SLAC/AP-7
September 1983
(AP)

ACCELERATOR PARAMETERS FOR AN e^+e^- SUPER LINEAR COLLIDER

RICHARD B. NEAL*
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

1. Introduction

This note summarizes the results of a brief study carried out in the Spring, 1983, concerning optimization of the parameters of a traveling-wave accelerator feeding an e^+e^- super linear collider. This work was part of a preliminary study of various aspects of super colliders by numerous SLAC people during the same period.

While the results obtained are generally valid for a wide range of accelerator characteristics and performance, the numerical examples given are based upon the assumptions shown in Table 1. A disk-loaded constant-gradient accelerator structure is assumed since its non-uniform radial dimensions along the length of each accelerator section make it less susceptible to transverse beam instabilities than a constant-impedance structure. Parameters of the accelerator sections used with the SLAC two-mile accelerator are used as the bases of design examples at the same frequency and scaling to other frequencies. The characteristics of four accelerator models meeting the beam requirements of Table 1 are given in Table 2.

2. Beam Energy and Beam Loading

The steady-state energy of a constant-gradient linac is given by[1]:

$$V = \left(P_{rf(PK)} L \bar{r}\right)^{1/2} \left(1 - e^{-2\tau}\right)^{1/2} - \frac{I \bar{r} L}{2} \left[1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}}\right] \tag{1}$$

where $P_{rf(PK)}$ is the total input peak rf power, L is the total length, $\bar{r}$ is the average shunt impedance of each accelerator section, τ (nepers) is the attenuation parameter of the accelerator sections, and I is the peak beam current. The first term on the right in Eq. (1) is the no-load energy, V_0, and the second term is the energy reduction, ΔV, due to beam loading.

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

In the design of linear colliders the beam pulse is generally short compared to the filling time, in which case the steady state condition represented by Eq. (1) is never reached. In this case, the beam loading is given by a transient term that replaces the beam loading term in Eq. (1). The transient beam loading is given by[2]:

$$\Delta V = \frac{I\bar{r}L}{2}\left[\frac{1 - 2\tau\frac{t_b}{t_F}\,e^{-2\tau} - e^{-2\tau\frac{t_b}{t_F}}}{1 - e^{-2\tau}}\right] . \tag{2}$$

For $t_b \geq t_F$, ΔV in Eq. (2) becomes identical to the steady-state beam loading term in Eq. (1). In this note, it is assumed that single bunch energy spread is negligible compared to the beam loading energy spread given in Eq. (2). Single bunch energy spread depends upon the bunch length, the number of particles per bunch, the accelerating gradient, and the wavelength.[3]

For $2\tau\frac{t_b}{t_F} \ll 1$, Eq. (2) can be written in the simplified form:

$$\Delta V \approx \frac{qL\omega}{2}\,\frac{\bar{r}}{\bar{Q}} \tag{3}$$

where $q = It_b$ is the charge accelerated in each beam pulse, which has the length t_b.

If it is desired that the design energy of the collider equal the fully beam-loaded energy V then the no-load energy of the accelerator must be

$$V_0 = V\left(1 + \frac{\Delta V}{V}\right) . \tag{4}$$

3. Peak rf Power Input to Accelerator

Substituting Eq. (4) in the no-load portion of Eq. (1) and rearranging we obtain the required peak rf power input to the accelerator:

$$P_{\substack{rf(PK) \\ (accel.)}} = \frac{V^2}{\bar{r}L(1 - e^{-2\tau})}\left[1 + \frac{\Delta V}{V}\right]^2 . \tag{5}$$

4. Beam Pulse Length and Bunch Separation

Two operating modes may be utilized depending upon the number of collider interaction regions used for conducting physics experiments.

If a single interaction region is used, beam transport to the experimental area is facilitated by having a narrow energy distribution in the output beam. A useful method of compensating for the beam-loading energy spread in the linac consists of injecting the beam before the accelerator is completely filled with rf energy. For a constant gradient accelerator, the energy decrement ΔV corresponding to beam injection at a time Δt before the accelerator is filled is given by (see Appendix A):

$$\frac{\Delta V}{V_0} = \left(\frac{2\tau}{e^{2\tau} - 1}\right) \frac{\Delta t}{t_F} \quad . \tag{6}$$

. Thus to compensate for beam loading ΔV, the beam can be injected Δt seconds before the accelerator is filled; the beam pulse continues until filling is complete and the increase in available voltage during the filling process compensates the energy loss due to beam loading. The beam pulse length is thus Δt and, if there are b bunches in the beam pulse, the bunch separation is $\Delta t/(b-1)$ seconds or $[\Delta t/(b-1)]\ f$ wavelengths, where f is the operating frequency.

An alternative operating mode is useful if the beam is to be divided and transported to several interaction regions. In this case, the design of the beam transport system is simplified by taking advantage of the beam-loading energy spread given in Eq. (2). The beam is injected at the end of the filling time and the rf pulse is extended beyond the filling time by an amount equal to the beam pulse length. The longer pulse requires more rf energy but the fractional increase is usually small since the beam pulse in collider applications is generally much shorter than the filling time. It is, of course, possible not to extend the rf pulse length, but in this case, the fact that the tail end of the rf pulse is moving away from the input end of the accelerator sections during the beam pulse will result in a further spread in beam energy by an amount (see Appendix B):

$$\frac{\delta V}{V_0} = \frac{2\tau}{1 - e^{-2\tau}} \frac{t_b}{t_F} \tag{7}$$

which is additive to the energy spread due to beam loading. Whether this total spread can be tolerated depends upon engineering and physics considerations. In energy calculations in this report, it will be assumed that the rf pulse has been lengthened by the amount t_b beyond that required to fill the accelerator.

It should be clear that either of the modes of operation described above can be selected simply by adjustment of the injector timing.

