

SLAC/AP - 51
April 1986
(AP)

SOME PROBLEMS ON RF BREAKDOWN IN
ROOM TEMPERATURE ACCELERATOR
STRUCTURE, A POSSIBLE CRITERION*

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* Work supported by the Department of Energy, contract DE - AC03 - 76SF00515.

1. Introduction

For many years, the accelerator community has been interested in knowing the high-breakdown limits of RF accelerator structures. In the absence of much experimental data, accelerator designers have relied on the well-known Kilpatrick criterion¹ which predicts that these limits are a function of the RF frequency and electrode geometries for the CW regime or very long pulses. Recent experimental results show that the Kilpatrick criterion indeed could be exceeded considerably. Experiments done at Varian^{2,3} on “half” single cavities and at SLAC⁴ on a standing-wave $2\pi/3$ mode linac structure indicate that an accelerating field of up to 150 MV/m and peak surface field of up to 300 MV/m can be obtained for a few μs pulses in s-band RF case. The evidence is that these limits do not seem to be very sensitive to choice of metal as long as reasonable quality control and cleanliness precautions of cavity surfaces are observed. Careful and gradual RF processing enables one to bring these structure up to a fairly well defined gradient limits above which breakdown invariably occurs. Also, the studies at Los Alamos,⁵ and in Germany⁶ reveal similar facts.

There are many factors to be discussed concerning the voltage breakdown phenomenon: field emission, absorption, positive ion bombardment, electron multipacting, microparticle charge acquisition, electromechanical detachment and non-metallic field emitting microregimes, etc. We will confine the discussion to high gradient, room-temperature accelerators, which have clean well-finished cavity surfaces and good vacuum conditions ($10^{-6} - 10^{-9}\tau$). We will simplify the analysis without loss of important physics effects.

At very low gas pressure electron ionization becomes inadequate to cause a breakdown; which means that to the first order, vacuum breakdown is pressure-independent. Furthermore, in an RF field the energy gain of an ion is very small. The equation of motion along the field direction x for a charged particle in an

RF field can be expressed as

$$m \frac{d^2 x}{dt^2} = eE_0 \sin(\omega t + \phi) , \quad (1)$$

where m and e are the mass and charge of the particle, E_0 and ω are the peak field strength and angular frequency of the RF field. By solving the equation (1), the maximum energy gain \mathcal{E}_{max} is

$$\mathcal{E}_{max} = \frac{2e^2}{m} \left(\frac{E_0}{\omega} \right)^2 . \quad (2)$$

For example, at 2856 MHz and $E_0 = 300$ MV/m the maximum energy gain of a hydrogen ion is only 0.05 MeV. For other types of ions and for more arbitrary starting phases. \mathcal{E}_{max} would be much less. Thus, ion bombardment can almost be ignored. The electron multipacting can be suppressed if the peak electrical field E_0 satisfies the following condition:⁷

$$E_0 > \frac{m\omega^2 d}{2e} , \quad (3)$$

where d is the cavity length. Actually the field gradients, that we are interested in are much higher than those shown in (3) above.

In this note we focus on the breakdown-initiating mechanism due to “cold” field electron emission occurring at isolated sites on broad-area cavity surfaces, where the field is locally enhanced. The influences of an alternating field and transition time of tunneling are taken into account. Finally the thermal instability resulting in vacuum voltage breakdown is hypothesized to derive a new criterion for room-temperature accelerator structure.

2. Kilpatrick Criterion; It's Derivation and Availability to Accelerator

In 1957, Kilpatrick summarized several investigators' experiences, data and theories. Considering the probability of field-emitted electrons together with the linear dependence of secondary-electron emission upon ion energy, the following equation was given to specify an upper limit for no vacuum sparking:

$$WE^2 \exp(-1.7 \times 10^5/E) = 1.8 \times 10^{14} , \quad (4)$$

where W is the maximum possible ionic energy in electron volt, and E is the electric cathode gradient in V/cm. The relationship of $W \sim E$ is presented graphically as Fig. 1.

