.0$$

$$p = \frac{1}{3} \boldsymbol{\sigma} : \mathbf{i} \quad \bar{\sigma} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}} \quad \mathbf{s} = \boldsymbol{\sigma} - p \mathbf{i}$$

$$\varphi = \sigma^* - a$$

$$\sigma^* = \sqrt{\bar{\sigma}^2 + \frac{a^2}{b^2} p^2}$$

$$\dot{\boldsymbol{\varepsilon}}^{in} = \begin{cases} \dot{\lambda} \mathbf{g} = e^h \left\langle \frac{\sigma^*}{a} - 1 \right\rangle^n \mathbf{g} & \text{when } \frac{\sigma^*}{a} - 1 > 0 \\ \mathbf{0} & \text{when } \frac{\sigma^*}{a} - 1 \leq 0 \end{cases}$$



# UCPD Model for Rigid Foam

$$\mathbf{g}_{associated} = \frac{\frac{\partial \phi}{\partial \boldsymbol{\sigma}}}{\left| \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right|} = \frac{\frac{3}{a^2} \mathbf{s} + \frac{2}{3b^2} p \mathbf{i}}{\left| \frac{3}{a^2} \mathbf{s} + \frac{2}{3b^2} p \mathbf{i} \right|}$$

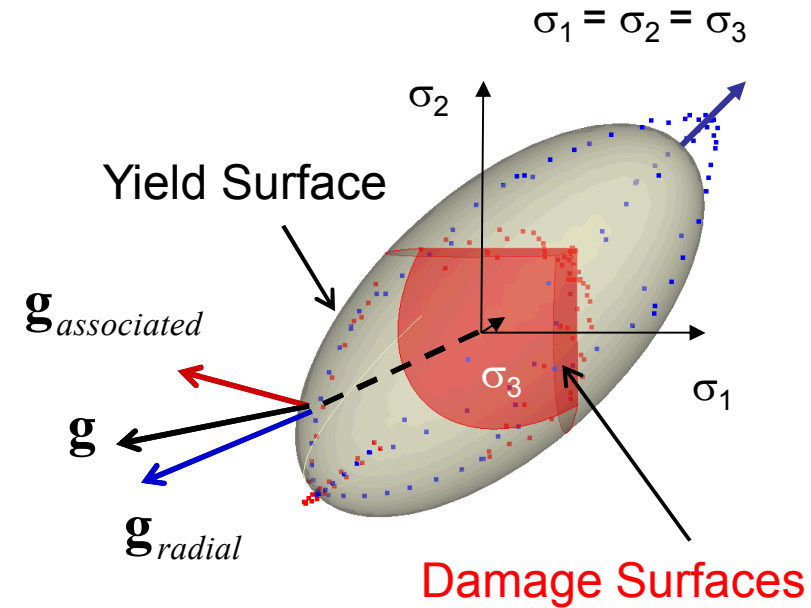
$$\mathbf{g}_{radial} = \frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}|} = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{\sigma} : \boldsymbol{\sigma}}}$$

$$\mathbf{g} = \frac{(1-\beta) \mathbf{g}_{associated} + \beta \mathbf{g}_{radial}}{|(1-\beta) \mathbf{g}_{associated} + \beta \mathbf{g}_{radial}|}$$

$$\phi = \frac{\phi_0 V_0}{V}$$

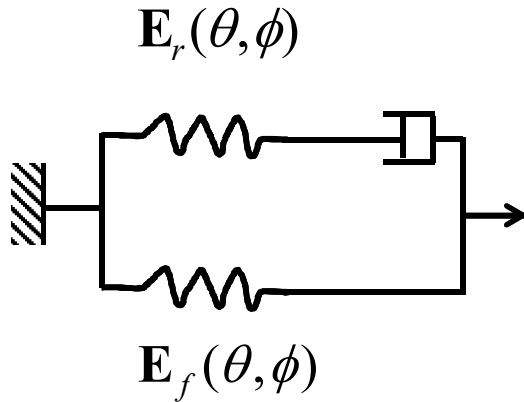
$$\varphi_{Damage}^i = \sigma^{**i} - c(1-w) = 0, i = 1,3$$

$$w = w(\varepsilon_{dam}) = w(a_{dam} \varepsilon_{max} + b_{dam} \varepsilon_{vol}^p)$$



# Flex Foam Model for Flexible and Rigid Foam

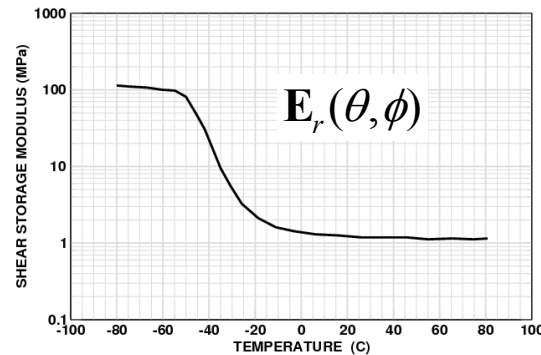
RIGID ~ UCPD MODEL



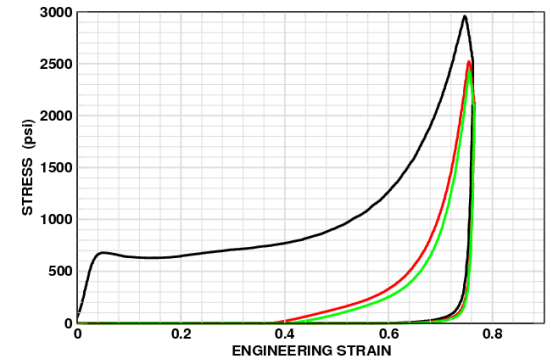
$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}_r + \dot{\boldsymbol{\sigma}}_f$$

$$\dot{\boldsymbol{\sigma}}_r = \mathbf{E}_r : \dot{\boldsymbol{\varepsilon}}^e = \mathbf{E}_r : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in})$$

$$\dot{\boldsymbol{\varepsilon}}^{in} = \dot{\lambda} \mathbf{g} = e^h \left( \frac{\sigma^*}{a} \right)^n \mathbf{g}$$



