

# Narrow-band Impedance of the PEP-II Collimators\*

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## Abstract

Collimators in the PEP-II High Energy Ring (HER) and Low Energy Ring (LER) may contribute large impedance to the rings because of close proximity to the beam. The collimators, which normally intrude into the vacuum chamber, surprisingly, support a novel type of localized modes that can generate large transverse narrow-band impedance. Numerical simulations show that the transverse impedance can be substantially reduced by appropriate choice of the collimator geometry. Another possibility is the excitation of trapped modes between two closely spaced collimators. The leakage of these modes through the collimator apertures into the beampipe are evaluated to determine their possible effects on beam instability. The analysis presented here can be applied to other beamline components intruding into the vacuum chamber such as masks and insertion devices.

## 1 Introduction

The PEP-II collimation system consists of different kinds of collimators in the HER and LER. The collimators are located in the straight sections, the vacuum chambers of which are made of circular, stainless steel pipes. Most of these collimators inserted into the vacuum chamber are close to the beam axis, raising concerns that wakefields generated by the beam may cause beam instabilities.

Previous studies show that collimators generate mainly broad-band impedance [1]. Since the number of collimators are normally small compared with those of other beamline components, for example, the BPM and bellows, their contribution to the impedance budget is relatively small. Clearly in cavity-type structures modes can exist below the beampipe cutoff frequency; that this arises also in collimator-type structures is not so obvious. Localized modes can also be excited between two collimators spaced close to each other so that they form a cavity-like structure. For collimators of small length, for example, the HER gasket collimators, these localized modes can tunnel through the collimator apertures at two ends into the beampipe and this reduces the  $Q$  factors of the modes. Thus, both the broad-band and narrow-band impedances of the collimators have not been major concerns for the PEP-II rings.

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Conventional wisdom suggests that trapped modes can exist in a region where the vacuum chamber bulges [2]. But, as will be shown later, hybrid TE-like trapped modes can also be excited in a constriction of the vacuum chamber. The constriction reduces the local TE cutoff frequency below that of the beampipe, in the same manner that a ridge waveguide lowers the TE mode cutoff frequency. In contrast to a pure TE mode, the hybrid mode possesses a longitudinal electric field component that can couple strongly to the beam. This type of modes appears at the PEP-II collimators, and they can have high impedance that may affect the transverse motion of the beam.

In the following, we use MAFIA [3] to evaluate the narrow-band impedance of the collimators built for the PEP-II commissioning. In section 2, we describe the method of calculating the transverse impedance of a non-cylindrically symmetric beamline component such as the collimator. In section 3, we determine the narrow-band impedance of the HER gasket collimators. In addition, we study the effect of cross-talk between two gasket collimators by calculating the coupling of trapped modes excited between the collimators to the beampipe. In section 4, we determine the narrow-band impedance of the LER collimators. We investigate the effect of the collimator geometry on the transverse narrow-band impedance. We give a summary in section 5.

## 2 Method of Calculations

For an arbitrary structure, the longitudinal impedance  $Z_l$  depends on the offsets of both the leading ( $x_l$ ) and the trailing ( $x_t$ ) particles. An exception is the round beampipe where the impedance is independent of the offset of the trailing particle. For small offsets,  $x_{l,t} \ll b$ , where  $b$  is the beampipe half aperture width,  $Z_l$  can always be expanded in a Taylor series:

$$Z_l(\omega, x_l, x_t) = Z_1 + (x_l + x_t)Z_2 + (x_l^2 + x_t^2)Z_3 + x_l x_t Z_4 + \dots, \quad (1)$$

where  $Z_m$ ,  $m = 1, 2, 3, \dots$ , are functions of  $\omega$ . Here we have used the symmetry of the impedance with respect to exchange of  $x_t$  and  $x_l$  and have not explicitly written similar terms depending on the offsets  $y_l$  and  $y_t$  in the  $y$ -direction. For structures with additional symmetry, some of the coefficients vanish. For example, if a structure is symmetric with respect to  $x \rightarrow -x$ , there are no linear terms in  $x_l$  and  $x_t$ .