For RF the maximum possible ionic energy is related to transit-time and phasing effects. Kilpatrick studied the case of parallel plates electrodes with gap g . Figure 2 shows how W depends on RF voltage V with frequency f . The quantity V^* is defined by

$$V^*(g/\lambda)^2 (m_0c^2/\pi q) \quad (5)$$

where $\lambda = \lambda/2\pi$ and q is the electron charge, M_0 is the mass of atomic hydrogen. The threshold vacuum sparking data for RF are in region $V/V^* \leq 1$. The slope in this region from Fig. 2 is approximately 0.61, then we have

$$W = \frac{0.153 q E^2}{M_0 \pi f^2} . \quad (6)$$

By substituting W of Eq. (6) into Eq. (4), the relationship of RF frequency in MHz and maximum electrical field in MV/m is built as follows:

$$f = 1.64 E^2 \exp\left(-\frac{8.5}{E}\right) . \quad (7)$$

From the derivation of the above formulae, they were not intended to be a criterion for RF breakdown in accelerators, which have varieties of cavity shapes.

Also the modern vacuum equipments cavity surface machining and processing techniques are much improved compared to those of thirty years ago. No doubt further study is necessary.

3. Field Electron Emission from Metal

Following Sommerfeld, Fowler and Nordheim formulated the quantum mechanical tunneling of conduction electrons through the modified potential barrier at the a surface of metal in an applied electric field.^{8,9} the one-dimensional barrier is illustrated in Fig. 3. The effective potential energy $V(x)$ is

$$V(x) = \begin{cases} -W_a & \text{where } x < 0 \\ -eEx - \frac{e^2}{4x} & \text{where } x > 0 \end{cases} , \quad (8)$$

the term $-\frac{e^2}{4x}$ is the contribution of the interaction with image charge. The tedious mathematical treatments can be outlined as follows. It is assumed that the conduction electrons in metal form a gas of free particles, which have Fermi-Dirac statistics. The number of electrons within volume v and with momenta within the indicated range $dP_x dP_y dP_z$ can be expressed by

$$dn = \frac{2v}{h^3} \frac{dP_x dP_y dP_z}{e^{\frac{\epsilon - \zeta}{kT}} + 1} , \quad (9)$$

where h is Planck's constant, k is Boltzmann's constant, ζ is called Fermi energy and T is temperature. The flux of electrons that have normal energy components between W_x and $W_x + dW_x$ and incident upon the barrier within the metal is $N(W_x)dW_x$. $N(W_x)$ is supply function which can be found by integrating Eq. (9) over all P_y and P_z . The probability of penetration of the barrier $D(W_x)$ is called the transmission coefficient. By WKB approximation, we can have the solution for the time-independent Schrödinger equation:

$$D(W_x) = e^{-\int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}[V(x)-W(x)]} dx} , \quad (10)$$

where x_1 and x_2 are the zeros of the radicand. The total number of electrons that tunnel through the barrier is obtained by integrating the product of these

functions $N(W_x)D(W_x)dW_x$ over all energies. Ordinarily the lowest energy $-W_a$ is far below the Fermi energy. By putting the lower limit at $-\infty$, the field emission current density is

$$j_F = e \int_{-\infty}^{\infty} D(W_x) N(W_x) dW_x . \quad (11)$$

For low temperature ($T \leq 300^\circ\text{K}$) it had the form:

$$j_F = \frac{1.54 \times 10^{-6} E^2}{\phi t^2(y)} \exp \left[\frac{-6.83 \times 10^9 \phi^{1.5} v(y)}{E} \right] \text{ (Amp/cm}^2\text{)} , \quad (12)$$

where E is the surface electrical field in V/m, ϕ is the work function of the emitting material in eV, $t(y)$ and $v(y)$ are tabulated dimensionless elliptic functions, where $y = 3.79 \times 10^{-5} E^{0.5}/\phi$ and $v(y) = 0.956 - 1.062 y^2$. Substituting them into (12) and let $t(y) \approx 1$. The Fowler-Nordheim equation has the following form:

$$j_F = \frac{1.54 \times 10^{-6} \times 10^{4.52} \phi^{-0.5} E^2}{\phi} \exp \left(-\frac{6.53 \times 10^9 \phi^{1.5}}{E} \right) \text{ (Amp/cm}^2\text{)} . \quad (13)$$

As a practical consideration we introduce ideas of localized emission sites and enhancement factors. Any metal surface in accelerator cavities is far from planar and clean, there are large variations in the microscopic surface field. Local enhanced field would be created on some protruding microstructures, e.g. microprotrusions. The idealized microprotrusion geometries can be ellipsoid cylinders, needle-like shapes or other forms. By convergence, the microscopic field E is geometrically enhanced to a higher microscopic value E_m at the tip of the protrusion:

$$E_m = \beta E . \quad (14)$$

The ratio of E_m and E is defined as the field-enhancement factor. Which depends on the geometries and dimensions of a given protrusion.