5. Total AC Power Requirement

The total AC power required is obtained by dividing the peak rf power given in Eq. (5) by the modulator, power source, and transmission line efficiency factors (M, K, and T, respectively) and multiplying by the duty factor, which is the product of the equivalent modulator pulse length and the pulse repetition rate, F. The equivalent modulator pulse length is the sum of the filling time, t_F, of the accelerator, the beam pulse length, t_b, (assuming that the beam pulse occurs *after* the accelerator is completely filled), and a time interval, t_r, which accounts for the rise and fall times of the modulator pulse.

The factor t_r is needed in determining total energy requirements to account for the fact that the modulator consumes energy during the rise and fall times * even though the rf pulse occurs only during the flat-top region of the pulse. A good approximation to the effective modulator pulse length can be obtained by measuring the pulse length at a voltage level that is $\sim$ 70% of the peak voltage. Thus, the factor t_r is $\sim$ 0.30 times the sum of the rise and fall times.

With the above considerations the total AC power required becomes:

$$P_{AC} = \frac{1}{MKT} \frac{V^2}{\bar{r} L(1 - e^{-2\tau})} \left(1 + \frac{\Delta V}{V}\right)^2 (t_r + t_b + t_F) F \quad . \tag{8}$$

We note that the filling time, t_F, in a constant-gradient structure is given by[1]:

$$t_F = \frac{2\bar{Q}}{\omega} \tau \quad . \tag{9}$$

6. rf Energy Disposition: Lost in Accelerator Walls; Converted to Beam Power; to rf Load

The rf energy stored in the accelerator after each filling time is given by[1]:

$$W_{st} = P_{rf(PK)}\, t_F \left(\frac{1 - e^{-2\tau}}{2\tau}\right) \quad . \tag{10}$$

---

* This is not necessarily true. For example, a grid-controlled or laser-controlled rf source could be designed to avoid such power consumption. However, no such devices exist at this time at the required power levels.

Thus the fraction of the input energy that is stored is:

$$F_{st} = \frac{1 - e^{-2\tau}}{2\tau} \quad . \tag{11}$$

Therefore, the fraction of the input energy that is lost in the accelerator walls during the filling time is:

$$F_{W1} = 1 - \frac{1 - e^{-2\tau}}{2\tau} \quad . \tag{12}$$

If there were no beam, the energy in the accelerator structure would propagate through the structure and the residual energy would enter the rf loads. In this case, the fraction of the input energy that is transmitted to the loads would be

$$F_L = e^{-2\tau} \quad . \tag{13}$$

After filling, in the process of passing through the structure, the fraction of the input energy that is lost in the accelerator walls is the fraction that is stored minus the fraction that goes to the load:

$$\begin{aligned} F_{W2} &= F_{st} - F_L \\ &= \frac{1 - e^{-2\tau}}{2\tau} - e^{-2\tau} \end{aligned} \tag{14}$$

The total fraction of the input energy that is lost in the walls is therefore

$$F_W = F_{W1} + F_{W2} = 1 - e^{-2\tau} \quad . \tag{15}$$

During actual operation, assume that the beam is turned on for a very brief period at the moment the accelerator is completely filled and that $t_b \ll t_F$. A fraction, $1 - [1 - (\Delta V/V_0)]^2$, of the stored energy is converted to beam energy. Thus in terms of the input rf energy

$$F_b \approx \left(\frac{1 - e^{-2\tau}}{2\tau}\right)\left[1 - \left(1 - \frac{\Delta V}{V_0}\right)^2\right] \quad . \tag{16}$$

The fraction of the energy lost in the walls (after filling) then becomes:

$$F_{W2} \approx \left[\frac{1 - e^{-2\tau}}{2\tau} - e^{-2\tau}\right]\left(1 - \frac{\Delta V}{V_0}\right)^2 \quad . \tag{17}$$

Thus, the total energy lost to the walls is:

$$F_W = F_{W1} + F_{W2} \approx 1 - \frac{1-e^{-2\tau}}{2\tau} + \left[\frac{1-e^{-2\tau}}{2\tau} - e^{-2\tau}\right]\left(1 - \frac{\Delta V}{V_0}\right)^2 \quad . \tag{18}$$

Similarly, the fraction of the energy transmitted to the load is:

$$F_L \approx e^{-2\tau}\left(1 - \frac{\Delta V}{V_0}\right)^2 \quad . \tag{19}$$

The addition of Eqs. (16), (18) and (19) shows that the necessary condition

$$F_W + F_b + F_L = 1$$

is satisfied. Examination of these equations indicates that for a given value of beam loading $\Delta V/V_0$, F_b and F_L are maximum and F_W is minimum as $\tau \to 0$. An accelerator design with a low τ is therefore advantageous in obtaining high efficiency. This is especially true if schemes for useful recovery of the energy passing into the rf loads can be devised. However, there are practical limitations in designing for a very low τ, as discussed later.

If the beam pulse length, t_b, is not negligible compared to the filling time, t_F, there will be a transient period of duration t_b during which the fields in the accelerator adjust themselves to the new beam loading conditions. In this case the term $[1-(\Delta V/V_0)]^2$ in Eqs. (16)-(19) should be replaced by the expression (see Appendix C):

$$\bar{R} = 1 - \left(\frac{2t_F}{t_F + t_b}\right) A + \left(\frac{t_F - \frac{t_b}{3}}{t_F + t_b}\right) A^2 \tag{20}$$

where

$$A = \frac{qL\omega}{2V_0}\,\frac{\bar{r}}{\bar{Q}} \approx \frac{\Delta V}{V_0} \quad .$$

7. Average Beam Power

The average beam power is the product of average beam energy and average beam current.

$$P_{b(AV)} \approx \frac{V_0 + V}{2}\, qF = V\left(1 + \frac{\Delta V}{2V}\right) qF \quad . \tag{21}$$

The same result may be obtained by multiplying F_b from Eq. (16) (corrected by using Eq. (20)) by the rf energy entering the accelerator per pulse and the pulse rate, F.