$$\sigma^* = \sqrt{\bar{\sigma}^2 + \frac{a^2}{b^2} p^2}$$

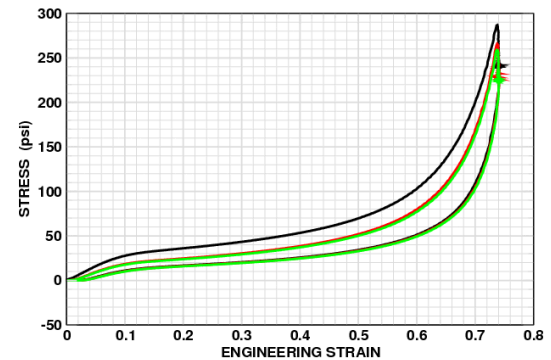


Rigid at -53.9 °C

FLEXIBLE ~ MOSTLY JUST NON-LINEAR ELASTIC

$$\dot{\boldsymbol{\sigma}}_f = \mathbf{E}_f(\theta, \phi) : \dot{\boldsymbol{\varepsilon}}$$

$$\mathbf{E}_f = 2G(\theta, \phi) \mathbf{P}^d + 3K(\theta, \phi) \mathbf{P}^{sp}$$



Flexible at 21.1°C

Next Step Include Effect of Damage on Elastic Moduli



# Flex Foam Model Parameters and State Variable Names

## Flex Foam Model Parameters

begin parameters for model flex\_foam

youngs modulus = 2400.0

poissons ratio = 0.050

phi = 0.520

flow rate = 1.000

power exponent = 3.000

dev multiplier = 0.2

tensile strength = 500.0

adam = 1.0

bdam = 0.5

....

$$E = E_r \cdot E(\theta) \cdot E(\phi)$$

$$v = v_r \cdot v(\theta) \cdot v(\phi)$$

youngs function = f\_Modulus

poissons function = f\_Constant

youngs phi function = f\_E

poissons phi function = f\_Constant

rate function = f\_Rate

exponent function = f\_Expo

shear hardening function = f\_Shear

hydro hardening function = f\_Hydro

beta function = f\_Beta

dmod function = d\_Modulus

dpr function = f\_Constant

dmod phi function = f\_E

dpr phi function = f\_Constant

damage function = f\_Damage

end parameters for model flex\_foam

## State Variables

damage

denergy

dstrain

emax

epvol

eqps

fa

fb

iter

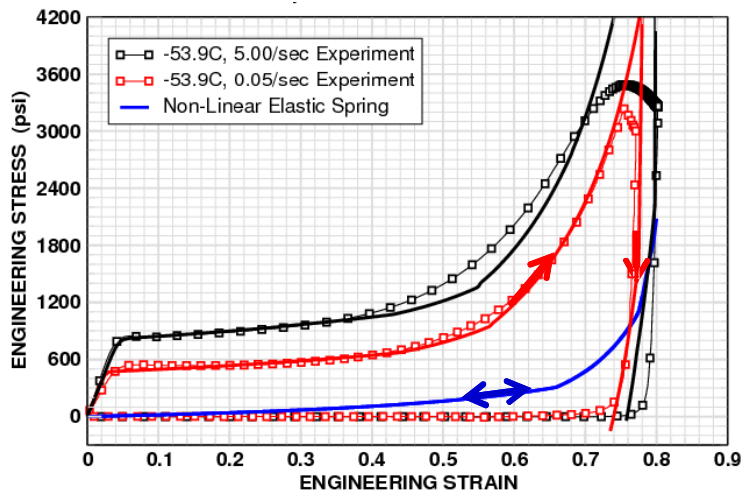
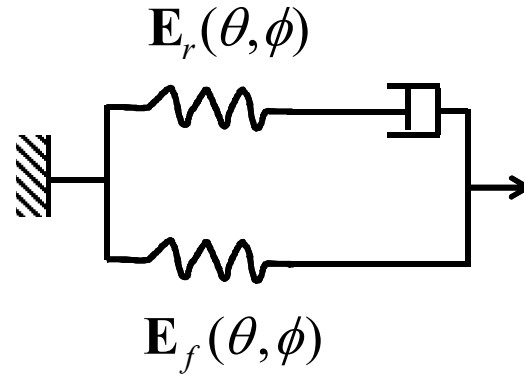
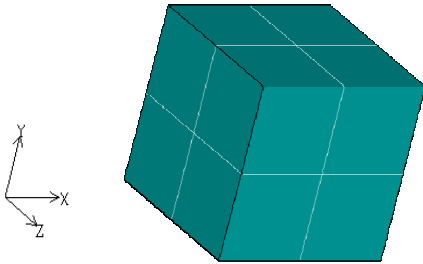
phi