The external measurable parameter is field emissions current I_F instead of current density j_F . By introducing the emitting area A_e , Eq. (13) can be written in the following form:¹⁰

$$\frac{d(\log_{10} I_F/E^2)}{d(1/E)} = -\frac{2.84 \times 10^9 \phi^{1.5}}{\beta} . \quad (15)$$

This is the slope for a Fowler-Nordheim plot of $\log_{10} I_F/E^2$ versus $1/E$, and the intercept is

$$\log_{10} (I_F/E^2)_{E \rightarrow \infty} = \log_{10} \left[\frac{1.54 \times 10^{-6} A_e \beta^2 10^{4.52\phi^{-0.5}}}{\phi} \right] . \quad (16)$$

Numerical calculation of the effects of random emitter arrays on measured Fowler-Nordheim plots based on the total field emission current have been carried out^{11,12} and these indicate that in general, a straight line fit can be expected, the slope of which approximates the β value of the sharpest emitter. The determined emitting area corresponds to a value somewhat less than that associated with a value computed for all the emitters.

There are more hypotheses proposed to explain the enhanced emissions: oxide films and associated trapped positive surface charges, partially embedded non-metallic impurity particles, absorbed particles on metal surface possessing a special band structure to enhance tunneling, etc. Some of these hypotheses predict that the field emission current would have the characteristic Fowler-Nordheim form while others predict a different relationship. But under clean condition the pre-breakdown current has been established as obeying the Fowler-Nordheim equation according to many experimental results. It is acceptable to use effective β as a judgment for the field enhancement degree of a certain accelerator cavity surface. This enhancement can be due to electrical field enhancement by microprotrusions or due to current increase based on other mechanisms mentioned above. Still the Eq. (15) is a very important theoretical basis for experimental measurements of β value.

4. Field Electron Emission for Alternating Field

We suppose the surface field in an accelerator cavity is $E = E_0 \sin \omega t$ and also assume that the field emission happens at a very short period of time compared to the RF period T . The average field emission current can be obtained by time-averaging:

$$\begin{aligned} \bar{I}_F &= \frac{1}{T} \int_0^T I_F(t) dt \\ &= \frac{1.54 \times 10^{-6} A_e \beta^2 10^{4.52} \phi^{-0.5}}{\phi} \frac{2}{T} \int_0^{\frac{T}{4}} \sin^2 \omega t e^{-\frac{6.53 \times 10^9 \phi^{1.5}}{\beta E_0 \sin \omega t}} dt . \end{aligned}$$

We integrate by approximation as Ref. 13, let

$$S(\mu) = \frac{2}{T} \int_0^{\frac{T}{4}} \sin^2 \omega t e^{-\frac{\mu}{\sin \omega t}} dt$$

with

$$\mu = \frac{6.53 \times 10^9 \phi^{1.5}}{\beta E_0} ,$$

and change the variables: $\sin \omega t = \frac{1}{x}$, then

$$\begin{aligned} S(\mu) &= \frac{1}{\pi} \int_1^{\infty} \frac{e^{\mu x}}{x^3 \sqrt{x^2 - 1}} dx \\ &= \frac{1}{\pi} \int_1^{\infty} R(x) \frac{e^{\mu x}}{\sqrt{x - 1}} dx \end{aligned}$$

with

$$R(x) = \frac{1}{x^3 \sqrt{x + 1}}$$

$$S(\mu) \approx \frac{1}{\pi} \tilde{R}(\mu) \int_1^{\infty} \frac{e^{\mu x}}{\sqrt{x-1}} dx = \frac{\tilde{R}(\mu)}{\sqrt{\pi\mu}} e^{-\mu} .$$

$\tilde{R}(\mu)$ is a slowly varying function of μ . For example, $\phi_{cu} = 4.65$ eV, $\beta E \sim 6 \times 10^6$ MV/m, $\mu \approx 11$, by using a computer the results of numerical integration show that corresponding to different conducting material and large variation of field strength, μ can change from 8 to 20, but $R(\mu)$ only changes from 0.55 to 0.57. So we can have the field-emission current for alternating field:

$$\bar{I}_f = \frac{6.02 \times 10^{-2} \times 10^{4.52\phi^{-0.5}} A_e (\beta E)^{2.5}}{\phi^{1.75}} e^{\frac{6.53 \times 10^9 \phi^{1.5}}{\beta E}} \text{ (Amp/cm}^2\text{)} , \quad (18)$$

where ϕ is the work function in eV, E is the surface field in V/m β is the enhancement factor and \bar{I}_F is the average field emission current on an emitting area A_e in m^2 .

