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High-Order Unstructured Entropy-Stable Methods for Compressible Fluid Dynamics Simulations

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Old Dominion Mathematics Seminar

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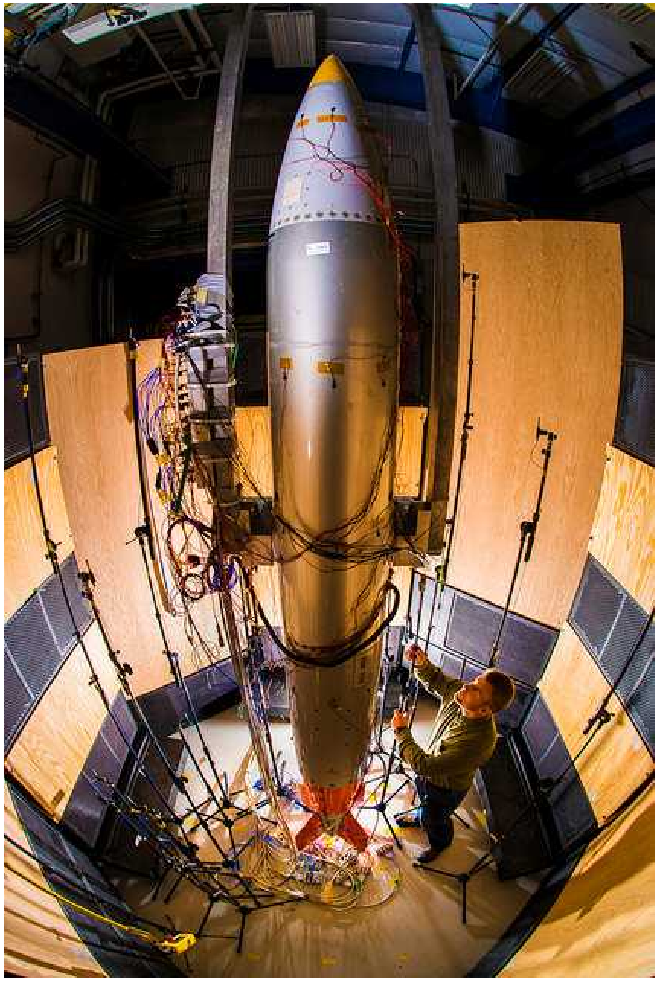


PROBLEMS OF INTEREST

DNS and LES

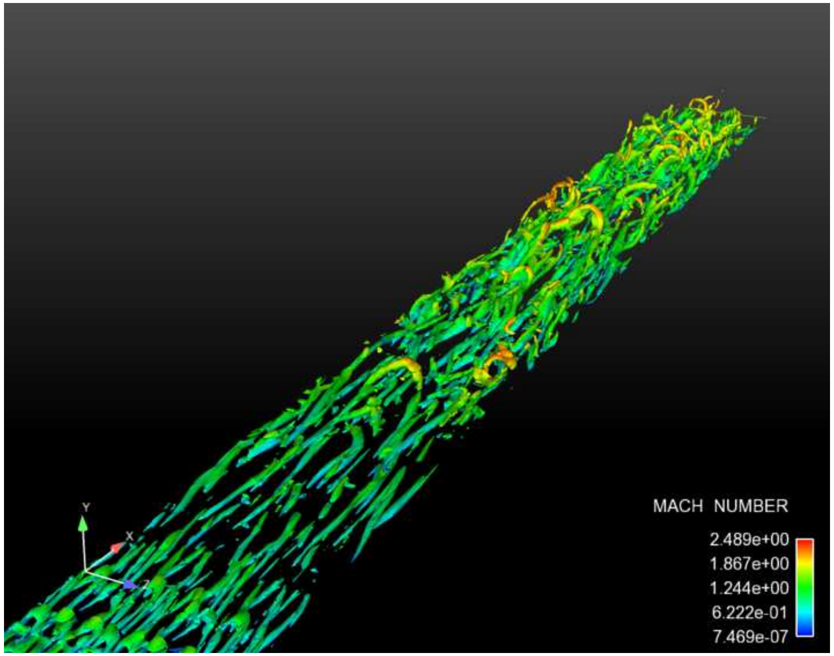


Figure: Mach 2.0 boundary layer DNS w/ p = 7 high-order elements on coarse mesh

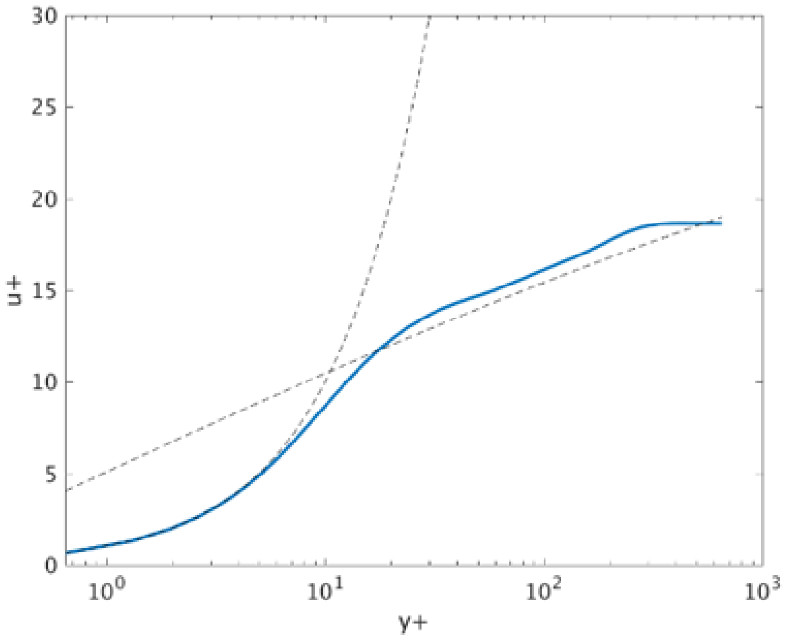


Figure: Coarse mesh misses profile

- Emphasizes explicit calculations
- Requires high resolution and low dissipation
- Most useful to inform uncertainties in engineering analyses
- Also useful for model development

DES and WMLES

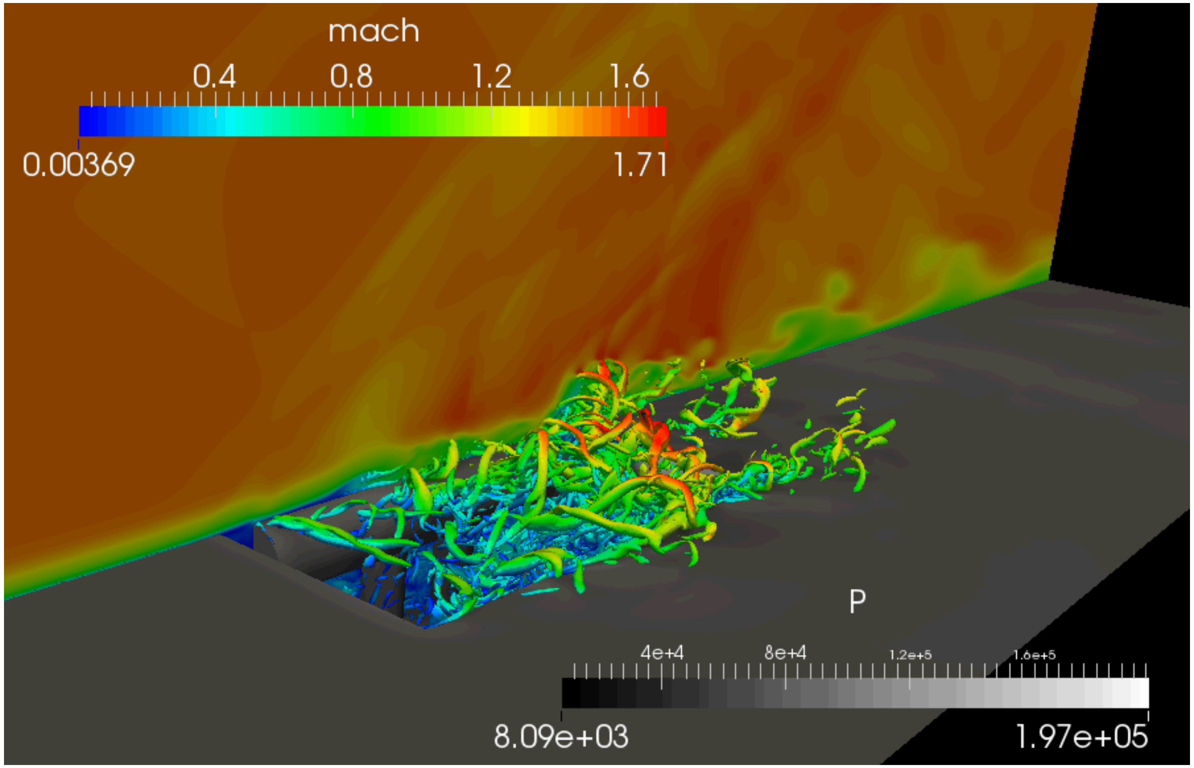
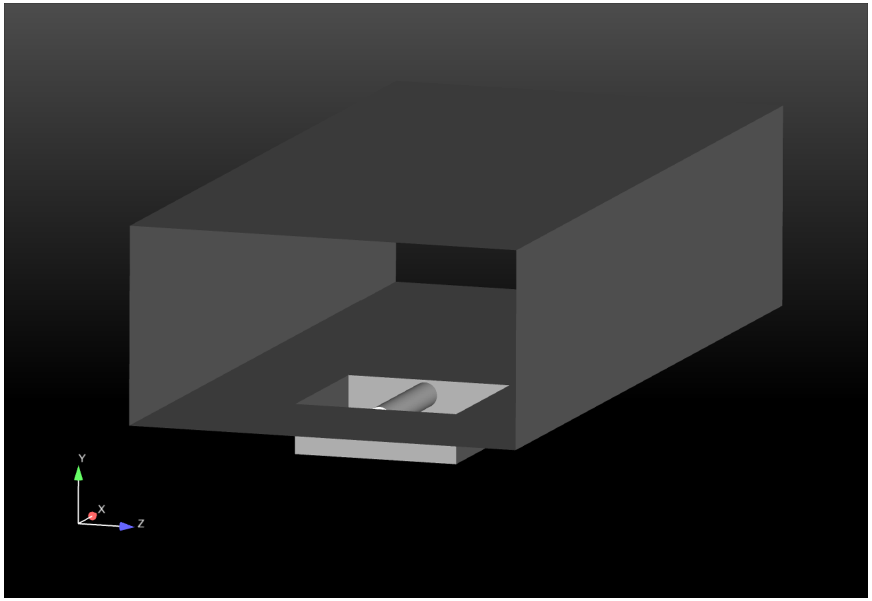
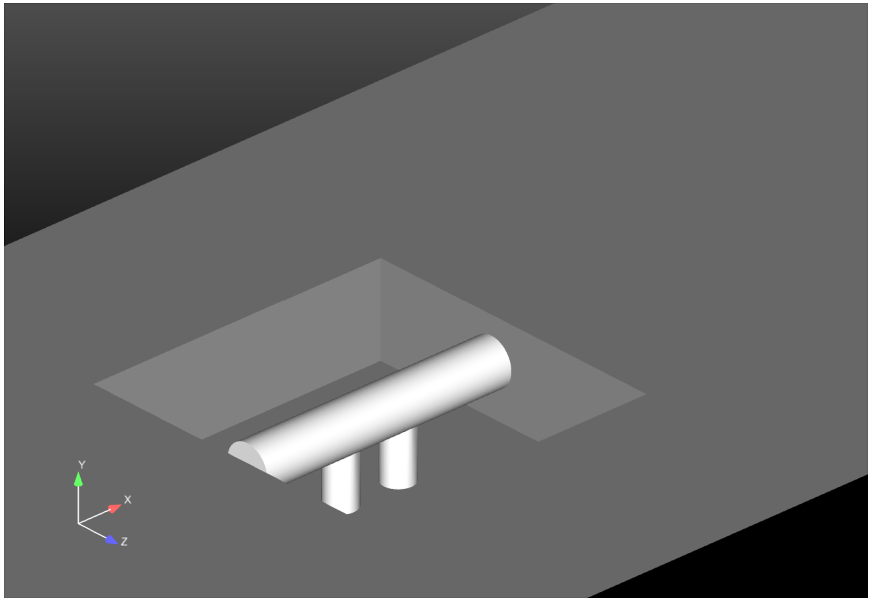


