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Consideration of SRS Decayed Sine Tone Frequencies

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1 ABSTRACT

In order to perform a shock test on a shaker, a time history must be synthesized that approximately matches the desired Shock Response Spectrum (SRS). The sum of decaying sinusoids method is commonly used for time domain synthesis, as it allows the engineer to accurately match SRS profiles of just about any shape. The method uses a summation of several sinusoid waves at discrete frequencies to produce a composite wave whose SRS closely matches the desired SRS. The frequencies of the sinusoid tones are typically chosen to have a prescribed octal spacing. In this paper we investigate the sensitivity of the response of a test article to the frequencies of the sinusoid tones, especially with respect to the test article's natural frequencies. It was discovered that there is a strong correlation between the sine tone frequencies and the natural frequencies of the test article.

2 INTRODUCTION

When an electrodynamic shaker is used to perform shock testing, an SRS specification alone cannot be used to control the shaker input. A time history must be synthesized to match the intent of the SRS specification. There are several methods available for synthesizing time histories to match SRS specifications for shaker shock testing. The sum of decaying sinusoids method is commonly used in nuclear weapon qualification testing, particularly because of its robustness for use with detailed SRS profiles. A time history is calculated with this method using

$$\ddot{x}(t) = \sum_{i=1}^{n} A_i e^{-\zeta_i \omega_i (t - t_{di})} \sin \omega_i (t - t_{di}) \quad (1)$$

where each of n sine tones are described with an amplitude A, frequency ω, damping ratio ζ, and time delay t_d. Additional details on this method are provided in Reference 1.

Using this method, the SRS of the synthesized time history will typically best match the SRS straight-line specification at the frequencies of the discrete sine tones. The frequencies of the sine tones are typically chosen by prescribing a certain number of tones per octave, typically somewhere in the range of 3 to 10, to ensure that the synthesized time history SRS is within some tolerance to the SRS specification across the bandwidth of interest while also maintaining a reasonable number of tones for the test engineer to enter into the shaker control software. Typically no information about the test article is considered when selecting the frequencies of the sine tones. However, all structures have certain frequencies where little force is required to excite vibrations as well as other frequencies where it is nearly impossible to make the structures vibrate. Thus, the following case studies were designed to investigate the relationship between a structure's natural frequencies and the frequencies of the sine tones in the SRS synthesis.

To study the effects of this relationship, we must first establish the response characteristics of interest. In these studies, the comparisons of interest were: SRS shape, maximum absolute response in the time domain, and time history energy. The time history energy, E, is the zero order moment of the time history and is calculated using

$$E = m_0 = \int_0^T x^2(t)dt \qquad (2)$$

where $x(t)$ is the time history that spans a time block from 0 to T. Additional details on temporal moments are provided in Reference 2. While other metrics may be of interest, especially for more complicated systems, these metrics were chosen to provide a straight-forward assessment of the response of the systems presented here.

3 CASE STUDIES

A unique Device Under Test (DUT) was modelled for each of the two case studies. Each DUT was a simple single-degree-of-freedom (SDOF) system with mass and stiffness properties chosen to result in a specific natural frequency. The dynamic characteristics of the parts are defined in the following sections for each case. Both systems had 3% of critical damping.

The basic process is outlined in Figure 1. A time history is input to the SDOF system, and the equations of motion are integrated using Newmark integration to calculate the response. The response characteristics are then compared to the input characteristics.