The transverse kick is given by the transverse wake potential  $W_x$  as  $\Delta p_x = N_b e^2 W_x$ , where  $N_b$  is the number of particles in the bunch. Panofsky-Wenzel theorem gives the Fourier component of the transverse wake potential  $\tilde{W}_x$  in terms of  $Z_l$ :

$$\tilde{W}_x(\omega, x_l, x_t) = \frac{1}{k} \frac{\partial Z_l}{\partial x_t}, \quad (2)$$

or

$$\tilde{W}_x(\omega, x_l, x_t) = \frac{1}{k} [Z_2 + 2x_t Z_3 + x_l Z_4 + \dots]. \quad (3)$$

Here  $k = \omega/c$ . The coefficients  $Z_2, Z_3$  and  $Z_4$  carry different physical meaning.  $Z_2$  generates an orbit distortion, but does not cause beam instability.  $Z_3$  is responsible for tune shift while  $Z_4$  for beam instability. Therefore, for an asymmetric structure, the transverse impedance in general has two components, namely,  $Z_x^{(l)} = Z_4/k$  and  $Z_x^{(t)} = 2Z_3/k$ . For the round beam pipe, the latter component is zero.

We want to calculate the transverse impedance in terms of the longitudinal impedance that has been evaluated with equal offsets, i.e.,

$$Z_l(\omega, x) = Z_1 + 2xZ_2 + x^2(2Z_3 + Z_4) + \dots \quad (4)$$

Choosing the nominal beam position at  $x = 0$ , we obtain

$$Z_1(\omega) = Z_l(\omega, x = 0), \quad (5)$$

$$Z_2(\omega) = \frac{1}{2} \frac{\partial Z_l(\omega, x, x)}{\partial x} \Big|_{x=0}, \quad (6)$$

$$2Z_3(\omega) + Z_4(\omega) = \frac{1}{2} \frac{\partial^2 Z_l(\omega, x)}{\partial x^2} \Big|_{x=0}. \quad (7)$$

Note that  $Z_3$  and  $Z_4$  enter into  $Z_l(\omega, x)$  as a combination of  $2Z_3 + Z_4$ , which describes the kick to a trailing particle when both the leading and trailing particles travel along the same trajectory.

There is no general way to define each of  $Z_3$  and  $Z_4$  in terms of  $Z_l(\omega, x)$  or to describe the transverse impedance by a single parameter. However, it is possible to do this for a narrow-band impedance dominated by a single resonant mode for a structure with the symmetry with respect to  $z \rightarrow -z$ . As is known [4], a longitudinal impedance that has resonances at eigenfrequencies  $\omega_n$ , can be represented by a sum of resonance terms:

$$Z_l(\omega, x_l, x_t) = \sum \Lambda_n(\omega) V_n^*(\omega, x_l) V_n(\omega, x_t), \quad V_n(\omega, x) = \int E_n(x, z) e^{ikz} dz. \quad (8)$$

Here  $E_n(x, z)$  are functions describing the spatial structure of the eigenmodes, and  $\Lambda_n(\omega) = \Lambda_n^*(-\omega^*)$ . For a high- $Q$  mode,  $\Lambda_n$  has a narrow peak at  $\omega = \omega_n$ . Hence, for  $\omega$  close to the resonant frequency  $\omega_n$ , only one term gives a dominant contribution. In this approximation, the longitudinal impedance can be factorized and written as

$$Z_l(\omega, x_l, x_t) = \Lambda_n(\omega) V_n^*(x_l, \omega) V_n(x_t, \omega). \quad (9)$$

The eigenfunction  $E_n$  for a trapped mode responsible for the narrow-band impedance can be chosen to be real. Then, for a structure with symmetry  $z \rightarrow -z$ , the  $V_n$  are also real.

