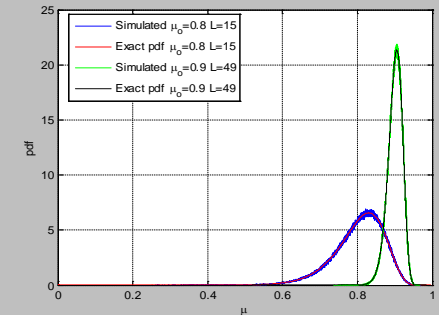
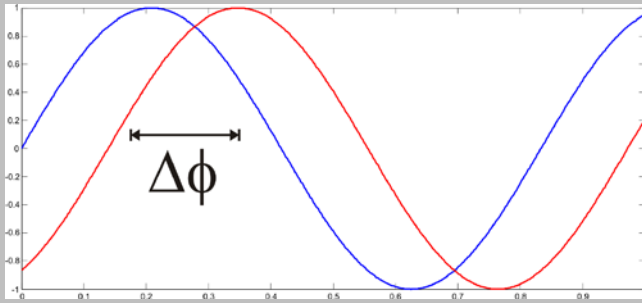


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# Coherence in Radar Processing for SPIE Radar Sensor Technology XXI

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# Overview

- Introduction
- Some coherence basics
- Sources of coherence loss
- Estimation of Coherence
- Conclusions

# Purpose

- Coherence is very important in radar imaging
  - Coherence within a pulse
  - Coherence between pulses
  - Coherence between images (which requires the previous two)
  - All of these are related subjects
- Although each of these are required and each are important the focus here is on developing a deeper background into SAR coherence between 2 images.
- In particular, want to emphasize radar effects on coherence for the radar system designer to maximize the radar coherence
  - Choices in system
  - Choices in processing
  - Choices in geometry

# Introduction

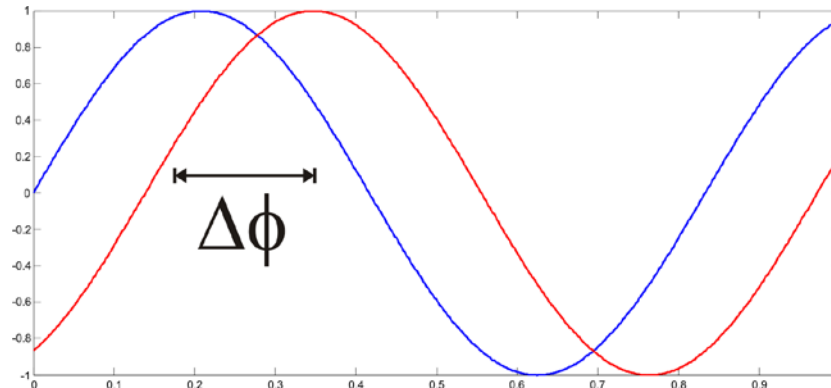
- Estimate of coherence attempts to measure the similarity between two (complex) synthetic aperture radar images.
  - Actually the measure of dissimilarities indicate potential change regions
- Coherence for change detection in synthetic aperture radar first discussed by Jakowatz et al 1996.
- Has been discussed and expanded by various authors into fields such as interferometric SAR and polarimetry
- History of estimating coherence has been around for a while:
  - The statistics of the estimator of coherence were developed early 1900s by Fisher
- Much of the coherence in radar we are interested in is strongly related to coherence topics in optics:
  - Young, van-Cittert, Zernike, Born, Wolf

# Introduction

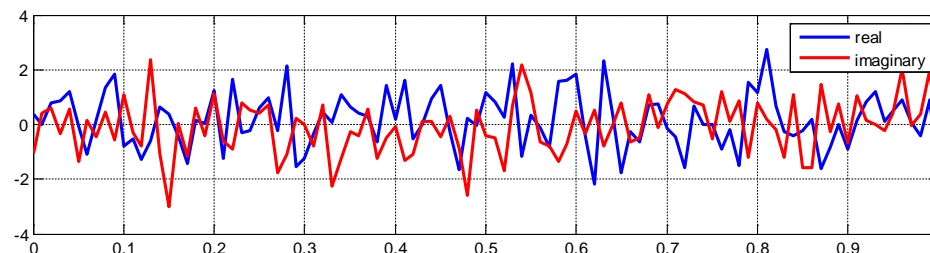
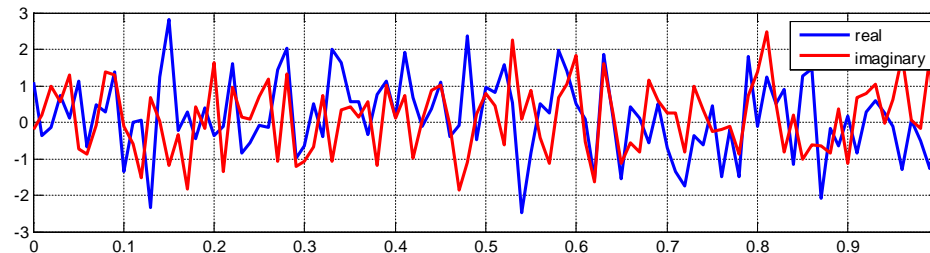
- (Complex) sinusoids can be described by amplitude frequency and phase
  - For given frequency, signal delays introduce phase shifts
  - These delays can be accounted for by adjusting the phase
  - Linear systems maintain the signals as superposition of (complex) sinusoids
- Maintaining coherence means we maintain consistency in the phase relationship between signals

# Coherent signals

- Sinusoids (just a phase difference):



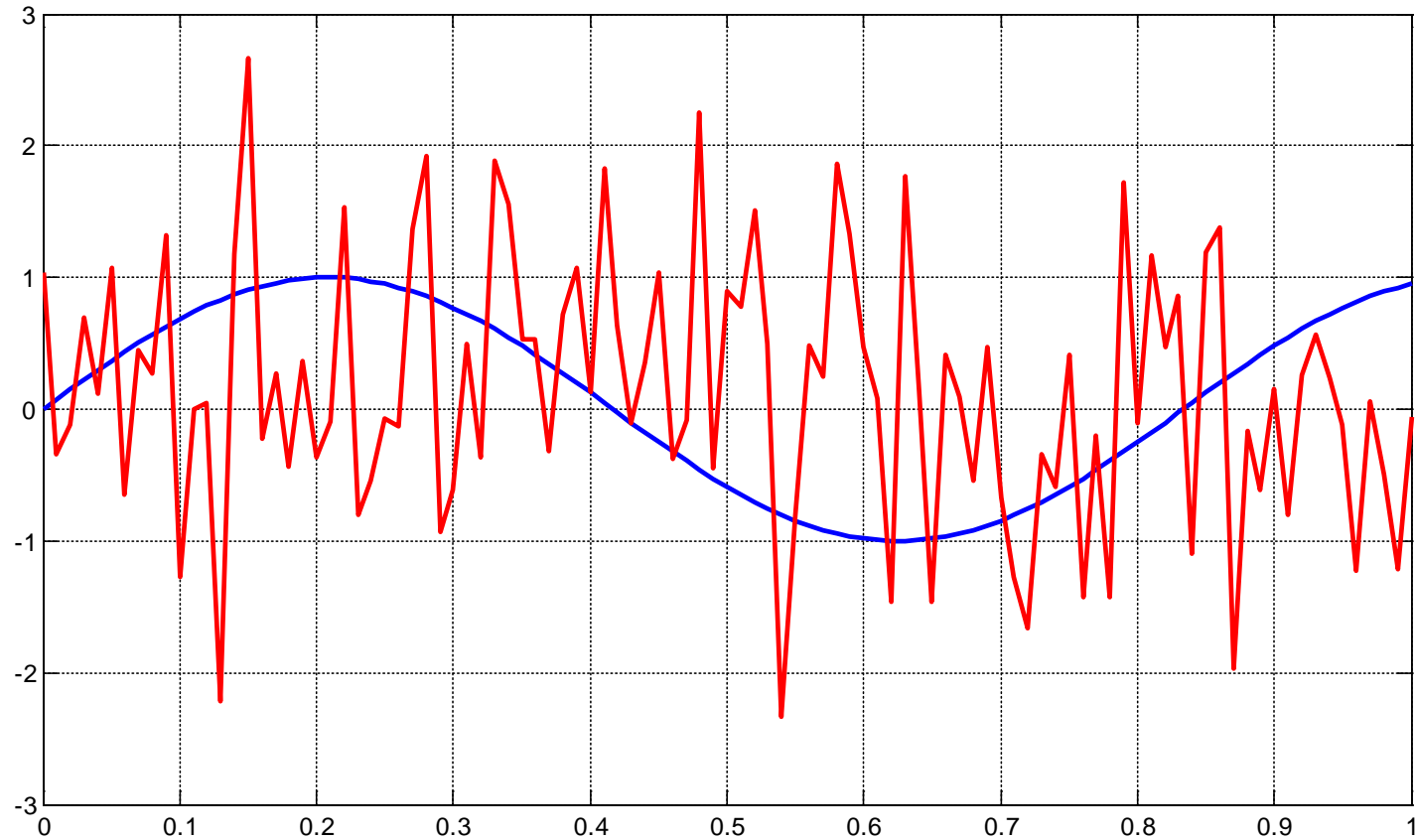
- Finite bandwidth random signals (just a phase difference):





