

Strain-rate-dependent Failure Criteria for Composite Laminates: Application of the Northwestern Failure Theory to Multiple Material Systems

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ABSTRACT

The strain-rate-dependent matrix-dominated failure of multiple fiber-reinforced polymer matrix composite systems was evaluated over the range of quasi-static (10^{-4}) to dynamic (10^3 s $^{-1}$) strain rates using available experimental data from literature. The strain rate dependent parameter, m , was found to relate strain-rate dependent lamina behavior linearly to the logarithm of strain rate. The parameter was characterized for a class of laminates comprised of epoxy-based matrices and either carbon or glass fibers, and determined to be approximately 0.055 regardless of fiber type. The strain-rate-dependent Northwestern Failure Criteria were found to fit all data in superior agreement to classical approaches across all strain rates evaluated based on solely lamina-level properties. It was determined that using the determined m value with the Northwestern Failure Criteria provided an accurate prediction of material behavior regardless of fiber type for the identified material class, which significantly reduces the material characterization testing required for the typical building block approach used by industry for computational analysis validation.

Keywords: Composites, Failure, Strain Rate, Northwestern Failure Theory, Verification and Validation

Introduction

The Northwestern University Failure Theory for composite lamina has previously been validated using the IM7/8552 material system for lamina and angle-ply laminates [1-6] from quasi-static to dynamic strain rates. This material system was also recently evaluated by Camanho et al. [7] in support of a proposed three-dimensional invariant-based failure criterion for composite lamina. The carbon-epoxy constituent combination provides high strain to failure and intermediate strength for aerospace structural applications. While the experimental results appear to fit the theoretical prediction quite well, they merely serve as preliminary validation (via redundancy) to the underlying theory. Of particular importance is the need to predict the influence of strain rate on composite material properties, the validation of which typically requires extensive experimental data. The objective of the current work is to highlight a simple modeling approach that uses the Northwestern Failure Theory to predict composite strain rate dependence for multiple material classes using reduced and practical data sets.

Strain-rate effects on matrix-dependent properties

To investigate a potential relationship between strain rate and matrix response, the experimental data from [1] is first used as an example for developing the strain-rate-dependence model based on IM7/8552. The strain rate dependent matrix dependent material properties are shown in Table 1.

Table 1 Strain rate dependent properties for IM7-8552 Lamina [1]

Property	Average Strain Rate, $\dot{\varepsilon}$ (s ⁻¹)		
	10 ⁻⁴	1	800
Transverse Modulus, E_2 (GPa)	9	10.6	11.2
Shear Modulus, G_{12} (GPa)	5.6	6.23	6.8
Transverse Tensile Strength, F_{2t} (MPa)	76	89.6	105
Transverse Compressive Strength, F_{2c} (MPa)	288	357	393
Shear Strength, F_6 (MPa)	89	109	122.5

The modulus and strength data were first normalized by the respective static values. The strain rates imparted span a range of six decades; thus, a logarithmic axis was used as the strain rate scale against which the normalized moduli/strength values were plotted. Important to note in the analysis is that the axial stress and strain are in relation to the global loading axes and not the lamina principal directions. Daniel et al. [7] previously addressed the conversion of these strains to principal coordinates, and noted that the final result was insignificantly different than simply using the loading axis strains as reference. The results are shown in Figure 1 – Figure 3. For the tested strain rates, the moduli in Figure 1 have been fitted with a relationship that is linear with strain rate. The relationship is defined as:

$$P(\dot{\varepsilon}) = P(\dot{\varepsilon}_o) \left[m \log_{10} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right) + 1 \right]$$

where P is the modulus (E_2 , G_{12}), m is 0.034, and $\dot{\varepsilon}_o$ is reference strain rate of 10⁻⁴ s⁻¹.

Figure 2 also shows that the lamina strengths F_{2c} , F_{2t} , and F_6 vary linearly with strain rate. The relationship is modified to reflect strength properties:

$$F(\dot{\varepsilon}) = F(\dot{\varepsilon}_o) \left[m \log_{10} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right) + 1 \right]$$

where F is the strength (F_{2c} , F_{2t} , F_6), m is 0.055, and $\dot{\varepsilon}_o$ is reference strain rate of 10⁻⁴ s⁻¹.

Figure 66 shows the transverse compressive strength variation with strain rate in relation to the several tests performed at each rate. The relationship and parameters are then:

$$F_{2c}(\dot{\varepsilon}) = F_{2c}(\dot{\varepsilon}_o) \left[m \log_{10} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right) + 1 \right]$$

where m is 0.054 and $\dot{\varepsilon}_o$ is reference strain rate of 10⁻⁴ s⁻¹.