Figure: Turbulent flow around store in cavity

- Scale necessitates wall modeling
- Need to accurately predict acoustic load
- Direct engineering applicability
- Requires RANS solution and implicit time integration

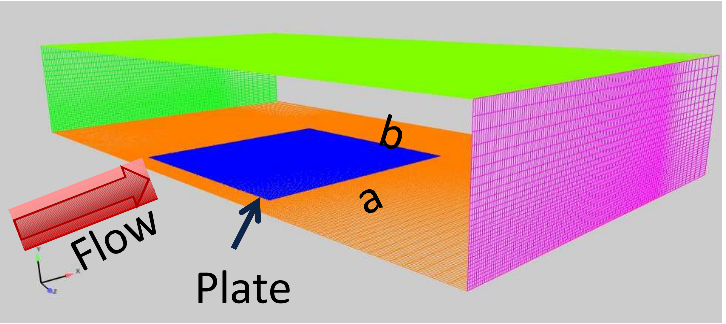
Geometric Complexity

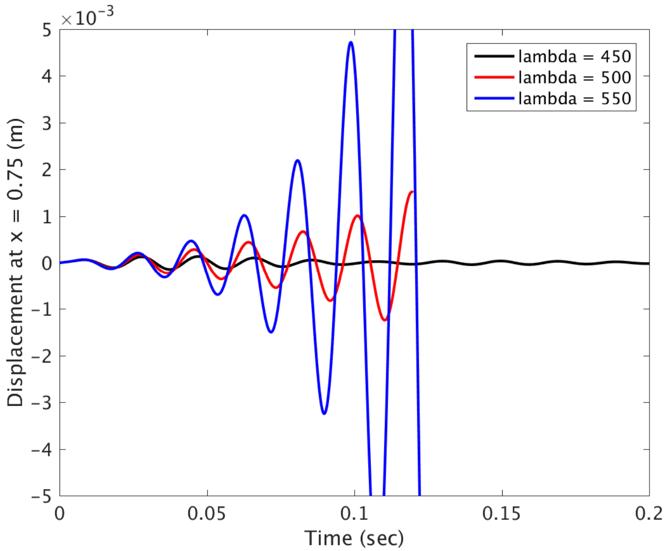




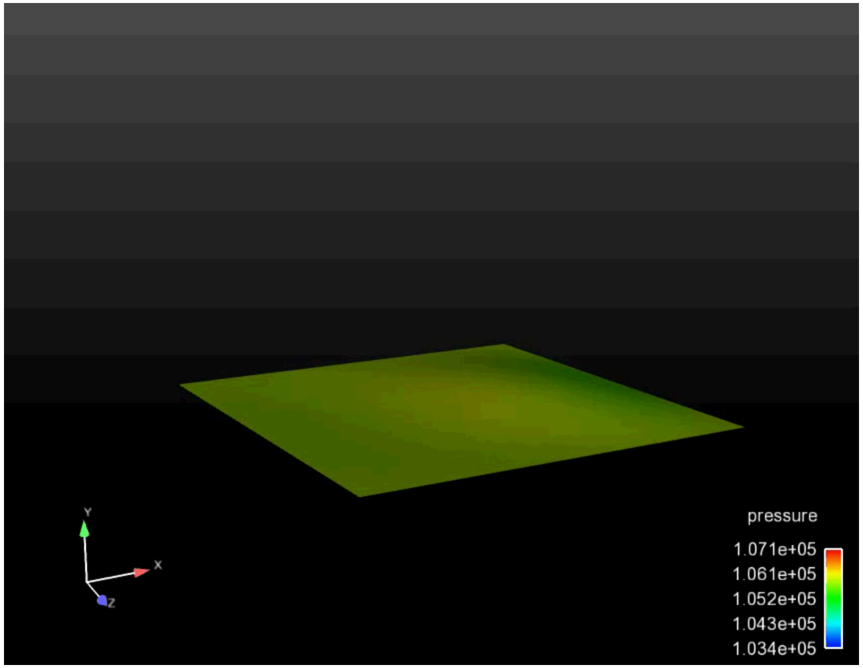
- Need to reduce mesh generation time
- Different methods have different quality requirements
- High-order unstructured requires smooth elements, but supports non-smooth mesh
- Unstructured has a more natural path to adaptivity

FSI and Moving Meshes





λ = 500



- Some problems require two-way coupling
- Often must combine low-order structure with high-order fluid

Efficiency and Robustness

- Efficiency
  - Strong argument for high-order in DNS
  - Lots of arguments about high-order in LES
  - Seemingly little success in DES for high-order
- What does robustness mean?
  - Doesn't blow up
  - Doesn't give unrealizable answers (expansion shocks, etc.)
  - Doesn't exhibit excessive spurious oscillations
  - Gives better answers with mesh refinement
- How do we get there?
  - Use the math Luke!
  - …and pray with RANS equations

VIEWS ON STABILITY AND OTHER REQUIREMENTS

Illustrative Example

$$\partial_t u + \partial_x f(u) = \partial_x g(u, \partial_x u),$$
$$f(u) = \frac{u^2}{2}, \quad g(u, \partial_x u) = \mu \partial_x u$$

- What do we care about?
  - Doesn't blow up and converges (stability)
  - Satisfies Rankine-Hugoniot shock relation

$$[f(u)]_{x_s^-}^{x_s^+} - \frac{dx_s}{dt}[u]_{x_s^-}^{x_s^+} = \frac{1}{2}\left(u_{x_s^+}^2 - u_{x_s^-}^2\right) - \frac{dx_s}{dt}\left(u_{x_s^+} - u_{x_s^-}\right)$$

  - Satisfies the entropy condition

$$\partial_t S + \partial_x F(u) = \partial_t\left(\frac{u^2}{2}\right) + \partial_x\left(\frac{u^3}{3}\right) \leq 0$$

Illustrative Example

$$\partial_t u + \partial_x f(u) = \partial_x g(u, \partial_x u),$$
$$f(u) = \frac{u^2}{2}, \quad g(u, \partial_x u) = \mu \partial_x u$$

$$u_h(x) = u_i \phi_i(x)$$

$$\int \phi_i \phi_j \partial_t u_j dV + \int \phi_i \partial_x f(u_h) dV = \phi_i \mu \partial_x \phi_j u_j |_{-1}^{1} - \int \partial_x \phi_i \mu \partial_x \phi_j u_j dV$$

How do we ensure stability of the nonlinear term?

$$\int u_i \phi_i f(u_h) dV \geq \left. \frac{u^3}{3} \right|_{-1}^{1}$$

Illustrative Example

How do we ensure stability (and entropy) of the nonlinear term?

$$\int u_i \phi_i f(u_h) dV \geq \left. \frac{u^3}{3} \right|_{-1}^{1}$$

Exact Integration

$$\int \phi_i \phi_k u_k \partial_x \phi_j u_j dV \rightarrow \int u_i \phi_i \phi_k u_k \partial_x \phi_j u_j dV = \left. \frac{u^3}{3} \right|_{-1}^{1}$$

Artificial Viscosity

$$\int \phi_i \partial_x \phi_j f(u_j) dV + \int \partial_x \phi_i \hat{\mu} \partial_x \phi_j dV \rightarrow \int u_i \phi_i \partial_x \phi_j f(u_j) dV + \int \partial_x \phi_i u_i \hat{\mu} \partial_x \phi_j dV \overset{?}{\geq} \left. \frac{u^3}{3} \right|_{-1}^{1}$$

Summation-by-parts

$$\frac{1}{3} \left([qu]_{ij} + [uq]_{ij} \right) u_j \rightarrow \frac{1}{3} u_i \left([qu]_{ij} + [uq]_{ij} \right) u_j = \left. \frac{u^3}{3} \right|_{-1}^{1},$$

$$[qu]_{i\alpha} = q_{i\alpha} u_\alpha, \quad [uq]_{\alpha i} = u_\alpha q_{\alpha i}$$

Illustrative Example

$$\partial_t u + \partial_x f(u) = \partial_x g(u, \partial_x u),$$
$$f(u) = \frac{u^2}{2}, \quad g(u, \partial_x u) = \mu \partial_x u$$

- Satisfying Rankine-Hugoniot
  - Prove it directly using weak form solution
    - Use integration by parts

$$[\psi f]_a^b + \psi|_{x_s} \left(\frac{dx_s}{dt} [u]_{x_s^-}^{x_s^+} - [f]_{x_s^-}^{x_s^+} \right) + \int_a^b \psi u_t - \psi_x f(u) dV$$

    - Be sure to keep everything in the space—need exact quadrature
  - For SBP operators, prove it on strong form using Lax-Wendroff Theorem

$$\mathcal{P} u_t + \Delta \bar{f} = 0 \qquad \bar{f}_i = \bar{f}_i(u_{i-\ell}, \ldots, u_{i+\ell}), \quad \bar{f}_i(v, \ldots, v) = f(v)$$

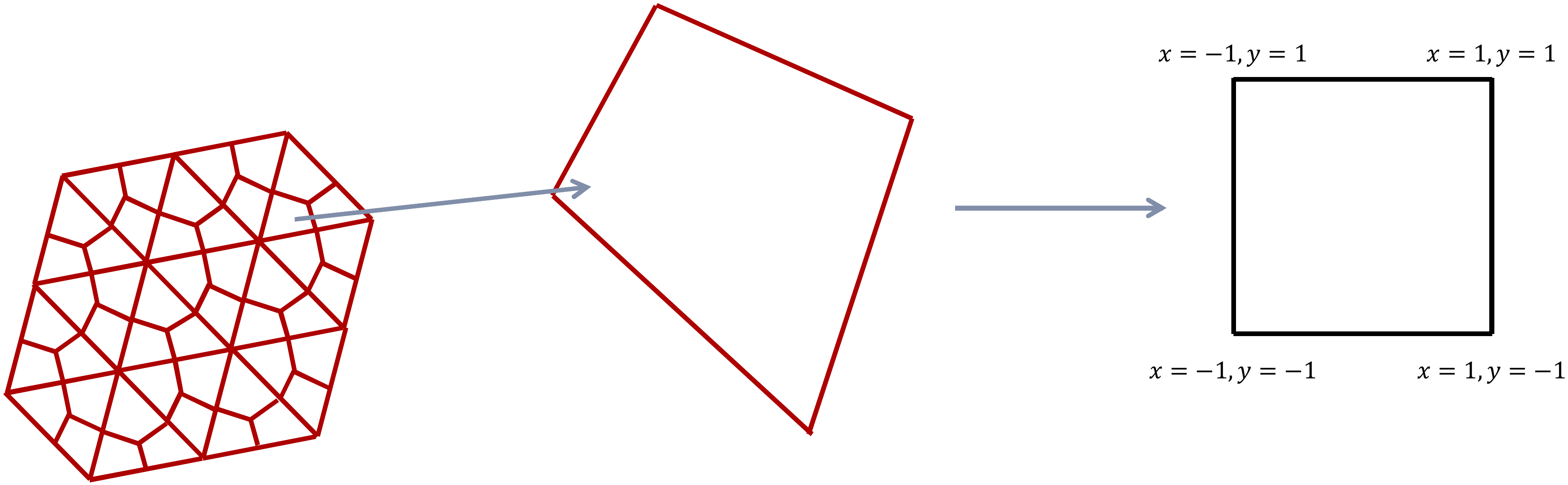
$$\psi^T \mathcal{P} \partial_t u_h + \psi^T \Delta \bar{f} = \psi_N f_N - \psi_1 f_1 + \psi^T \mathcal{P} \partial_t u_h - \psi_x^T \bar{f}$$

