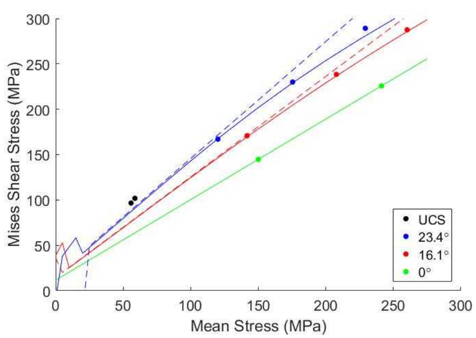
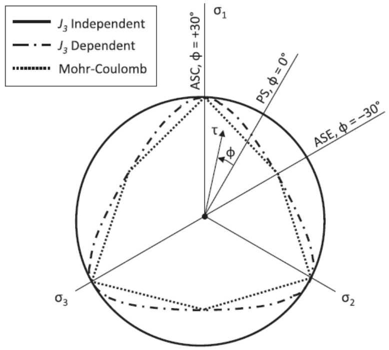
SAND2017-6155C





Bifurcation theory applied to granite under general states of stress

M.D. Ingraham, T.A. Dewers, M. Williams, C.S.N. Cheung, B.C. Haimson

U.S. DEPARTMENT OF ENERGY NNSA

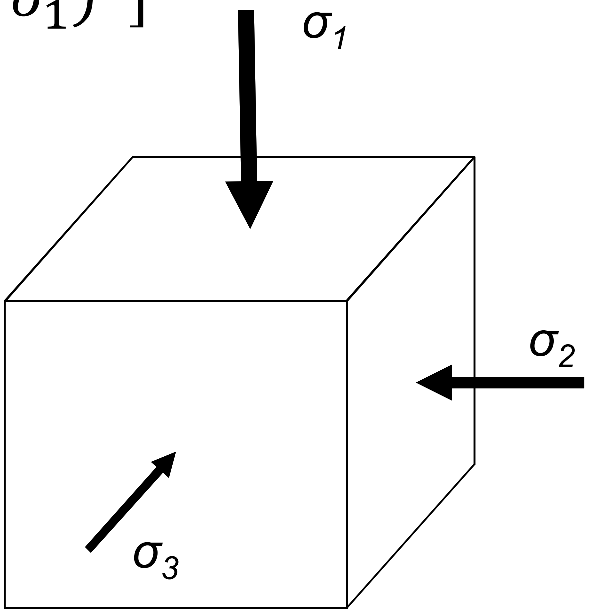
Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Introduction

- 3D stress state
  - Invariants
  - 3D Failure Surfaces
- Stress-Strain response
  - Rock Type
  - Response to 3D stresses
  - σ_2 dependence
- Bifurcation condition
  - Assumptions
  - Results
- Constitutive model for failure
  - mMNLD
  - Mogi
- Conclusions

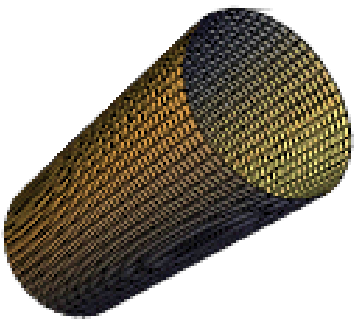
3D Stress State: Invariants

- $\theta = \tan^{-1}\left[\frac{\sigma_1 - 2\sigma_2 + \sigma_3}{\sqrt{3}(\sigma_1 - \sigma_3)}\right]$
- $\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$
- $\tau = \sqrt{\frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$
- $I_1 = 3\sigma_m$
- $\tau = \sqrt{J_2}$
- $\sin 3\theta = -\frac{\sqrt{27}J_3}{2\tau^3}$

