

# Development of a wide-range electrical and thermal conductivity model for ICF applications

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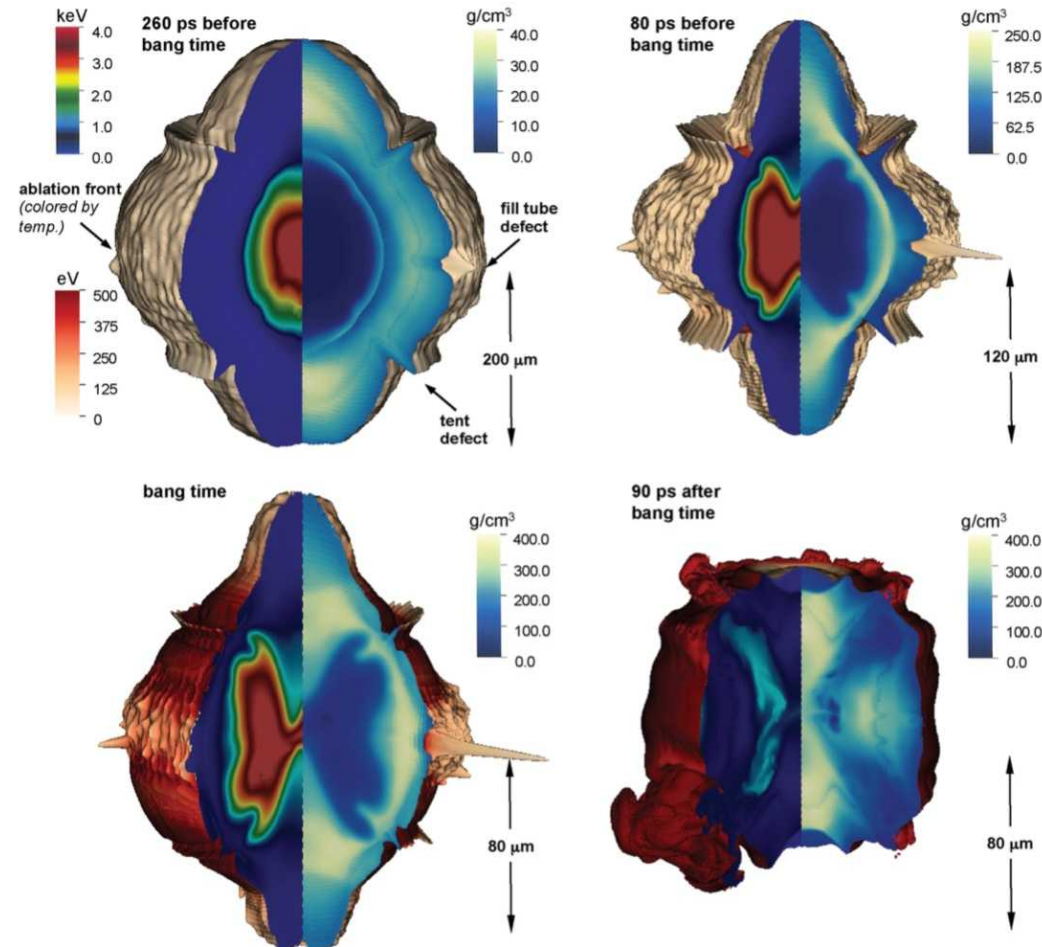
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# Simulations of ICF capsule implosions have reached a high level of sophistication

## 3D HYDRA simulation of a high-foot ICF capsule implosion

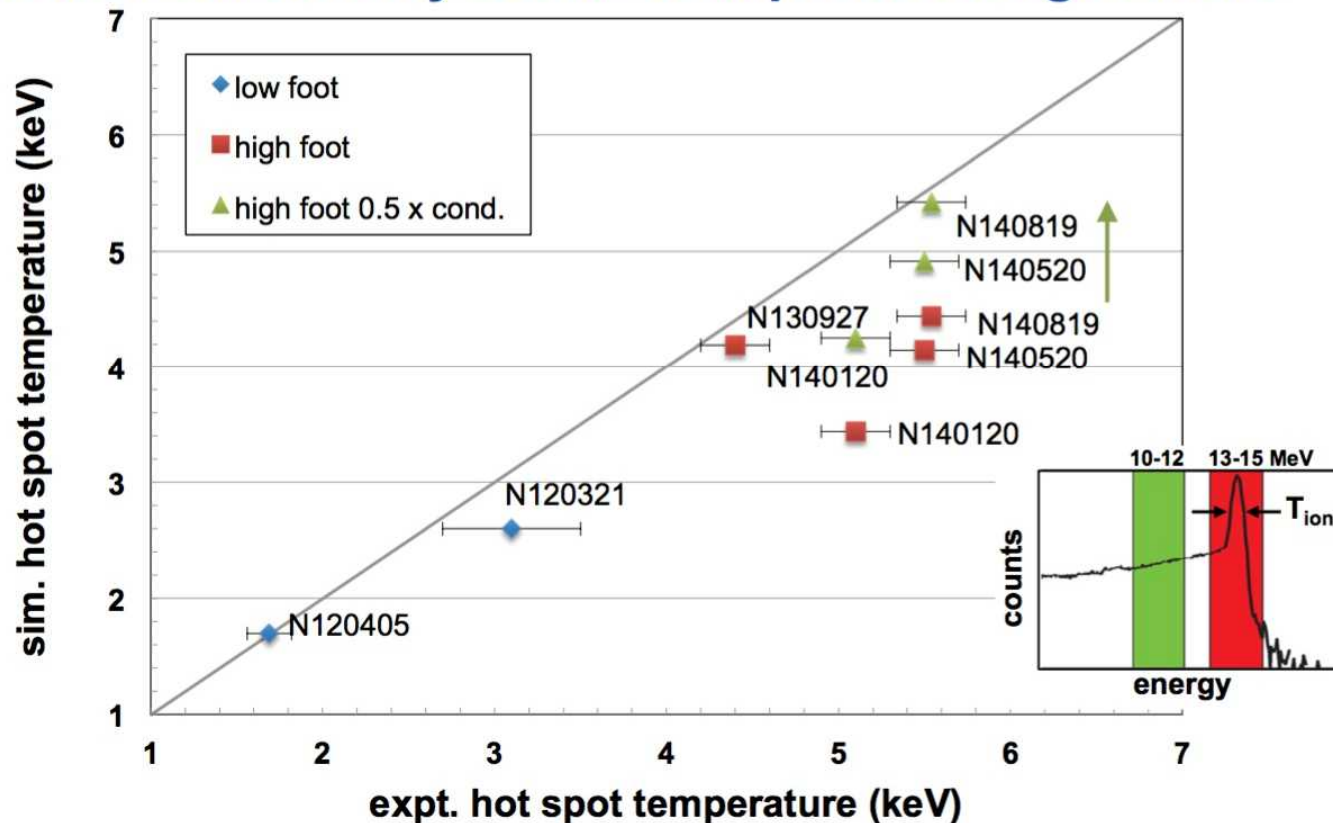
056302-9 Clark *et al.*

Phys. Plasmas **23**, 056302 (2016)



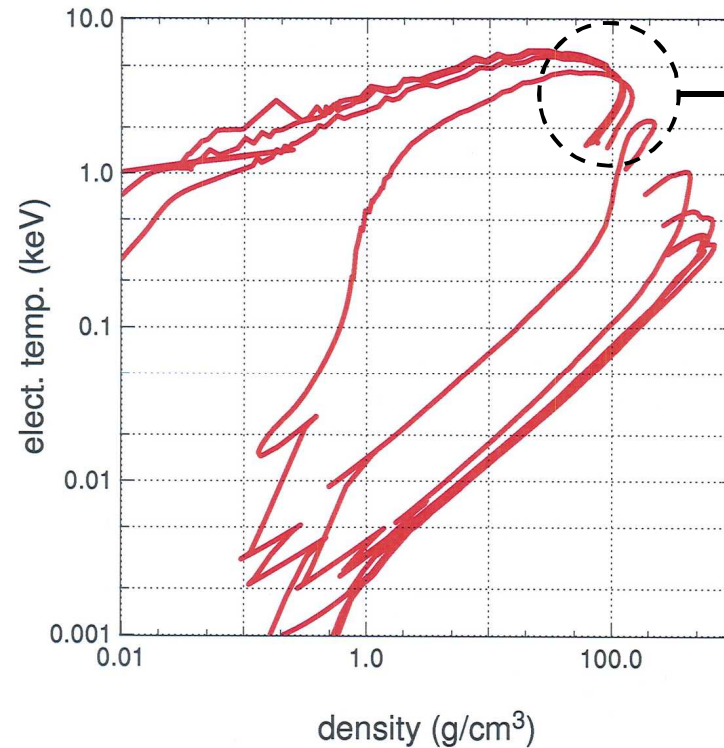
# Simulations of NIF high-foot capsule implosions with lowered thermal conductivity are in better agreement with experiment

Applying a simple multiplier of one half to the DT thermal conductivity seems to improve the agreement



Slide courtesy of Dan Clark (LLNL)

# ICF capsule implosions traverse a broad region of phase space, from condensed matter to WDM to ideal plasma



Hot spot boundary –  
region of greatest yield  
sensitivity to thermal  
conductivity (Steve Hahn)

Capsule phase space trajectories courtesy of Dan Clark (LLNL)

# We appeal to three different computational frameworks for a wide-range description of hydrogen transport

- First-principles calculations with DFT-MD and Kubo-Greenwood transport calculations
  - No assumption of ionization level or electron distribution
  - Bare proton potential – no pseudopotential issues
  - Particularly well suited to warm dense matter
  - Very high temperatures are computationally difficult – must resort to extrapolations for converged quantities
- Quantum Lenard-Balescu calculations\*
- Particularly well suited to the ideal plasma limit: weak coupling, small angle scattering
- Includes electron diffraction and dynamic screening
- Does not include electron degeneracy – no Pauli blocking
- Assumes full ionization

\* See Whitley, Scullard, Benedict, *et al.*, *Contrib. Plasma, Phys.* **55**, 192 (2015).