8. Overall Conversion Efficiency

The overall efficiency of converting the input AC power to average beam power is obtained by dividing Eq. (21) by Eq. (8). The result is:

$$\eta = \frac{P_{b(AV)}}{P_{AC}} = \frac{(1+\frac{\Delta V}{2V})\; q\,\bar{r}\;(1-e^{-2\tau})\; MKT}{(1+\frac{\Delta V}{V})^2\;(\frac{V}{L})\;(t_r+t_b+t_F)} \quad . \tag{22}$$

9. Optimization of Accelerator Design

The choice of design parameters of the accelerator structure can be based upon various criteria. Assuming that the basic parameters (V, q, and F) of the collider are fixed by particle physics considerations, the remaining accelerator parameters can be selected on the basis of appropriate technical and cost factors. The minimization of cost is clearly a primary goal but this obviously requires detailed design and cost information which is not available until later in the design program. However, both construction and operating costs are strongly influenced by the total power requirements and thus the maximization of the conversion efficiency η given in Eq. (22) is certain to be a very important factor in controlling overall costs.

In maximizing η, it is instructive to consider the idealized case where the rise-fall time factor, t_r, and the beam pulse length, t_b, are negligible compared to the filling time, $t_F = (2\,\bar{Q}\,/\omega)\;\tau$. If beam loading is light the expressions in parentheses in Eq. (22) containing $\Delta V/V$ in the numerator and denominator are close to unity and the conversion efficiency has the simple dependency:

$$\eta \propto \frac{q\omega}{(\frac{V}{L})}\,\frac{\bar{r}}{\bar{Q}}\left(\frac{1-e^{-2\tau}}{2\tau}\right) \quad . \tag{23}$$

Some comments on Eq. (23) are in order. Since $\bar{r}\,/\,\bar{Q}\;\propto \omega$, η varies as ω^2 and thus, for highest efficiency, the frequency should be as high as practicable. However, the use of higher frequencies is limited by the availability of adequate power sources, the decreased transverse size of the accelerator structure, heating effects, more stringent alignment tolerances, and possible beam dynamics difficulties due to enhanced wake field phenomena. At this time it is not possible to state precisely at what frequency level one or more of these effects will limit performance. However, it is clear that the frequency should be as high as the various limiting factors permit. Moreover, at a given frequency, efficiency is improved by choosing a structure with as high a value of $\bar{r}/\bar{Q}$ as feasible. The gradient V/L should be as low as practicable in order to maximize η but a gradient that is too low will result in excessive accelerator length and higher than optimal costs. The efficiency is improved as the attenuation parameter τ becomes smaller and is maximum as $\tau \to 0$. However, this process cannot be taken too far since, for a fixed feed interval, decreasing τ will result in rapidly decreasing $\bar{r}/\bar{Q}$; alternately, for fixed $\bar{r}/\bar{Q}$, decreasing τ can result in an excessive number of feeds.

We will now return to consideration of the general relation for η (Eq. (22)). The existence of the parenthetical factors containing $\Delta V/V$ in the numerator and denominator result in decrease of conversion efficiency as the beam loading increases. For example, at a beam loading of 20%, these factors together serve to reduce the efficiency by $\sim$ 25%. Also, values of t_r and t_b that are not negligible compared to the filling time t_F cause the optimum value of τ to rise.

The attenuation parameter, τ, appears explicitly in Eq. (22) as a factor in the numerator term $(1 - e^{-2\tau})$ and in t_F in the denominator (since $t_F = 2\bar{Q}\ \tau/\omega$). τ appears implicitly in the quantities $\bar{r}$ and $\bar{Q}$. We can examine the implicit relationship for $\bar{r}$ and $\bar{Q}$ by reference to the SLAC constant-gradient accelerator structure. The group velocity of this structure varies between the values .0204 c at the input end of each section to .0065 c at the output end. The shunt impedance, r, per unit length and the Q vary almost linearly (but with opposite slopes) over the same structure, r varying from 53 to 60 $M\Omega/m$ and Q varying from 14160 to 13220. Thus r and Q can be closely approximated by the expressions:

$$r\left(\frac{M\Omega}{m}\right) = 53 + 7\ \frac{z}{\ell} \tag{24}$$

$$Q = 14160 - 940\ \frac{z}{\ell}\ . \tag{25}$$

Thus the average values of r and Q over the entire length of the section are:

$$\bar{r}\left(\frac{M\Omega}{m}\right) = \frac{53+60}{2} = 56.5 \tag{26}$$

$$\bar{Q} = \frac{14160+13220}{2} = 13690 \ . \tag{27}$$

If a portion of the section of length ℓ' (measured from the input end) is used as a "new" accelerator section the average r and Q of that section would be

$$\bar{r} = 53 + 3.5 \frac{\ell'}{\ell} \tag{28}$$

$$\bar{Q} = 14160 - 470 \frac{\ell'}{\ell} \tag{29}$$

But the ratio ℓ'/ℓ can be written in terms of the attenuation factors τ' and τ of the "new" and original sections, respectively, as:

$$\frac{\ell'}{\ell} = \frac{1-e^{-2\tau'}}{1-e^{-2\tau}} \ . \tag{30}$$

Thus, after noting that $\tau = 0.57$ and $1 - e^{-2\tau} = .6802$ for the existing SLAC sections we may express (28) and (29) as follows:

$$\bar{r}\left(\frac{M\Omega}{m}\right) = 53 + 5.15(1-e^{-2\tau'}) \tag{31}$$

$$\bar{Q} = 14160 - 691(1-e^{2\tau'}) \tag{32}$$

(The prime will be dropped in later applications of these equations.)

It should be emphasized that Eqs. (31) and (32) are valid at a frequency of 2856 MHz. At other frequencies $\bar{r}$ scales as $\omega^{1/2}$ and Q as $\omega^{-1/2}$ assuming that the same v_g schedule is maintained between input and output ends of the section in all cases. For the same value of τ the length of the accelerator section and the filling time both scale as $\omega^{-3/2}$.

With the above factors and considerations, a value of τ can be chosen which maximizes the conversion efficiency η in Eq. (22). A further discussion of this process and some numerical results are given in Section 14 below.