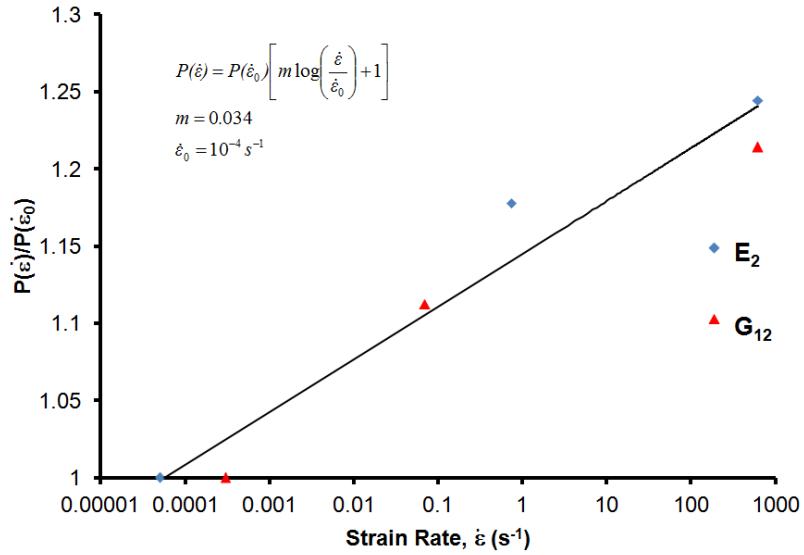


Figure 1 Variation of transverse and shear moduli with strain rate

From Figure 1, the moduli appear to be affected similarly by increasing strain rate. Further, the assumption may be made that since these material stiffnesses behave similarly, the ratio of them may be considered a constant. Therefore, the ratio (defined below) may be obtained by simply performing the static characterization.

$$\frac{E_2}{G_{12}} = \alpha$$

Also important to note is that the shear modulus increases similarly to what is predicted by the theoretical model.

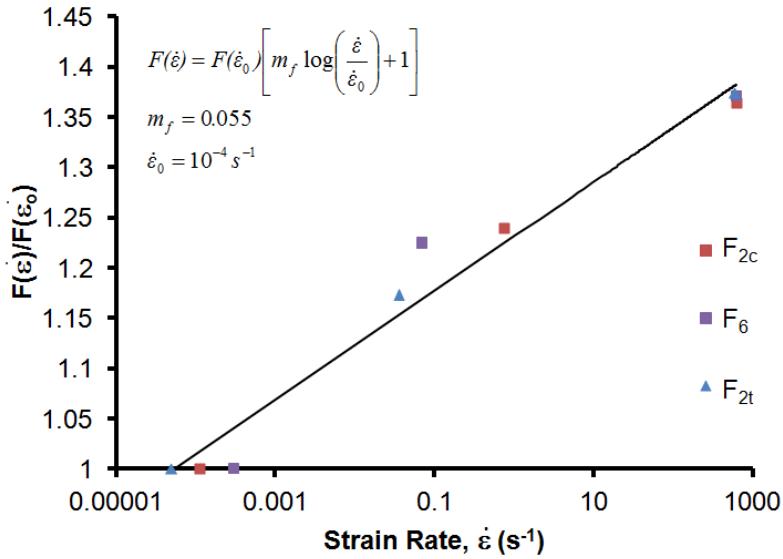


Figure 2 Variation of transverse compressive and shear strength versus strain rate

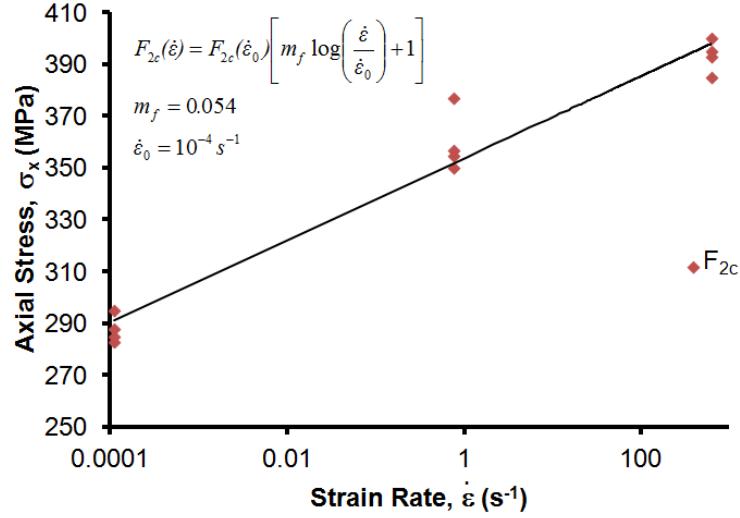


Figure 3 Variation of transverse compressive strength with strain rate

The figures provide an intriguing opportunity for modeling the lamina behavior. For the measured strain range, the strain-rate-sensitive properties have a linear response to the logarithm of strain rate. Using the presented relationship, the properties may be transformed from the current strain rate to that of the reference strain rate, $\dot{\varepsilon}_0$, according to the factor:

$$f(\dot{\varepsilon}) = m \log_{10} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) + 1$$

The values for σ_2 and τ_6 may then be determined for these rates through

$$\dot{\sigma}_i = \sigma_i f(\dot{\varepsilon})^{-1}$$

$$\sigma_i = \sigma_2, \tau_6$$

The above relations may then be used to map failure envelope data across the applicable strain rates into a master envelope based on the 'm' parameter, as shown in Figure 4 and Figure 5.