# We appeal to three different computational frameworks for a wide-range description of hydrogen transport (cont.)

- Linear response theory in the Zubarev framework\*
  - Boltzmann collision operator
  - Particularly well suited to the expanded, partially ionized plasma
  - T-matrix scattering cross sections – strong and weak scattering
  - Pauli blocking included
  - Static screening lengths for ion and electron potentials are assumed
  - Ionization state provided by auxiliary models

\* See H. Reinholz, R. Redmer, and S. Nagel, Phys. Rev. E **52**, 5368 (1995)

All calculations and results in this talk are for pure hydrogen (H)

# The CEA group demonstrated the viability of *ab initio* thermal conductivity calculations for very dense hydrogen

PRL 102, 075002 (2009)

PHYSICAL REVIEW LETTERS

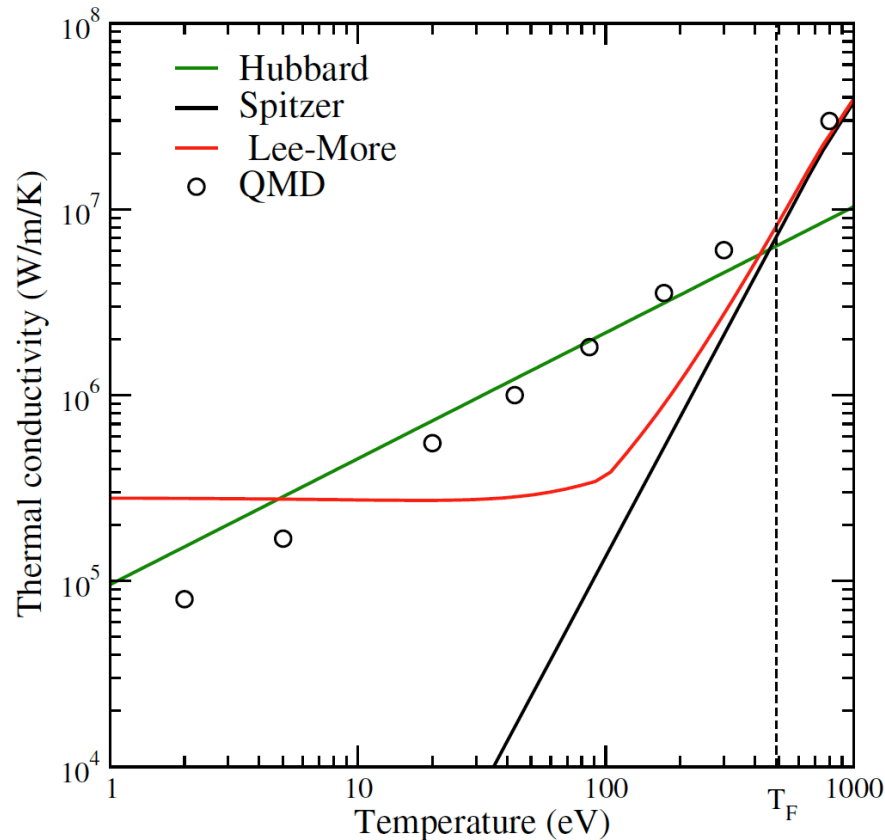
week ending  
20 FEBRUARY 2009

## *Ab Initio* Determination of Thermal Conductivity of Dense Hydrogen Plasmas

Vanina Recoules, Flavien Lambert, Alain Decoster, Benoit Canaud, and Jean Cl  rouin

CEA, DAM, DIF, F-91297 Arpajon, France

(Received 4 November 2008; published 20 February 2009)



# First-principles calculations of transport quantities are carried out in the Kubo – Greenwood / Chester – Thellung formalism

- For these dense conditions we abandon the pseudopotential approach and use a bare proton (this forces high plane wave cutoff energies)
- We calculate the full set of Onsager transport coefficients and calculate the thermal conductivity directly (no Wiedemann-Franz law assumptions)

$$L_{mn}(\omega) = \frac{2\pi q^{4-m-n}}{3V m_e^2 \omega} \sum_{\mathbf{k}\nu\mu} \overset{\text{Fermi weights}}{(f_{\mathbf{k}\nu} - f_{\mathbf{k}\mu})} \overset{\text{Dipole matrix elements}}{\langle \mathbf{k}\nu | \hat{\mathbf{p}} | \mathbf{k}\mu \rangle \langle \mathbf{k}\mu | \hat{\mathbf{p}} | \mathbf{k}\nu \rangle} \\ \cdot \left( \frac{E_{\mathbf{k}\nu} + E_{\mathbf{k}\mu}}{2} - h \right)^{m+n-2} \underset{\text{Onsager weights}}{\delta(E_{\mathbf{k}\mu} - E_{\mathbf{k}\nu} - \hbar\omega)} \underset{\text{Energy conservation}}{\quad} .$$

$$\omega \rightarrow 0 \quad K = \frac{1}{T} \left( \mathcal{L}_{22} - \frac{\mathcal{L}_{12}^2}{\mathcal{L}_{11}} \right),$$



# Larger boxes (more atoms) are necessary to converge the degenerate limit

1024 H atoms

$0.0208 \text{ \AA}^3/\text{atom}$

$0.14 a_B^3/\text{atom}$

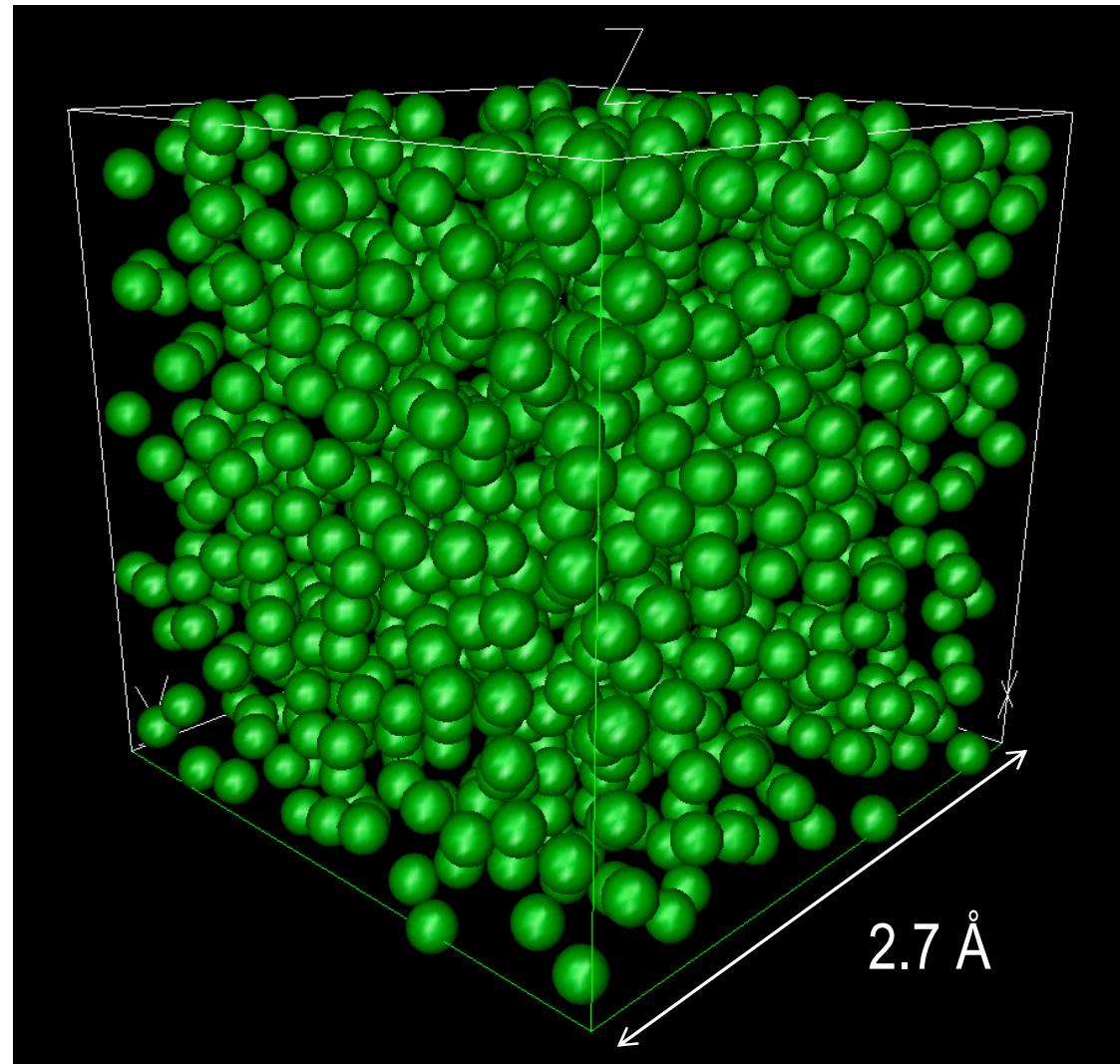
$T = 10 \text{ eV}$

$80 \text{ g/cc H}$

or

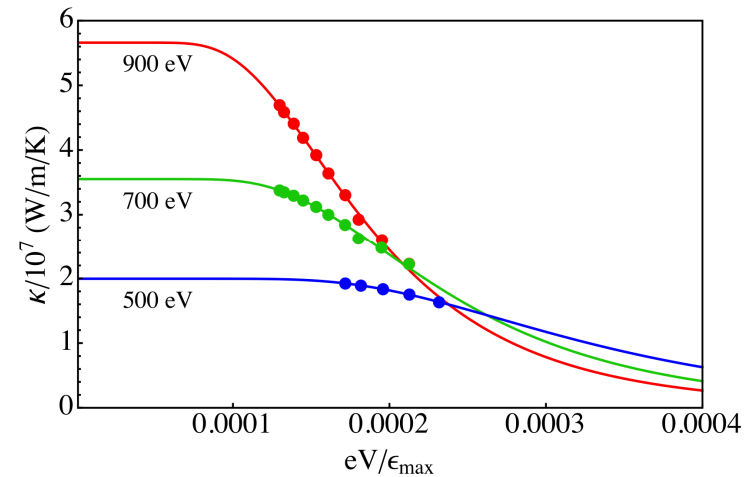
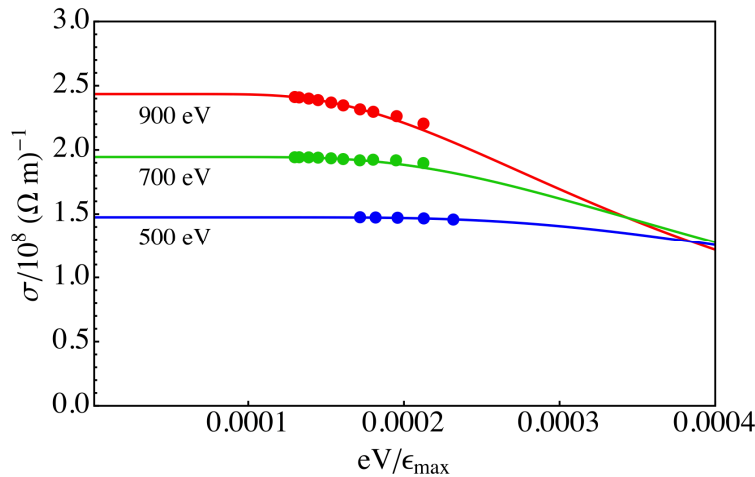
$200 \text{ g/cc DT}$