Expanding  $V_n(x_l), V_n(x_t)$  in Taylor series:  $V_n(x) = a + bx + cx^2 + \dots$ , it is easy to see that the four parameters  $Z_k, k = 1, 2, 3, 4$ , are given by  $\Lambda = \Lambda_n$  and three real parameters  $a, b$ , and  $c$ :

$$Z_1 = \Lambda a^2, \quad Z_2 = \Lambda ab, \quad Z_3 = \Lambda ac, \quad Z_4 = \Lambda b^2. \quad (10)$$

At resonance,  $Z_l = R_s$ , where  $R_s$  is the shunt impedance of the mode. Then,  $b = Z_2/\sqrt{\Lambda Z_1}$ , and

$$Z_4 = \frac{Z_2^2}{Z_1}, \quad (11)$$

in which coefficients  $Z_1$  and  $Z_2$  can be found from  $Z_l(\omega, x)$  (Eqs. (5)-(6)). The remaining parameter  $Z_3$  can be determined using  $Z_4$  and Eq. (7). For the case having  $x_t = 0$ , the transverse impedance becomes

$$Z_x = \frac{1}{kZ_1} \left( \frac{1}{2} \frac{\partial Z_l}{\partial x} \right)^2. \quad (12)$$

### 3 HER Gasket Collimators

Gasket collimators have been installed in the IR-12 section of the HER to reduce backgrounds. The gasket collimators, made of copper, are essentially thin one-sided irises inserted into the vacuum chamber either in the transverse horizontal or transverse vertical direction. Each collimator has a length of 1 cm and a height of 3.5 cm, and they are about 6 m apart. The edge of the collimator is at a distance of about 1 cm from the axis of the beampipe with a radius of 4.45 cm. Two kinds of narrow-band impedance are addressed here. First, we identify trapped modes at an individual collimator and determine their impedance. Second, since two collimators are not isolated from each other, trapped modes can exist between them. We determine the  $Q$  factors of these modes by computing their coupling into the beampipe through the collimator apertures.

#### (a) *Narrow-band Impedance at a Collimator*

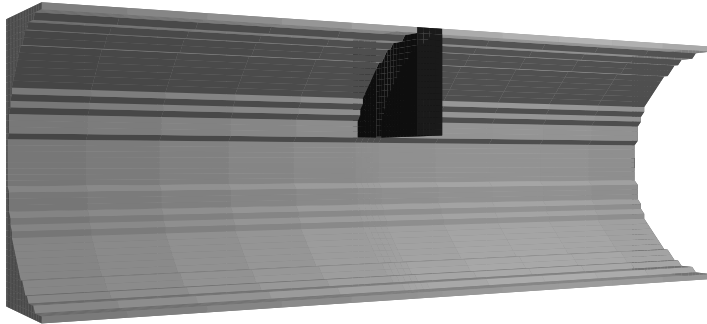


Figure 1: MAFIA model of the HER gasket collimator.

The MAFIA model of the gasket collimator is shown in Fig. 1. Because of symmetry, only half of the geometry is simulated. In Fig. 2, we show a trapped mode found at the collimator. It is a  $TE_{111}$ -like mode with a frequency of 1.943 GHz, which is slightly below the  $TE_{11}$  cutoff frequency of 1.967 GHz of the beampipe. Since the mode frequency is close to the beampipe cutoff frequency, it extends into the beampipe a distance much longer than the collimator length. Thus the wall loss power due to the mode is predominately