10. Length of Accelerator Section

The length, ℓ, of the accelerator section, i.e., the distance between feeds if there are no gaps between sections, can be determined from the equation for group velocity in a constant-gradient section[1]:

$$v_g = \frac{\omega \ell}{\bar{Q}} \frac{1 - \frac{z}{\ell}\left(1 - e^{-2\tau}\right)}{1 - e^{-2\tau}} \quad . \tag{33}$$

At $z = 0$, Eq. (33) becomes (after rearranging):

$$\ell = v_g \, \frac{\bar{Q}}{\omega} \left(1 - e^{-2\tau}\right) \quad . \tag{34}$$

Inserting the known value of v_g at the input end of the section (e.g., $v_g = .0204$ c for the standard SLAC section at 2856 MHz) and the previously determined values of $\bar{Q}$ and τ, the section length ℓ can be calculated.

11. Number of rf Feeds

The number of rf feeds in each accelerator is equal to the total accelerator length divided by the length of each accelerator section:

$$N_F = \frac{L}{\ell} \quad . \tag{35}$$

Appropriate adjustments have to be made to this equation if there are gaps between some or all of the sections, say, for beam focusing or monitoring.

12. Peak and Average rf Power from Power Sources

The total peak rf power input to the accelerator is given by Eq. (5). The total peak rf power that must be provided by the rf sources is the amount given by Eq. (5) divided by the rf transmission efficiency, T.

The average rf power from the rf sources is the peak power from the sources multiplied by the rf duty cycle, which is the product of the rf pulse length, $t_b + t_F$, and the pulse repetition rate, F. Thus

$$\underset{(source)}{P_{rf(AV)}} = \underset{(source)}{P_{rf(PK)}} \times (t_b + t_F)\, F \quad . \tag{36}$$

Note that no allowance has been made for pulse rise and fall times since it is assumed that the rf pulse is precisely defined by the rf drive pulse and is perfectly rectangular.

13. Number of rf Sources and Peak Power Per Source

Having determined the peak rf power required from the rf sources, the number of rf sources can be estimated by projecting the peak power per source that will be available at the time the super-collider is constructed. Based upon presently achievable performance and the continuation of R&D already in progress, it seems reasonable to expect that, say, ten years in the future, rf power sources will be available that will individually produce 300-400 MW peak at S band and 150-200 MW peak at C band.

Another consideration is that the rf transmission system is greatly simplified if the number of accelerator feeds per rf source is a power of 2. In other words, the power from the source should be split in half an integral number of times. Thus, we may write:

$$N_s P_{s(PK)} = P_{\substack{rf(PK) \\ (source)}} \tag{37}$$

$$N_s = \frac{N_F}{2^n} \tag{38}$$

where N_s is the number of rf sources, $P_{s(PK)}$ is the peak power per source, N_F is the number of rf feeds, and n is the number of times the power from each source is split.

Inserting Eq. (38) in Eq. (37) we obtain:

$$\frac{P_{s(PK)}}{2^n} = \frac{P_{rf(PK)}}{N_F} \quad . \tag{39}$$

Thus, for given values of peak rf power and number of feeds, Eq. (39) can be used to determine the best "fit" to the power per rf source and the number of power "splits" in the transmission system.

14. Characteristics of Accelerator Design Models

To illustrate the design principles discussed above, the parameters of four accelerator models have been calculated. All of these models meet the basic collider requirements given in Table 1. The designs are based on two frequencies: 1) 2856 MHz (the frequency of the present SLAC accelerator); and 2) 5712 MHz (twice the frequency of the SLAC accelerator). At each frequency two gradients have been considered: 1) 10 MeV/m; and 2) 20 MeV/m. The average shunt impedance, $\bar{r}$, and average $\bar{Q}$ used for these calculations are given in Eqs. (31) and (32) for the 2856 MHz case. At 5712 MHz, the following equations, based on scaling $\bar{r}$ as $\omega^{1/2}$ and $\bar{Q}$ as $\omega^{-1/2}$, have been used

$$\bar{r} = \left[74.95 + 7.283(1 - e^{-2\tau})\right] 10^6 \quad \text{ohms/m} \tag{40}$$

$$\bar{Q} = 10,013 - 488.6(1 - e^{-2\tau}) \ . \tag{41}$$

The modulator pulse rise and fall times have been assumed to be 0.1 μs and 0.15 μs, respectively, for all cases. This leads to the value $t_r = 0.075 \times 10^{-6}$ seconds. Equation (9) has been used to represent the value of t_F.

In these models, it has been assumed that beams will be delivered to several experimental areas rather than to just one area. As discussed previously in Section 4, the design of the beam transport system is simplified in this case by taking advantage of the beam-loading energy spread given by Eq. (2). Thus, the bunch separation and resulting beam pulse length can be chosen simply on the basis of beam transmission considerations in the accelerator. A bunch separation of ten wavelengths has been chosen for all of the models studied.

As discussed in Section 4, delivery of the beam to a single experimental area would be facilitated by minimizing the energy spread of the accelerated beam. This could be accomplished by the early injection technique described in that section. In this case the beam pulse length would be (from Eqs. (6), (9) and (3))

$$t_b = \frac{(e^{2\tau} - 1)qL\bar{r}}{2V_0} \ . \tag{42}$$

Since this equation has a τ dependence it would have to be incorporated into Eq. (22) for the τ optimization process. As noted above, the experimental area alternative used does not require such a procedure.

In calculating overall AC power requirements the efficiency factors have been assumed to be

$$M = .82$$

$$K = .61$$

$$T = .90$$

for the modulator, rf source, and rf transmission systems, respectively. The product of these factors leads to a combined efficiency of 45%.

The results of these calculations are given in Table 2. The conversion efficiencies η are plotted versus τ for the 2856 MHz cases in Fig. 1 and for the 5712 MHz cases in Fig. 2. The results shown in Table 2 are based upon the value of τ giving maximum η in each case.