Illustrative Example

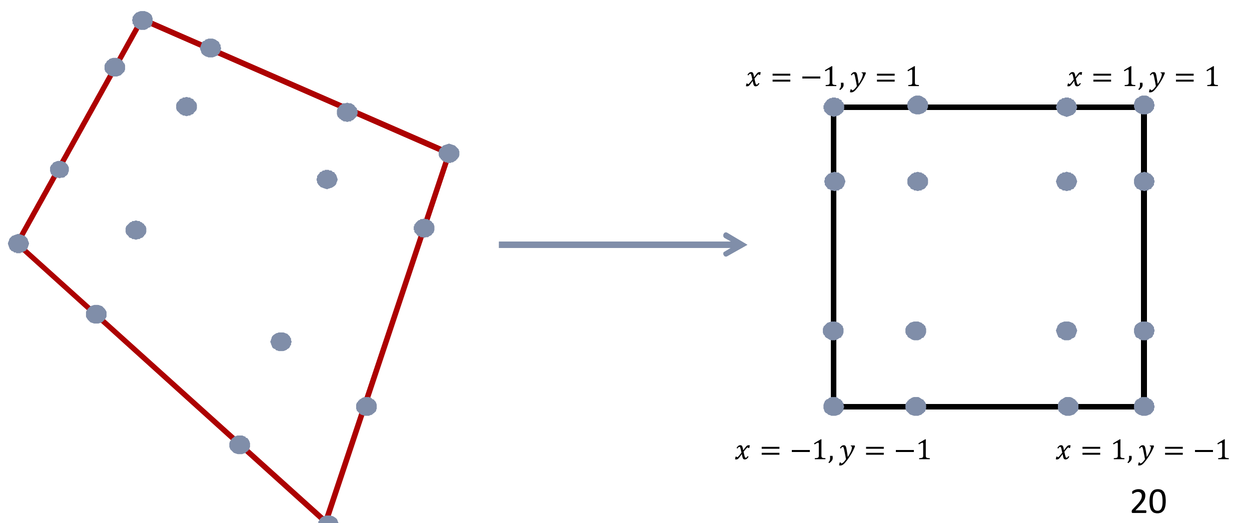
- Stability
  - Nonlinear energy/entropy condition on advective term bounds solution in L2
  - Ensures errors do not grow nonlinearly
- Weak solution
  - Want to ensure if we converge, we converge to weak solution
  - Wrong shock speeds have bad implications
- Entropy Stability
  - Ensures a weak solution is the physically realizable solution
  - Satisfies the second law of thermodynamics
  - Can be satisfied locally

OUR APPROACH

High-Order Collocation Elements



Enrich to p = 3



High-Order Collocation Elements

$$\int_\Omega \phi_i \phi_j \partial_t u_j dV + \int_\Omega \phi_i \partial_{x_k} \phi_j f_k(u_j) dV = g_i^{(b)} + g_i^{(I)} + g_i^{(S)} - \int_\Omega \partial_{x_\ell} \phi_i \hat{c}^{\ell k} \partial_{x_k} \phi_j w(u_j) dV$$

$$\mathcal{P}\partial_t \mathbf{u} + \mathcal{Q}_k \mathbf{f}_k = \mathbf{g}^{(b)} + \mathbf{g}^{(I)} + \mathbf{g}^{(S)} + \mathcal{Q}_\ell^T \hat{C}^{\ell k} \mathcal{D}_k \mathbf{w}$$

- Entropy conservation correction (no integral form)

$$g_i^{(S)} = [q_k]_{ij} f_k(u_j) - [q_k f_k^{(S)}]_{ij} 1_j, \quad [q_\beta f_\beta^{(S)}]_{i\alpha} = [q_\beta]_{i\alpha} f_\beta^{(S)}(u_i, u_\alpha)$$

$$\mathbf{g}^{(S)} = \mathcal{Q}_k \mathbf{f}_k - [\mathcal{Q}_k \mathcal{F}_k]\mathbf{1}$$

- Entropy stable inter-element penalty

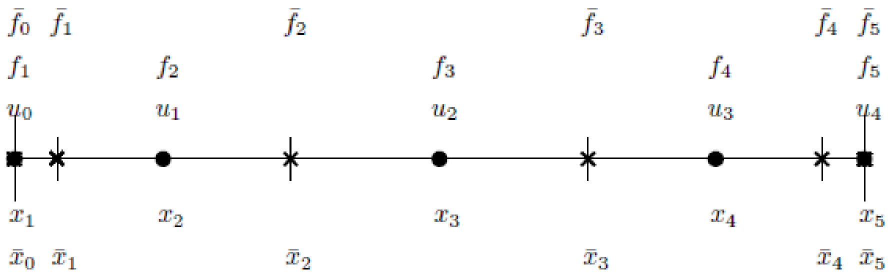
$$\begin{aligned} g_i^{(I)} = & [l_k f_k^{(S)}]_{ij}^- 1_j - [l_k f^{(S)}]_{i\ell}^+ 1_\ell + [l_k |\bar{A}|_k]_{ij}^- ([i]_{j\ell}^- u_\ell^- [i]_{jm}^+ u_m^+) \\ & + [r_k f_k^{(S)}]_{ij}^- 1_j - [r_k f^{(S)}]_{i\ell}^+ 1_\ell + [r_k |\bar{A}|_k]_{ij}^- ([i]_{j\ell}^- u_\ell^- [i]_{jm}^+ u_m^+) \end{aligned}$$

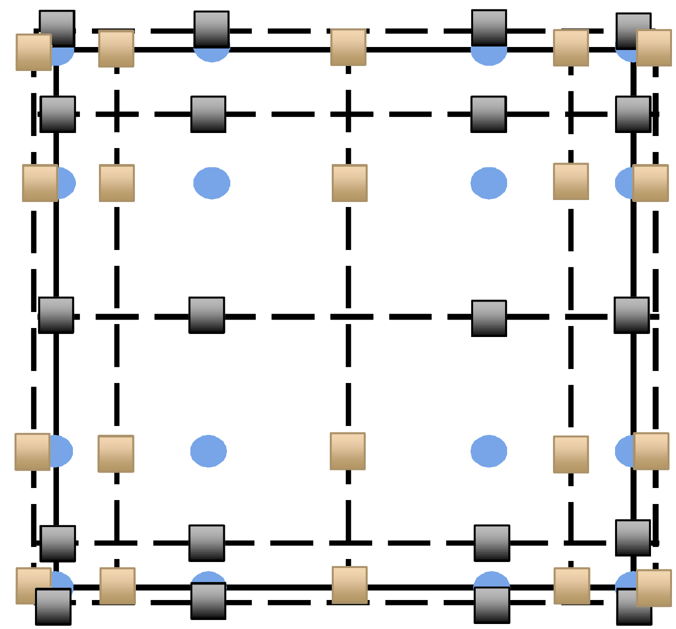
$$[l_k f^{(S)}]_{i\alpha}^+ = l_{i\alpha}^+ f_k^{(S)}(u_i^-, u_\alpha^+), \quad [l_k |\bar{A}|_k]_{i\alpha}^- = [l_k]_{i\alpha} |\bar{A}|([i]_{\alpha\ell}^- u_\ell, [i]_{\alpha m}^+ u_m^+)$$

$$\mathbf{g}^{(I)} = [L_k^- \mathcal{F}_k^-]\mathbf{1} - [L_k^+ \mathcal{F}_k^+]\mathbf{1} + [L_k^- |\mathcal{A}|_k]\mathbf{1} + [R_k^- \mathcal{F}_k^-]\mathbf{1} - [R_k^+ \mathcal{F}_k^+]\mathbf{1} + [R_k^- |\mathcal{A}|_k]\mathbf{1}$$

High-Order Collocation Elements

$$[q_k f_k^{(S)}]_{ij} 1_j = \bar{f}_i^{(S)k} - \bar{f}_{i-1}^{(S)k}, \quad \bar{f}_{i+1} = \sum_{k=i+1}^{N} \sum_{\ell=1}^{i} 2 q_{\ell k} f_k^{(S)}(u_\ell, u_k)$$





Entropy Stability

$$S = -\rho s \qquad F_i = -\rho u_i s \qquad w^T = \partial_u S$$

$$S_{uu} = S_{uu}^T, \quad \xi^T S_{uu} \xi > 0 \quad \forall \xi \neq 0$$

$$f_u = f_w w_u, \quad f_w = f_w^T, \quad \hat{c}_{ij} = c_{ij} w_u = \hat{c}_{ji}^T, \quad \xi_{x_i} \hat{c}_{ij} \xi_{x_j} \geq 0$$

$$\partial_t u + \partial_{x_i} f_i(u) = \partial_{x_i} c_{ij} \partial_{x_j} u$$

$$\partial_u S \partial_t u + \partial_u S \partial_{x_i} f_i(u) = \partial_u S \partial_{x_i} c_{ij} \partial_{x_j} u$$

$$\partial_t S + \partial_{x_i} F_i = w^T \partial_{x_i} \hat{c}_{ij} \partial_{x_j} w = \partial_{x_i} \left(w^T \hat{c}_{ij} \partial_{x_j} w \right) - \partial_{x_i} w^T \hat{c}_{ij} \partial_{x_j} w$$

$$\frac{d}{dt} \int_\Omega S dV + \int_{\partial\Omega} F_i dS_i = \int_{\partial\Omega} w^T \hat{c}_{ij} \partial_{x_j} w dS_i - \int_\Omega \partial_{x_i} w^T \hat{c}_{ij} \partial_{x_j} w dV$$

Entropy Stability

$$\frac{d}{dt}\int_{\Omega} S dV + \int_{\partial\Omega} F_i dS_i \leq 0$$

- Satisfy entropy conservation by construction

$$w^T[\mathcal{Q}_k \mathcal{F}_k]\mathbf{1} = \mathbf{1}^T \mathcal{Q}_k F_k = F_k|_b - F_k|_a$$

- Sufficient condition is to satisfy locally

$$w_i^T(\bar{f}_i^{(S)k} - \bar{f}_{i-1}^{(S)k}) = \bar{F}_i - \bar{F}_{i-1}$$

$$\bar{f}_{i+1} = \sum_{k=i+1}^{N} \sum_{\ell=1}^{i} 2 q_{\ell k} f_k^{(S)}(u_\ell, u_k) \qquad (w_\ell - w_k)^T f_k^{(S)}(u_\ell, u_k) = \psi_\ell - \psi_k$$

- Need diagonal norm for time term

$$w_i^T p_{ij} \partial_t u_j = 1_i p_{ij} \partial_t S(u_j)$$

Discontinuity Capturing

- Artificial viscosity based on Hughes, Mallet, and Shakib

$$\hat{\mu} = \left(\frac{(\mathcal{L}u)^T(\mathcal{L}u)}{w^T u + \partial_{x_i} w^T g^{ij} u_w \partial_{x_j} w} \right)^{\frac{1}{2}}$$

$$\hat{\mu}_c = \max_k \frac{|T_k^{-1}\mathcal{L}u|}{(w^T u + \partial_{x_i} w^T g^{ij} u_w \partial_{x_j} w)^{\frac{1}{2}}}, \quad M_\alpha = T_\alpha \mathrm{diag}(\hat{\mu}_c) T_\alpha^T$$

$$\mathcal{L}u = \partial_t u + A_k \partial_{x_k} u - \partial_{x_i} \hat{c}_{ij} \partial_{x_j} w, \quad \partial_t u = -\partial_{x_k} f_k + \partial_{x_i} \hat{c}_{ij} \partial_{x_j} w$$

$$\mathcal{L}u = A_k \partial_{x_k} u - \partial_{x_k} f_k$$

- Need to Regularize
  - Project onto linear element
  - Take elemental max per node
  - Linearly interpolate back onto high-order element

Discontinuity Capturing

- Artificial viscous term

$$\int_{\partial\Omega} \phi_i g^{\ell m} \hat{\mu} u_w \partial_{x_m} \phi_j w(u_j) dS_\ell - \int_{\Omega} \partial_{x_\ell} \phi_i g^{\ell m} \hat{\mu} w_u \partial_{x_m} \phi_j w(u_j) dV$$

$$\int_{\partial\Omega} \phi_i g^{\ell m} M \partial_{x_m} \phi_j w(u_j) dS_\ell - \int_{\Omega} \partial_{x_\ell} \phi_i g^{\ell m} M \partial_{x_m} \phi_j w(u_j) dV$$