Furthermore, we should consider the retardation of electrons in tunneling through a barrier. For quasi-stationary state single particle tunneling, there exists much discussion about this retardation or tunneling life time.¹⁴ The tunneling situation can be modeled as a time-dependent two-state system, in which the time-dependent basis states were taken to be localized states, ψ_s and ψ_k on each side of the barrier. When the perturbation V added, the two states couple through the overlap of their exponential tails in the barrier region. The system, known to have been in the initial discrete state s at $t = 0$, will be in the unperturbed final eigenstate $k \neq s$ at time t . The probability is then given by¹⁵

$$|C_k(t)|^2 = 2 |\langle \psi_k | V | \psi_s \rangle|^2 \frac{1 - \cos \omega_{ks} t}{(E_k - E_s)^2} , \quad (19)$$

where E_s , E_k are energies for states ψ_s , ψ_k and $\omega_{ks} = (E_k - E_s)/\hbar = \delta E/\hbar$. Let $D^*(t) = 1 - \cos \omega_{ks} t = 1 - \cos \frac{\delta E}{\hbar} t$, after a time $\tau = \frac{\pi \hbar}{2\delta E}$, we have $D^*(\tau) = 1$. It means that the electron would become a free electron. The δE is proportional

to e^{-kw} ,¹⁶ where w is the barrier width and k in the decay constant:

$$\delta E \propto e^{-\frac{2\sqrt{2m\phi}}{3k}} = e^{-\frac{3.42 \times 10^9 \phi^{1.5}}{\beta E}} .$$

For $E_k \approx E_s$ or $\omega_{ks}t \ll 1$,

$$D^*(t) \approx \frac{1}{2} \left(\frac{\pi t}{2\tau} \right)^2 , \quad (20)$$

i.e. the transition probability increases quadratically with time. In an actual RF field the tunneling is concentrating in the flat top near E_{max} , for instance, with an effective time for tunneling $\Delta t \approx 0.1/f$.¹⁷ By using Eqs. (18, 20)

$$\bar{I}_F = C^* \frac{(\beta E)^{2.5}}{f^2} e^{-\frac{1.34 \times 10^{10} \phi^{1.5}}{\beta E}} , \quad (21)$$

where C^* is a constant.

For a certain structure the slope for a plot of $\log_{10} I_F/E^{2.5}$ versus $1/E$ has the following form:

$$\frac{d(\log_{10} I_F/E^{2.5})}{d(1/E)} = -\frac{5.8 \times 10^9 \phi^{1.5}}{\beta} . \quad (22)$$

Therefore, this important parameter of β value can be obtained by performing field emission current measurements.

5. Breakdown Mechanisms and Criteria

The very high field emission currents flowing through the microprotrusions may cause excessive resistive heating. Then the localized melting or vaporization processes at the tip areas lead to the creation of a microplasma. This is the essential prerequisite for a breakdown event.¹⁰ Under good vacuum conditions this is the only origin of an ionizable medium. Several energy exchanges are involved by a high field emission current flowing into the tip of the emitter, thermal cooling by heat conduction down the shank of the protrusion into the “cold” reservoir provided by the bulk of the cavity body, thermal cooling by radiation losses from the surface of the emitter (which plays an insignificant role). Also there exist internal heating or cooling, the so called Nottingham effect. This effect, which can cause heating or cooling, arises because of a difference between the average energy of emission and the average energy of the electrons replacing the emitted electrons. In order to simplify the problem, consider a thermally steady-state solution for the truncated cone type of emitter as shown in Fig. 4. The difference between the top temperature T_r and the base temperature T_b of the emitter is¹⁸

$$\Delta T = T_r - T_b = \left[\Omega(T)/2K\pi^2 \right] (I_F/\alpha r)^2 + (kT_r I_F / K\alpha r) \cos \pi P \quad , \quad (23)$$

where I_F is the total emission current, $\Omega(T)$ is the electrical resistivity and K is the thermal conductivity of the emitter, k is Boltzman’s constant, $P \approx 9.3 \times 10^5 \phi^{0.5} T/E$. The first term of Eq. (22) is due to resistive heating, it increases rapidly with emission and temperature, the second term represents Nottingham effect. For less refractory metal such as copper, whose melting point is $\sim 1360^\circ\text{K}$, Nottingham heating is the dominating factor determining the temperature of microprotrusions.

