

HOM LOSSES AT THE INTERACTION REGION OF THE B FACTORY*

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Masking at the Interaction Region (IR) will presumably reduce the synchrotron radiation background in the detector. One possible layout of the IR for the B factory, depicted in Fig. 1, shows a rather complicated system of masks. A bunch passing each mask will generate RF waves. These waves (usually called higher order modes, HOMs) will be absorbed in the beam pipe wall producing additional heating and, interacting with the beam, kicking particles in the radial and azimuthal directions. This may change the bunch motion and its emittance. These effects are estimated in the present note.

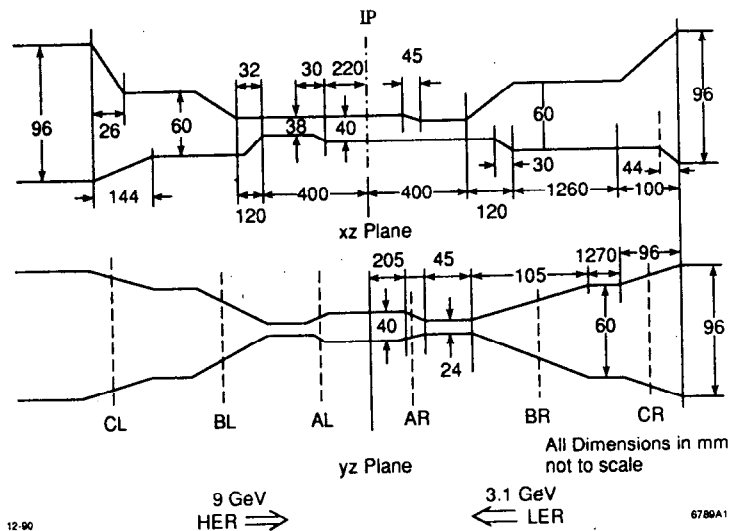


Fig. 1. Layout of the interaction region.

To start with, there are a few general comments. Masking is achieved by a system of asymmetric tapers (see Fig. 1 where all dimensions are given in millimeters, and all the taper angles equal 10°). Studying such a structure is

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rather difficult due to the lack of analytical and numerical methods. However, the additional effect of tapers cannot be too large, since the bunches have a substantial offset in the horizontal $x-z$ plane anyway. Hence, the real structure may be approximated by a cylindrically symmetric structure for which several analytical results are known and the numerical code TBCI is available. The cylindrically symmetric structure is henceforth assumed to have the dimensions of the real structure in the vertical $y-z$ plane.

Even after such a simplification, several difficulties still remain.

- TBCI, having a limited number of mesh points, cannot handle a structure with a very large aspect ratio (the ratio of the length to the radial dimensions). To get around this difficulty, a real structure is usually split into several sections which are then studied independently.
- However, that leaves the problem of taking into account the interference of the waves generated at different sections. The interference of waves tends to reduce the total loss. An example of that is an array of cavities for which the impedance per cavity is smaller than the impedance of a single cavity. Figure 2 illustrates this phenomena for two steps. The total loss decreases when the distance between the steps decreases. However, this effect is small provided that the distance between the steps is large in comparison with the bunch length. This condition is fulfilled in the case under consideration.
- TBCI has limited accuracy especially for a structure in which the radius of the downstream pipe is larger than that of the upstream pipe. We call such a structure "a taper-out" (in the opposite case, it is called "a taper-in"). To reduce errors in the calculations for a "step-out" structure, we notice that the difference of the longitudinal losses for a taper-out and a taper-in is a constant that depends only on the beam pipe radii:

$$k_i^{\text{out}} - k_i^{\text{in}} = \frac{2}{\sigma\sqrt{\pi}} \ln \frac{b}{a} \quad (1)$$

This constant is related to the difference of the energy of the field of a particle in pipes of different radii. Thus, it suffices to calculate the loss for a taper-in, and then use Eq. (1) to calculate the loss for the taper-out. Such an estimate is less sensitive to the uncertainty of the length of the beam pipe than that obtained by direct calculations for a taper-out.