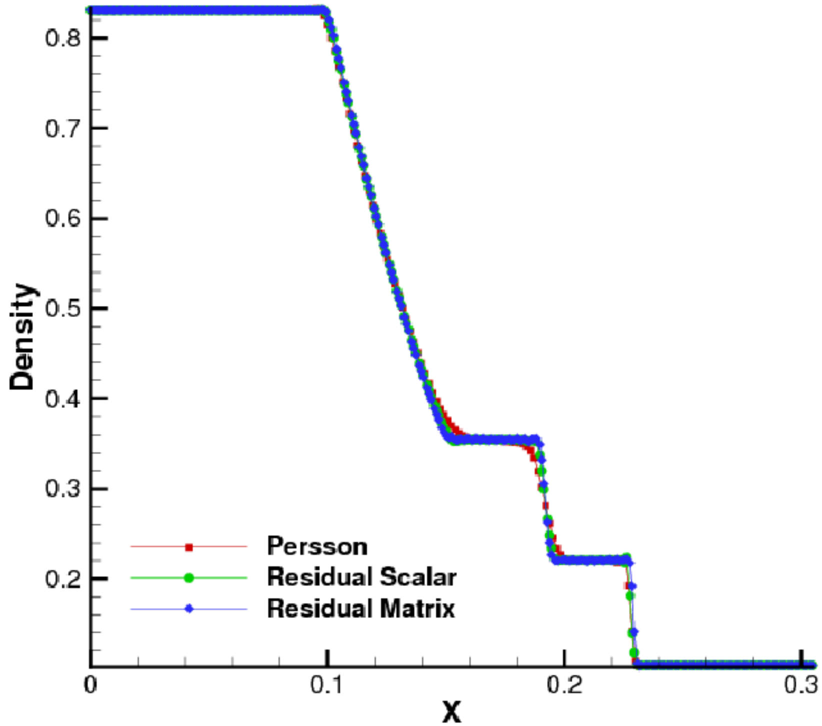
- Entropy Stability

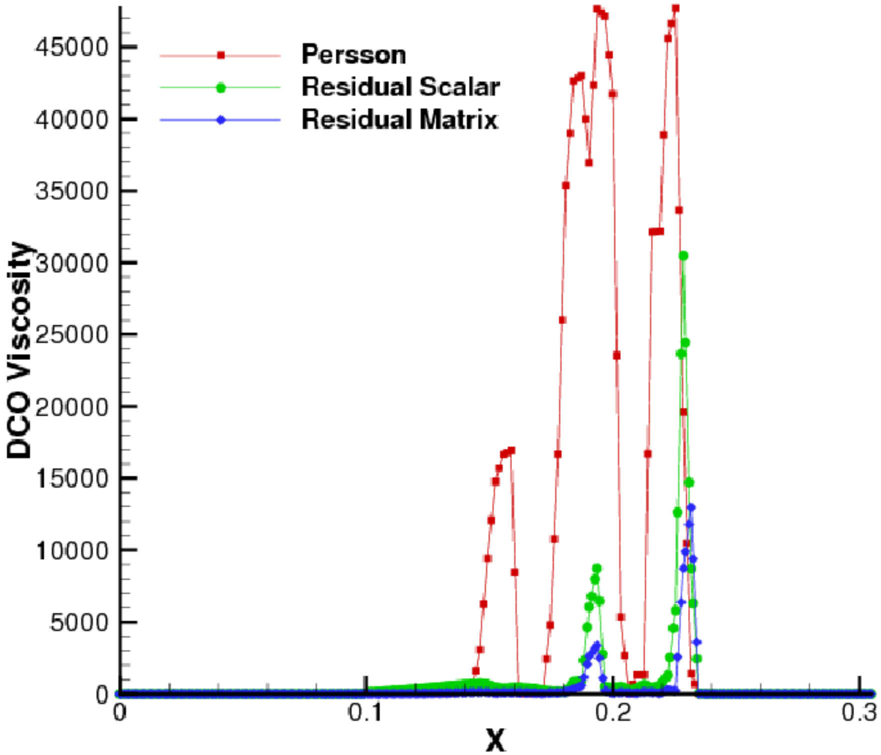
$$\int_{\partial\Omega} \phi_i w(u_i)^T g^{\ell m} \hat{\mu} u_w \partial_{x_m} \phi_j w(u_j) dS_\ell - \int_{\Omega} \partial_{x_\ell} \phi_i w(u_i)^T g^{\ell m} \hat{\mu} w_u \partial_{x_m} \phi_j w(u_j) dV$$

$$\int_{\partial\Omega} w(u_i)^T \phi_i g^{\ell m} M \partial_{x_m} \phi_j w(u_j) dS_\ell - \int_{\Omega} \partial_{x_\ell} \phi_i w(u_i)^T g^{\ell m} M \partial_{x_m} \phi_j w(u_j) dV$$