# Incoherent signals

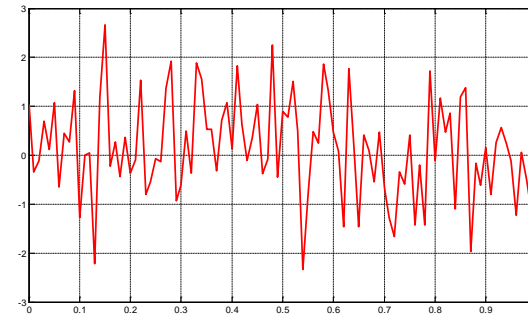
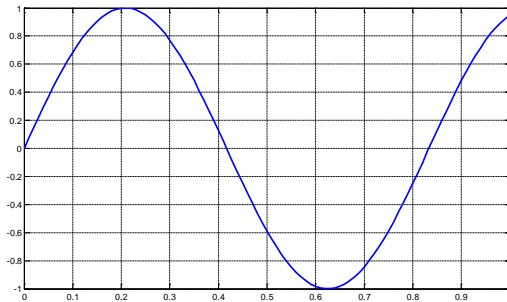
- No simple phase difference between:





# Introduction

- In general, signals are combinations of coherent and incoherent parts (partial coherence)



- We want to have a good estimator for the fraction of signal content that is coherent
- *NOTE that the fact that a signal is not coherent with another can be important information*
  - E.g., can be an indicator of improper performance of a radar system
  - We glossed over change indicator previously



# Introduction

- Brief summary
  - Coherence in complex sinusoids is a measure of phase consistency in time and/or space
  - Coherence applies to tones and random signals
    - Generally we are comfortable with coherence of pure tones
    - We also need to consider effectively a bandwidth of complex sinusoids with random amplitudes and phases
  - In general, coherence is a measure of statistical similarity
  - Permits observation of constructive and destructive interference
  - High coherence permits the observation of constructive and destructive interference fringes
  - Low coherence or loss of coherence destroys the ability to observe these fringes
  - We will see that the magnitude (modulus) of the coherence is the is an indication of the fraction of coherence between two signals



# Introduction

- Phase information in radars
  - The far-field return phase from a radar system has a term proportional to the range  $\propto \exp(j4\pi/\lambda \cdot r)$ 
    - i.e., contains relative geometry information
    - This geometry term is valuable in determining target location
  - The radar phase also captures information about the phase of the target; hence, valuable information about the target
    - Ideal point targets have a target phase that is constant over frequency and radar geometry
    - “Ideal” clutter has a phase that is consistent for a given ground location, at a given frequency when viewed from a specific geometry (unless disturbed), but is uncorrelated otherwise
- Coherence in radars
  - Hence we want to carefully control waveforms, oscillators, sampling, etc. to maintain the radar phase information

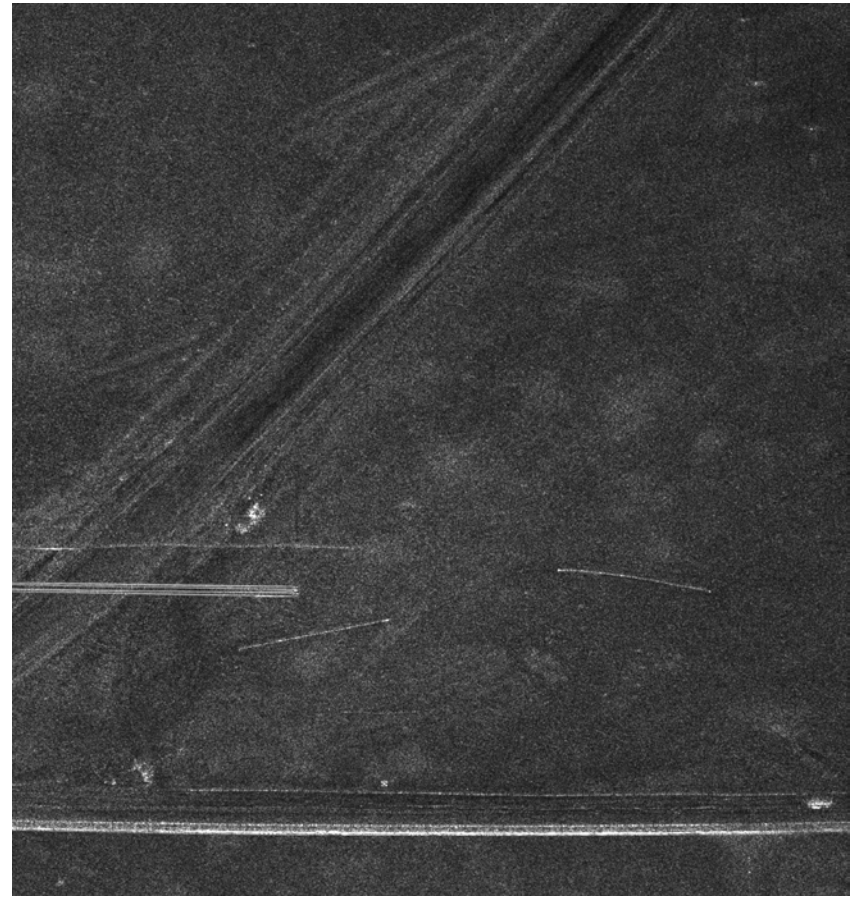


# Introduction

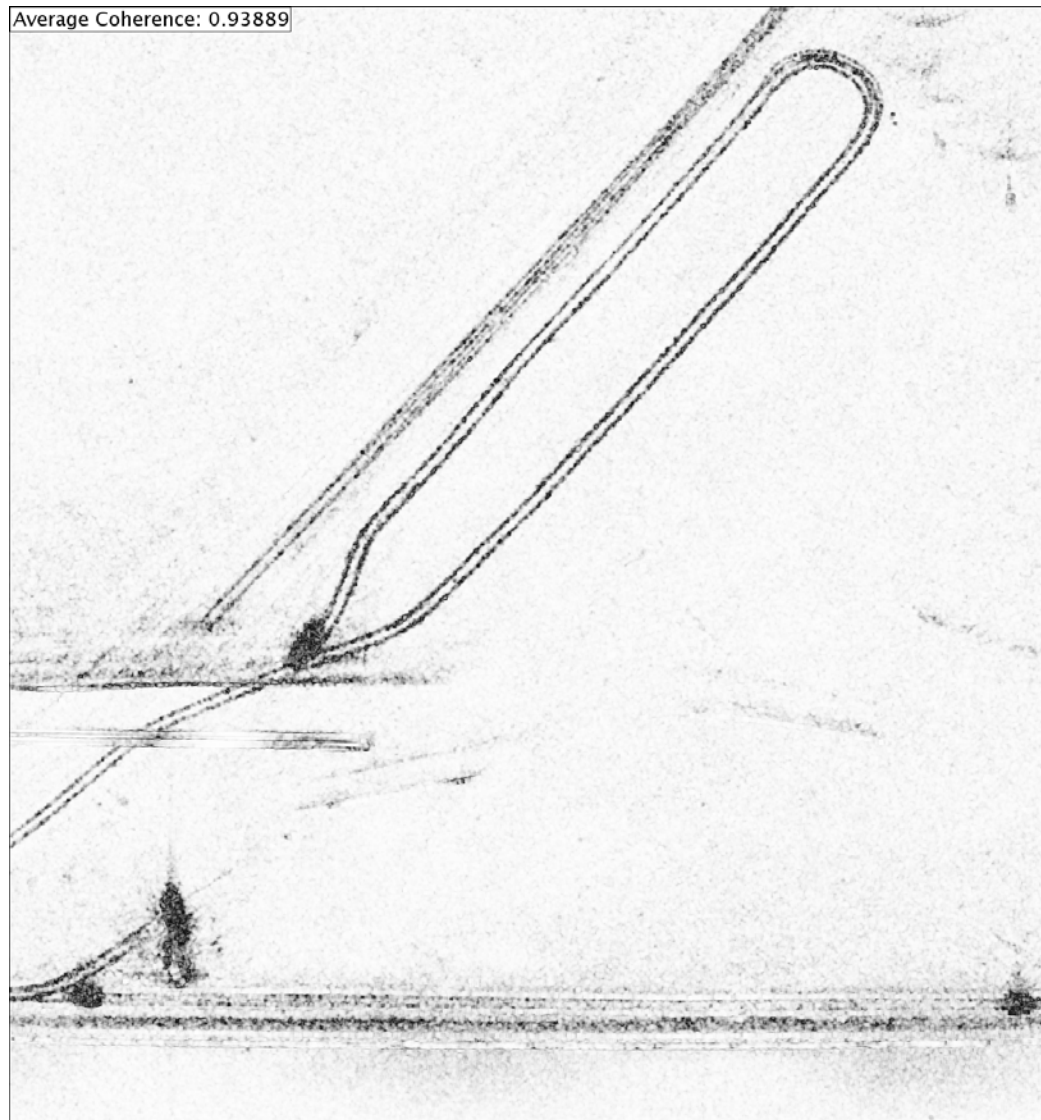
- Why is this coherence important?
- *If we know the phase difference* between signals we can adjust the phase and add them together
  - This results in “constructive interference” for the signal of interest (coherent part) and “destructive interference” (in the mean) between the unwanted signals (incoherent part)
  - Make use of this between pulses for image formation in synthetic aperture radar (SAR)
  - Make use of this between images for tomographic (3D) processing in SAR
  - Can purposely add images out of phase for clutter cancellation in moving target detection
- If we do not know the phase difference, we can estimate it:
  - Between pulses for autofocus
  - Between images for interferometric SAR (InSAR or IFSAR)
  - Between images for polarimetric SAR
  - Between images for direction-of-arrival estimation