The relation Eq. (1) is illustrated in Fig. 3. It has been shown in Ref. 1 and can be obtained analytically.²

- Finally, one needs to know if the radiated waves are absorbed in the vicinity of the interaction point (IP) or far away from it, where there is more room for cooling. Strictly speaking, waves which are generated at a discontinuity propagate both upstream and downstream of it. However, the diffraction model^{3,4} and numerical simulations⁵ show that the amplitude of a wave propagating upstream (downstream) from a step-out (step-in) is very small. This means that waves of the outward tapers will be absorbed outside of the middle Be pipe of the IP, and therefore are not dangerous. From this point of view, it would be possible to make the outward tapers steeper, but that would increase transverse kicks.

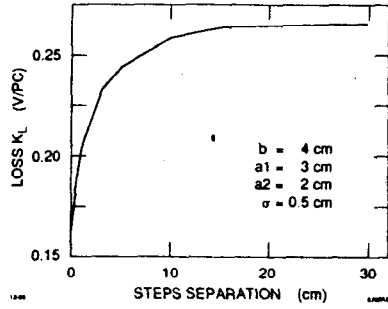


Fig. 2. Longitudinal loss for two steps $r = b$, if $z < 0$; $r = a_1$, if $0 < z < g$; $r = a_2$, if $g < z$; and $b > a_1 > a_2$ versus g .

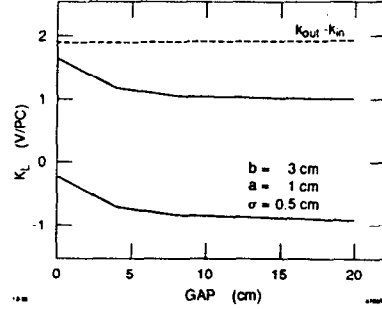


Fig. 3. The loss of a taper-out (upper solid line) and a taper-in (bottom solid line), and the difference (dashed line) $k_l^{out} - k_l^{in}$ versus the length of a taper.

Note that the constant in Eq. (1) is independent of the angle α of a taper. However, the sum

$$k_l^{out} + k_l^{in} = 2\Delta(\alpha) \quad (2)$$

related to the radiated energy, depends on α . For $\alpha = \pi/2$,

$$\Delta\left(\frac{\pi}{2}\right) \simeq \frac{1}{\sigma} \ln \frac{b}{a}$$

$\Delta(\alpha)$ goes to zero for small α . Thus, even a very long taper-out can reduce the loss by a factor two at most. The loss of a very long cavity tapered symmetrically on both sides goes to zero with $\alpha \rightarrow 0$. A substantial reduction of the loss can be achieved with a taper angle of order

$$\alpha \simeq \frac{\sigma}{b-a}$$

or less. The angle 10° used in the design (Fig. 1) satisfies this criterion.

The loss k_l defines the RF power radiated by the beam:

$$P = \sum_{\pm} 1.6 \times 10^{-7} N_B^{\pm} \left(\frac{I_{av}^{\pm}}{A}\right) \left(\frac{k_l}{V/pC}\right) [W] \quad (3)$$

For the B factory parameters

$$N_B = 7.88 \times 10^{10}, \quad I_{av} = 2.23 \text{ A [LER]} \quad ,$$

$$N_B = 5.414 \times 10^{10}, \quad I_{av} = 1.54 \text{ A [HER]} \quad ,$$

the power in the HOMs is

$$P = 41.5 \frac{k_l}{V/pC} [\text{kW}] \quad (4)$$

The calculations performed by TBCI give $k_l = 2.85 \times 10^{-3} \text{ V/pC}$ or $P = 120 \text{ W}$ for the middle tapered cavity with radius 2 cm; $k_l = 0.044 \text{ V/pC}$ or $P = 610 \text{ W}$ for the two inward tapers connecting pipes with radii $a = 1.2 \text{ cm}$ and $b = 3 \text{ cm}$; and $k_l = 0.044 \text{ V/pC}$ or $P = 1.8 \text{ kW}$ for the two outward tapers connecting pipes with radii $a = 3 \text{ cm}$ and $b = 4.8 \text{ cm}$. The total radiated RF power per IR is $P = 2.6 \text{ kW}$, and the power absorbed in the Be pipe is $P = 120 \text{ W}$. That corresponds to an energy loss per particle of 0.6 KeV for LER, and 0.4 KeV for HER. The most dangerous is power deposition into Be pipe which should not exceed 500 W. Table 1 gives parameters of the modes found by URMEL for the middle tapered cavity. Here are several modes with rather high R/Q [note that the loss $k_l = \frac{\sigma}{2}(R/Q)$] taken from this table.