It may be observed that, as previously noted, the overall conversion efficiency improves with the higher frequency and with the lower gradient. However, it is expected that wakefield phenomena will make it increasingly difficult to accelerate the assumed number (1.4×10^{10}) of particles per bunch with increasing frequency and decreasing gradient. The number of accelerator feeds also increases as the frequency becomes higher and/or as the gradient becomes lower. A large number of feeds could become a significant cost factor. When detailed costs of hardware components are available it may prove to be more economical to reduce the tabulated number of feeds somewhat. Figures 1 and 2 indicate that the curve of conversion efficiency versus τ has a relatively flat maximum. For example, in Case 1 (2856 MHz; 20 MeV/m) maximum η/MKT ($=$ .054) is obtained at $\tau = 0.27$, but doubling the value of τ to 0.54 (thus, increasing the feed interval from 1.98 m to 3.09 m, i.e., 56%) would decrease η/MKT to 0.050, i.e., only 7.4%. In using this procedure to optimize overall costs not only hardware costs but also power costs should be considered and the optimization process should cover a suitable operating period, say $\sim$ 10 years.

The same values of modulator pulse rise and fall times (0.10 and 0.15 μs, respectively) have been used for all cases examined. These values are considerably less than those obtained with present systems. If the assumed rise and fall times cannot be achieved, the optimum value of τ will increase and the overall conversion efficiency will decrease in each case.

The exact relation (Eq. (2)) was used to calculate the beam loading factor, ΔV, in all cases. Use of the approximate relation (Eq. (3)) would have resulted in a value of ΔV about 7% above the correct value in the 5712 MHz cases and thus would have required about 14% more power.

Acknowledgement

I would like to thank Gregory Loew for helpful comment and discussion during the preparation of this report.

References

1. R. B. Neal, Report No. ML513, Microwave Laboratory, Stanford University (1958).
2. J. Haimson, IEEE Trans. on Nucl. Sci., Vol. NS-12, No. 3, 996 (1965).
3. P. B. Wilson, SLAC-PUB-3227 (1983).

Table 1

Assumed Beam Requirements*
For a Super Linear Collider

V	=	1 TeV
b	=	12 bunches per pulse
n	=	1.4×10^{10} particles per bunch
s	=	10 λ bunch separation
q	=	26.9×10^{-9} coulombs per pulse
F	=	180 pulses per second

*For each of two linacs.

Table 2

Parameters of Four Design Models*

	No. 1	No. 2	No. 3	No. 4
Accelerator Frequency (MHz)	2856	2856	5712	5712
Gradient (MeV/m)	20	10	20	10
Attenuation Parameter (τ)	0.27	0.27	0.38	0.37
Filling Time (μs)	0.418	0.418	0.207	0.202
Length of Accelerator Section (m)	1.98	1.98	0.78	0.87
Number of rf Feeds	25253	50505	64127	114,653
Total Peak rf Power (MW)	1055×10^3	574×10^3	740×10^3	501×10^3
Total Average rf Power (MW)	86.8	47.2	30.1	19.9
Number of Feeds per rf Source	8	32	16	32
Number of rf Sources	3157	1578	4008	3583
Peak rf Power per rf Source (MW)	334	364	185	140
Average rf Power per rf Source (kW)	27.5	29.9	7.5	5.6
Peak rf Power per Feed (MW)	37.6	10.2	10.4	3.9
Bunch Separation (λ)	10	10	10	10
Beam Pulse Length (ns)	38.5	38.5	19.2	19.2
Peak Beam Current (A)	0.70	0.70	1.40	1.40
Average Beam Power (MW)	4.95	5.06	5.28	5.72
rf Input Energy to Accelerator per Pulse (J)	4.34×10^5	2.36×10^5	1.50×10^5	1.00×10^5
Fraction of rf Input Energy Stored in Accelerator	0.77	0.77	0.70	0.71
Fraction of rf Input Energy Transferred to Beam	0.06	0.12	0.19	0.32
Fraction of rf Input Energy Lost in Accelerator Walls	0.40	0.39	0.47	0.42
Fraction of rf Input Energy Entering rf Loads	0.54	0.49	0.34	0.26
Beam Loading ($\Delta V/V_0$)	0.043	0.083	0.154	0.266
Maximum Conversion Efficiency (η/MKT)	0.054	0.102	0.146	0.238
Total AC Power for Accelerator (MW)	202	110	80	53

*All figures are for one accelerator producing the beam parameters given in Table 1.