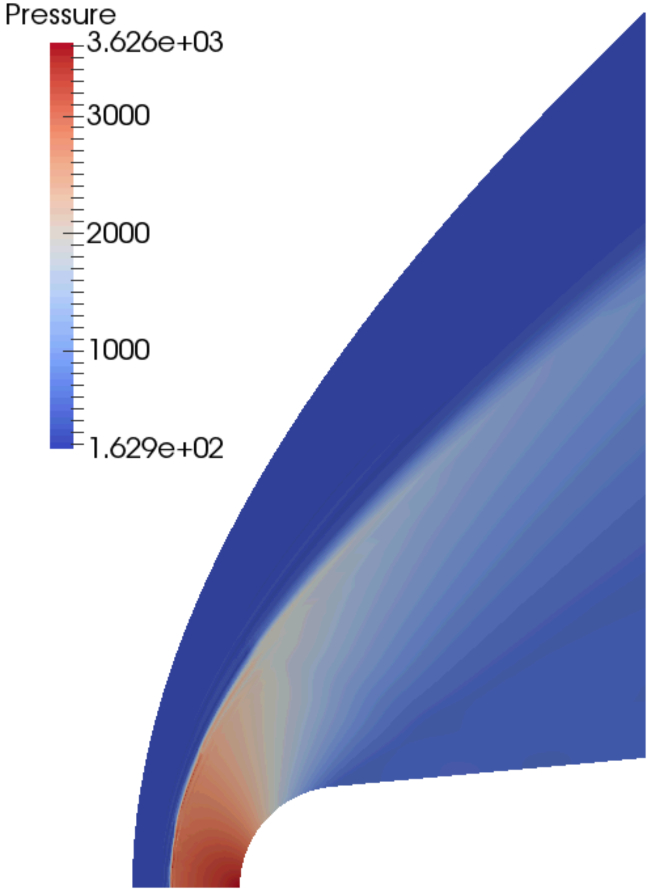
Discontinuity Capturing

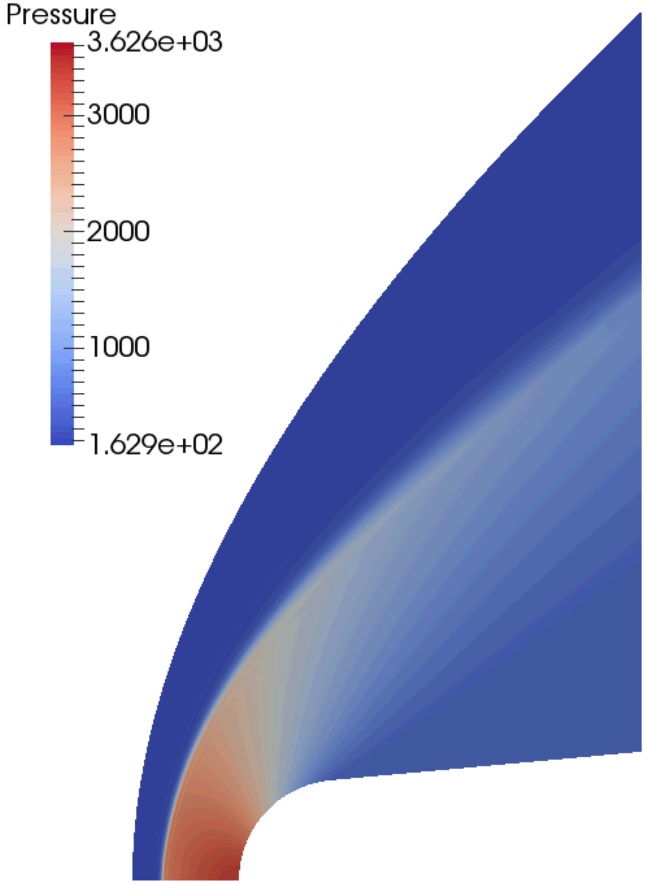
p = 3





Discontinuity Capturing



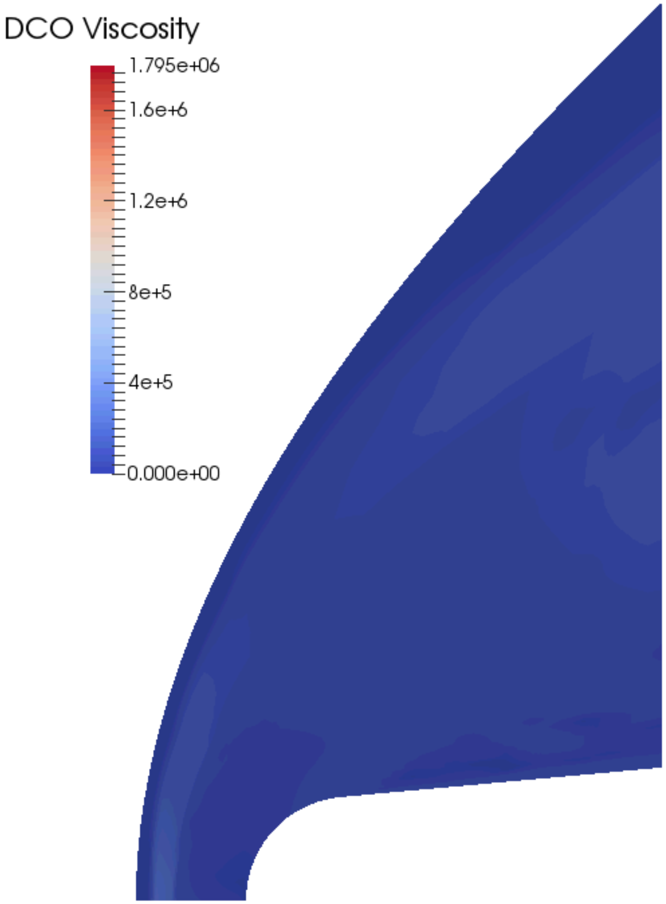


p = 2

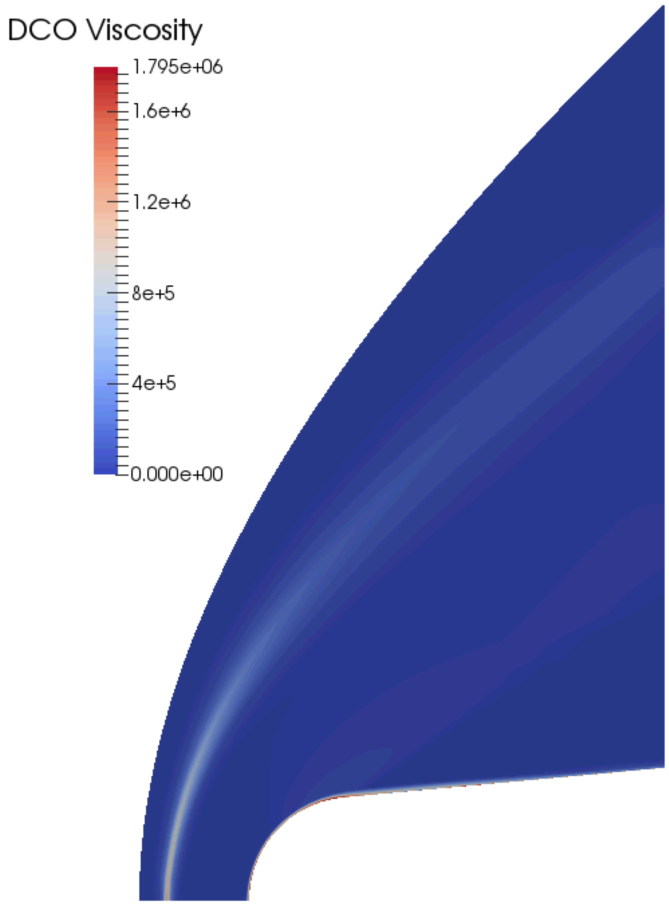
Persson and Peraire

Linear Residual Matrix

Discontinuity Capturing



Persson and Peraire



Linear Residual Matrix

Continuous Formulation

- Initially discontinuous nodes have identical solution

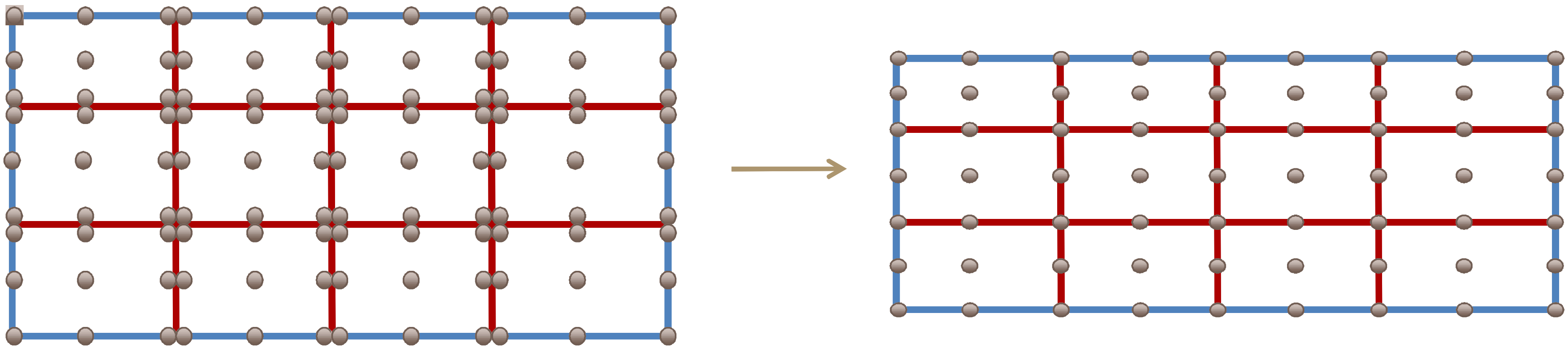
$$u(x_L) = u(x_R) = u(x_i)$$

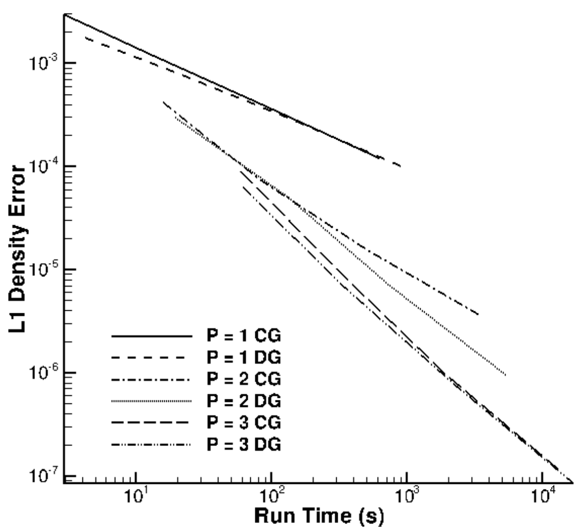
- Add residuals such that discontinuous nodes have same residuals
- Condense identical residuals

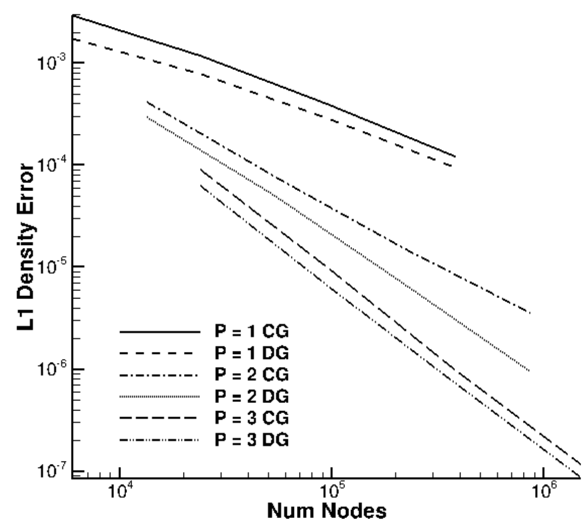
$$\partial_t u(x_L) = \partial_t u(x_R) = \partial_t u(x_i)$$

- Maintains entropy stability
- Most useful for mesh motion

Continuous Formulation







Accuracy Degradation

- "Conventional" scheme shows design order on smooth meshes
- Source terms (MMS) are underintegrated and show less than design order
- Canonically bad meshes also show degradation of accuracy

Cartesian

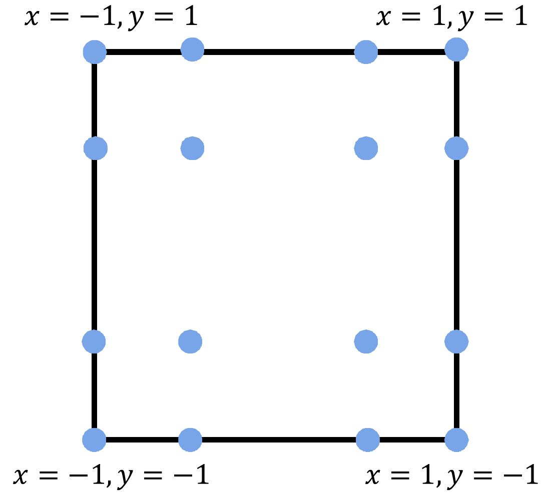
Mesh	L1 error	L1 rate	Linf error	Linf rate
1	1.74E-007		5.92E-004	
2	1.16E-008	3.90	7.30E-005	3.01
3	7.13E-010	4.02	5.11E-006	3.83
4	4.15E-011	4.10	3.98E-007	3.68

TQuad

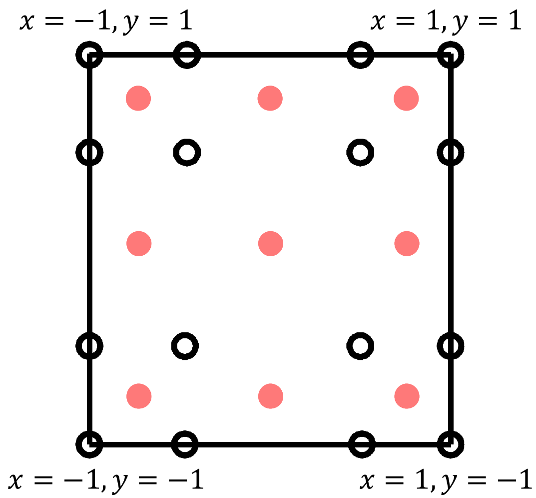
Mesh	L1 error	L1 rate	Linf error	Linf rate
1	1.25E-006		5.02E-003	
2	8.68E-008	3.84	1.16E-003	2.10
3	5.66E-009	3.93	1.62E-004	2.84
4	3.68E-010	3.94	1.52E-005	3.41

Other Formulations

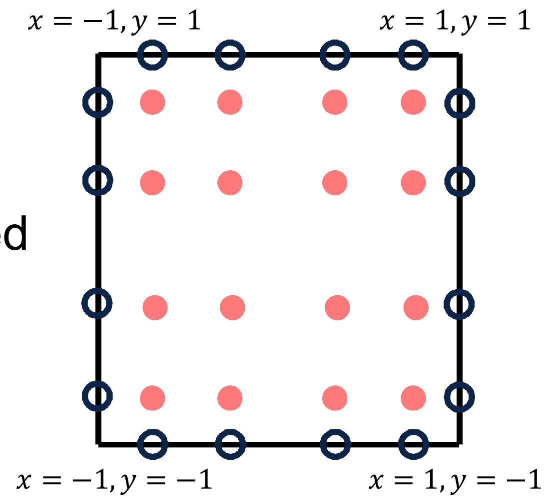
Conventional



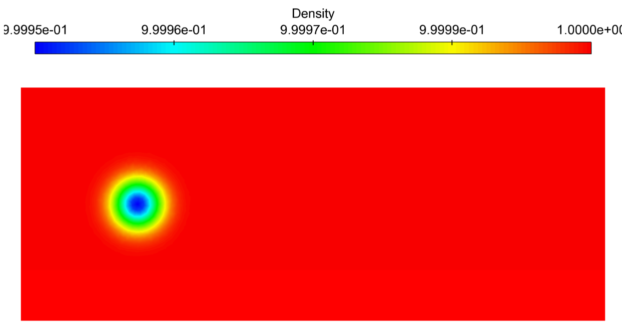
Fully Staggered

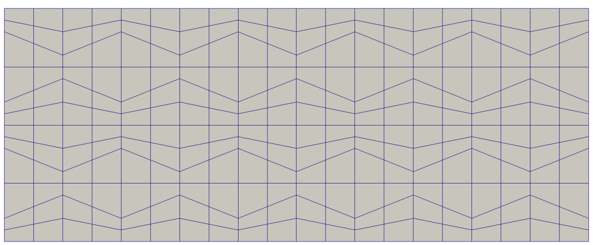


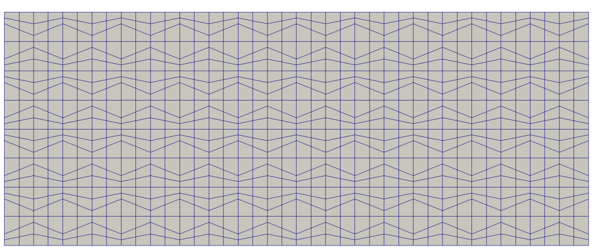
Generalized



Other Formulations







p = 4

Conventional

Mesh	L2 error	L2 rate
1	3.17E-005	
2	1.19E-006	4.74
3	6.12E-008	4.28
4	2.66E-009	4.52

Fully Staggered

Mesh	L2 error	L2 rate
1	1.01E-005	
2	2.07E-007	5.60
3	6.82E-009	4.93
4	2.46E-010	4.79

Generalized

Mesh	L2 error	L2 rate
1	8.32E-006	
2	3.19E-007	4.71
3	1.04E-008	4.94
4	3.66E-010	4.83

Other Formulations

- Based on exact diagonal mass matrix
  - "Fixes" rely on moving solution points to Legendre-Gauss points
- Does Lax-Wendroff work with staggering

$$\tilde{u}_t + \tilde{\mathcal{I}}\mathcal{P}^{-1}\Delta\bar{f} = 0$$

$$\tilde{\psi}^T\tilde{\mathcal{P}}\tilde{u}_t + \tilde{\psi}^T\tilde{\mathcal{P}}\tilde{\mathcal{I}}\mathcal{P}^{-1}\Delta\bar{f} = \tilde{\psi}^T\tilde{\mathcal{P}}\tilde{u}_t + \psi^T\Delta\bar{f} = 0, \quad \tilde{\mathcal{P}}\tilde{\mathcal{I}} = \mathcal{I}^T\mathcal{P}$$

$$\tilde{\psi}^T\tilde{\mathcal{P}}\tilde{u}_t + \psi^T\Delta\bar{f} = \psi_N f_N - \psi_1 f_1 + \tilde{\psi}^T\tilde{\mathcal{P}}\partial_t\tilde{u}_h - \psi_x^T\bar{f}$$

- Does it make a difference for real problems?
  - For p = 1, it matters
  - For p = 3, it may matter
  - For p = 7 in turbulent flow, probably doesn't matter

