

# Single Bunch Stability in LER of PEP-II\*

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## Abstract

The note describes results of studies of the single bunch stability in the low energy ring (LER) of the PEP-II B-factory[1]. Simulations describe the potential well distortion (PWD) obtained by numerical solution of the Haisinski equation[2] and results on the beam stability obtained with the code TRISIM[3]. Both longitudinal and transverse wake fields are taken into account.

## 1 Introduction

Preliminary estimates[1] indicate that single bunch in the LER of the PEP-II B-factory has to be stable, both longitudinally and transversely, at the maximum design bunch current 1.8 mA (beam current 3A). However, realistic wakes of the machine has been constructed only recently using results of the extensive numerical simulations of the vacuum components of the ring[4]. Additional to that, the code TRISIM[3], a simulation program for single-bunch collective effects written by one of the authors (G. S.), became recently available. This allows us to study beam stability in a more reliable way than it is possible analytically.

In the beginning, we discuss shortly how the wake fields responsible for the effects under consideration have been constructed. Then we present results on the PWD and, later, results on the beam stability obtained with the code TRISIM. The input parameters are based on the nominal parameters of the LER: energy 3.105 GeV, revolution period 7.336  $\mu$ s, rf voltage 5.1 MeV, RF frequency 476 MHz, SR energy loss 0.75 MEV/turn, momentum compaction  $\alpha = 1.23 \times 10^{-3}$ , average beta function  $\beta_y = 14.5$  m, betatron tune  $\nu_y = 36.642$ , horizontal emittance  $\epsilon_x = 65.58$  nm, coupling  $\epsilon_y/\epsilon_x = 0.03$ , damping times  $\tau_E = 30.2$  ms,  $\tau_y = 61.1$  ms. The nominal bunch length of 33.1 ps and the rms energy spread of 2.394 MeV were used to generate the initial distribution of a bunch.

## 2 Wake Fields

The longitudinal wake field of a point-like charge  $W_l^s(z)$  was constructed[5] from the combination of a narrow-band impedances of the higher order modes of the RF cavities and 290 BPMs, and of the broad-band impedances of the resistive walls, high-frequency tail of the RF cavities impedance, and the impedance of the vacuum components of the

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ring. Parameters of the HOMs were taken from the results of the measurements[6]. The conservative scenario of 8 cavities in the ring was taken for calculations. Because of the large number of different vacuum components in the ring, the impedances of these components were calculated with the codes ABCI and MAFIA only for a Gaussian bunch with the nominal rms length  $\sigma = 1$  cm. For such a bunch, the longitudinal components of the ring have mostly inductive impedances with the total inductance of all components  $L = 87$  nH. However, the loss factor of these components is not zero, and the total loss factor was estimated  $\kappa = 2.9$  V/pC at  $\sigma = 1$  cm. (The loss  $\kappa$  is reduced here by the contribution of the high-frequency tail of the cavities).

To reconstruct the wake of a point-like charge, we use the modified inductive impedance (in CGS units,  $c_0$  is the velocity of light)

$$Z(\omega) = -\frac{i\omega L}{c_0^2(1 - i\omega a/c_0)^{3/2}}. \quad (1)$$

which is pure inductive at small frequencies  $\omega a/c_0 \ll 1$  and rolls-off as  $\omega^{-1/2}$  at high frequencies. The corresponding wake at  $z > 0$  is

$$W_l^\delta(z) = \frac{L}{a\sqrt{\pi z a}}(1 - 2\frac{z}{a})e^{-z/a}, \quad (2)$$

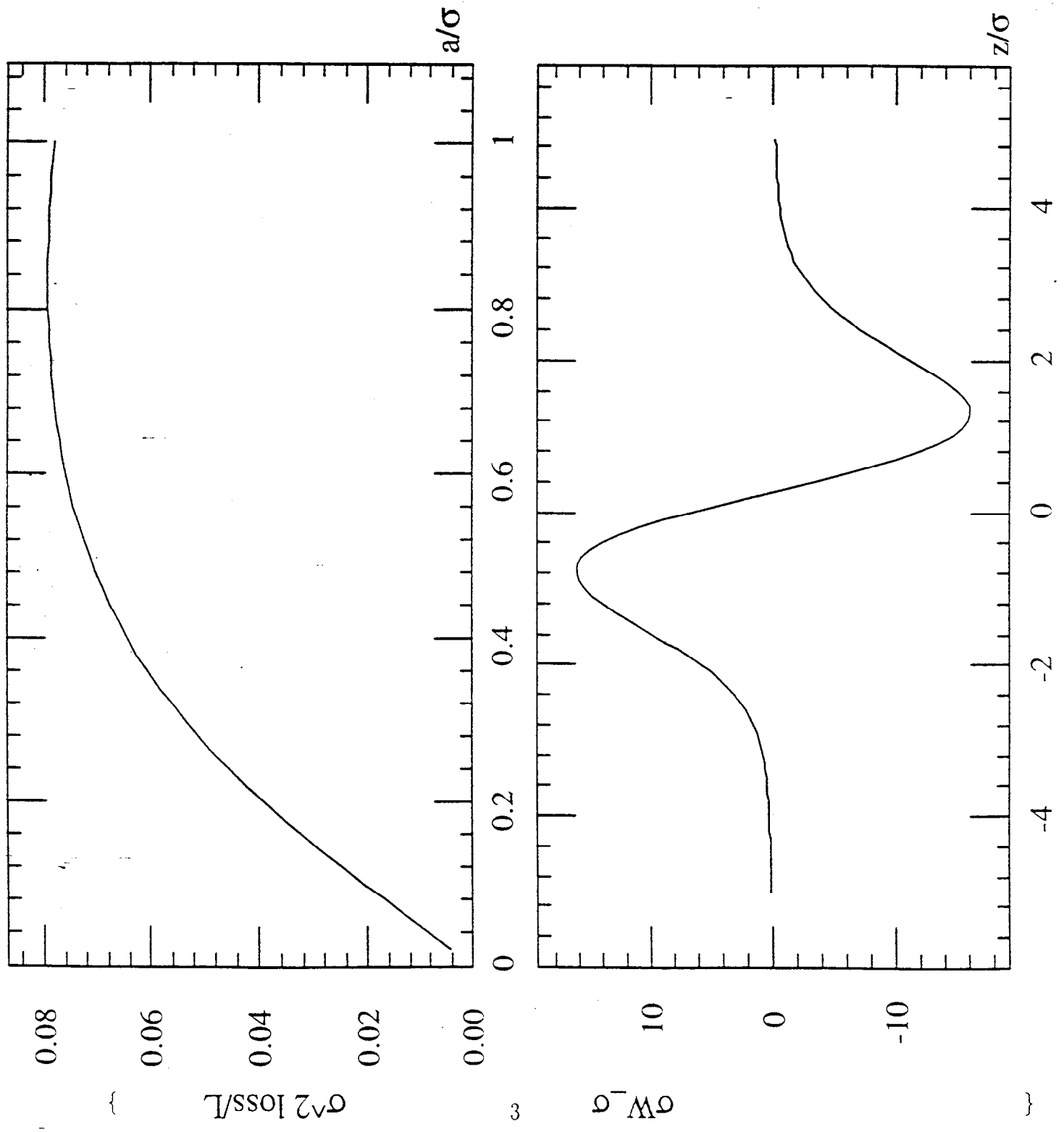
and is equal to zero at  $z < 0$ . In the limit  $a \rightarrow 0$  it corresponds to the usual inductive wake  $W_l^\delta(z) = L\delta(z)/\partial z$ . The roll-off parameter  $a$  can be defined from the loss

$$\kappa(\sigma) = \int \frac{d\omega}{2\pi} Z(\omega) e^{-\frac{1}{2}(\omega\sigma/c_0)^2} \quad (3)$$

provided  $\kappa(\sigma)$  is known at some  $\sigma$ . Parameter  $\sigma^2\kappa(\sigma)/L$  vs  $a/\sigma$  is plotted in Fig. 1. For small  $a/\sigma \ll 1$ ,

$$\kappa(\sigma) = \frac{3La}{8\sqrt{\pi}\sigma^3} \quad (4)$$

Fig 1



Dependence of the loss on  $\sigma$  was checked[5] for a typical inductive components and is in a good agreement with results of the code ABCI. Using parameters  $L$  and  $\kappa$  for  $\sigma = 1$  cm. we obtain  $a = 0.185$  cm. To check the result, we plot in the Fig. 1 also the wake field

$$W_l^\sigma(z) = \int dz' \rho_\sigma(z') W_l^\delta(z' - z) \quad (5)$$

of a Gaussian bunch with the density  $\rho_\sigma$  and parameters  $\sigma = 1$  cm,  $a = 0.185$  cm. The wake is clearly mostly inductive.

The transverse wake  $W_\perp^\delta(z)$  was calculated from measured dipole modes of the RF cavities, impedance of the resistive walls, and the impedance of the vacuum components of the ring. The later was defined from the Wenzel-Panofsky theorem assuming that the dipole longitudinal wake  $W_l^{m=1}(z)$  is related to the monopole longitudinal wake,  $W_l^{m=1} \simeq 2W_l^{m=0}/b^2$  where  $b$  is the average beam pipe radius. This assumption was confirmed by ABCI.