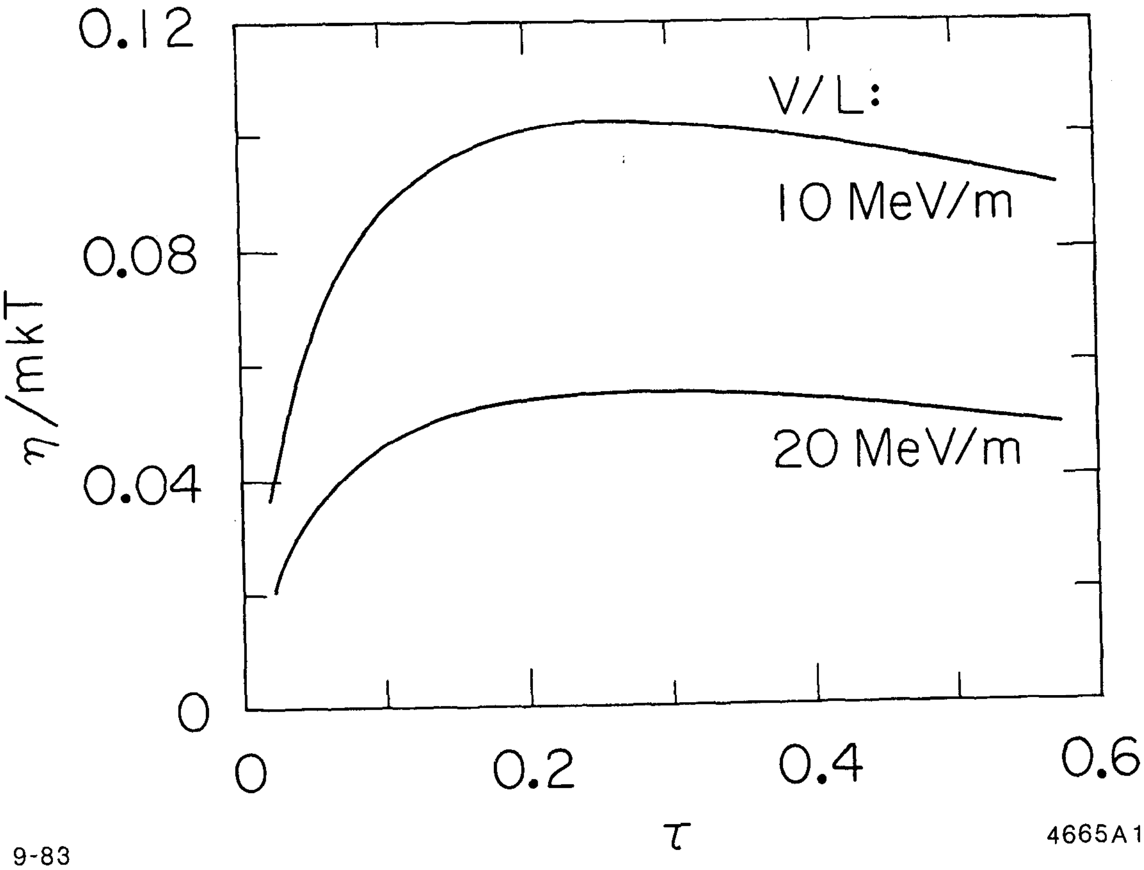


Fig. 1. Conversion efficiency versus τ ($f = 2856$ MHz).

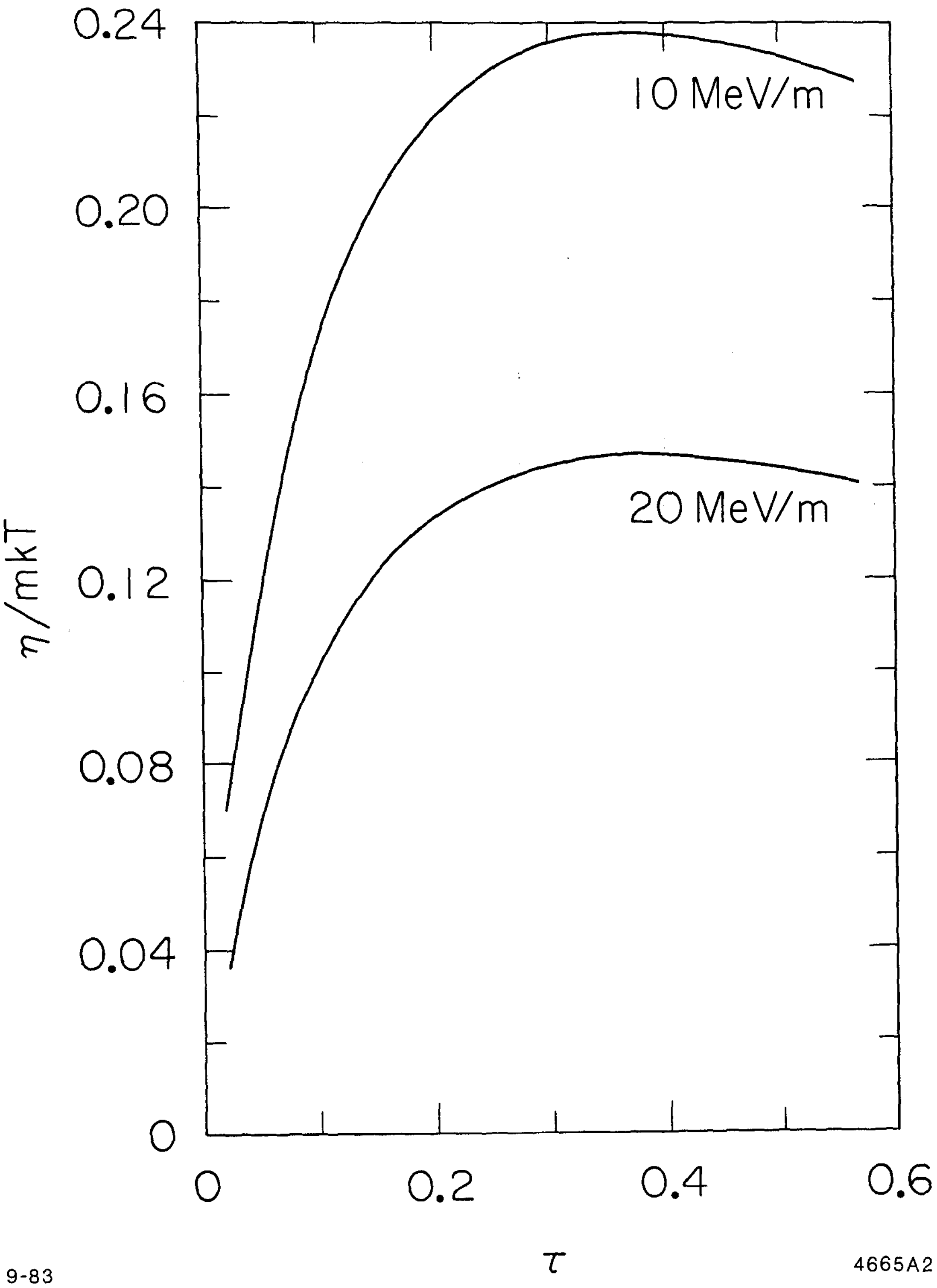


Fig. 2. Conversion Efficiency versus τ ($f = 5712$ MHz).

APPENDIX A

Early Injection to Compensate for Beam Loading

This appendix calculates the amount of time that the beam should be injected early in a constant-gradient accelerator section to compensate for beam loading.