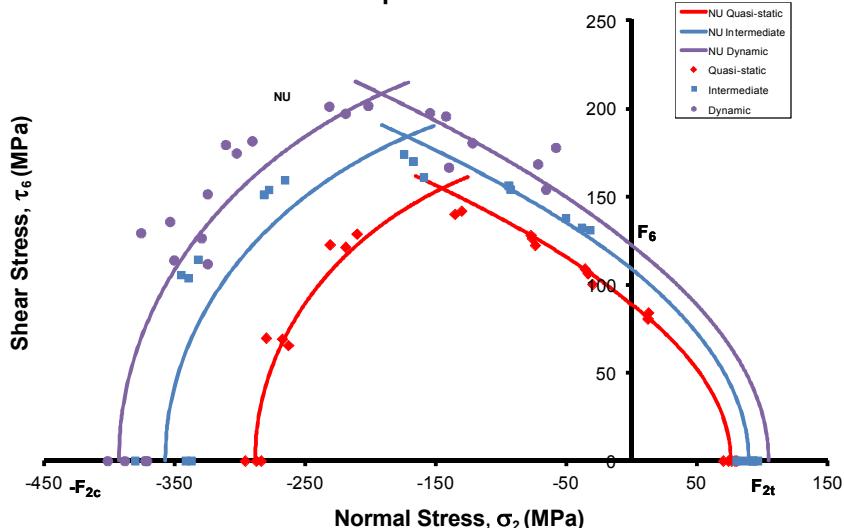


Figure 4 Quasi-static, intermediate, and dynamic strain rate data plotted with Northwestern Failure Theory failure envelopes [1]

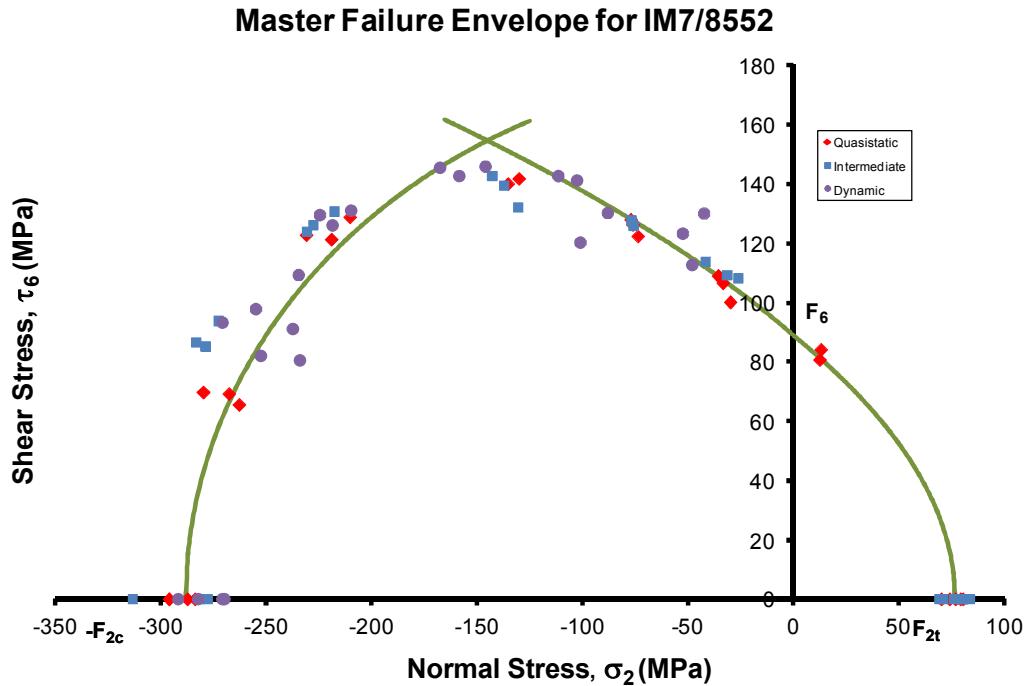


Figure 5 Strain-rate-dependent master failure envelope for IM7/8552 composite lamina [1]

The above methodology is next applied to available literature data for a variety of material systems to investigate and evaluate strain rate dependence based on material class.

Application to Additional Carbon-Epoxy Material Systems

The behavior of IM7/8552 at varying strain rates was recently investigated by Raimondo et al. [8]. As shown in Figure 6, the data matches well to that obtained in [1], and the NU Theory provides an excellent fit. The data was used to determine a rate dependence parameter (m_f) of 0.061.