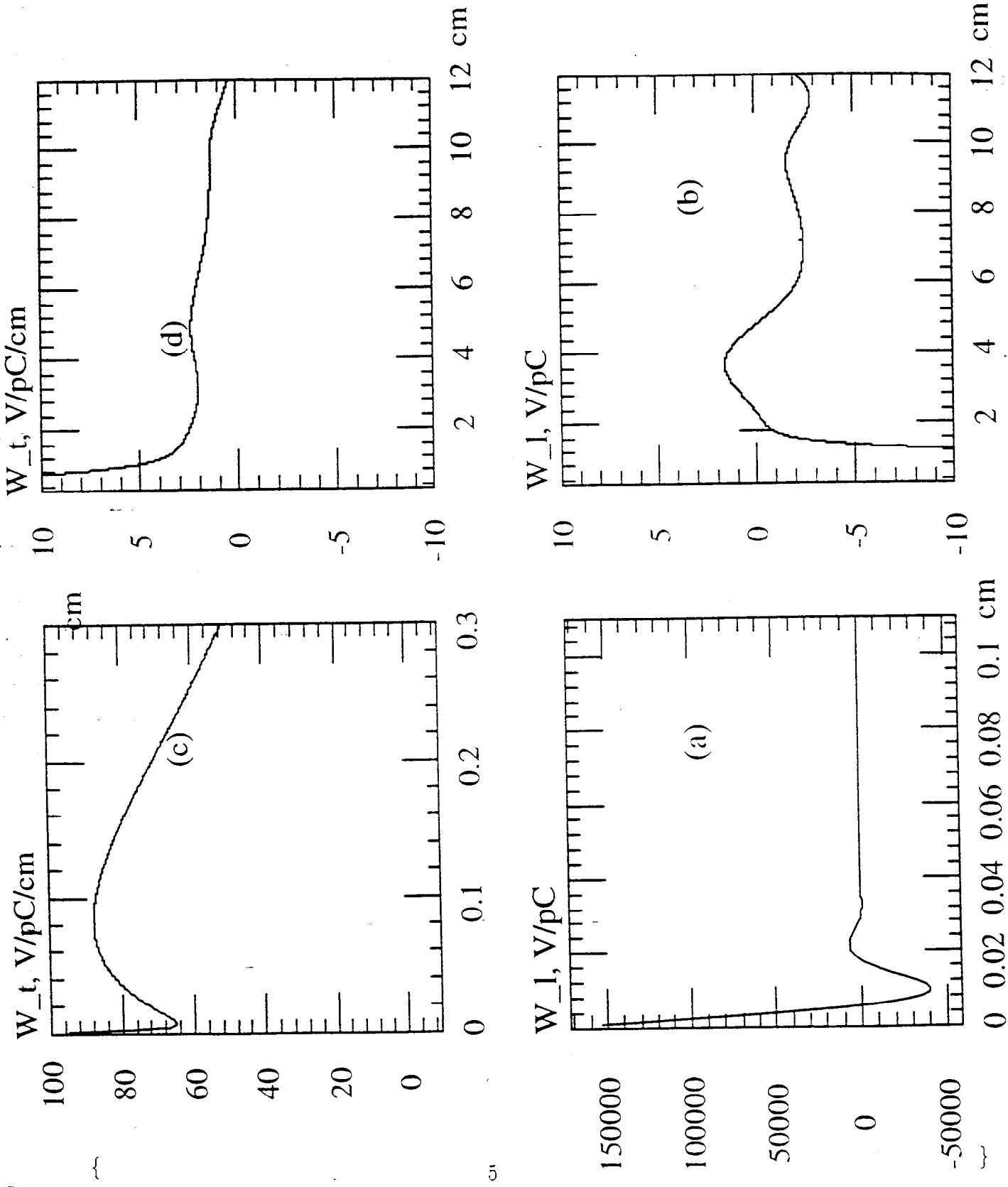
The longitudinal wake  $W_l^{m=0}(z)$  is shown in Fig. 2 at small and large distances. At small distances it is dominated by the contribution of the resistive walls and the inductive impedance, at large distance the main contribution comes from a few modes of the RF cavities.

The wake of the triangular bunch  $W_l^\Delta(z)$ , which is needed as input for TRISIM, was calculated then by convolution of the  $W_l^\delta(z)$  with the density

$$\rho_\Delta(z) = \frac{1}{\Delta} [1 - |z/\Delta|], \quad |z| < \Delta. \text{eq : 6} \quad (6)$$

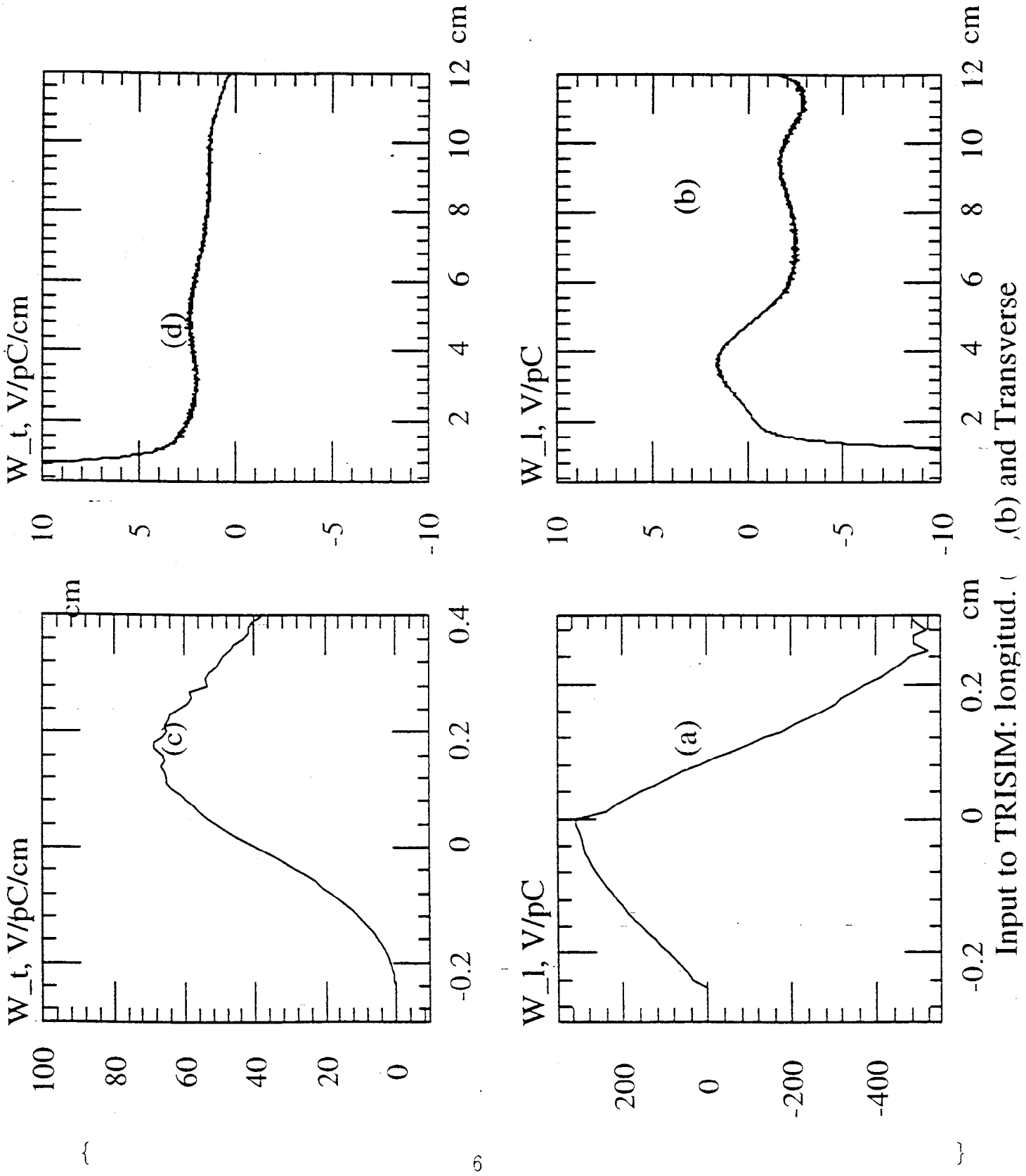
Parameter  $\Delta = \sigma/4 = 0.25$  cm was used in the simulations. Note that the wake  $W_l^\Delta$  of a triangular bunch located at  $z_k$  is  $W_l^\Delta(z - z_k)$  and is defined provided  $W_l^\Delta(z)$  is known. The main difference of the wakes of a triangular bunch shown in Fig. 3 in comparison with the wakes of the point-like charge in Fig.2 is that  $W_l^\Delta(z) \neq 0$  at  $z < 0$  and the short wave oscillations of the point-like wakes at the very small distances are averaged out in the input to TRISIM. Such an averaging corresponds to the wake averaging in a real bunch.

Fig. 2



Longitud. (a),(b) and Transverse (c), (d) wakes

Fig. 3



### 3 Potential Well Distortion

The steady-state PWD is described by a solution of the Haisinski equation[2] for the density of a bunch  $\rho(x)$ ,

$$\rho_H(x) = \frac{1}{Z_H} e^{-U(x)}, \quad \int \rho_H(x) dx = 1. \quad (7)$$

The total potential  $U(x)$  is the sum of the rf potential  $U_{rf} \simeq x^2/2$  and the self-consistent potential  $U_W$ :

$$U_W = \lambda \int_x^\infty dx' \rho(x') S[(x' - x)]. \quad (8)$$

Here  $x = z/\sigma_0$ , and  $S(z)$  is an integral of the wake function  $W(z)$ ,  $S(z) = \int_0^z dz' W(z')$ . The factor  $\lambda$  depends on the number of particles per bunch  $N_b$ ,

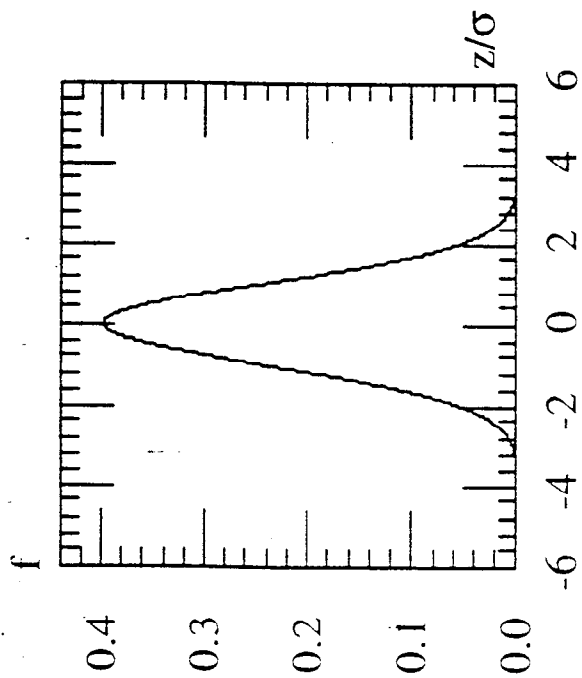
$$\lambda = \frac{N_b \tau_0}{2\pi R \alpha \gamma \delta^2}. \quad (9)$$

At the zero-current,  $\rho_H(x)$  describes a Gaussian bunch with the rms  $\sigma_0$ .