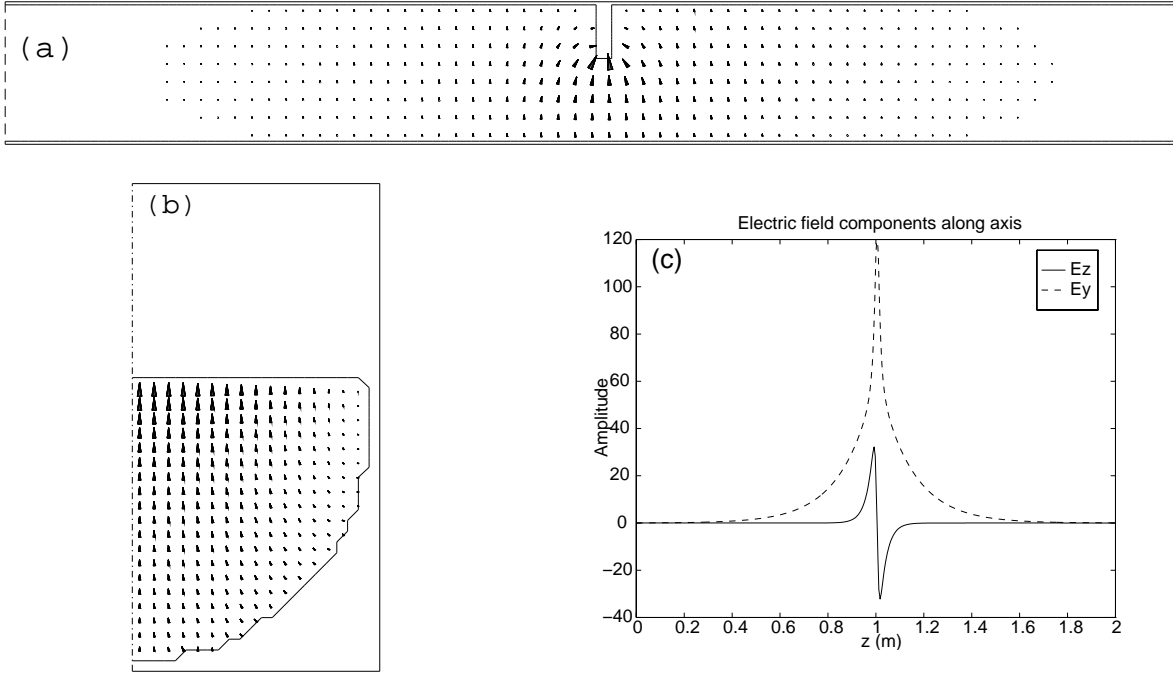


Figure 2: TE-like mode at the HER gasket collimator: (a)-(b) Electric field patterns at different cross sections; (c) Electric field components along the axis.

determined by the material property of the stainless steel beam pipe. Furthermore, unlike a pure TE mode, the mode has a noticeable longitudinal electric field component along the beam axis which can produce an appreciable transverse impedance as a result of its transverse gradient.

In Table 1, we list some calculated RF parameters of the trapped mode. It has a  $Q$  of 2980 and a transverse impedance of  $1.29 \text{ M}\Omega/\text{m}$  in the direction toward the collimator. The corresponding transverse impedance responsible for beam instability in the direction parallel to the collimator vanishes due to symmetry. In comparison, the severest  $m = 1$  transverse mode at  $1.413 \text{ GHz}$  of the RF cavity, has a  $Q$  of 1140 and a transverse impedance of  $0.13 \text{ M}\Omega/\text{m}$  [5]. Thus, some feedback gain may be required to suppress the gasket mode.

Frequency	1.943 GHz
Quality factor, $Q$	2980
Shunt impedance, $R_s$	30.7 k $\Omega$
Transverse impedance: $Z_{3,x}$	-1.08 M $\Omega/\text{m}$
$Z_{4,x}$	0 M $\Omega/\text{m}$
$Z_{3,y}$	1.06 M $\Omega/\text{m}$
$Z_{4,y}$	1.29 M $\Omega/\text{m}$

Table 1: RF parameters of the trapped mode at the HER gasket collimator.

(b) *Trapped Modes between Two Collimators*

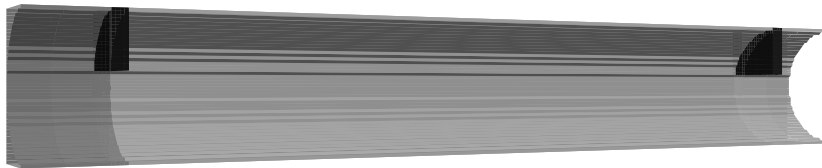


Figure 3: MAFIA model of two closely spaced collimators.