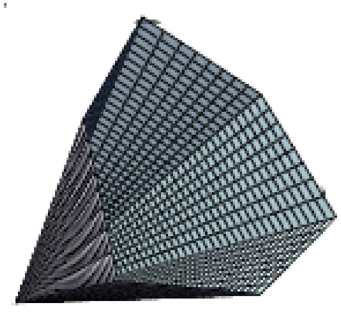


3D Failure Surfaces

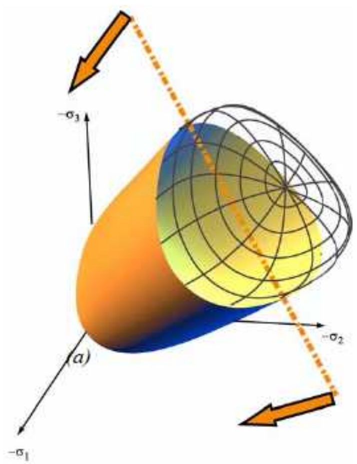
Von Mises

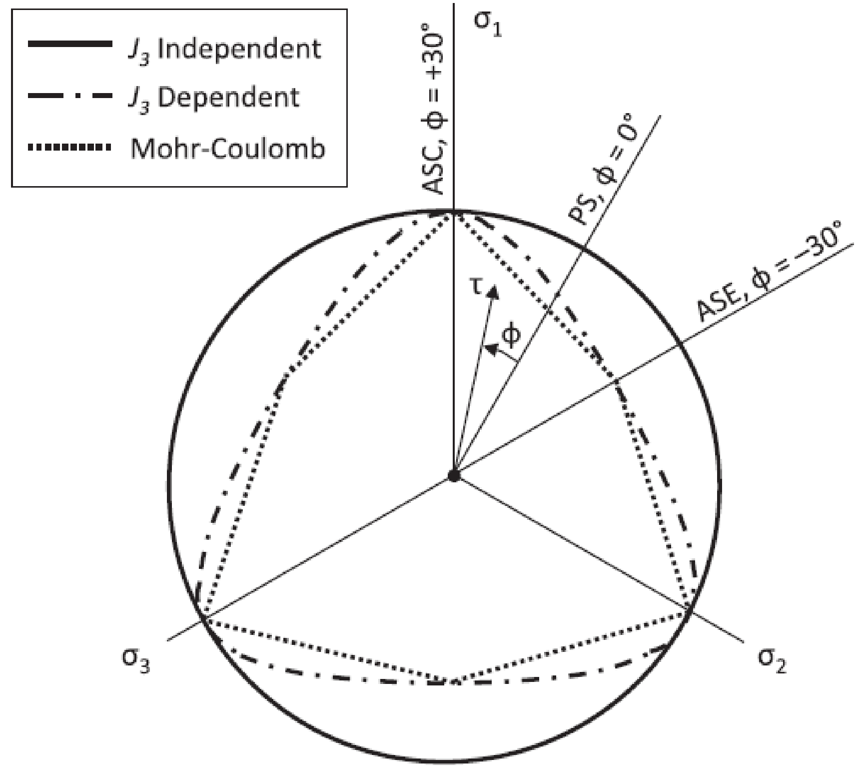