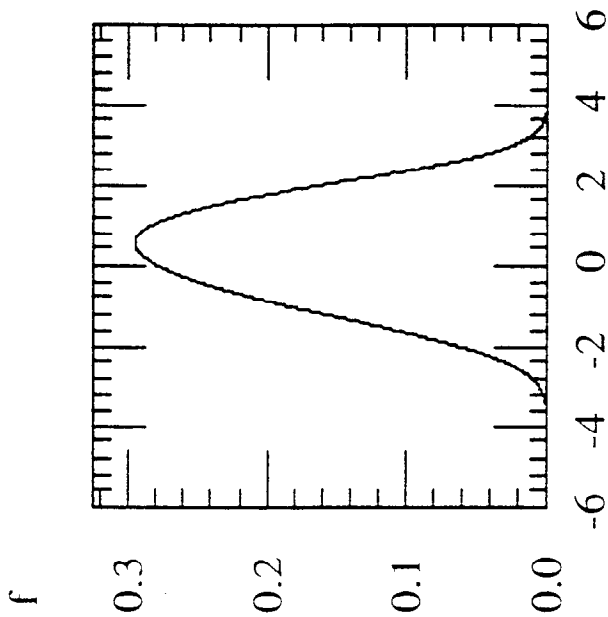
Eq. (7) can be solved numerically defining  $\rho(x)$  and  $U(x)$ . The later can be used to find the action variable  $I = \int (dx/2\pi) \sqrt{2(H - U(x))}$  and the synchrotron frequency  $\frac{\omega(I) = \partial H}{\partial I}$ . Examples of the density profile and the shape of the potential well are shown in Fig. 4 for two cases,  $N_B = 0.154 \times 10^{10}$  and  $N_B = 17.174 \times 10^{10}$ .

The bunch lengthening is clear visible, but even in the last high current case (total current 6.2 A, bunch current 3.75 mA) the potential well does not show a second minimum. Variation of the rms bunch length and the position of bunch centroid is shown in Fig. 5. At the maximum nominal machine current of 3 A bunch lengthening is small, less than 10%. Fig. 6 depicts the dependence of the synchrotron frequency on the amplitude  $I$  of oscillations for various bunch currents. At small amplitudes, the frequency  $\omega(0)$  decreases with  $N_B$  while the frequency spread increases and is about 20% at 3 A.

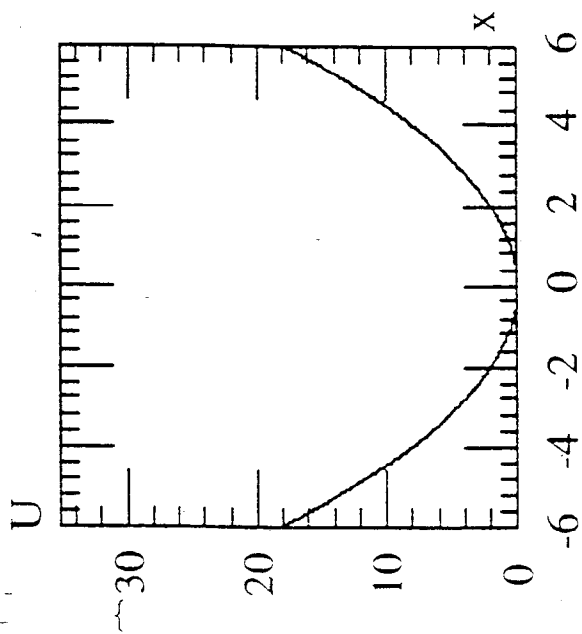
Fig. 4



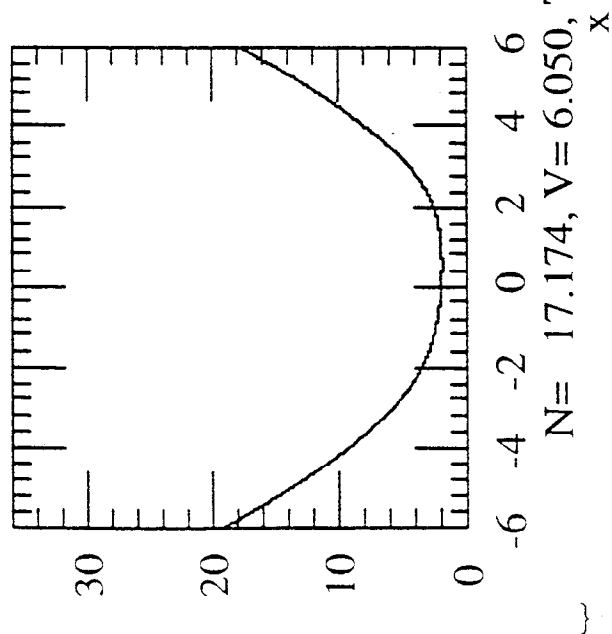
$N = .154, V = 6.050, T = 1.0, \lambda = .153E-03$