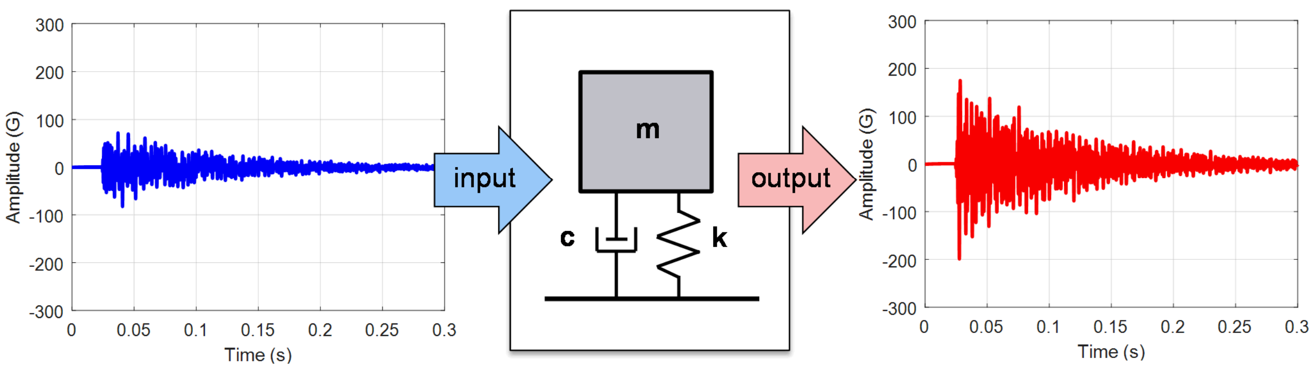

Figure 1. Basic process used in case studies to determine response to shock input

3.1 Case 1

Case 1 was generated to study the most basic relationship between test article resonance and sine tone frequency. In this case, the DUT resonance was set to 1500 Hz. This resonant frequency was chosen somewhat arbitrarily, but is within the range of typical resonances seen in aerospace components.

The input for case 1 was a single decaying sine tone, and the frequency of this sine tone was varied from 600 Hz to 3500 Hz. For each sine wave input, the decay rate was chosen such that the signal length was always 0.3 seconds. Some sample SRS plots are shown in Figure 2, Figure 3, and Figure 4 for cases where the input frequency was lower than, equal to, and higher than the DUT resonant frequency, respectively.

As seen, no matter the input frequency, there is always some energy at the resonant frequency of the DUT. However, there is a massive amplification of the SRS when the two frequencies are equal.

For all input frequency cases, the maximum response amplitude was taken and normalized by the maximum input amplitude. This normalized response amplitude is plotted against the normalized input frequency in Figure 5. Also, the time history energy was calculated for all input frequency cases, again with the response normalized by the input, and the results are shown in Figure 6. In both Figure 5 and Figure 6, there is a strong amplification of response when the normalized frequency is 0 dB, where the input frequency equals the DUT resonant frequency. As the input frequency diverges from the resonant frequency, the response amplification decreases. Interestingly, the plots are not symmetric about the 0 dB frequency line. As the input frequency gets higher than the resonant frequency, there is less amplification than if the input frequency is lower than the resonant frequency. In fact, once the input frequency is approximately 3 dB higher than the resonant frequency, there is no amplification of the time history energy.

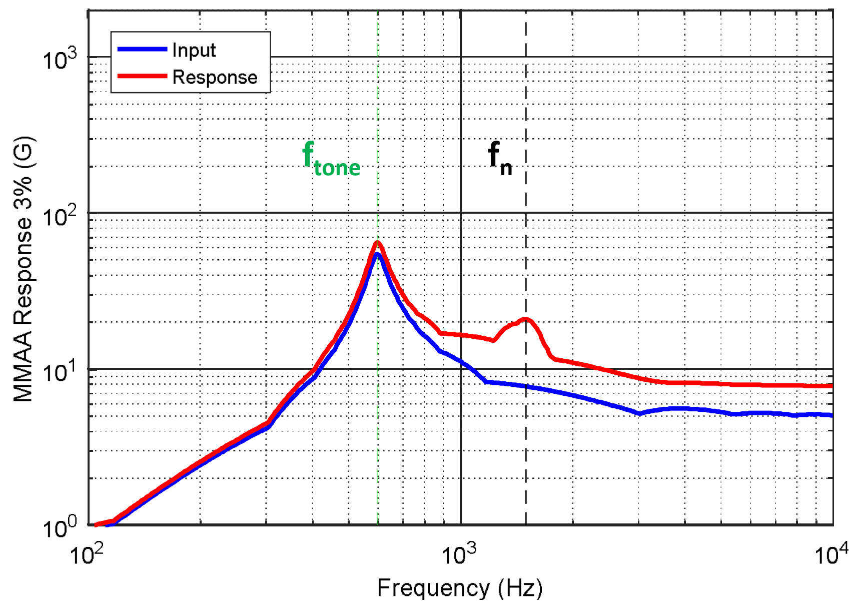

Figure 2. Case 1, input frequency significantly lower than DUT resonant frequency