Mohr-Coulomb



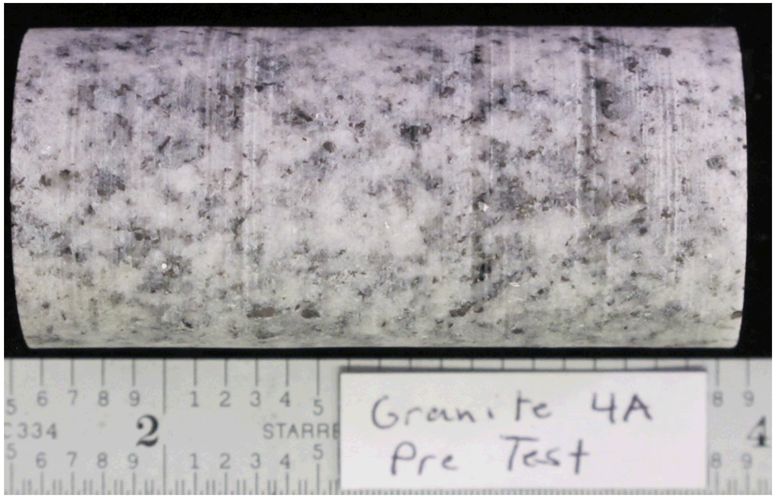
J_3 Dependent





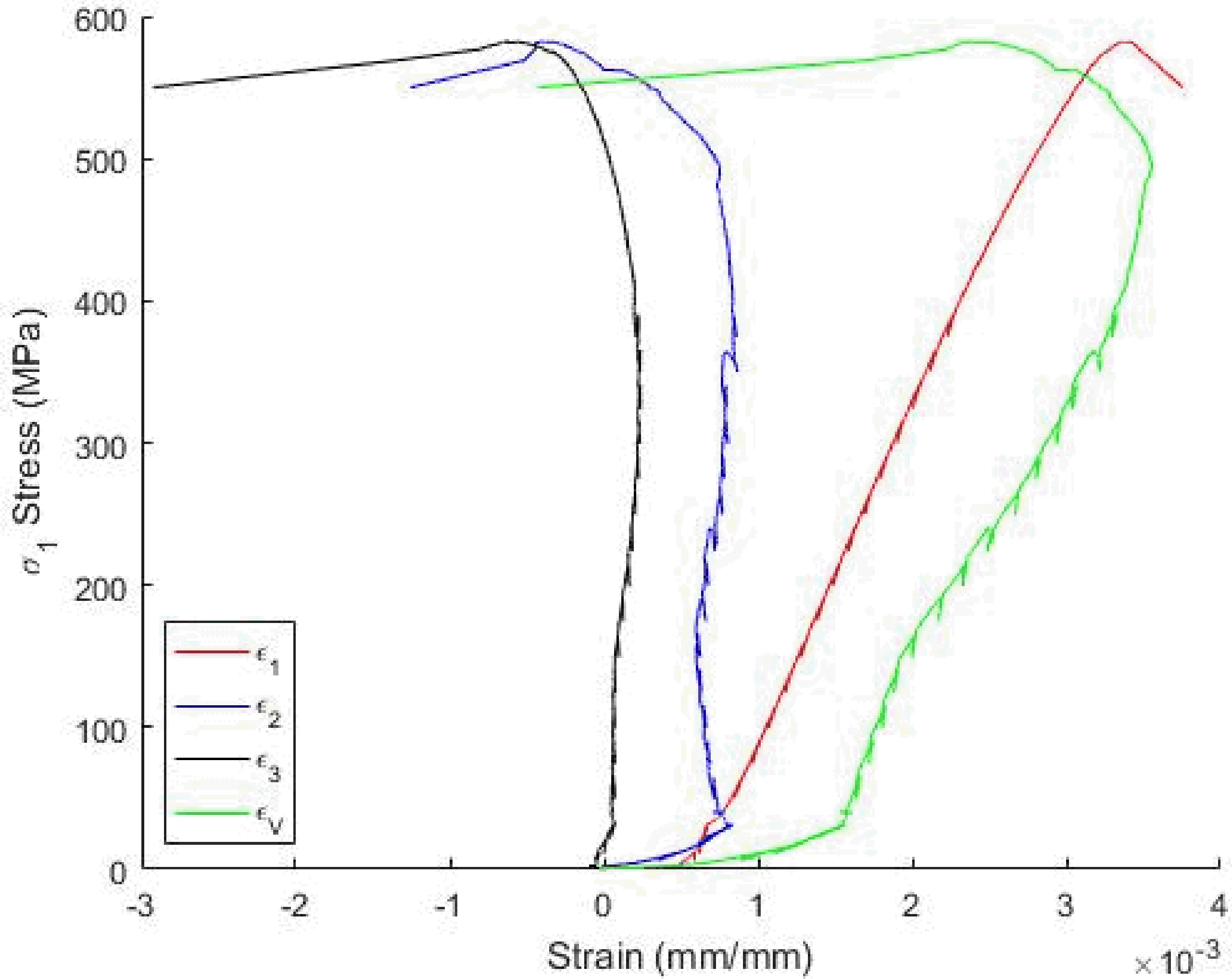
3D Diagrams from Brannon et al. SAND2009-2282