$N = 17.174, V = 6.050, T = 1.0, \lambda = .918E-01$

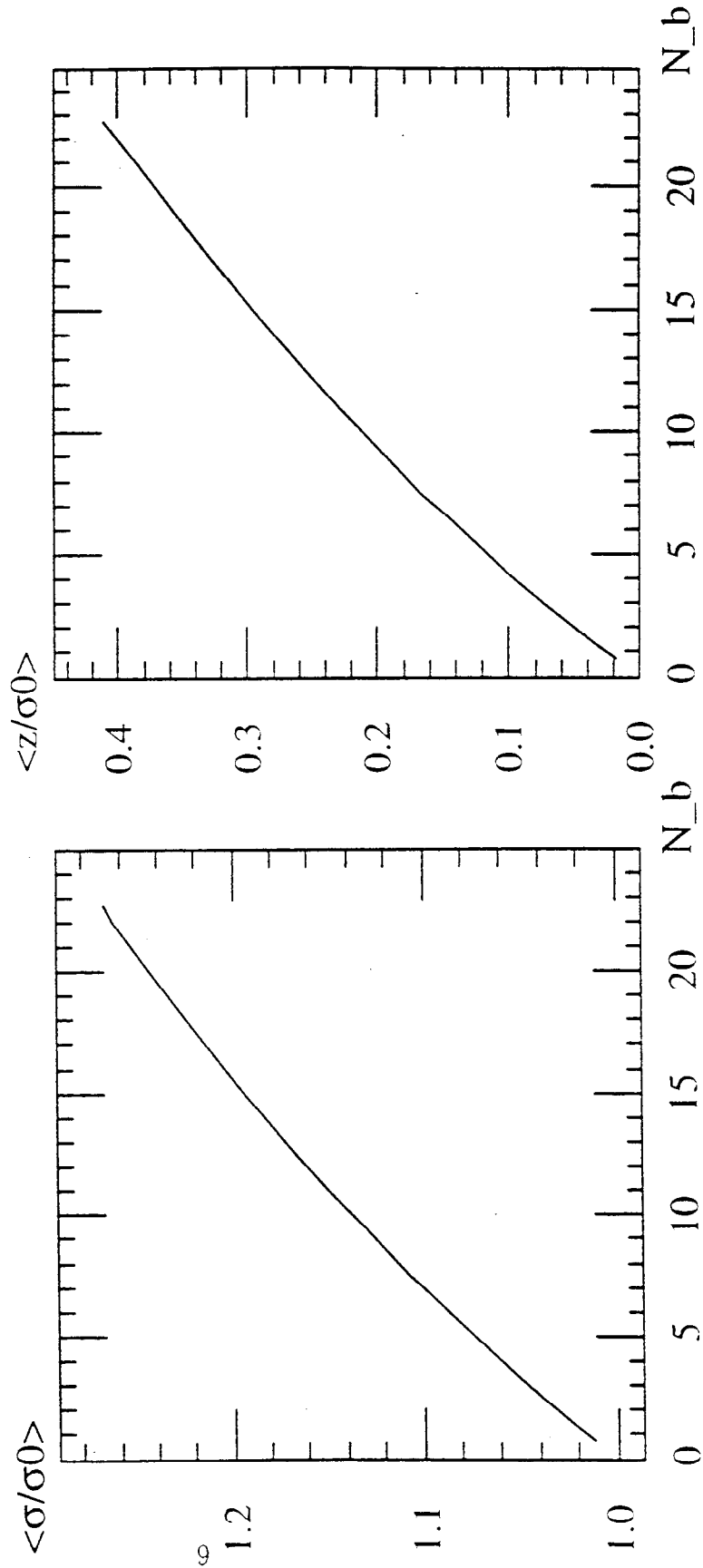


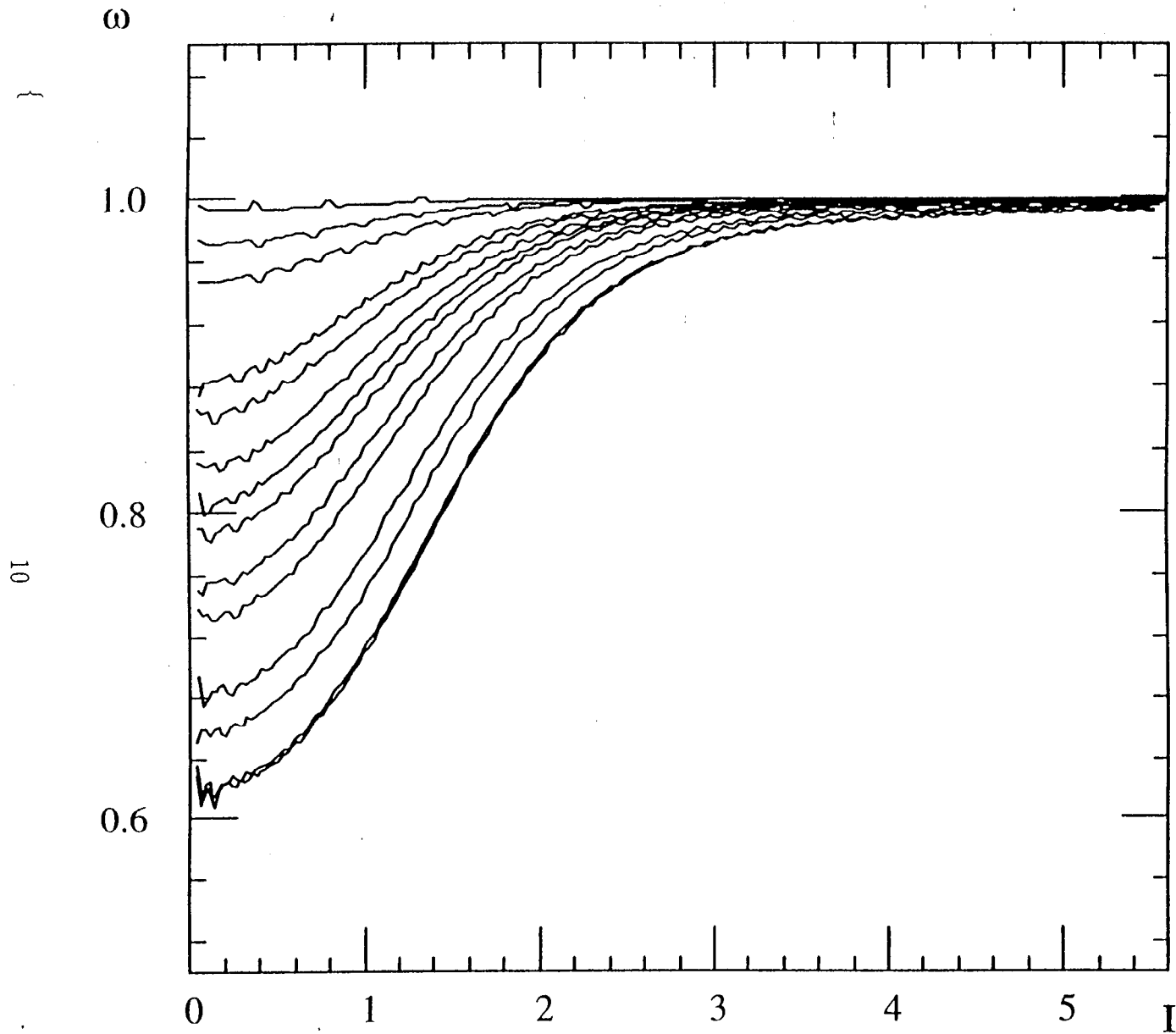
$U$



$x$

Fig. 5





$N = .154$	$\omega_0 = 0.993$
.725	0.970
1.36	0.946
3.35	0.881
4.14	0.861
5.46	0.830
6.56	0.804
7.50	0.784
9.40	0.751
10.9	0.732
15.0	0.690
17.7	0.653
22.0	0.652
22.7	0.652

Fig. 5

## 4 Bunch Stability

The bunch stability was studied with the code TRISIM. The typical results of TRISIM are shown in Fig. 7 and Fig. 8 at the bunch current  $I_b = 1.8$  mA (total current 3 A). Both longitudinal and transverse wakes were taken into account. No feedback system was turned on. The chromaticity of the ring  $\xi = (1/\nu_y)d\nu_y/d\delta$  was set to  $\xi = 0.05$ .

INPUT FILE NAME Iersum (part 1) RUN DATE 7/ 6/ 96 TIME 12. 28. 48

Distributions are approximated by linear interpolation (8.333 ps step)  
 Longitudinal wake switched ON - Transverse wake switched ON

Number of particles 2000 Damping time (s) 0.03 Beam energy (GeV) 3.105  
 Number of turns 20000 Energy spread (MeV) 2.394 Radiation loss (MeV) 0.75

Bunch current (mA)	1.8	Betatron tune	36.642
Total RF Voltage (MV)	5.1	Synchrotron tune	0.034

Equilibrium values (averaged over 5000 turns)

Bunch center (ps) -90.126 Bunch length (ps) 34.428 Bunch width (mm) 0.164  
 Mean energy (MeV) -0.375 Energy spread (MeV) 2.546 Total losses (MeV) 1.358

Total CPU time (s) 620.476 Number of particles lost 0

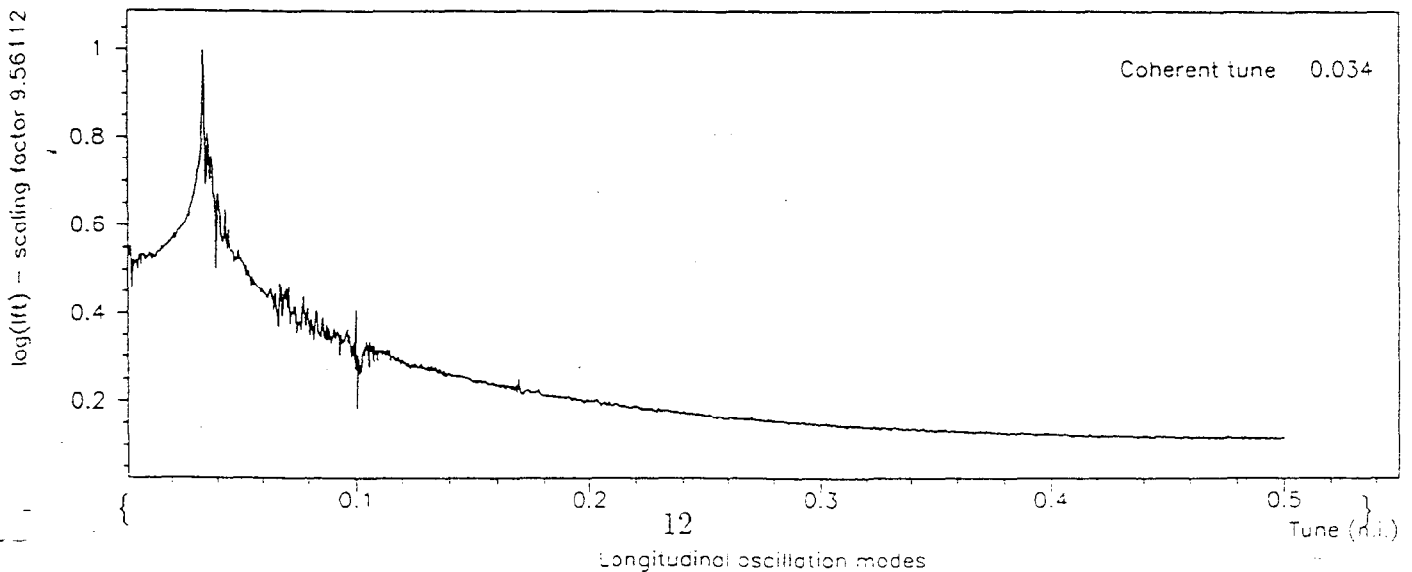
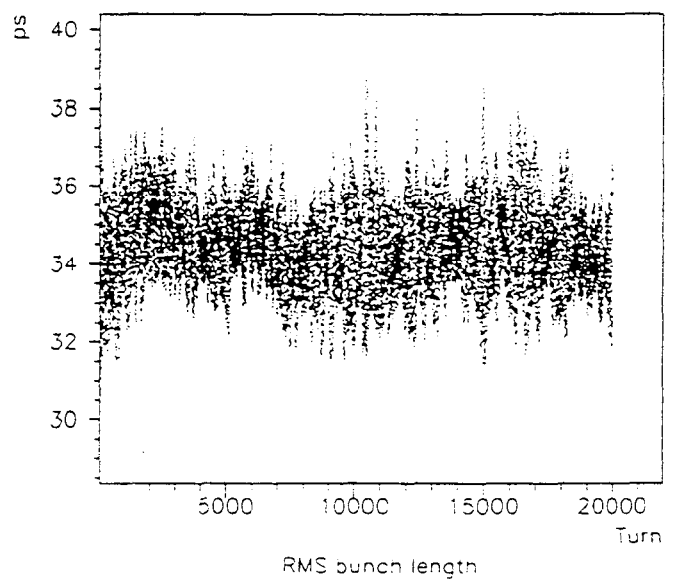
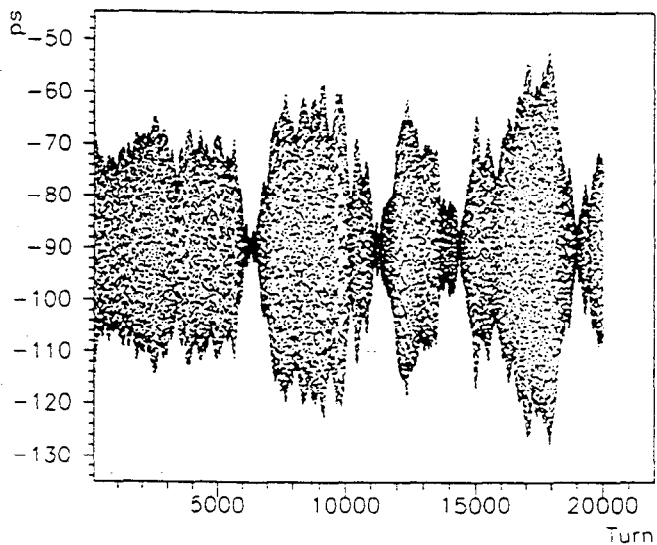
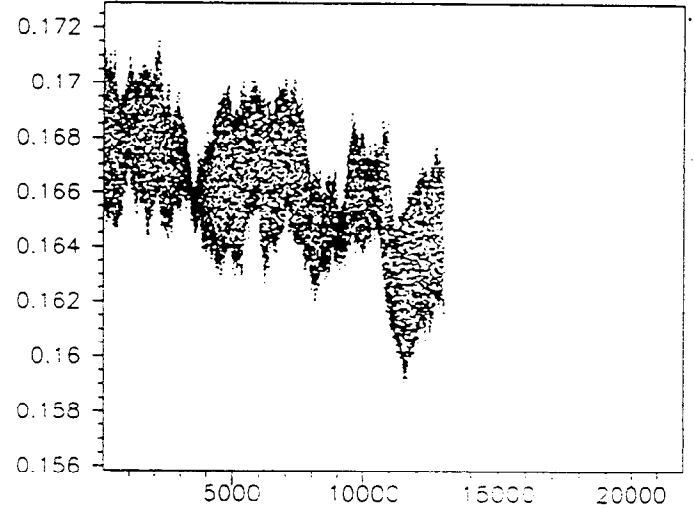
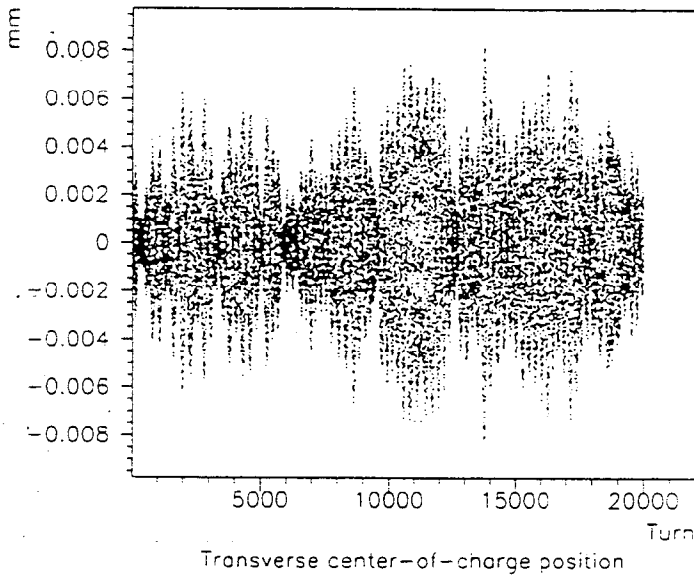