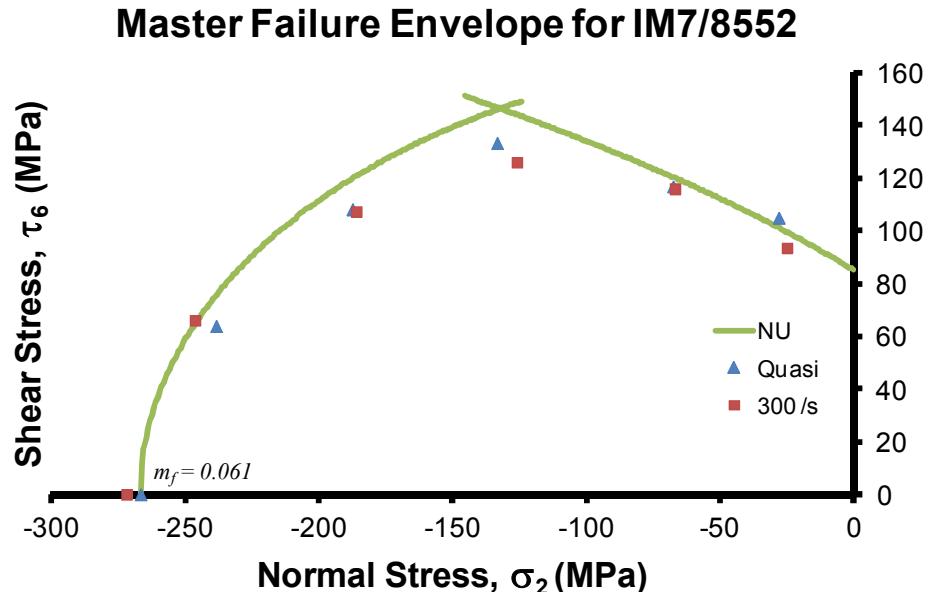


Figure 6 Master NU Lamina Failure Envelope for IM7/8552; data from [8]

The IM7/8552 lamina combines stiff carbon fibers with a toughened epoxy matrix. The fibers are considered to exhibit linear elastic behavior that is independent of strain rate, while the matrix has been shown to undergo significant property variation with strain rate. Therefore, it is generally proposed that similar composite systems-wherein the fiber stiffness greatly exceeds that of the matrix- should be governed by the same behavior. For example, the lamina system IM7/977-2 was investigated by Gilat et al. [10] to determine the biaxial tensile strain rate dependence. 977-2 is a toughened-epoxy formulation that has similar properties (Table 2) to those of 8552, except with lower shear strength; thus, this provides a fitting opportunity by which to compare the NU Theory fit. The experimental comparison to the NU Theory prediction is shown in Figure 7.

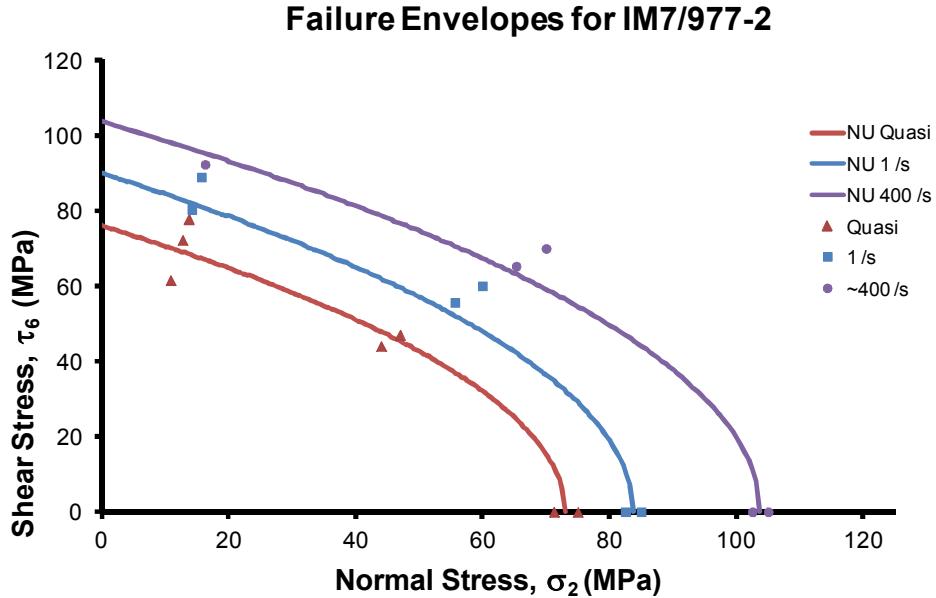


Figure 7 NU Lamina Failure Envelopes for IM7/977-2; data from [10]

The matrix dependent properties are clearly influenced by strain rate. The rate dependent parameter, m_f , was determined and the IM7/977-2 data was recast into a master failure envelope, shown in Figure 8.