# Coherent change detection (CCD)

- Another important application is the detection of change between images (coherent change detection – CCD)
  - Application of detection of loss of coherence between images



# Coherent change detection (CCD)



# Introduction

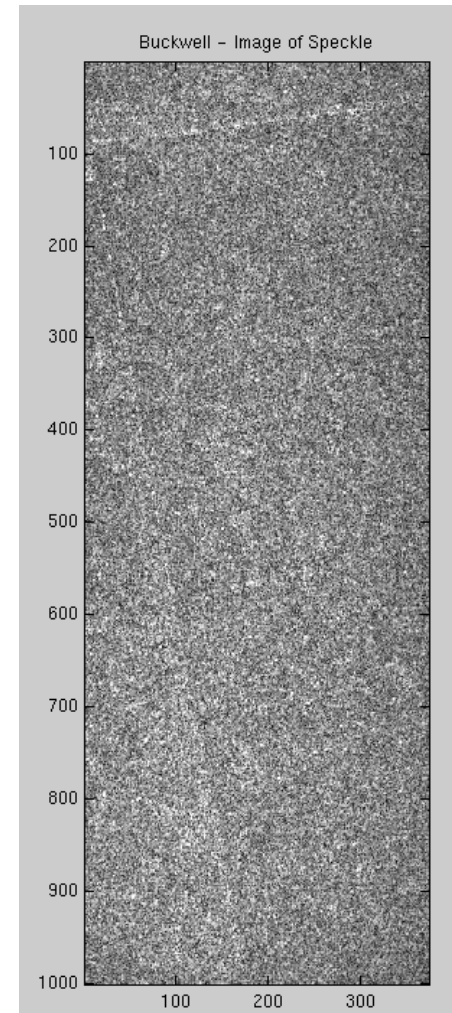
- ***All of these applications drive the radar system engineer to require that the radar system itself maintain high coherence***
- The magnitude of the coherence becomes important
- Most of the rest of the discussion will be on this fact

# Coherence and speckle

- Our signal of interest is coherent radar “speckle”
  - This is related to the subject of mutual coherence from incoherent sources in optics
- Since this is the case, a very powerful tool in optics co-opted for radar coherence is the van-Cittert Zernike theorem
  - Refer to Rodriguez, et al. for application to radar
  - Relates the spatial coherence to the Fourier transform of the intensity distribution of the target (i.e., the resolution cell)
  - Although we will not go into detail, various results presented here can be derived from the van-Cittert Zernike theorem

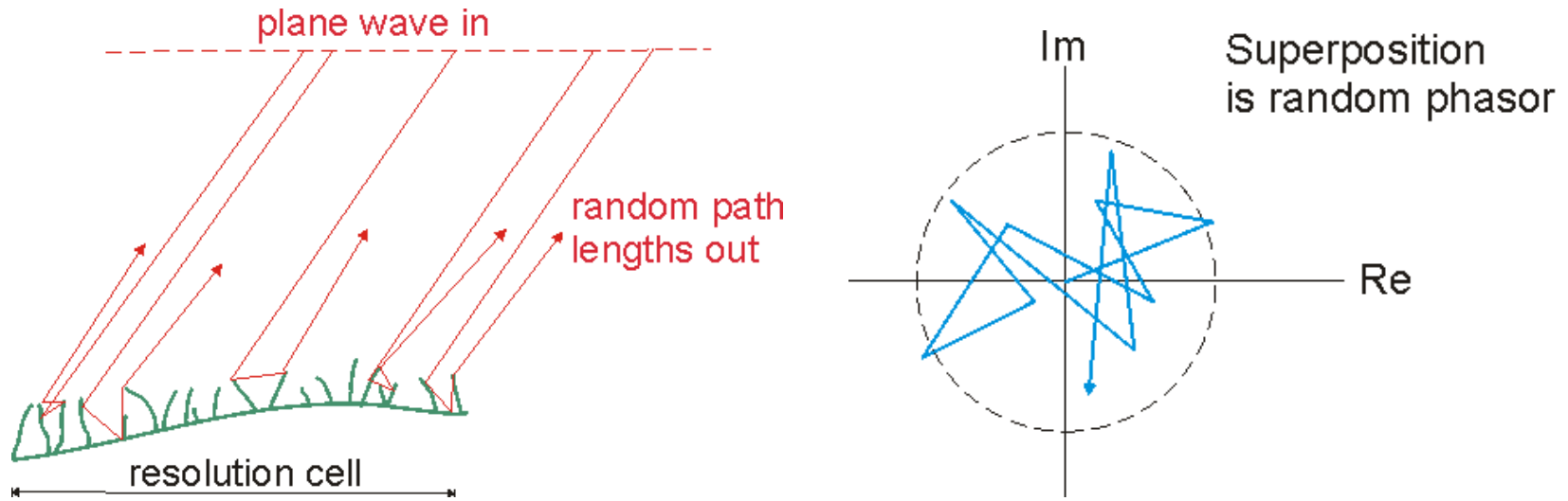
# Coherence basics - speckle

- Coherence depends greatly upon the target characteristics
- Our target of interest today is so called “random clutter”, aka “uniform clutter”, “speckle”
- Speckle derives from coherent imaging systems
- It gives the familiar “salt and pepper” look to SAR images
- Speckle is actually a signal and not a nuisance, particularly in coherence processing between 2 images



# Coherence basics - speckle

- Speckle comes from the superposition of coherent (complex) signals scattered from “rough” objects on the order of a wavelength within a single resolution cell.



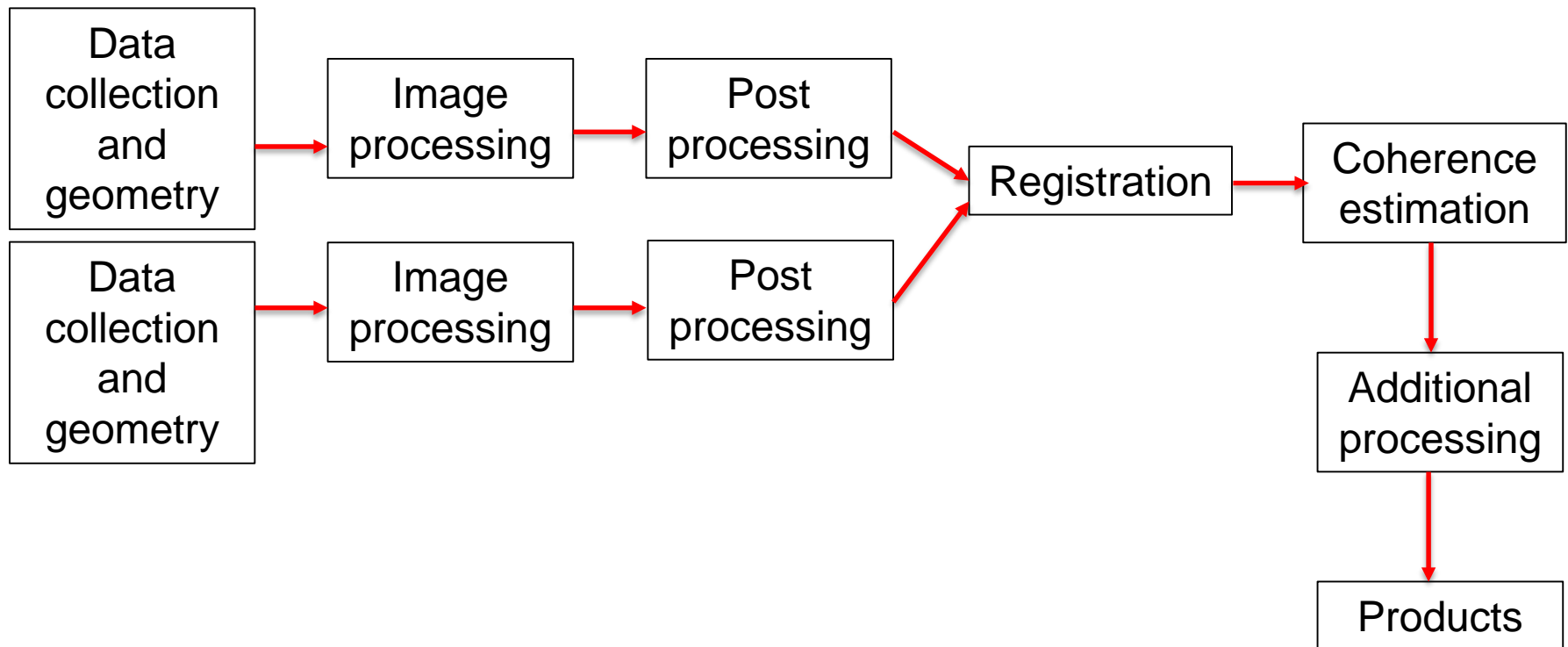
- NOTE that if we look at the resolution cell from same direction should get the same phasor result if the cell is not disturbed
- No two adjacent resolution cells should have the same phasor result
- The combination of these two things makes the complex coherence between two images interesting

# Coherence basics - speckle

- “Disturbances” due to loss of coherence in the radar, the collection geometries, and processing “corrupt” the speckle phasors between the 2 image which introduces decorrelation
- This “corruption” shifts part of the correlated part of the speckle signal to decorrelated speckle leading to coherence loss
- This loss is a serious issue because the corrupted “signal” results in “noise” that is the same strength as the desired “signal”
- Sources of coherence loss are presented later

# Coherence basics - processing

- The diagram below shows basic processing to estimate coherence between two images.