Sierra White Granite

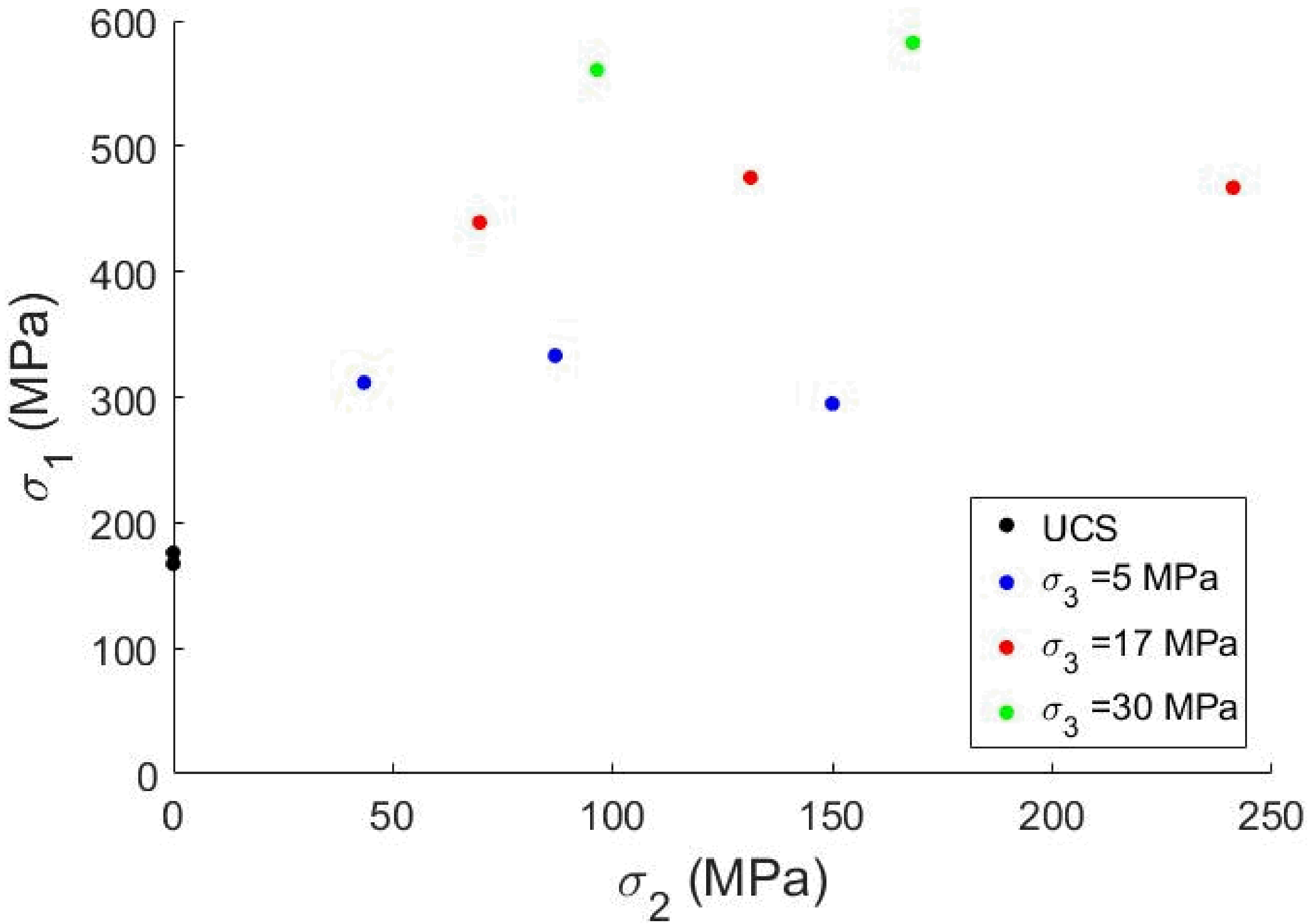


- UCS – 176.2 Mpa
- Density – 2.64 g/cc
- Grain size 1-3 mm
- Youngs Modulus – 48.5 Gpa
- Poisson's Ratio – 0.22
- Comprised of oligoclase, quartz, orthoclase, biotite, muscovite, and other trace.

Stress-strain response



σ_2 Dependence



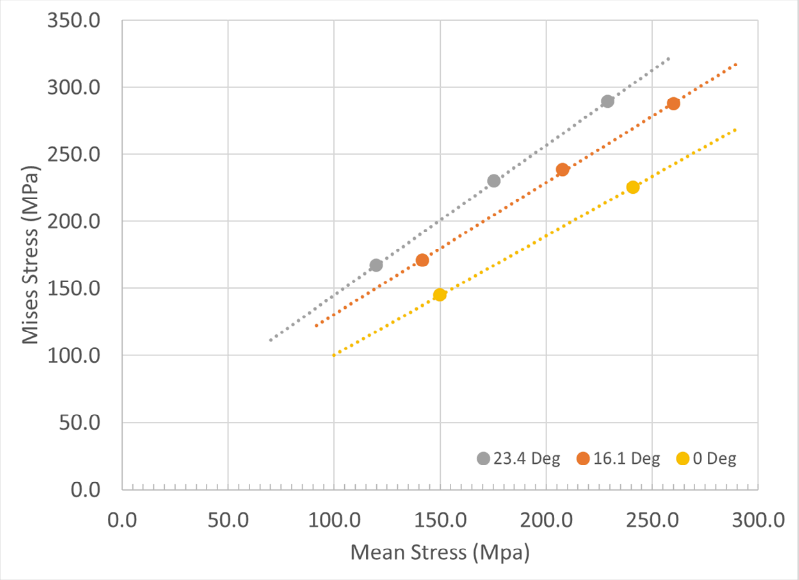
Bifurcation Condition

$$\alpha = \frac{\pi}{4} + \frac{1}{2}\sin^{-1}\left[\frac{\frac{2}{3}(1+\nu)(\beta+\mu) - N_{II}(1-2\nu)}{\sqrt{4-3N_{II}^2}}\right]$$