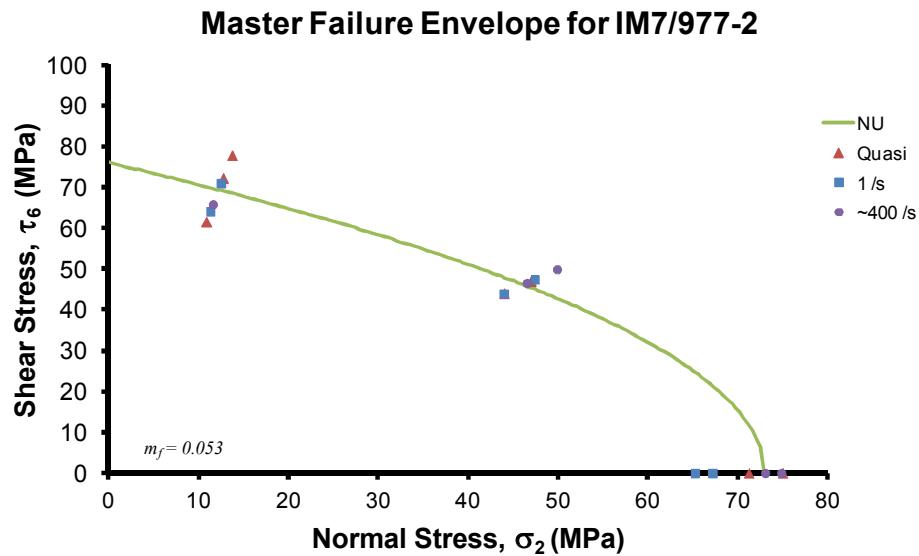


Figure 8 Master NU Lamina Failure Envelope for IM7/977-2; data from [10]

The resulting fit to the IM7/977-2 system is excellent – indicating that when the fiber type remains the same and the matrix properties (formulation) are similar, the NU Theory aptly predicts failure. This again holds true in another similar composite system, IM7/8551-7, the data for which was obtained by Vinson et al. [11] and is presented graphically in Figure 9.

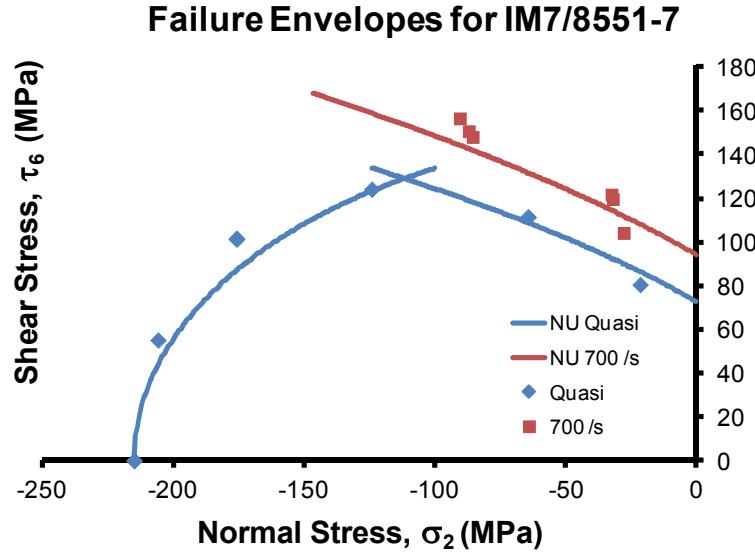


Figure 9 NU Lamina Failure Envelopes for IM7/8551-7; data from [11]

The quasi-static material properties for IM7/8551-7 are provided in Table 2. The rate dependence was determined based on the available data, and the master failure curves are presented in Figure 10.

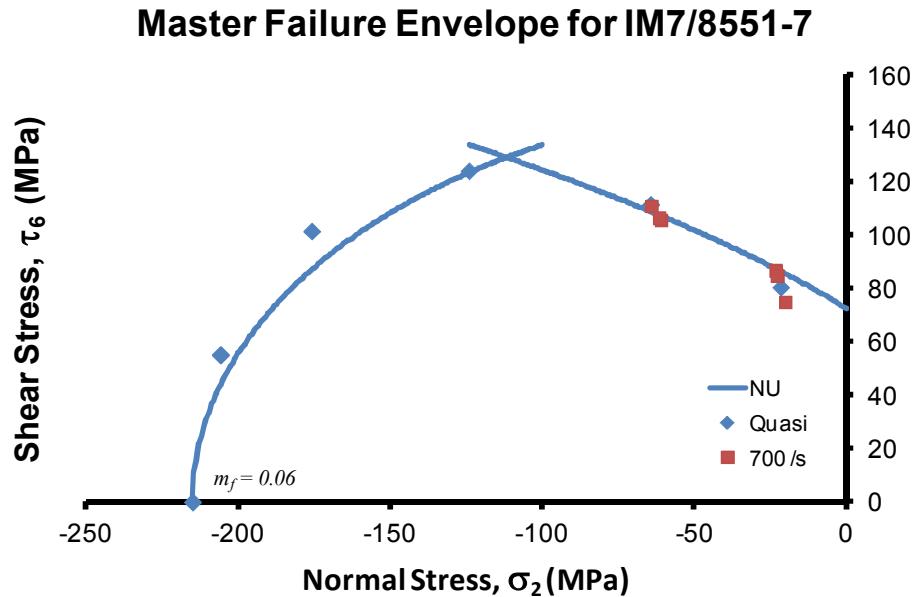


Figure 10 Master NU Lamina Failure Envelope for IM7/8551-7; data from [11]