Moving Geometry

- Solve ALE equations

$$\partial_t(Ju) + \partial_{\xi_k}\left(a_{k\ell}f_\ell - \partial_t x_\ell a_{k\ell}u\right) = \partial_{x_k} a_{k\ell}\hat{c}_{\ell j}\partial_{x_j} w$$
$$\partial_t J - \partial_{\xi_k} a_{k\ell}\partial_t x_\ell = 0, \quad \partial_{\xi_k} a_{k\ell} = 0, \quad \ell = 1,2,3$$

$$a_{k\ell} = J\partial_{x_\ell}\xi_k$$

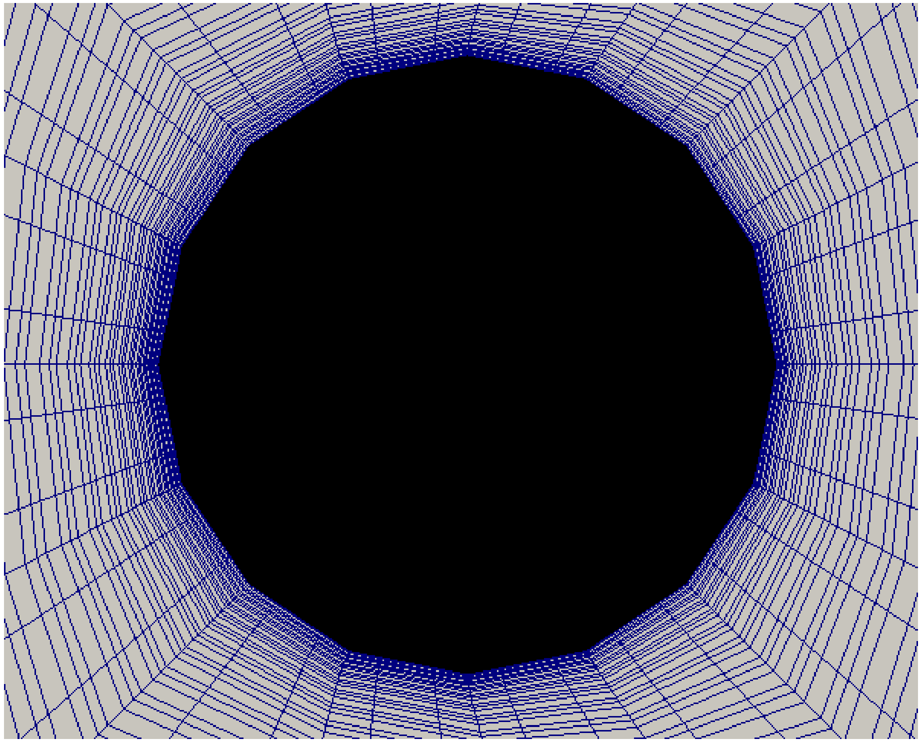
- Solve linear elasticity for mesh

$$\partial_{x_j}\left(\lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij}\right) = f_i$$