pwork

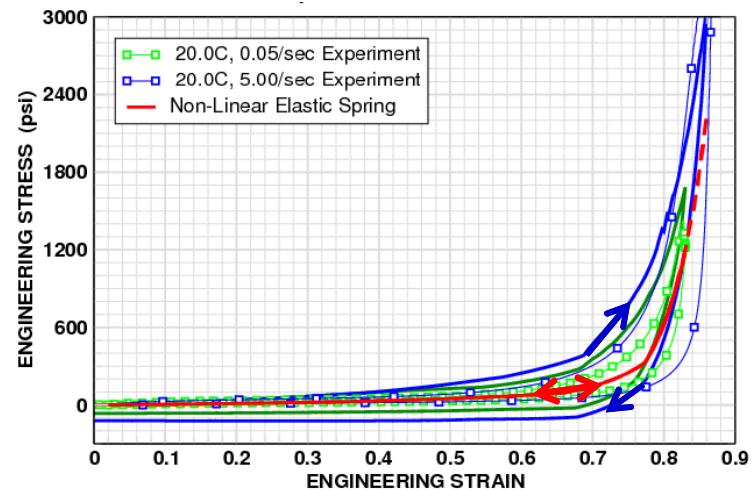
vstrain

# 15 pcf Flexible Polyurethane Foam

## Comparison of Model Predictions with Experiments



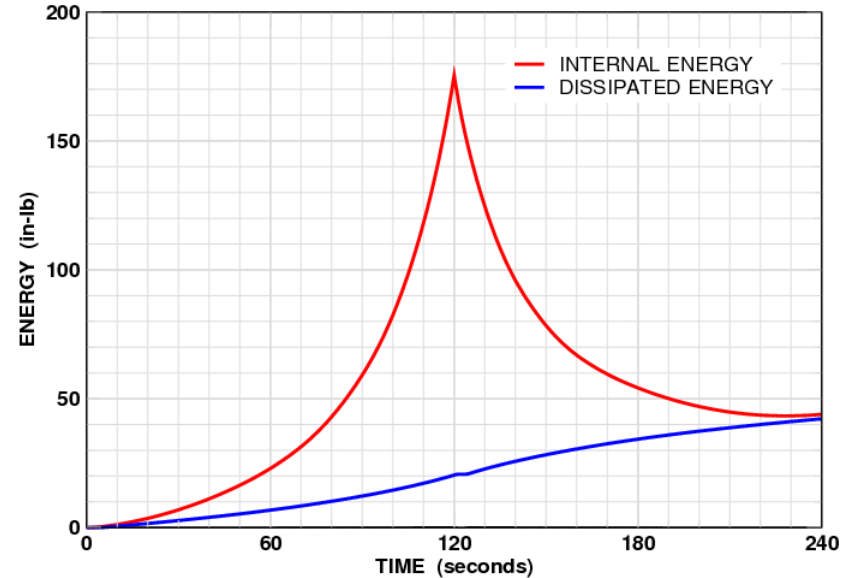
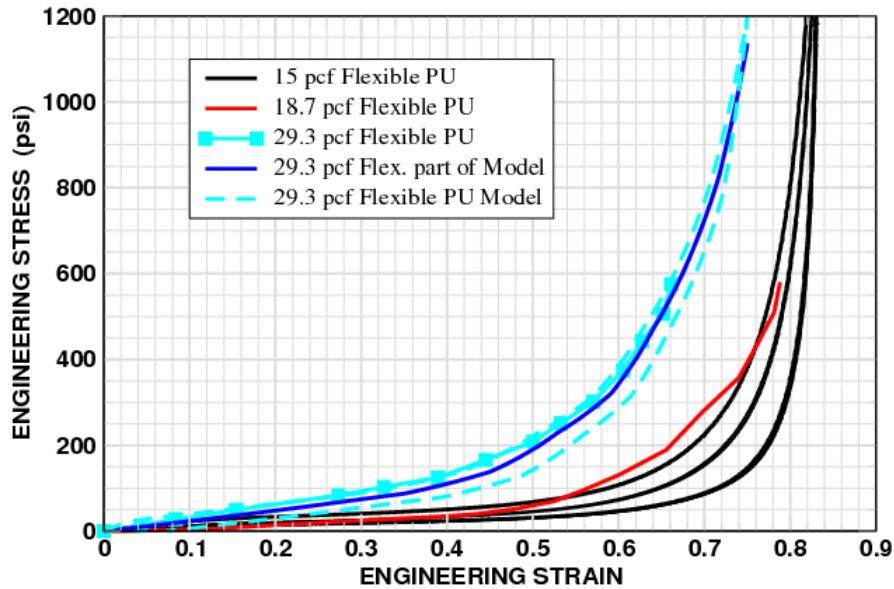
-53.9 °C



20 °C

Uniaxial Compression

# Predicted Energy Dissipation with Flex Foam Model



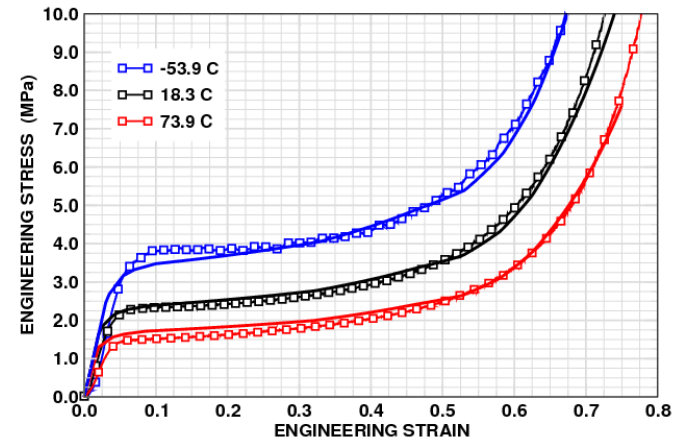
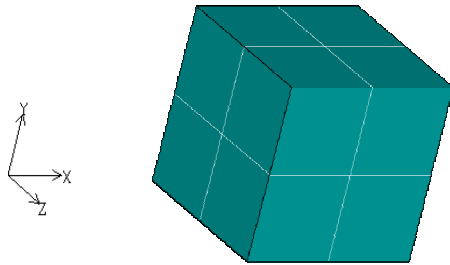
$$P_{work} = \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{in}$$

$$D_{energy} = \iint \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{in} dV dt$$

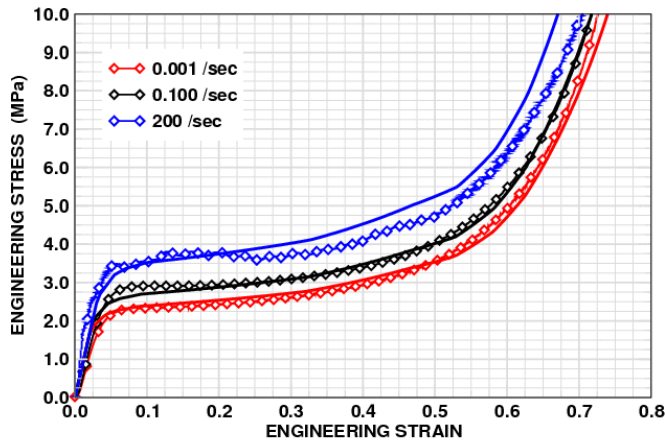
in Algebra compute:  
Diss. Energy = SUM(DENERGY)