The fit to experimental data is again quite good; further, the rate dependence was determined to be similar to that of the IM7/8552 and IM7/977-2 systems. The analysis verifies that the NU Theory is well-suited for predicting failure of toughened epoxy matrix/high stiffness fiber lamina composites. A critical consideration is to investigate the predictive capability of the NU Theory for less ductile matrices partnered with a different carbon fiber. Daniel et al. [7] previously determined the strain rate dependent behavior of AS4/3501-6 composite lamina (Table 2), and the results are shown in Figure 11.

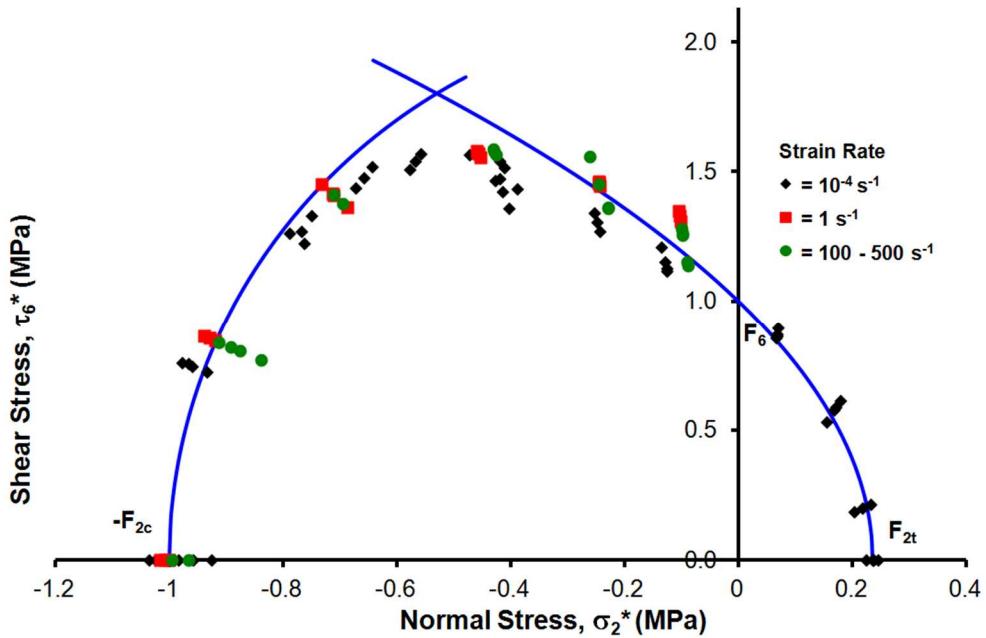


Figure 11 Normalized Master NU Lamina Failure Envelope for AS4/3501-6; data from [7]

The strain rate dependence parameter (m) was determined to be 0.057 (~0.06). The AS4/3501-6 lamina has higher stiffness and behaves in a more brittle fashion compared to the 8552, 8551-7, and 977-2 laminae. However, the NU Theory is also shown to predict failure quite well for this system. Bing and Sun [12] also performed an extensive set of tests on the AS4/3501-6 system at varying strain rates. The data from the authors' investigation indicated a strain rate dependence parameter of 0.06, and the fit is shown in Figure 12.