# Coherence basics – estimation

- Important to recall that we are *estimating* the coherence between two images
- Distinguish between:
  - the population coherence (true value if infinite number of samples available) – will use symbol  $\mu_0$
  - the sample coherence (estimate from limited data) – will use symbol  $\hat{\mu}$

- The population coherence is given by: 
$$\mu_0 = \frac{\langle x_1 x_2^* \rangle}{\sqrt{\langle |x_1|^2 \rangle \langle |x_2|^2 \rangle}}$$

- The sample coherence is from the MLE of the population coherence given by:

$$|\hat{\mu}| \approx \frac{\left| \sum_{n=1}^{L-1} x_{1,n} x_{2,n}^* \right|}{\sqrt{\sum_{n=1}^{L-1} |x_{1,n}|^2 \sum_{n=1}^{L-1} |x_{2,n}|^2}}$$

- As with MLE's in general, in the limit as  $L \rightarrow \infty$ , the sample coherence goes to the population coherence

# Coherence loss - overview

- Many errors lead to decorrelation (not exhaustive list follows):
  - Thermal noise
  - Temporal decorrelation
  - Observation geometry differences
  - Uncompensated motion errors
  - Registration errors
  - Processing errors
  - Interpolation errors
  - Multiplicative noise
  - Polarization mismatch
  - Errors in the estimator
  - System trade-off errors – e.g., ambiguities allowed
- We will consider some of these in the following slides
- Not all coherence errors lead to “loss” – there are insidious errors that lead to false coherence gain

# Coherence loss - basics

- Coherence is a linear measure of the consistency between the 2 images
- We divide the information content between 2 images into 2 parts: 1) a correlated part “ $c$ ”; 2) an uncorrelated part “ $d$ ”:

$$\mu = \frac{c}{c + d}$$

- For coherence loss, we are interested in the terms that decrease  $c$  and increase  $d$  – i.e., convert  $c$  into  $d$
- Generally, this follows a product form:

$$\mu = \frac{c_1}{c_1 + d_1 + d_2} = \left[ \frac{(c_1 + d_2)}{(c_1 + d_2) + d_1} \right] \left( \frac{c_1}{c_1 + d_2} \right) = \mu_1 \cdot \mu_2$$

# Coherence basics - accumulation

- One of the important points in coherence is that various errors that cause decorrelation between images accumulate (and rather quickly)
- It is NOT sufficient to limit decorrelation in only one or even a few areas, such as registration errors, or flight geometry restrictions – you need to consider all the potential error sources
- The total accumulation of coherence is given as the product of coherences due to individual error sources
  - For example, if our SNR is such that we would get 0.9 coherence and we set our registration accuracy in range to 0.9 and registration in azimuth to 0.9, then the best coherence possible is 0.73 (as we will see, this is the equivalent of 4 dB SNR and perfect registration)



# Coherence loss – thermal noise

- The coherence due to thermal noise is given by (assuming equal SNR in both images):

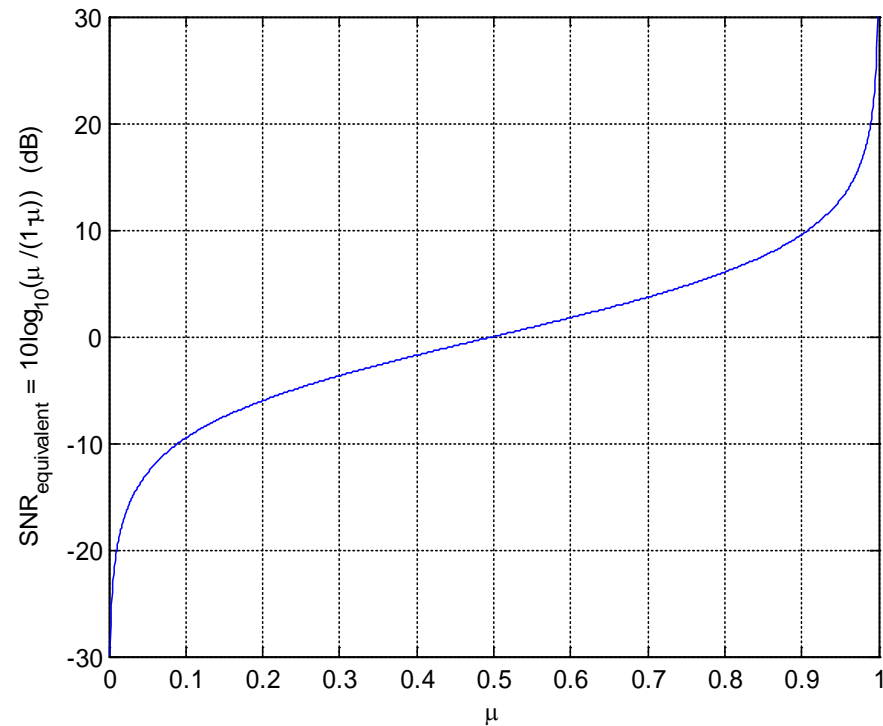
$$\mu_{snr} = \frac{s}{s+n} = \frac{snr}{snr+1}$$

- If the SNRs differ between images:

$$\mu_{snr} = \sqrt{\frac{snr_1}{snr_1+1}} \sqrt{\frac{snr_2}{snr_2+1}}$$

# Coherence loss – thermal noise

- Since radar engineers tend to be comfortable with SNR, we can invert the equations given a total coherence to an equivalent SNR.



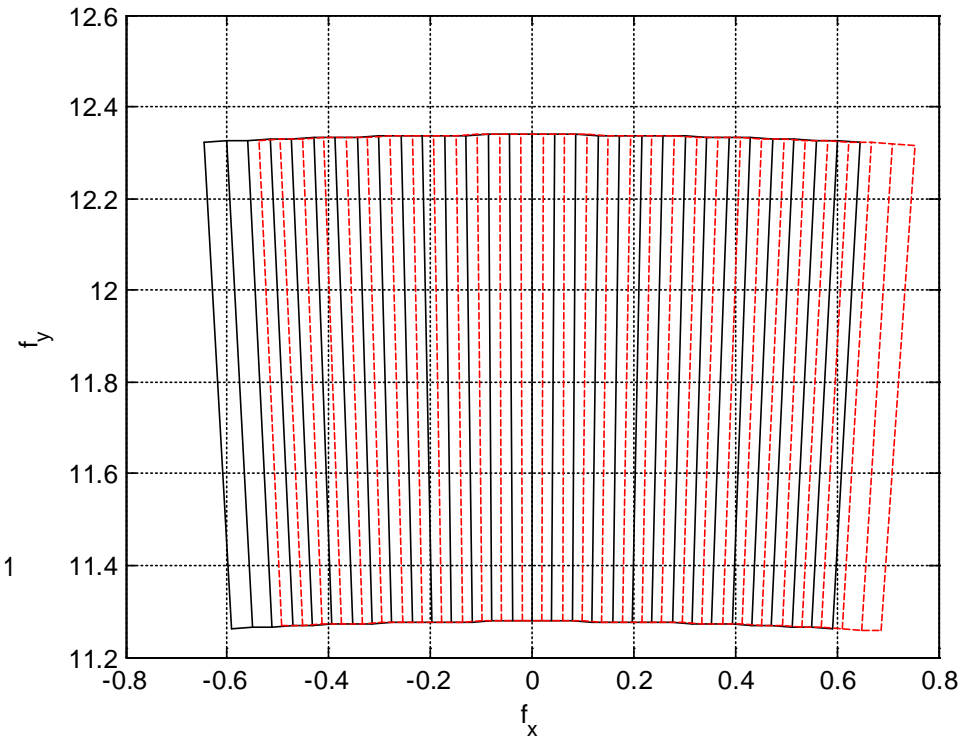
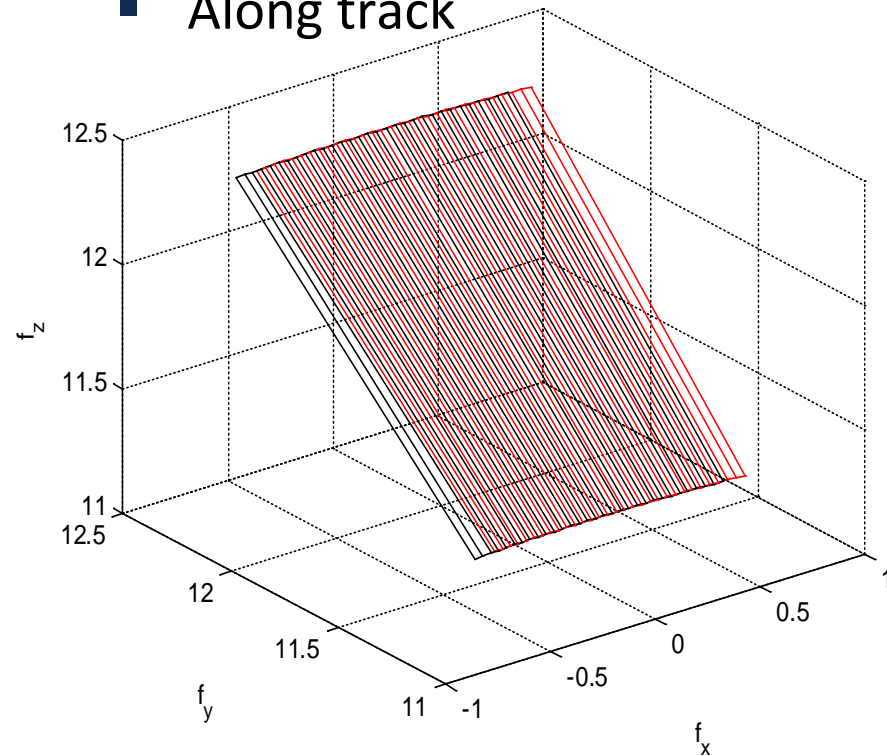
# Coherence loss - geometry