$$N_{II} = \frac{\sigma_m - \sigma_2}{\tau}$$

$\nu =$ Poisson's Ratio
$\alpha =$ Band angle
$\beta =$ Plastic potential
$\mu =$ Slope of the yield surface

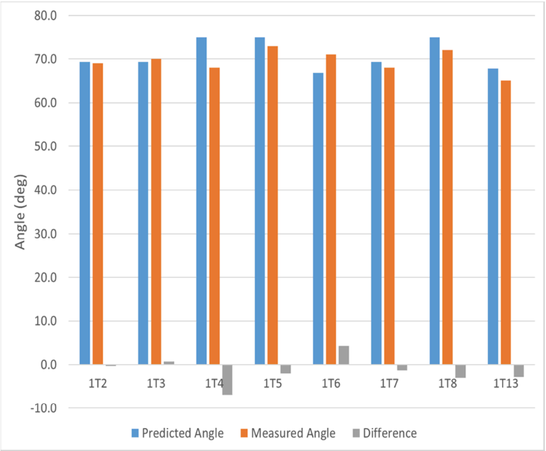
Bifurcation in this form developed by Rudnicki and Olsson (1998)



- Assumptions
  - Normality of the yield surface ($\beta = \mu$)
  - Failure surface is an acceptable proxy for yield surface
- In the region of interest the yield surface was represented by a straight line for each Lode angle