Let T_r equal melting temperature and reasonably to assume αr proportional to $1/\beta$. According to the discussions above we can derive an RF breakdown

criterion as follows:

$$f = C \beta^{1.75} E^{1.25} \exp \left(\frac{-6.68 \times 10^9 \phi^{1.5}}{\beta E} \right), \quad (24)$$

C is a constant related to material characteristics such as work function, resistivity and thermal conductivity. The building materials for accelerator cavities and RF components usually are Cu, Al or material coated with Ag or Au, all of which have very common characteristics. For SLAC well-RF processed cavities the measured $\beta = 102$ and breakdown limit $E_{max} = 300$ MV/m were reached for 2856 MHz RF field. A possible RF breakdown criterion then can be written as:

$$f = 6.55 \times 10^{-3} \beta^{1.75} E^{1.25} \exp \left(\frac{-6.68 \times 10^9 \phi^{1.5}}{\beta E} \right) \quad (25)$$

where f is RF frequency in MHz. E is surface electrical field limit in V/m. ϕ is work function of cavity material in eV, and β is measured enhancement factor. For different β value, a group of curves are plotted in Fig. 5.

The Eq. (25) and curves in Fig. 5 have very clear physics meanings. For a certain RF frequency when RF processing was performed, the field emission current would be reduced gradually. Often β value is getting smaller, especially in the beginning of the processing. The sparking or breakdown threshold can be increased. This progress corresponds to a horizontal dashed line in Fig. 5. According to the microwave scaling law, lower frequency accelerators have relatively larger cavities, and condeminational areas are larger. Many experimental data showed this situation. Thus we can explain why the lower frequency accelerators have lower RF breakdown thresholds.

Acknowledgements

The author gratefully acknowledges the valuable advice and encouragement of Prof. G. A. Loew. Thanks are also due to Profs. P. B. Wilson, V. Nguyen-Tong and R. H. Miller for useful discussions.

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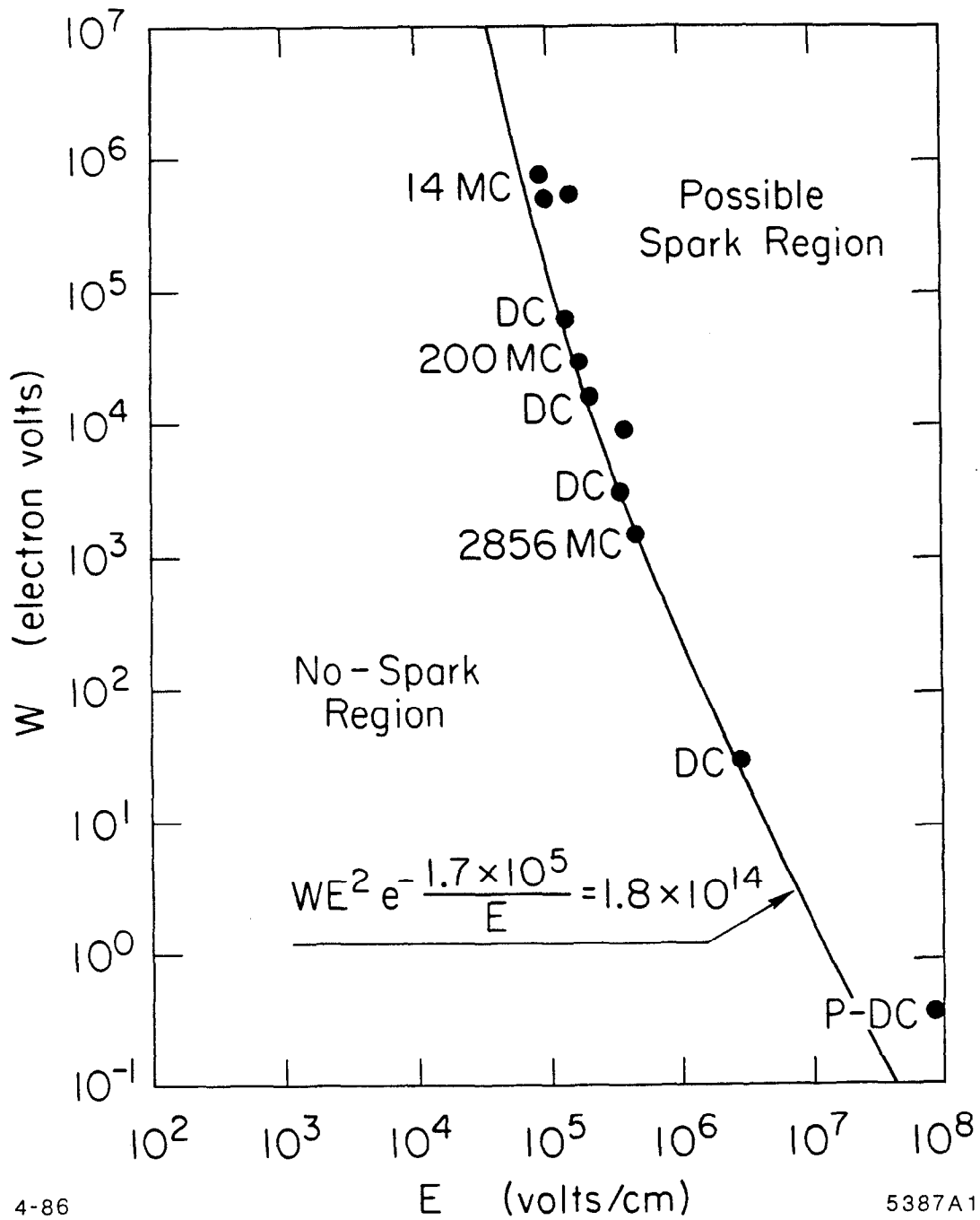