$P = 13 \text{ Gbar}$



The transport properties are also difficult to converge in the high temperature limit, particularly the thermal conductivity

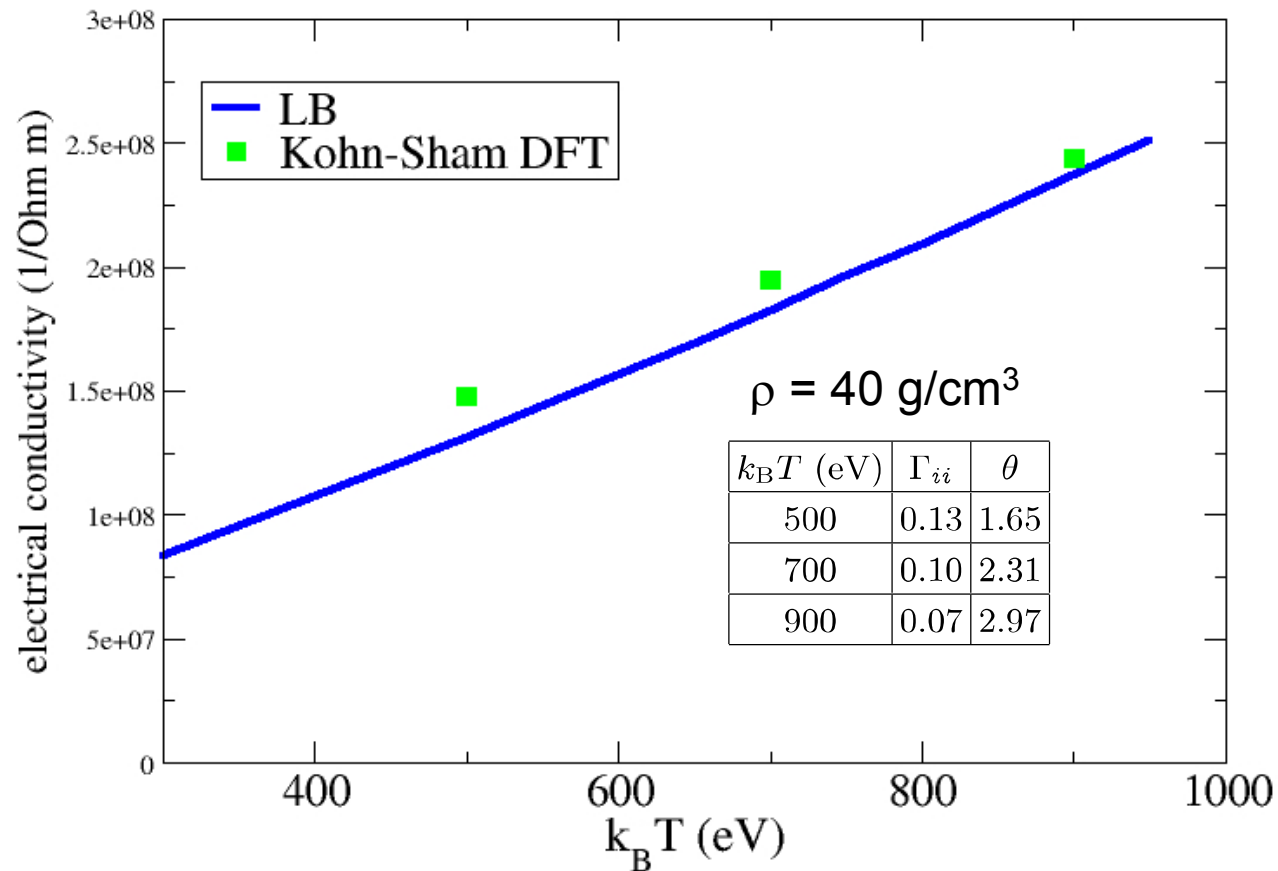
$$\rho = 40 \text{ g/cm}^3$$



$$\kappa(\epsilon_{\text{max}}) = \kappa_{\infty} \frac{\int_{-\infty}^{\epsilon_{\text{max}}} E^2 E^{\gamma} \frac{\partial f}{\partial E} dE}{\int_{-\infty}^{\infty} E^2 E^{\gamma} \frac{\partial f}{\partial E} dE}$$

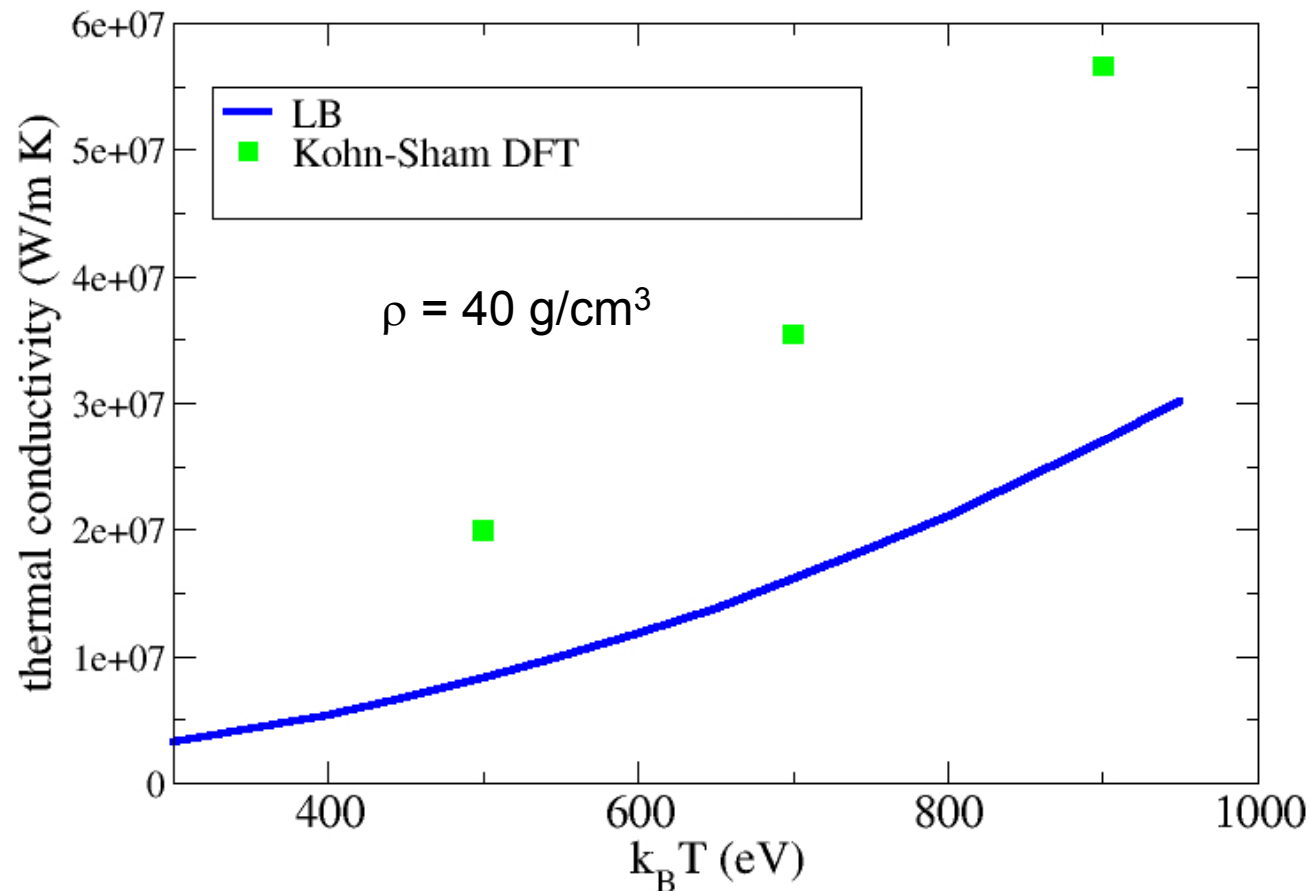
We extrapolate to infinite bands through the assumption of a power law scaling of the dipole matrix elements

The electrical conductivities from the Kubo-Greenwood calculations agree well with our quantum Lenard-Balescu results



Residual differences are due to small degeneracy effects included in the DFT but not the QLB. The sum rule on  $\sigma(\omega)$  is well satisfied.