TABLE 1

TM0-EE-5	$f = 5946.21 \text{ MHz}$	$R/Q = 0.144$	$k_l = 2.75 \times 10^{-3}$
TM0-EE-8	$f = 6283.47 \text{ MHz}$	$R/Q = 0.195$	$k_l = 3.83 \times 10^{-3}$
TM0-EE-9	$f = 6425.07 \text{ MHz}$	$R/Q = 0.171$	$k_l = 3.45 \times 10^{-3}$

These frequencies are well below the cutoff frequency 9561.88 MHz. The total loss estimated by URMEL for these modes alone exceeds by a large factor the loss that is found by TBCI for all modes. This is probably the result of the low accuracy of the calculations with URMEL for such a long structure. However, URMEL indicates that all these modes are trapped and will be absorbed in the Be pipe.

It is well known that the wake field of a train of equally spaced bunches can be substantially different (enhanced under the resonance conditions) from the wake field of a single bunch. That is true also for the energy loss. The enhancement factor for a mode with the wave length λ and the quality factor Q depends on the bunch spacing s_B :

$$F(x, y) = \frac{\sinh x}{\cosh x - \cos y}$$

where $x = ks_B = 2\pi s_B/\lambda$ and $y = ks_B/Q$.

In the resonance where $ks_B/2\pi \approx$ integer, the enhancement factor has the resonance structure:

$$F = \frac{(n\pi/Q)}{\left(\frac{ks_B}{2\pi} - n\right)^2 + \left(\frac{n}{2Q}\right)^2}$$

Q factor of the trapped HOMs can be very high, of the order of $Q \approx (a/\delta)$ where δ is the skin depth. For example, the parameter $ks_B/2\pi = (25 - 0.016)$ is close to an integer $n = 25$ for the TM0-EE-5 mode ($f = 5946.21$ MHz) and for the train of bunches filling each other bucket at $f_{RF} = 476$ MHz.

Taking $\delta = 1.17 \times 10^{-4}$ cm, we obtain $Q \approx 1.7 \times 10^4$ for $a = 2$ cm Be pipe. That only gives $F = 0.9$. However, the enhancement can be large in the unlucky situation with a mode hitting a resonance. The problem has to be reconsidered carefully when final design of the masks will be set.

The power deposited at the IR should be compared with the average power deposited in the ring. Some of this power can propagate to the IR, producing additional heating unless special absorbers preventing propagation of the modes to the IR are installed. To estimate this loss, we use the following model of the frequency dependence of the longitudinal impedance:

$$\frac{Z(n)}{n} = \begin{cases} R_l, & \text{if } \omega \leq \omega_{cutoff}; \\ R_l \left(\frac{\omega_{cutoff}}{\omega}\right)^{3/2}, & \text{otherwise.} \end{cases}$$

Here $n = \omega/\omega_{rev}$, where $\omega_{rev} = c/R$ is the revolution frequency, and the cutoff frequency $\omega_{cutoff} = c/a$ depends on the beam pipe radius a . The loss

$$k_l = 4 \int_0^{\infty} dk e^{-k^2 \sigma^2} \operatorname{Re} \left(\frac{Z(\omega)}{Z_0} \right), \quad Z_0 = 377 \Omega,$$

is dominated by the contribution of the high frequency tail:

$$k_l = \frac{1}{60} \left(\frac{R_l}{\Omega} \right) \left(\frac{R}{a} \right) \frac{1}{\sqrt{a\sigma/\text{cm}^2}} \left[\frac{\text{V}}{\text{pC}} \right]$$

Take $R_l = 0.5 \Omega$, $R = 350$ m, $\sigma = 1$ cm, $a = 5$ cm. Then $k_l = 26$ V/pC, which corresponds to the power $P = 1.08$ MW/ring. Multiplying this power by the ratio of the length of the Be pipe (40 cm) to the circumference of the ring $2\pi R = 2200$ m, we get 200 W.