Fig. 1: W plotted against E , the cathode gradient. For DC, W corresponds to the applied voltage; for RF, W is a function of frequency and gap. (From Kilpatrick [1]).

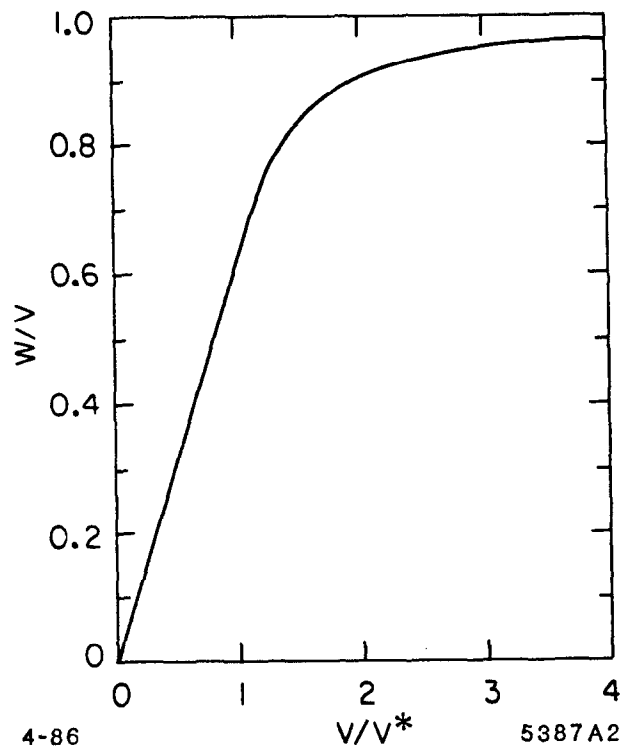
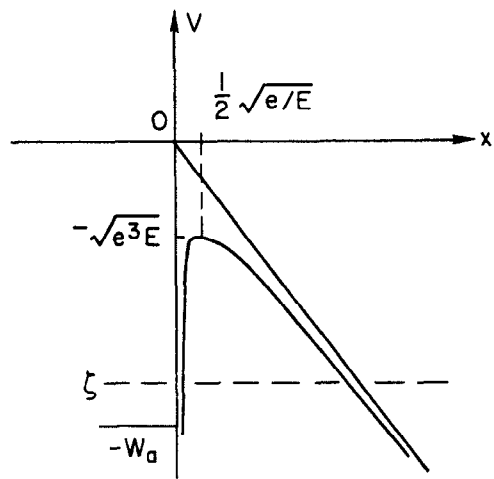


Fig. 2 : Graph for ion transit-time correction. For plane parallel fields, $V^* = (g/\lambda)^2 M_0 c^2 / \pi q$. For computing the maximum cathode ion energy, where $\nabla \cdot E$ is not ≈ 0 , the calculation of W or V^* is without reference. (From Kilpatrick [1]).



