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RF Group

Power Dissipation in Rectangular S-Band Waveguide

The magnetic fields for the TE_{10} mode are:

$$H_z = j \frac{E_0}{2a}\left(\frac{\lambda_0}{2a}\right) \cos \frac{\pi x}{a} \cos \omega t$$

$$H_x = - \frac{E_0}{Z_{TE}} \sin \frac{\pi x}{a} \cos \omega t$$

$$H_y = 0$$

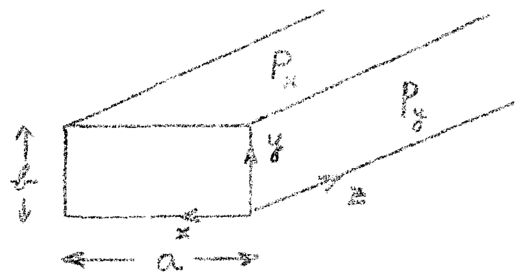
where

$$Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda_0}{2a}\right)^2}} \qquad \text{and} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

The power dissipation per unit area of waveguide wall is given by

$$P = \frac{1}{2} R_s \left| H_t \right|^2$$

where H_t is the tangential magnetic field and R_s is the surface resistivity.



a = 2.840" = 7.22 cm
b = 1.340" = 3.41 cm

Let P_x and P_y be the average powers dissipated in the top or bottom and side walls respectively. One then obtains:

$$P_x = \frac{R_s}{2} \frac{E_0^2}{\eta^2} \left(\frac{\lambda_0}{2a}\right)^2 \left[1 - \left(1 - \left(\frac{2a}{\lambda_g}\right)^2 \right) \sin^2 \frac{\pi x}{a} \right]$$

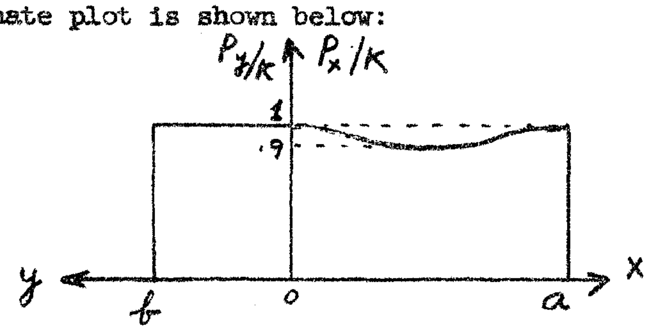
$$P_y = \frac{R_s}{2} \frac{E_0^2}{\eta^2} \left(\frac{\lambda_0}{2a}\right)^2$$

For standard S-band guide, these expressions yield for $\lambda_0 = 10.5$ cm

$$P_x = K \left[1 - 0.116 \sin^2 \frac{\pi x}{a}\right]$$

$$P_y = K$$

An approximate plot is shown below:



The total powers P_x and P_y are obtained by integration:

$$\left(P_x\right)_{\text{total}} = \frac{aR_s}{4} \frac{E_0^2}{\eta^2}$$

$$\left(P_y\right)_{\text{total}} = \frac{bR_s}{2} \frac{E_0^2}{\eta^2} \left(\frac{\lambda_0}{2a}\right)^2$$

The ratio is

$$\frac{P_x}{P_y} = \frac{2a^3}{\lambda_0^2 b} = 2 \text{ for standard S-band guide, at } \lambda_0 = 10.5 \text{ cm}$$

Thus, the power dissipation appears to be practically constant around the guide.

It can easily be shown that for any T_{n0} mode, the above ratio is

$$\frac{P_x}{P_y} = \frac{2a^3}{n^2 \lambda_0^2 b}$$