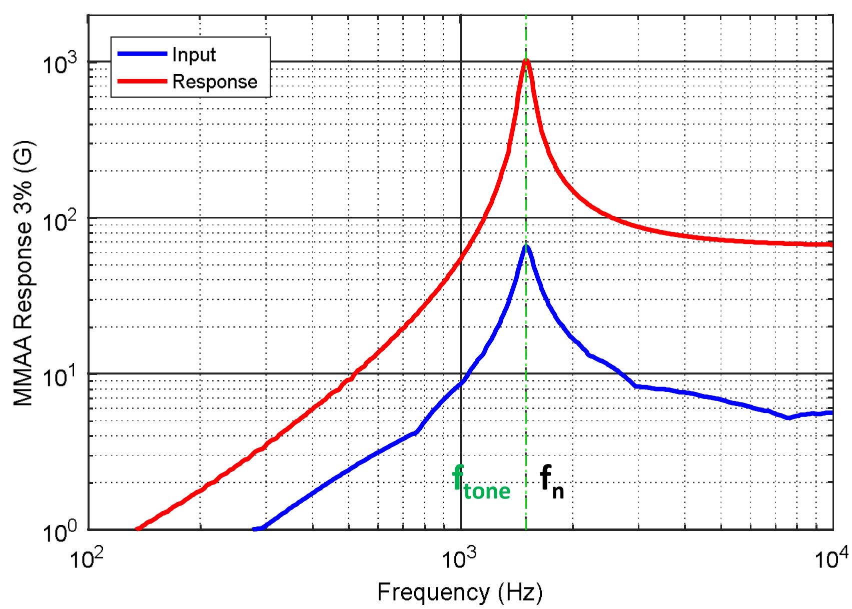

Figure 3. Case 1, input frequency equal to DUT resonant frequency

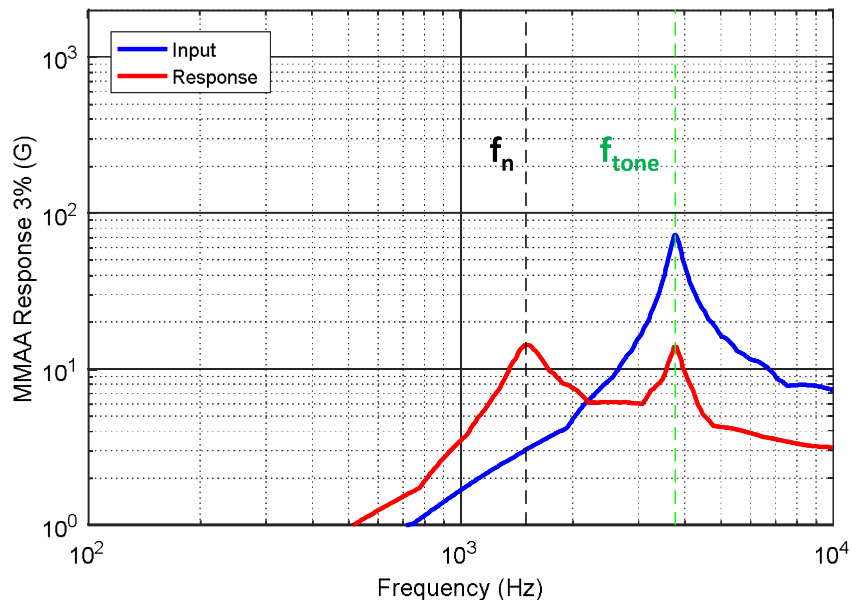

Figure 4. Case 1, input frequency significantly higher than DUT resonant frequency

