SLAC-92
UC-34, Physics
(EXPI)

ANALYSIS OF MUON HYDROGEN
BUBBLE CHAMBER EXPERIMENTS

by

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December 1968

Technical Report
Prepared Under
Contract AT(04-3)-515
for the USAEC
San Francisco Operations Office

Printed in USA. Available from CFSTI, National Bureau of Standards, U. S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.65

TABLE OF CONTENTS

I. INTRODUCTION

The first purpose of this report is to present the basic data for the consideration of muon exposures in liquid hydrogen bubble chambers at the Stanford Linear Accelerator Center. The object of the exposures would be to study muon + proton inelastic reactions. A second purpose is to show the improvement in the data rate if a very large hydrogen bubble chamber is used. First, the relevant properties of the SLAC muon beam are discussed. Then the inelastic event rate to be expected in a SLAC bubble chamber is given. Finally, the event rate is given for a very large hydrogen bubble chamber such as the Argonne National Laboratory 12-foot chamber or the 25-foot chamber proposed by Brookhaven National Laboratory.

All of this report is based on our current knowledge and experience obtained from the muon spark chamber experiment now being conducted at SLAC. Therefore, one should keep in mind that it is possible that more ingenious or more persistent experimenters may make major improvements in the situation as described here. This is not the place to present the detailed physics arguments for carrying out the muon exposures in a bubble chamber. The general argument is simply that at present there is very little known about muon + proton (or electron + proton) inelastic interactions, compared to pion + proton or kaon + proton--or even photon + proton--inelastic interactions. Some aspects of the electron inelastic interaction are now being studied with great care by spectrometer and counter experiments. Spark chamber muon experiments here at SLAC will add yet more information. But there will still be large areas of ignorance, which can be most fruitfully explored by the bubble chamber technique, with its capability of complete analysis of events. The basic limitation is the small number of events which will be obtained in the bubble chamber. No conclusion is drawn in this note as to whether the complete analysis of events compensates for the small number of events.

II. MUON BEAM PROPERTIES AT SLAC

The present muon beam at SLAC at 10 GeV/c can yield about 500 muons per pulse in a 30-cm^2 area with a 17-GeV/c electron beam of 20-mA peak current. For 2-pulse-per-second operation, one can go to 40-mA peak current. This yields about 30 muons per cm^2 per pulse. The momentum resolution is ± 1.5%. Further details of the beam are given in Ref. 1, and we only note that the intensity falls off a factor of 2 at 4 or 12 GeV/c compared to 10 GeV/c. Also, the momentum resolution can be improved to ± 0.8% with an intensity loss of 50%.

There are two ways of using the muon beam in a chamber, as a dispersed beam or as a "pencil" beam. As will be seen in Sect. III below, we need as much flux as possible, and a pencil beam might provide more flux. To consider the pencil beam in more detail we need to estimate its maximum diameter, d. Suppose the minimum scattering angle of the muon which we need to observe is θ_m (rad). Allowing a maximum 100-cm path length, d (cm) = 100 θ_m. If we take $\theta_m = 0.02$ rad, we may lose some useful events, whereas $\theta_m = 0.01$ is certainly safe. We use $\theta_m = 0.015$ or d = 1.5 cm. Now d = 1.5 cm means that recoil protons which must completely cross the pencil beam (at 45°) must have a momentum greater than 170 MeV/c. In a 1.5-cm-diameter pencil beam we can obtain about 50 muons per pulse. A substantial increase in the diameter, to say 3 cm, would seem to lead to too great a loss of forward particles and of slow recoil protons. For a particular experiment this point might be reconsidered and one might use a 3-cm diameter pencil beam with 200 muons per pulse.

Now a dispersed beam might have anywhere from 20 to 80 muons per pulse,[2] depending on the chamber width. For the SLAC 40-inch chamber I have used 40 muons per pulse. Thus, a dispersed beam has as much flux as the 1.5-cm diameter pencil beam.

III. EVENT RATE FOR INELASTIC REACTIONS AT SLAC

We shall calculate the event rate for 10-GeV/c muons and for a 0.5-meter interaction length in the chamber. The differential inelastic cross section can be written:

$$\frac{d^2\sigma}{dq^2 dp_2} = \frac{\alpha}{2\pi q^2}\left(\frac{p_2|\vec{q}|}{E_2 p_1^2}\right)\left[1 - \frac{2m_\mu^2}{q^2} + \frac{2E_1E_2 - q^2/2}{|\vec{q}|^2}\right]\left[\sigma_T\left(q^2, q_0\right) + \epsilon\sigma_L\left(q^2, q_0\right)\right]$$

$$\epsilon = \left(\frac{2E_1\ E_2 - q^2/2}{|\vec{q}|^2}\right) \quad \left(1 - \frac{2m_\mu^2}{q^2} + \frac{2E_1\ E_2 - q^2/2}{|\vec{q}|^2}\right)$$

where, $\vec{p}_1$, E_1 is the the initial muon momentum, energy

$\vec{p}_2$, E_2 is the final muon momentum, energy

$\vec{q}$, q_0 is the virtual photon momentum, energy

$\vec{q} = \vec{p}_1 - \vec{p}_2$

$q_0 = E_1 - E_2$

$q^2 = \left|q_0^2 - |\vec{q}|^2\right|$

m_μ = muon mass

$$\sigma_T\left(q^2, q_0\right) \xrightarrow[q^2 \to 0]{} \sigma_{\text{real photons}}\left(q_0\right)$$

$$\sigma\left(q^2, q_0\right) \xrightarrow[q^2 \to 0]{} 0$$

We use our current spark chamber measurements of σ_T and σ_L and some electron measurements to get the event rate given in Table 1. The number of events in each interval are per 1,000,000 exposures, with 20 muon-meters of track length per picture. Table 1 uses q_0 intervals (note that q_0 is the laboratory energy of the virtual photon) and q^2 intervals. We use the rough approximations:

For $q^2 \leq 0.4\ (\text{GeV/c})^2$, $\left(\sigma_T + \epsilon\sigma_L\right) = 100\,\mu\text{b}$

For $q^2 > 0.4\ (\text{GeV/c})^2$, $\left(\sigma_T + \epsilon\sigma_L\right) = 300\left(\frac{1}{1 + q^2/M_\rho^2}\right)^2\mu\text{b}$

Table 1

NUMBER OF EVENTS, 10 GeV/c INCIDENT MUONS

(40 Muons per Exposure
0.5 M Interaction Length Per Muon
1,000,000 Exposures)

q^2 $(\text{GeV/c})^2$ / q_0 GeV	0.05 to 0.10	0.1 to 0.2	0.2 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1.0	Above 1.0	Total all q^2
1 to 2	9	8	8	3	1.2	0.6	0.3	30
2 to 4	7	7	7	3	1.4	0.8	0.3	27
4 to 6	3	3	3	1.6	0.8	0.4	0.15	12
6 to 8	2	2	2	1.0	0.6	0.3	0.10	8
8 to 10	1.2	1.3	1.4	0.7	0.4	0.2	0.05	5
Total all q_0	22	21	21	9	4	2	0.9	

We find the total number of events for $q_0 > 1$ GeV and $q^2 > 0.05\ (\mathrm{GeV}/c)^2$ is about 82 in 10^6 pictures.

We can make a rough estimate from real photon bubble chamber exposures[3] as to the kind of events we will find. This is given below.

Reaction Products	Percent of all Events
μ + All strange particle events	3 to 6
$\mu + Y + K$	0.5 to 1
$\mu + Y + K + \pi$	1
$\mu + p + \pi^+ + \pi^-$	20
$\mu + p + \pi + \pi^-$ (ρ^0 only)	15
$\mu + p + \omega^0$	1 to 2
$\mu + p + \pi^+ + \pi^+ + \pi^- + \pi^-$	4 to 5
$\mu + N^* + \pi$	≈ 2
μ + (p or n) + π	≈ 2

Thus, except for the $p + \pi^+ + \pi^-$ channel, we can only expect a few events or one event per channel in 10^6 exposures. Clearly then 10^7 to 10^8 exposures is the number which must be considered.

IV. OTHER INCIDENT MOMENTA

It is easy to derive a crude equation for use at other momenta, neglecting the muon mass and taking $2E_1E_2 \gg q^2/2$

$$\frac{d^2\sigma}{dp^2 dp_2} = \left(\frac{\alpha}{\pi q^2}\right)\left(\frac{p_2}{p_1 q_0}\right)\Sigma$$

where $\Sigma = \sigma_T + \epsilon\sigma_L$

For a q_0 range from q_0 min $= r_1 p_1$ to q_0 max $= r_2 p_1$ with $0 < r_1 L r_2 L_1$

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{\pi q^2} \int_{r_1 p_1}^{r_2 p_1} \frac{(p_1 - q_0)\Sigma}{p_1 \, q_0} \, d\, q_0$$

$$\frac{d\sigma}{dq^2} = \frac{\alpha \overline{\Sigma}}{\pi q^2} \left[- (r_2 - r_1) + \ln \frac{r_2}{r_1} \right]$$

and $\overline{\Sigma}$ is almost independent of P_1 for estimation purposes.

Thus, in any q^2 interval the total number of events is independent of P_1. For most experiments we take $r_2 = 1$, $r_1 = 0.1$ because $q_0 < 0.1\, P_1$ events are not of interest relative to the entire q_0 range. Then

$$\frac{d\sigma}{dq^2} \approx \frac{1.4\, \alpha}{\pi q^2}$$

where we can use for $\overline{\Sigma}$ the approximations used before for $(\sigma_T + \epsilon \sigma_L)$. For $q^2 \leq 0.4\ (\text{GeV/c})^2$ we get $\sigma(0.05 \leq q^2 \leq 0.4) = 0.7\ \mu b$ independent of energy.

We do not get a substantial increase of events as the energy increases, unless we use very low q_0 events.

V. VERY LARGE HYDROGEN CHAMBERS

The only way to increase the event rate is to increase the number of muons or to use a longer chamber. Both of these conditions are met by the Argonne 12-foot hydrogen chamber now being built, or by the 25-foot chamber proposed by BNL.

Because these chambers are wider we might allow 60 muons per pulse. Then using 200 inches interaction length as an average betweeen the 12-foot and 25-foot chambers:

$$\text{Events per } 10^6 \text{ exposures} = 1200 \ .$$

This is a much better rate per exposure. However, the events per second are not so different. At SLAC the 40-inch chamber can go twice per second. The large chambers may go, say, every 2.5 seconds. The large chamber event rate per second is then three times the small chamber rate. The relative event rate per exposure of the large chambers versus the small chambers is 15.

VI. TRIGGERING THE CHAMBER

It is evident that a great saving in film can be made by triggering the chamber lights and camera on inelastic muon interactions. It is possible to insert a counter hodoscope in the muon beam before and after the chamber to trigger on muons which have deflected in the chamber by at least 10 or 15 mrad. False events will come from elastic scattering and interactions in the entrance and exit windows. The hodoscope design depends on the particular chamber and muon beam and will not be discussed here.

ACKNOWLEDGMENT

The author wishes to thank the Aspen Center for Physics for its hospitality during the time this report was written.

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