The cross-talk between two collimators separated at a close distance from each other allows the excitation of modes when a beam passes through the structure. The schematic of the model for two gasket collimators in the vacuum chamber is shown in Fig. 3. In MAFIA calculations, only one half of the structure shown in the figure needs to be simulated by using an appropriate boundary condition at the midplane between the collimators. As the separation of the collimators is long compared with the chamber radius, a set of modes with different longitudinal variations can be found in the structure. Here, we are interested in investigating TM-like modes as they usually have stronger coupling to the beam than TE-like modes. Fig. 4 shows a TM-like mode from a MAFIA frequency domain calculation. It can be seen that considerable field intensity appears in the continuing beampipe, indicating that the result depends on the boundary condition set at the beampipe end. This makes it difficult to determine accurately the RF parameters of the mode.

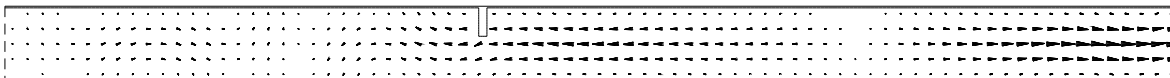


Figure 4: A TM-like mode between two collimators. The left of the collimator is the beampipe and the right a fraction of the chamber between the collimators.

The frequency domain calculation suggests that there is large leakage of resonance fields through the collimator apertures. To quantify the field leakage, we have carried out a MAFIA time domain calculation to determine the  $Q_{ext}$  of the modes. A dipole signal driven, slightly above the  $TM_{01}$  cutoff frequency of the beampipe, in the chamber between the collimators excites electromagnetic fields which can then couple into the beampipe. The leakage fields propagating in the beampipe can be decomposed into TE and TM modes. Figs. 5(a)-(b) show the amplitudes of the TE and TM modes as a function of time, respectively. The long tail of the oscillations represents some high  $Q$  resonances in the structure. Figs. 5(c)-(d) show the Fourier spectra of the TE and TM modes, respectively. A series of peaks can be identified as the  $TM_{01n}$  modes between the collimators. They are slightly above the  $TM_{01}$  cutoff frequency of 2.58 GHz of the beampipe, and have a frequency separation of about 30-40 MHz. Some modes have broad peaks, implying that they strongly couple into the beampipe and hence have small  $Q_{ext}$ 's. The highest  $Q_{ext}$  among the resonances is found to be about 1500. Together with the fact that the shunt impedance is normally small for a long structure, this may not pose serious problems to the beam stability.