Fig 8



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TRISIM - SIMULATION OF COHERENT INSTABILITIES AT LER

Fig. 9a

Offset-mm vs time-ps

RUN DATE 6/ 6/ 96 TIME 20. 45. 13

Distributions are approximated by linear interpolation in steps of 8.333 ps

Longitudinal wake switched ON

Number of particles

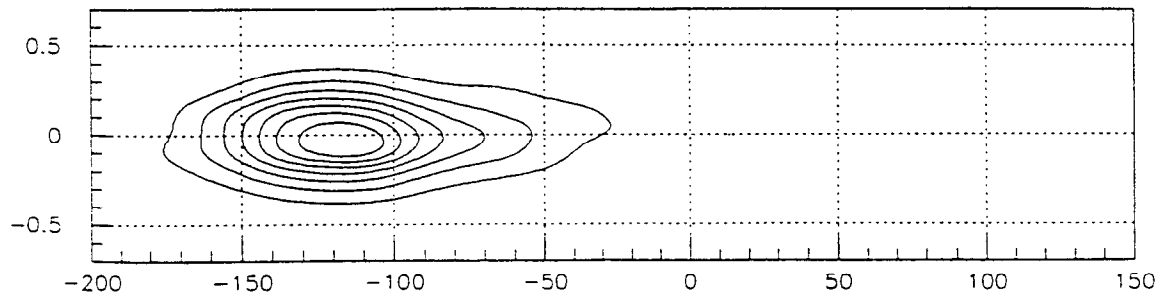
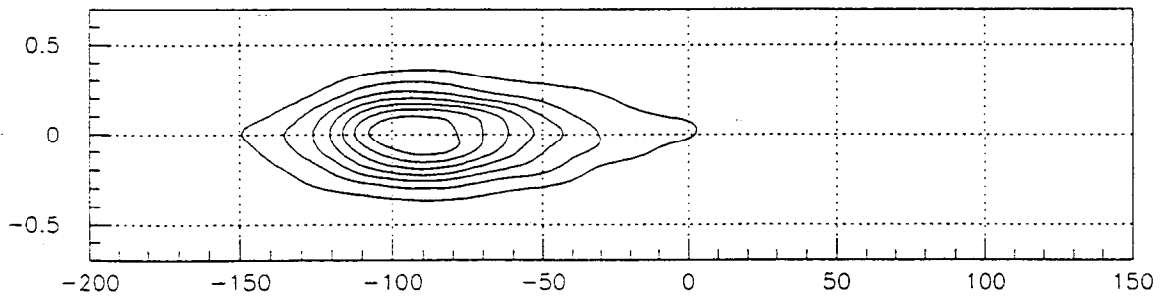
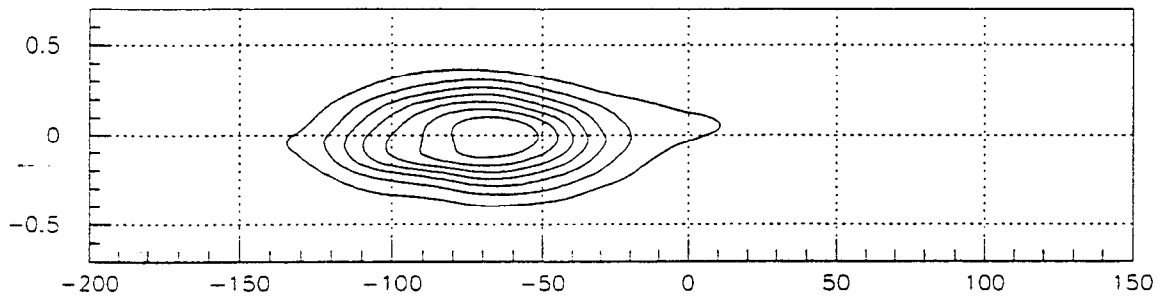
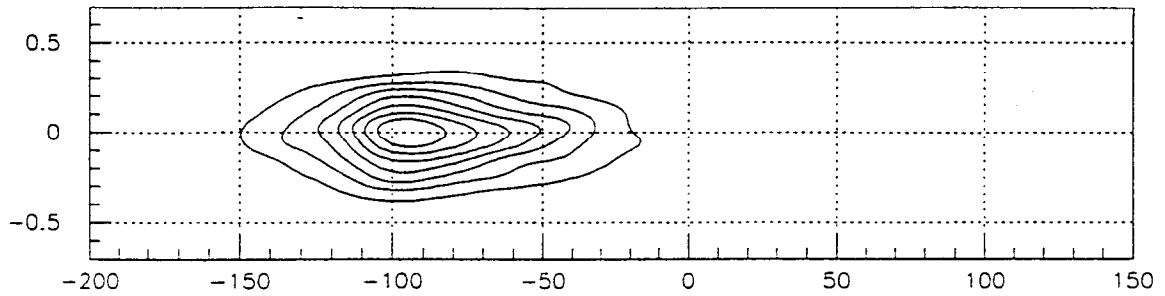
2000

Transverse wake switched ON

Current per bunch (mA)

1.8

BUNCH CONFIGURATION AT TURNS 9950 9955 9960 9965



TRISIM - SIMULATION OF COHERENT INSTABILITIES AT LER

Fig 96

Offset-mm vs time-ps

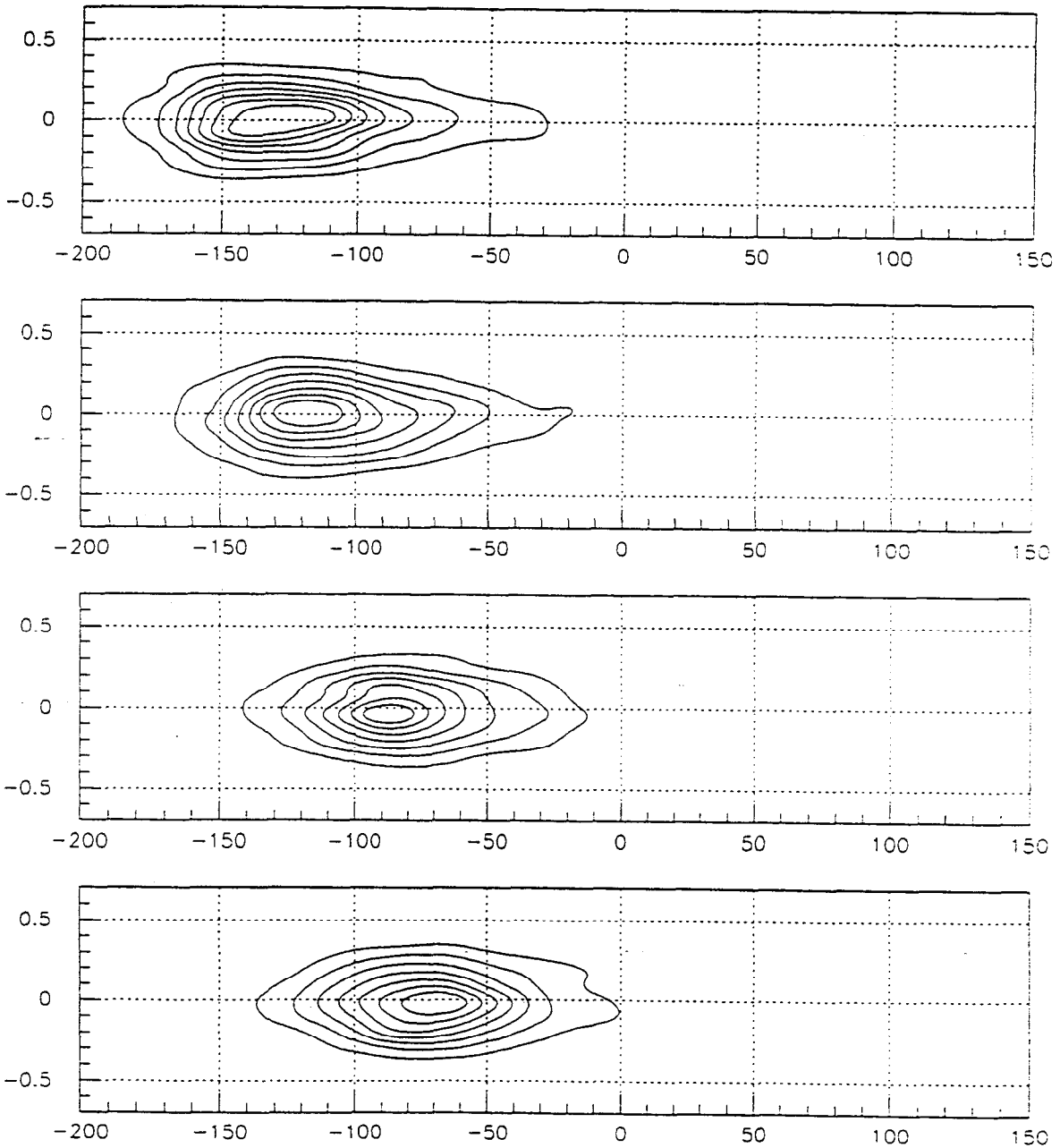
RUN DATE 6/ 6/ 96 TIME 20.45.13

Distributions are approximated by linear interpolation in steps of 8.333 ps

Longitudinal wake switched ON Number of particles 2000

Transverse wake switched ON Current per bunch (mA) 1.8

BUNCH CONFIGURATION AT TURNS 9970 9975 9980 9985



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The bunch is stable: no particles were lost after  $2 \times 10^4$  revolutions (about  $5\tau_E$ ) but the bunch centroid oscillates with irregular and large amplitude variations which reaches sometimes 40 ps. The rms bunch length increases at this current to  $34.22 \pm 1.0$  ps, or by 8%, and the energy spread grows to 2.54 MeV. The spectrum of the longitudinal motion has a large peak at the synchrotron frequency  $\nu_s = 0.034$  and a smaller peak at  $2\nu_s$ . The spectrum of the transverse motion, see Fig. 8, indicates a large betatron peak at the fractional betatron tune  $0.64 = 1 - 0.363$ . At the positive chromaticity there are synchrotron sidebands with the amplitude strongly dependent on  $\xi$ . Fig. 9 shows bunch profiles in  $(y,z)$  plane at each 5-th revolution after few damping times. The oscillations of the bunch centroid with synchrotron period are quite noticeable, but the distortion of the bunch remains relatively small.