The thermal conductivities from the Kubo-Greenwood calculations (■) do not agree with our quantum Lenard-Balescu results



Density functional theory provides an accurate description of the electrical conductivity in the non-degenerate limit, but only partially captures the thermal transport

No momentum transport for e-e

$$\frac{1}{\sigma} = \frac{1}{S_{\sigma}\sigma_{ei}} + \cancel{\frac{1}{\sigma_{ee}^{\text{OCP}}}} = \frac{1}{S_{\sigma}\sigma_{ei}},$$

DFT

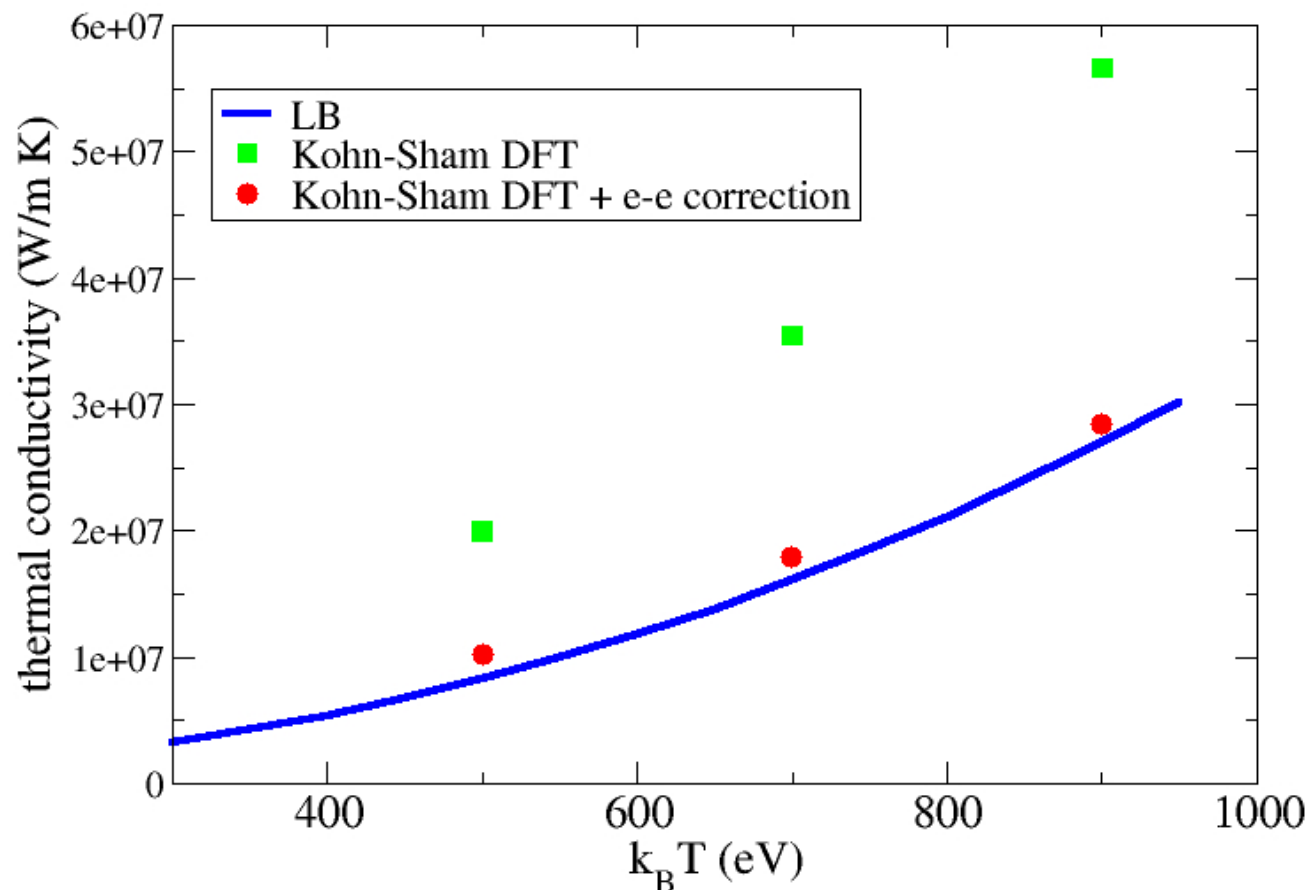
$$\frac{1}{\kappa} = \frac{1}{S_{\kappa}\kappa_{ei}} + \boxed{\frac{1}{\kappa_{ee}^{\text{OCP}}}},$$

DFT

Explicit e-e collisions,  
missing from DFT

See: Desjarlais, Scullard, Benedict, Whitley, and Redmer, Phys. Rev. E **95**, 033203 (2017)

The thermal conductivities from the Kubo-Greenwood calculations with an e-e correction (•) agree with our quantum Lenard-Balescu results



The red points show the agreement when an explicit e-e scattering correction, calculated in the Zubarev formalism (T-matrix, Boltzmann collision operator) is added to the thermal.

Small differences due to the treatment of degeneracy remain

# We build a wide-range model with a suite of semi-classical algorithms tuned to the results of our calculations

- Lee-More-Desjarlais algorithms\*
  - Non-ideal Saha ionization model
  - Electron-ion and electron-neutral scattering
  - Degeneracy dependent reduction factors for electron-electron scattering
  - Wide-range Coulomb logarithm model
    - Depends on  $\Gamma_{ij}$  in the degenerate limit
    - Includes  $b_{\min C}$ ,  $b_{\min Q}$  for the classical or quantum minimum impact parameter in the plasma limit
    - Includes  $b_{\max}$  tuned to capture effects of dynamic screening
  - Several additional minor tuning parameters

\* MPD, Contrib. Plasma Phys. **41**, 267 (2001)

# The plasma literature suggests a variety of choices for the minimum impact parameter in the quantum limit

$$b_{minQ} = \frac{h}{2m_e \bar{V}_e} \quad \text{e.g. Lee-More}$$

$$b_{minQ} = \frac{\hbar}{m_e \bar{V}_e} \quad \text{e.g. Jackson}$$

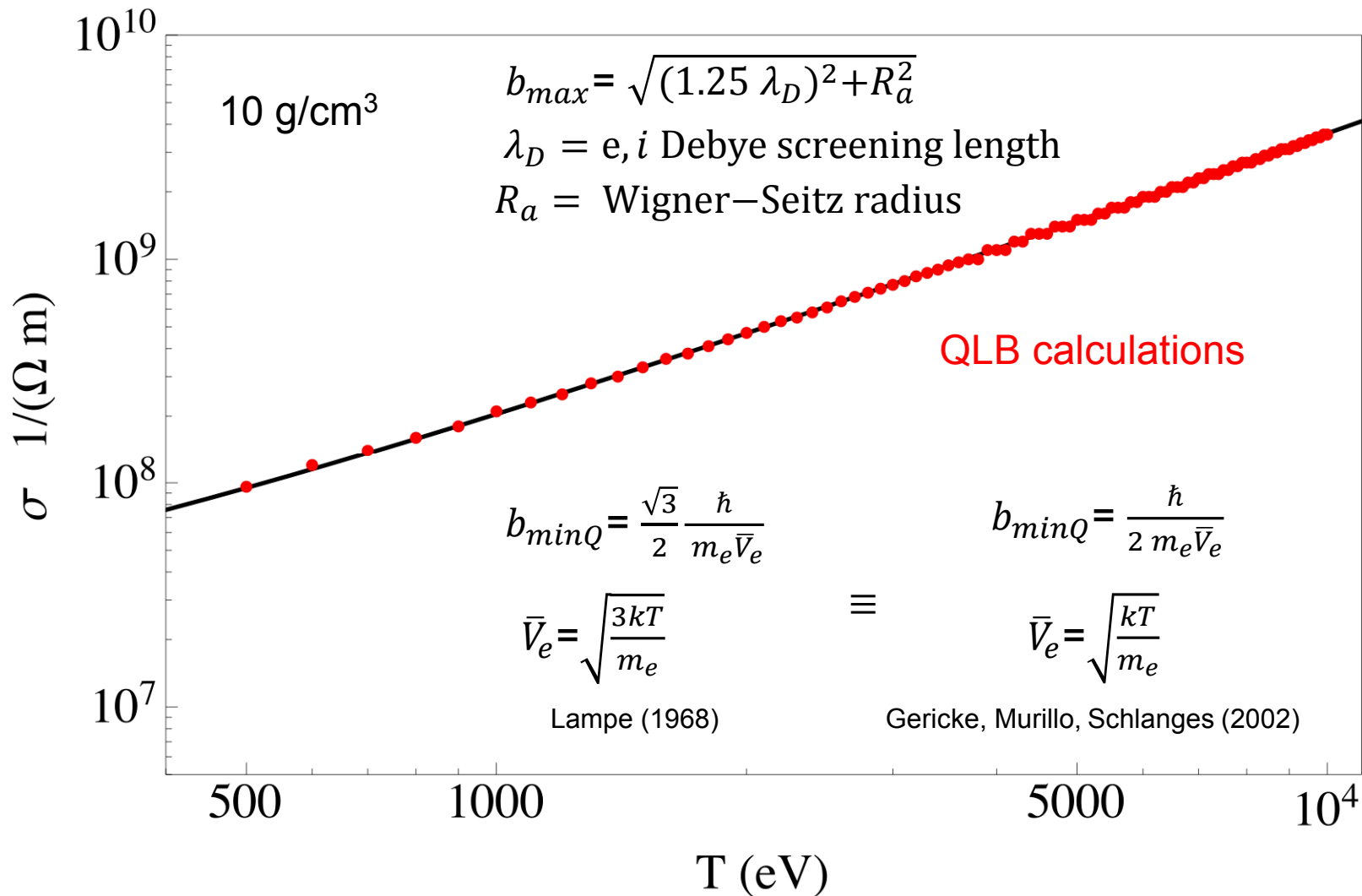
$$b_{minQ} = \frac{\hbar}{2 m_e \bar{V}_e} \quad \text{e.g. Brysk}$$

$$b_{minQ} = \frac{\sqrt{3}}{2} \frac{\hbar}{m_e \bar{V}_e} \quad \text{e.g. Lampe}$$