The group velocity in a constant gradient accelerator section is given by[1]:

$$v_g = \frac{\omega \ell}{\bar{Q}} \frac{1 - \frac{z}{\ell}\left(1 - e^{-2\tau}\right)}{1 - e^{-2\tau}} \tag{A.1}$$

where the symbols have been previously defined in the body of this report. The time required for the rf energy to travel distance z in the accelerator structure is then:

$$\begin{aligned} t(z) &= \int_0^z \frac{dz}{v_g} \\ &= -\frac{\bar{Q}}{\omega}\, \ell n\left[1 - \left(1 - e^{-2\tau}\right) \frac{z}{\ell}\right] \end{aligned} \tag{A.2}$$

or, solving for z:

$$z = \left(\frac{1 - e^{-\omega t/\bar{Q}}}{1 - e^{-2\tau}}\right) \ell \ . \tag{A.3}$$

Since the no-load accelerating field in the structure is given by:

$$E = \frac{V_0}{\ell} = \left(\frac{P_0\, \bar{r}}{\ell}\right)^{1/2} \left(1 - e^{-2\tau}\right)^{1/2} \tag{A.4}$$

the beam energy obtained by injecting when the rf energy has reached point z is given by:

$$V(z) = Ez = z\left(\frac{P_0\, \bar{r}}{\ell}\right)^{1/2} \left(1 - e^{-2\tau}\right)^{1/2} \tag{A.5}$$

or, in terms of t:

$$V(t) = \left(P_0\, \bar{r}\, \ell\right)^{1/2} \frac{\left(1 - e^{-\omega t/\bar{Q}}\right)}{\left(1 - e^{-2\tau}\right)^{1/2}} \ . \tag{A.6}$$

The increment, ΔV, by which $V(t)$ has not reached full value (i.e., at $t = t_F$) is:

$$\Delta V = (P_0 \bar{r} \ell)^{1/2} \left[(1 - e^{-2\tau})^{1/2} - \frac{1 - e^{-\omega t/\bar{Q}}}{(1 - e^{-2\tau})^{1/2}} \right]$$

$$= V_0 \left[1 - \frac{1 - e^{\omega t/\bar{Q}}}{1 - e^{-2\tau}} \right] \qquad (A.7)$$

or, since $t_F = (2\bar{Q}/\omega)\tau$:

$$\frac{\Delta V}{V_0} = \left[1 - \frac{1 - e^{-(t/t_F)2\tau}}{1 - e^{-2\tau}} \right] \ . \qquad (A.8)$$

To compensate for beam loading of $\Delta V/V_0$ the beam should be injected at a time $\Delta t = t_b$ before the accelerator is filled, i.e., at time $t = t_F - t_b$. Thus:

$$\frac{\Delta V}{V_0} = 1 - \frac{1 - e^{(t_b/t_F)\,2\tau}\, e^{-2\tau}}{1 - e^{-2\tau}} \ . \qquad (A.9)$$

Rearranging (A.9) we obtain:

$$e^{\frac{t_b}{t_F}2\tau} = 1 + (e^{2\tau} - 1)\,\frac{\Delta V}{V_0} \ . \qquad (A.10)$$

For $(t_b/t_F)\,2\tau \ll 1$ Eq. (A.10) becomes:

$$t_b \approx \frac{e^{2\tau} - 1}{2\tau}\,\frac{\Delta V}{V_0}\,t_F \ . \qquad (A.11)$$

APPENDIX B

Energy Spread Caused by Beam Injection at End of Filling Time

If a beam pulse of length t_b is injected immediately after the accelerator is filled, the first electrons injected will be accelerated to full energy but the last electrons which are injected at a time $t = t_b$ later will receive less energy because the rf energy will have moved downstream during that period leaving no accelerating fields in the front end of the accelerator sections. Thus, there will be a beam energy spread due to this effect that is additive to the energy spread due to beam loading.

The beam energy obtained by injecting when the rf energy has moved downstream by a distance z is given by:

$$V = E(\ell - z) = \left(\frac{P_0 \bar{r}}{\ell}\right)^{1/2} (1 - e^{-2\tau})^{1/2} (\ell - z) \ . \qquad (B.1)$$

As shown in Appendix A, the relationship between the distance z traveled by the rf wave and the time required for this travel is:

$$z = \left(\frac{1 - e^{-\omega t/\bar{Q}}}{1 - e^{-2\tau}}\right) \ell \ . \qquad (B.2)$$

Inserting Eq. (B.2) in Eq. (B.1) yields:

$$V = (P_0 \bar{r} \ell)^{1/2} \frac{e^{-\omega t/\bar{Q}} - e^{-2\tau}}{(1 - e^{-2\tau})^{1/2}} \ . \qquad (B.3)$$

The energy gain versus time given by Eq. (B.3) can be expressed as a fraction of the maximum no-load energy gain V_0:

$$V = \left(\frac{e^{-\omega t/\bar{Q}} - e^{-2\tau}}{1 - e^{-2\tau}}\right) V_0 \ . \qquad (B.4)$$

Thus the spread in energies of an electron accelerated at $t = 0$ and an electron accelerated at $t = t_b$ is:

$$\delta V = V_0 \left(1 - \frac{e^{-\omega t_b/\bar{Q}} - e^{-2\tau}}{1 - e^{-2\tau}}\right) \qquad (B.5)$$

or,

$$\frac{\delta V}{V_0} = \frac{1 - e^{-\omega t_b/\bar{Q}}}{1 - e^{-2\tau}} \tag{B.6}$$

which, for $\omega t_b / \bar{Q} \ll 1$, becomes:

$$\frac{\delta V}{V_0} \approx \frac{2\tau}{1 - e^{-2\tau}} \frac{t_b}{t_F} \quad . \tag{B.7}$$

APPENDIX C

Power Flow to rf Loads During Transient Beam Loading

This appendix derives the expression given in Eq. (20) that may be used in Eqs. (16), (18) and (19), respectively, to determine the fractions of the rf energy transferred to the beam, lost in the accelerator walls, and transmitted to the rf loads during transient beam loading.