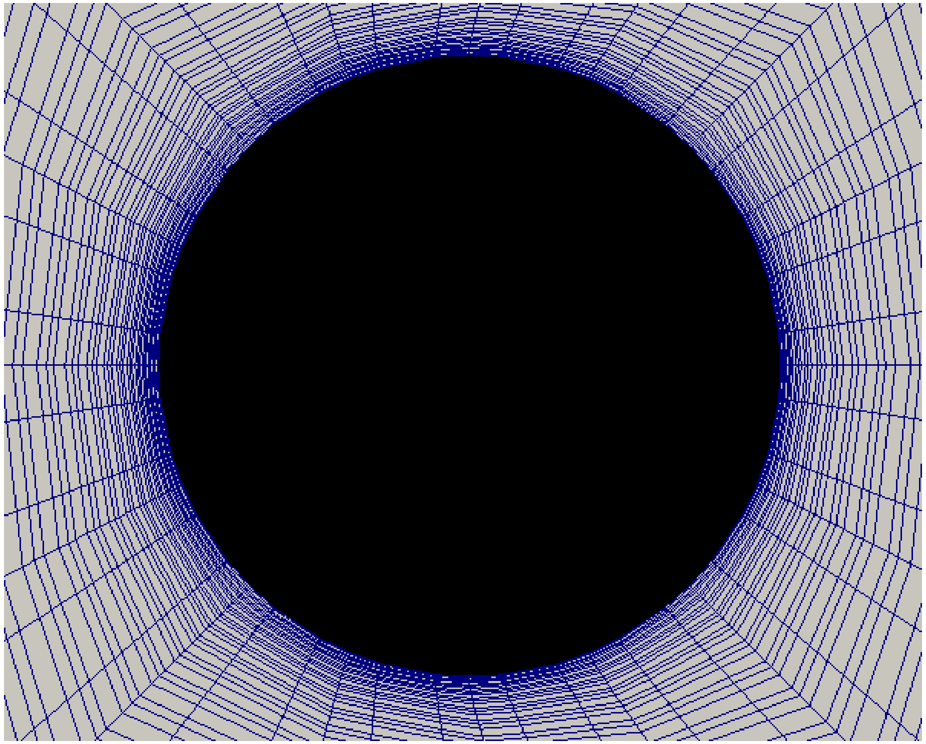
$$E|_{x_i} = \frac{\max(v)}{v|_{x_i}}$$

Geometry Fitting

- Surface Fitting
  - p = 3

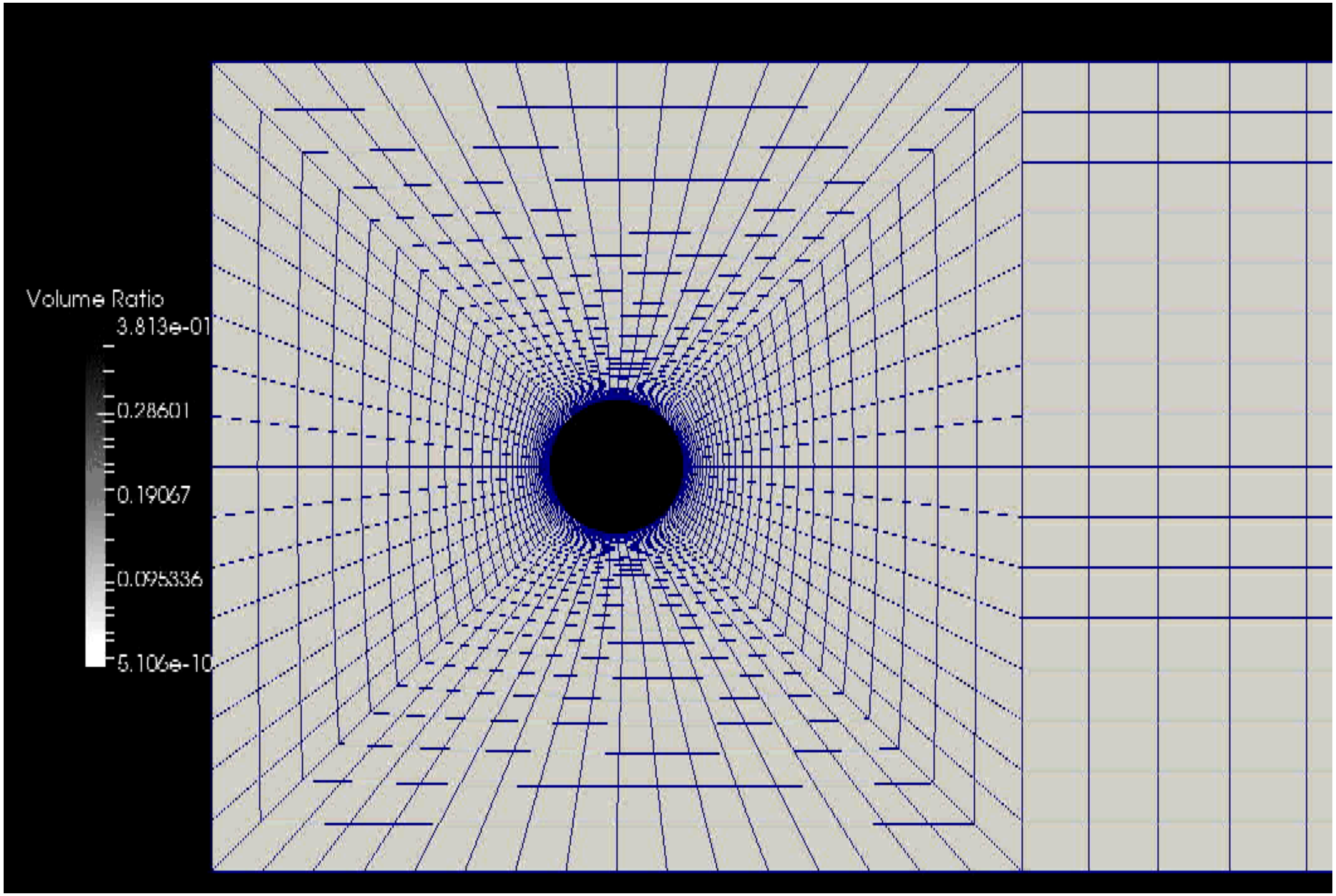


Faceted



Surface Fit

Moving Geometry

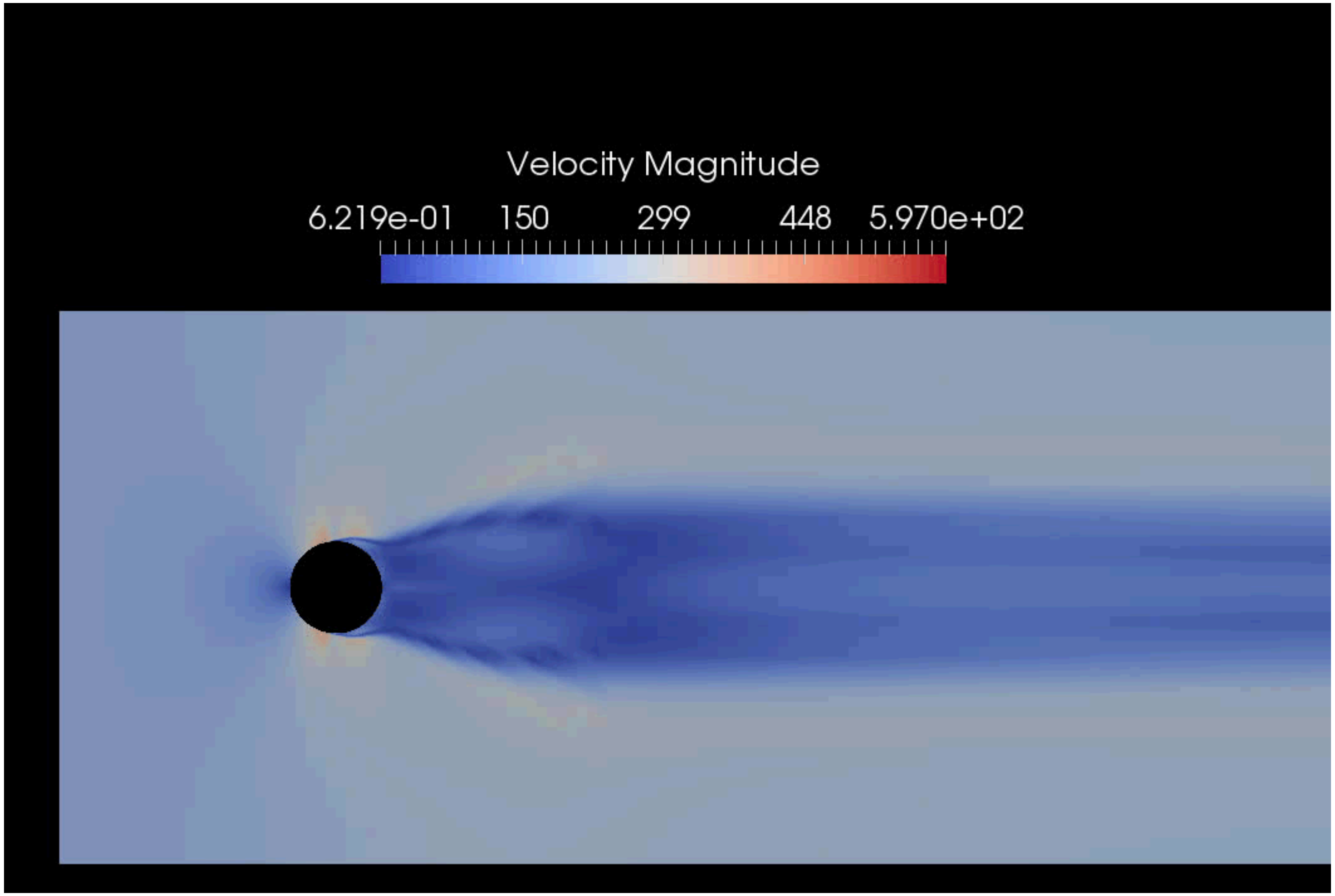


Moving Geometry

Cylinder in inviscid crossflow in a 3D channel

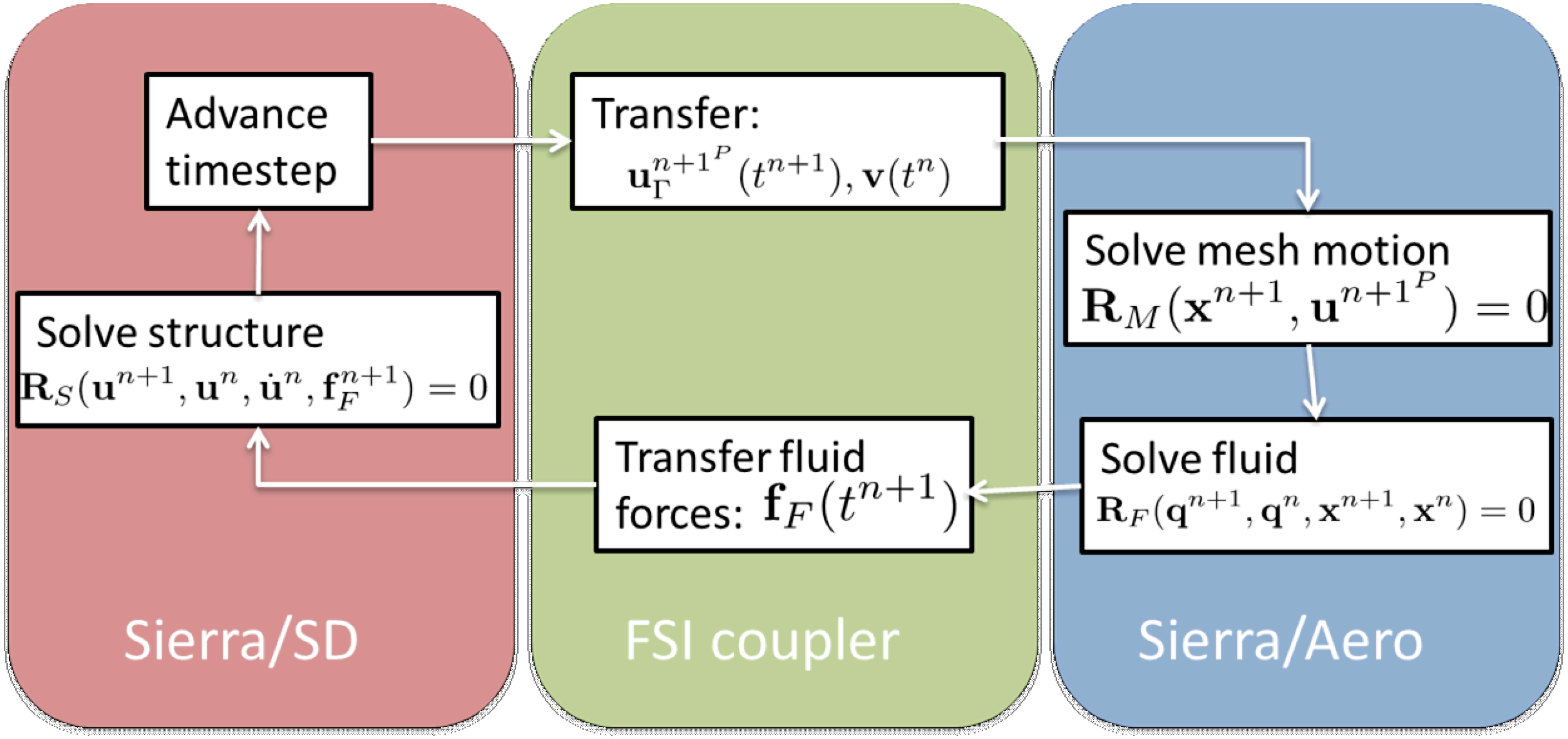
Coarse simulation shows oscillating cylinder doesn't exhibit any spurious behavior

M = 0.5
P = 2
A = 0.25 m
D = 1.0 m



Fluid-Structure Interaction

- Time steps are sufficiently small for accuracy
  - No need for sub-iterations between physics solvers



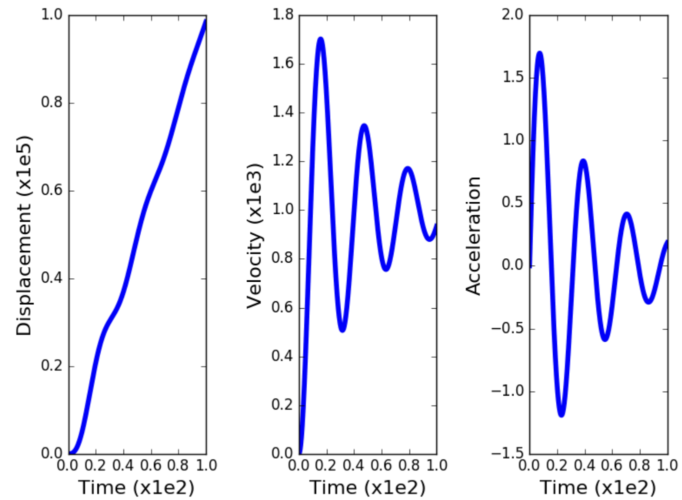
Predictor: $$\mathbf{u}_\Gamma^{n+1^P} = \mathbf{u}_\Gamma^n + \alpha_0 \Delta t \dot{\mathbf{u}}_\Gamma^n + \alpha_1 \Delta t \left(\dot{\mathbf{u}}_\Gamma^n - \dot{\mathbf{u}}_\Gamma^{n-1}\right)$$

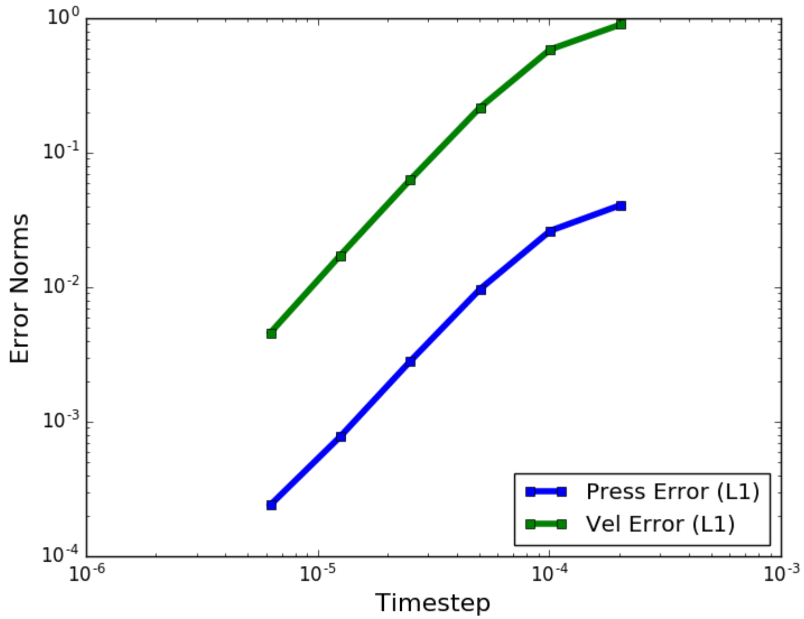
Fluid-Structure Interaction

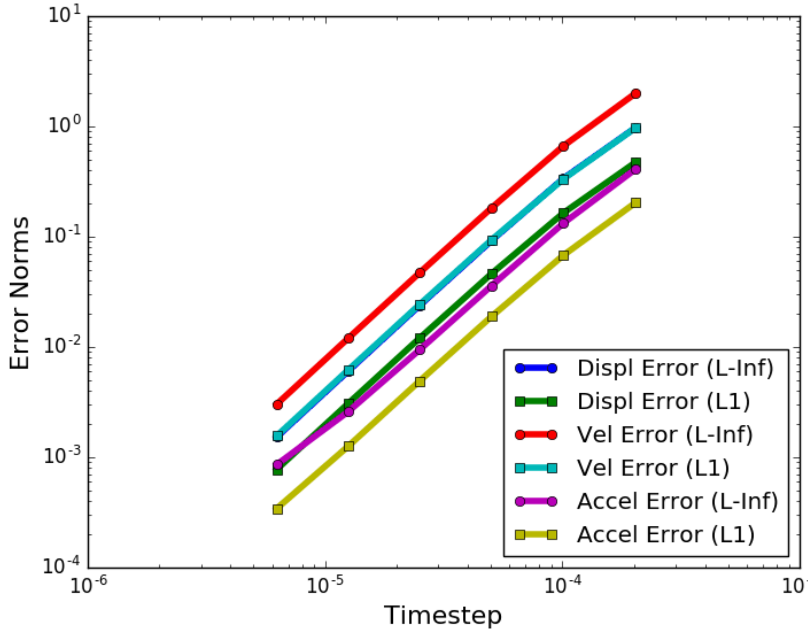
$$ma + cv + ku = \bar{v}kt$$

$$v(0) = u(0) = 0$$

$$u(t) = e^{-dt}\left(a\cos(\omega t) + b\sin(\omega t)\right) + \bar{v}(t - \beta)$$







ENTROPY STABILITY AND RANS

RANS

SST K-Omega

$$\begin{pmatrix} \rho \\ \rho u_i \\ \rho \tilde{E} \\ \rho k \\ \rho \omega \end{pmatrix} + \partial_{x_k} \begin{pmatrix} \rho u_k \\ \rho u_i u_k + p\delta_{ik} \\ \rho u_k \tilde{E} + u_k p \\ \rho u_k k \\ \rho u_k \omega \end{pmatrix} = \partial_{x_k} \begin{pmatrix} 0 \\ \sigma_{ik} + \tau_{ik} \\ u_\ell(\tau_{\ell k} + \sigma_{\ell k}) + \kappa \partial_{x_k} T \\ (\mu + \frac{\mu_T}{c_k})\partial_{x_k} k \\ (\mu + \mu \frac{\mu_T}{c_\omega})\partial_{x_k} \omega \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\sigma_{ij}\partial_{x_j} u_i + \beta^* \rho k \omega \\ \sigma_{ij}\partial_{x_j} u_i - \beta^* \rho k \omega \\ \gamma \frac{\omega}{k} \sigma_{ij} \partial_{x_j} u_i - \beta^* \rho \omega^2 \end{pmatrix}$$

$$\tilde{E} = c_v T + \frac{1}{2} u_k u_k = E - k$$

Spalart Allmaras

$$\begin{pmatrix} \rho \\ \rho u_i \\ \rho E \\ \rho \hat{\nu} \end{pmatrix} + \partial_{x_k} \begin{pmatrix} \rho u_k \\ \rho u_i u_k + p\delta_{ik} \\ \rho u_k E + u_k p \\ \rho u_k \hat{\nu} \end{pmatrix} = \partial_{x_k} \begin{pmatrix} 0 \\ 0 \\ u_\ell(\tau_{\ell k} + \sigma_{\ell k}) + \kappa \partial_{x_k} T \\ \frac{\mu + \rho \hat{\nu}}{\sigma} \partial_{x_k} \hat{\nu} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho c_{b1} \hat{s} \hat{\nu} - \rho c_{w1} f_w \left(\frac{\hat{\nu}}{d} \right)^2 \\ + \frac{\rho c_{b2}}{\sigma} \partial_{x_j} \hat{\nu} \partial_{x_j} \hat{\nu} \end{pmatrix}$$

RANS

SST K-Omega

$$\eta = -\rho s + \rho R \frac{k^2}{\phi_k^2} - \rho R \log \frac{\omega}{\phi_\omega}$$

$$w = \begin{pmatrix} \frac{h+RT-u_k u_k}{T} - s - R\frac{k^2}{\phi_k^2} - R\log\frac{\omega}{\phi_\omega} \\ \frac{u_1}{T} \\ \frac{u_2}{T} \\ \frac{u_3}{T} \\ \frac{-1}{T} \\ R\frac{k}{\phi_k^2} \\ -\frac{R}{\omega} \end{pmatrix}$$