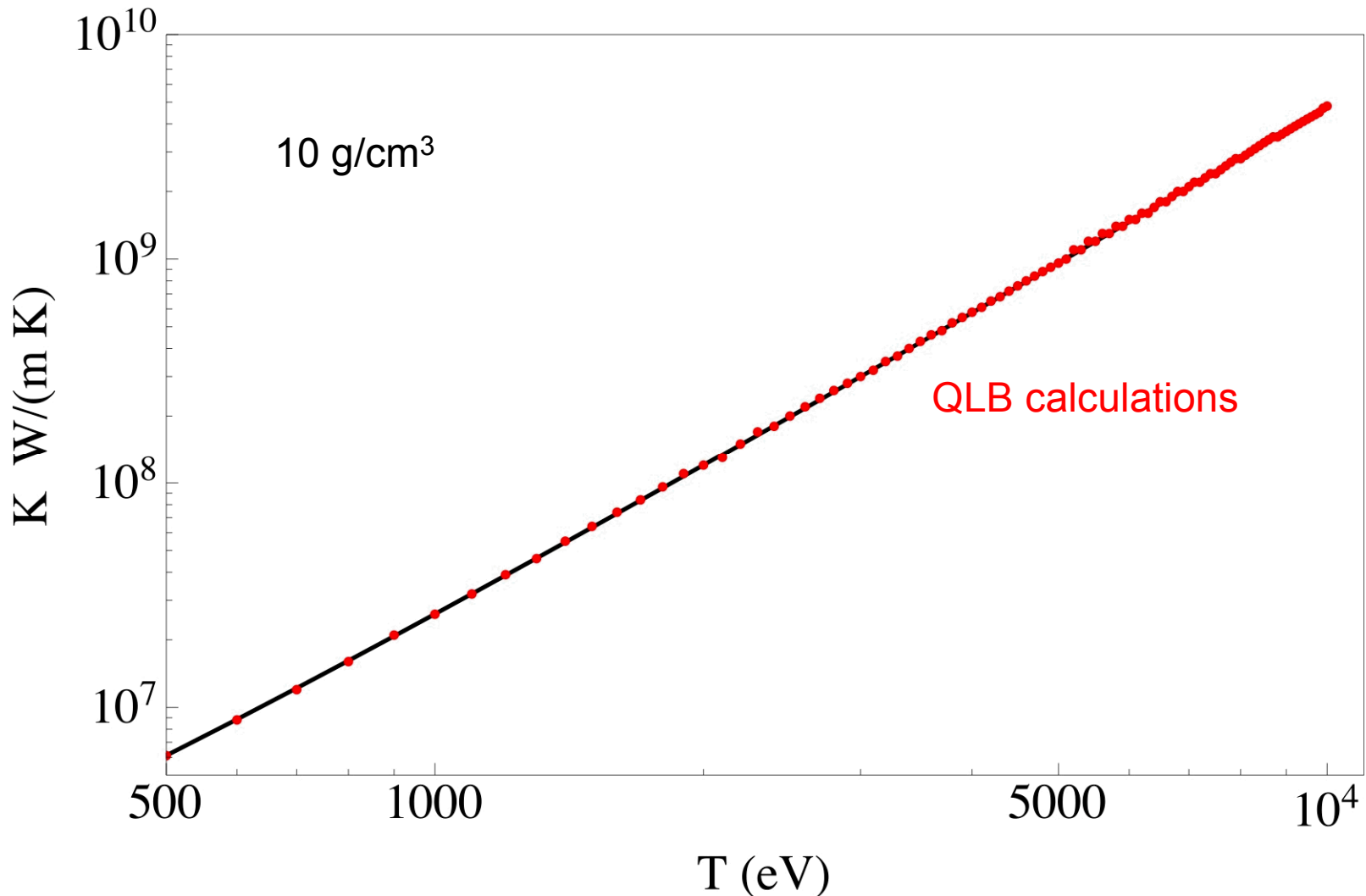
Combined with  $\bar{V}_e = \sqrt{\frac{3kT}{m_e}}$  or in some cases  $\bar{V}_e = \sqrt{\frac{kT}{m_e}}$  or  $\bar{V}_e = \sqrt{\frac{2kT}{m_e}}$



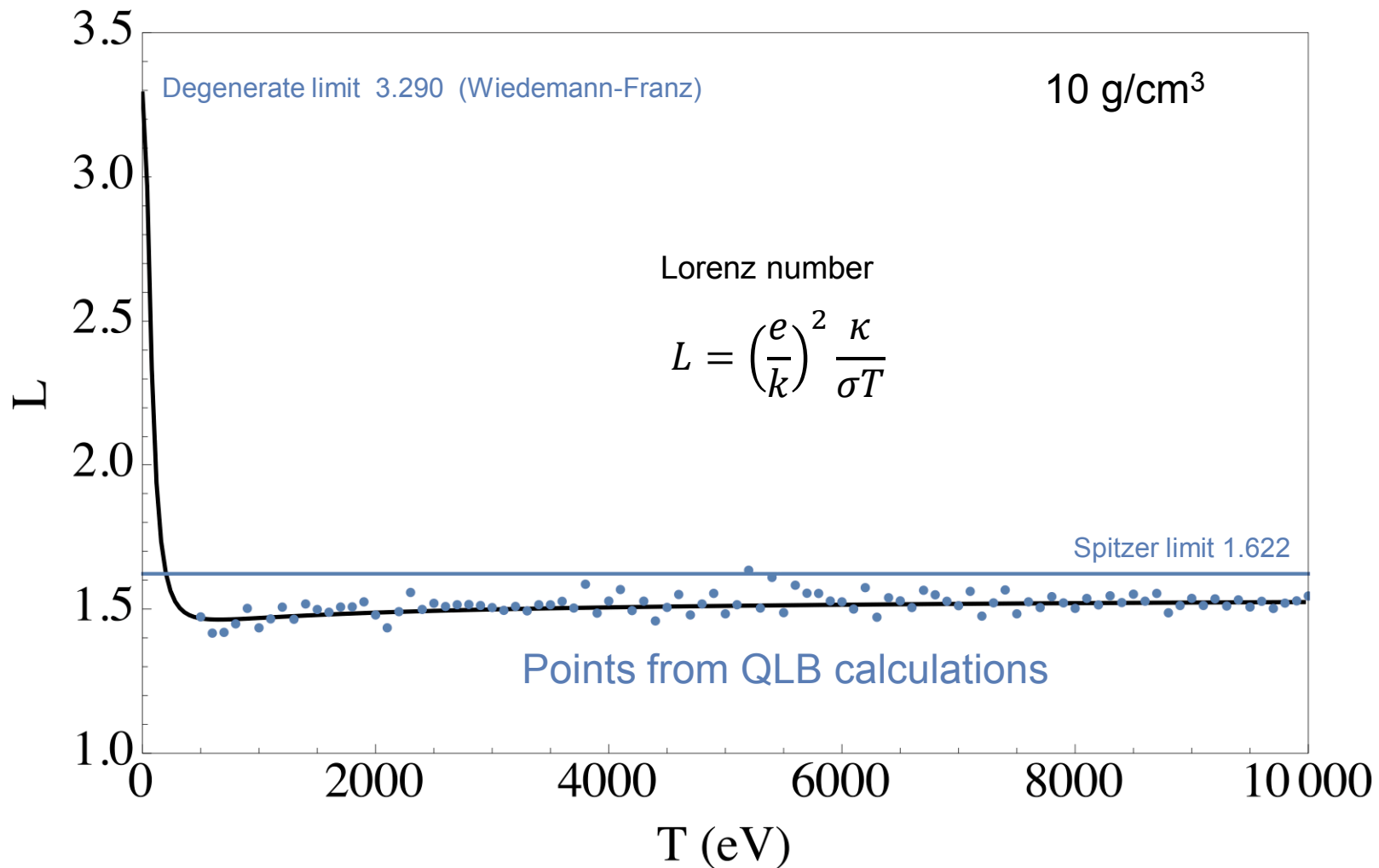
We found optimum choices of  $b_{max}$  and  $b_{minQ}$



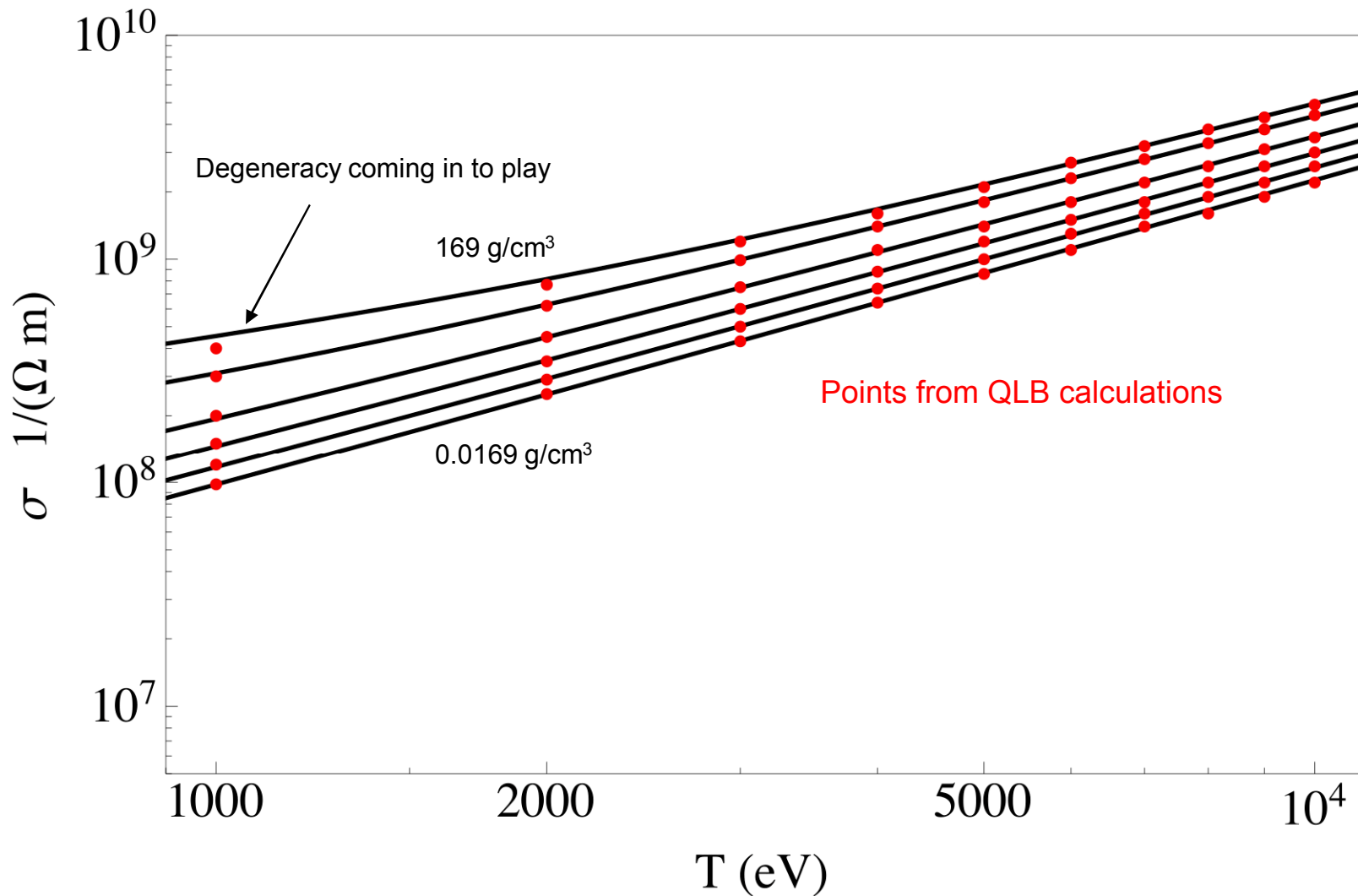
The thermal conductivity is equally well fit with the same parameters