A bunch remains stable at  $I_b = 1.9$  mA but the amplitude of the centroid oscillations increases. Finally, at  $I_b = 2.0$  mA, bunch becomes unstable due to rapidly growth of the centroid oscillations with simultaneous growth of the rms bunch length, see Fig. 10. The spectrum of the transverse motion in this case is shown in Fig. 11

Fig. 10

INPUT FILE NAME lersum (part 1) RUN DATE 5/ 6/ 96 TIME 15. 40. 26

Distributions are approximated by linear interpolation (8.333 ps step)  
 Longitudinal wake switched ON - Transverse wake switched ON

Number of particles 2000 Damping time (s) 0.03 Beam energy (GeV) 3.105  
 Number of turns 12000 Energy spread (MeV) 2.394 Radiation loss (MeV) 0.75

Bunch current (mA)	2	Betatron tune	36.642
Total RF Voltage (MV)	5.1	Synchrotron tune	0.034

Equilibrium values (averaged over 2000 turns)

Bunch center (ps) -92.8 Bunch length (ps) 70.402 Bunch width (mm) 0.165  
 Mean energy (MeV) -0.401 Energy spread (MeV) 4.777 Total losses (MeV) 1.397

Total CPU time (s) 268.019 Number of particles lost 0

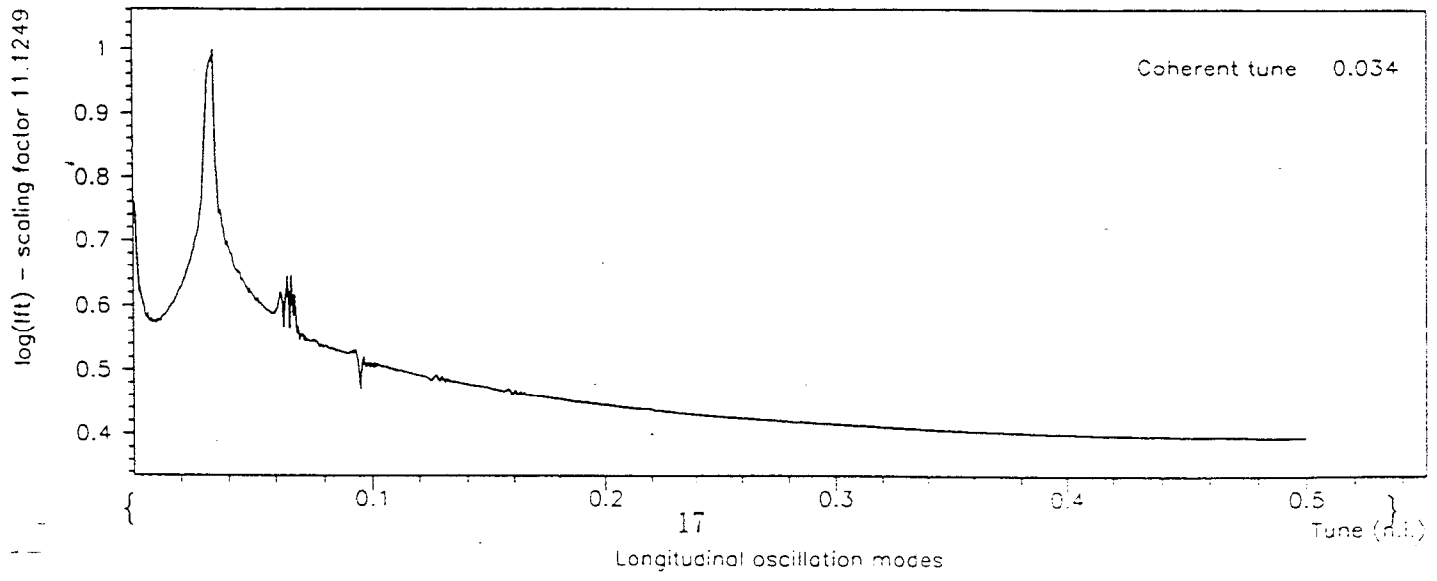
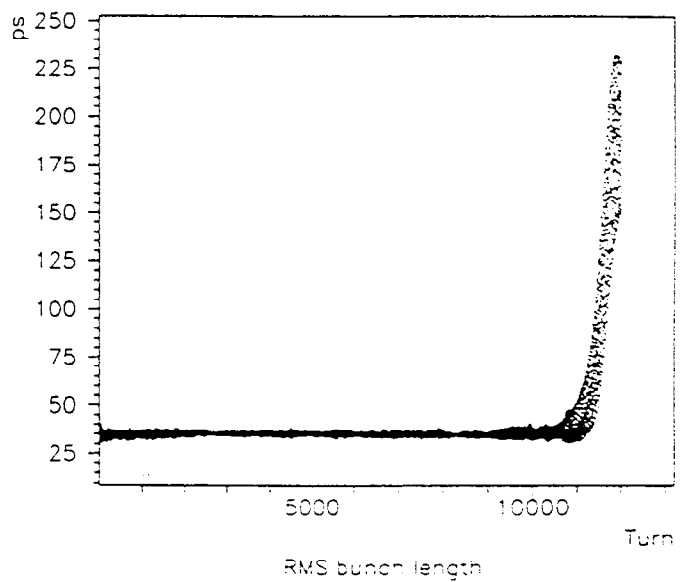
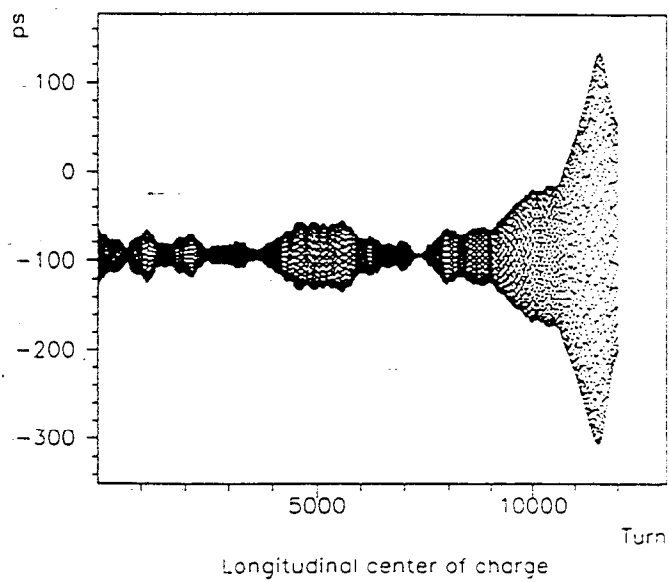


Fig. 11

INPUT FILE NAME lersum (part 1) RUN DATE 5/ 6/ 96 TIME 15. 40. 26

Distributions are approximated by linear interpolation (8.333 ps step)  
 Longitudinal wake switched ON – Transverse wake switched ON

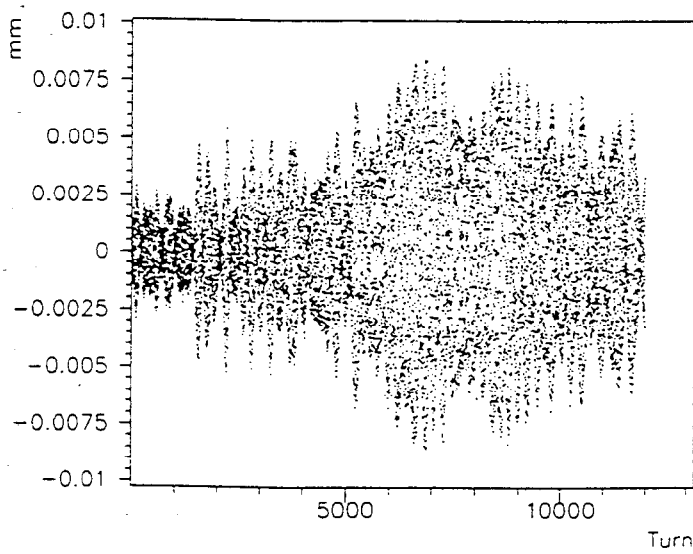
Number of particles 2000 Damping time (s) 0.03 Beam energy (GeV) 3.105  
 Number of turns 12000 Energy spread (MeV) 2.394 Radiation loss (MeV) 0.75

Bunch current (mA)	2	Betatron tune	36.642
Total RF Voltage (MV)	5.1	Synchrotron tune	0.034

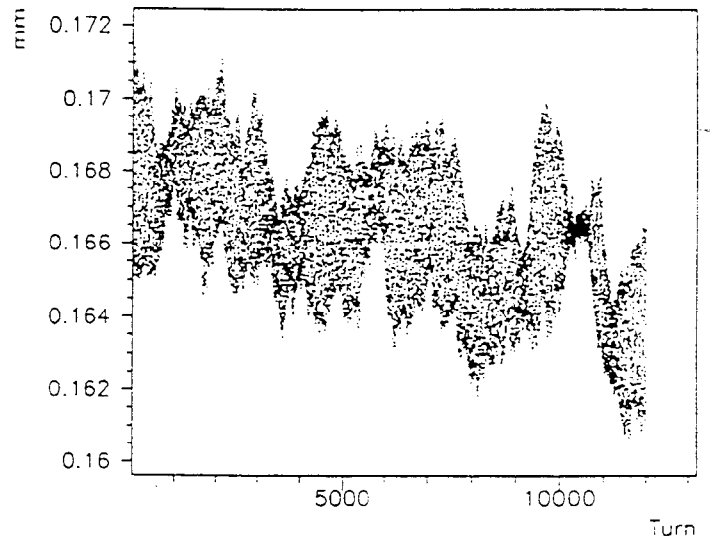
Equilibrium values (averaged over 2000 turns)

Bunch center (ps) -92.8 Bunch length (ps) 70.402 Bunch width (mm) 0.165  
 Mean energy (MeV) -0.401 Energy spread (MeV) 4.777 Total losses (MeV) 1.397