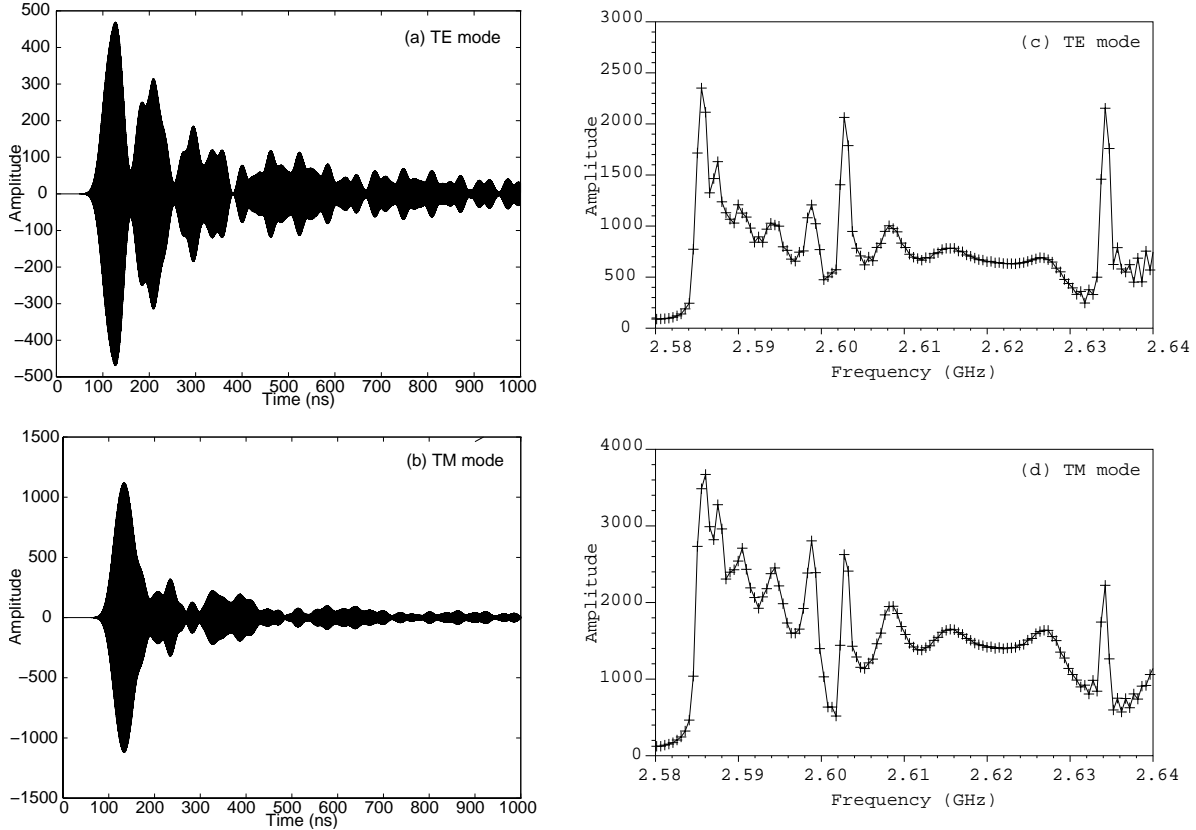


Figure 5: (a)-(b) Signals of TE and TM modes coming out from collimators; (c)-(d) Fourier transform of the signals.

## 4 LER Collimators

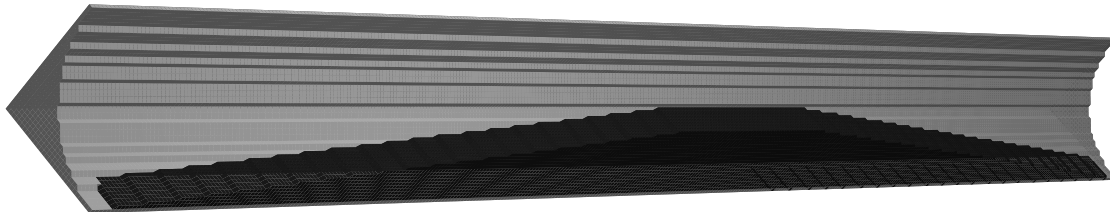


Figure 6: MAFIA model of the LER collimator.

Each of the LER collimators (see Fig. 6) consists of a central flat region with tapers at both ends. It is made of GlidCop, which has a similar conductivity as copper. The length of the central flat region is 9 cm, the taper length 26 cm, and the thickness of the collimator 1 inch. The offsets of the collimator edges from the beam axis are 9 mm, 11.5 mm or 16 mm, corresponding to different taper angles.

Carrying out analysis similar to that described in the last section, a trapped mode found at the LER collimator with an offset of 9 mm is shown in Fig. 7. Again, it is a TE-like mode with a frequency of 1.464 GHz. In contrast to the short iris structure of the HER