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Fig. 3 : Potential energy diagram.

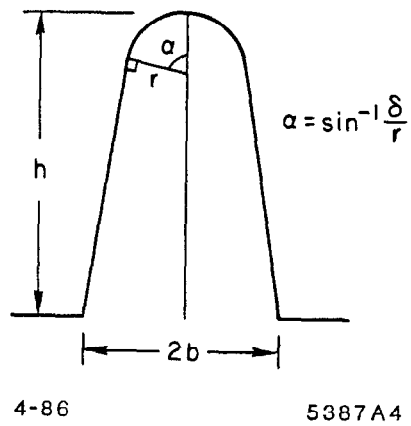


Fig. 4 : An idealized field-emitting microprotrusion.

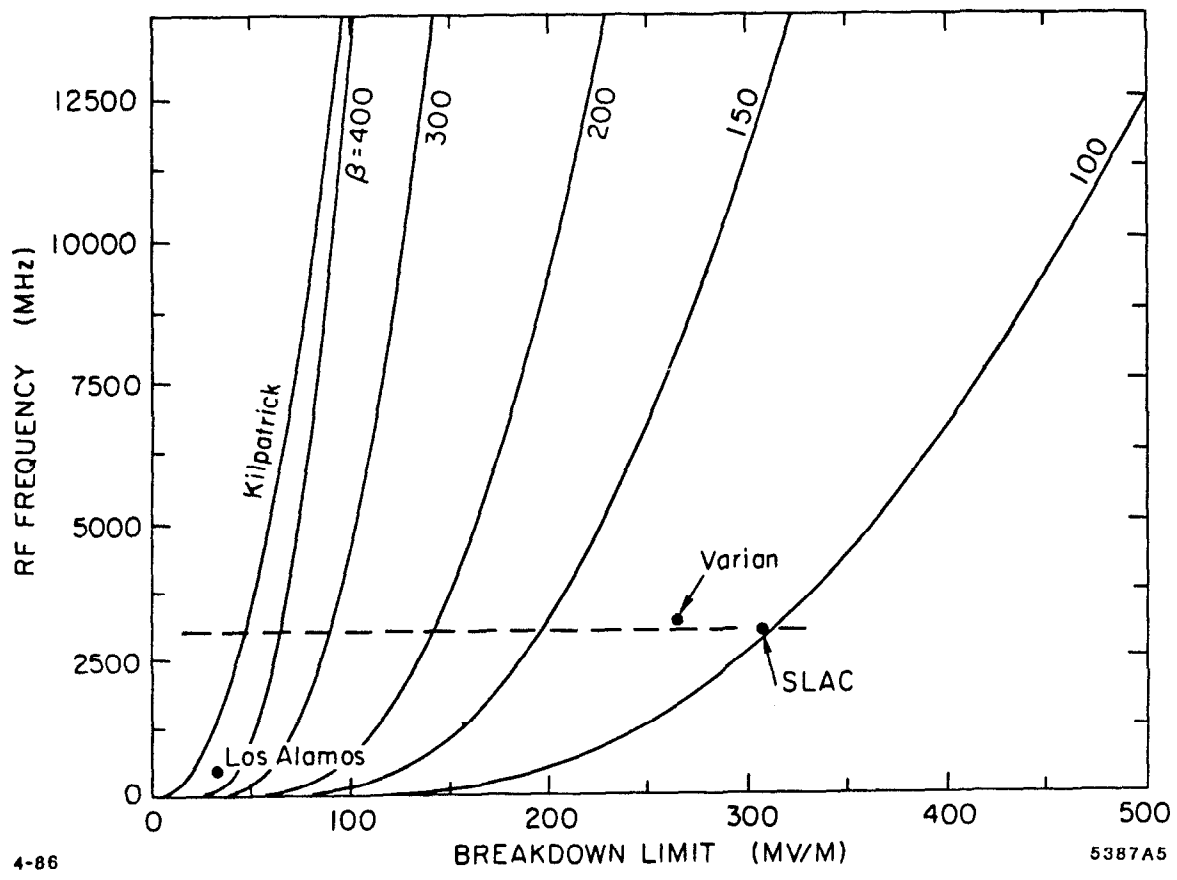


Fig. 5 : RF breakdown criteria and some experimental results.