- We need to collect (or process to) very similar geometries in order to achieve high coherence (i.e., to limit the effects of geometry decorrelation on CCD images)
- This is the first of a list things we consider for coherence loss issues which convert speckle “signal” to speckle “noise”

# Coherence Loss – Geometry

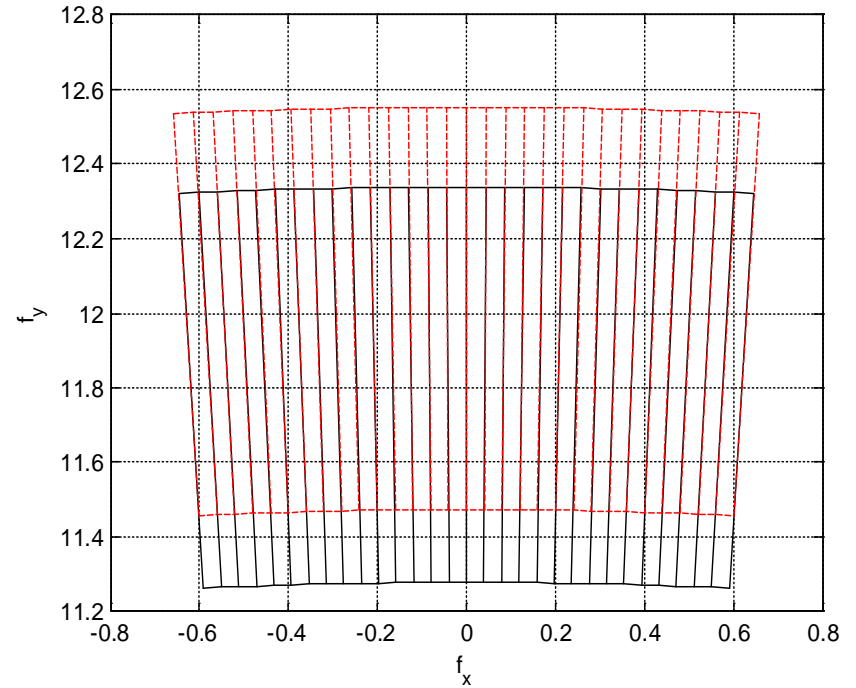
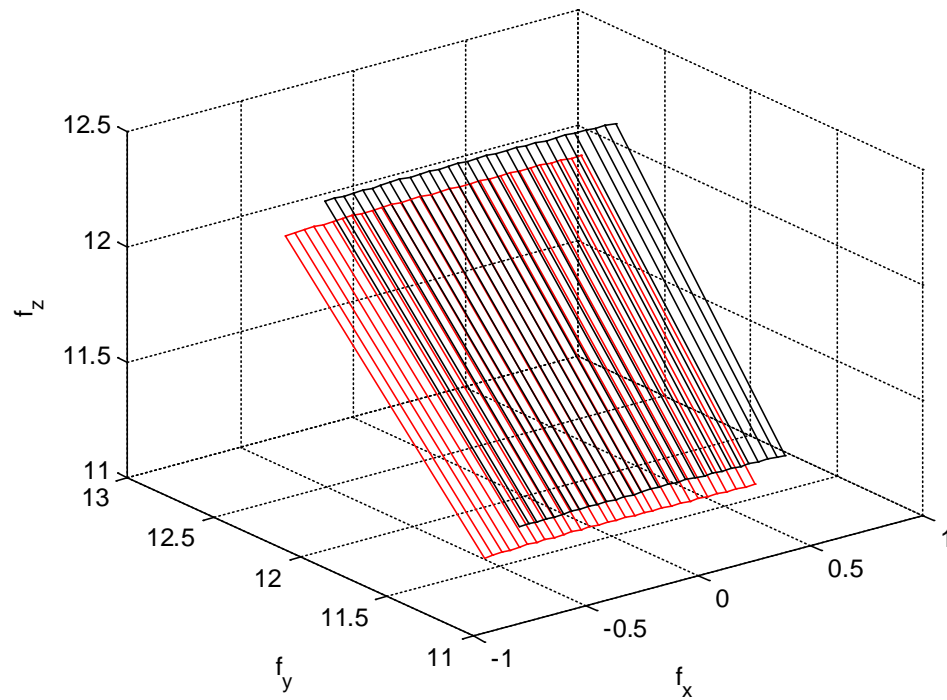
(spatial frequency viewpoint)

- A good way to look at the geometry issue is as overlap of spatial frequencies
- Along track



# Coherence Loss – Geometry

- Cross track





# Coherence Loss – Geometry

(flight path match)

- NOTE that alignment of the spatial frequencies are necessary in both cross-track and along track
- Alignment of the spatial frequencies can be accomplished by careful control of the repeat passes
- Alignment of the spatial frequencies can also be controlled to some extent by motion compensation (if the radar is designed and set up properly, e.g., using the same nominal grazing angles for each pass)
- Alignment of the spatial frequencies can also be controlled to some extent by processing
- The latter two are affected by terrain

# Coherence Loss – Geometry

(coherence equations)

- The following equation shows the variables that affect cross-track geometric correlation (under several assumptions and caveats):

$$\mu_{geom,xtrk} = \begin{cases} 1 - \frac{2\rho_r \tan(\psi + \eta) \Delta\psi}{\lambda} & \frac{2\rho_r \tan(\psi + \eta) \Delta\psi}{\lambda} \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- The following equation shows the variables that affect along-track geometric correlation (under several assumptions and caveats):

$$\mu_{geom,atrk} = \begin{cases} 1 - \frac{2\Delta\theta_{offset} \rho_a \cos\psi}{\lambda} & \frac{2\Delta\theta_{offset} \rho_a \cos\psi}{\lambda} \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Resolutions, aperture offsets, and grazing angles all affect this coherence

# Coherence Loss – Geometry

(aperture trimming)