Internal Energy is global variable  
from ADAGIO

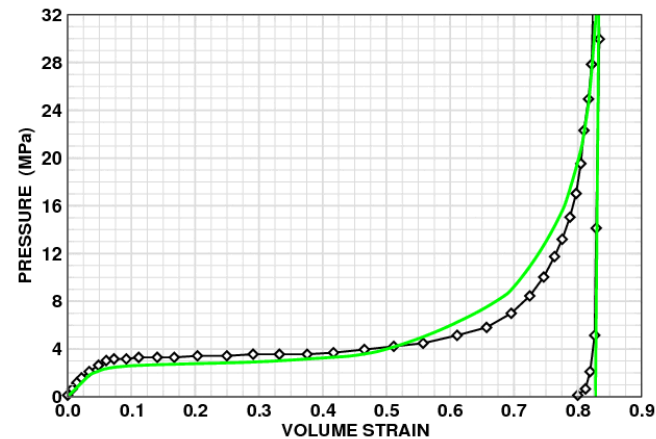
# FR3712 Comparison of Model Predictions with Experiments



Uniaxial Compression at 0.001/second

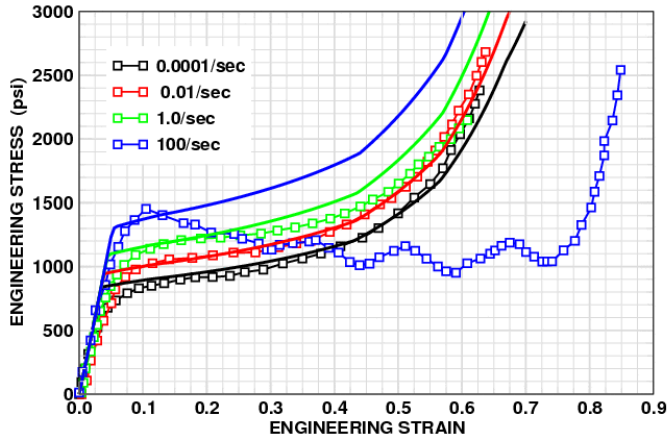


Uniaxial Compression at 18.3 C

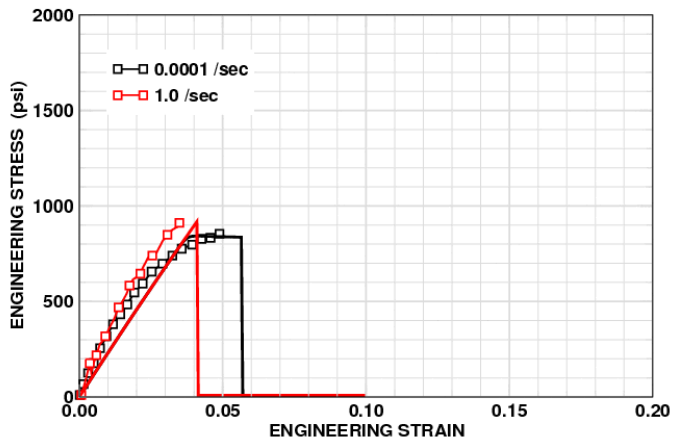


Hydrostatic Compression at 20.0 C

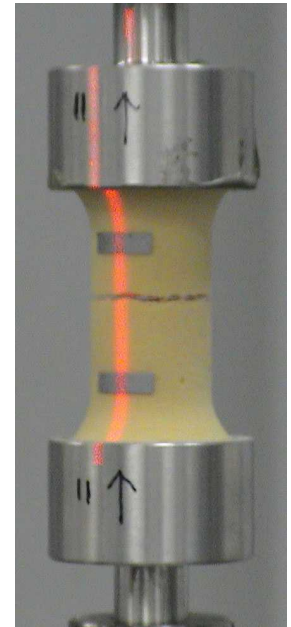
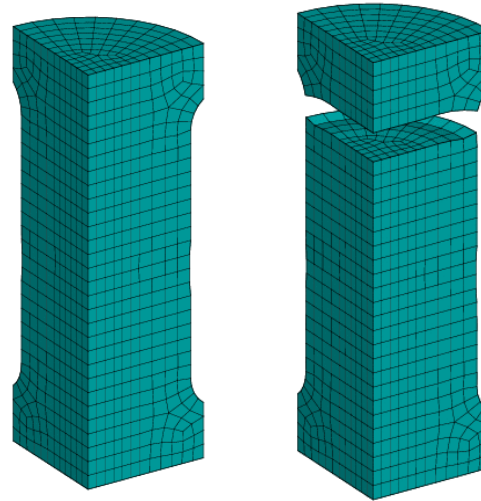
# PMDI20 Comparison of Model Predictions with Experiments