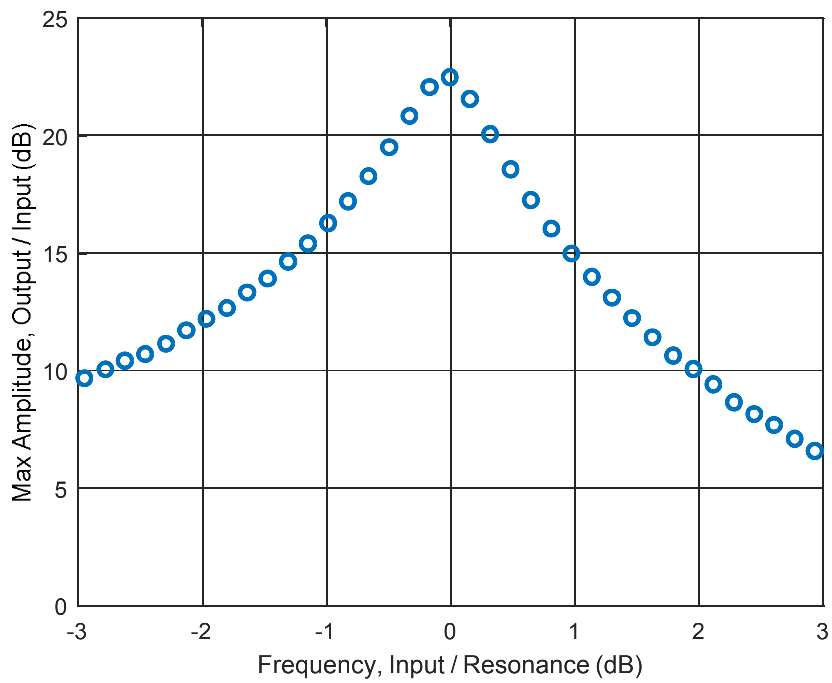

Figure 5. Case 1, normalized maximum amplitude as a function of normalized frequency

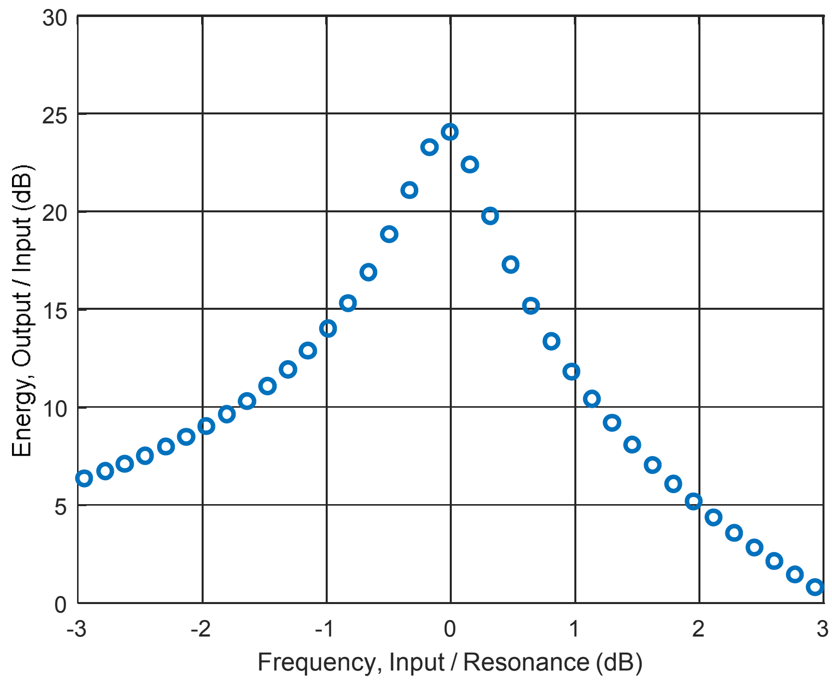

Figure 6. Case 1, normalized time history energy as a function of normalized frequency

3.2 Case 2

Case 2 was generated to study a more realistic case where the input is comprised of many decaying sinusoids. The SRS specification along with its decayed sine representation is shown in Figure 7. Only 3 points per octave were used to synthesize the time history. As seen, this allows the valleys of the SRS to fall right around -3dB of the specification.

For case 2, the nominal resonant frequency of the SDOF system was set to 850 Hz. However, for this case, the stiffness was altered to cause changes in the resonant frequency, as is typical in real manufactured parts. Note that mass alterations could also be used to change the resonant frequency with seemingly similar results. The resonant frequency was varied from 650 Hz up to 1050 Hz in 5 Hz increments. For reference, the input shock included two sine tones in this bandwidth: 760 Hz and 965 Hz.