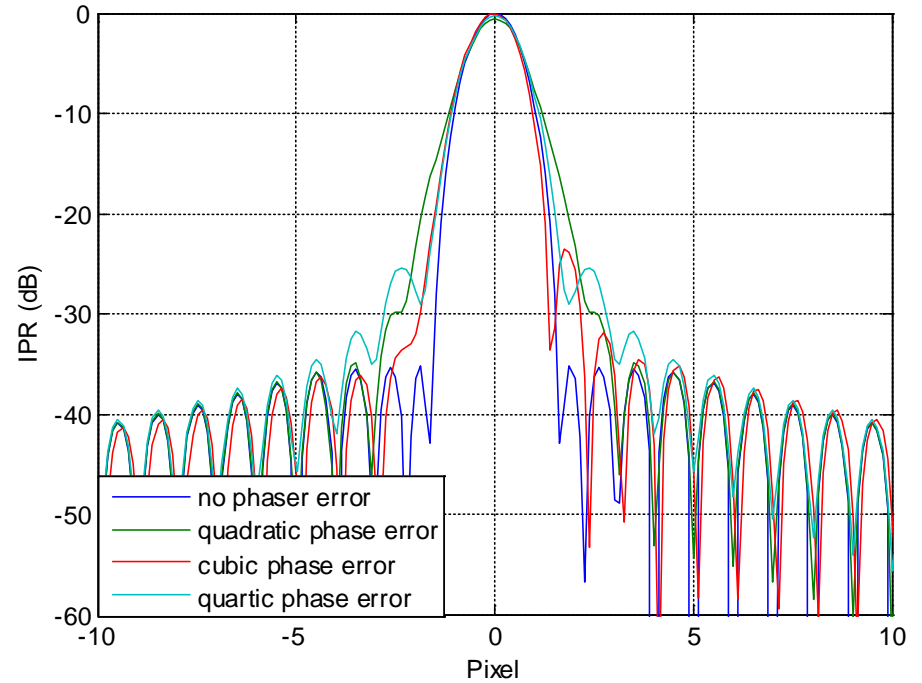
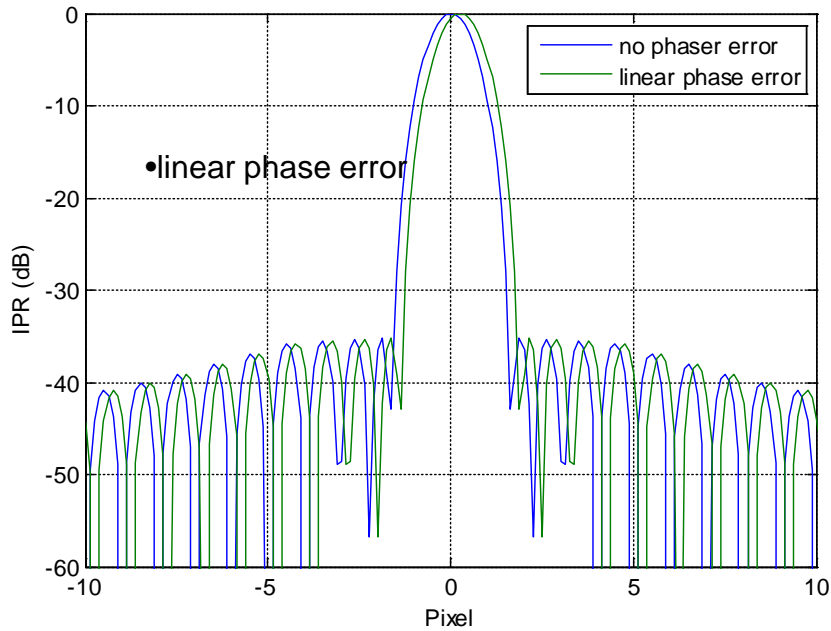
- Coherence can be improved in processing by “aperture trimming”
  - Aperture trimming reduces resolution
- There are actually two coherence loss terms for the coherence (if you do not trim):
  - The terms due to spatial frequency (aperture) mismatch
  - Reduction in coherence in estimation with linear phase term

# Coherence Loss – Motion Errors

- Coherence is strongly affected by uncompensated motion error differences between the apertures
- The difference in uncompensated motion errors results in decorrelating the speckle “signal” into speckle “noise”
- Errors include, for example:
  - Linear (registration errors)
  - Quadratic
  - Cubic
  - Quartic
  - Sinusoidal
  - Random

# Coherence Loss – Motion Errors

(Examples for IPRs)



# Coherence Loss – Motion Errors

(equation)

- The coherence equation for the errors is rather simple:

$$|\mu_{IPR}| = \frac{\int x_1(t) x_2^*(t) dt}{\sqrt{\left(\int x_1(t) x_1^*(t) dt\right) \left(\int x_2(t) x_2^*(t) dt\right)}}$$

- This equation is a very powerful equation and has many uses beyond CCD
- Lower order errors tend to be more significant (e.g., registration and quadratic errors)

# Coherence Loss – Motion Errors

(linear or registration)

- The coherence equation in this case falls out to the simple correlation equation with the independent variable as the shift:

$$|\mu_{reg}| = \frac{\int x_1(t) x_1^*(t - \tau) dt}{\left| \int x_1(t) x_1^*(t) dt \right|}$$

- As an example, assuming rectangular IPRs:

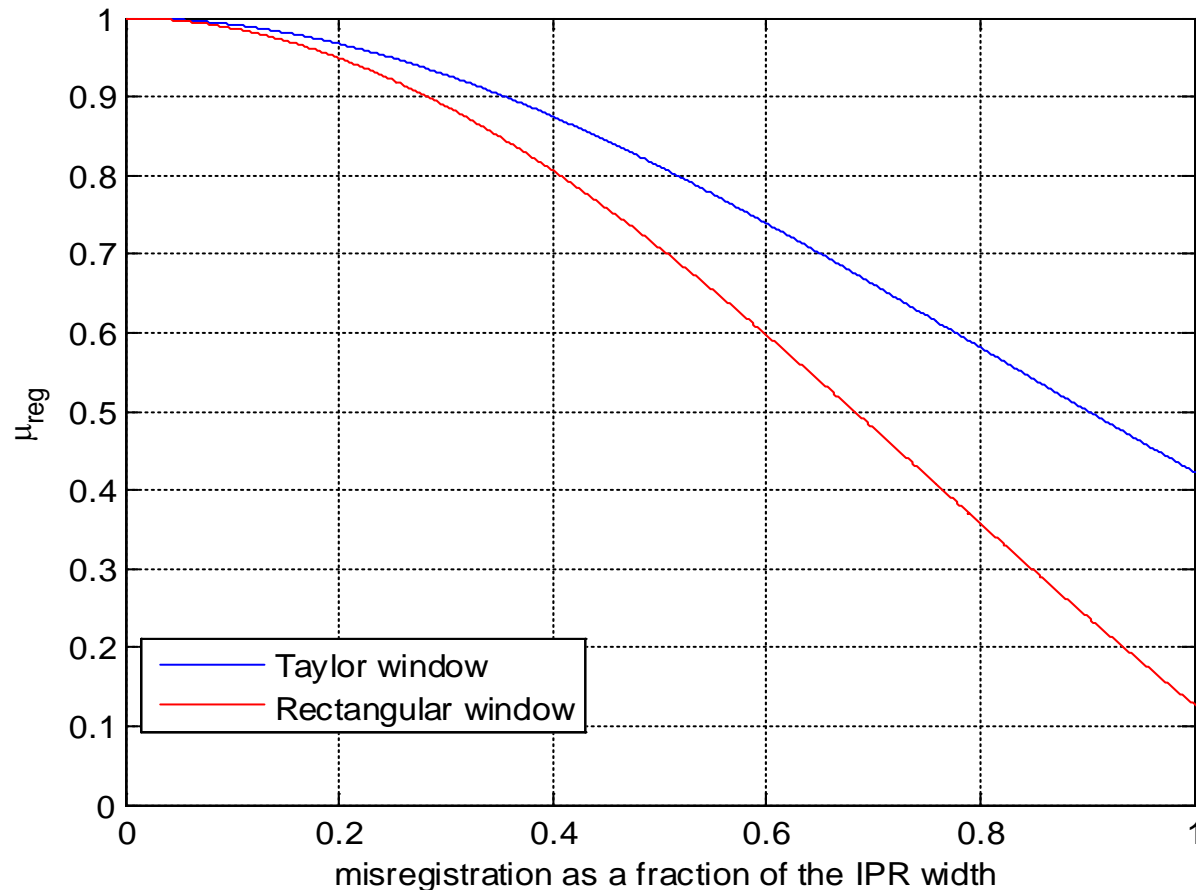
$$|\mu_{reg}| = \begin{cases} 1 - \left| \frac{\delta}{\rho} \right| & \text{for } \left| \frac{\delta}{\rho} \right| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- We can see from this equation that the important quantity is the misregistration as a fraction of the resolution cell size, for example, for about 0.9 coherence from registration we want registration less than  $1/10^{\text{th}}$  of a resolution cell!
- NOTE that we have to worry about registration in both range and azimuth

# Coherence Loss – Motion Errors

(linear or registration)