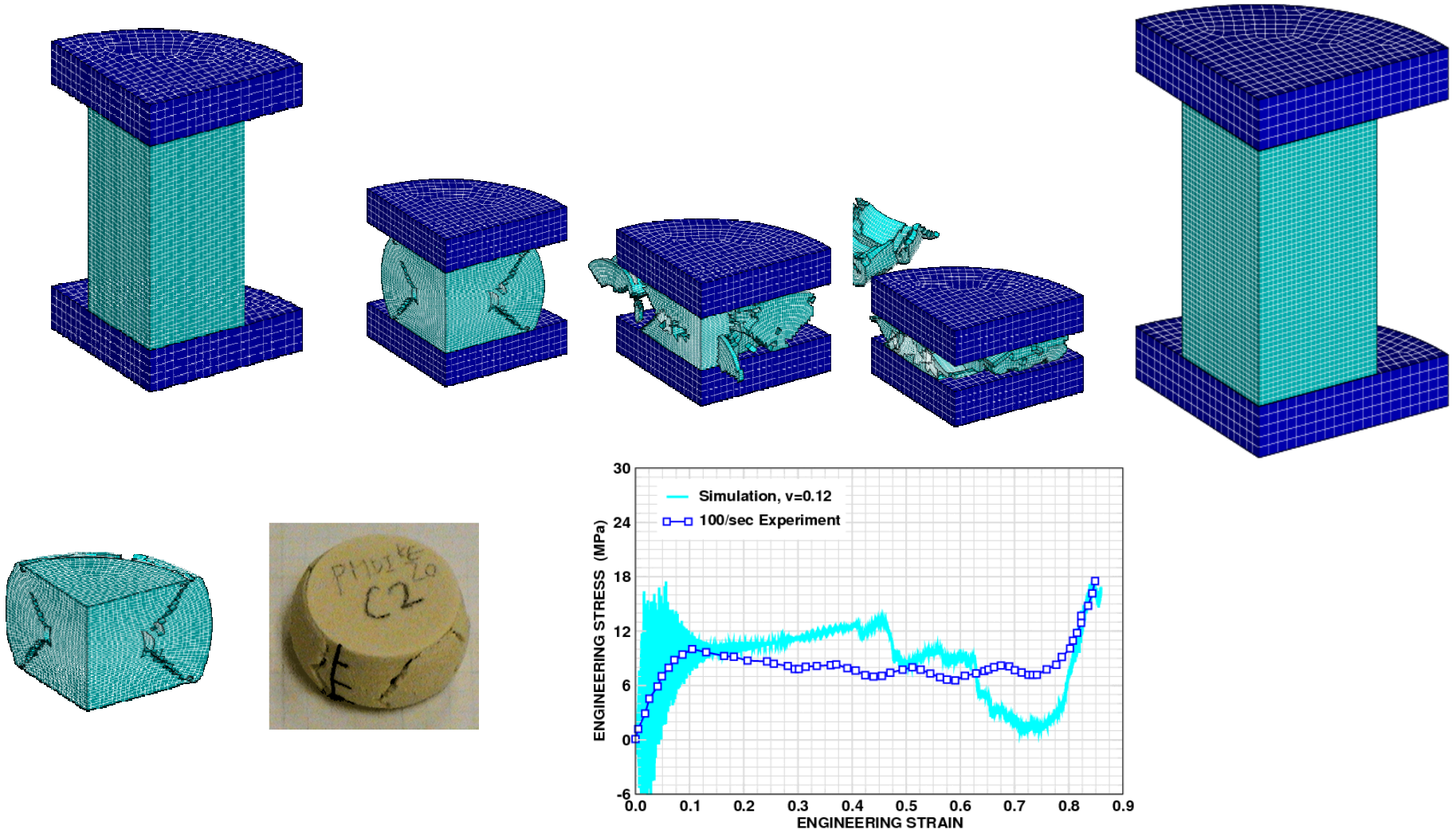
Uniaxial Compression at 21.1 C



Uniaxial Tension at 21.1 C

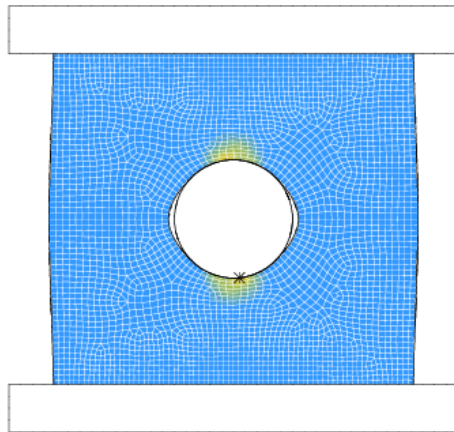
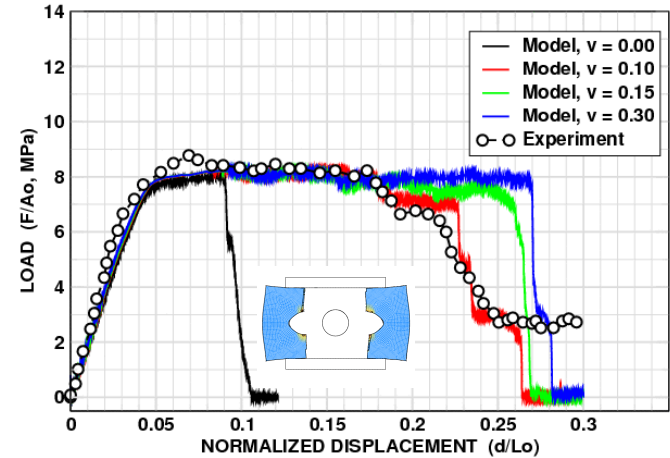
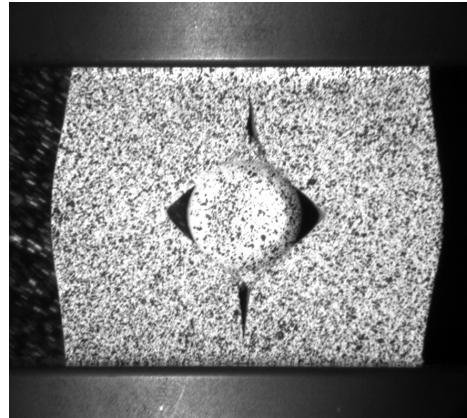
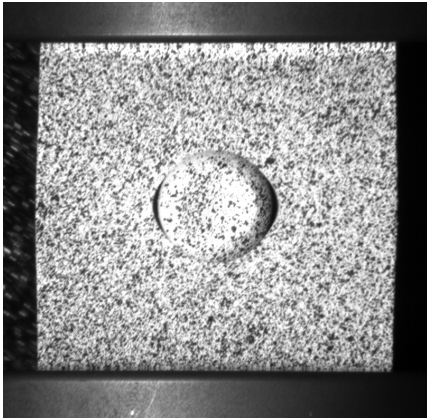


# PMDI20 Comparison of Model Predictions with Experiments

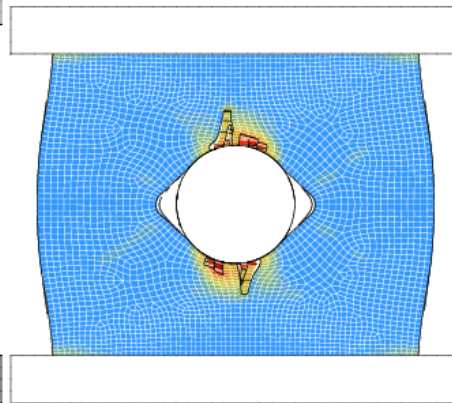




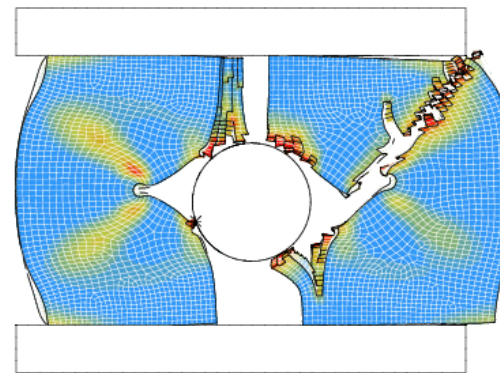
# Uniaxial Compression of PMDI Foam Block with Steel Rod