Master Failure Envelope for AS4/3501-6

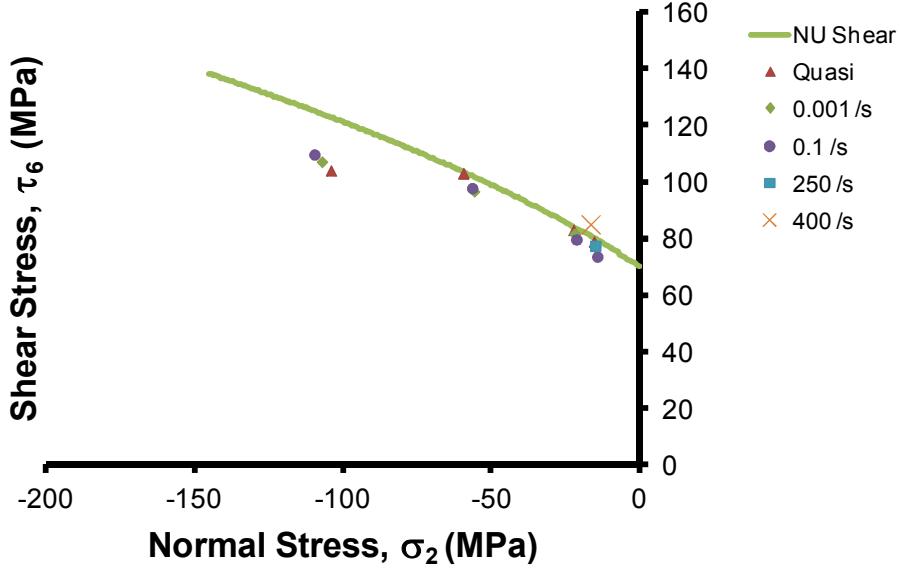


Figure 12 Master NU Failure Envelope for AS4/3501-6; data from [12]

A further comparison is made for the 3501-6 matrix combined with IM6G fibers [13, 14]. The rate dependence was determined to be $m_f = 0.063$. Only data for transverse compression was available in the study; however, the strain-rate-dependence is almost the same as that for the AS4/3501-6 system. This provokes an intriguing question: What role, if any,

do the fibers play in matrix-dominated failure behavior? The strain-rate-dependence of glass fiber and epoxy systems is next evaluated.

Application to Glass-Epoxy Systems

Tsai and Sun [15, 16] previously determined the strain rate dependence of an S2/8552 (glass fiber) lamina. S2 glass fibers are considerably less brittle than carbon fibers. The results are plotted with the NU failure envelope in Figure 13, and the properties are listed in Table 2.

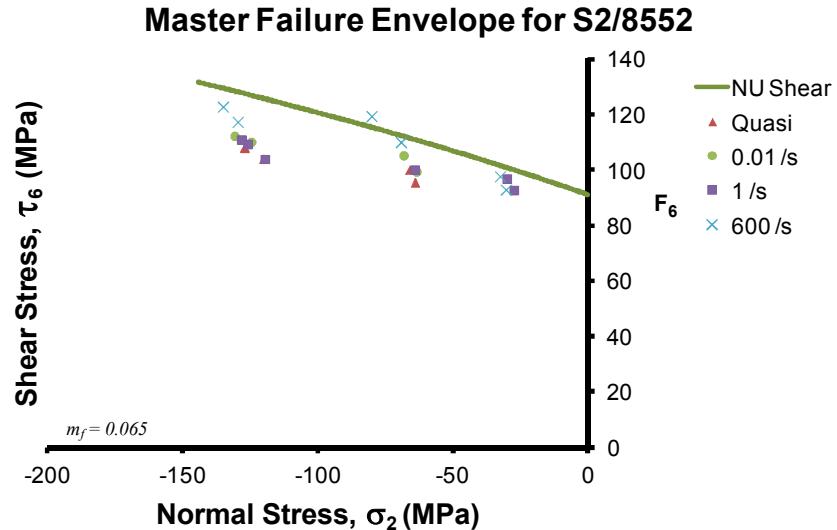


Figure 13 Master NU Failure Envelope for S2/8552; data from [15, 16]

Interestingly, the rate dependence based on the shear strength (F_6) is similar to the previous values determined for lamina utilizing the 8552 system (although, the value for E_2 appears to be inaccurate). This indicates that for the range of fiber stiffness and fiber volume fractions considered, the strain rate dependence of the matrix-dominated material properties is dependent solely on the matrix constituent. Significantly, the analysis indicates that once the strain rate dependence is determined for a given fiber/matrix combination, in this case IM7/8552, that same rate dependence holds when the matrix is used with other similar fiber types. Thirupukuzhi and Sun [17] obtained static and intermediate strain rate tensile data for the S2/8553-40 lamina system. The NU Theory envelope is presented in Figure 14.

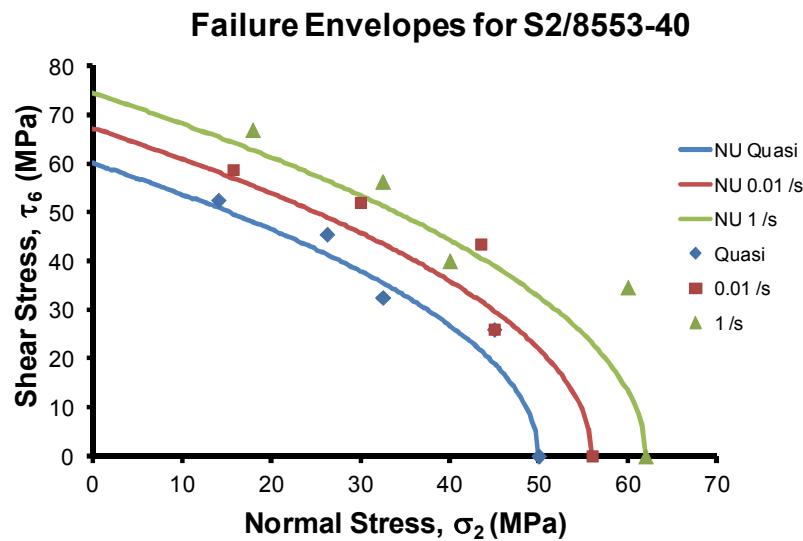


Figure 14 NU Lamina Failure Envelopes for S2/8553-40; data from [17]

The rate dependence was determined to be $m_f = 0.06$, which was similar to the other epoxy systems. The master failure envelope is provided in Figure 15. An overall comparison of the evaluated lamina rate dependencies is provided in Table 3.