Figure 8 shows a sample response where the system had a 760 Hz resonance, coincident with a sine tone frequency, whereas Figure 9 shows the response of the system with the nominal 850 Hz resonance. Note that although there is only around a 10% difference in the resonances, as is typical in many manufactured parts, the difference in response is significant. When the resonance lines up with an input sine tone frequency, as in Figure 8, the response is amplified not only at that frequency but across the entire bandwidth. Conversely when the resonance occurs at a frequency approximately halfway between two tones, as in Figure 9, the response is much flatter and less amplified.

Again, the maximum normalized response was calculated for all frequency cases and is plotted in Figure 10. The normalized time history energy was also calculated for all frequency cases and is shown in Figure 11. For reference, the sine tone frequencies in this bandwidth are shown with dashed green lines.

In both Figure 10 and Figure 11, there is a strong amplification of response when the resonant frequency lines up with an input sine tone frequency, and less amplification between frequencies. However, the character of the plots is notably different. The shape of Figure 10 matches that of the input specification in Figure 7 and therefore has a local minima around 830 Hz. However the shape of Figure 11 is much more smooth and symmetric and has a local minima halfway between the two sine tones, around 860 Hz.

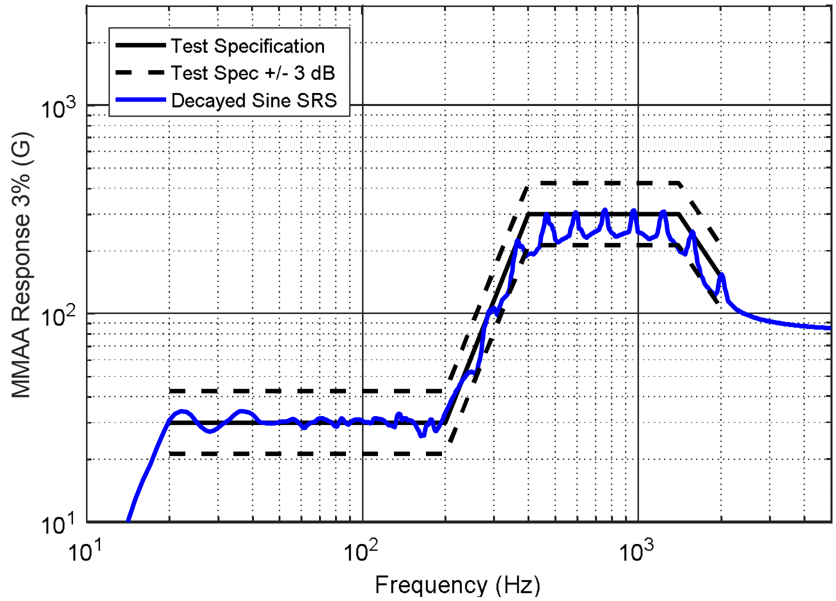

Figure 7. SRS specification and corresponding decayed sine input for Case 2

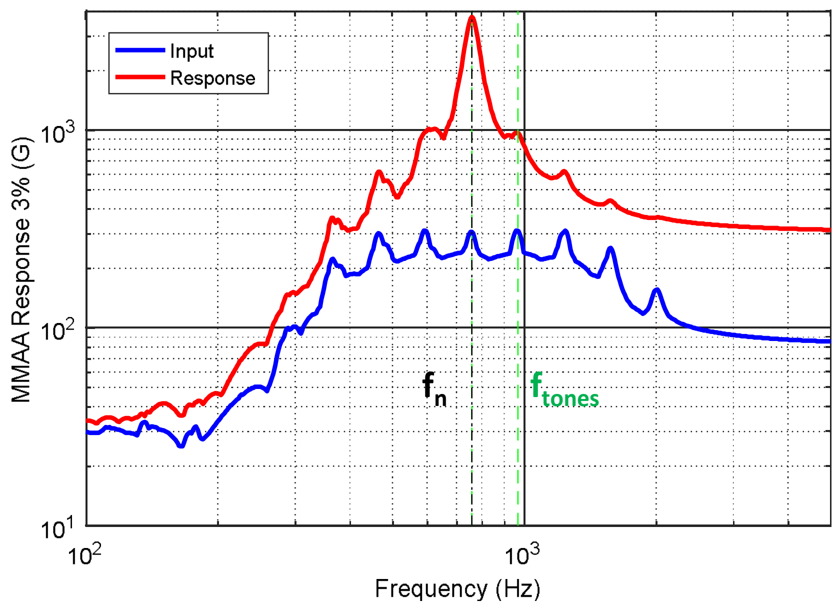

Figure 8. Case 2, low DUT resonant frequency equal to 760 Hz sine tone frequency

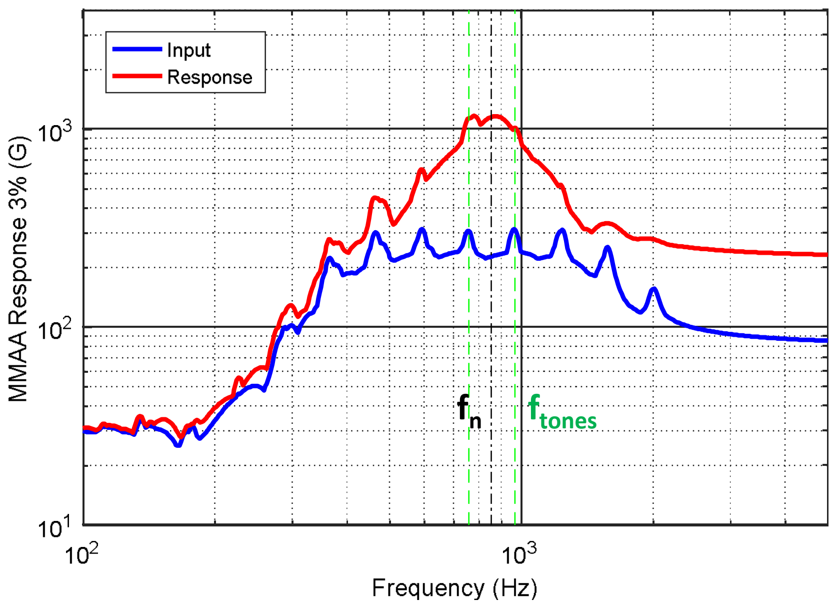

Figure 9. Case 2, intermediate DUT resonant frequency, between two sine tone frequencies