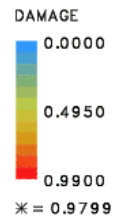
7% compression  
Peak load



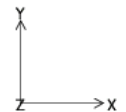
15% compression



24% compression

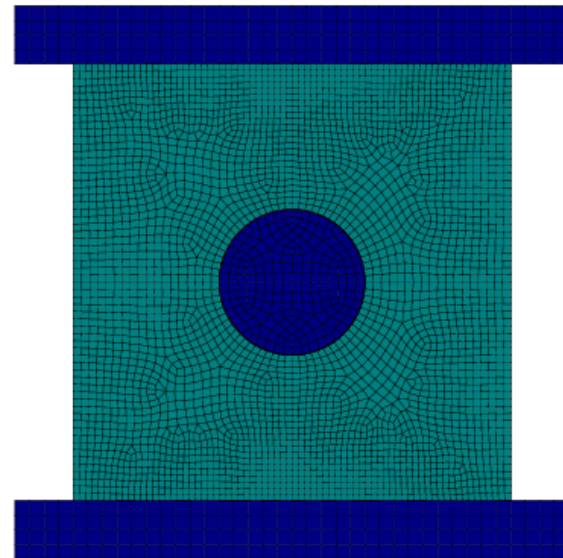
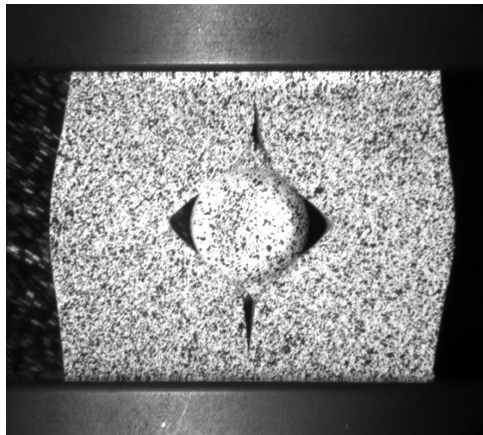
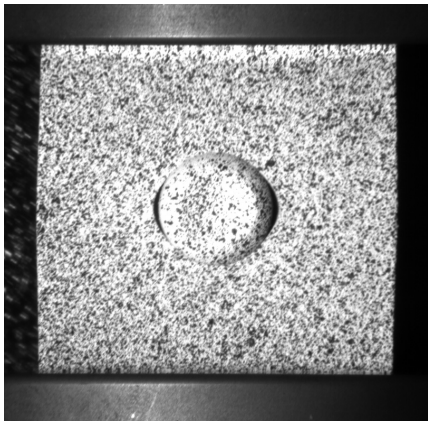


TIME 36.00E-3



Prediction Depends on Friction

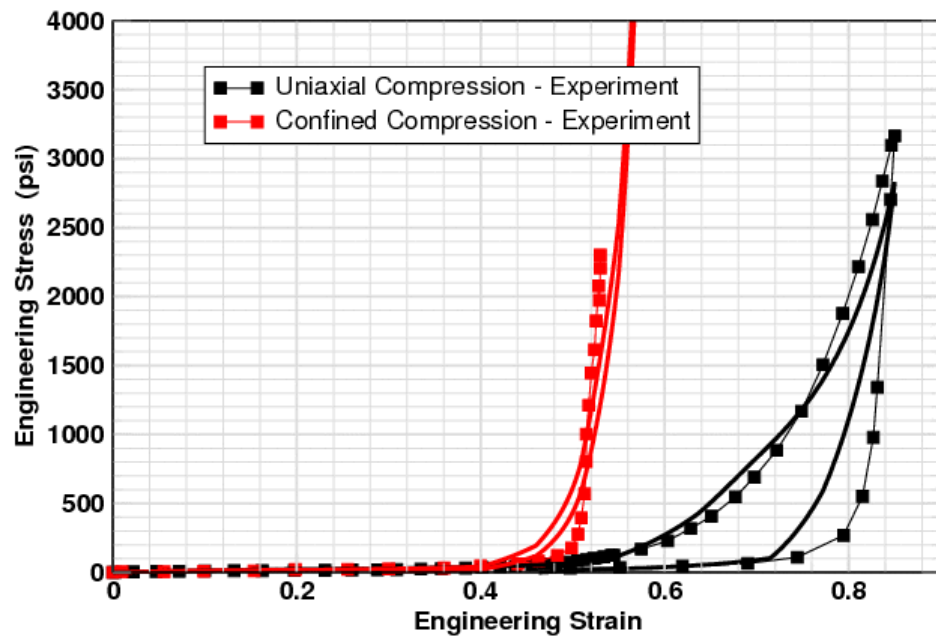
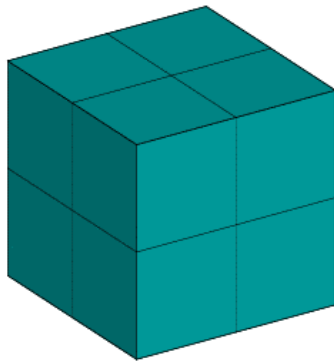
# Uniaxial Compression of PMDI Foam Block with Steel Rod