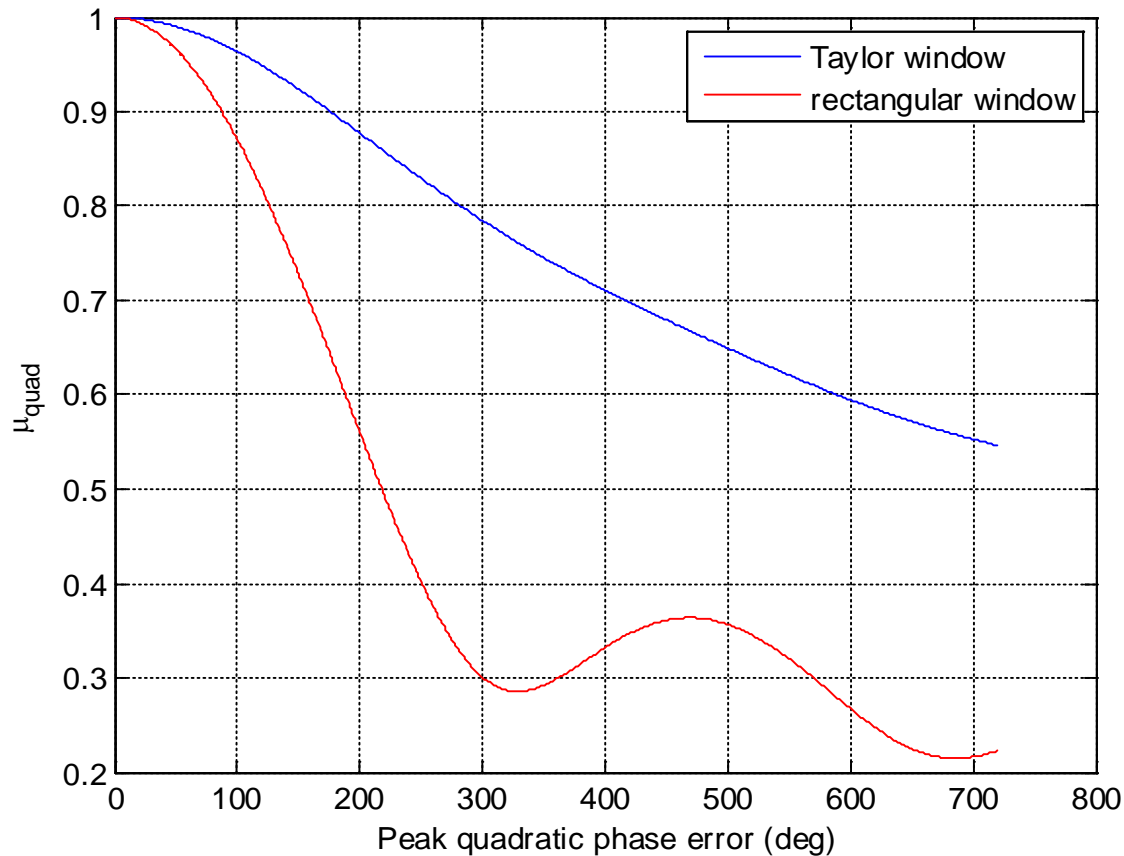
- Coherence for the Taylor and rectangular windows:



# Coherence Loss – Motion Errors

(quadratic phase error)

- Coherence for the Taylor and rectangular windows:



# Coherence Loss – Processing Errors

- As noted with focus processing errors can lead to the phase error differences between
  
- Other sources of processing errors include:
  - Range-walk differences
  - Interpolation errors
  - Multiplicative noise (MNR)
  
- Will briefly discuss interpolation errors and MNR

# Coherence Loss – Processing Errors

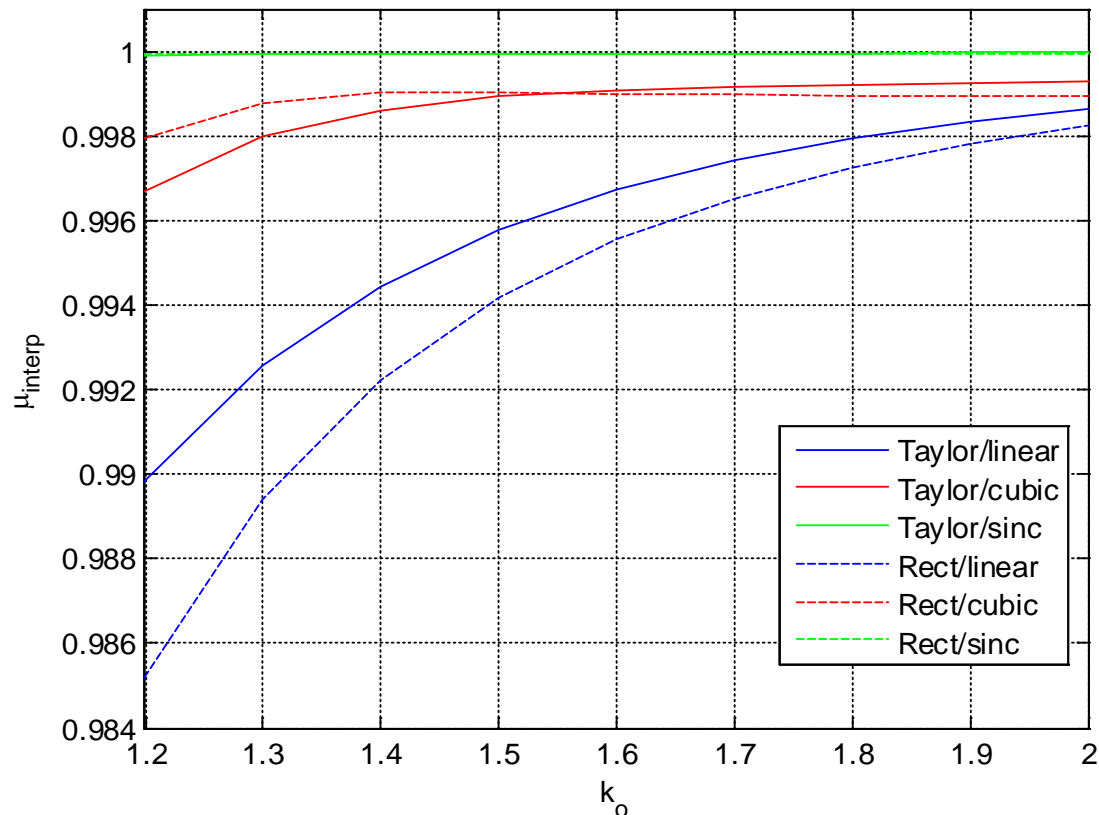
(interpolation)

- Interpolation to warp one image to match the other is required to maintain coherence between two images from different collections
  - Necessary because motion is not known precisely for both images
- Errors in interpolation is another source of differences between the images that can lead to decorrelation
- Need to be wise about interpolation choices
  - The choice of interpolation kernel and length are important in avoiding coherence loss
  - Oversampling of the image is also important!

# Coherence Loss – Processing Errors

(interpolation)

- These values are optimistic
- Recall that registration needs to be accounted for in both range and azimuth



# Coherence Loss – Processing Errors

(MNR)

- It is assumed in this analysis that the multiplicative noise is random, in this case the approximate coherence equation is:

$$|\mu_{mult}| \approx \frac{1}{(1 + mnr)}$$

- However, sidelobes, ambiguities, and quantization noise, etc., *can be correlated* which itself disturbs coherence change detection
- The next viewgraph shows ambiguity errors that lead to BOTH correlated and decorrelated errors!

# Coherence Loss – Processing Errors

(MNR – ambiguities example – at the right)



# Coherence Loss – other errors

- Polarization mismatch between passes can cause coherence loss
  - Usually can be kept small
  - Might have to watch for 2-axis gimbal
- Temporal decorrelation (e.g., due to weather, time, etc.) can be significant but only recently has there been good work characterizing its effect
  - It is very dependent on the scene content
  - Is very dependent upon wavelength (worse at higher frequencies)
- RFI

# Coherence Loss – accumulation

- Recall that the total coherence is the product of the result of all of these coherence sources

$$\mu = \mu_{snr} \cdot \mu_{temporal} \cdot \mu_{ipr} \cdot \mu_{geom} \cdot \mu_{interp} \cdot \mu_{mnr} \cdot \mu_{pol}$$

- This is the important equation for system designers to keep in mind!
- Coherence loss adds up through this accumulation
- Coherence can be dominated by a single variable, or by the accumulation of losses
- Therefore the radar system designer must be careful in choices throughout the system design and processing chains!

# Coherence Estimation

- Recall that we are estimate the (true) coherence using:

$$|\hat{\mu}| \approx \frac{\left| \sum_{n=1}^{L-1} x_{1,n} x_{2,n}^* \right|}{\sqrt{\sum_{n=1}^{L-1} |x_{1,n}|^2 \sum_{n=1}^{L-1} |x_{2,n}|^2}}$$

- Where  $L$  is the number of independent samples, and  $x_{i,n}$  is the  $n^{\text{th}}$  complex pixel for the  $i^{\text{th}}$  image
- We get looks by averaging over adjacent pixels (assuming they are independent)
- There is “loss” in the coherence estimator
  - These “losses” are in the form of bias and noise in the estimation
  - The assumption behind coherence estimation is that the statistics used in the estimation are constant, therefore large estimation boxes can actually cause loss of coherence (as well as loss of resolution in the CCD image)
  - Small estimation boxes are noisy and can have high bias