Spalart Allmaras

$$\eta = -\rho s + \rho R \frac{\hat{\nu}^2}{\phi_{\hat{\nu}}^2}$$

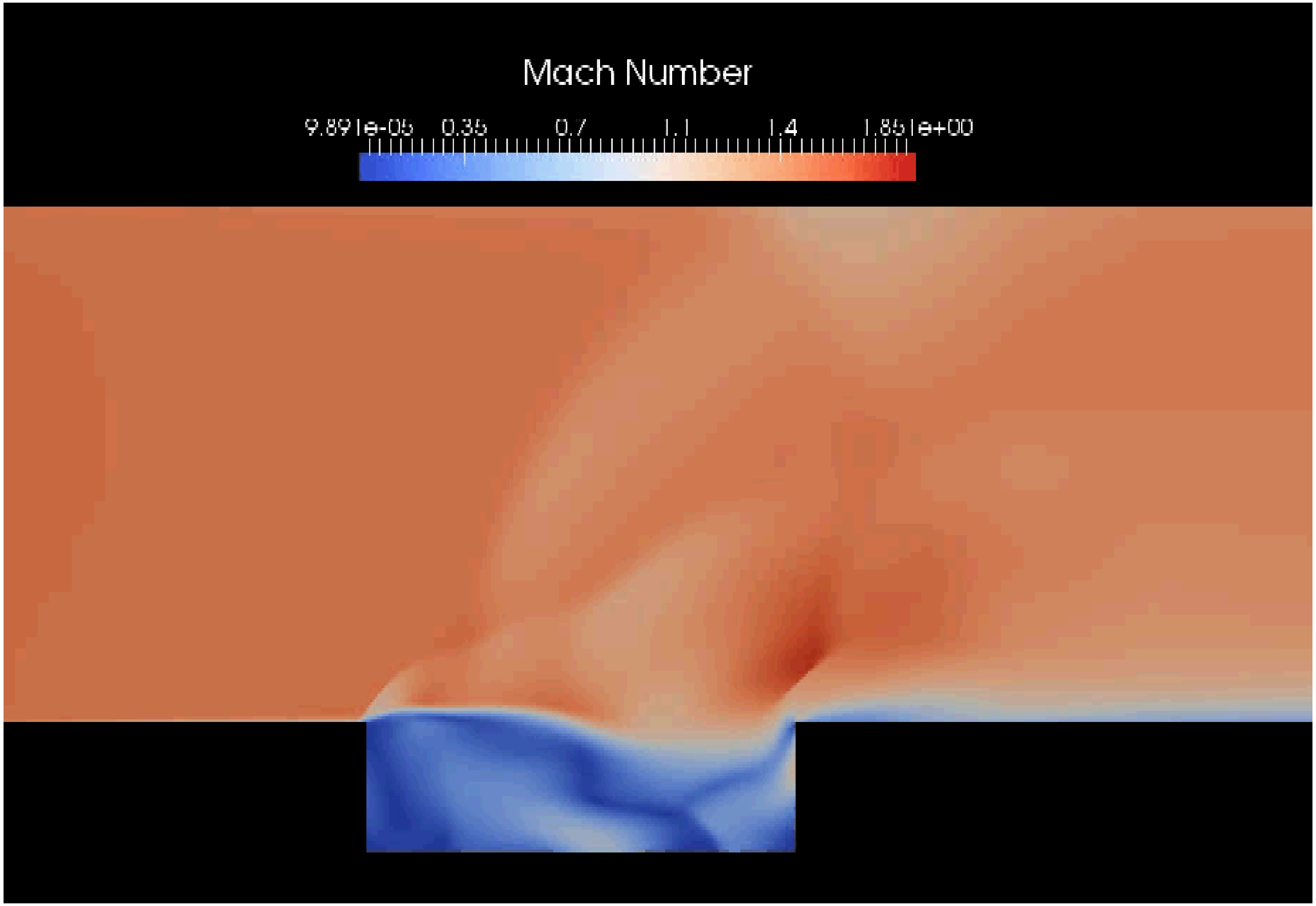
$$w = \begin{pmatrix} \frac{h+RT-u_k u_k}{T} - s - R\frac{\hat{\nu}^2}{\phi_{\hat{\nu}^2}} \\ \frac{u_1}{T} \\ \frac{u_2}{T} \\ \frac{u_3}{T} \\ \frac{-1}{T} \\ R\frac{\hat{\nu}}{\phi_{\hat{\nu}}^2} \end{pmatrix}$$

Entropy Stability and RANS

- Can satisfy:
  - Symmetrize equations
  - Convex entropy variables
  - Compatible with DCO stability
- Don’t know:
  - Actual bounds
  - How to deal with LW-satisfying form of energy for SST
  - How to show boundedness of source terms

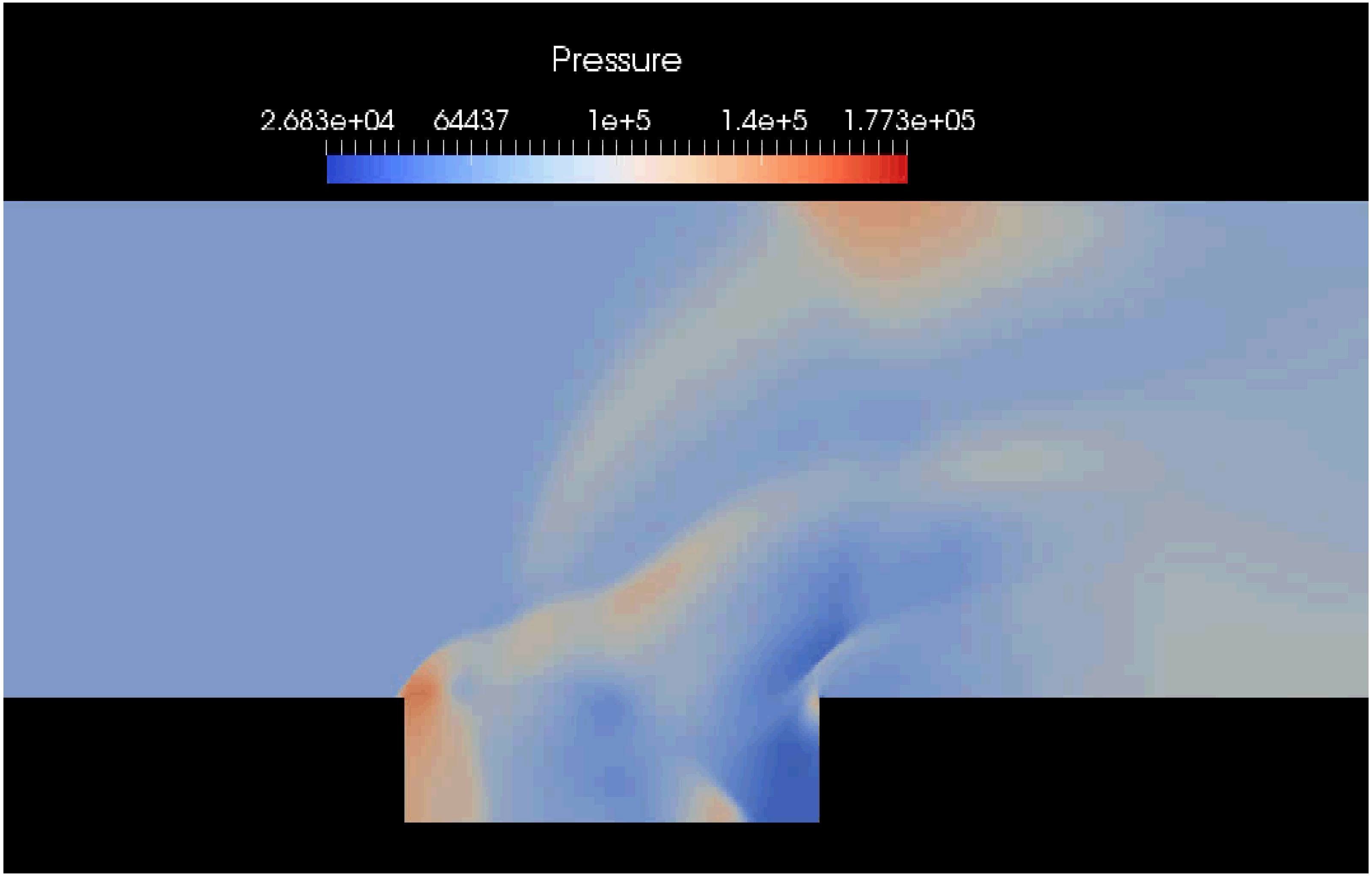
DES

p = 3, SA DES



DES

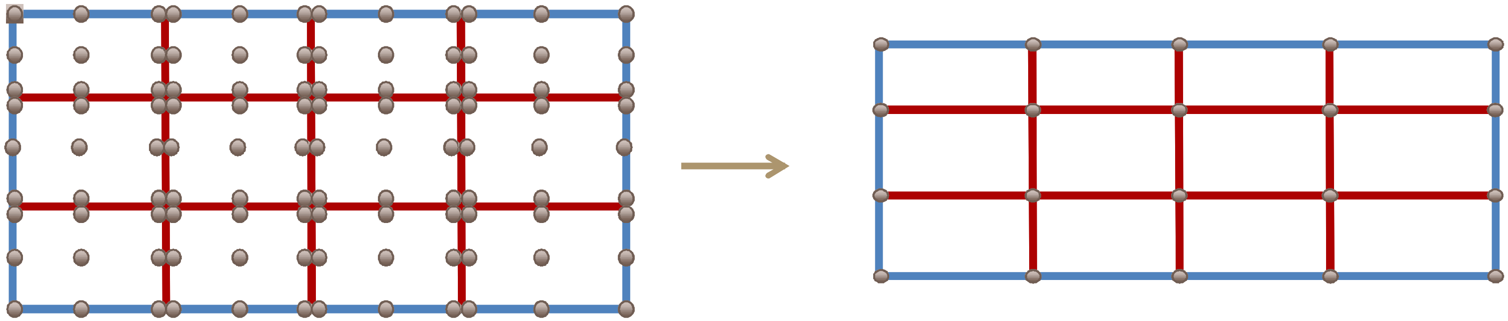
p = 3, SA DES



Linear Solvers

- Fluid solver
  - Block SGS
  - Block ILU0
  - Uses Trilinos/Tpetra and Trilinos/Ifpack2
- Elasticity Solver
  - Multigrid
  - SGS on fine, high-order mesh

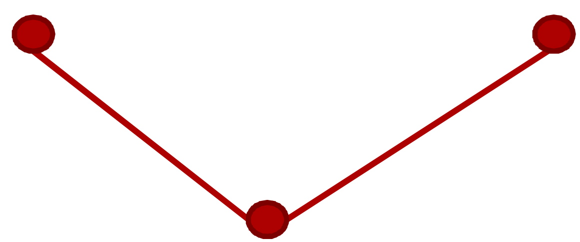
Linear Solvers



$$\text{solve } F(q) = 0 \qquad A_{\text{fine}} \approx \frac{\partial F(q)}{\partial q}$$

$$\text{iterate } A_{\text{fine}} \delta q^k = -F(q^k), \quad q^{k+1} = q^k + \delta q^k$$

$A_{\text{fine}} x = b$



$x = x + R^T y$

$$A_{\text{coarse}} y = Rb$$

$$A_{\text{coarse}} = R A_{\text{fine}} R^T$$

Future Needs

- More on shock capturing
- Reacting flows
- More on linear/nonlinear solvers
- LES models for high-order methods
- More friendly turbulence models or different solution approaches
- hp-Adaptivity
- Comparisons to SBP finite-difference

Future Directions at Sandia

- Solve real problems
  - Move toward wall-modeled LES
  - Simulation time comparison to "optimal" low order methods
  - Continue addressing workflow issues
- Requires application framework to do it right
  - Need exact same physics routines
  - Hard to make valid comparison between codes
  - Need flexibility to code each algorithm optimally
  - Take disruptive changes from many core as opportunity
  - Requires serious investment/commitment