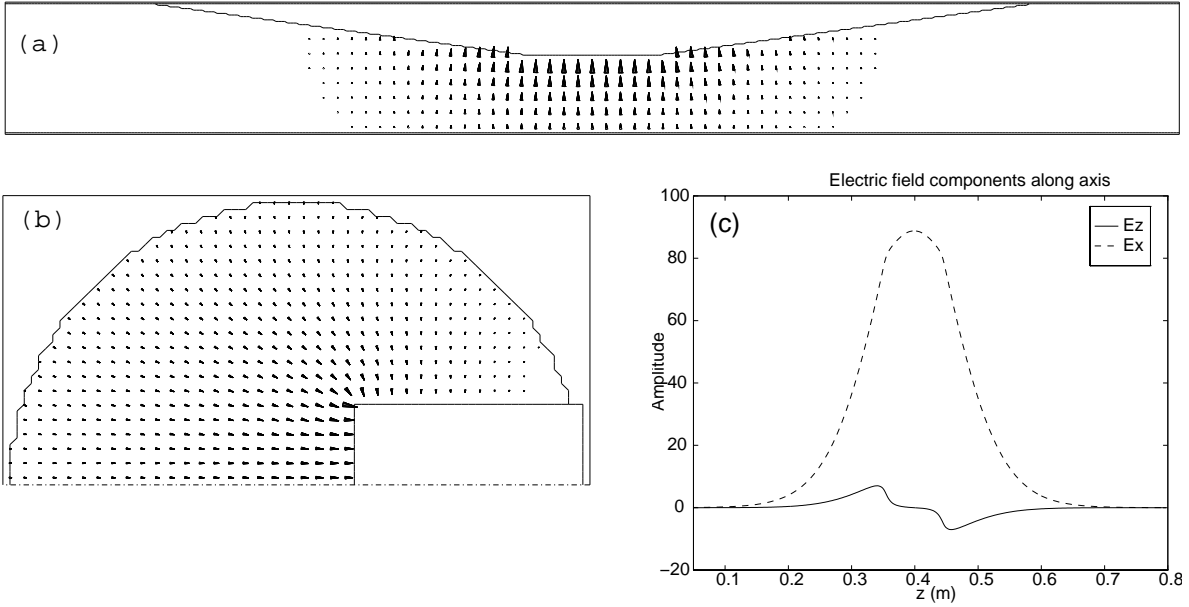


Figure 7: TE-like mode at the LER ollimator: (a)-(b) Electric field patterns at different cross sections; (c) Electric field components along the axis.

gasket collimator in which the localized mode extends into the upstream and downstream beampipes, this mode is more confined to the collimator region. The mode frequency lies between the  $TE_{11}$  cutoff frequencies of the beampipe and of the central region of the collimator which are 1.957 GHz and 1.423 GHz respectively. The longitudinal electric field component is small compared with the transverse component, and is noticeable only around the beginnings of the tapers. Using the procedure mentioned in section 2, for a collimator with 9 mm offset, the transverse impedance contributing to beam instability in the direction toward the collimator is found to be  $124.3 \text{ k}\Omega/\text{m}$ , and that in the direction parallel to the collimator vanishes because of symmetry. This relatively small transverse impedance should be readily handled by the feedback system.

Offset (mm)	$f$ (GHz)	$Q$	$R_s$ ( $\text{k}\Omega$ )	$Z_{3,x}$ ( $\text{k}\Omega/\text{m}$ )	$Z_{4,x}$ ( $\text{k}\Omega/\text{m}$ )	$Z_{3,y}$ ( $\text{k}\Omega/\text{m}$ )	$Z_{4,y}$ ( $\text{k}\Omega/\text{m}$ )
9	1.464	2340	2.04	47.8	124.3	-48.0	0
11.5	1.518	2350	0.98	39.0	61.9	-39.0	0
16	1.618	2400	0.24	13.1	15.2	-13.1	0

Table 2: RF parameters of the trapped mode at the LER collimator.

The RF parameters of the trapped mode for different offsets of the collimator from the beam axis are listed in Table 2. The frequency shift of the trapped mode is about 50 MHz and 100 MHz for different offsets. The bandwidth of an individual mode is about 650 kHz. Therefore, the modes due to different offsets do not overlap and their contributions to the impedance can be treated separately. Considering that there are six

identical RF cavities in the LER, the impedance due to the collimators is an order of magnitude smaller than that contributed by the severest transverse mode of the cavities. Also, the transverse impedance of the collimator is reduced almost by a factor of 10 when the offset is 16 mm. In this case, the taper has a more gentle slope and this reduces the field transverse gradient and hence the transverse impedance.

## 5 Summary

It has been shown that collimator-type structures can support a novel type of localized TE-like modes that may generate large transverse narrow-band impedance. Numerical calculations using MAFIA for the HER gasket collimators give moderately high transverse impedance which nevertheless can be handled by the PEP-II feedback system. Long tapers at the ends of the LER collimators can reduce the transverse impedance because of the decrease in the TM content of the localized mode, and in the transit time factor typical of long structures. Furthermore, resonances excited between two HER gasket collimators are able to couple out into the beampipe and hence have a relatively small  $Q_{ext}$ . The method described in this note can be applied to similar beamline components that intrude into the vacuum chamber.

## Acknowledgements

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