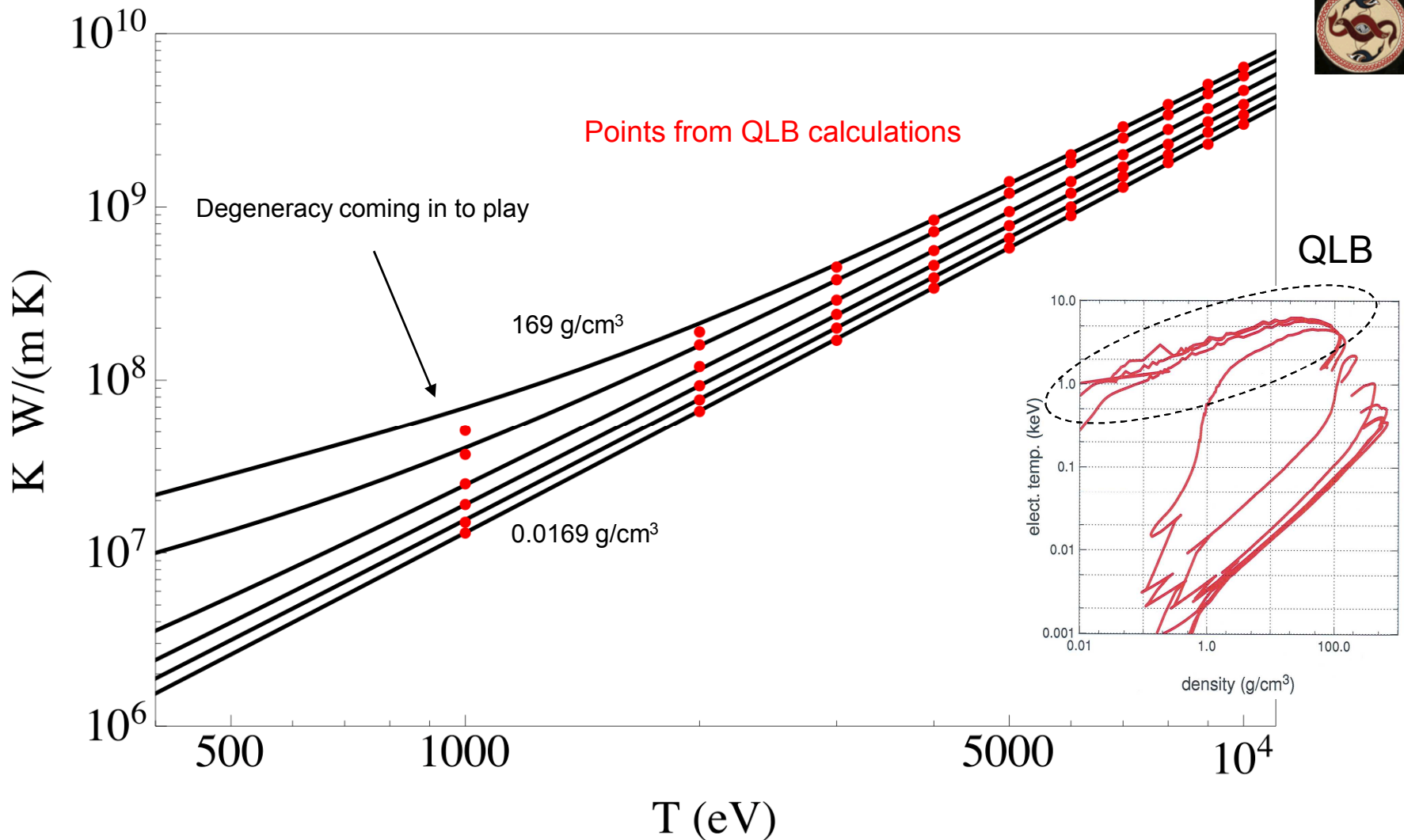
Dynamic screening has a greater effect in reducing the thermal conductivity, pushing L below the Spitzer limit



The model accurately represents the QLB electrical conductivity results over many decades of  $\rho$  and  $T$

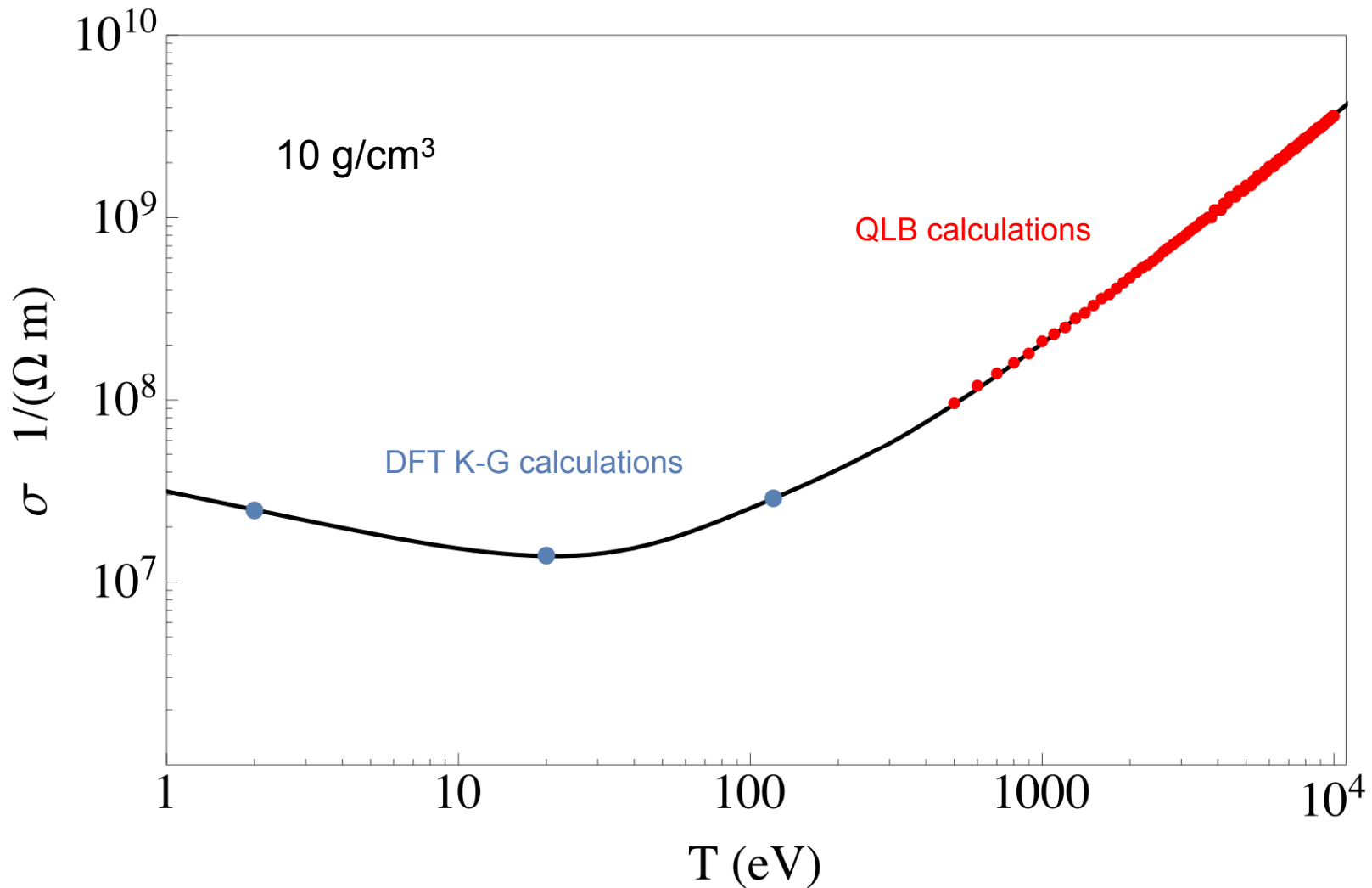


The model accurately represents the QLB thermal conductivity results over many decades of  $\rho$  and  $T$