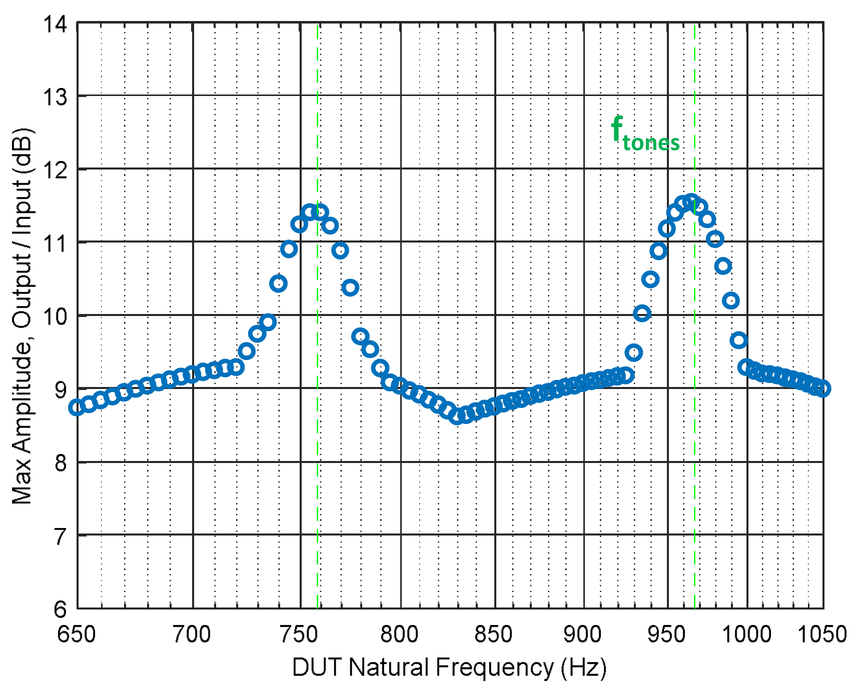

Figure 10. Case 2, normalized maximum amplitude as a function of frequency

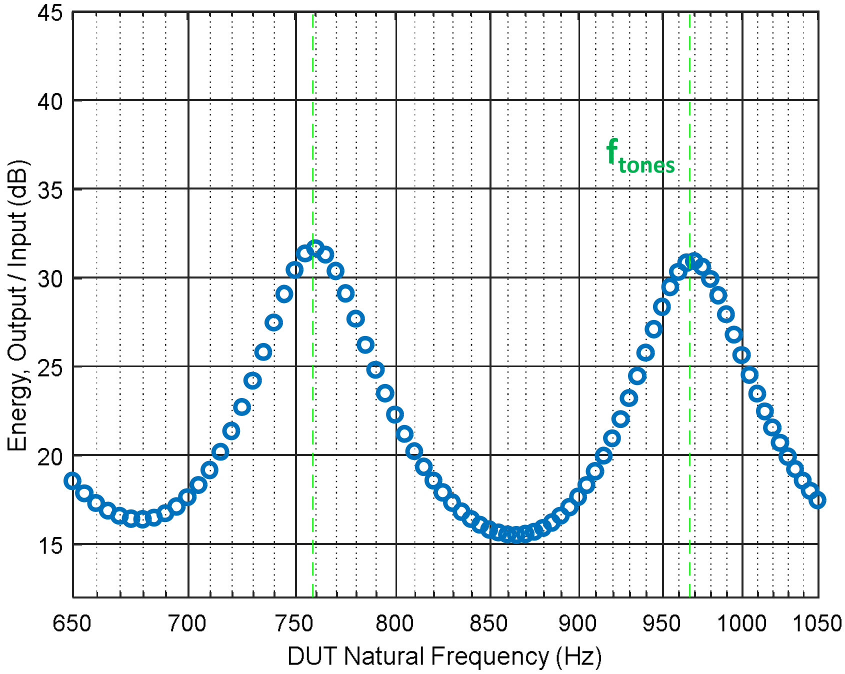

Figure 11. Case 2, normalized time history energy as a function of frequency

4 CONCLUSIONS AND FUTURE WORK

The standard approach for synthesizing a decaying sinusoid shock does not take into account any information about the part being tested. This work explored the relationship between the natural frequencies of the part and the frequency content of the decayed sinusoid input shock. In all cases, response was amplified when the system resonant frequency was equal to that of a sine tone. This demonstrates the importance of comparing the exposure that a structure sees in the test lab to the exposure it sees in the service environment. The standard synthesis approach in conjunction with part-to-part variability could manifest different stress and strain states during component testing and qualification.

Analytical models can help expedite this process without wasting expensive test time and hardware. Additionally, although only amplitude and energy were considered here, it is important to consider what other metrics might be relevant. For example, higher order modes may contribute less to the overall response, and it may only be necessary to consider the frequencies of the first few modes with respect to the sine tone frequencies.