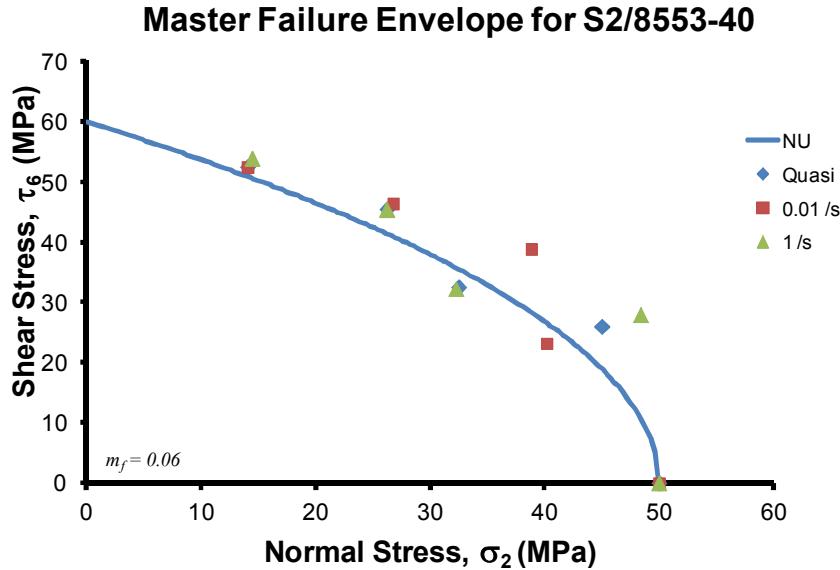


Figure 15: Master NU Lamina Failure Envelope for S2/8553-40; data from [17]

The individual envelope fits appear to be quite good for the experimental data. Thus, the NU Theory is found to be useful for predicting the strain rate dependent failure of a given lamina for the domain of stiff fibers combined with comparatively ductile matrices. For a given matrix, it was determined that the fiber type was insignificant to the material strain rate dependence, which indicates that considerable testing time may be saved in characterizing material performance across different lamina types utilizing the same matrix but different fibers. Critically, the rate dependence parameter of each system was shown to be ~ 0.06 regardless of fiber or matrix type. For the general class of laminae investigated, the matrices range quite substantially in relative strength and strain to failure, but this bears little reflection in strain rate dependence. It is proposed that the chemical crosslinking of the epoxy polymer chains during cure are a driver of this similar behavior, but it is outside the scope of the current work. Significantly, the above analysis indicates that for the general combination of stiff fibers with epoxy matrix, the lamina strain rate dependence parameter may potentially be assumed to be 0.06, which greatly reduces the number of characterization tests required.

Table 2 Quasi-static lamina properties from literature

Fiber	Matrix	E_2	G_{12}	F_6	F_{2t}	F_{2c}	V_f
AS4	3501-6	10300	7	76	57	228	0.63
IM6G	3501-6	10000	--	--	--	267	--
IM6G	3501-6	10000	--	--	--	267	--
IM7	8552	--	--	--	--	267	--
IM7	977-2	8570	--	76	73	267	--
IM7	8551-7	8343	5860	72.6	76	215	--
S2	8552	20000	6900	87.3	--	--	0.56
S2	8552	20000	6900	87.3	--	--	0.56
S2	8553-40	12730	4460	60	50	--	--
IM7	8552	9000	5600	89	76.4	288	0.58

Table 3 Lamina rate dependence determine from literature data

Fiber	Matrix	m_r
AS4	3501-6	0.06
AS4	Epoxy	0.05
AS4	Epoxy	0.052
IM6G	3501-6	0.065
IM7	8552	0.061
IM7	977-2	0.053
IM7	8551-7	0.06
S2	8552	0.065
S2	8553-40	0.06

Summary and Conclusions

The NU Failure Theory has been verified by an investigation of the IM7/8552 lamina rate dependence, validated by investigation of the AS4/3501-6 lamina system, and further validated for stiff fiber/epoxy lamina systems of varying strength and strain at failure by several additional comparisons using data independently obtained from the current work. The methodology was shown to support the concept of matrix strain-rate-dependence based on material formulation, which may enable reduction of required test data to characterize strain-rate-dependent properties.

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