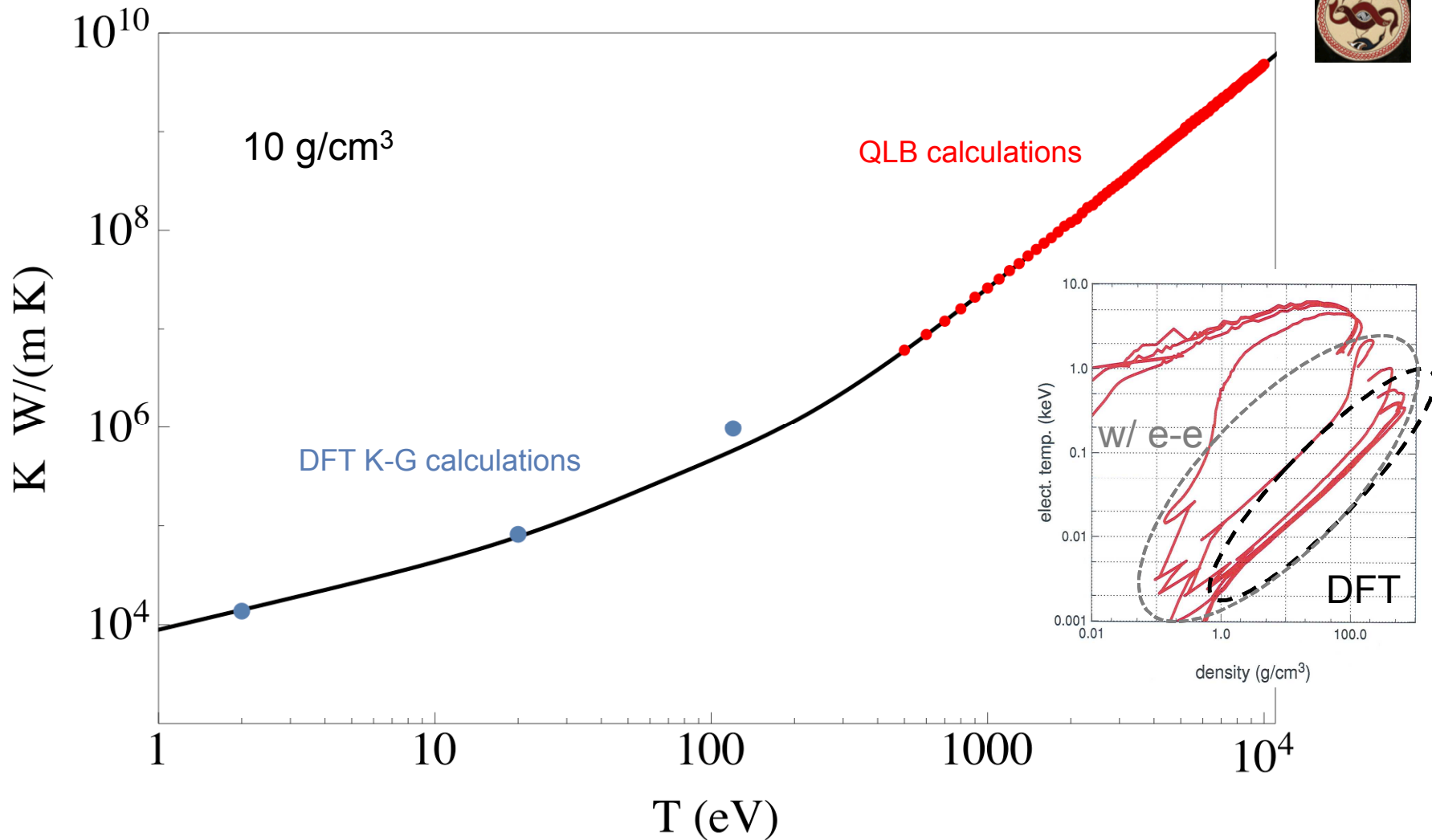


These QLB results are lower than the current LLNL thermal conductivity model by about 15%

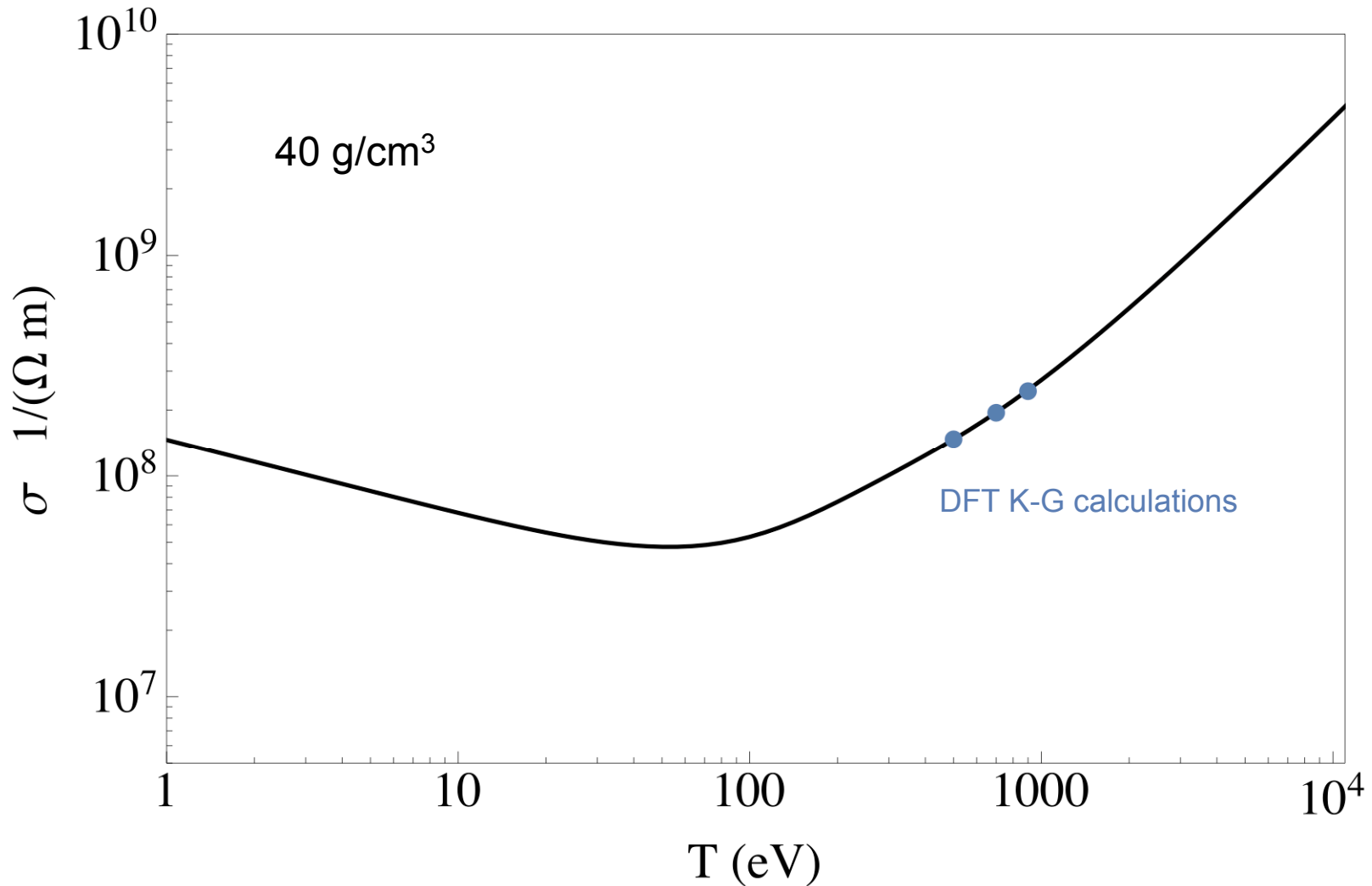
The model is tuned to our DFT Kubo-Greenwood electrical conductivity calculations in the degenerate limit



The model is tuned to our DFT Kubo-Greenwood thermal conductivity calculations in the degenerate limit

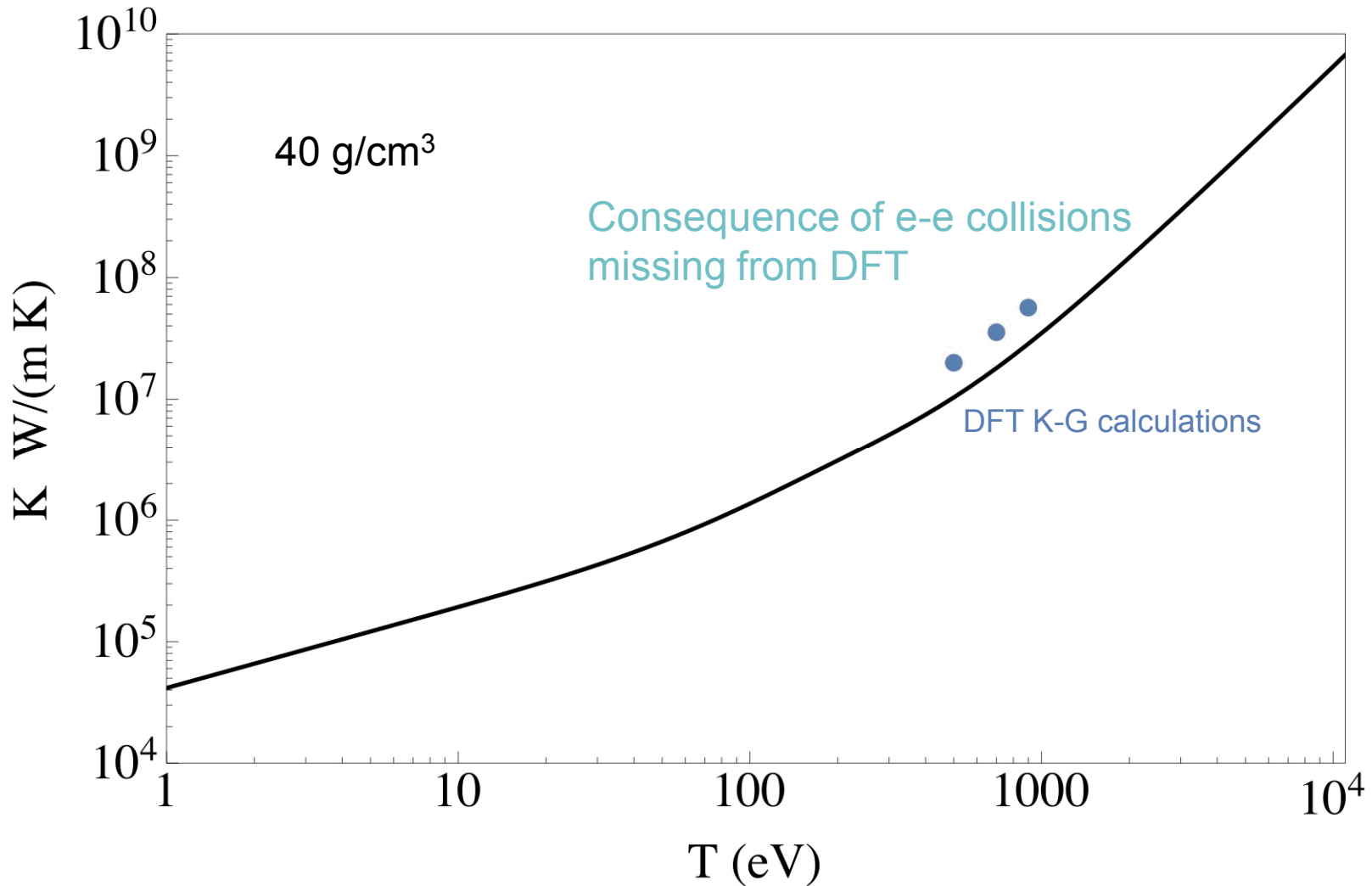


We accurately fit DFT-KG electrical conductivities calculated at higher density in the non-degenerate limit