The results presented here are relevant to several other areas within shock testing. For example, resonant plate testing involves exciting a structure at primarily one frequency – a resonance of the plate. This scenario is essentially identical to that explored in Case 1. Thus, a part with a resonant frequency matching the knee frequency of the resonant plate test will have its response significantly amplified. Additionally, in shaker shock testing, if a test fixture has a mode whose frequency lines up with a decayed sine tone frequency, the input to the component would be greatly amplified. It is important to evaluate all aspects of the test setup and ensure the desired response is being achieved with the shock application technique used.

The amplifications observed here are likely related to the fact that the SRS of a decayed sinusoid shock matches the SRS specification best at the sine tone frequencies, with a scalloping effect in between tones. Hence with either increased sine tone density or increased damping, the system response is expected to be less sensitive to frequency. However, the response may not accurately represent the service environment because the service environment will likely consist of broadband exposure rather than energy concentrated at discrete frequencies, as is the case with the decayed sinusoid synthesis. Future work will explore these hypotheses.

5 REFERENCES

1. D. O. Smallwood, “Time History Synthesis for Shock Testing on Shakers,” Seminar on Understanding Digital Control and Analysis in Vibration Test Systems, A Publication of The Shock and Vibration Information Center, Naval Research Laboratory, Washington, D.C., 1976.
2. D. O. Smallwood, “Characterization and Simulation of Transient Vibrations Using Band Limited Temporal Moments,” Proceedings of the 60th Shock and Vibration Symposium, Volume III, pp 93–112, Portsmouth, Virginia, Nov. 1989.