Bifurcation Results

	σ_m (MPa)	τ (MPa)	θ°	σ_3	α_m	α_p
1T2	260.2	287.5	16.1	30.1	69	69.4
1T3	141.7	170.8	16.1	5	70	69.3
1T4	120.1	167.0	23.4	5.1	68	75.0
1T5	175.4	230.0	23.4	17.2	73	75.0
1T6	150.0	144.7	0.0	5.3	71	66.7
1T7	207.8	238.4	16.1	17.0	68	69.3
1T8	229.2	289.2	23.4	30.2	72	75.0
1T13	241.2	225.7	0.0	15.5	65	67.8

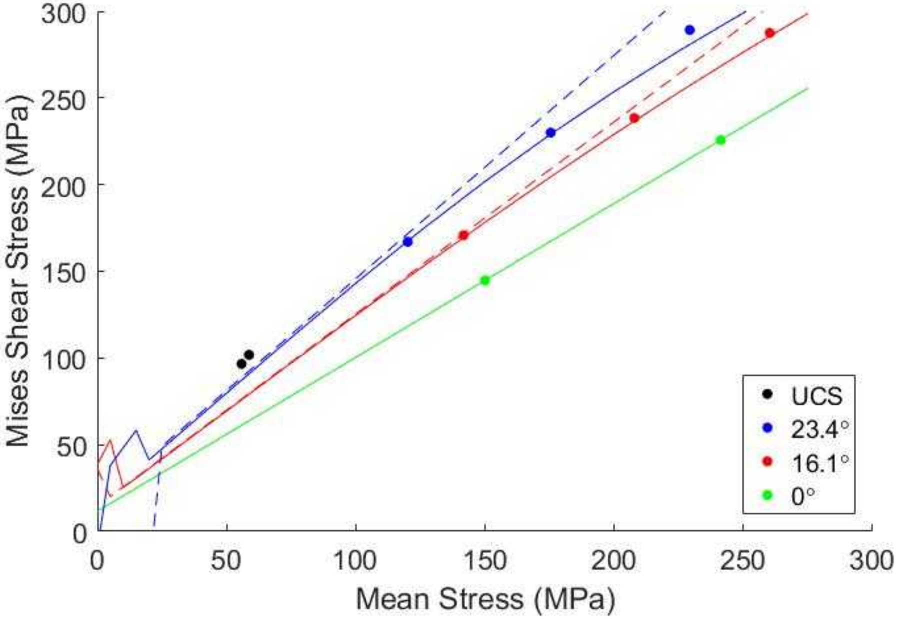


Modified Matsuoka-Nakai-Lade-Duncan

$$\sqrt{\frac{4}{27}A(\sigma)\sin(3)}\left(\frac{\tau}{\tau_0(\sigma)}\right)^3 + \left(\frac{\tau}{\tau_0(\sigma)}\right)^2 - 1 = 0$$