# Coherence Estimation

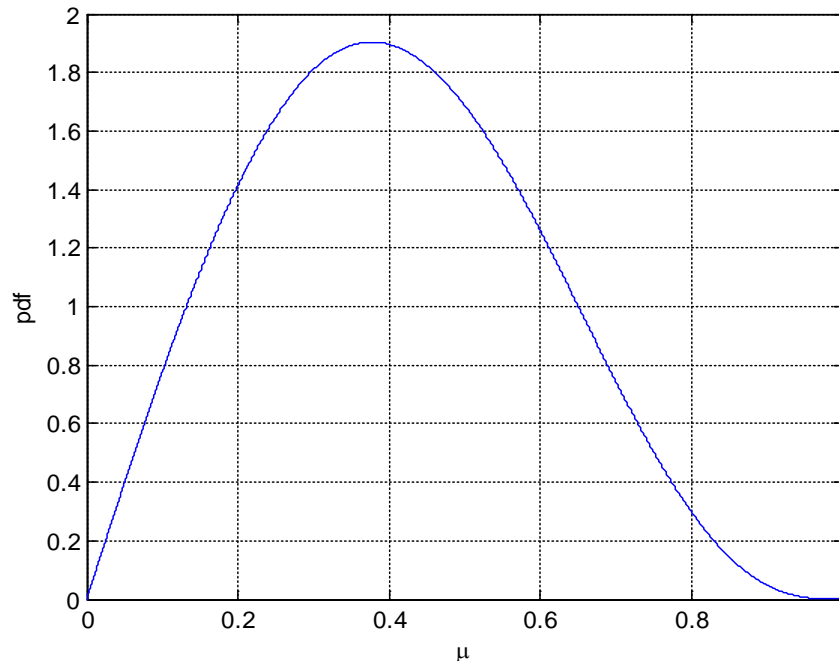
- As with MLEs, the estimator of the sample coherence asymptotically approaches the true population coherence in the limit as the number of independent looks averaged goes to infinity

$$\hat{\mu} \rightarrow \mu_0 = \frac{|\langle x_1 x_2^* \rangle|}{\sqrt{\langle |x_1|^2 \rangle \langle |x_2|^2 \rangle}}$$

- The probability density function (pdf) for the sample coherence is NOT Gaussian!
- Like Gaussian, the pdf is fully described by only 2 parameters, but unlike Gaussian, the 2 parameters are the population coherence and the number of independent looks

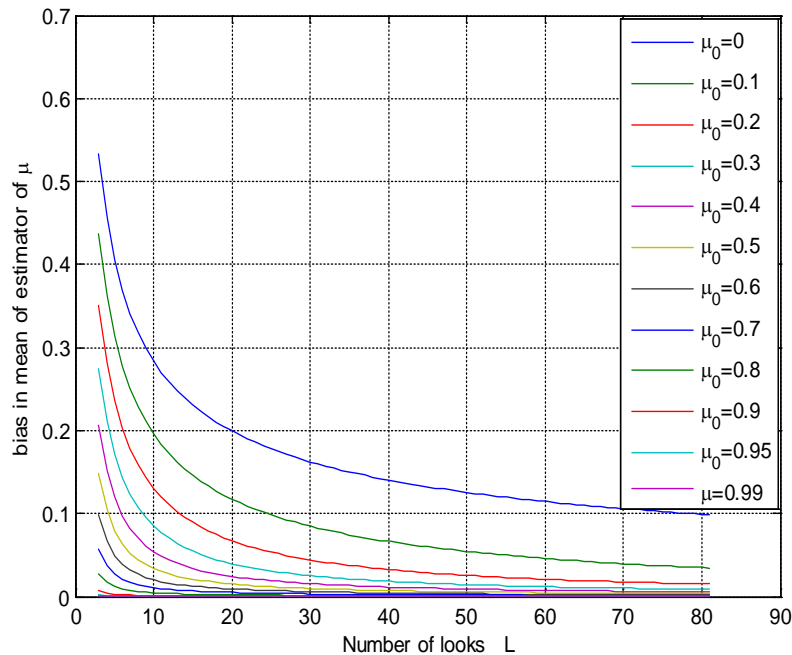
# Coherence Estimation

- The coherence estimator is highly biased for low coherence, especially for low numbers of looks
  - Look at the pdf for zero coherence ( $L=5$ ) to see why bias is an issue
  - Also, note that since coherence estimate values can only be from 0 to 1, the mean value has to be between these values, therefore noise can only add bias

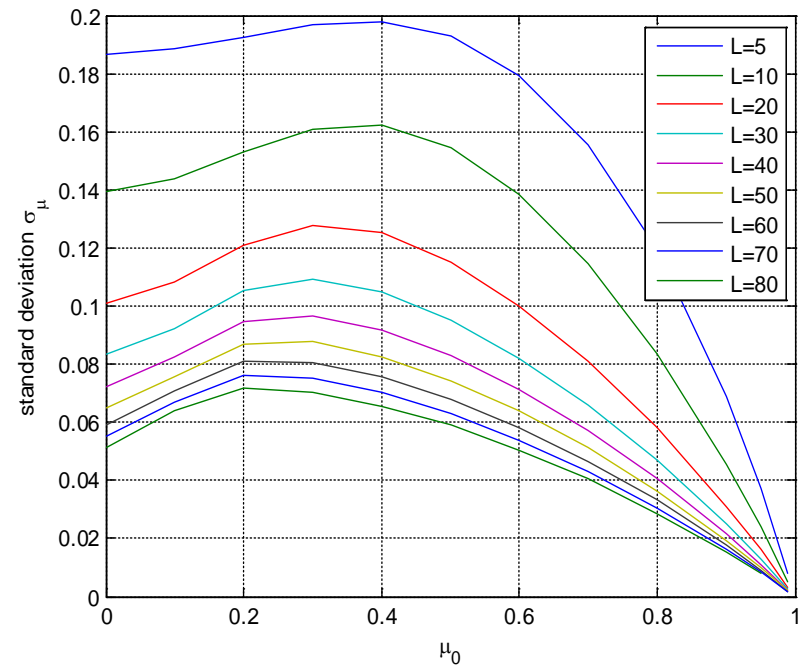


# Coherence Estimation

- Estimator bias and variance



Bias



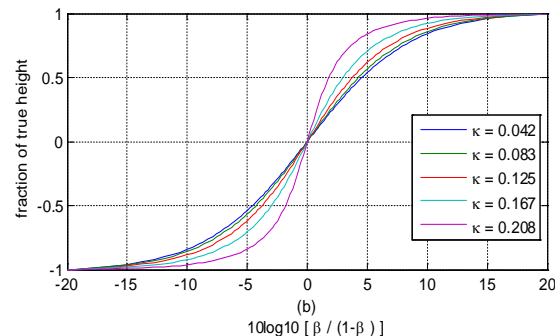
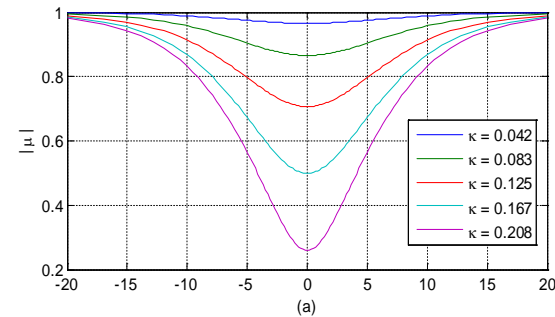
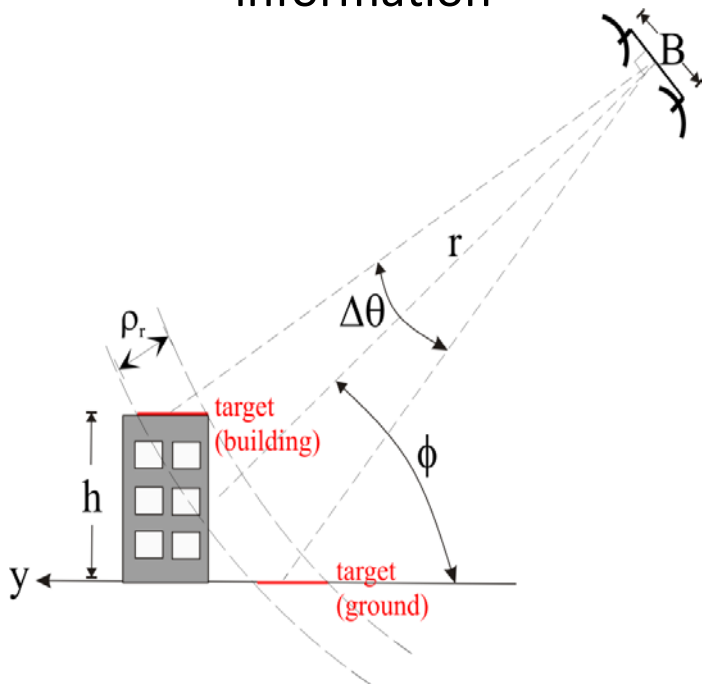
Standard deviation

# Coherence estimation - L

- In practical coherence estimation, adjacent pixels are not independent
  - The number of looks,  $L$ , will be less than the number of pixels used by around 1.5 times the resolution oversample in range times the resolution oversample in azimuth (for our standard Taylor window)
  - This means higher variance and bias in the coherence estimator than we expect if the looks were all independent

# Another interesting application

- The application coherence in building layover in interferometric SAR is an interesting application
  - van Cittert-Zernike theorem is very useful in this case
  - In InSAR, the phase contains the geometry information but in this case, the magnitude of the coherence actually also contains geometry information



# Conclusion

- Radar systems are very sensitive to coherence and this makes it very powerful tool in radar detection and estimation
- Although we focused on coherence between images, coherence between pulses and for image processing is related
- Coherence between images is important in many areas
  - Interferometric SAR
  - Tomography
  - Polarimetric SAR
  - Clutter cancellation in moving target detection
  - Direction-of-arrival estimation
  - Coherent change detection
- We must be cognizant of the factors affecting coherence if we want good coherence in radar systems

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