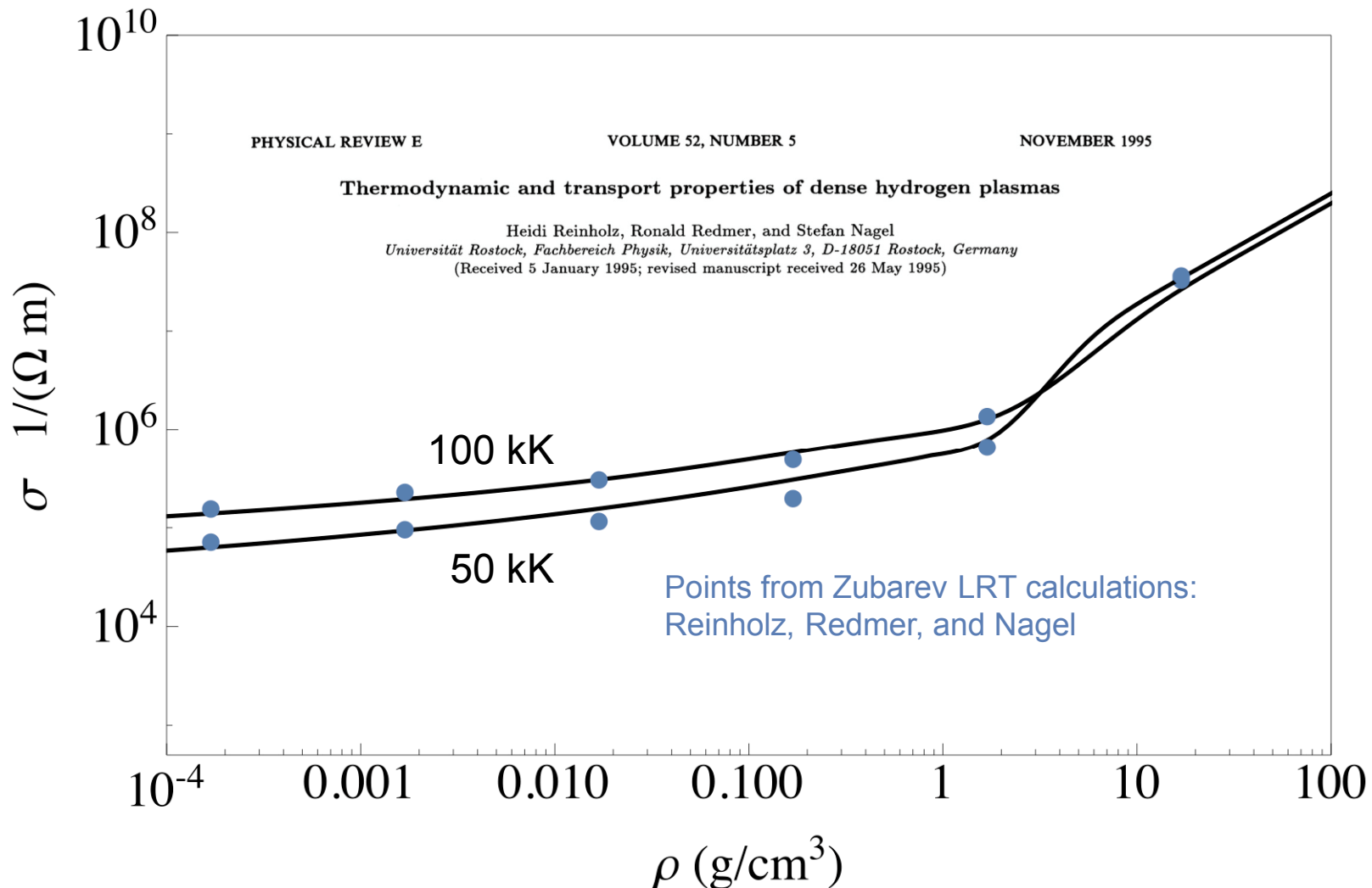




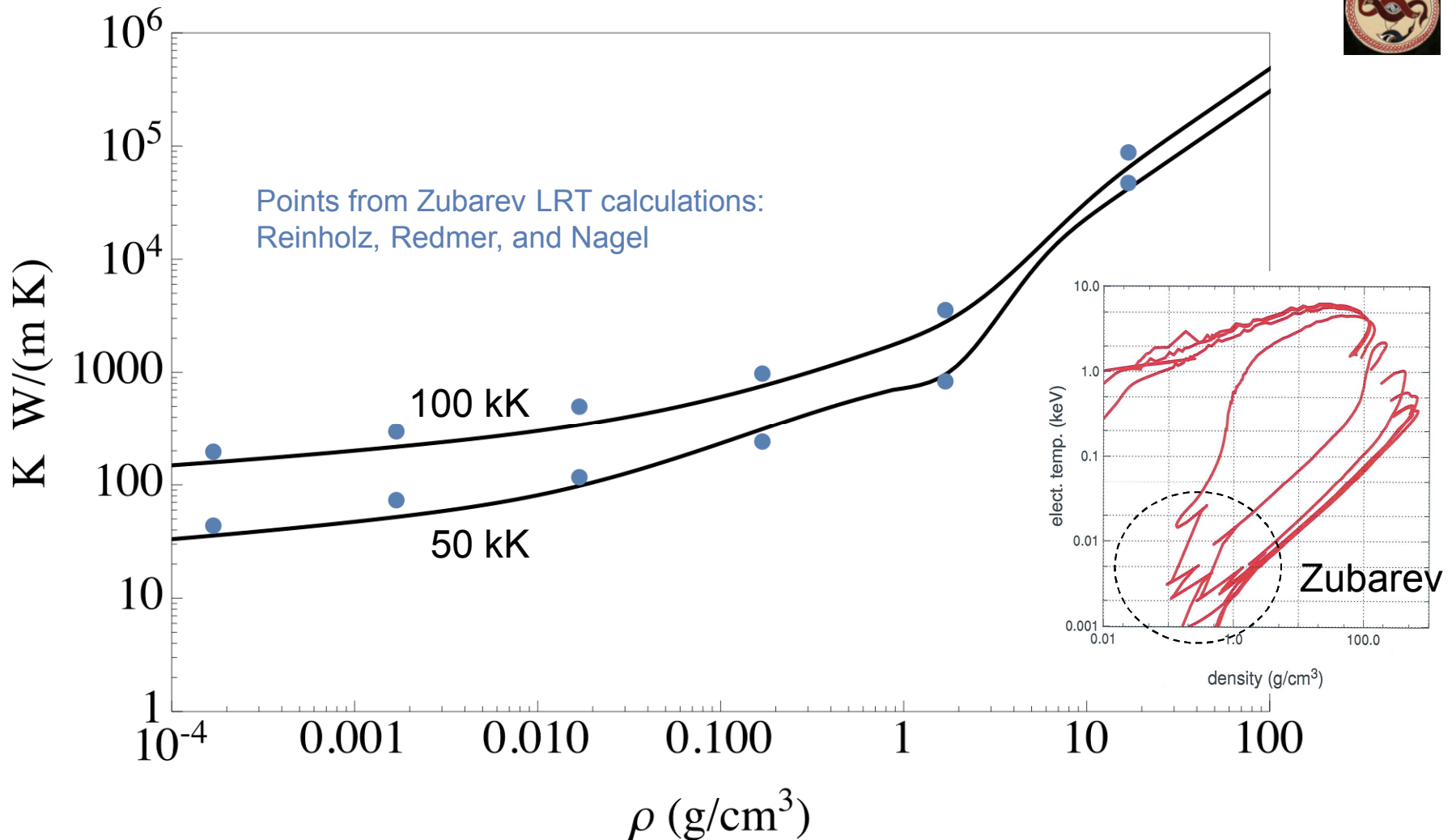
The DFT-KG thermal conductivities in the non-degenerate limit are missing the explicit e-e scattering contribution



# The wide-range model is in good agreement with Zubarev electrical conductivity calculations for hot expanded plasmas



# The wide-range model is in good agreement with Zubarev thermal conductivity calculations for hot expanded plasmas



We will revisit the Zubarev calculations with static screening lengths matched to dynamic screening 27

# Summary



- Comparisons between state-of-the-art 3D simulations and data from NIF experiments are in better agreement when the current model for thermal conductivity is reduced
- We have combined results from 3 different calculational frameworks to tune a wide-range model for the electrical and thermal conductivity of hydrogen emphasizing ICF conditions
- The model accurately captures the results of the calculations, particularly in the region of the hot-spot boundary where sensitivity is greatest, and provides for a 15% lower thermal conductivity