$$\tau_0(\sigma) = 0.8878\sigma + 11.604$$

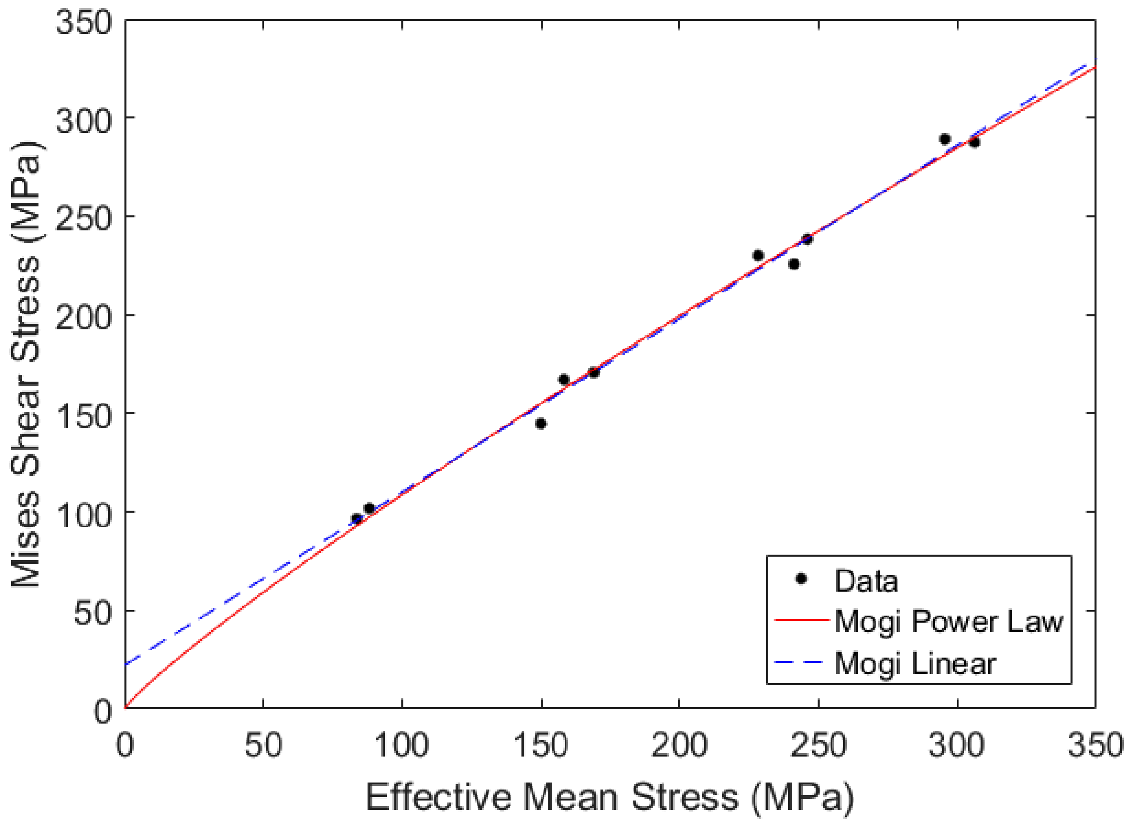
$$A(\sigma) = 5E^{-6}\sigma^2 - 0.0008\sigma - 0.9541$$



	Calculated A	Fit A	τ_0	Calculated τ	Actaual τ	τ Difference
1A	-0.989	-0.984	63.76	100.2	101.8	1.6
3A	-0.989	-0.983	61.13	95.9	96.6	0.7
1T2	-0.844	-0.824	242.65	285.7	287.5	1.8
1T3	-0.987	-0.967	137.40	169.5	170.8	1.3
1T4	-0.975	-0.978	118.20	167.6	167.0	-0.6
1T5	-0.945	-0.941	167.32	229.0	230.0	1.0
1T7	-0.925	-0.905	196.07	236.8	238.4	1.6
1T8	-0.918	-0.875	215.07	281.0	289.2	8.2

Mogi Criterion

- $\sigma_{me} = \frac{(\sigma_1 + \sigma_3)}{2}$
- $\tau = a + b\sigma_{me}$
  - a=22.21
  - b=0.880
- $\tau = \alpha\sigma_{me}^{n}$
  - α=1.923
  - n=0.8762



Correlation coefficient is nearly the same for both methods (0.993 and 0.992 respectively)

Conclusions

- Failure depends on σ_2
- Band angle predictions match well with experiments
  - Noteworthy considering assumptions made
- Results are well modeled by both mMNLD model, and both Mogi criteria
- Selection of model depends on data available and intended use for model (are invariants required?)

Acknowledgements

- Moo Lee, Mike Hileman, Perry Barrow, Aron Robbins, R. Charles Choens
- Work was funded under Sandia National Labs: Laboratory Directed Research and Development Program
- SAND2017-XXXX

Publication of failure data is in review: Ingraham, MD, Dewers, TA, Lee, M, Cheung, CSN, Haimson, B “Response of Sierra White Granite subjected to true triaxial states of stress” Acta Geotechnica (2017) Submitted.