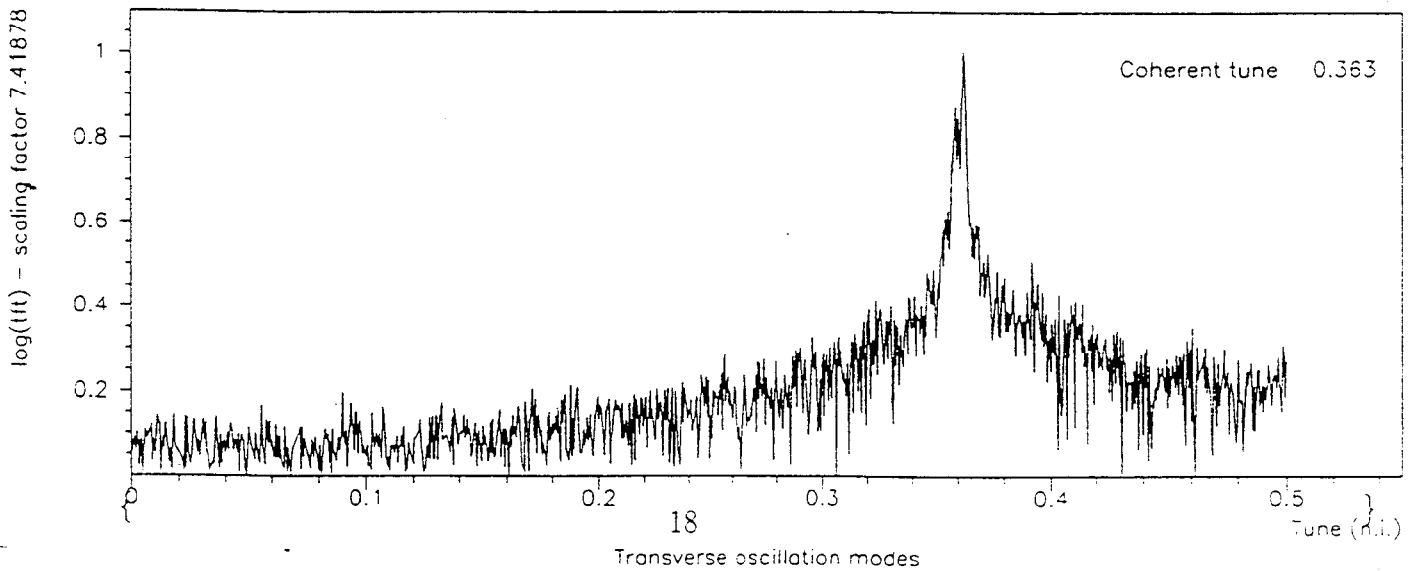
Total CPU time (s) 268.019 Number of particles lost 0



Transverse center-of-charge position



Vertical bunch width RMS



Transverse oscillation modes

Fig. 12 shows the dependence of the position of the bunch centroid  $\langle x \rangle$  in ps, of the rms bunch length  $\sigma$  in ps, and the energy spread  $\langle \delta E \rangle$  in MeV vs bunch current in mA. Each point corresponds to one run of TRISIM. Fluctuations increase at large currents. The growth of the energy spread is noticeable above 1 mA indicating onset of the microwave instability.

These results has been checked and confirmed with the number of macro-particles increased up to  $10^4$ . The instability occurs as a result of increasing longitudinal oscillations of the bunch centroid rather than due to oscillations of the bunch shape. Tracking with large aperture shows recurrences in the bunch blow-up: at the 2 mA bunch current, bunch slowly blows up during 8000-9000 turns, then shrinks in 100-200 turns to a small size, and blows up again. The process repeats itself several times during 40000 turns.

## 5 Transverse Stability

If the longitudinal wake is turned off and the chromaticity is positive, bunch remains stable up to very high currents but, at 12.5 mA, becomes unstable indicating the onset of the (strong) head-tail instability. The situation is drastically different with a negative chromaticity. At the zero or negative chromaticity and  $I_b = 1.8$  mA bunch is unstable. The growth rate at the same current and  $\xi = -0.05$  is shown in Fig. 13. The snapshots of the bunch profile for the same 1.8 mA and  $\xi = -0.1$  are shown in Fig. 14. {

Fig. 12

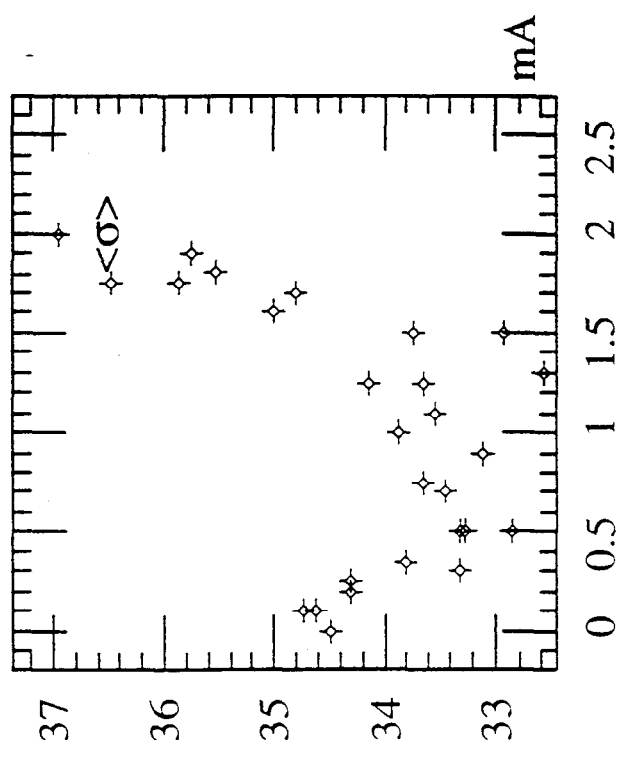
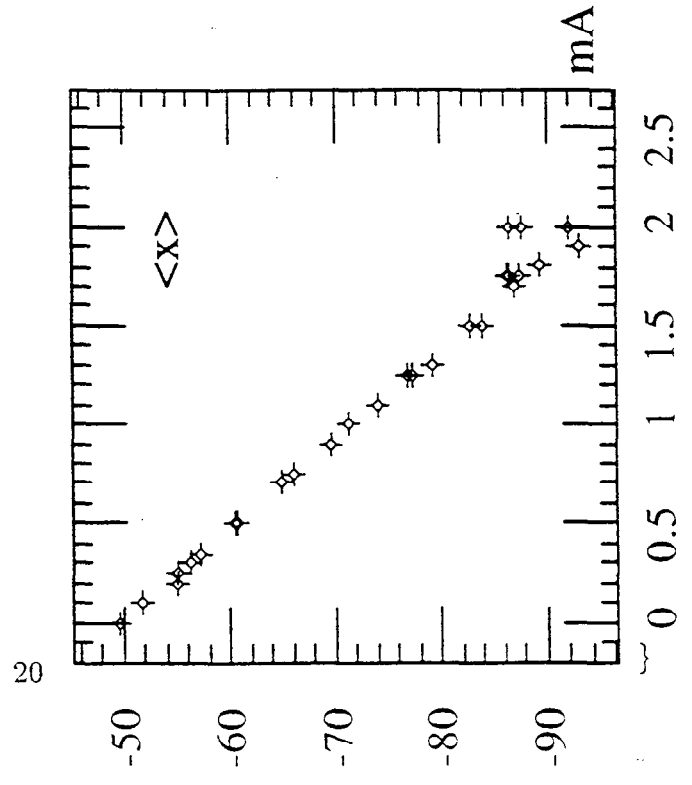
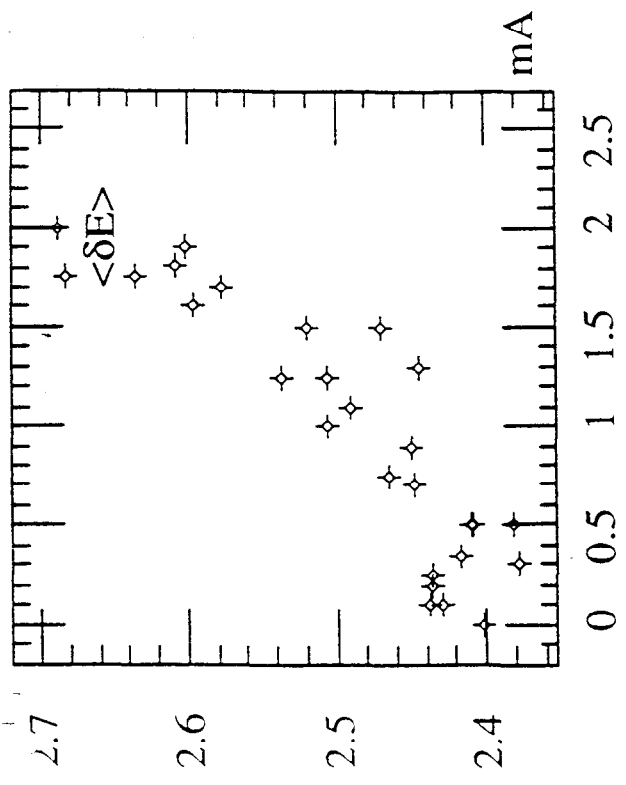


Fig. 13

INPUT FILE NAME lersum (part 1) RUN DATE 6/ 6/ 96 TIME 21. 0. 54

Distributions are approximated by linear interpolation (8.333 ps step)  
 Longitudinal wake switched OFF – Transverse wake switched ON