The steady-state power flow in a constant gradient accelerator structure is given by[1]:

$$\frac{P_z}{P_0 - Y_0 z} = \left[1 - \frac{I}{2}\left(\frac{r}{Y_0}\right)^{1/2} \ell n \, \frac{P_0}{P_0 - Y_0 z}\right]^2 \qquad (C.1)$$

where P_z is the power flow at a point distant z from the input end of the structure and Y_0 is the power loss per unit length in the structure in the absence of beam loading. Other symbols have been previously defined.

Equation (C.1) can be expressed in terms of the time t required for the wave to travel at group velocity to point z. The relation between t and z derived in Appendix A is:

$$\frac{z}{\ell} = \frac{1 - e^{-\omega t/\bar{Q}}}{1 - e^{-2\tau}} \quad . \qquad (C.2)$$

Inserting (C.2) and the relation

$$Y_0 = \frac{P_0 - P_\ell}{\ell} = \frac{P_0(1 - e^{-2\tau})}{\ell} \qquad (C.3)$$

in Eq. (C.1) we obtain:

$$P(z,t) = P_0 e^{-2\tau(t/t_F)}\left[1 - \frac{Ir\ell\tau}{V_0}\,\frac{t}{t_F}\right]^2 \quad . \qquad (C.4)$$

Equation (C.4) gives the power flow at a point z defined by Eq. (C.2). For example, at $t = 0$, $z = 0$ and $P(z,t) = P_0$; at $t = t_F$, $z = \ell$ and $P(z,t) = P_0\, e^{-2\tau}[1 - (Ir\ell\tau/V_0)]^2$. In the latter case, the beam has been on for a time t_F and steady state power flow has been established at all points in the accelerator.

We are interested in the power flow at $z = \ell$ in the case where the beam is injected at the end of the filling time for a period, t_b, that is short compared to the filling time, t_F. At $z = \ell$, Eq. (C.4) becomes:

$$P_\ell = P_0\, e^{-2\tau} \left[1 - \frac{Ir\ell\tau}{V_0} \frac{t}{t_F} \right]^2 \tag{C.5}$$

where t is now measured from the instant that the beam is first turned on and $0 \leq t \leq t_b$.

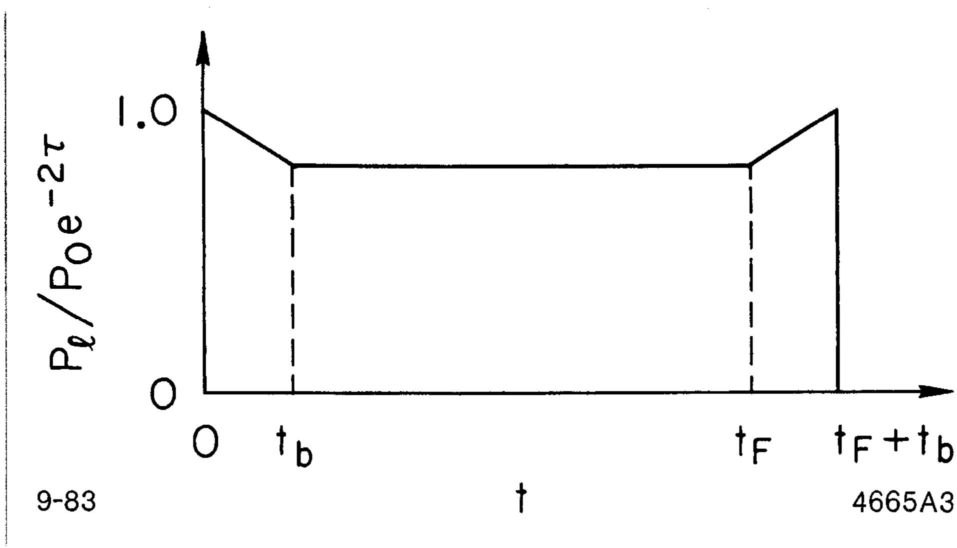
From Eq. (C.5) we note that P_ℓ decreases with increasing t and reaches the minimum value

$$P_{\ell(min)} = P_0\, e^{-2\tau} \left[1 - \frac{Ir\ell\tau}{V_0} \frac{t_b}{t_F} \right]^2$$

or, since $q = It_b$ and $t_F = 2\bar{Q}\,\tau/\omega$,

$$P_{\ell(min)} = P_0\, e^{-2\tau} \left[1 - \frac{q\ell\omega}{2V_0} \frac{\bar{r}}{\bar{Q}} \right]^2 \tag{C.6}$$

P_ℓ holds steady at that minimum value for an additional time period $t_F - t_b$ and then increases to the value $P_0\, e^{-2\tau}$ during a period t_b. (The transient behavior at the end of the rf pulse can be shown by a treatment similar to that given above for the initial part of the pulse.) Thus, the power output to the rf load has the following appearance:



To determine the total rf energy passing into the rf loads we find the average value of the expression in brackets in Eq. (C.6) over the region $0 \leq t \leq (t_F + t_b)$. We designate this expression R.

$$R = \left[1 - A\,\frac{t}{t_b}\right]^2 \quad \text{for} \quad 0 \leq t \leq t_b$$

$$R = [1 - A]^2 \quad \text{for} \quad t_b \leq t \leq t_F$$

$$R = \left[1 - A\left(\frac{t_b + t_F - t}{t_b}\right)\right]^2 \quad \text{for} \quad t_F \leq t \leq (t_F + t_b)$$

where $A = (qL\omega/2V_0)\,(\bar{r}\,/\,\bar{Q})$.

The average value of R is found to be:

$$\bar{R} = 1 - \left(\frac{2t_F}{t_F + t_b}\right) A + \left(\frac{t_F - \frac{t_b}{3}}{t_F + t_b}\right) A^2 \ . \tag{C.7}$$

We note that, as $t_b \to 0$, $\bar{R} \to (1 - A)^2 \approx [1 - (\Delta V/V_0)]^2$. The expression for $\bar{R}$ given in Eq. (C.7) should be used in Eqs. (16), (18) and (19) in place of the quantity $[1 - (\Delta V/V_0)]^2$ to obtain precise values of f_b, f_W, and f_L, respectively. This exchange is especially important in those cases where t_b is not negligible compared to t_F.