Comparison of Observed and Predicted Deformation

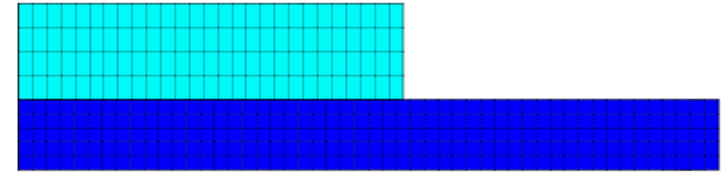
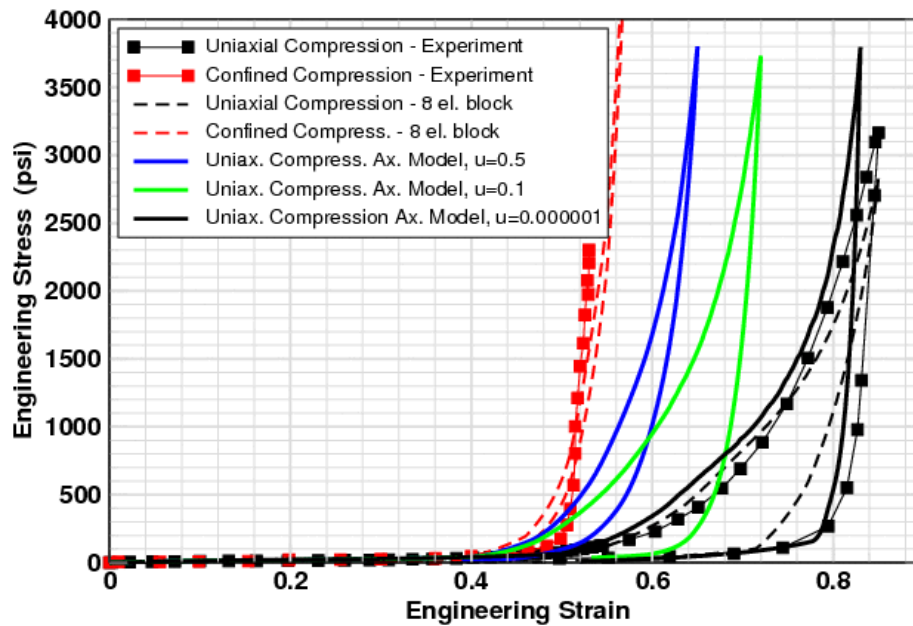
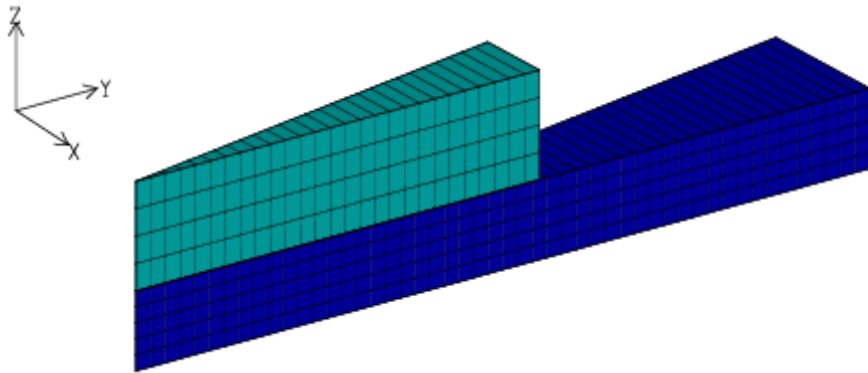
# Uniaxial Compression Cellular Silicone Foam

Simulation with 8-element block model

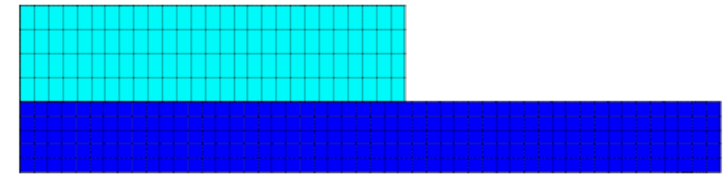


Experiment: Uniaxial Compression? of 1.10 inch diameter, 0.275 inch thick sample

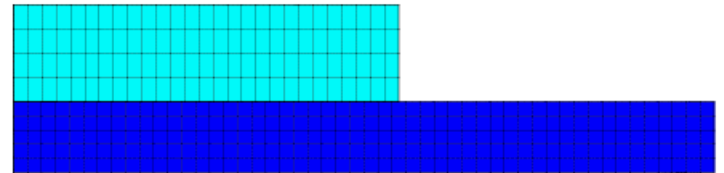
# Uniaxial Compression Cellular Silicone Foam



$\mu = 0.000001$  (black curve)



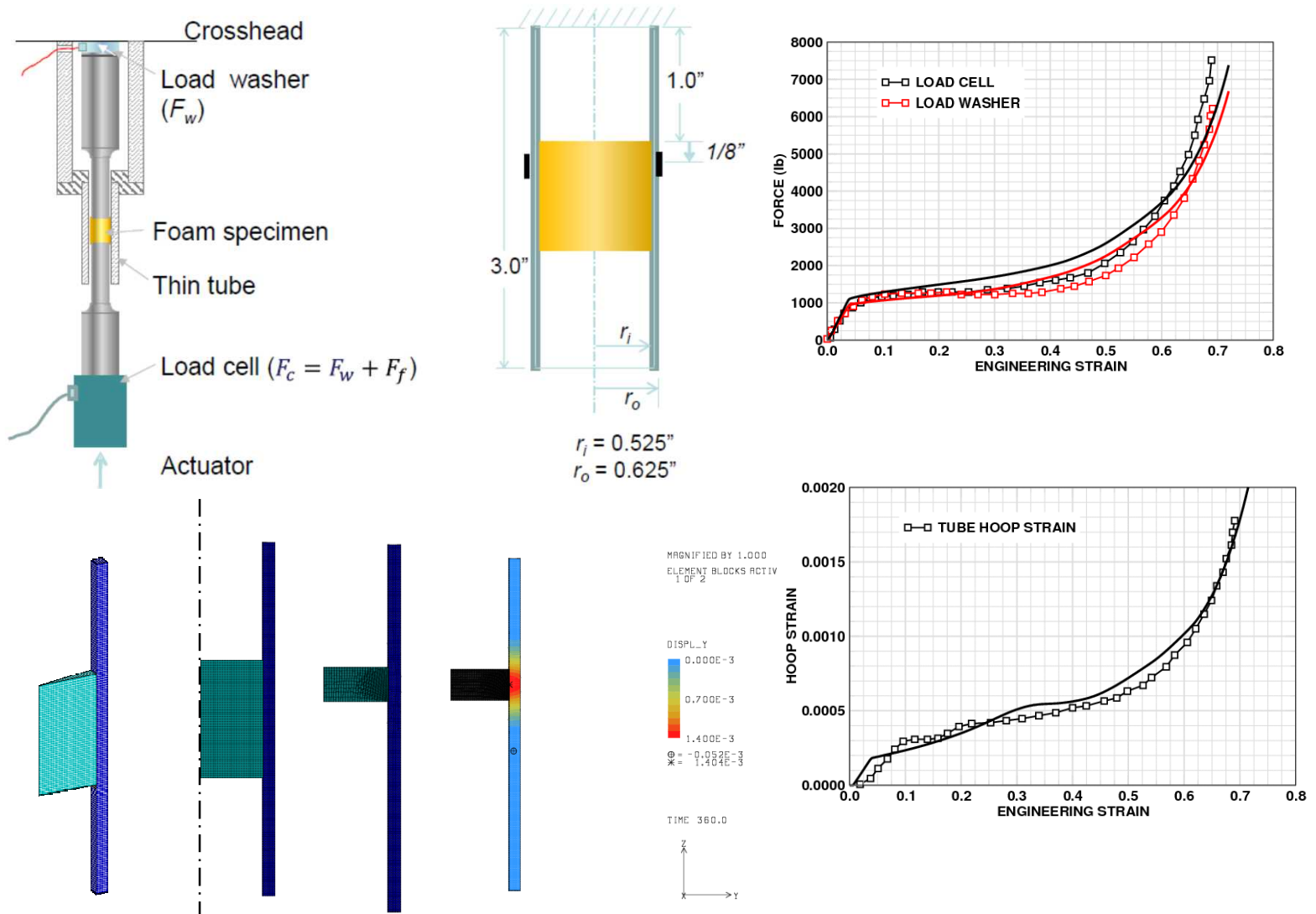
$\mu = 0.10$  (green curve)