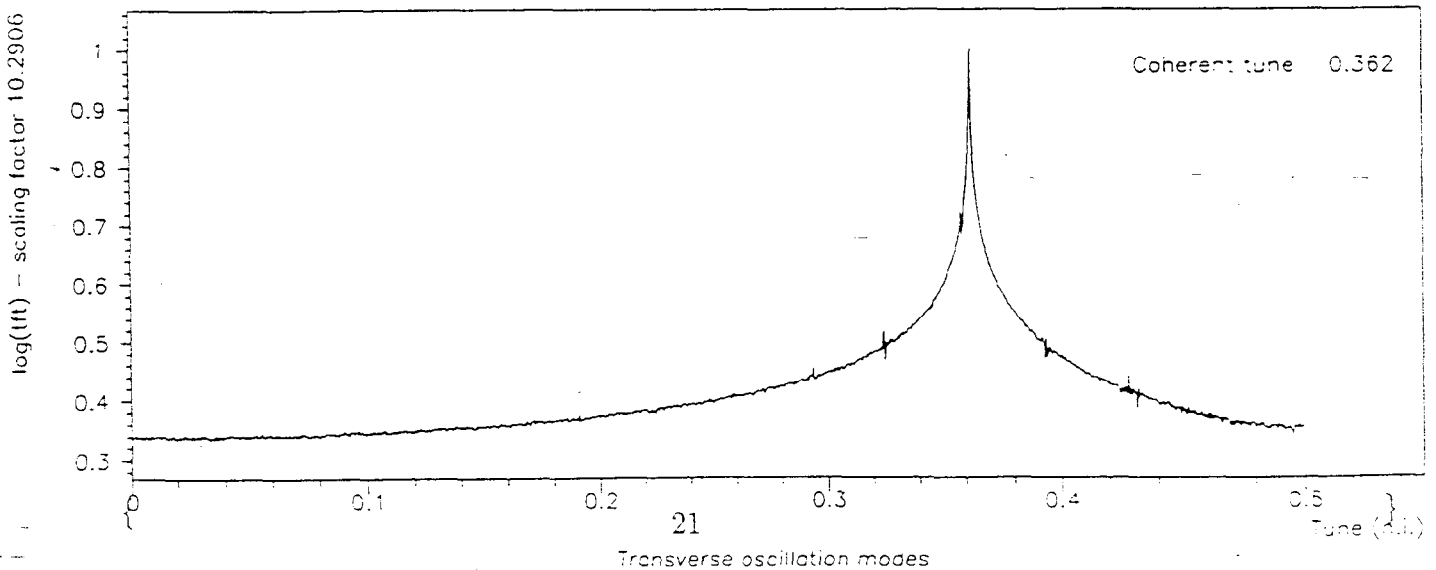
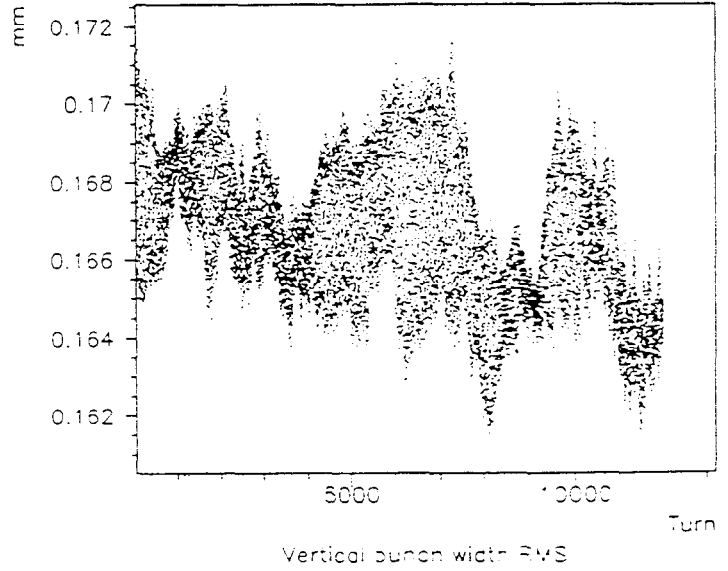
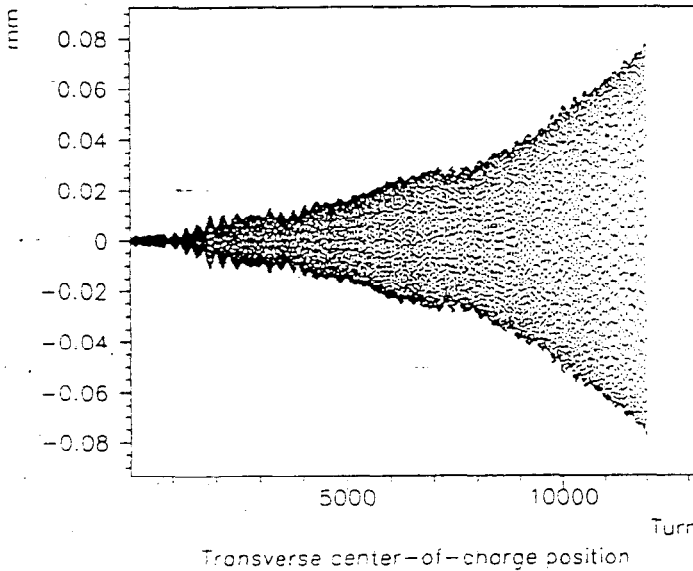
Number of particles 2000 Damping time (s) 0.03 Beam energy (GeV) 3.105  
 Number of turns 12000 Energy spread (MeV) 2.394 Radiation loss (MeV) 0.75

Bunch current (mA)	1.8	Betatron tune	36.642
Total RF Voltage (MV)	5.1	Synchrotron tune	0.034

Equilibrium values (averaged over 2000 turns)

Bunch center (ps) -49.626 Bunch length (ps) 34.324 Bunch width (mm) 0.165  
 Mean energy (MeV) -0.375 Energy spread (MeV) 2.39 Total losses (MeV) 0.754

Total CPU time (s) 363.899 Number of particles lost 0



TRISIM - SIMULATION OF COHERENT INSTABILITIES AT LER

Fig. 14a

Offset-mm vs time-ps

RUN DATE 6/ 6/ 96 TIME 21.37.49

Distributions are approximated by linear interpolation in steps of 8.333 ps

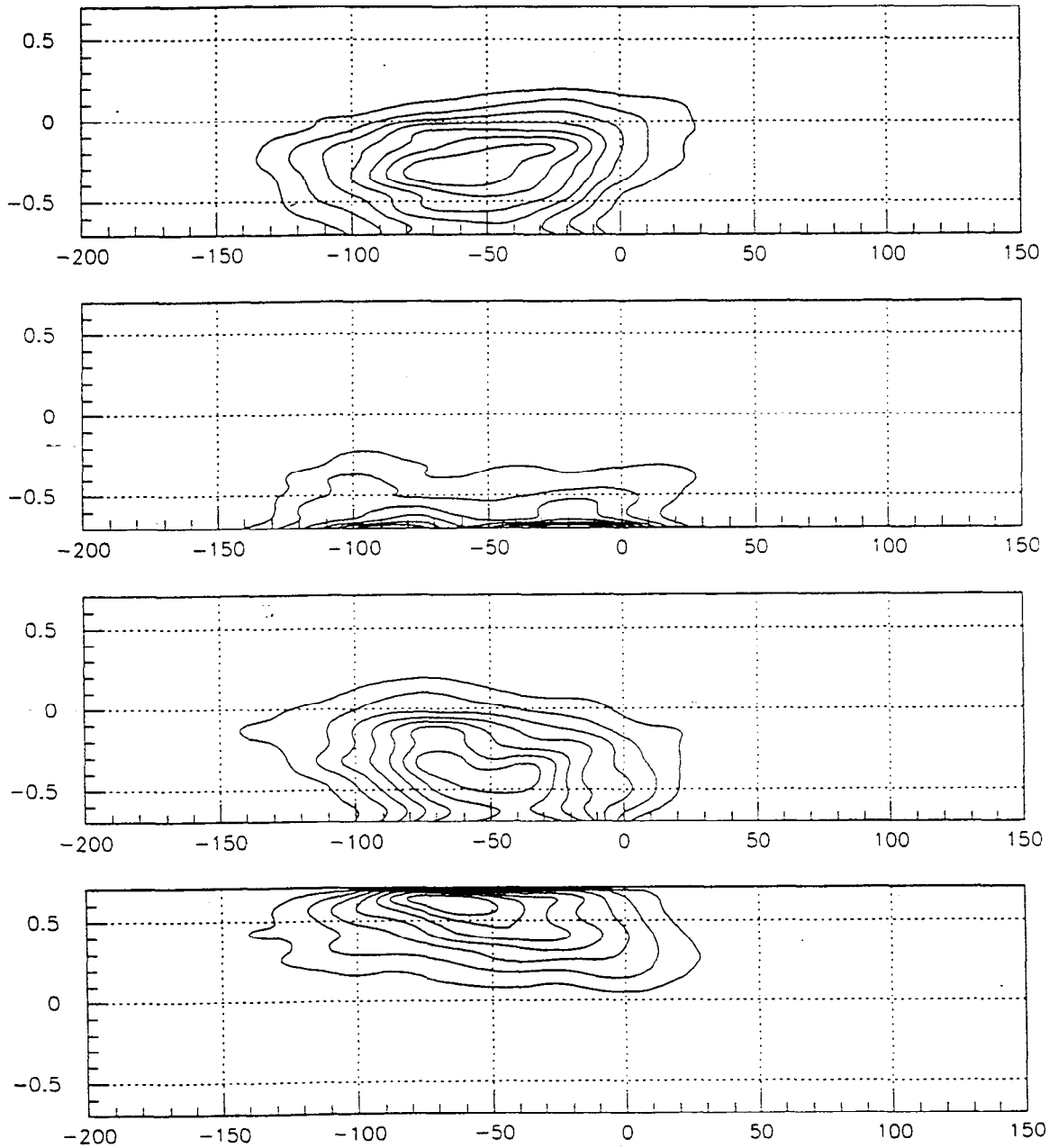
Longitudinal wake switched OFF

Number of particles 2000

Transverse wake switched ON

Current per bunch (mA) 1.8

BUNCH CONFIGURATION AT TURNS 9970 9975 9980 9985



## 6 Discussion

The growth of the energy spread, indicating the onset of the microwave instability, is noticeable at the bunch current above 1 mA. The growth of the energy spread and the rms bunch length is relatively small and not much different from the PWD results up to the bunch current 1.8 mA. However, the amplitude of the longitudinal bunch centroid oscillations increases and, at  $I_b = 2$  mA, bunch is already unstable, rms becomes very large, and particles get lost. It is interesting, that  $\omega(I)$  dependence at this current becomes flat at small amplitudes  $I$ . This might indicate that microwave instability starts when  $d\omega/dI = 0$ . It would be interesting to understand whether the longitudinal feedback system can increase the threshold of the microwave instability. The bunch remains transversely stable up to the  $I_b = 10$  mA at the zero or positive chromaticity. This corresponds to the previous estimates of the threshold of the head-tail instability. However, at the negative chromaticity the (weak) head-tail threshold is very low: bunch is unstable at  $I_b = 1.8$  mA and  $\xi = -0.05$ .

## 7 Acknowledgments

One of us (S.H.) thanks Cho Ng for his help with setting up the computer.

## References

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