The ohmic loss is much smaller:

$$\frac{dP}{dz} = \frac{2}{3\pi} \left(\frac{e^2}{a} \right) \frac{N_B^2 f}{\sigma_{Be} Z_0 \sigma \delta},$$

where δ is the skin depth at the maximum bunch frequency $c/2\pi\sigma$, σ is the rms bunch length, $\sigma_{Be} = 3.1 \times 10^5 \Omega^{-1} \text{cm}^{-1}$ is the conductivity of the Be. That gives $dP/dz = 0.187$ W/cm, or 7.5 W deposited in the Be pipe.

The transverse kick k_{\perp} defines the average transverse kick for a particle with an offset r_{off} :

$$\langle \Delta p_{\perp} \rangle = N_B e^2 r_{off} k_{\perp}$$

This can be translated as the focal length F of an effective quadrupole:

$$\frac{1}{F} = \frac{(lB')_{eff}}{B\rho},$$

where

$$(lB')_{eff} = eN_B c k_{\perp} = 5.4 \times 10^{-13} N_B \left(\frac{k_{\perp}}{\text{V/pC/cm}} \right) [\text{T}],$$

or to a deflection angle

$$\theta = \frac{\langle \Delta p_{\perp} \rangle}{p} = \frac{r_{off}}{B\rho} (lB')_{eff} \quad (5)$$

The horizontal offset of the reference trajectory at the position $l_z = 1830$ mm from the IP is 3 mm for HER and 8 mm for LER. The transverse kicks for each taper are given in Table 2. Eq. (5) gives the deflection angle $\theta = 3.6 \times 10^{-6}$, which is smaller than the angular divergence of the beam

$$\theta_x = \frac{\sigma_x}{l_z} = 10^{-4}, \quad \theta_y = 4. \times 10^{-6}, \quad (6)$$

provided that the beam offset in the vertical plane is much smaller than that in the horizontal plane.

TABLE 2

Tapers	Cl	Bl	Al	Ar	Br	Cr
k_r	-0.048	-0.483	+0.49	-0.38	+0.063	+0.077
k_{ϕ}	-0.014	-0.084	-0.056	-0.058	-0.074	-0.017

The transverse wake function found by TBCI is equal to zero at the head of a bunch, and reaches a maximum at the tail of the bunch. The maximum value of the wake function in all cases is not more than two times larger than the average value which defines the transverse kick. Therefore, according to Eq. (6), the effect of the transverse kick on the beam emittance is small. The azimuthal kick is smaller than the radial: $k_{\phi}/k_r = 1/5$. The data on the transverse wakes at each taper are summarized in Table 3.

TABLE 3

	W_ϕ min/max	W_r min/max	k_ϕ	k_r	k_l
Cl	-0.066/ 3.1×10^{-3}	-0.374/ 9.49×10^{-7}	-0.0146	-0.048	0.290
Bl	-0.16/0.008	-1.31/ 5.48×10^{-6}	-0.084	-0.483	+0.427
Al	-0.109/0.018	-1.1 $\times 10^{-5}$ /1.015	-0.056	0.49	-0.33
Ar	-0.11/0.007	-1.02/ 3.9×10^{-6}	-0.058	-0.381	0.331
Br	-0.149/0.0132	2.2×10^{-6} /1.3	-0.0744	0.63	-0.43
Cr	-0.08/0.027	4.0×10^{-5} /0.384	-0.0167	0.0769	-0.33
Al+Ar	-0.25/0.024	-0.013/0.21	-0.128	0.114	-4.8×10^{-3}

CONCLUSION

The present design of the IR masking is acceptable from the point of view of the HOM loss.

I appreciate the useful discussions with H. DeStaebler.

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