$\mu = 0.50$  (blue curve)

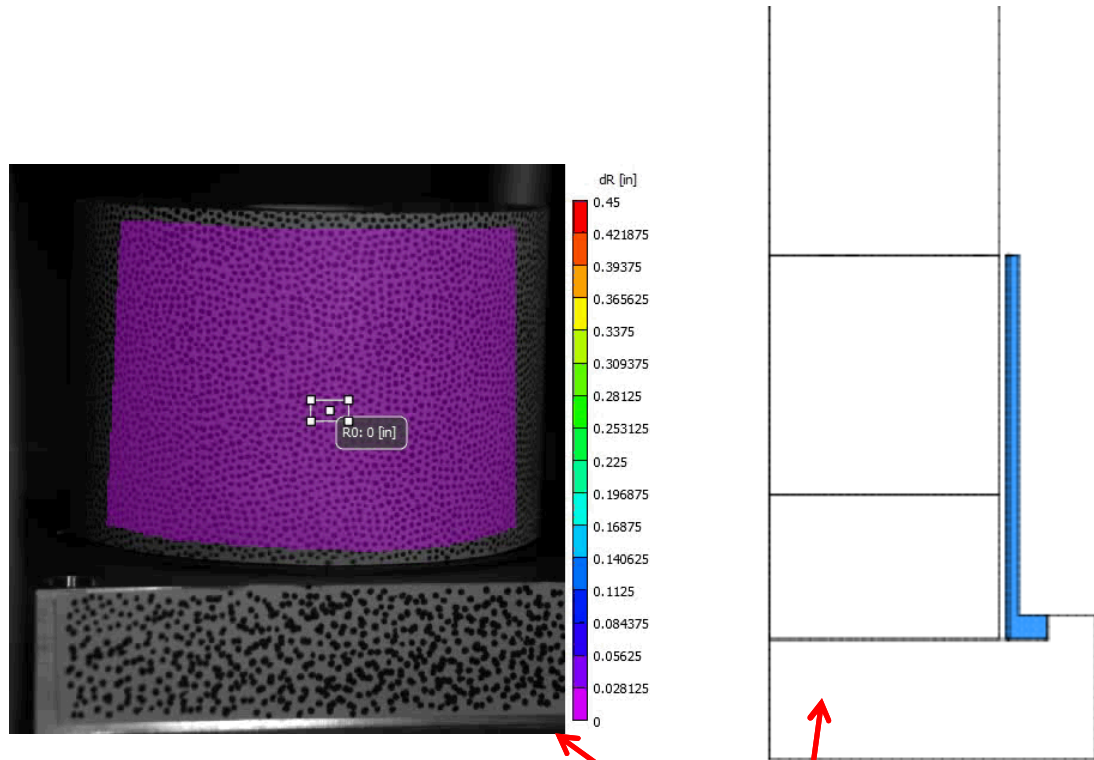
Uniaxial Compression of 1.10 inch diameter, 0.275 inch thick sample

# PMDI20 Comparison of Model Predictions with Experiments



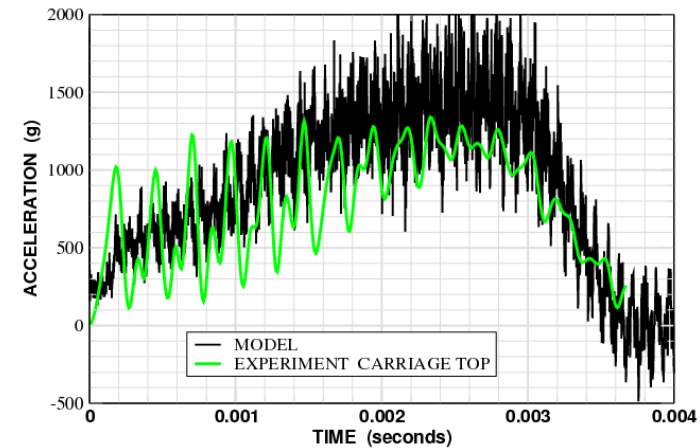


# TufFoam35 Validation Experiment – Drop Table Crush Test



Experiment:  
Matthew Spletzer, 1528  
Wei-Yang Lu, 8343

block underneath moves but  
NOT in model.



model a bit high on peak acceleration  
but pulse duration/shape is good.



- ❑ Polyurethane foam response depends on temperature and strain-rate
- ❑ Polyurethane foam that is Flexible at room temperature can become rigid at cold temperatures.
- ❑ Flex Foam model captures change from Flexible to Rigid
- ❑ Both Flexible and Rigid foams exhibit damage when crushed
- ❑ Flex Foam model is work in progress. Future work will be to capture effects of damage on foam moduli